$$\lim_{h\to 0}\frac{(x+h)^2-x^2}{h} = \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} \quad (4)$$
$$= \lim_{h\to 0}\frac{2xh+h^2}{h} \quad (5)$$
$$= \lim_{h\to 0}\frac{h(2x+h)}{h} \quad (6)$$
$$= \lim_{h\to 0} 2x+h \quad (7)$$
$$= 2x \quad (8)$$

Derive on the board.

Derivate of a parabola:

$$\lim_{h\to 0}\frac{(x+h)^2-x^2}{h} = \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} \quad (9)$$
$$= \lim_{h\to 0}\frac{2xh+h^2}{h} \quad (10)$$
$$= \lim_{h\to 0}\frac{h(2x+h)}{h} \quad (11)$$
$$= \lim_{h\to 0} 2x+h \quad (12)$$
$$= 2x \quad (13)$$

What is the derivative of the function $f(x) = x^n$?

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = nx^{n-1} \tag{14}$$

Derivate of a polynomial $f(x) = x^n$ [DFO20]:

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} \tag{15}$$

$$= \lim_{h\to 0} \frac{\sum_{i=0}^{n} \binom{n}{i} x^{n-1} h^i - x^n}{h} \tag{16}$$

$$= \lim_{h\to 0} \frac{\sum_{i=1}^{n} \binom{n}{i} x^{n-1} h^i}{h} \tag{17}$$

$$= \lim_{h\to 0} \sum_{i=1}^{n} \binom{n}{i} x^{n-1} h^{i-1} \tag{18}$$

$$= \lim_{h\to 0} (\binom{n}{1} x^{n-1}) + \sum i = 2n \binom{n}{i} x^{n-i} h^{i-1} \tag{19}$$

$$= \frac{n!}{1!(n-1)!} x^{n-1} = nx^{n-1}. \tag{20}$$

The gradient

The gradient lists partial derivatives with respect to all inputs in a vector. For a function $f : \mathbb{R}^n \to \mathbb{R}$ of n variables the gradient $\nabla f : \mathbb{R}^n \to \mathbb{R}^n$ is defined as

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}. \tag{46}$$

- Gradients point in the steepest ascent direction.
- To find the gradient, we must compute the partial derivate with respect to every input.
- A vector collects all derivates.

Recall the Rosenbrock function:

$$f(x,y) = (a-x)^2 + b(y-x^2)^2 \tag{52}$$

$$\nabla f(x,y) = \begin{pmatrix} -2a + 2x - 4byx + 4bx^3 \\ 2by - 2bx^2 \end{pmatrix} \tag{53}$$

On the board, derive:

$$f(x,y) = (a-x)^2 + b(y-x^2)^2 \tag{54}$$

$$= a^2 - 2ax + x^2 + b(y^2 - 2yx^2 + x^4) \tag{55}$$

$$= a^2 - 2ax + x^2 + by^2 - 2byx^2 + bx^4 \tag{56}$$

$$\Rightarrow \frac{\partial f(x,y)}{\partial x} = -2a + 2x - 4byx + 4bx^3 \tag{57}$$

$$\Rightarrow \frac{\partial f(x,y)}{\partial y} = 2by - 2bx^2 \tag{58}$$

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