Derivation of the parabola derivative

$$
\begin{aligned}
\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} && (4) \\
&= \lim_{h \to 0} \frac{2xh + h^2}{h} && (5) \\
&= \lim_{h \to 0} \frac{h(2x+h)}{h} && (6) \\
&= \lim_{h \to 0} 2x + h && (7) \\
&= 2x && (8)
\end{aligned}
$$

Derive on the board.

Derivate of a parabola:

$$
\begin{aligned}
\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} && (9) \\
&= \lim_{h \to 0} \frac{2xh + h^2}{h} && (10) \\
&= \lim_{h \to 0} \frac{h(2x+h)}{h} && (11) \\
&= \lim_{h \to 0} 2x + h && (12) \\
&= 2x && (13)
\end{aligned}
$$

The derivate of a polynomial

What is the derivative of the function $f(x) = x^n$?

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = nx^{n-1} \tag{14}$$

Derivate of a polynomial $f(x) = x^n$ [DFO20]:

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} \tag{15}$$

$$= \lim_{h\to 0} \frac{\sum_{i=0}^{n} \binom{n}{i} x^{n-1} h^i - x^n}{h} \tag{16}$$

$$= \lim_{h\to 0} \frac{\sum_{i=1}^{n} \binom{n}{i} x^{n-1} h^i}{h} \tag{17}$$

$$= \lim_{h\to 0} \sum_{i=1}^{n} \binom{n}{i} x^{n-1} h^{i-1} \tag{18}$$

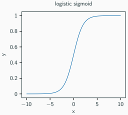
$$= \lim_{h\to 0} (\binom{n}{1} x^{n-1}) + \sum i = 2n \binom{n}{i} x^{n-i} h^{i-1} \tag{19}$$

$$= \frac{n!}{1!(n-1)!} x^{n-1} = nx^{n-1}. \tag{20}$$

The logistic sigmoid [GBC16]

The sigmoid function $\sigma(x)$ is a common activation function.

$$\sigma(x) = \frac{1}{1+e^{-x}} \tag{25}$$



$$\begin{aligned}
\frac{\mathrm{d}\sigma(x)}{\mathrm{d}x} &= \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{1+e^{-x}} && (26)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{1+e^{-x}} \cdot 1 && (27)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{1+e^{-x}} \cdot \frac{e^x}{e^x} && (28)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\frac{e^x}{e^x+1} && (29)
\end{aligned}$$

$$g(x) = e^x, h(x) = e^x + 1 \tag{30}$$

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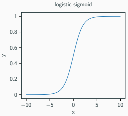

$$\frac{\mathrm{d}\sigma(x)}{\mathrm{d}x} = \frac{e^x \cdot (e^x+1) - e^x \cdot e^x}{(e^x+1)^2} \tag{31}$$

$$= \frac{e^x \cdot e^x + e^x - e^x \cdot e^x}{(e^x+1)^2} \tag{32}$$

$$= \frac{e^x}{(e^x+1)^2} = \frac{e^x}{(e^x+1)}\frac{1}{(e^x+1)} \tag{33}$$

$$= \frac{e^x}{(e^x+1)}\left(\frac{1+e^x-e^x}{(e^x+1)}\right) \tag{34}$$

$$= \frac{e^x}{(e^x+1)}\left(\frac{1+e^x}{(e^x+1)} - \frac{e^x}{(e^x+1)}\right) \tag{35}$$

$$= \frac{e^x}{(e^x+1)}\left(1 - \frac{e^x}{(e^x+1)}\right) \tag{36}$$

$$= \sigma(x)(1-\sigma(x)) \tag{37}$$

The gradient

The gradient lists partial derivatives with respect to all inputs in a vector. For a function $f : \mathbb{R}^n \to \mathbb{R}$ of n variables the gradient $\nabla f : \mathbb{R}^n \to \mathbb{R}^n$ is defined as

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}. \tag{45}$$

- Gradients point in the steepest ascent direction.
- To find the gradient, we must compute the partial derivate with respect to every input.
- A vector collects all derivates.

The gradient of the Rosenbrock function

Recall the Rosenbrock function:

$$f(x, y) = (a - x)^2 + b(y - x^2)^2 \tag{51}$$

$$\nabla f(x, y) = \begin{pmatrix} -2a + 2x - 4byx + 4bx^3 \\ 2by - 2bx^2 \end{pmatrix} \tag{52}$$

On the board, derive:

$$\begin{aligned}
f(x, y) &= (a - x)^2 + b(y - x^2)^2 && (53)\\
&= a^2 - 2ax + x^2 + b(y^2 - 2yx^2 + x^4) && (54)\\
&= a^2 - 2ax + x^2 + by^2 - 2byx^2 + bx^4 && (55)\\
\Rightarrow \frac{\partial f(x, y)}{\partial x} &= -2a + 2x - 4byx + 4bx^3 && (56)\\
\Rightarrow \frac{\partial f(x, y)}{\partial y} &= 2by - 2bx^2 && (57)
\end{aligned}$$

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