1. Vectors

A **Vector** is made up of **Elements** aka entries, components, or coefficients

An **n-vector** has n elements, or has a size of n.

indexed from 1 (not 0)

if a is an n-vector, it has elements $a_1, a_2, \ldots, a_n$

$a = b$ if a and b have the same size and $a_1 = b_1, a_2 = b_2, \ldots, a_n = b_n$

Example Vectors:

$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $a = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ $b = (-1, -2, 0)$

$\begin{bmatrix} a \\ b \end{bmatrix} = (4, 5, 6, -1, -2, 0)$ ← This is a block, or stacked vector.

A **Sparse** vector has many zeros. $nnz(x)$ is the number of non-zero elements in x.

✳ Some notational conventions are used to distinguish vectors from scalars, but nothing is standardized.

Subvectors or slices are a subset of a vector.

$a = (1, 2, 3)$

$a_{1:2} = (1, 2)$

: is colon notation

1:2 is the index range

A **Unit-vector** consists of a 1 and the rest are 0s.

$(1, 0, 0)$ This is a sparse vector.

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

A **Zero-vector** is a vector of all zeros. This is the sparsest possible vector.

A **Ones-vector** is a vector of all ones.

Vectors: Operations

Vector Addition

$$\begin{bmatrix}1\\2\\3\end{bmatrix} + \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}5\\7\\9\end{bmatrix}$$

Complexity: n

if $c = a + b$, then $c_i = a_i + b_i$

Vector-scalar multiplication

$$4\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}4\\8\\12\end{bmatrix}$$

Complexity: n

for a constant α, if $b = \alpha a$, then $b_i = \alpha a_i$.

Inner Product

Complexity: $2n-1$

$$\begin{bmatrix}1\\2\\3\end{bmatrix}^T \begin{bmatrix}-4\\1\\0\end{bmatrix} = 1(-4) + 2(1) + 3(0) = -1$$

CAUTION

Don't confuse math notation with programming language syntax!

Vectors a, b, c

Scalars α, β

$a - a = 0$]

$a + 0 = a$] here 0 is a zero vector

associative $(a+b)+c = a+(b+c)$

$(\alpha a)^T b = \alpha(a^T b)$ $\alpha(\beta b) = (\alpha\beta)b$

commutative $a + b = b + a$

$a^T b = b^T a$ $\alpha b = b\alpha$

$(a+b)^T c = a^T c + b^T c$

$\alpha(a+b) = \alpha a + \alpha b$

$b(\alpha+\beta) = b\alpha + b\beta$

Complexity

number of flops required to complete an operation

8 Bytes for a floating point number

8n Bytes for an n-vector

Flop: floating point operation; $+ - \times \div$

Gflop: Giga-flop; 1 billion flops

The order of an operation is the complexity, ignoring any constants. All of the operations on this page have an order of n.