ME4020: Machine Dynamics Lab

Jumping Impulse

Spring 2018

1 Objective

This lab has two major objectives: 1) to measure and understand impulse and how it relates to force and change in velocity, and 2) understand how multiple linear regression works for calibrating a force plate.

1.1 Calibration Using Multiple Regression

Often real applications require using multiple variables and simple line fitting is insufficient. This lab introduces the student to the concept of multiple linear regression.

1.2 Measure Force and Determine Impulse

After completing this lab, students should have first hand experience relating force, impulse and velocity for straight line motion.

2 Theory

2.1 Determining Jump Height Based on Impulse

Jumping requires a person to exert a force on the ground (or platform) over some time. This force starts with the static weight of the jumper, then reduces as the jumper squats during the counter movement. The legs then begin pushing back on the floor as the jumper accelerates his/her mass upward. This force builds during the jump then falls to zero as the shoes leave the platform.

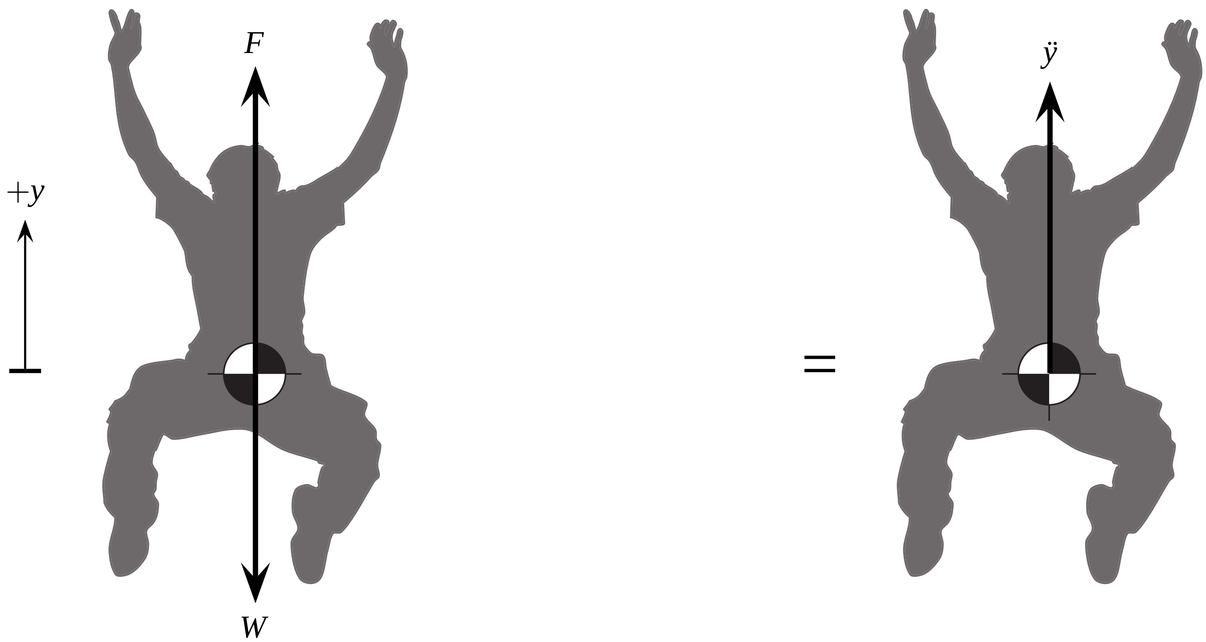


Figure 1: The Free body Diagram (FBD) and the Inertial Response Diagram (IRD) as it relates to a jumper.

If we assume the goal of jump height measurements are to determine the height of the center of mass, then the body can be treated as a point mass. Newton's 2nd Law says:

$$\sum \vec{F} = m\vec{a} \tag{1}$$

but for this case, the motion of interest is only in the vertical direction and the forces consist of gravity and contact force. Therefore, the free body diagram along with the inertial response diagram is shown in Fig. 1. Newton's Second Law can be rewritten in differential form as

$$\sum \vec{F} = m\frac{dv}{dt} \tag{2}$$

where the sum of the forces is $\sum \vec{F} = F - W$ in the y direction. Eq. (2) can be integrated over some time:

$$\begin{aligned}
\int_{t_0}^{t} F(t) - W\,dt &= \int_{v(t_0)}^{v(t)} m\,dv \\
\int_{t_0}^{t} F(t) - W\,dt &= m[v(t) - v(t_0)] \\
\int_{t_0}^{t} F(t) - W\,dt &= \frac{W}{g}[v(t) - v(t_0)] \\
\underbrace{\frac{g}{W}\int_{t_0}^{t} F(t) - W\,dt}_{\text{external impulse}} &= \underbrace{v(t) - v(t_0)}_{\text{change in velocity}}
\end{aligned} \tag{3}$$

which means a change in velocity can be determined if the external impulse can be measured. If the initial force measurement is taken while the jumper is still, then the static weight W is known. The time before the jump maneuver, when the jumper is still, will be denoted at t_0 and can be any time when the jumper is still. Since the jumper is still, $v(t_0) = 0$.

The time history of force is measured and recorded as a series of forces in time. Since the goal is to determine the integral quantity of impulse, a numerical approximation based on the trapezoidal rule will be used. A formula for determining the impulse from $t = t_0$ to $t = t_n$ is as follows:

$$\int_{t_0}^{t_n} F(t) - W\,dt \approx \frac{\Delta t}{2}\sum_{i=1}^{n}(F_i + F_{i-1} - 2W) \tag{4}$$

where there are $n+1$ data points because 0 is used as for the first point. The formula in Eq. (4) can be easily implemented in a spreadsheet or computer program. Once the impulse is estimated, the velocity at the end of the impulse can be determined as

$$v(t_n) = \frac{g}{W}\int F\,dt. \tag{5}$$

This velocity will determine the take off velocity if the time t_n corresponds to the time when the nor more force is applied to the platform. In other words, the jumper has taken off. Once initial take-off velocity is known, the only force applied to the body is weight. Application of Newton's Law gives

$$\sum F = -W = ma \qquad \text{or} \qquad -W = \frac{W}{g}\ddot{y} \tag{6}$$

which leads to a simple 2nd order differential equation: $\ddot{y} = -g$, $y(0) = 0$, and $\dot{y}(0) = v(t_n) = v_o$. Since gravity imposes a constant acceleration, the following kinematic relationship exists:

$$v^2 = v_o^2 - 2gh \tag{7}$$

where g is the gravitational constant and h is the height change from the initial position. At the peak height, the velocity is zero, so combining Eqs. (7) and (5) gives and estimate for jump height:

$$h = \frac{g\left(\int F\, dt\right)^2}{2W^2} \tag{8}$$

2.2 Jump Height from Hang Time

If the jumper takes off and lands at the same height, then the time traveling up and down are equal. A solution to the constant acceleration differential equation can be obtained through integration and results in a quadratic polynomial:

$$y(t) = y_o + v_o t - \frac{g}{2}t^2 \tag{9}$$

Since the times splits between up and down are even, and the time is 1/2 the hang time, the time equation can be written as

$$h = \frac{g}{2}\left(\frac{\text{hang time}}{2}\right)^2. \tag{10}$$

Hang-time is easily measured off the time history of force. Keep in mind that the assumption is that the mass center starting height is the same as the landing height. This may be violated if the jumper tuck his/her knees and falls farther than they took off.

2.3 Multiple Regression for Calibration

The platform used for this experiment has three load cells at the corners of the triangular structure. Since there are three supports, the sum of the three reaction forces is equal to the applied weight less the weight of the platform W_p. The weight of the platform is unknown and will be included in the measurement as an offset. In equation form, this is written as:

$$F = G_A V_A + G_B V_B + G_C V_C - W_p + \varepsilon \tag{11}$$

This is a linear equation and can be written in vector form:

$$\{F\} = \begin{bmatrix} V_A \; V_B \; V_C \; -1 \end{bmatrix} \begin{Bmatrix} G_a \\ G_B \\ G_C \\ W_p \end{Bmatrix} + \{\varepsilon\} \tag{12}$$

where the gain vector is constant regardless of the forces and voltages. The residual term ε exists because no measurement system is perfect. There will be slight errors in the process so the linear relationship will not be perfect. The goal of the calibration is to determine the gain constants by minimizing the residual based on multiple measurements. If n measurements are available, then a series of linear equations can be combined into one equation as follows:

$$\underbrace{\begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{Bmatrix}}_{\mathbb{Y}} = \underbrace{\begin{bmatrix} V_{A,1} & V_{B,1} & V_{C,1} & -1 \\ V_{A,2} & V_{B,2} & V_{C,2} & -1 \\ \vdots & \vdots & \vdots & \vdots \\ V_{A,n} & V_{B,n} & V_{C,n} & -1 \end{bmatrix}}_{\mathbb{X}} \underbrace{\begin{Bmatrix} G_a \\ G_B \\ G_C \\ W_p \end{Bmatrix}}_{\beta} + \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix} \tag{13}$$

or

$$\mathbb{Y} = \mathbb{X}\beta + \vec{\varepsilon} \tag{14}$$

If n is less than 4, then the system cannot be solved because it is under-determined. If $n = 4$ then the system gives a unique solution, but if $n > 4$, then the system is overdetermined and a solution can be obtained by minimizing the sum of the square of the residuals. This is called the least-squares technique. The sum of squares of the residuals is computed by

$$SSE = \vec{\varepsilon}^T \vec{\varepsilon} = \begin{Bmatrix} \varepsilon_1 \; \varepsilon_2 \; \cdots \; \varepsilon_n \end{Bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix} \tag{15}$$

Equation 14 is known as the normal equation and the solution of the overdetermined system that minimizes SSE is

$$\beta = (\mathbb{X}^T \mathbb{X})^{-1} \mathbb{X}^T \mathbb{Y} \tag{16}$$

Once β is determined with Eq. (16), the gain constants are known as well as the platform weight. Further statistical analysis can be performed on the residuals to ensure that the linear model is the most appropriate, but that is beyond the scope of the course.

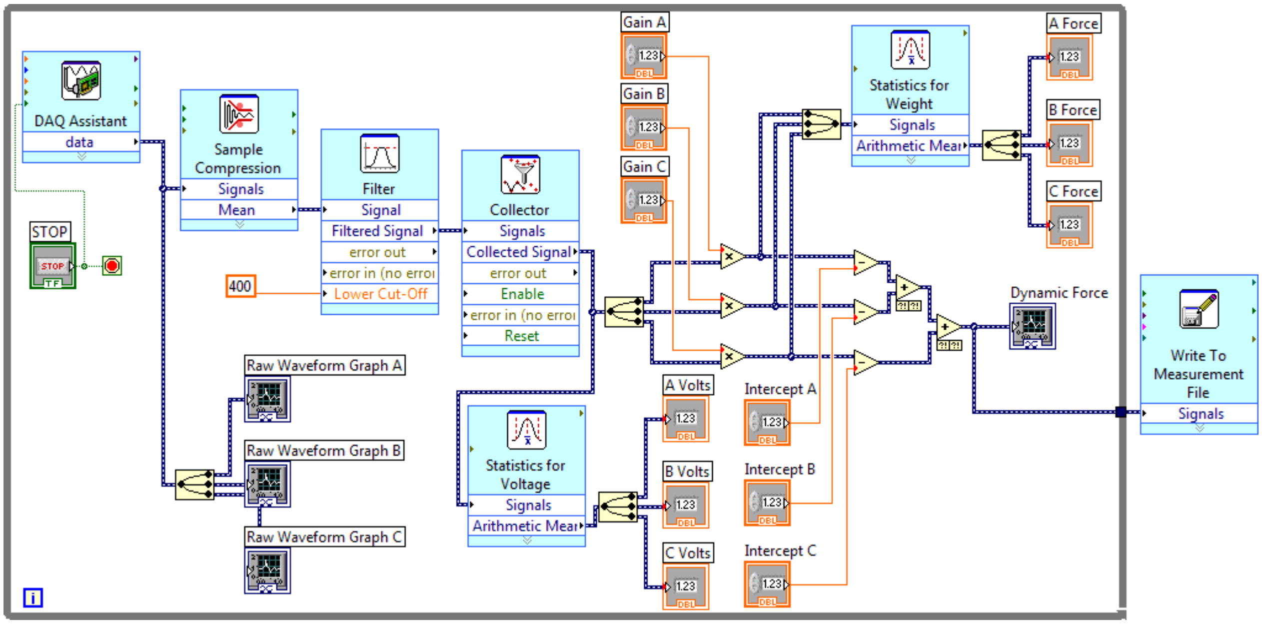


Figure 2: LabVIEW Block Diagram for acquiring data from the three independent load cells.

3 Procedure

This lab requires four parts to complete: 1) Set up the computer DAQ system, 2) Calibrate the force platform, 3) Obtain the data, and 4) analyze the data to determine jump heights. This section will provide the information needed to accomplish these tasks.

3.1 Set Up LabVIEW for Data Acquisition

Open LabVIEW and create a VI capable of measuring the raw voltage from the load cells. The load cells are 2500# shear-beam load cells with a full bridge strain gauge sensor grid. The output on the sensors is 3mV/V at full scale. To use these sensors, ensure they are plugged into the NI 9237 module of the CompactDAQ and the CompactDAQ is on. Once the physical hardware is connected, build a VI similar to the ones shown in Figures 2 and 3. It is recommended to build and test the DAQ Assistant and Raw Waveform Graphs first.

When starting, use a value of 1 for all the gain constants and a value of 0 for all the intercepts. This will make the outputs simply the raw voltage.

3.2 Determine Gain Constants

Once the VI works and displays the average voltage over a period of time, the calibration can commence. Start by obtaining baseline data and record the voltage outputs for no load. Then take a 65-lb dumbbell and place it anywhere on the platform. Record the new voltages. Move the weight to another location (as far away as possible) and repeat recording the voltages. A table for gathering

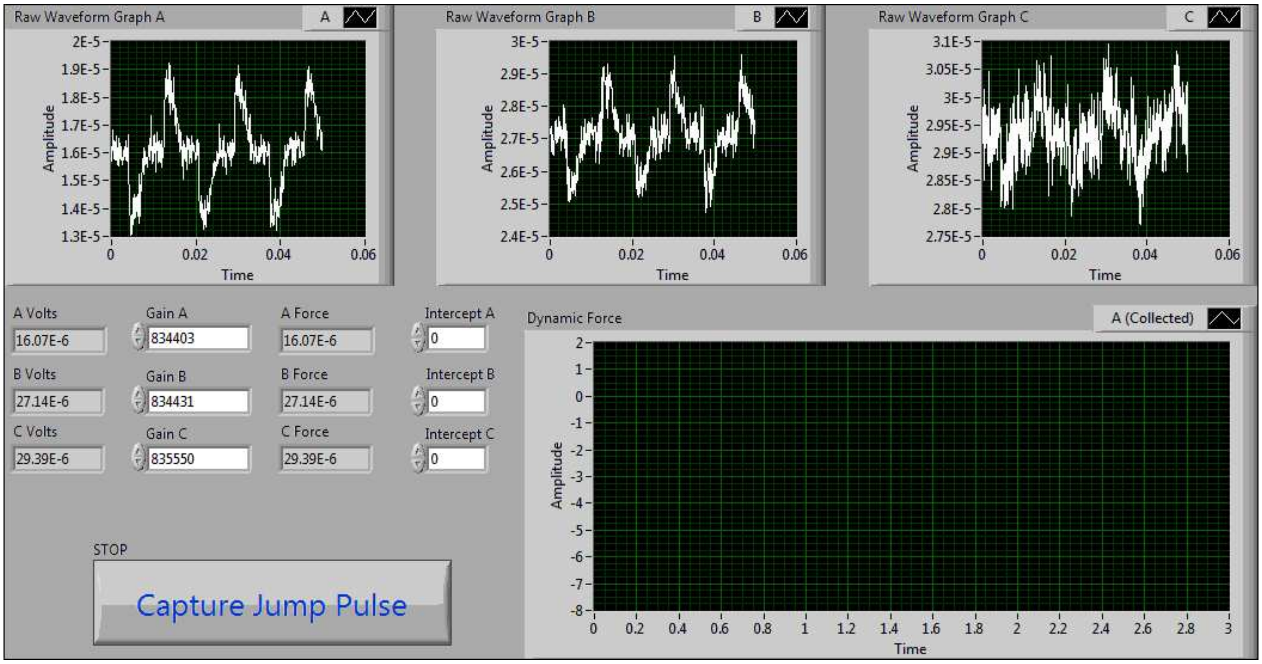


Figure 3: LabVIEW Front Panel for acquiring data from the three independent load cells.

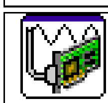
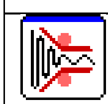
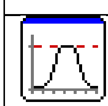
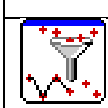
Icon	Tool	Properties
	DAQ Assistant	Sample Rate: 5k Samples to read: 1000 (continuously) Full Bridge with 5V internal excitation
	Sample Compression	Reduction Factor: 5 Reduction Method: Mean
	Filter	Filter Type: Lowpass Cut-Off: 1000 IIR/FIR: Infinite Impulse Response (IIR) Filter Topology: Butterworth Order: 4
	Collector	Number of Points to Collect: 20000

Table 1: LabVIEW VI Configuration Properties

Total Weight (lb)	Location 1	Location 2	Voltage A	Voltage B	Voltage C
0	N/A	N/A			
65	middle	N/A			
65	over A	N/A			
65	over B	N/A			
65	over C	N/A			
130	middle	over A			
130	middle	over B			
130	middle	over C			
130	middle	middle			
195	anywhere				
260	anywhere				

Table 2: Data collection table for calibration. Please ask for assistance when moving weights if you are uncomfortable with the task.

the data may look like the on shown in Table 2.

Use the table to build the $\mathbb{X}$ matrix and $\mathbb{Y}$ vector in Eq. 14 and solve for β to get the gains. Next, enter the calculated gain values into the LabVIEW program to get a signal output in pounds.

Finally, each channel needs to be zeroed. Therefore, run the instrument with no weight on the platform. Record the steady output for the individual channels and use those for intercept values in the LabVIEW vi. Be sure that the dynamic with is zero when no objects are on it. The platform should be calibrated at this time.

3.3 Measure Impulse and Height

In this section, the process of jumping and recording data are given. This section is optional and not required for the lab. Example data, as shown in Figure 4, are available for download from the course website in the form of a Matlab .m file from JumpExample.m on Harvey.

1. Have the jumper mount the platform and remain still. Obtain a static weight.
2. Have the jumper reach as high as possible and touch the slap sticks. Be sure that the maximum reach is near the bottom of the stack of sticks by adjusting the slap-stick height.
3. Perform a couple practice jumps to get the feel of the platform and timing on the data acquisition. Jump using two feet and reach as high as possible on the slap-sticks.

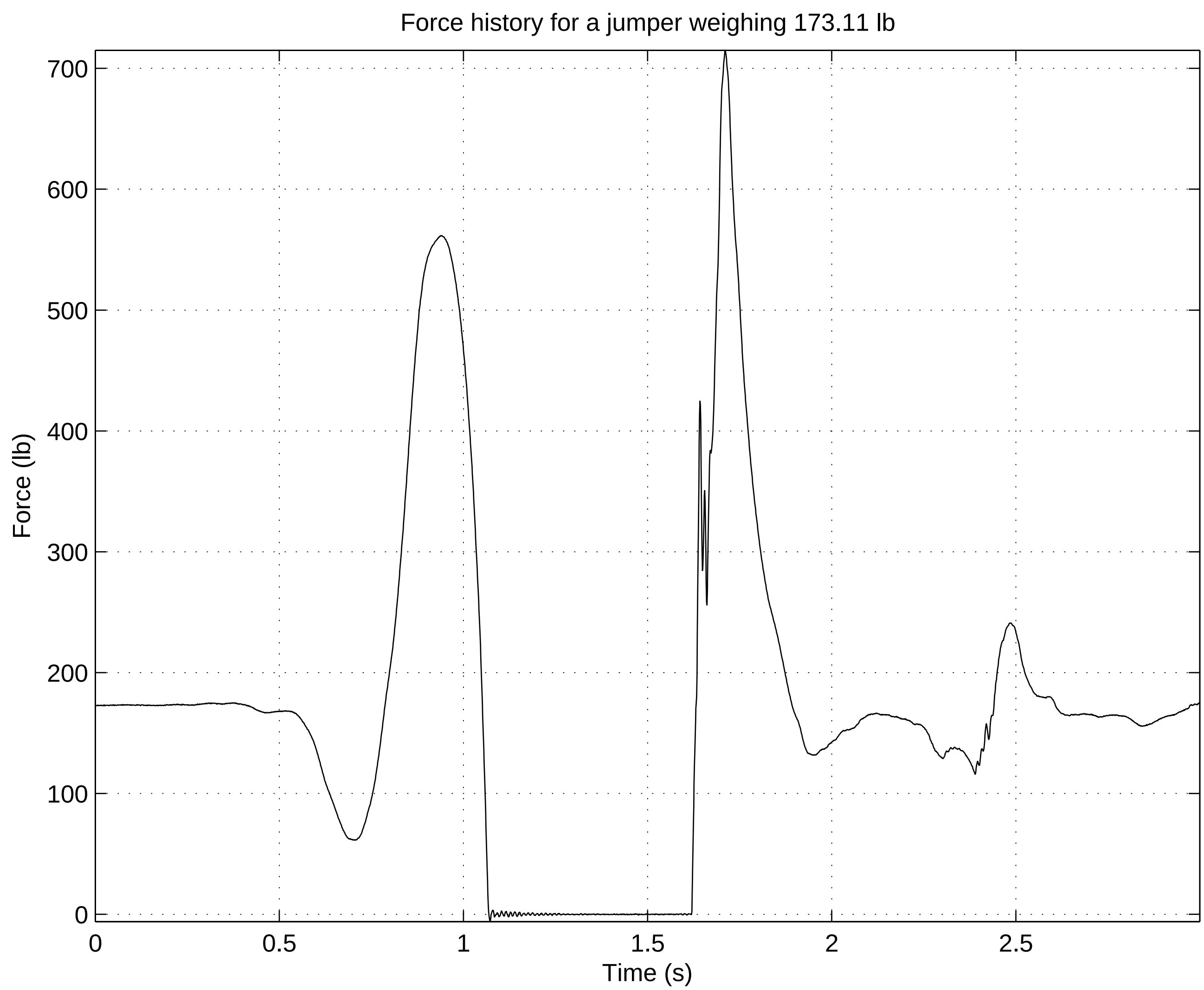


Figure 4: Example jump profile data including three phases: take-off impulse, hang-time, and landing impulse.

4. Save the time history of force to a data file and import the data into Excel or Matlab for further analysis.

3.4 Data Analysis

There are three different segments to measure jump height from the impulse: the take off impulse, time in the air, and landing impulse. The force vs. time history is capable of providing enough information to perform the analysis. Use the example file or your own file to perform the analysis based on the Theory section of the lab. In Matlab, the `cumtrapz` command can be useful for determining impulse.

4 Reporting Requirements

Please turn in the following as a single PDF file uploaded to Harvey two weeks after you conduct the lab.

1. A coversheet with your name, lab section, date the lab was performed, and your group member names.
2. A print of your LabVIEW Front panel
3. A print of your LabVIEW Block Diagram
4. Raw Data from your calibration procedure in table form
5. Determination of your gain constants
6. A record of the offsets needed to zero the platform
7. Determination of the jump height from the slap-sticks
8. A graph of the jump and landing profile (i.e. the force time history)
9. Determination of the takeoff impulse
10. Determination of the jump height from takeoff impulse
11. Determination of the hang time
12. Calculation of the jump height given hang time
13. A table summarizing the jump height measures from 1) slap sticks, 2) take-off impulse, and 3) hang-time. Also include relative differences assuming the slap-sticks as the standard.
14. Please note the similarities and differences between the measures. Comment on which one you think is most reliable.