Isaiah 3:10 "Tell the righteous it will be well with them, for they will enjoy the fruit of their deeds".
Proverbs 10:28 "The hope of the righteous will be gladness, But the expectation of the wicked will perish".

STA 2200 CAT 2 MARKING SCHEME

Question One

a) $P(|Z|>1.45)=P(Z>1.45 \text{ or } Z<-1.45)=2P(Z<-1.45)=2(0.07353)=0.14706$

b) Let T be the thickness of a randomly chosen silicon wafers. $T\sim N(1\,,0.1^2) \Longrightarrow Z=\frac{T-1}{0.1}\sim N(0\,,1)$

$P(0.85<T<1.1)=P(-1.5<Z<1)=\Phi(1)-\Phi(-1.5)$

$=0.84134-0.06681=0.77454$

Let t be the required thickness, then $F(t)=\Phi\left(\frac{t-1}{0.1}\right)=0.9972=\frac{t-1}{0.1}=2=t=1.2$ mm

Let X be the number of acceptable wafers then

$$X\sim Bin(200\,,0.77454) \Longrightarrow X\approx N(154.908\,,34.9258) \Longrightarrow Z=\frac{X-154.908}{\sqrt{34.9258}}\sim N(0\,,1)$$

$$\underbrace{P(140<X<170)=P(141\le X\le 169)}_{Binomial} \approx \underbrace{P(140.5<X<169.5)}_{Normal\ approximation} \approx \underbrace{P(-2.44<Z<1.62)}_{Standardized\ Normal}$$

$$=\Phi(1.62)-\Phi(-2.44)=0.94738-0.00734=0.94004$$

c) Let A be the amount of credit card debt of a randomly choose household, then

$$A\sim N\left(15{,}250\ ,\ 7{,}150^2\right) \Longrightarrow \bar{A}\sim N\left(15{,}250\ ,\frac{7{,}150^2}{1600}\right) \Longrightarrow Z=\frac{\bar{A}-15{,}250}{7{,}150/40}\sim N(0\,,1)$$

$$P(\mu-300<\bar{A}<\mu+300)=P(14{,}950<\bar{A}<15{,}550)=P(-1.68<Z<1.68)$$

$$=\Phi(1.68)-\Phi(-1.68)=0.95352-0.04648=0.90704$$

d) $J\sim N(46\,,36) \Longrightarrow Z=\frac{J-46}{6}\sim N(0\,,1) \therefore P(J<49)=P(Z<0.5)=0.69146$

Required is $P(P-10<J<P+10)=P(-10<J-P<10)$ but $J-P\sim N(-9\,,100)$

$$\therefore P(-10<J-P<10)=P\left(\frac{-10+9}{10}<Z<\frac{10+9}{10}\right)=\Phi(1.9)-\Phi(-0.1)$$

$$=0.97128-0.46017=0.51111$$

Question Two

a) The sample came from a **normally** distributed population

b) The observed value lies outside the critical region so do not reject H_0

For a 1 tailed test p-value = Area beyond the test statistic = 0.0429

Since $0.01<0.0429<0.05$, we reject H_0 at 5% but we fail to reject H_0 at 1%

Romans 12:2 "Do not conform to the pattern of this world, but be transformed by the renewing of your mind. Then you will be able to test and approve what God's will is—his good, pleasing and perfect will".

Isaiah 3:10 "Tell the righteous it will be well with them, for they will enjoy the fruit of their deeds".
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For a 2 tailed test $p - value = 2(\text{Area beyond the test statistic}) = 2(0.00705) = 00141.$

Since $0.01 < 0.0141 < 0.05$, we reject H_0 at 5% but we fail to reject H_0 at 1%

c) 20% and the test is a 2 tailed

d)

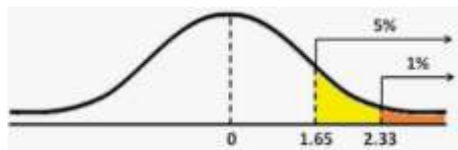
$\text{p} - \text{value} = 2\text{P}(\text{X} \leq 14) = 0.16088 > \alpha = 0.10 \therefore$ we fail to reject H_0

Cathy's test is a 1 tailed while Owen's test is 2 tailed

Cathy's $\text{p} - \text{value} = 1\text{P}(\text{X} \leq 14) = 0.08044 < \alpha = 0.10 \therefore$ she should reject H_0

e) Hypothesis $\text{H}_0: \mu = 110 \; vs \; \text{H}_1: \mu > 110 \quad \alpha = 0.01$

Assuming H_0 is true $\bar{b} \sim N\left(110, \frac{15^2}{40}\right)$



Critical region (based on $\alpha = 0.01$)

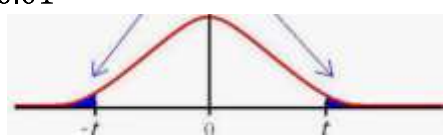
Reject H_0 if $|Z_{stat}| > 2.33$

Test statistic $Z_{stat} = \frac{114.5 - 110}{15/\sqrt{40}} \approx 6.114$

Decision: since the calculated $Z_{stat} = 6.114 > 2.33$, we reject the H0

Conclusion: there is sufficient evidence to show that the

f) Hypothesis $\text{H}_0: \mu = 650$ vs $\text{H}_1: \mu \neq 650 \quad \alpha = 0.01$



σ is unknown $\Longrightarrow$ a t test

Critical values (df=11 and $\alpha = 0.01$)
$t_{(11,0.005)} = \pm 3.106$.
From the calculator $\overline{\text{X}} = 672.5$ and $s = 43.72$

The 99% CI for μ is $\bar{X} \pm \left(\frac{s}{\sqrt{n}}\right) t_{(11,0.005)} = 672.5 \pm 3.106 \left(\frac{43.72}{\sqrt{12}}\right)$ ie $633.3 \leq \mu \leq 711.7$

Decision: Since our hypothesized value lies inside this interval, we fail to reject H_0

Conclusion: no sufficient evidence to show that the yield of the new process are significantly different from that of the old process

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