rk2log

Deriving the transfer function for a bridged t-network pt. 1

This post will derive the transfer function for a bridged t-network, expressed using impedances, and will be the first of a series of posts I'll be making as I'm learning about, and working on, a design for an analog drum machine similar to that of the famous Roland machines.

The post will begin with a short background for why the bridged t-network in particular will be examined. Then, the derivation of the transfer function will be given, followed by a simulation of the circuit to validate that the derived expression is correct.

I. Background

In the service manual for the tr-808 drum machine, the following principal diagram for the basic pulse-triggered sound generator can be found:

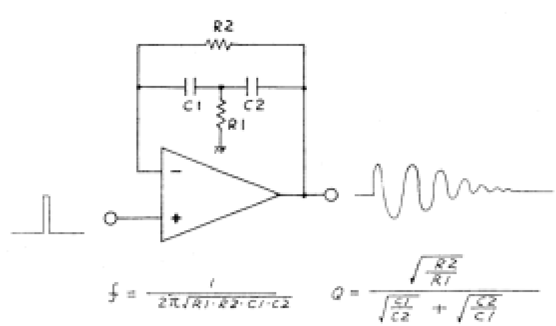

Fig 1, representative bridged t-network schematic from tr-808 service manual

While the circuitry in the tr-808 is not as straight forward as in Fig 1, it is an excellent starting point for a study. By understanding the principal schematic, it creates the foundation for learning about the more complex and nuanced circuits in the actual hardware.

The sound generator consists of an operational amplifier (op-amp) and a feedback circuit consisting of the bridged t-network, which forms a bandpass filter with a high resonance. The filter acts as a damped oscillator (see Fig 2), and will self-oscillate at the resonance-frequency for a short time after being stimulated by an input-pulse. This is makes up the basic drum sound.

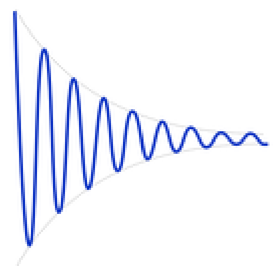

Fig 2, example of a damped oscillation

The circuit above (Fig 1) will be able to create a tom-like sound or a kick-like sound depending on the resonant frequency's value, which is determined by the valeus of the circuit components.

II. Transfer function for the bridged t-network

The transfer function describes the relation between the components and the circuits' behavior, and will be derived from the following diagram:

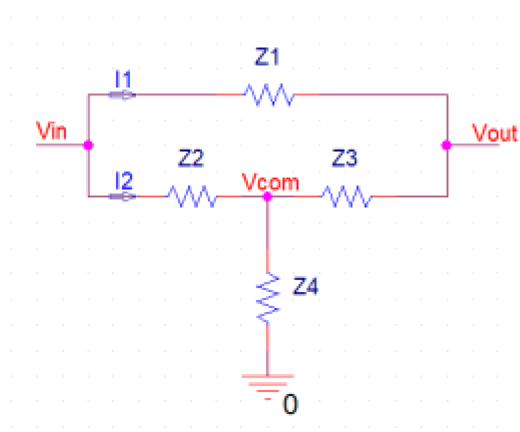

Fig 3, bridged t-network with currents labeled I1 and I2

The transfer function is an expression for the ratio between the output voltage V_o and a given input voltage V_i:

$$\text{transfer function} = \frac{V_o}{V_i}$$

Deriving the transfer function from the schematic becomes a process of finding an expression for the output voltage as as a function of the input voltage, and factoring that expression.

The output voltage node and input voltage node are separated by the impedance Z_1. The current I_1 via Z_1 causes a voltage drop, so that the output voltage can be expressed by:

$$V_o = V_i - I_1 Z_1$$

I_1 can be expressed by introducing a common-voltage V_{com} at the junction between Z_2, Z_3 and Z_4:

$$I_1 = \frac{V_i - V_{com}}{Z_1 + Z_3}$$

To express the common-voltage we notice that Z_1, Z_2 and Z_3 form an equivalent resistor $Z_{(1+3)//2}$ that together with Z_4 form a voltage divider with V_{com} as its output:

$$V_{com} = V_i \frac{Z_4}{Z_{tot}}$$

where the total impedance Z_{tot} is:

$$Z_{tot} = Z_{(1+3)//2} + Z_4$$

The current I_1 can now be rewritten as:

$$
\begin{aligned}
I_1 &= V_i \frac{1}{Z_1 + Z_3} - V_{com} \frac{1}{Z_1 + Z_3} \\
&= V_i \frac{1}{Z_1 + Z_3} - V_i \frac{Z_4}{Z_{tot}} \frac{1}{Z_1 + Z_3}
\end{aligned}
$$

and then factored:

$$
\begin{aligned}
&= \frac{V_i}{Z_1 + Z_3}(1 - \frac{Z_4}{Z_{tot}}) \\
&= \frac{V_i}{Z_1 + Z_3}(\frac{Z_{tot} - Z_4}{Z_{tot}}) \\
&= \frac{V_i}{Z_1 + Z_3}(\frac{Z_{(1+3)//2}}{Z_{tot}})
\end{aligned}
$$

By calculating the equivalent impedance $Z_{(1+3)//2}$ and then Z_{tot}, the expression for the current can be made explicit:

$$
Z_{(1+3)//2} = \frac{(Z_1 + Z_3)Z_2}{(Z_1 + Z_3) + Z_2}
$$

$$
\begin{aligned}
Z_{tot} &= \frac{(Z_1 + Z_3)Z_2}{(Z_1 + Z_3) + Z_2} + Z4 \\
&= \frac{(Z_1 + Z_3)Z_2}{Z_1 + Z_2 + Z_3} + Z4 \frac{(Z_1 + Z_2 + Z_3)}{(Z_1 + Z_2 + Z_3)} \\
&= \frac{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)}{Z_1 + Z_2 + Z_3}
\end{aligned}
$$

The impedance-quotient then becomes:

$$
\begin{aligned}
\frac{Z_{(1+3)//2}}{Z_{tot}} &= \frac{\frac{(Z_1 + Z_3)Z_2}{Z_1 + Z_2 + Z_3}}{\frac{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)}{Z_1 + Z_2 + Z_3}} \\
&= \frac{(Z_1 + Z_3)Z_2}{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)}
\end{aligned}
$$

The current I_1 can now be expressed with the known impedances as:

$$
\begin{aligned}
I_1 &= \frac{V_i}{Z_1 + Z_3}\left(\frac{Z_{(1+3)//2}}{Z_{tot}}\right) \\
&= \frac{V_i}{(Z_1 + Z_3)} \frac{(Z_1 + Z_3)Z_2}{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)} \\
&= V_i \frac{Z_2}{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)}
\end{aligned}
$$

Finally, we can return to the output voltage expression, and factor out V_i:

$$
\begin{aligned}
V_o &= V_i - I_1 Z_1 \\
&= V_i - V_i \frac{Z_2}{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)} Z_1 \\
&= V_i \left(1 - \frac{Z_1 Z_2}{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)}\right)
\end{aligned}
$$

and then calculate the difference inside the parentheses by expanding the leading 1-term:

$$
\begin{aligned}
&= V_i \frac{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3) - Z_1 Z_2}{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)} \\
&= V_i \frac{Z_1 Z_2 + Z_2 Z_3 + Z_4(Z_1 + Z_2 + Z_3) - Z_1 Z_2}{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)} \\
&= V_i \frac{Z_2 Z_3 + Z_4(Z_1 + Z_2 + Z_3)}{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)}
\end{aligned}
$$

With the original expression fully factorized, the transfer function can now be expressed:

$$
\frac{V_o}{V_i} = \frac{Z_2 Z_3 + Z_4(Z_1 + Z_2 + Z_3)}{(Z_1 + Z_3)Z_2 + Z_4(Z_1 + Z_2 + Z_3)}
$$

And the parenthesises expanded:

$$
\frac{V_o}{V_i} = \frac{Z_1 Z_4 + Z_2 Z_3 + Z_2 Z_4 + Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_4 + Z_2 Z_3 + Z_2 Z_4 + Z_3 Z_4}
$$

The transfer function for the schematic has now been derived.

III. Validating the transfer function with a simulation

In order to validate that the transfer function is correct, it will be tested using resistive values and compared to the output from the simulation software PSpice.

A simple straight forward test is to simply define all the impedances to 1kΩ:

$$\frac{V_o}{V_i}(Z_i = 1k) = \frac{(1+1+1+1)k}{(1+1+1+1+1)k} = \frac{4k}{5k} = 0.8$$

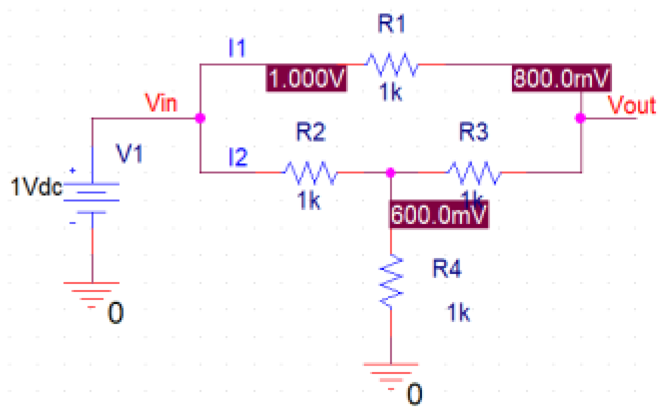

Fig 4, simulation result from PSpice for test 1

With the input voltage V_i set to 1V, the output voltage V_o is 0.8V as expected.

In order to make sure this isn't just a fluke result, another set of test-values can be defined:

$$\begin{aligned}\frac{V_o}{V_i}(V_i = ik) &= \frac{(1\cdot4+2\cdot3+2\cdot4+3\cdot4)k}{(1\cdot2+1\cdot4+2\cdot3+2\cdot4+3\cdot4)k} \\ &= \frac{4+6+8+12}{2+4+6+8+12} = \frac{30}{32} = 0.9375\end{aligned}$$

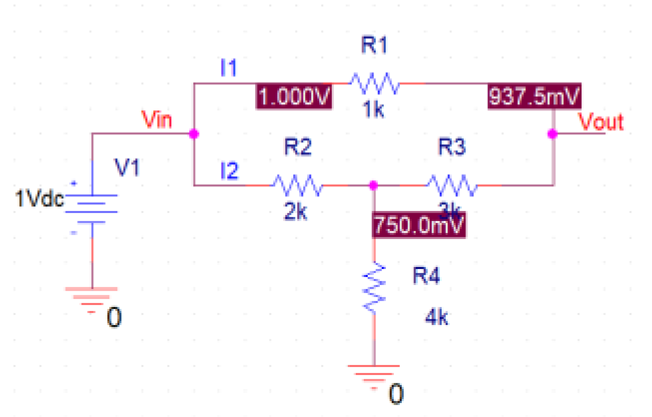

Fig 5, simulation result from PSpice for test 2

For this second test the result is also the expected one, with an input voltage V_i of 1 giving 937.5 mV output.

IV. Conclusion

A valid transfer function for the bridged t-network has been derived for impedances. In order to analyze its oscillation properties and bode diagram, those impedances need to be defined using resistors R and capacitors C.

This will be explored in the next part.

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HC

February 7, 2019 at 9:26 am

Thanks dude

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