

# I. VECTORS

<sup>A</sup>  
Vector

is made up of

Elements

aka entries,  
components,  
or coefficients

<sup>An</sup>  
 $n$ -vector

has  $n$  elements,

or has a SIZE of  $n$ .

indexed from 1  
(not 0)

if  $a$  is an  $n$ -vector,  
it has elements  
 $a_1, a_2, \dots, a_n$

$a = b$  if  $a$  and  $b$  have the same  
size and  $a_1 = b_1, a_2 = b_2, \dots,$   
 $a_n = b_n$

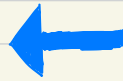
Example Vectors:

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad a = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \quad b = (-1, -2, 0)$$

<sup>A</sup>  
Sparse

$\text{nnz}(x)$  is the  
number of non-zero  
elements in  $x$ .

$$\begin{bmatrix} a \\ b \end{bmatrix} = (4, 5, 6, -1, -2, 0)$$



This is a block, or  
stacked vector.

\* Some notational conventions are used to distinguish  
vectors from scalars, but nothing is standardized.

Subvectors

or

slices are a subset of  
a vector.

$$a = (1, 2, 3)$$

$$a_{1:2} = (1, 2)$$



is colon notation  
 $1:2$  is the index range

<sup>A</sup>  
Unit-vector

consists of a 1 and  
the rest are 0s.

$$(1, 0, 0)$$

This is a  
sparse  
vector.

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

<sup>A</sup>  
Zero-vector

is a vector of all zeros

This is the  
sparsest  
possible  
vector.

<sup>A</sup>  
Ones-vector

is a vector of all ones.

# Vectors: OPERATIONS

## Vector Addition

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

Complexity:  $n$   
if  $c = a + b$ , then  $c_i = a_i + b_i$

## Vector-scalar multiplication

$$4 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 12 \end{bmatrix}$$

Complexity:  $n$   
for a constant  $\alpha$ , if  $b = \alpha a$ , then  $b_i = \alpha a_i$

## Inner Product

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}^T \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix} = 1(-4) + 2(1) + 3(0) = -1$$

Complexity:  $2n-1$

## CAUTION

Don't confuse math notation with programming language syntax!

<p>Vectors <math>a, b, c</math> Scalars <math>\alpha, \beta</math></p> <p><math>a - a = 0</math> <math>a + 0 = a</math></p> <p>associative <math>(\alpha a)^T b = \alpha (a^T b)</math></p> <p>commutative <math>a^T b = b^T a</math></p>	<p><math>(a+b)^T c = a^T c + b^T c</math> <math>\alpha(a+b) = \alpha a + \alpha b</math> <math>b(\alpha + \beta) = b\alpha + b\beta</math></p> <p>here 0 is a zero-vector</p> <p><math>(a+b) + c = a + (b+c)</math> <math>\alpha(\beta b) = (\alpha\beta)b</math> <math>a+b = b+a</math> <math>\alpha b = b\alpha</math></p>
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Complexity: number of flops required to complete an operation

8 Bytes for a floating point number  
8n Bytes for an n-vector

Flop: floating point operation;  $+$   $-$   $\times$   $\div$   
Gflop: Giga-flop; 1 billion flops

The order of an operation is the complexity, ignoring any constants.  
All of the operations on this page have an order of  $n$ .