Algorithm Specification – VIC State Updating

Version 1.0.0

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List of Symbols

Symbols	Description
Main Variables	
A	Grid cell area fraction [-]
H	Number of HRUs per band
S	State variable (generic)
b	Band index
t	Initial model state prior to updating
t^*	Final model state after updating
t'	Intermediate model state
Subscripts	
g	Glacier HRU
i	Non-glacier HRU (can also include Open Ground)
2	Open Ground HRU

1 BACKGROUND

This specification details the method for updating the VIC state file following glacier updating. One of the main features in the coupling of the VIC model of the UBC Regional Glaciation model (RGM) is the feedback if glacier area and surface elevation from the RGM to VIC. Changes in glacier area (passed from RGM to VIC as an updated glacier mask) and surface elevation are incorporated into the VIC model via updating of the vegetation parameter file and the elevation band file. A side-effect of this updating step is the need to adjust certain state variables to ensure conservation of mass within the individual VIC cells as a result of area and elevation updating. Conceptually, this entails the redistribution of water and energy between individual HRUs. For example, Goal of water re-distribution between HRUs is to converse the volume of water within a grid cell. In general, the following must hold for a given cell:

$$\sum_{i=1}^{H} S_i(t) \cdot A_i(t) = \sum_{i=1}^{H^*} S_i(t^*) \cdot A_i(t^*) \qquad 1\text{-}(1)$$

where H and H^* are the number of HRUs before and after glacier updating, respectively, S_i is a given state variable (e.g. water equivalent depth of snow) and A_i is the area fraction of HRU number i, and t and t^* represent the model state before and after glacier updating, respectively. The same concept holds for the conservation of energy. However, glaciers (more specifically, glacier HRUs) don't exist in every elevation band, furthermore, glacier changes also don't occur in every elevation band. Hence, state updating is more likely to occur on a band-by-band basis, and equation 1-(1) is re-written as

$$\sum_{b=1}^{B} \sum_{i=1}^{H_b} S_{i,b}(t) \cdot A_{i,b}(t) = \sum_{b=1}^{B^*} \sum_{i=1}^{H_b^*} S_{i,b}(t^*) \cdot A_{i,b}(t^*) \qquad 1\text{-}(2)$$

where B and B* are the number of elevation bands before and after glacier updating, and H_b and H_b^* are the number of HRUs in band b before and after glacier updating. Hence, state updating need only occur in elevation bands in which a glacier HRU undergoes an area change (i.e. $A_{i,b}(t) \neq A_{i,b}(t^*)$). On exception is the case where an elevation bands is created or disappears as a result of glacier HRU being created or removed, respectively; in these cases HRUs in both band b and it's neighboring band (b-1 or b+1, depending on situation), may also be affected.

The specifications described in the following sections for state updating is considered within the context of the pseudo-code shown in the text box below. The pseudo-code describes an algorithm for looping through cells and elevation bands within cells, and then checking within bands for different cases, or contexts, in which the area fraction of glacier HRUs can change, including the disappearance and initiation of glaciers. These are referred to as cases, of which seven have been indicated (which should cover all possibilities). The type of state updating required will depend on the context within which glacier area fractions change (the case). Of course, if a glacier HRU does not change its area,

then no state updating is required for the HRUs in that particular elevation band.

```
for (cell in cells) {
    for (band in bands) {
        glac_area_change <- area_frac_glac[cell,band,t] - area_frac_glac[cell,band,t-1]
        if (glac_area_change != 0) {
            if (glac_area_change < 0) { //Glacier HRU in band shrinks
                if (H[cell,band,t]>1) { //Band with glacier plus one or more additional HRUs
                    #CASE 1
                } else if (H[cell,band,t]==1) { //Band with single glacier HRU
                    #CASE 2
                } else if (H[cell,band,t]==0) { //Band, along with glacier, no longer, exists
                    if (area_frac_glac[cell,band-1,t ] > 0) { //glacier HRU exists in next lower band
                        #CASE 3a
                    } else { //glacier HRU does not exist in next lower band
                        #CASE 3b
                    }
                }
            } else { //Glacier HRU expands
                if (H[cell,band,t]>1) { //Band with new or pre-existing glacier plus one or more additional HRUs
                    #CASE 4
                } else if (H[cell,band,t]==1) { //Band with single glacier HRU
                    #CASE 5
                } else if (H[cell,band,t-1]==0) { //New band with single glacier HRU
                    #CASE 6
                }
            } else {
                #NO CHANGE IN STATE FOR HRUs IN THIS BAND
            }
    }
}
```

The state variables being considered are summarized in Table 1. Note that lake variables (i.e. LAKE_XXXX) are not being considered in this specification.

Table 1. State Variable Summary

State Variable	Description
ENERGY_TCANOPY_FBCOUNT	
ENERGY_T_FBCOUNT	
ENERGY_TFOLIAGE_FBCOUNT	
ENERGY_TSURF_FBCOUNT	
ENERGY_T	
GLAC_CUM_MASS_BALANCE	Glacier cumulative mass balance
GLAC_SURF_TEMP_FBCOUNT	
GLAC_SURF_TEMP	Temperature of glacier surface Layer
GLAC_WATER_STORAGE	Water stored in the glacier
GRID_CELL	Grid cell ID number
HRU_BAND_INDEX	Band index
HRU_VEG_INDEX	HRU index
HRU_VEG_VAR_DEW	Water stored on surface/vegetation
LAYER_ICE_CONTENT	Ice in each soil layer
LAYER_MOIST	xxx in each soil layer
NUM_BANDS	Number of bands (set in global file)
SNOW_ALBEDO	Albedo of snow
SNOW_CANOPY_ALBEDO	Albedo of snow stored in the canopy
SNOW_CANOPY	Snow stored in the canopy
SNOW_COLD_CONTENT	Cold content of snow surface layer
SNOW_COVERAGE	Snow coverage fraction
SNOW_DENSITY	Snow density
SNOW_DEPTH	Snow depth
SNOW_LAST_SNOW	Days since last snowfall
SNOW_MELTING	Snow melting indicator [0 or 1]
SNOW_PACK_TEMP	Temperature of snow pack layer
SNOW_PACK_WATER	Water stored in snow pack layer
SNOW_SURF_TEMP_FBCOUNT	
SNOW_SURF_TEMP	Temperature of snow surface layer
SNOW_SURF_WATER	Water stored in snow surface layer
SNOW_SWQ	Total snow water equivalent
SOIL_DZ_NODE	
SOIL_ZSUM_NODE	
VEG_TYPE_NUM	Vegetation classification ID

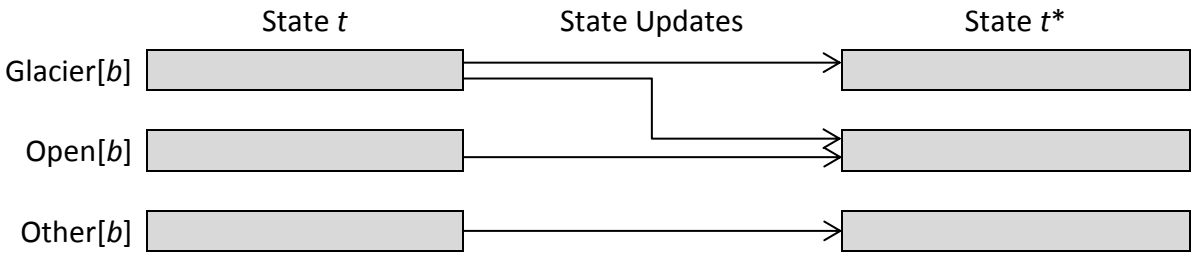
2 CASE 1

2.1 Description

Glacier HRU in band *b* shrinks by ΔA and is replaced by open ground HRU in band *b*. This case specification is motivated by the following area changes:

HRU	State *t*	State *t**
Glacier[*b*]	$A_g(t) > 0$	$0 < A_g(t^*) < A_g(t)$
Open[*b*]	$A_2(t) \geq 0$	$A_2(t^*) > A_2(t)$
Other[*b*]	$A_i(t) \geq 0$	$A_i(t^*) = A_i(t)$

HRU state updates map between the HRUs as follows:



2.2 State Variables Water Balance

Source HRU	Source State Variable, *S*(*t*)	Target HRU	Target State Variable, *S*(*t**)
Glacier	LAYER_MOIST (N layers)	Open	LAYER_MOIST (N layers)
Glacier	LAYER_ICE_CONTENT (N layers)	Open	LAYER_ICE_CONTENT (N layers)
Glacier	HRU_VEG_VAR_WDEW	Open	HRU_VEG_VAR_WDEW
Glacier	SNOW_SWQ	Open	SNOW_SWQ
Glacier	SNOW_SURF_WATER	Open	SNOW_SURF_WATER
Glacier	SNOW_PACK_WATER	Open	SNOW_PACK_WATER

2.2.1 Objective(s):

The following constraints apply for these state variables:

1. Conserve water storage depth, *d*, (specific mass) in glacier HRU; i.e. decrease water volume proportional to $d \cdot \Delta A$
2. Add lost glacier water storage volume $d \cdot \Delta A$ to open ground HRU

2.2.2 Specification for Glacier and Open HRUs:

Ensure conservation of mass between the glacier and open ground HRU such that

$$A_g(t) \cdot S_g(t) + A_2(t) \cdot S_2(t) = A_g(t^*) \cdot S_g(t^*) + A_2(t^*) \cdot S_2(t^*).$$

Therefore, the updated water storage depth in the open ground HRU is

$$S_2(t^*) = \left[A_g(t) \cdot S_g(t) + A_2(t) \cdot S_2(t) - A_g(t^*) \cdot S_g(t^*)\right] \cdot \frac{1}{A_2(t^*)}.$$

However, according to objective 1

$$\boxed{S_g(t^*) = S_g(t)} \qquad 2\text{-}(3)$$

such that

$$\boxed{S_2(t^*) = \left\{S_g(t)\left[A_g(t) - A_g(t^*)\right] + A_2(t) \cdot S_2(t)\right\} \cdot \frac{1}{A_2(t^*)}.} \qquad 2\text{-}(4)$$

2.2.3 Specification for Remaining HRUs:

No change to values; i.e.

$$\boxed{S_i(t^*) = S_i(t)} \qquad 2\text{-}(5)$$

2.3 State Variables Glacier Water Storage

Source HRU	Source State Variable, *S*(*t*)	Target HRU	Target State Variable, *S*(*t**)
Glacier	GLAC_WATER_STORAGE	Glacier	GLAC_WATER_STORAGE

2.3.1 Objective(s):

The following constraint applies for these state variables:

1. Conserve mass within the glacier HRU; i.e. GLAC_WATER_STORAGE stays with glacier

2.3.2 Specification for Glacier HRU:

Ensure conservation of mass in glacier HRU between step *t* and *t** such that

$$A_g(t) \cdot S_g(t) = A_g(t^*) \cdot S_g(t^*)$$

and

$$S_g(t^*) = S_g(t)\frac{A_g(t)}{A_g(t^*)} \qquad \text{2-(6)}$$

2.3.3 Specification for Remaining HRUs:

Not applicable

2.4 State Variables Glacier Mass Balance

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier	GLAC_CUM_MASS_BALANCE	Glacier	GLAC_CUM_MASS_BALANCE

2.4.1 Objective(s):

The following constraint applies for these state variables:

1. Conserve depth (specific mass) within the glacier HRU

2.4.2 Specification for Glacier HRU:

Ensure conservation of specific mass in glacier HRU between step t and t^* such that

$$S_g(t^*) = S_g(t) \qquad \text{2-(7)}$$

2.4.3 Specification for Remaining HRUs:

Not applicable

2.5 State Variables Snow Properties

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Both[a]	SNOW_DEPTH	Same	SNOW_DEPTH
Both	SNOW_DENSITY	Same	SNOW_DENSITY

[a] Both glacier and open ground HRUs

2.5.1 Objective(s):

The following constraints apply for these state variables:

1. Conserve snow depth
2. Conservation of snow mass

2.5.2 Specification for Glacier and Open HRUs:

By objective 1, snow depth remains constant in all HRUs such that

$$SNOW\_DEPTH(t^*) = SNOW\_DEPTH(t) \quad \text{2-(8)}$$

In order to ensure conservation of snow mass (objective 2), the new density is calculated as

$$SNOW\_DENSITY(t^*) = [SNOW\_SWQ(t^*) \cdot 1000]/SNOW\_DEPTH(t^*) \quad \text{2-(9)}$$

2.5.3 Specification for Remaining HRUs:

No change to values; same as equation 2-(5).

2.6 State Variables Snow Energy

Source HRU	Source State Variable, *S*(*t*)	Target HRU	Target State Variable, *S*(*t**)
Both	SNOW_COLD_CONTENT[a]	Same	SNOW_COLD_CONTENT
Both	SNOW_SURF_TEMP	Same	SNOW_SURF_TEMP
Both	SNOW_PACK_TEMP	Same	SNOW_PACK_TEMP

[a] Cold content of snow surface layer only

2.6.1 Objective(s):

The following constraints apply for these state variables:

1. Conservation of energy; i.e. same cold content in HRU after re-adjustment area and water storage

2.6.2 Specification for Glacier and Open HRUs:

Cold content [J/m^2] is calculated as

$$SNOW\_COLD\_CONTENT = CH_{ice} \cdot SNOW\_DEPTH \cdot SNOW\_SURF\_TEMP.$$

As cold content must be conserved for any given HRU, we have

$$SNOW\_COLD\_CONTENT(t^*) = SNOW\_COLD\_CONTENT(t)\frac{A(t)}{A(t^*)} \quad \text{2-(10)}$$

Given the updated cold content, the snow surface layer temperature is updated as

$$SNOW\_SURF\_TEMP(t^*) = \frac{SNOW\_COLD\_CONTENT(t^*)}{CH_{ice} \cdot SNOW\_DEPTH(t^*)}. \quad \text{2-(11)}$$

By the same arguments, snow pack temperature is updated using

$$SNOW\_PACK\_TEMP_p(t^*) = \frac{SNOW\_DEPTH(t) \cdot A(t)}{SNOW\_DEPTH(t^*) \cdot A(t^*)} SNOW\_PACK\_TEMP(t),$$

which, as *SNOW_DEPTH*(*t**) = *SNOW_DEPTH*(*t*), simplifies to

$$SNOW\_PACK\_TEMP(t^*) = \frac{A(t)}{A(t^*)} SNOW\_PACK\_TEMP(t). \quad \text{2-(12)}$$

2.6.3 Specification for Remaining HRUs:

No change to values; same as equation 2-(5).

2.7 State Variables Miscellaneous

Source HRU	Source State Variable, *S*(*t*)	Target HRU	Target State Variable, *S*(*t**)
Both	ENERGY_T	Same	ENERGY_T
Both	GRID_CELL	Same	GRID_CELL
Both	HRU_BAND_INDEX	Same	HRU_BAND_INDEX
Both	HRU_VEG_INDEX	Same	HRU_VEG_INDEX
Both	NUM_BANDS	Same	NUM_BANDS
Both	SNOW_ALBEDO	Same	SNOW_ALBEDO
Both	SNOW_CANOPY_ALBEDO	Same	SNOW_CANOPY_ALBEDO
Both	SNOW_COVERAGE	Same	SNOW_COVERAGE
Both	SNOW_LAST_SNOW	Same	SNOW_LAST_SNOW
Both	SNOW_MELTING	Same	SNOW_MELTING
Both	SOIL_DZ_NODE	Same	SOIL_DZ_NODE
Both	SOIL_ZSUM_NODE	Same	SOIL_ZSUM_NODE
Both	ENERGY_T_FBCOUNT	Same	ENERGY_T_FBCOUNT
Both	ENERGY_TCANOPY_FBCOUNT	Same	ENERGY_TCANOPY_FBCOUNT
Both	ENERGY_TFOLIAGE_FBCOUNT	Same	ENERGY_TFOLIAGE_FBCOUNT
Both	ENERGY_TSURF_FBCOUNT	Same	ENERGY_TSURF_FBCOUNT
Both	GLAC_SURF_TEMP_FBCOUNT	Same	GLAC_SURF_TEMP_FBCOUNT
Both	SNOW_SURF_TEMP_FBCOUNT	Same	SNOW_SURF_TEMP_FBCOUNT

2.7.1 Objective(s):

The following constraints apply for these state variables:

1. Values remain constant between step *t* and *t**

2.7.2 Specification for Glacier and Open HRUs:

By objective 1,

$$\boxed{S(t^*) = S(t) \quad for\ all\ HRUs.} \qquad \text{2-(13)}$$

2.7.3 Specification for Remaining HRUs:

As per equation 2-(13).

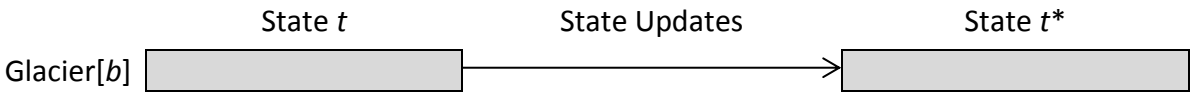
3 CASE 2

3.1 Description

The glacier HRU shrinks by ΔA in band b and only a single HRU exists in band b. This case specification is motivated by the following area changes:

HRU	State t	State t^*
Glacier[b]	$A_g(t) > 0$	$0 < A_g(t^*) < A_g(t)$

HRU updates map as follows:



3.2 State Variables Water Balance

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier	LAYER_MOIST (N layers)	Glacier	LAYER_MOIST (N layers)
Glacier	LAYER_ICE_CONTENT (N layers)	Glacier	LAYER_ICE_CONTENT (N layers)
Glacier	HRU_VEG_VAR_WDEW	Glacier	HRU_VEG_VAR_WDEW
Glacier	SNOW_SWQ	Glacier	SNOW_SWQ
Glacier	SNOW_SURF_WATER	Glacier	SNOW_SURF_WATER
Glacier	SNOW_PACK_WATER	Glacier	SNOW_PACK_WATER
Glacier	GLAC_WATER_STORAGE	Glacier	GLAC_WATER_STORAGE

3.2.1 Objective(s):

The following constraints apply for these state variables:

1. Conserve mass within glacier HRU

3.2.2 Specification for Glacier HRU:

Ensure conservation of mass in glacier HRU between step t and t^* such that

$$A_g(t) \cdot S_g(t) = A_g(t^*) \cdot S_g(t^*)$$

and

$$S_g(t^*) = S_g(t)\frac{A_g(t)}{A_g(t^*)} \qquad 3\text{-}(1)$$

3.3 State Variables Glacier Mass Balance

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier	GLAC_CUM_MASS_BALANCE	Glacier	GLAC_CUM_MASS_BALANCE

3.3.1 Objective(s):

The following constraint applies for these state variables:

1. Conserve depth (specific mass) within the glacier HRU

3.3.2 Specification for Glacier HRU:

As per equation 2-(7).

3.4 State Variables Snow Properties

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier	SNOW_DEPTH	Glacier	SNOW_DEPTH
Glacier	SNOW_DENSITY	Glacier	SNOW_DENSITY

3.4.1 Objective(s):

The following constraints apply for these state variables:

1. Conserve snow depth
2. Conservation of snow mass

3.4.2 Specification for Glacier HRU:

As per equations 2-(8) and 2-(9)

3.5 State Variables Snow Energy

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier	SNOW_COLD_CONTENT[a]	Glacier	SNOW_COLD_CONTENT
Glacier	SNOW_SURF_TEMP	Glacier	SNOW_SURF_TEMP
Glacier	SNOW_PACK_TEMP	Glacier	SNOW_PACK_TEMP

[a] Cold content of snow surface layer only

3.5.1 Objective(s):

The following constraints apply for these state variables:

1. Conservation of energy; i.e. same cold content in HRU after re-adjustment area and water storage

3.5.2 Specification for Glacier HRU:

As per equations 2-(10), 2-(11) and 2-(12)

3.6 State Variables Miscellaneous

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier	ENERGY_T	Glacier	ENERGY_T
Glacier	GRID_CELL	Glacier	GRID_CELL
Glacier	HRU_BAND_INDEX	Glacier	HRU_BAND_INDEX
Glacier	HRU_VEG_INDEX	Glacier	HRU_VEG_INDEX
Glacier	NUM_BANDS	Glacier	NUM_BANDS
Glacier	SNOW_ALBEDO	Glacier	SNOW_ALBEDO
Glacier	SNOW_CANOPY_ALBEDO	Glacier	SNOW_CANOPY_ALBEDO
Glacier	SNOW_COVERAGE	Glacier	SNOW_COVERAGE
Glacier	SNOW_LAST_SNOW	Glacier	SNOW_LAST_SNOW
Glacier	SNOW_MELTING	Glacier	SNOW_MELTING
Glacier	SOIL_DZ_NODE	Glacier	SOIL_DZ_NODE
Glacier	SOIL_ZSUM_NODE	Glacier	SOIL_ZSUM_NODE
Glacier	ENERGY_T_FBCOUNT	Glacier	ENERGY_T_FBCOUNT
Glacier	ENERGY_TCANOPY_FBCOUNT	Glacier	ENERGY_TCANOPY_FBCOUNT
Glacier	ENERGY_TFOLIAGE_FBCOUNT	Glacier	ENERGY_TFOLIAGE_FBCOUNT
Glacier	ENERGY_TSURF_FBCOUNT	Glacier	ENERGY_TSURF_FBCOUNT
Glacier	GLAC_SURF_TEMP_FBCOUNT	Glacier	GLAC_SURF_TEMP_FBCOUNT
Glacier	SNOW_SURF_TEMP_FBCOUNT	Glacier	SNOW_SURF_TEMP_FBCOUNT

3.6.1 Objective(s):

The following constraints apply for these state variables:

1. Values remain constant between step t and t^*

3.6.2 Specification for Glacier HRU:

As per equation 2-(13).

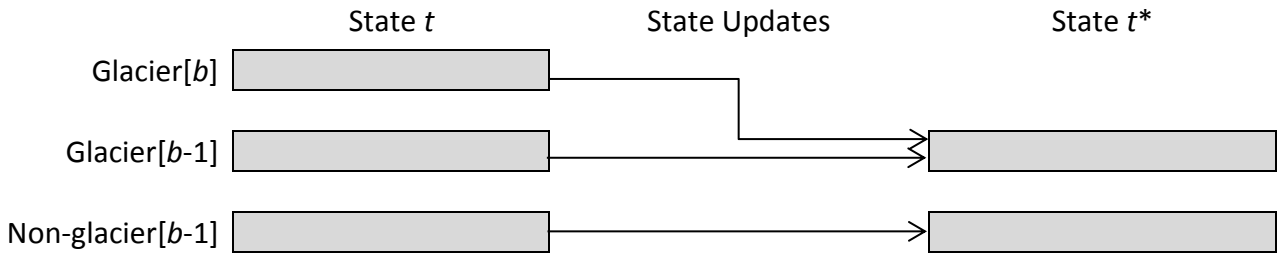
4 CASE 3a - Incomplete

4.1 Description

Glacier HRU shrinks and disappears from band, *b*, and band *b* disappears as well and a glacier HRU exists in next lower band, *b*-1. In this case we assume that the glacier in band *b* "shrinks into" the glacier in band *b*-1. This case specification is motivated by the following area changes:

VIC Element	State t	State t'	State t^*
		HRU Area	
Glacier[b]	$A_g(t) > 0$		$A_g(t^*) = 0$
Glacier[b-1]	$A_g(t) \geq 0$	$0 < A_g(t') = \neq A_i(t)$	$A_g(t^*) = A_g(t')$
Non-glacier[b-1]	$A_i(t) > 0$	$A_i(t') = \neq A_i(t)$	$A_i(t^*) = A_i(t')$
		Band Area	
b	$A_b(t) = 0$		$A_b(t^*) > 0$
b-1	$A_{b-1}(t) > 0$	$A_{b-1}(t') = \neq A_{b-1}(t)$	$A_{b-1}(t^*) = A_{b-1}(t')$

HRU updates map as follows:



4.2 State Variables Water Balance

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier[b]	LAYER_MOIST (N layers)	Glacier[b-1]	LAYER_MOIST (N layers)
Glacier[b]	LAYER_ICE_CONTENT (N layers)	Glacier[b-1]	LAYER_ICE_CONTENT (N layers)
Glacier[b]	HRU_VEG_VAR_WDEW	Glacier[b-1]	HRU_VEG_VAR_WDEW
Glacier[b]	SNOW_SWQ	Glacier[b-1]	SNOW_SWQ
Glacier[b]	SNOW_SURF_WATER	Glacier[b-1]	SNOW_SURF_WATER
Glacier[b]	SNOW_PACK_WATER	Glacier[b-1]	SNOW_PACK_WATER
Glacier[b]	GLAC_WATER_STORAGE	Glacier[b-1]	GLAC_WATER_STORAGE

4.2.1 Objective(s):

The following constraints apply to these state variables:

1. Conserve mass between neighboring glacier HRUs; i.e. all water stored in glacier from upper band *b* added to glacier HRU in next lower band *b*-1
2. Water from source glacier HRU is zeroed and/or record removed from state file

4.2.2 Specification for Glacier HRU in band b-1:

By conservation of mass (objective 1)

$$A_g(b,t') \cdot S_g(b,t') = A_g(b-1,t^*) \cdot \Delta S_g(b-1,t^*)$$

and

$$\Delta S_g(b-1,t^*) = S_g(b,t') \frac{A_g(b,t')}{A_g(b-1,t^*)}$$

such that

$$\boxed{S_g(b-1,t^*) = S_g(b-1,t') + \Delta S_g(b-1,t^*).} \quad \text{4-(1)}$$

4.2.3 Specification for Glacier HRU in band b:

Values for the glacier HRU in band *b* are zeroed (and/or removed) as this HRU no longer exists, hence

$$\boxed{S_g(b,t^*) = 0.} \quad \text{4-(2)}$$

4.2.4 Specification for Non-glacier HRUs in band b-1:

By conservation of mass

$$\boxed{S(t^*) = S(t') \quad for\ all\ HRUs.} \quad \text{4-(3)}$$

.

4.3 State Variables Glacier Mass Balance

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
n/a	n/a	Glacier[*b*-1]	GLAC_CUM_MASS_BALANCE

4.3.1 Objective(s):

The following constraint applies for these state variables:

1. Conserve depth (specific mass) within the target glacier HRU
2. Variable from source glacier HRU is zeroed and/or record removed from state file

4.3.2 Specification for Glacier HRU in band b-1:

Ensure conservation of specific mass in glacier HRU between step *t'* and *t** such that

$$S_g(t^*) = S_g(t')$$ 4-(4)

4.3.3 Specification for Glacier HRU in band b:

All values zeroed as per equation 4-(2).

4.3.4 Specification for Non-glacier HRUs in band b-1:

No change, as per equation 4-(3).

4.4 State Variables Snow Properties

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier[b-1]	SNOW_DEPTH	Glacier[b-1]	SNOW_DEPTH
Glacier[b-1]	SNOW_DENSITY	Glacier[b-1]	SNOW_DENSITY

4.4.1 Objective(s):

The following constraints apply for these state variables:

1. Conserve snow depth
2. Conservation of snow mass
3. Water from source glacier HRU is zeroed and/or record removed from state file

4.4.2 Specification for Glacier HRU in band b-1:

By objective 1, snow depth remains constant in all HRUs such that

$$SNOW\_DEPTH(t^*) = SNOW\_DEPTH(t)$$ 4-(5)

Density is updated as per equation 2-(9).

4.4.3 Specification for Glacier HRU in band b:

All values zeroed as per equation 4-(2).

4.4.4 Specification for Non-glacier HRUs in band b-1:

Update as per equation 4-(3).

4.5 State Variables Snow Energy

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier[b-1]	SNOW_COLD_CONTENT[a]	Glacier[b-1]	SNOW_COLD_CONTENT[a]
Glacier[b-1]	SNOW_SURF_TEMP	Glacier[b-1]	SNOW_SURF_TEMP
Glacier[b-1]	SNOW_PACK_TEMP	Glacier[b-1]	SNOW_PACK_TEMP

[a] Cold content of snow surface layer only

4.5.1 Objective(s):

The following constraints apply for these state variables:

1. Conservation of energy; i.e. same cold content in HRU after re-adjustment area and water storage
2. Values from source glacier HRU are zeroed and/or record removed from state file

4.5.2 Specification for Glacier HRU in band b-1:

Snow cold content is updated as per

$$SNOW\_COLD\_CONTENT(t^*) = SNOW\_COLD\_CONTENT(t')\frac{A(t')}{A(t^*)} \quad \text{4-(6)}$$

Snow surface temperature is updated as equations 2-(11) and snow pack temperature is updated using

$$SNOW\_PACK\_TEMP(t^*) = \frac{A(t')}{A(t^*)}SNOW\_PACK\_TEMP(t'). \quad \text{4-(7)}$$

4.5.3 Specification for Glacier HRU in band b:

All values zeroed as per equation 4-(2).

4.5.4 Specification for Non-glacier HRUs in band b-1:

No change, as per equation 4-(3).

4.6 State Variables Miscellaneous

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier[b-1]	ENERGY_T	Glacier[b-1]	ENERGY_T
Glacier[b-1]	GRID_CELL	Glacier[b-1]	GRID_CELL
Glacier[b-1]	HRU_BAND_INDEX	Glacier[b-1]	HRU_BAND_INDEX
Glacier[b-1]	HRU_VEG_INDEX	Glacier[b-1]	HRU_VEG_INDEX
Glacier[b-1]	NUM_BANDS	Glacier[b-1]	NUM_BANDS
Glacier[b-1]	SNOW_ALBEDO	Glacier[b-1]	SNOW_ALBEDO
Glacier[b-1]	SNOW_CANOPY_ALBEDO	Glacier[b-1]	SNOW_CANOPY_ALBEDO
Glacier[b-1]	SNOW_COVERAGE	Glacier[b-1]	SNOW_COVERAGE
Glacier[b-1]	SNOW_LAST_SNOW	Glacier[b-1]	SNOW_LAST_SNOW
Glacier[b-1]	SNOW_MELTING	Glacier[b-1]	SNOW_MELTING
Glacier[b-1]	SOIL_DZ_NODE	Glacier[b-1]	SOIL_DZ_NODE
Glacier[b-1]	SOIL_ZSUM_NODE	Glacier[b-1]	SOIL_ZSUM_NODE

4.6.1 Objective(s):

The following constraints apply for these state variables:

1. Values remain constant between step t and t^*
2. Values from source glacier HRU are zeroed and/or record removed from state file

4.6.2 Specification for Glacier HRU in band b-1:

As per equation 4-(3).

4.6.3 Specification for Glacier HRU in band b:

All values zeroed as per equation 4-(2).

4.6.4 Specification for Non-glacier HRUs in band b-1:

No change, as per equation 4-(3).

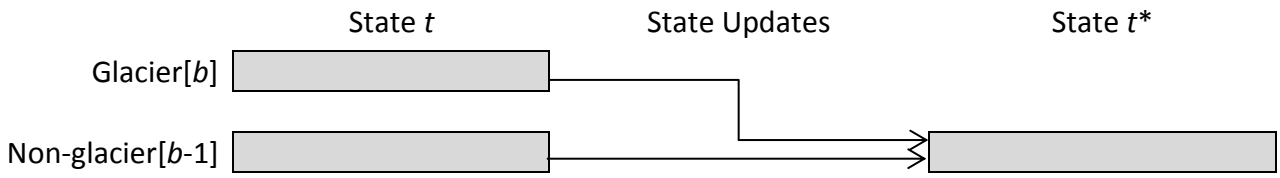
5 CASE 3b - Incomplete

5.1 Description

The Glacier HRU shrinks and disappears from band *b*, and band *b* disappears as well and no glacier HRU exists in next lower band, *b*-1. This case specification is motivated by the following area changes:

VIC Element	State t	State t'	State t^*
		HRU Area	
Glacier[*b*]	$A_g(t) > 0$		$A_g(t^*) = 0$
Glacier[*b*-1]	$A_g(t) \geq 0$	$A_g(t') = 0$	
Non-glacier[*b*-1]	$A_i(t) > 0$	$A_i(t') = \mid \neq A_i(t)$	$A_i(t^*) = A_i(t)$
		Band Area	
b	$A_b(t) = 0$		$A_b(t^*) > 0$
b-1	$A_{b-1}(t) > 0$	$A_{b-1}(t') = \mid \neq A_{b-1}(t)$	$A_{b-1}(t^*) = A_{b-1}(t')$

HRU updates map as follows:



5.2 State Variables Water Balance

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier[*b*]	LAYER_MOIST (N layers)	All[*b*-1]	LAYER_MOIST (N layers)
Glacier[*b*]	LAYER_ICE_CONTENT (N layers)	All[*b*-1]	LAYER_ICE_CONTENT (N layers)
Glacier[*b*]	HRU_VEG_VAR_WDEW	All[*b*-1]	HRU_VEG_VAR_WDEW
Glacier[*b*]	SNOW_SWQ	All[*b*-1]	SNOW_SWQ
Glacier[*b*]	SNOW_SURF_WATER	All[*b*-1]	SNOW_SURF_WATER
Glacier[*b*]	SNOW_PACK_WATER	All[*b*-1]	SNOW_PACK_WATER
Glacier[*b*]	GLAC_WATER_STORAGE	All[*b*-1]	LAYER_MOIST [Nth layer]

5.2.1 Objective(s):

The following constraints apply to these state variables:

1. Conserve of mass between disappearing glacier HRU and non-glacier HRUs in next lower band
2. Increase water storage in non-glacier HRUs proportional to HRU area; i.e. more water added

to larger HRUs, less water to smaller HRUs

3. Water from source glacier HRU is zeroed and/or record removed from state file

5.2.2 Specification for All HRUs in band b-1:

By conservation of mass (objective 1)

$$A_g(b,t) \cdot \Delta S_g(b,t) = -\sum_{i=1}^{H} A_i(b-1,t') \cdot \Delta S_i(b-1,t'). \quad \text{5-(1)}$$

Also, according to objective 2

$$\frac{\Delta S_i(b-1,t')}{A_i(b-1,t')} = \frac{\Delta S_{i+1}(b-1,t')}{A_{i+1}(b-1,t')} = \cdots = \frac{\Delta S_H(b-1,t')}{A_H(b-1,t')}. \quad \text{5-(2)}$$

Hence, we must solve for ΔS_i by solving equations 4-(1) and 4-(2) as a system of H linear equations. Finally, the final water storage depth in the H HRUs in band b-1 is given by

$$S_i(b-1,t^*) = S_i(b-1,t') + \Delta S_i(b-1,t'), \quad for\ i = 1,\dots,H. \quad \text{5-(3)}$$

5.2.3 Specification for Glacier HRU in band b:

As per equation 4-(2).

5.3 State Variables Glacier Mass Balance

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier[b-1]	GLAC_CUM_MASS_BALANCE	Glacier[b-1]	GLAC_CUM_MASS_BALANCE

5.3.1 Objective(s):

The following constraint applies for these state variables:

1. Variable not applicable to target HRUs; variable is zeroed and/or record removed from state file

5.3.2 Specification for All HRUs in band b-1:

Not applicable.

5.3.3 Specification for Glacier HRU in band b:

As per equation 4-(2).

5.4 State Variables Snow Properties

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
All[b-1]	SNOW_DEPTH	All[b-1]	SNOW_DEPTH
All[b-1]	SNOW_DENSITY	All[b-1]	SNOW_DENSITY

5.4.1 Objective(s):

The following constraints apply for these state variables:

1. Conserve snow depth in target HRUs
2. Conservation of snow mass in target HRUs
3. Water from source glacier HRU is zeroed and/or removed from state file

5.4.2 Specification for All HRUs in band b-1:

As per equations 2-(8) and 2-(9)

5.4.3 Specification for Glacier HRU in band b:

As per equation 4-(2).

5.5 State Variables Snow Energy

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
All[b-1]	SNOW_COLD_CONTENT[a]	All[b-1]	SNOW_COLD_CONTENT[a]
All[b-1]	SNOW_SURF_TEMP	All[b-1]	SNOW_SURF_TEMP
All[b-1]	SNOW_PACK_TEMP	All[b-1]	SNOW_PACK_TEMP

[a] Cold content of snow surface layer only

5.5.1 Objective(s):

The following constraints apply for these state variables:

1. Conservation of energy; i.e. same cold content in HRU after re-adjustment area and water storage
2. Values from source glacier HRU are zeroed and/or record removed from state file

5.5.2 Specification for All HRUs in band b-1:

As per equations 2-(10), 2-(11) and 2-(12)

5.5.3 Specification for Glacier HRU in band b:

As per equation 4-(2).

5.6 State Variables Miscellaneous

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
All[b-1]	ENERGY_T	All[b-1]	ENERGY_T
All[b-1]	GRID_CELL	All[b-1]	GRID_CELL
All[b-1]	HRU_BAND_INDEX	All[b-1]	HRU_BAND_INDEX
All[b-1]	HRU_VEG_INDEX	All[b-1]	HRU_VEG_INDEX
All[b-1]	NUM_BANDS	All[b-1]	NUM_BANDS
All[b-1]	SNOW_ALBEDO	All[b-1]	SNOW_ALBEDO
All[b-1]	SNOW_CANOPY_ALBEDO	All[b-1]	SNOW_CANOPY_ALBEDO
All[b-1]	SNOW_COVERAGE	All[b-1]	SNOW_COVERAGE
All[b-1]	SNOW_LAST_SNOW	All[b-1]	SNOW_LAST_SNOW
All[b-1]	SNOW_MELTING	All[b-1]	SNOW_MELTING
All[b-1]	SOIL_DZ_NODE	All[b-1]	SOIL_DZ_NODE
All[b-1]	SOIL_ZSUM_NODE	All[b-1]	SOIL_ZSUM_NODE

5.6.1 Objective(s):

The following constraints apply for these state variables:

1. Values remain constant between step t and t^*
2. Values from source glacier HRU are zeroed and/or record removed from state file

5.6.2 Specification for All HRUs in band b-1:

As per equation 4-(3).

5.6.3 Specification for Glacier HRU in band b:

As per equation 4-(2).

6 CASE 4 - Incomplete

6.1 Description

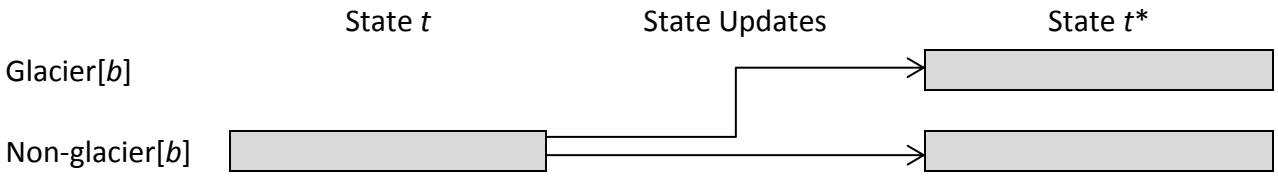
A *new* or *pre-existing* glacier HRU in band *b* expands by ΔA into one or more existing non-glacier HRUs in band *b*. This case specification is motivated by the following area changes for a *new* glacier:

HRU	State *t*	State *t**
Glacier[*b*]	$A_g(t) = 0$	$A_g(t^*) > 0$
Non-glacier[*b*]	$A_i(t) > 0$	$0 \leq A_i(t^*) < A_i(t)$

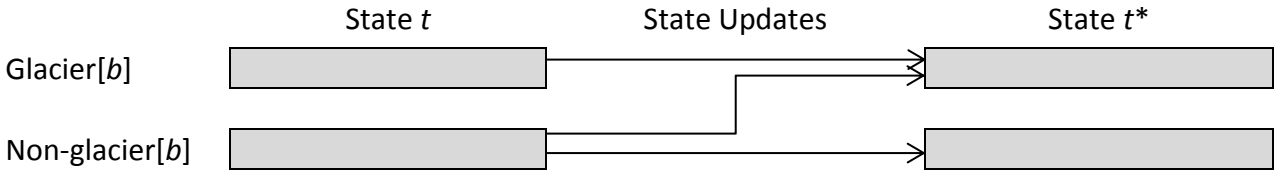
and by the following area changes for a *pre-existing* glacier:

HRU	State *t*	State *t**
Glacier[*b*]	$A_g(t) > 0$	$A_g(t^*) > 0$
Non-glacier[*b*]	$A_i(t) > 0$	$0 \leq A_i(t^*) < A_i(t)$

HRU updates map as follows for a *new* glacier:



HRU updates map as follows for a *pre-existing* glacier:



6.2 State Variables Water Balance

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Non-Glacier	LAYER_MOIST (N layers)	Glacier	LAYER_MOIST (N layers)
Non-Glacier	LAYER_ICE_CONTENT (N layers)	Glacier	LAYER_ICE_CONTENT (N layers)
Non-Glacier	HRU_VEG_VAR_WDEW	Glacier	GLAC_WATER_STORAGE
Non-Glacier	SNOW_SWQ	Glacier	SNOW_SWQ
Non-Glacier	SNOW_SURF_WATER	Glacier	SNOW_SURF_WATER
Non-Glacier	SNOW_PACK_WATER	Glacier	SNOW_PACK_WATER
Non-Glacier	SNOW_CANOPY	Glacier	SNOW_SWQ

6.2.1 Objective(s):

The following constraints apply for these state variables:

1. Conserve water storage depth (specific mass) in all non-glacier HRUs; i.e. water volume (and mass) in each non-glacier HRU decreases proportional to $d \cdot \Delta A$.
2. Add volume to glacier HRU; i.e. $\Delta V_g = -\sum_{i=2}^{H} d_i \cdot \Delta A_i$.

6.2.2 Specification when pre-existing Glacier HRU:

By conservation of mass we have

$$A_g(t) \cdot S_g(t) + \sum_{i=2}^{H} A_i(t) \cdot S_i(t) = A_g(t^*) \cdot S_g(t^*) + \sum_{i=2}^{H} A_i(t^*) \cdot S_i(t^*).$$

However, for the non-glacier HRUs (see objective 1) we set

$$\boxed{S_i(t^*) = S_i(t)} \qquad 6\text{-}(1)$$

such that for the glacier HRU

$$\boxed{S_g(t^*) = \left\{A_g(t) \cdot S_g(t) + \sum_{i=2}^{H} S_i(t)[A_i(t) - A_i(t^*)]\right\} \Big/ A_g(t^*).} \qquad 6\text{-}(2)$$

6.2.3 Specification when new Glacier HRU added:

We again set

$$\boxed{S_i(t^*) = S_i(t).} \qquad 6\text{-}(3)$$

For the glacier HRU, modify equation 6-(2) to account for $A_g(t) = 0$ and $d_g(t) = 0$, such that

$$S_g(t^*) = \left\{\sum_{i=2}^{H} S_i(t)[A_i(t) - A_i(t^*)]\right\} \Big/ A_g(t^*). \quad \text{6-(4)}$$

6.3 State Variables Glacier Mass Balance

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier	GLAC_CUM_MASS_BALANCE	Glacier	GLAC_CUM_MASS_BALANCE

6.3.1 Objective(s):

The following constraint applies for these state variables:

1. Initialize values at zero for new glacier
2. Conserve depth (specific mass) within the glacier HRU for pre-existing glacier

6.3.2 Specification when new Glacier HRU added:

$$S_g(t^*) = 0. \quad \text{6-(5)}$$

6.3.3 Specification when pre-existing Glacier HRU:

$$S_g(t^*) = S_g(t). \quad \text{6-(6)}$$

6.4 State Variables Snow Properties

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
All	SNOW_DEPTH	Same	SNOW_DEPTH
All	SNOW_DENSITY	Same	SNOW_DENSITY

6.4.1 Objective(s):

The following constraints apply for these state variables:

1. Conserve snow depth in non-glacier HRUs
2. Snow depth for new glacier is weighted average snow depth from pre-existing non-glacier HRUs
3. Conserve snow depth for pre-existing glacier
4. Conservation of snow mass

6.4.2 Specification when new Glacier HRU added:

As snow depth remains constant in all non-glacier HRUs, the new snow depth is simply

$$SNOW\_DEPTH_i(t^*) = SNOW\_DEPTH_i(t) \qquad \text{6-(7)}$$

The updated snow depth for the new glacier HRU is then the weighted average

$$SNOW\_DEPTH_g(t^*) = \left[\sum_{i=2}^{H} SNOW\_DEPTH_i(t) \cdot A_i(t)\right] \Big/ \sum_{i=2}^{H} A_i(t)\,. \qquad \text{6-(8)}$$

In order to ensure conservation of snow mass (objective 4), the new snow density for all HRUs is calculated as

$$SNOW\_DENSITY(t^*) = [SNOW\_SWQ(t^*) \cdot 1000]/SNOW\_DEPTH(t^*)\,. \qquad \text{6-(9)}$$

6.4.3 Specification when pre-existing Glacier HRU:

As snow depth remains constant in all HRUs, the new snow depth is simply

$$SNOW\_DEPTH_i(t^*) = SNOW\_DEPTH_i(t) \qquad \text{6-(10)}$$

and

$$SNOW\_DEPTH_g(t^*) = SNOW\_DEPTH_g(t)\,. \qquad \text{6-(11)}$$

In order to ensure conservation of snow mass (objective 4), the new snow density for each HRU is calculated as per 2-(8).

6.5 State Variables Snow Energy

Source HRU	Source State Variable, *S*(*t*)	Target HRU	Target State Variable, *S*(*t**)
All	SNOW_COLD_CONTENT[a]	Same	SNOW_COLD_CONTENT
All	SNOW_SURF_TEMP	Same	SNOW_SURF_TEMP
All	SNOW_PACK_TEMP	Same	SNOW_PACK_TEMP

[a] Cold content of snow surface layer only

6.5.1 Objective(s):

The following constraints apply for these state variables:

1. Conservation of energy; i.e. same cold content in HRU after re-adjustment area and water storage

6.5.2 Specification when new Glacier HRU added:

For the non-glacier HRUs cold content, snow surface temperature and snow pack temperature are updated as per equations 2-(10), 2-(11) and 2-(12).

For the new glacier HRU, cold content is simply

$$SNOW\_COLD\_CONTENT_g(t^*) = 0 \qquad \text{6-(12)}$$

and the snow surface and snow pack temperature at t^* are set to zero; i.e.

$$SNOW\_SURFACE\_TEMP(t^*) = 0 \; and \; SNOW\_PACK\_TEMP(t^*) = 0. \qquad \text{6-(13)}$$

6.5.3 Specification when pre-existing Glacier HRU:

For all HRUs cold content, snow surface temperature and snow pack temperature are updated as per equations 2-(10), 2-(11) and 2-(12).

6.6 State Variables Energy and Snow Properties

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
All	ENERGY_T	Same	ENERGY_T
All	SNOW_ALBEDO	Same	SNOW_ALBEDO
All	SNOW_CANOPY_ALBEDO	Same	SNOW_CANOPY_ALBEDO
All	SNOW_COVERAGE	Same	SNOW_COVERAGE
All	SNOW_LAST_SNOW	Same	SNOW_LAST_SNOW
All	SNOW_MELTING	Same	SNOW_MELTING

6.6.1 Objective(s):

The following constraints apply for these state variables:

1. Values initialized at zero for new glacier
2. Values remain constant between step t and t^* for pre-existing glacier

6.6.2 Specification when new Glacier HRU added:

For the new glacier HRU

$$S_g(t^*) = 0.$$ 6-(14)

For the non-glacier HRUs, updating is as per equation 2-(13).

6.6.3 Specification when pre-existing Glacier HRU:

For the all HRUs, updating is as per equation 2-(13).

6.7 State Variables Miscellaneous

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
All	GRID_CELL	Same	GRID_CELL
All	HRU_BAND_INDEX	Same	HRU_BAND_INDEX
All	HRU_VEG_INDEX	Same	HRU_VEG_INDEX
All	NUM_BANDS	Same	NUM_BANDS
All	SOIL_DZ_NODE	Same	SOIL_DZ_NODE
All	SOIL_ZSUM_NODE	Same	SOIL_ZSUM_NODE

6.7.1 Objective(s):

The following constraints apply for these state variables:

1. Values remain constant between step *t* and *t** for pre-existing glacier
2. Initialized with relevant values for new glacier

6.7.2 Specification when new Glacier HRU added:

For new glacier HRU, new values are initialized as follows:

$GRID\_CELL_g = GRID\_CELL_i$
$HRU\_BAND\_INDEX_g = b$;
$HRU\_VEG\_INDEX_g$ = Glacier_ID;
$NUM\_BANDS_g = NUM\_BANDS_i$;
$SOIL\_DZ\_NODE_g = SOIL\_DZ\_NODE_i$
$SOIL\_ZSUM\_NODE_g = SOIL\_ZSUM\_NODE_i$

For the non-glacier HRUs, values remain the same, i.e. as per equation 2-(13).

6.7.3 Specification when pre-existing Glacier HRU:

Updated values calculated as per equation 2-(13).

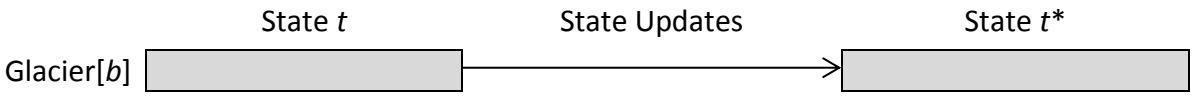
7 CASE 5

7.1 Description

Pre-existing glacier HRU in band *b* expands by Δ*A* and only the glacier HRU exists in current band. This case specification is motivated by the following area changes:

HRU	State *t*	State *t**
Glacier[*b*]	$A_g(t) > 0$	$A_g(t^*) > A_g(t)$

HRU state updates map as follows:



The remainder of this specification is identical to that of Case 2 (see §3).

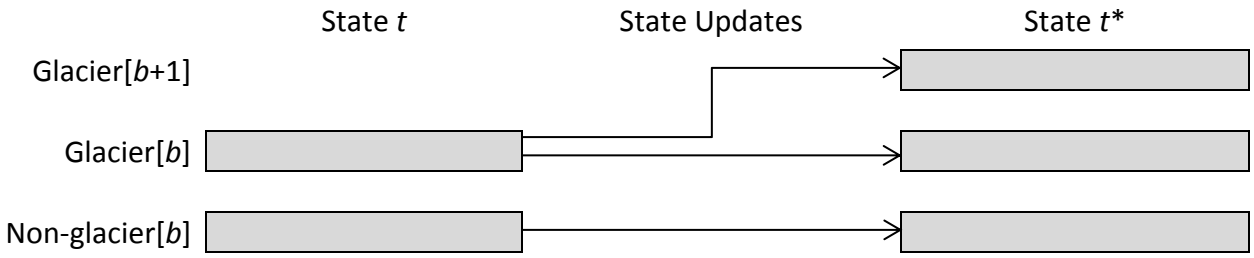
8 CASE 6 - Incomplete

8.1 Description

A new glacier HRU is created in new band *b*+1, and band *b* contains a pre-existing glacier HRU. In this case we assume the new glacier 'grows out of' or 'thickens' from the pre-existing glacier. This case specification is motivated by the following area changes:

VIC Element	State t	State t'	State t^*
		HRU Area	
Glacier[b+1]	$A_g(t) = 0$		$A_g(t^*) > 0$
Glacier[b]	$A_g(t) \geq 0$	$A_g(t) \neq \mid = A_g(t') > 0$	$A_g(t^*) = A_g(t')$
Non-glacier[b]	$A_i(t) \geq 0$	$A_i(t') \neq \mid = A_i(t)$	$A_i(t^*) = A_i(t')$
		Band Area	
b+1	$A_{b+1}(t) = 0$		$A_{b+1}(t^*) > 0$
b	$A_b(t) > 0$		$A_b(t^*) = A_b(t))$

HRU state updates map as follows:



8.2 State Variables Water Balance

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier[b]	LAYER_MOIST (N layers)	Glacier[b+1]	LAYER_MOIST (N layers)
Glacier[b]	LAYER_ICE_CONTENT (N layers)	Glacier[b+1]	LAYER_ICE_CONTENT (N layers)
Glacier[b]	HRU_VEG_VAR_WDEW	Glacier[b+1]	HRU_VEG_VAR_WDEW
Glacier[b]	SNOW_SWQ	Glacier[b+1]	SNOW_SWQ
Glacier[b]	SNOW_SURF_WATER	Glacier[b+1]	SNOW_SURF_WATER
Glacier[b]	SNOW_PACK_WATER	Glacier[b+1]	SNOW_PACK_WATER
Glacier[b]	GLAC_WATER_STORAGE	Glacier[b+1]	GLAC_WATER_STORAGE

8.2.1 Objective(s):

The following constraints apply to these state variables:

1. Conserve mass between neighboring glacier HRUs
2. Specific depth in band *b* glacier is constant
3. Non-glacier HRUs unaffected

8.2.2 Specification for Glacier HRU in band b:

As per objective 2,

$$S_g(b, t^*) = S_g(b, t'). \quad \text{8-(1)}$$

8.2.3 Specification for Glacier HRU in band b and b+1:

If, by conservation of mass (objective 1)

$$A_g(b+1, t^*) \cdot S_g(b+1, t^*) + A_g(b, t^*) \cdot S_g(b, t^*) = A_g(b, t') \cdot S_g(b, t')$$

Then, given equation 8-(1),

$$S_g(b+1, t^*) = \{S_g(b, t')[A_g(b, t') - A_g(b, t^*)]\} / A_g(b+1, t^*). \quad \text{8-(2)}$$

8.2.4 Specification for Non-glacier HRUs in band b:

No change, as per equation 2-(13).

8.3 State Variables Glacier Mass Balance

Source HRU	Source State Variable, *S*(*t*)	Target HRU	Target State Variable, *S*(*t**)
n/a	n/a	Glacier[*b*+1]	GLAC_CUM_MASS_BALANCE

8.3.1 Objective(s):

The following constraint applies for these state variables:

1. Initialize value for new glacier at zero cumulative mass balance
2. Conserve value for pre-existing glacier

8.3.2 Specification for Glacier HRU in band b:

Updated as per equation 4-(4).

8.3.3 Specification for Glacier HRU in band b+1:

GLAC_CUM_MASS_BALANCE = 0.

8.3.4 Specification for Non-glacier HRUs in band b:

Not applicable.

8.4 State Variables Snow Properties

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
All	SNOW_DEPTH	Same	SNOW_DEPTH
All	SNOW_DENSITY	Same	SNOW_DENSITY

8.4.1 Objective(s):

The following constraints apply for these state variables:

1. Conserve snow depth
2. Conservation of snow mass

8.4.2 Specification for Glacier HRU in band b:

As per equations 2-(8) and 2-(9)

8.4.3 Specification for Glacier HRU in band b+1:

The snow depth in the new glacier HRU is set equivalent to that in the pre-existing glacier HRU, such that

$$SNOW\_DEPTH_g(b+1, t^*) = SNOW\_DEPTH_g(b, t') \,. \qquad \text{8-(3)}$$

The density is then updated as per equation 2-(9).

8.4.4 Specification for Non-glacier HRUs in band b:

No change, as per equation 2-(13).

8.5 State Variables Snow Energy

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
All	SNOW_COLD_CONTENT[a]	Same	SNOW_COLD_CONTENT[a]
All	SNOW_SURF_TEMP	Same	SNOW_SURF_TEMP
All	SNOW_PACK_TEMP	Same	SNOW_PACK_TEMP

[a] Cold content of snow surface layer only

8.5.1 Objective(s):

The following constraints apply for these state variables:

1. Conservation of energy; i.e. same cold content in HRU after re-adjustment of area and water storage
2. …

8.5.2 Specification for Glacier HRU in band b:

Update as per equations 2-(10), 2-(11) and 2-(12)

8.5.3 Specification for Glacier HRU in band b+1:

…

8.5.4 Specification for Non-glacier HRUs in band b-1:

No change, as per equation 4-(3).

8.6 State Variables Miscellaneous

Source HRU	Source State Variable, $S(t)$	Target HRU	Target State Variable, $S(t^*)$
Glacier[b]	ENERGY_T	Glacier[b+1]	ENERGY_T
Glacier[b]	GRID_CELL	Glacier[b+1]	GRID_CELL
Glacier[b]	HRU_BAND_INDEX	Glacier[b+1]	HRU_BAND_INDEX
Glacier[b]	HRU_VEG_INDEX	Glacier[b+1]	HRU_VEG_INDEX
Glacier[b]	NUM_BANDS	Glacier[b+1]	NUM_BANDS
Glacier[b]	SNOW_ALBEDO	Glacier[b+1]	SNOW_ALBEDO
Glacier[b]	SNOW_CANOPY_ALBEDO	Glacier[b+1]	SNOW_CANOPY_ALBEDO
Glacier[b]	SNOW_COVERAGE	Glacier[b+1]	SNOW_COVERAGE
Glacier[b]	SNOW_LAST_SNOW	Glacier[b+1]	SNOW_LAST_SNOW
Glacier[b]	SNOW_MELTING	Glacier[b+1]	SNOW_MELTING
Glacier[b]	SOIL_DZ_NODE	Glacier[b+1]	SOIL_DZ_NODE
Glacier[b]	SOIL_ZSUM_NODE	Glacier[b+1]	SOIL_ZSUM_NODE

8.6.1 Objective(s):

The following constraints apply for these state variables:

1. Values remain constant between step t and t^*
2. …

8.6.2 Specification for Glacier HRU in band b:

No change, as per equation 2-(13).

8.6.3 Specification for Glacier HRU in band b+1:

All values set equal to those for the glacier HRU in band b, i.e.

$$S_g(b+1, t^*) = S_g(b, t) \qquad \text{8-(4)}$$

8.6.4 Specification for Non-glacier HRUs in band b:

No change, as per equation 4-(3).