ME4020: Machine Dynamics Lab

Bicycle Moment of Inertia

January 16, 2018

1 Objective

The purpose of this lab is to familiarize students with taking dynamic vibration measurements and determining inertial properties of arbitrary shapes. For this lab, the arbitrary shape is a bicycle.

2 Background Information

2.1 Coil Spring Stiffness

The stiffness of a helical coil spring is determined based on the angle of twist of the coils. Calculating the stiffness of a helical coil extension springs uses the following formula:

$$k = \frac{d^4 G}{8 D^3 N} \tag{1}$$

where d is the wire diameter, $G = 11.9$ Mpsi is the shear modulus, D is the mean helix diameter and N is the number of active coils. The dimensions (in inches) for the spring in this lab is presented in

2.2 Mass Moment of Inertia From Pendulum

The bicycle and test assembly can be modeled as a rigid pendulum. The resistance to rotation of a rigid pendulum about point O is quantified by the polar moment of inertia. If the pendulum is free

Spring	Length	Outer Diameter	Inner Diameter	Wire Diameter	N
1	10.866	1	0.788	0.104	104

Table 1: Coil spring dimensions in inches. Note: The outer and inner diameter are not d and D; instead, they are dimensions as measured by a caliper.

to rotate with no applied force, the equation of motion from summing the moments about point O is

$$I_O\ddot{\theta} + wL_{cg}\sin\theta = 0 \tag{2}$$

where I_O is the mass moment of inertia, θ is the angle rotated from rest, w is the total weight of the assembly, and L_{cg} is the distance from the point of rotation to the center of mass of the assembly. If small angles are used, then $\sin\theta \approx \theta$ when measured in radians. Making this substitution gives a constant coefficient, 2nd order, differential equation:

$$I_O\ddot{\theta} + wL_{cg}\theta = 0 \tag{3}$$

which can be rearranged to the form:

$$\ddot{\theta} + \omega_n^2\theta = 0 \tag{4}$$

where $\omega_n^2 = \frac{wL_{cg}}{I_O}$ is the square of the natural frequency. For small damping, the observed frequency is similar to the natural frequency and can be determined by measuring the period of oscillation. The period relates to the natural frequency as

$$T = \frac{2\pi}{\omega_n} \tag{5}$$

Therefore, the mass moment of inertia can be determined as

$$I_O = \frac{wL_{cg}T^2}{4\pi^2} \tag{6}$$

where I_O is the polar moment of inertia about the pivot point,
w is the total weight of the rotating bicycle assembly,
L_{cg} is the length from the pivot point to the center of mass, and
T is the period of oscillation.

Units of I_O should be equivalent to mass-length^2. The moment of inertia as measured is a result of the parallel axis theorem and is calculated as

$$I_O = I_{COM} + \frac{w}{g}L_{cg}^2 \tag{7}$$

where I_{COM} is the moment of inertia of the assembly about its center of mass and g is the acceleration due to gravity. If L_{cg} is in inches, then g =386.4 inches/second^2. The value of L_{cg} can be determined in a similar way as Homework 1, Problem 5. However, this is for the assembly only. It will need to be broken into pieces.

The total moment of inertia, which is determined from measurements is made up from the following contributions:

$$I_O = I_{O,support} + I_{G,bike} + \frac{w_{bike}}{g}L_{cg,bike}$$

The goal is to determine the moment of inertia of the bike, $I_{G,bike}$. The moment of inertia of the support, $I_{O,support}$ can be determined by modeling it as a thin rod where the formula is readily available. The weight of the bike is given in Table 3. The distance from the pivot point to the center of mass of the bike is determined using a static analysis.

2.3 Mass Moment of Inertia From Spring

A free body diagram of the bicycle and apparatus can be based on the sketch shown in 1. All important points of measurement should be labeled. The Y-axis is the earth-fixed axis that is pointing in the direction of gravity. The composite center of mass is not shown on the sketch. Using the sketch for inspiration, draw a free body diagram of the rotating apparatus. Then determine the equations of motion assuming no damping or velocity dependent terms.

$$\sum M_O = I_O \ddot{\phi}$$

where ϕ is the small angle that the apparatus rotates about equilibrium. It is important to realize that the weight terms and the average spring force will cancel each other out.

Manipulate the equations of motion to obtain an expression that has the following form:

$$\ddot{\phi} + \lambda \phi = 0 \tag{8}$$

where ϕ is the change in angle from the resting position and λ is a constant that is determined by manipulating the equations derived from the free body diagram. The expression for λ will have the moment of inertia of the rotating masses, I_o in the denominator. The numerator will have a value determined from geometry spring constant, k. In order for the equations to simplify, a small angle approximation may have to be made. This assumption says that any angle $\theta \approx \sin\theta$ for small angles. Similarly, the small angle assumption says that $\cos\theta \approx 1$ when $\theta \lll 1$. Also, the following identities may be helpful:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

The solution to the differential equation shown in Eq. 8 is a harmonic equation with a frequency of oscillation of

$$\omega_n = \sqrt{\lambda} = \sqrt{\frac{\text{stiffness}}{\text{inertia}}} \tag{9}$$

where ω_n is in radians per second. The frequency and period of oscillation can be measured and is recorded in Table 4. Some assistance in this derivation is provided in Fig. **??**.

3 Laboratory Procedure

Build a LabVIEW interface to report angle from the quadrature encoder. A partially built VI can be obtained from Harvey.

1. Create a DAQ MX tasks to acquire a signal from the quadrature encoder using ctr0.

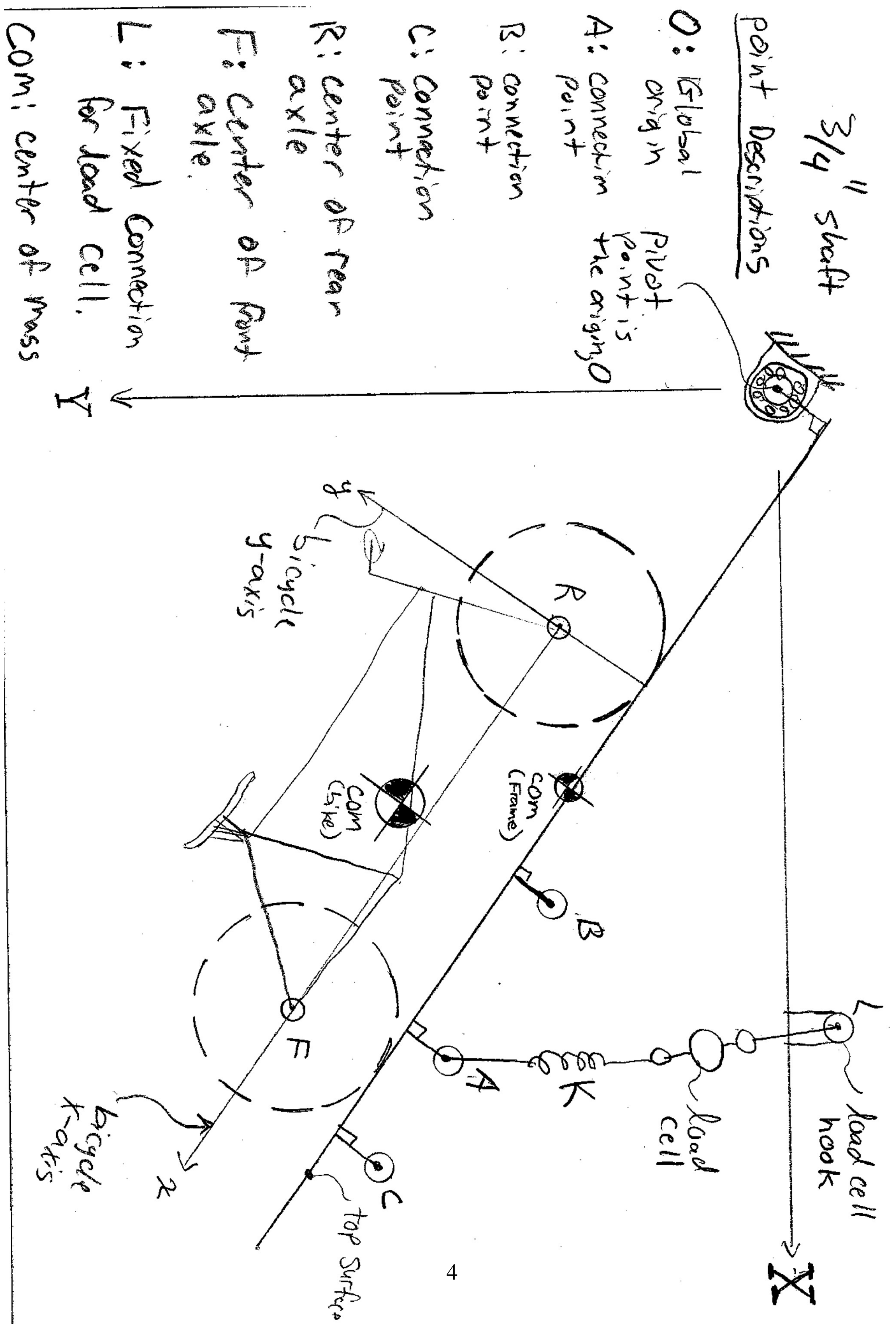
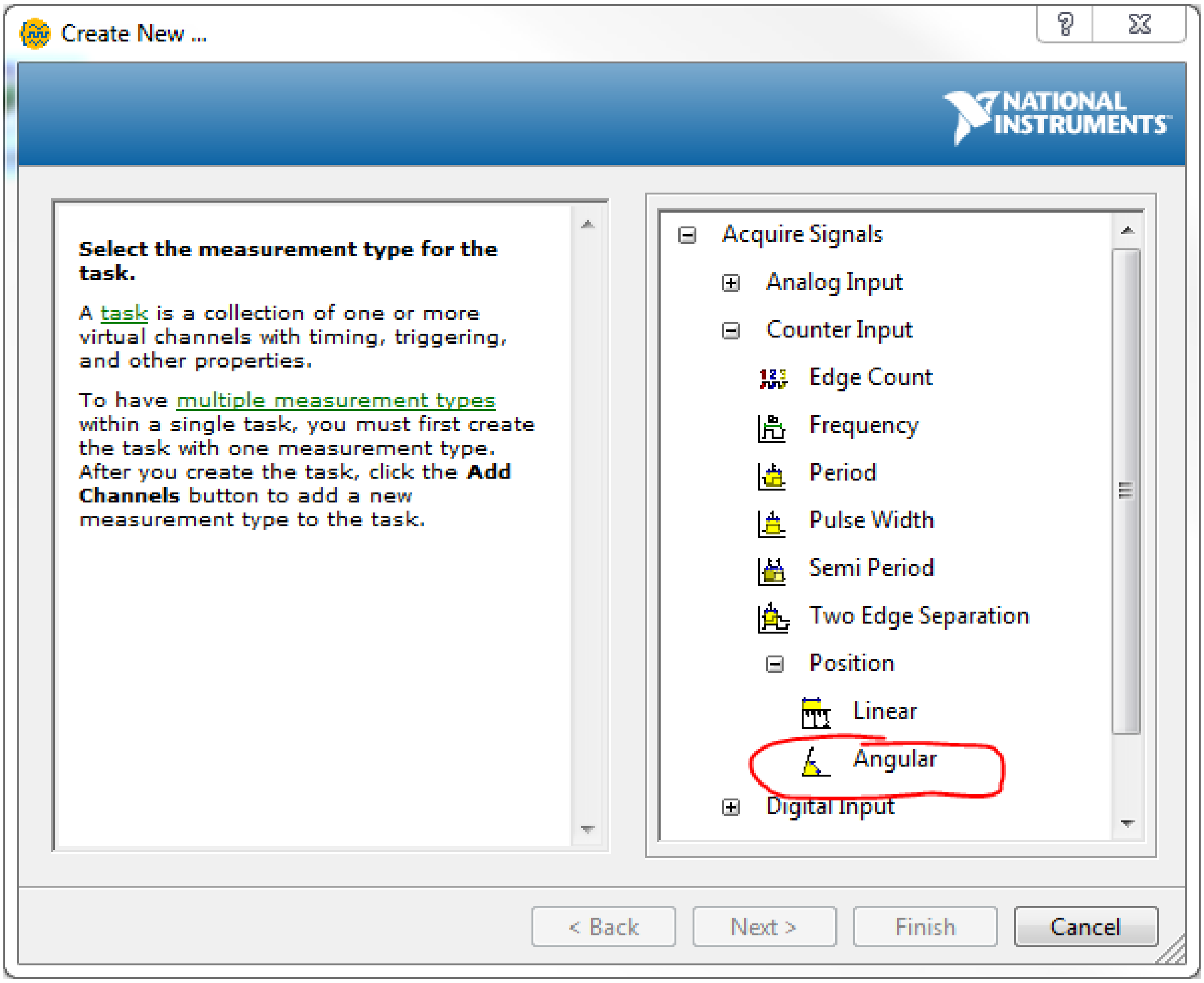


Figure 1: Sketch of test apparatus with a bicycle.

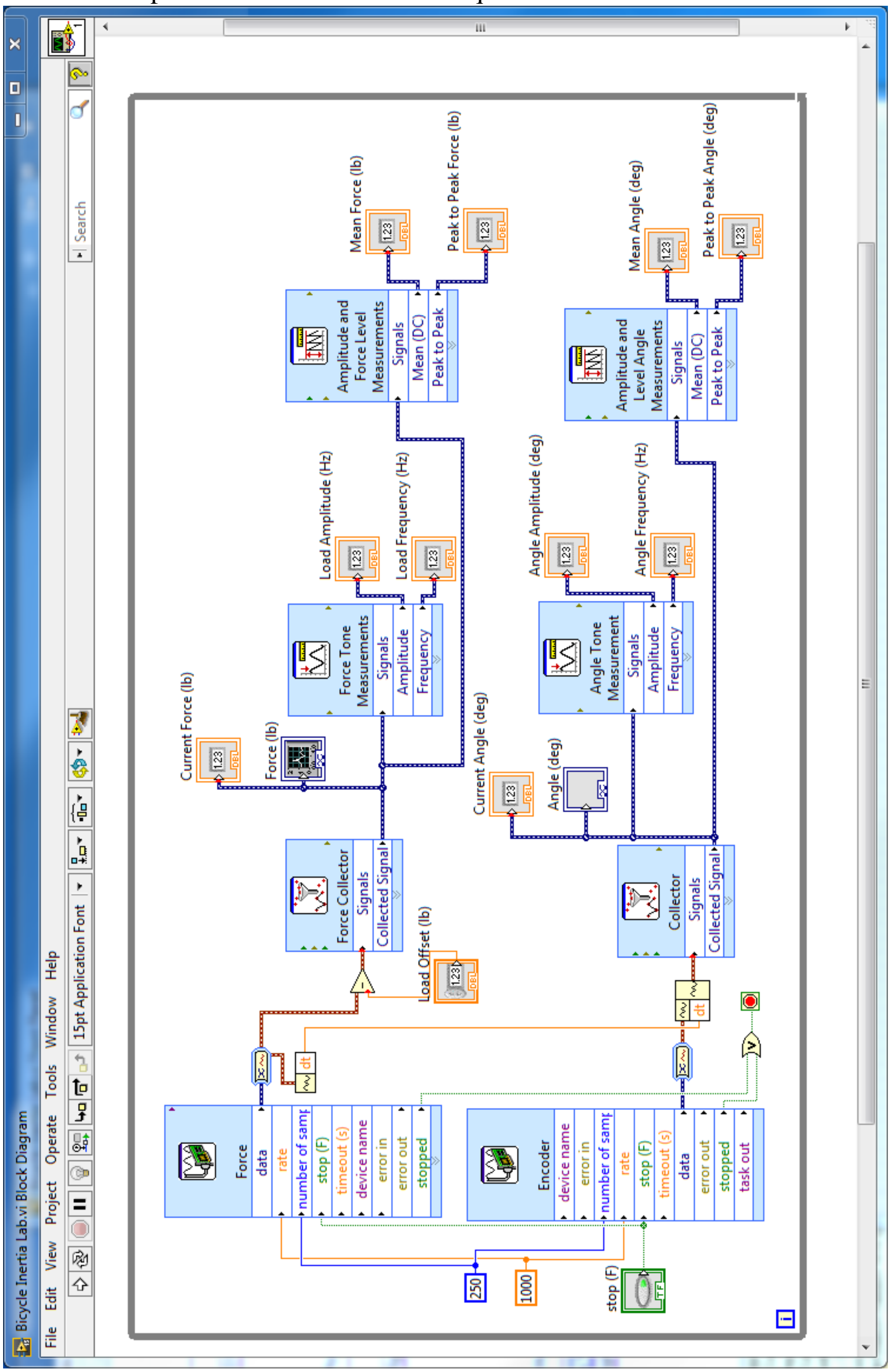


a) Configure the quadrature encoder to have 2048 pulses per turn and X4 encoding. Select continuous sampling at 1000 Hz and 250 samples. Be sure to change the inputs to PFI0 and PFI1 to match the wiring of the lab. Be sure to confirm the settings by rotating the apparatus through about 90 degrees.

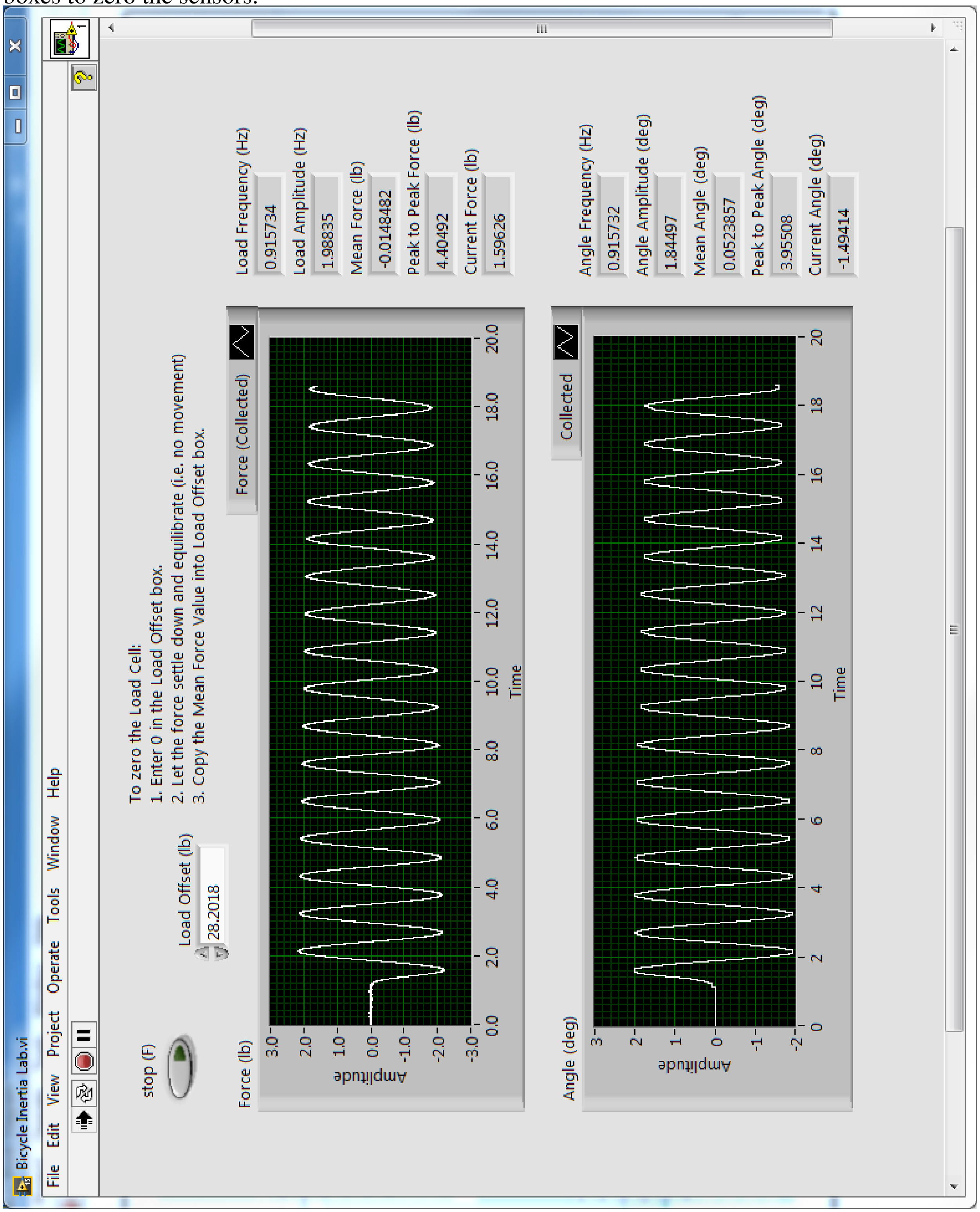
b) Change the timing to external and use the Sample Clock from the NI9237 module.

2. Add collectors with 30,000 samples.

3. Add other express VIs to assist in data acquisition and reduction.



4. The front panel will have the data of interest. Copy the stable mean values into the Offset boxes to zero the sensors.



5. Test the VI by hanging a weight from the load cell and moving the apparatus through a known angle.

3.1 Spring Stiffness Determination

1. Determine the spring stiffness analytically using Eq. 1.
2. Connect the spring to the load cell.
3. Hang a chain from the spring so you can connect weights to the chain while measuring the spring deflection.
4. Measure a position of the rig hanging from the bottom of the spring. Pick an easy and accurate measurement location that you will use each time.
5. Hang some weights from the chain. For four separate mass totals, do the following and record the results in Table 2.
    a) Record the load from the load cell.
    b) Measure the deflection of the spring.
    c) Determine the frequency of oscillation when the mass is free to vibrate.
6. Determine the spring stiffness based on determining the slope of a line fitted to the force vs deflection data.
7. Determine the spring stiffness based on the natural frequency of vibration using the following formula:

$$k = m\omega_n^2 \tag{10}$$

   where ω_n is the observed natural frequency in radians per second. Report the stiffness in pounds per inch.

Name:		
Description	Value	Units
Weight of the bicycle	26.90	pounds
Weight of the apparatus (without bicycle)	13.36	pounds
Diameter of the pivot	0.75	inches
Angle from vertical of free hanging apparatus[3]	-1.758	degrees
Horizontal distance from load cell support to pivot	51.5	inches
Vertical distance from load cell support to pivot	0.5	inches
Distance from pivot to Point A		inches
Distance of rear axle from pivot		inches
Distance of front axle from pivot		inches
Distance of rear axle from back side of Unistrut		inches
Distance of front axle from back side of Unistrut		inches
Wheelbase of the bicycle		inches
Load when spring is connected to Point A (unistrut only)	6.657	pounds
Load when spring is connected to Point A (combination)		pounds
Angle of pivot when spring is connected to Point A		degrees
Distance from load cell support to Point A with chain		inches

Table 3: Data table for static measurements.

Name:		
Description	Value	Units
Calculation of stiffness for Spring #1 from Eq. 1		lb/in
Load cell reading just the spring attached.[1]	0	lb
Initial deflection with Spring #1[2]		inches
Load cell reading with added mass #1		lb
Deflection with added mass #1		inches
Load cell reading with added mass #2		lb
Deflection with added mass #2		inches
Load cell reading with added mass #3		lb
Deflection with added mass #3		inches
Load cell reading with added mass #4		lb
Deflection with added mass #4		inches
Time for 30 vibration cycles mass #4		seconds
Frequency of oscillations mass #4		Hz

Table 2: Spring stiffness measurements

3.2 Static Measurements

1. Weigh the bicycle with support assembly (This was done by the TA).
2. Hang the setup from the 50-lb load cell with the spring.
3. Measure static distances between key points in the setup.
4. Establish a coordinate system to describe positions on the bicycle.
5. Determine the center of mass location of the bicycle (in a plane only) using concepts in statics (i.e. sum of moments as done on HW 1)

3.3 Dynamic Measurements

1. Let the bike and frame assembly hang freely.
2. Measure the angle of the support assembly with an angle finder or digital protractor.
3. Zero the encoder by restarting the LabVIEW program.
4. Excite the system by giving it a gentle nudge.
5. Let the system oscillate for 30 seconds until the LabVIEW graphs are full of consistent harmonic data.
6. Record the numerical output from the LabVIEW instruments. Ensure that both measurement systems are giving consistent data.
7. Align the support rod with the vertical axis and record the angle change from the encoder.
8. Ensure the spring is attached to the load cell.
9. With just a spring and clasps hanging, zero the load cell.
10. Connect the chain and spring assembly to the apparatus.
11. Stabilize the system and record the static weight and angle from LabVIEW.
12. Measure the distance from the load cell attachment to the attachment point on the apparatus.
13. Excite the system by giving it a gentle nudge. Be sure to keep the angle changes small.
14. After the system oscillates for 30 seconds and the harmonic data fills the LabVIEW graphs, record the data from LabVIEW.

Name:		
Description	Value	Units
Length from load cell support to Point A		inches
Static load when attached to Point A		pounds
Frequency of free oscillation		Hz
Average angle of pivot		degrees
Angle change amplitude		degrees
Load amplitude		pounds
Time elapsed for 30 cycles		seconds

Table 4: Data table for dynamic free vibration measurements.

4 Reporting Requirements

Individually submit the following:

1. A 2-page summary report with the following:
   a) Title of the lab
   b) Group member names and e-mail addresses.
   c) Date of the lab.
   d) A graph of the time history of the angular velocity in the pitch direction showing harmonic motion. This graph must be properly labeled with the duration of 1 period annotated. You can start with a screenshot of the LabVIEW front panel.
   e) Determine the stiffness of the spring using three different methods (strength of materials equation, force vs deflection slope, and natural frequency).
   f) Determine at least 2 values (based on different experimental setups) for the bicycle pitch moment of inertia. Estimate a level of precision in your estimates. In other words, what is the percentage difference of your estimates? What is the average value? Be sure these numbers are for the bicycle only and not the combination.
   g) Center of mass location of the bicycle with the origin at the rear axle and the x-axis going from the rear axle through the front axle.
   h) Mass and weight of the bicycle.
   i) How small is a small angle? Plot the value of θ vs θ and $\sin\theta$vs θ on the same graph for $-25^\circ < \theta < 25^\circ$. Be sure to convert the angle θ into radians. Comment on your observations of this graph

2. A completed data table for spring stiffness measurements (Table 2).

3. A completed data table for static measurements (Table 3).

4. A completed data table for dynamic measurements (Table 4).

5. Hand calculations and hand written explanations of the calculations used to arrive at the mass properties of the bicycle and apparatus.