Expert Systems with Applications 37 (2010) 7895–7900

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

A method for multiple attribute group decision making based on the ET-WG and ET-OWG operators with 2-tuple linguistic information

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ARTICLE INFO

Keywords:
Multiple attribute group decision making (MAGDM)
The extended 2-tuple weighted geometric (ET-WG)
The extended 2-tuple ordered weighted geometric (ET-OWG) operator
2-tuple
Aggregation

ABSTRACT

With respect to multiple attribute group decision-making problems with linguistic information of attribute values and weight values, a group decision analysis is proposed. Some new aggregation operators are proposed: the extended 2-tuple weighted geometric (ET-WG) and the extended 2-tuple ordered weighted geometric (ET-OWG) operator and properties of the operators are analyzed. Then, A method based on the ET-WG and ET-OWG operators for multiple attribute group decision-making is presented. In the approach, alternative appraisal values are calculated by the aggregation of 2-tuple linguistic information. Thus, the ranking of alternative or selection of the most desirable alternative(s) is obtained by the comparison of 2-tuple linguistic information. Finally, a numerical example is used to illustrate the applicability and effectiveness of the proposed method.

1. Introduction

Multiple attribute decision-making problems are wide spread in real-life decision-making situation (Arrow, 1963). A multiple attribute decision making problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative. In order to choose a desirable solution, decision maker often provide his/her preference information which takes the form of numerical values, such as exact values, interval number values and fuzzy numbers. However, under many conditions, numerical values are inadequate or insufficient to model real-life decision problems. Indeed, human judgments including preference information may be stated in linguistic terms. Thus, multiple attribute decision-making problems under linguistic environment is an interesting research topic having received more and more attention from researchers during the last several years. In the process of multiple attribute decision-making, the linguistic decision information needs to be aggregated by means of some proper approaches so as to rank the given decision alternatives and then to select the most desirable one. Several methods have been proposed for dealing with linguistic information. These methods are mainly as follows:

(1) The method based on extension principle, which makes operations on the fuzzy numbers that support the semantics of the linguistic terms (Degani & Bortolan, 1988).
(2) The method based on symbols, which makes computations on the indexes of the linguistic terms (Delgado, Verdegay, & Vila, 1993).
(3) The method based on 2-tuple linguistic representation model, which composed by a linguistic term and a real number (Herrera & Martínez, 2000a, 2000b, 2001; Herrera, Martínez, & Sánchez, 2005). 2-tuple linguistic model has exact characteristic in linguistic information processing. It avoided information distortion and losing which occur formerly in the linguistic information processing. In recent years, 2-tuple linguistic model has been widely used in group decision making problems (Herrera & Martínez, 2001; Herrera et al., 2005; Herrera-Viedma, Martinez, Mata, & Chiclana, 2005; Jiang & Fan, 2003a, 2003b; Jiang, Fan, & Ma, 2008; Kim, Choi, & Kim, 1999; Martínez, 2007; Martínez, Liu, Ruan, & Yang, 2007; Tang & Zheng, 2006; Wei, 2008; Wei & Lin, 2008). Herrera and Martínez (2000a) developed 2-tuple arithmetic averaging (TAA) operator, 2-tuple weighted averaging (TWA) operator, 2-tuple ordered weighted averaging (TOWA) operator and extended 2-tuple weighted averaging (ET-WA) operator. Zhang and Fan (2006) proposed the extended 2-tuple ordered weighted averaging (ET-OWA) operator. Herrera et al. (2005) presented a group decision-making process for managing non-homogeneous information. The non-homogeneous information can be represented as values belonging to domains with different nature as linguistic, numerical and interval valued or can be values assessed in label sets with different granularity, multi-granular linguistic information. Herrera-Viedma et al. (2005) presented a model of consensus support system to assist the experts in all phases of the consensus reaching

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doi:10.1016/j.eswa.2010.04.047

process of group decision-making problems with multi-granular linguistic preference relations. Wang and Fan (2003b) proposed a TOPSIS method for solving multiple attribute group decision-making problems with linguistic assessment information. Liao, Li, and Lu (2007) presented a model for selecting an ERP system based on linguistic information processing. Wei and Lin (2008) developed GRA (grey relational analysis) method for multiple attribute group decision-making based on 2-tuple linguistic information. Wei (2008) utilized the maximizing deviation method to solve the 2-tuple linguistic multiple attribute group decision making with incomplete attribute weight information. Jiang and Fan (2003b) proposed the 2-tuple weighted geometric (TWG) operator and 2-tuple ordered weighted geometric (TOWG) operator. Herrera, Herrera-Viedma, and Martínez (2008) developed a fuzzy linguistic methodology to deal with unbalanced linguistic term sets. Wang (2009) presented a 2-tuple fuzzy linguistic evaluation model for selecting appropriate agile manufacturing system in relation to MC production.

In this paper, for the group decision-making problems, in which both the weights and the attribute preference values take the form of 2-tuple linguistic information, we have developed some new geometric aggregation operators: the extended 2-tuple weighted geometric (ET-WG) and the extended 2-tuple ordered weighted geometric (ET-OWG) operator and properties of the operators are analyzed. Then, A method based on the ET-WG and ET-OWG operators for multiple attribute group decision-making is presented. The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts and operational laws of 2-tuple linguistic variables. In Section 3 we develop some geometric aggregation operators with 2-tuple linguistic assessment information. In Section 4 we develop a new approach based on the ET-WG and ET-OWG operators to multiple attribute group decision-making with 2-tuple linguistic information processing. In Section 5, we give an illustrative example to verify the developed approach and to demonstrate its feasibility and practicality. In Section 6 we conclude the paper and give some remarks.

2. Preliminaries

Let $S = \{s_i | i = 1,2,\ldots,t\}$ be a linguistic term set with odd cardinality. Any label, s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics (Herrera & Martínez, 2000a, 2000b, 2001; Herrera et al., 2005):

(1) The set is ordered: $s_i > s_j$, if $i > j$; (2) Max operator: $\max(s_i, s_j) = s_i$, if $s_i \geqslant s_j$; (3) Min operator: $\min(s_i, s_j) = s_i$, if $s_i \leqslant s_j$. For example, S can be defined as

$$S = \{s_1 = \textit{extremely poor},\ s_2 = \textit{very poor},\ s_3 = \textit{poor},\ s_4 = \textit{medium},\ s_5 = \textit{good},\ s_6 = \textit{very good},\ s_7 = \textit{extremely good}\}$$

Herrera and Martínez (2000a, 2000b) developed the 2-tuple fuzzy linguistic representation model based on the concept of symbolic translation. It is used for representing the linguistic assessment information by means of a 2-tuple (s_i, α_i), where s_i is a linguistic label from predefined linguistic term set S and α_i is the value of symbolic translation, and $\alpha_i \in [-0.5, 0.5)$.

Definition 1. Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation, $\beta \in [1,t]$, being t the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [1,t]$ and $\alpha \in [-0.5, 0.5)$ then α is called a Symbolic Translation (Herrera and Martínez, 2000a, 2000b, 2001; Herrera et al., 2005).

Definition 2. Let $S = \{s_1, s_2, \ldots, s_t\}$ be a linguistic term set and $\beta \in [1,t]$ is a number value representing the aggregation result of linguistic symbolic. Then the function Δ used to obtain the 2-tuple linguistic information equivalent to β is defined as:

$$\Delta : [1,t] \rightarrow S \times [-0.5, 0.5) \tag{1}$$

$$\Delta(\beta) = \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5) \end{cases} \tag{2}$$

where round $(\cdot)$ is the usual round operation, s_i has the closest index label to β and α is the value of the symbolic translation (Herrera & Martínez, 2000a, 2000b, 2001; Herrera et al., 2005).

Definition 3. Let $S = \{s_1, s_2, \ldots, s_t\}$ be a linguistic term set and (s_i, α_i) be a 2-tuple. There is always a function Δ^{-1} can be defined, such that, from a 2-tuple (s_i, α_i) it return its equivalent numerical value $\beta \in [1,t] \subset R$, which is (Herrera & Martínez, 2000a, 2000b, 2001; Herrera et al., 2005).

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [1,t] \tag{3}$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \tag{4}$$

From Definitions 1 and 2, we can conclude that the conversion of a linguistic term into a linguistic 2-tuple consists of adding a value 0 as symbolic translation:

$$\Delta(s_i) = (s_i, 0) \tag{5}$$

Definition 4. Let (s_k, a_k) and (s_l, a_l) be two 2-tuple, they should have the following properties (Herrera & Martínez, 2000a, 2000b, 2001; Herrera et al., 2005).

(1) If $k < l$ then (s_k, a_k) is smaller than (s_l, a_l),
(2) If $k = l$ then,
  (a) if $a_k = a_l$, then (s_k, a_k), (s_l, a_l) represents the same information,
  (b) if $a_k < a_l$ then (s_k, a_k) is smaller than (s_l, a_l),
  (c) if $a_k > a_l$ then (s_k, a_k) is bigger than (s_l, a_l).

Definition 5. A 2-tuple negation operator.

$$neg(s_i, \alpha) = \Delta\Big(t + 1 - \Big(\Delta^{-1}(s_i, \alpha)\Big)\Big) \tag{6}$$

where t is the cardinality of S, $S = \{s_1, s_2, \ldots, s_t\}$ (Herrera & Martínez, 2000a, 2000b, 2001; Herrera et al., 2005).

Up to now, many operators have been proposed for aggregating information. Two of the most common operators for aggregating arguments are the weighted averaging operator and the ordered weighted averaging operators which are defined as follows, respectively.

Definition 6. Let WA: $R^n \rightarrow R$, if WA

$$WA_\omega(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} \omega_j a_j \tag{7}$$

Then WA is called a weighted averaging operator, where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $(a_1, a_2, \ldots, a_n)$, with $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$, R is the set of all real numbers (Harsanyi, 1995).

Definition 7. An ordered weighted averaging operator of dimension n is a mapping OWA: $R^n \rightarrow R$ that has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\mathrm{OWA}_w(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j a_{\sigma(j)} \tag{8}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$, such that $\alpha_{\sigma(j-1)} \geqslant \alpha_{\sigma(j)}$ for all $j = 2, \ldots, n$ (Yager, 1988).

From Definitions 6 and 7, we know that the WA operator first weights all the given arguments and then aggregates all these weighted arguments into a collective one. The fundamental aspect of the OWA operator is the reordering step; it first reorders all the given arguments in descending order and then weights these ordered arguments, and finally aggregates all these ordered weighted arguments into a collective one. The WA and OWA operators, however, have usually been used in situations where the input arguments are the exact values. Herrera and Martínez (2000a, 2000b) extended the WA and OWA operators to accommodate the situations where the input arguments are 2-tuple linguistic assessment information.

Definition 8. Let $x = \{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple, the 2-tuple arithmetic averaging is computed as (Herrera & Martínez, 2000a, 2000b)

$$(\bar{r}, \bar{a}) = TAA\left(\frac{1}{n}\sum_{j=1}^{n}\Delta^{-1}(r_j, a_j)\right), \quad \bar{r} \in S,\ \bar{a} \in [-0.5, 0.5) \tag{9}$$

Definition 9. Let $x = \{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weighting vector of 2-tuple (r_j,a_j) $(j = 1,2,\ldots,n)$ and $\omega_j \in [0,1]$, $\sum_{j=1}^{n}\omega_j = 1$, The 2-tuple weighted average is (Herrera & Martínez, 2000a, 2000b)

$$\begin{aligned}(\tilde{r}, \tilde{a}) &= TWA_{\omega}((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)) \\ &= \Delta\left(\sum_{j=1}^{n}\omega_j\Delta^{-1}(r_j, a_j)\right)\tilde{r} \in S, \quad \tilde{a} \in [-0.5, 0.5)\end{aligned} \tag{10}$$

Definition 10. Let $x = \{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple, A 2-tuple ordered weighted averaging operator of dimension n is a mapping $TOWA$: $R^n \to R$ that has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\begin{aligned}(\hat{r}, \hat{a}) &= \mathrm{TOWA_w}((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)) \\ &= \Delta\left(\sum_{j=1}^{n} w_j\Delta^{-1}(r_{\sigma(j)}, a_{\sigma(j)})\right), \quad \hat{r} \in S,\ \hat{a} \in [-0.5, 0.5)\end{aligned} \tag{11}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$, such that $(r_{\sigma(j-1)}, a_{\sigma(j-1)}) \geqslant (r_{\sigma(j)}, a_{\sigma(j)})$ for all $j = 2, \ldots, n$ (Herrera & Martínez, 2000a, 2000b).

The ordered weighted geometric (OWG) operator is an aggregation operator that Chiclana, Herrera, and Herrera-Viedma (2000) defined and characterized to design multiplicative decision-making models (Chiclana, Herrera, & Herrera-Viedma, 2001; Herrera, Herrera-Viedma, & Chiclana, 2001, 2003). It is based on the ordered weighted averaging (OWA) operator (Yager, 1988) and on the geometric mean. The OWG operator can only be used in situations where the input arguments are the exact numerical values. Recently, Jiang and Fan (2003b) extended the OWG operator to accommodate the situations where the input arguments are linguistic variables.

Definition 11. Let $x = \{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weighting vector of 2-tuple (r_j,a_j) $(j = 1,2,\ldots,n)$ and $\omega_j \in [0,1], \sum_{j=1}^{n}\omega_j = 1$, The 2-tuple weighted geometric operator is (Jiang & Fan, 2003b)

$$\begin{aligned}(\tilde{r}, \tilde{a}) &= TWG_{\omega}((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)) \\ &= \Delta\left(\prod_{j=1}^{n}\left(\Delta^{-1}(r_j, a_j)\right)^{\omega_j}\right)\tilde{r} \in S, \quad \tilde{a} \in [-0.5, 0.5)\end{aligned} \tag{12}$$

Definition 12. Let $x = \{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple, A 2-tuple ordered weighted geometric operator of dimension n is a mapping TOWG: $R^n \to R$ that has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$\begin{aligned}(\hat{r}, \hat{a}) &= \mathrm{TOWG}_w((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)) \\ &= \Delta\left(\prod_{j=1}^{n}\left(\Delta^{-1}(r_{\sigma(j)}, a_{\sigma(j)})\right)^{w_j}\right), \quad \hat{r} \in S,\ \hat{a} \in [-0.5, 0.5)\end{aligned} \tag{13}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$, such that $(r_{\sigma(j-1)}, a_{\sigma(j-1)}) \geqslant (r_{\sigma(j)}, a_{\sigma(j)})$ for all $j = 2, \ldots, n$ (Jiang & Fan, 2003b).

3. Some extended geometric aggregation operators with 2-tuple linguistic assessment information

Herrera and Martínez (2000a, 2000b) extended the TWA operators to accommodate the situations where the input arguments (including the attribute values and the attribute weight) are 2-tuple linguistic assessment information.

Definition 13. Let $x = \{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple and $C = ((c_1,\beta_1),(c_2,\beta_2),\ldots,(c_n,\beta_n))$ be the linguistic weighting vector of 2-tuple (r_j,a_j) $(j = 1,2,\ldots,n)$, The extended 2-tuple weighted average is (Herrera & Martínez, 2000a, 2000b)

$$\begin{aligned}(\tilde{r}', \tilde{a}') &= \mathrm{ET-WA}_C((s_1, a_1), (s_2, a_2), \ldots, (s_n, a_n)) \\ &= \Delta\left(\sum_{j=1}^{n}\frac{\Delta^{-1}(c_j, \beta_j)\Delta^{-1}(r_j, a_j)}{\sum_{j=1}^{n}\Delta^{-1}(c_j, \beta_j)}\right), \quad \tilde{r}' \in S,\ \tilde{a}' \in [-0.5, 0.5)\end{aligned} \tag{14}$$

Zhang and Fan (2006) extended the TOWA operators to accommodate the situations where the input arguments (including the attribute values and the attribute weight) are 2-tuple linguistic assessment information.

Definition 14. Let $x = \{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple, A extended 2-tuple ordered weighted average operator of dimension n is a mapping ETOWA: $R^n \to R$ that has an associated linguistic weighting vector $S = ((s_1,\eta_1),(s_2,\eta_2),\ldots,(s_n,\eta_n))$. Furthermore,

$$\begin{aligned}(\hat{r}', \hat{a}') &= \mathrm{ET-OWA}_S((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)) \\ &= \Delta\left(\sum_{j=1}^{n}\frac{\Delta^{-1}\left(s_j, \eta_j\right)\Delta^{-1}\left(r_{\sigma(j)}, a_{\sigma(j)}\right)}{\sum_{j=1}^{n}\Delta^{-1}(s_j, \eta_j)}\right), \quad \hat{r}' \in S,\ \hat{a}' \in [-0.5, 0.5)\end{aligned} \tag{15}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$, such that $(r_{\sigma(j-1)}, a_{\sigma(j-1)}) \geqslant (r_{\sigma(j)}, a_{\sigma(j)})$ for all $j = 2, \ldots, n$.

In the following, we shall propose some extended geometric aggregation operators to accommodate the situations where both the attribute values and the attribute weight are 2-tuple linguistic assessment information.

Definition 15. Let $x = \{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple and $C = ((c_1,\beta_1),(c_2,\beta_2),\ldots,(c_n,\beta_n))$ be the linguistic weighting vector of 2-tuple (r_j,a_j) $(j = 1,2,\ldots,n)$, The extended 2-tuple weighted geometric (ET-WG) operator is defined as follows:

$$\begin{aligned}(\tilde{r}', \tilde{a}') &= \mathrm{ET-WG}_C((s_1, a_1), (s_2, a_2), \ldots, (s_n, a_n)) \\ &= \Delta\left(\prod_{j=1}^{n}\left(\Delta^{-1}(r_j, a_j)\right)^{\frac{\Delta^{-1}\left(c_j, \beta_j\right)}{\sum_{j=1}^{n}\Delta^{-1}\left(c_j, \beta_j\right)}}\right), \quad \tilde{r}' \in S,\ \tilde{a}' \in [-0.5, 0.5)\end{aligned} \tag{16}$$

In the following, we shall study some desirable properties of the ET-WG operator.

Theorem 1 (Idempotency). *If $(s_j, a_j) = (s, a)$, $j = 1,2,\ldots,n$, then*

$$\mathrm{ET-WG}_C((s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)) = (s,a)$$

Theorem 2 (*Bounded*).

$$\begin{aligned} &Min((s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)) \\ &\quad \leqslant \mathrm{ET-WG}_C((s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)) \\ &\quad \leqslant Max((s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)) \end{aligned}$$

Definition 16. Let $x = \{(r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)\}$ be a set of 2-tuple, A extended 2-tuple ordered weighted geometric operator of dimension n is a mapping ET-OWG: $R^n \to R$ that has an associated linguistic weighting vector $S = ((s_1,\eta_1),(s_2,\eta_2),\ldots,(s_m,\eta_n))$. Furthermore,

$$\begin{aligned} (\hat{r}', \hat{a}') &= \mathrm{ET-OWG}_S((r_1,a_1),(r_2,a_2),\ldots,(r_n,a_n)) \\ &= \Delta\left(\prod_{j=1}^{n}\left(\Delta^{-1}\left(r_{\sigma(j)}, a_{\sigma(j)}\right)\right)^{\frac{\Delta^{-1}(s_j,\eta_j)}{\sum_{j=1}^{n}\Delta^{-1}(s_j,\eta_j)}}\right), \quad \hat{r}' \in S,\ \hat{a}' \in [-0.5, 0.5) \end{aligned} \tag{17}$$

where $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $(r_{\sigma(j-1)}, a_{\sigma(j-1)}) \geqslant (r_{\sigma(j)}, a_{\sigma(j)})$ for all $j = 2,\ldots,n$.

In the following, we shall study some desirable properties of the ET-OWG operator.

Theorem 3 (Idempotency). *If $(s_j, a_j) = (s, a)$, $j = 1,2,\ldots,n$, then*

$$\mathrm{ET-OWG}_S((s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)) = (s,a)$$

Theorem 4 (*Commutativity*).

$$\begin{aligned} &\mathrm{ET-OWG}_S((s_1',a_1'),(s_2',a_2'),\ldots,(s_n',a_n')) \\ &\quad = \mathrm{ET-OWG}_S((s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)) \end{aligned}$$

where $(s_1',a_1'),(s_2',a_2'),\ldots,(s_n',a_n')$ is any permutation of $(s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)$.

Theorem 5 (monotonicity). *If $(s_j, a_j) \leqslant \left(s_j', a_j'\right)$, for all j, then*

$$\begin{aligned} &\mathrm{ET-OWG}_S((s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)) \\ &\quad \leqslant \mathrm{ET-OWG}_S((s_1',a_1'),(s_2',a_2'),\ldots,(s_n',a_n')) \end{aligned}$$

Theorem 6 (*Bounded*).

$$\begin{aligned} &Min((s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)) \\ &\quad \leqslant \mathrm{ET-OWG}_S((s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)) \\ &\quad \leqslant Max((s_1,a_1),(s_2,a_2),\ldots,(s_n,a_n)) \end{aligned}$$

From Definitions 15 and 16, we know that the ET-WG operator weights the given arguments, and the ET-OWG operator first reorders the weighted arguments in descending order and weights these ordered arguments by the ET-OWG weights, and finally aggregates all the weighted arguments into a collective one.

4. An approach to multiple attribute group decision-making with 2-tuple linguistic assessment information

For the multiple attribute group decision making problems, in which both the weights and the attribute preference values take the form of linguistic variables, we shall develop a new approach based on the ET-WG and ET-OWG operators to multiple attribute group decision-making with 2-tuple linguistic information processing.

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of alternatives, and $G = \{G_1, G_2, \ldots, G_n\}$ be the set of attributes, $H^T = (h_1, h_2, \ldots, h_n)$ is the linguistic weighting vector of the attributes $G_j (j = 1,2,\ldots,n)$, where $h_j \in S$, $j = 1,2,\ldots,n$. Let $D = \{D_1, D_2, \ldots, D_t\}$be the set of decision makers. Suppose that $\widetilde{R}_k = (\tilde{r}_{ij}^{(k)})_{m\times n}$ is the decision matrix, where $\tilde{r}_{ij}^{(k)} \in \widetilde{S}$ is a preference value, which takes the form of linguistic variables, given by the decision maker $D_k \in D$, for the alternative $A_i \in A$ with respect to the attribute $G_j \in G$.

To get the best alternative(s), the follows steps are involved:

Step 1. Transforming linguistic decision matrix $R_k = \left(r_{ij}^{(k)}\right)_{m\times n}$ into 2-tuple linguistic decision matrix $R_k = \left(r_{ij}^{(k)}, 0\right)_{m\times n}$ and transforming linguistic weighting vector $H^T = (h_1, h_2, \ldots, h_n)$ into 2-tuple linguistic weighting vector $H^T = ((c_1, \beta_1), (c_2, \beta_2), \ldots, (c_n, \beta_n)) = ((h_1, 0), (h_2, 0), \ldots, (h_n, 0))$.

Step 2. Utilize the decision information given in matrix R_k, and the ET-WG operator

$$\begin{aligned} &z_i^{(k)} = \left(r_i^{(k)}, a_i^{(k)}\right) = \Delta\left(\prod_{j=1}^{n}\left(\Delta^{-1}\left(r_{ij}^{(k)}, 0\right)\right)^{\frac{\Delta^{-1}\left(c_j,\beta_j\right)}{\sum_{j=1}^{n}\Delta^{-1}\left(c_j,\beta_j\right)}}\right), \\ &r_i^{(k)} \in S,\ a_i^{(k)} \in [-0.5, 0.5) \end{aligned} \tag{18}$$

to derive the individual overall preference value $\tilde{r}_i^{(k)}$ of the alternative A_i.

Step 3. Utilize the ET-OWG operator:

$$\begin{aligned} &z_i = (r_i, a_i) = \Delta\left(\prod_{k=1}^{t}\left(\Delta^{-1}\left(\hat{r}_i^{\sigma(k)}, \hat{a}_i^{\sigma(k)}\right)\right)^{\frac{\Delta^{-1}\left(s_j,\eta_j\right)}{\sum_{j=1}^{n}\Delta^{-1}\left(s_j,\eta_j\right)}}\right), \\ &r_i \in S,\ a_i \in [-0.5, 0.5) \end{aligned} \tag{19}$$

to derive the collective overall preference values $z_i = (r_i, a_i) (i = 1,2,\ldots,m)$ of the alternative A_i, where $\left(\hat{r}_i^{\sigma(k)},\ \hat{a}_i^{\sigma(k)}\right)$ is the kth largest of the 2-tuple linguistic weighted arguments $\left(r_i^{(k)},\ a_i^{(k)}\right) (k = 1,2,\ldots,t)$, $S^T = ((s_1,\eta_1),(s_2,\eta_2),\ldots,(s_n,\eta_n))$ is the associated 2-tuple linguistic weighting vector of the ET-OWG operator.

Step 4. Rank all the alternatives $A_i (i = 1,2,\ldots,m)$ and select the best one(s) in accordance with $z_i (i = 1,2,\ldots,m)$. If any alternative has the highest z_i value, then, it is the most important alternative.

Step 5. End.

5. Numerical example

Let us suppose there is an investment company, which wants to invest a sum of money in the best option (adapted from Herrera & Herrera-Viedma (2000b)). There is a panel with five possible alternatives to invest the money: A_1 is a car company; A_2 is a food company; A_3 is a computer company; A_4 is an arms company; A_5 is a TV company. The investment company must take a decision according to the following four attributes: G_1 is the risk analysis; G_2 is the growth analysis; G_3 is the social-political impact analysis; G_4 is the environmental impact analysis. $H^T = (P, EP, VP, M)$ is the linguistic weighting vector of the attributes G_j $(j = 1,2,3,4)$. The five possible alternatives A_i $(i = 1,2,\ldots,5)$ are to be evaluated using the linguistic term set

$S = \{s_1 = extremely\ poor,\ s_2 = very\ poor,\ s_3 = poor,\ $
$s_4 = medium,\ s_5 = good,\ s_6 = very\ good,\ s_7 = extremely\ good\}$

by the three decision makers D_k ($k = 1,2,3$) under the above four attributes, and construct, respectively, the decision matrices as follows $\widetilde{R}_k = \left(\tilde{r}_{ij}^{(k)}\right)_{5\times 4} (k = 1,2,3)$:

$$R_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \end{array} \\ \begin{pmatrix} M & G & P & P \\ P & VP & M & P \\ G & M & G & EP \\ VG & P & P & G \\ EG & EP & VP & M \end{pmatrix} \end{array}$$

$$R_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \end{array} \\ \begin{pmatrix} P & M & VP & VP \\ VP & EP & G & G \\ M & G & P & EG \\ EG & VP & VP & M \\ P & VP & M & VP \end{pmatrix} \end{array}$$

$$R_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \end{array} \\ \begin{pmatrix} G & P & VP & VG \\ VP & G & P & G \\ VG & VP & G & P \\ G & VG & EG & VP \\ M & VP & M & G \end{pmatrix} \end{array}$$

In the following, we shall utilize the proposed approach in this paper getting the most desirable alternative(s):

Step 1. Transforming linguistic decision matrix $R_k = \left(r_{ij}^{(k)}\right)_{m\times n}$ into 2-tuple linguistic decision matrix $R_k = \left(r_{ij}^{(k)}, 0\right)_{m\times n}$ as follows:

$$R_1 = \begin{pmatrix} (M,0) & (G,0) & (P,0) & (P,0) \\ (P,0) & (VP,0) & (M,0) & (P,0) \\ (G,0) & (M,0) & (G,0) & (EP,0) \\ (VG,0) & (P,0) & (P,0) & (G,0) \\ (EG,0) & (EP,0) & (VP,0) & (M,0) \end{pmatrix}$$

$$R_2 = \begin{pmatrix} (P,0) & (M,0) & (VP,0) & (VP,0) \\ (VP,0) & (EP,0) & (G,0) & (G,0) \\ (M,0) & (G,0) & (P,0) & (EG,0) \\ (EG,0) & (VP,0) & (VP,0) & (M,0) \\ (P,0) & (VP,0) & (M,0) & (VP,0) \end{pmatrix}$$

$$R_3 = \begin{pmatrix} (G,0) & (P,0) & (VP,0) & (VG,0) \\ (VP,0) & (G,0) & (P,0) & (G,0) \\ (VG,0) & (VP,0) & (G,0) & (P,0) \\ (G,0) & (VG,0) & (EG,0) & (VP,0) \\ (M,0) & (VP,0) & (M,0) & (G,0) \end{pmatrix}$$

Step 2. Utilize the decision information given in matrix $\widetilde{R}_k$, and the ET-WG operator to derive the individual overall preference value $z_i^{(k)} = \left(r_i^{(k)},\ a_i^{(k)}\right)$ of the alternative A_i.

$$z_1^{(1)} = (P, 0.44), \quad z_2^{(1)} = (P, 0.05), \quad z_3^{(1)} = (P, -0.43)$$
$$z_4^{(1)} = (G, -0.47), \quad z_5^{(1)} = (M, -0.41), \quad z_1^{(2)} = (VP, 0.42)$$
$$z_2^{(2)} = (P, 0.23), \quad z_3^{(2)} = (G, -0.17), \quad z_4^{(2)} = (M, -0.16)$$
$$z_5^{(2)} = (P, -0.41), \quad z_1^{(3)} = (G, -0.4), \quad z_2^{(3)} = (M, -0.3)$$
$$z_3^{(3)} = (M, 0.2), \quad z_4^{(3)} = (M, 0.3), \quad z_5^{(3)} = (M, 0.2)$$

Step 3. Utilize the ET-OWG operator to derive the collective overall preference values $z_i = (r_i, a_i)$ ($i = 1,2,3,4,5$) of the alternative A_i, $V^T = (HZ, YB, HC)$ is the associated weighting vector of the ET-OWG operator.

$$z_1 = (M, -0.25), \quad z_2 = (P, 0.43), \quad z_3 = (M, 0.15)$$
$$z_4 = (M, 0.33), \quad z_5 = (M, -0.32)$$

Step 4. Ranking all the alternatives A_i ($i = 1,2,\ldots,5$) in accordance with the z_i ($i = 1,2,\ldots,5$): $A_4 \succ A_3 \succ A_1 \succ A_5 \succ A_2$, and thus the most desirable alternative is A_4.

6. Conclusion

For the multiple attribute group decision making problems, in which both the weights and the attribute preference values take the form of 2-tuple linguistic information, we have developed some new aggregation operators: the exteded 2-tuple weighted geometric (ET-WG) and exteded weighted geometric (ET-OWG) operator and properties of the operators are analyzed. Then, a method based on the ET-WG and ET-OWG operators for multiple attribute group decision-making is presented. In this approach, alternative appraisal values are calculated by the aggregation of 2-tuple linguistic information. Thus, the ranking of alternative or selection of the most desirable alternative(s) is obtained by the comparison of 2-tuple linguistic information. Finally, a numerical example is used to illustrate the applicability and effectiveness of the proposed method. Theoretical analyses and numerical results all show that the method is straightforward and has no loss of information. In the future, we shall continue working in the application of the geometric aggregation operators with 2-tuple linguistic assessment information to other domains.

References

Arrow, K. J. (1963). *Social choice and individual values*. New York: Wiley.

Chiclana, F., Herrera, F. & Herrera-Viedma, E. (2000). The ordered weighted geometric operator: Properties and application. In *Proceedings of the eighth international conference on information processing and management of uncertainty in knowledge-based systems* (pp. 985–991). Madrid, Spain.

Chiclana, F., Herrera, F., & Herrera-Viedma, E. (2001). Integrating multiplicative preference relations in a multipurpose decision-making model based on fuzzy preference relations. *Fuzzy Sets and Systems, 112*, 277–291.

Degani, R., & Bortolan, G. (1988). The problem of linguistic approximation in clinical decision-making. *International Journal of Approximate Reasoning, 2*, 143–162.

Delgado, M., Verdegay, J. L., & Vila, M. A. (1993). On aggregation operations of linguistic labels. *International Journal of Intelligent Systems, 8*, 351–370.

Harsanyi, J. C. (1995). Cardinal welfare, individualistic ethics, and interpersonal comparisons of utinity. *Journal of Political Economy, 63*, 309–321.

Herrera, F., & Herrera-Viedma, E. (2000b). Linguistic decision analysis: Steps for solving decision problems under linguistic information. *Fuzzy Sets and Systems, 115*, 67–82.

Herrera, F., Herrera-Viedma, E., & Chiclana, F. (2001). Multiperson decision-making based on multiplicative preference relations. *European Journal of Operational Research, 129*, 372–385.

Herrera, F., Herrera-Viedma, E., & Chiclana, F. (2003). A study of the origin and uses of the ordered weighted geometric operator in multicriteria decision-making. *International Journal of Intelligent Systems, 18*, 689–707.

Herrera, F., Herrera-Viedma, E., & Martínez, L. (2008). A fuzzy linguistic methodology to deal with unbalanced linguistic term sets. *IEEE Transactions on Fuzzy Systems, 16*(2), 354–370.

Herrera, F., & Martínez, L. (2000a). A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems, 8*, 746–752.

Herrera, F., & Martínez, L. (2000b). An approach for combining linguistic and numerical information based on 2-tuple fuzzy linguistic representation model in decision-making. *International Journal of Uncertainty, Fuzziness, Knowledge-Based Systems, 8*, 539–562.

Herrera, F., & Martínez, L. (2001). A model based on linguistic 2-tuple for dealing with multi-granular hierarchical linguistic contexts in multi-expert decision-making. *IEEE Transactions on Systems, Man, and Cybernetics, 31*, 227–234.

Herrera, F., Martínez, L., & Sánchez, P. J. (2005). Managing non-homogeneous information in group decision-making. *European Journal of Operational Research, 166*(1), 115–132.

Herrera-Viedma, E., Martinez, L., Mata, F., & Chiclana, F. (2005). A consensus support system model for group decision-making problems with multi-granular linguistic preference relations. *IEEE Transactions on Fuzzy Systems, 13*, 644–658.

Jiang, Y. P., & Fan, Z. P. (2003a). An approach to group decision-making problems based on 2-tuple linguistic symbol operation. *Systems Engineering and Electronics, 25*(11), 1373–1376.
Jiang, Y. P., & Fan, Z. P. (2003b). Property analysis of the aggregation operators for 2-tuple linguistic information. *Control and Decision, 18*(6), 754–757.
Jiang, Y. P., Fan, Z. P., & Ma, J. (2008). A method for group decision-making with multi-granularity linguistic assessment information. *Information Sciences, 178*(4), 1098–1109.
Kim, S. H., Choi, S. H., & Kim, J. K. (1999). An interactive procedure for multiple attribute group decision-making with incomplete information: Range-based approach. *European Journal of Operational Research, 118*, 139–152.
Liao, X. W., Li, Y., & Lu, B. (2007). A model for selecting an ERP system based on linguistic information processing. *Information Systems, 32*(7), 1005–1017.
Martínez, L. (2007). Sensory evaluation based on linguistic decision analysis. *International Journal of Approximate Reasoning, 44*(2), 148–164.
Martínez, L., Liu, J., Ruan, D., & Yang, J. B. (2007). Dealing with heterogeneous information in engineering evaluation processes. *Information Sciences, 177*(7), 1533–1542.
Tang, Y. C., & Zheng, J. C. (2006). Linguistic modeling based on semantic similarity relation among linguistic labels. *Fuzzy Sets and Systems, 157*, 1662–1673.
Wang, W. P. (2009). Evaluating new product development performance by fuzzy linguistic computing. *Expert Systems with Applications, 36*(6), 9759–9766.
Wang, X. R., & Fan, Z. P. (2003b). Method for group decision-making based on 2-tuple linguistic information processing. *Journal of Management Science in China, 6*(5), 1–5.
Wei, G. W. (2008). 2-tuple linguistic multiple attribute group decision-making with incomplete attribute weight information. *Systems Engineering and Electronics, 30*(2), 273–277.
Wei, G. W., & Lin, R. (2008). Method of grey relational analysis for multiple attribute group decision-making based on 2-tuple linguistic information. *Systems Engineering and Electronics, 30*(9), 1686–1689.
Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multicriteria decision-making. *IEEE Transactions on Systems, Man, and Cybernetics, 18*, 183–190.
Zhang, Y., & Fan, Z. P. (2006). An approach to linguistic multiple attribute decision-making with linguistic information based on ELOWA operator. *Systems Engineer, 24*(12), 98–101.

Further Reading

Ben-Arieh, D., & Chen, Z. F. (2006). Linguistic-labels aggregation and consensus measure for autocratic decision-making using group recommendations. *IEEE Transactions on Systems, Man and Cybernetics. Part A, 36*, 558–568.
Bordogna, G., Fedrizzi, M., & Passi, G. (1997). A linguistic modeling of consensus in group decision-making based on OWA operator. *IEEE Transactions on Systems, Man, and Cybernetics, 27*, 126–132.
Chang, S. L., Wang, R. C., & Wang, S. Y. (2007). Applying a direct multi-granularity linguistic and strategy-oriented aggregation approach on the assessment of supply performance. *European Journal of Operational Research, 177*(2), 1013–1025.
Chen, Z. F., & Ben-Arieh, D. (2006). On the fusion of multi-granularity linguistic label sets in group decision-making. *Computers & Industrial Engineering, 51*(3), 526–541.
Cordón, O., Herrera, F., & Zwir, I. (2002). Linguistic modeling by hierarchical systems of linguistic rules. *IEEE Transactions on Fuzzy Systems, 10*, 2–20.
Delgado, M., Herrera, F., Herrera-Viedma, E., Martin-Bautista, M. J., Martínez, L., & Vila, M. A. (2002). A communication model based on the 2-tuple fuzzy linguistic representation for a distributed intelligent agent system on internet. *Soft Computing, 6*, 320–328.
Fan, Z. P., Ma, J., & Zhang, Q. (2002). An approach to multiple attribute decision-making based on fuzzy preference information alternatives. *Fuzzy Sets and Systems, 131*(1), 101–106.
Fan, Z. P., Feng, B., Sun, Y. H., & Ou, W. (2009). Evaluating knowledge management capability of organizations: A fuzzy linguistic method. *Expert Systems with Applications, 36*(2), 3346–3354.
Halouani, N., Chabchoub, H., & Martel, J. M. (2009). PROMETHEE-MD-2T method for project selection. *European Journal of Operational Research, 195*(3), 841–849.
Herrera, F. (1995). A sequential selection process in group decision-making with linguistic assessment. *Information Sciences, 85*, 223–239.
Herrera, F., & Herrera-Viedma, E. (2000a). Choice functions and mechanisms for linguistic preference relations. *European Journal of Operational Research, 120*, 144–161.
Herrera, F., Herrera-Viedma, E., & Martínez, L. (2000). A fusion approach for managing multi-granularity linguistic term sets in decision-making. *Fuzzy Sets and Systems, 114*, 43–58.
Herrera, F., & Martínez, L. (1991). The 2-tuple linguistic computational model: Advantages of its linguistic description, accuracy and consistency. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 9*, 33–49.
Herrera, F., Herrera-Viedma, E., & Verdegay, J. (1996a). A linguistic decision process in group decision-making. *Group Decision and Negotiation, 5*(1), 165–176.
Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1996b). Direct approach processes in group decision-making using linguistic OWA operators. *Fuzzy Sets and Systems, 79*, 175–190.
Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1996c). A model of consensus in group decision-making under linguistic assessments. *Fuzzy Sets and Systems, 78*, 73–87.
Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1997). A rational consensus model in group decision-making using linguistic assessments. *Fuzzy Sets and Systems, 88*, 31–49.
Huynh, A. V., & Nakamori, Y. (2005). A satisfactory-oriented approach to multi-expert decision-making with linguistic assessments. *IEEE Transactions on Systems, Man and Cybernetics. Part B, 35*, 184–196.
Li, H. Y., & Fan, Z. P. (2003). Multi-criteria group decision-making method based on 2-tuple linguistic information processing. *Journal of Northeastern University (Natural Science), 24*(5), 495–498.
Liao, X. W., Li, Y., & Dong, G. M. (2006). A multi-attribute group decision-making approach dealing with linguistic assessment information. *System Engineering-Theory & Practice, 26*(9), 90–98.
Liu, X. W., & Chen, L. (2004). On the properties of parametric geometric OWA operator. *International Journal of Approximate Reasoning, 35*, 163–178.
Liu, X. W. (2006a). On the properties of equidifferent OWA operator. *International Journal of Approximate Reasoning, 43*, 90–107.
Liu, X. W. (2006b). Some properties of the weighted OWA operator. *IEEE Transactions on Systems, Man and Cybernetics, 36*(1), 118–127.
Liu, X. W. (2007). The solution equivalence of minimax disparity and minimum variance problems for OWA operators. *International Journal of Approximate Reasoning, 45*(1), 68–81.
Roubens, M. (1997). Fuzzy sets and decision analysis. *Fuzzy Sets and Systems, 90*, 199–206.
Tai, W. S., & Chen, C. T. (2009). A new evaluation model for intellectual capital based on computing with linguistic variable. *Expert Systems with Applications, 36*(2), 3483–3488.
Yager, R. R. (1993). Families of OWA operators. *Fuzzy Sets and Systems, 59*, 125–148.
Torra, V. (1996). Negation functions based semantics for ordered linguistic labels. *International Journal of Intelligent Systems, 11*, 975–988.
Wang, J. Q. (2006). Multi-criteria decision-making approach with incomplete certain information based on ternary AHP. *Journal of Systems Engineering and Electronics, 17*(1), 109–114.
Wang, X. R., & Fan, Z. P. (2003a). A method for group decision-making problems with different forms of preference information. *Journal of Northeastern University (Natural Science), 24*(2), 178–181.
Wei, F., Liu, C. A., & Liu, S. Y. (2006). A method for group decision-making with linguistic information based on uncertain information processing. *Operational Research and management Science, 15*(3), 31–35.
Yager, R. R. (1996). Quantifier guided aggregation using OWA operators. *International Journal of Intelligent Systems, 11*, 49–73.
Zadeh, L. A. (1975/1976). The concept of a linguistic variable and its application to approximate reasoning, Part 1–3. *Information Sciences, 8*, 199–249 [301–357; 9, 43–80].
Zadeh, L. A. (1983). A computational approach to fuzzy quantifiers in natural languages. *Computers and Mathematics with Applications, 9*, 149–184.
Zadeh, L. A., & Kacprzyk, J. (1999). *Computing with words in information/intelligent systems-Part 1: Foundations: Part 2: Applications* (vol. 1). Heidelberg: Physica-Verlag.
Zhang, Z. F., & Chu, X. N. (2009). Fuzzy group decision-making for multi-format and multi-granularity linguistic judgments in quality function deployment. *Expert Systems with Applications, 36*(5), 9150–9158.
Zimmermann, H. J. (1991). *Fuzzy set theory and its applications* (2nd ed.). Kluwer Academic Publishers.