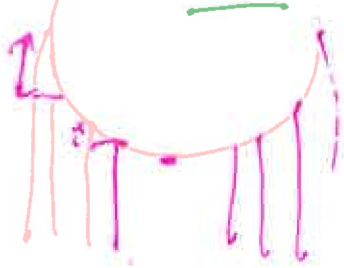
Lecture 2

①

$$\sum_{i=1}^{2} x_i = x_1 + x_2$$

$$\frac{\partial}{\partial x_1} = 1$$



$$\min \underbrace{\sum_{i=1}^{N} (y_i - (\theta_0 + \theta_1 x_i))^2}_{RSS}$$

$$\frac{\partial RSS}{\partial \theta_0} = \sum_{i=1}^{N} \left(2 (y_i - \theta_0 - \theta_1 x_i) \cdot \frac{\partial}{\partial \theta_0} (y_i - \theta_0 - \theta_1 x_i)\right)$$

$$= 2 \sum_{i=1}^{N} (y_i - \theta_0 - \theta_1 x_i)(0 - 1 - 0)$$

$$0 \leftarrow \frac{\partial RSS}{\partial \theta_0} = 2 \sum_{i=1}^{N} -(y_i - \theta_0 - \theta_1 x_i)$$

$$0 = -2 \sum_{i=1}^{N} (y_i - \theta_0 - \theta_1 x_i)$$

$$\sum_{i=1}^{2} 2x_i$$

$$= 2x_1 + 2x_2$$

$$= 2(x_1 + x_2)$$

$$\frac{0}{-2} = \sum_{i=1}^{N} (y_i - \theta_0 - \theta_1 x_i)$$

$$0 = \sum_{i=1}^{N} y_i - \sum_{i=1}^{N} \theta_0 - \sum_{i=1}^{N} \theta_1 x_i$$

$$\sum_{i=1}^{N} \theta_0 = \sum_{i=1}^{N} y_i - \sum_{i=1}^{N} \theta_1 x_i$$

$$\theta_0 \sum_{i=1}^{N} (1) = \sum_{i=1}^{N} y_i - \theta_1 \sum_{i=1}^{N} x_i$$

$$N\theta_0 = \sum_{i=1}^{N} y_i - \theta_1 \sum_{i=1}^{N} x_i$$

$$\theta_0 = \frac{1}{N}\left(\sum_{i=1}^{N} y_i - \theta_1 \sum_{i=1}^{N} x_i\right) \quad - \; 1$$

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$$\sum_{i=1}^{3} (x_i + y_i)$$

$$= x_1 + y_1 + x_2 + y_2 + x_3 + y_3$$

$$= (x_1 + x_2 + x_3) + (y_1 + y_2 + y_3)$$

$$= \sum_{i=1}^{3} x_i + \sum_{i=1}^{3} y_i$$

$$\sum_{i=1}^{3} 1$$

$$= 1 + 1 + 1$$

$$= 3$$

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$$RSS = \sum_{i=1}^{N} (y_i - (\theta_0 + \theta_1 x_i)^2)$$

$$0 = \frac{\partial RSS}{\partial \theta_1} = \frac{\partial}{\partial \theta_1} \left(\sum_{i=1}^{N} (y_i - (\theta_1 + \theta_1 x_i)^2 \right)$$

$$\theta_1 \sum_{i=1}^{N} x_i^2 = \sum_{i=1}^{N} y_i x_i - \theta_0 \sum_{i=1}^{N} x_i \quad - (2)$$

Putting (1) in (2)

$$\theta_1 \sum_{i=1}^{N} x_i^2 = \sum_{i=1}^{N} y_i x_i - \left(\frac{1}{N} \left[\sum_{i=1}^{N} y_i - \theta_1 \sum_{i=1}^{N} x_i \right) \sum_{i=1}^{N} x_i$$

$$\theta_1 \sum_{i=1}^{N} x_i^2 = \sum_{i=1}^{N} y_i x_i - \left(\frac{1}{N} \left[\sum_{j=1}^{N} y_j - \theta_1 \sum_{j=1}^{N} x_j \right] \right) \sum_{i=1}^{N} x_i$$

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<u>Digression</u>

$$\left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} x_i\right) \neq \sum_{i=1}^{N} x_i^2$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j$$

$N=2$

$$\sum_{i=1}^{2} x_i^2 = x_1^2 + x_2^2$$

$$\left(\sum_{i=1}^{2} x_i\right)\left(\sum_{i=1}^{2} x_i\right)$$

$$= (x_1 + x_2)(x_1 + x_2)$$

$$= \underline{x_1^2 + x_1 x_1 + x_1 + x + x_2^2}$$

$$=$$

$N=2$

$$\sum_{i=1}^{2}\sum_{j=1}^{2} x_i x_j$$

$$= \sum x_1 x_1 + x_1 x_2 + x_1 x_2 + x_2 x_2$$

$$\left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} x_i\right) = \left(\sum_{i=1}^{N} x_i\right)^2$$

$$\sum_{i=1}^{2} x_i = x_1 + x_2$$

$$\sum_{j=1}^{2} x_j = x_1 + x_2$$

Coming back

Solution on slides

Multiple Linear Regression:

$$y = \theta_0 + \sum_{i=1}^{p} \theta_i x_i .$$

$$\theta = \underset{(p+1)\times 1}{\begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_p \end{bmatrix}} \qquad x = \underset{(p+1)\times 1}{\begin{bmatrix} 1 \\ x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix}}$$

$$\theta^T = \underset{1\times(p+1)}{[\theta_0 \; \theta_1 \; \theta_2 \dots \theta_p]} \times \begin{bmatrix} 1 \\ x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix} = x$$

$$\begin{bmatrix} \text{CRIM} & \text{ZOOL} & \cdots \\ \vdots & & \end{bmatrix}_{506 \times 13}$$

$\downarrow$

X