

A reference dataset for astronomical transient event recognition I: lightcurves and tests on classical machine learning algorithms

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Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

We introduce ATRANCCATA (Annotated TRANSient Catalina CATALog) an annotated dataset of 4869 transient and 16940 non-transient object lightcurves built from the Catalina Real Time Transient Survey. We provide public access to this dataset as a plain text file to facilitate standardized quantitative comparison of astronomical transient event recognition algorithms. Some of the classes included in the dataset are: supernovae, cataclismic variables, active galactic nuclei, high proper motion stars, blazars and flares. As a complement to the dataset, we experiment with multiple data pre-processing methods, feature selection techniques and popular machine learning algorithms (Support Vector Machines, Random Forests and Neural Networks). We assess quantitative performance in two classification tasks: binary (transient/non-transient) and eight-class classification. The best performing algorithm is a Random Forest Classifier for both classification experiments. The next release of ATRANCCATA will include images and benchmarks with deep learning models. All our code and data is available to the community at <https://github.com/MachineLearningUniandes/ATRANCCATA>.

Key words: methods: data analysis, statistical

1 INTRODUCTION

The study and detection of astronomical variable sources is expected to occur at unprecedented scales with the new generation of forthcoming multi-epoch and multi-band (synoptic) astronomical surveys. For instance, projects like the Large Synoptic Survey Telescope (LSST) (Ivezić et al. 2008; Jurić et al. 2015) are expected to generate exuberant daily data-streams of about 20 petabytes every night.

One of the main challenges that these datasets want to address is Real-Time Transient classification (Kaiser 2004; Law et al. 2009; Shappee et al. 2014; Tonry et al. 2018; Bellm et al. 2019) including phenomena such as supernovae (SN), novae, neutron stars, blazars, pulsars, cataclysmic variable stars (CV), gamma ray bursts (GRB) and active galaxy nuclei (AGN). The time-domain dependency of these objects is one of the reasons why they are hard to classify: their data is usually heterogeneous, unbalanced, sparse, unevenly sampled and with missing information. These challenges have

motivated the use of Machine Learning (ML) algorithms to recognize and classify transient events.

There have been successful attempts to implement these algorithms using images as input. For instance, data from the SkyMapper Supernova and Transient Survey and the High cadence Transient Survey (HiTS) have been used as input to automatic detection algorithms (Gieseke et al. 2017; Cabrera-Vives et al. 2017). Convolutional Neural Networks (CNN) have also achieved high accuracy in this binary classification task. Other studies have shown that artifacts can be detected using features extracted from raw images. Klencki et al. (2016) achieved reliable classification by transforming transient data from the OGLE-IV data-reduction pipeline and training it with machine learning algorithms such as Artificial Neural Networks, Support Vector Machines, Random Forests, Naive Bayes, K-Nearest Neighbors and Linear Discriminant Analysis. Similarly, Wright et al. (2015) used images from Pan-STARRS1 Medium Deep Surveys, and du Buisson et al. (2015) processed single-epoch multi-band images from the SDSS supernova survey for the same purpose.

A complementary approach uses the lightcurves computed from the images to perform the classification task. For instance [D’Isanto et al. \(2016\)](#) used classical machine learning algorithms such as Random Forest, MultiLayer Perceptron and K-Nearest Neighbours lightcurves to classify transients from the Catalina Real Time Transient Survey; [Lochner et al. \(2016\)](#) used the same approach to find supernovas from the Supernova Photometric Classification Challenge.

A crucial element in the development of any ML algorithm is the compilation of the training dataset. In astronomy this task has been facilitated by the publication of large databases from different observational projects. However, the publications that make use of these datasets still make extensive use of the internal know-how of the scientific collaboration. It is still difficult for other scientists to rebuild a training dataset and perform comparisons with published results.

To address this issue, we compile and publish in easy-to-access files a dataset that can be used to train and test different ML algorithms for transient detection. We use public data from the Catalina Real-Time Transient Survey (CRTS) ([Drake et al. 2012](#)), an astronomical survey searching transient and highly variable objects as base for the dataset. In this paper we present lightcurve data, in a future publication we will present an imaging dataset.

In Section 2 we present the CRTS and the steps we follow to build the dataset. Then, in Section 3 we describe its main features together with the repository structure gathering the files and Python code to explore it. In Section 4 we show how this dataset can be used to perform tests using ML methods following a similar approach as [D’Isanto et al. \(2016\)](#), and the experiments that we perform. We finalize in Section 5 with a summary of the main features of our dataset and the results of our experiments.

2 THE LIGHTCURVE DATASET

We use public data from the Catalina Real-Time Transient Survey (CRTS) ([Drake et al. 2009](#)), an astronomical survey searching transient and highly variable objects. The CRTS covered 33000 squared degrees of sky and took data since 2007. Three telescopes were used: Mt. Lemmon Survey (MLS), Catalina Sky Survey (CSS), and Siding Spring Survey (SSS). So far, CRTS has discovered more than 15000 transient events. We use data from the CSS telescope, which is an f/1.8 Schmidt telescope located in the Santa Catalina Mountains in Arizona. The telescope is equipped with a 111-megapixel detector, and covered 4000 square degrees per night, with a limiting magnitude of 19.5 in the V band.

Putting together the lightcurves for ATRANCATA implies cross-matching different files in the legacy CRTS webpage: http://nesssi.cacr.caltech.edu/DataRelease/CRTS-I_transients.html. The photometry is stored in two different kinds of files: **phot** that come from the main photometry database and **orphan** that correspond to transients not associated with the 500 million sources in the main photometry database. There are also **out** files that must be used to link transient IDs to database IDs.

For each one of the 5540 transients reported and classified in the archival webpage <http://nesssi.cacr.caltech.edu/catalina/All.arch.html>

Class	Object Count
SN	1723
CV	988
HPM	640
AGN	446
SN?	319
Blazar	243
Unknown	228
Flare	219
AGN?	138
CV?	77

Table 1. Top 10 transient classes in the CRTS with their respective number of lightcurves.

<http://nesssi.cacr.caltech.edu/catalina/All.arch.html> we use its transient IDs and its database IDs to look for the lightcurves in the **phot** and **orphan** files. Only 4982 transients can be linked to available data to reconstruct their lightcurves. Furthermore, some of these lightcurves are duplicated, i.e. they had the same number of observations, Modified Julian Date (MJD) and magnitude measurement. We ignore the duplicates to end up with 4869 unique transients with an associated lightcurve. Figure 1 summarizes this process.

The CRTS dataset already provides a classification. The most numerous classes are: supernovae, cataclysmic variable stars, blazars, flares, asteroids, active galactic nuclei, and high-proper-motion stars (HPM). Though most objects in the transient object catalogue belong to a single class, there is some uncertainty in the categorization of some of them. In this case, an interrogation sign is used when a class is not clear e.g. SN? or sometimes multiple possible classes are found for a single event e.g. SN/CV. Table 1 summarizes the number of objects in each class.

To compile the non-transient lightcurves we retrieve sources in the dataset from the CRTS online catalogue by retrieving objects within a 20 arcsecond radius from CRTS detected transients, and removing known transient lightcurves from that set. We end up with 16940 unique non-transient lightcurves. Figure 2 illustrates this process.

Figure 3 shows the number of lightcurves as a function of average magnitude (left panel) and as a function of the number of points in the lightcurve (right panel). We show separately the whole data set and three most representative classes: supernova, cataclysmic variables and active galactic nuclei. For these four sets, the median magnitude is in the range 18 – 20. The number of points in the lightcurve has a larger variability. The median for all the curves is close to 30, while for SN, CV and AGN it is close to 15, 50 and 180, respectively. We provide sample lightcurves of the most represented transient classes and non-transient sources in Figure 4 and Figure 5, respectively. The brightness evolution of non-transient sources is more stable over time, while transient objects present non-periodical changes at different time scales.

The challenges in the classification of lightcurves include the inherent nature of transient events, which is reflected in different brightness behaviors, their evolution over time, and the nonuniform sampling of observations at sequential dates. Besides, there is a large class imbalance to localize transient events, and perform their subsequent classification.

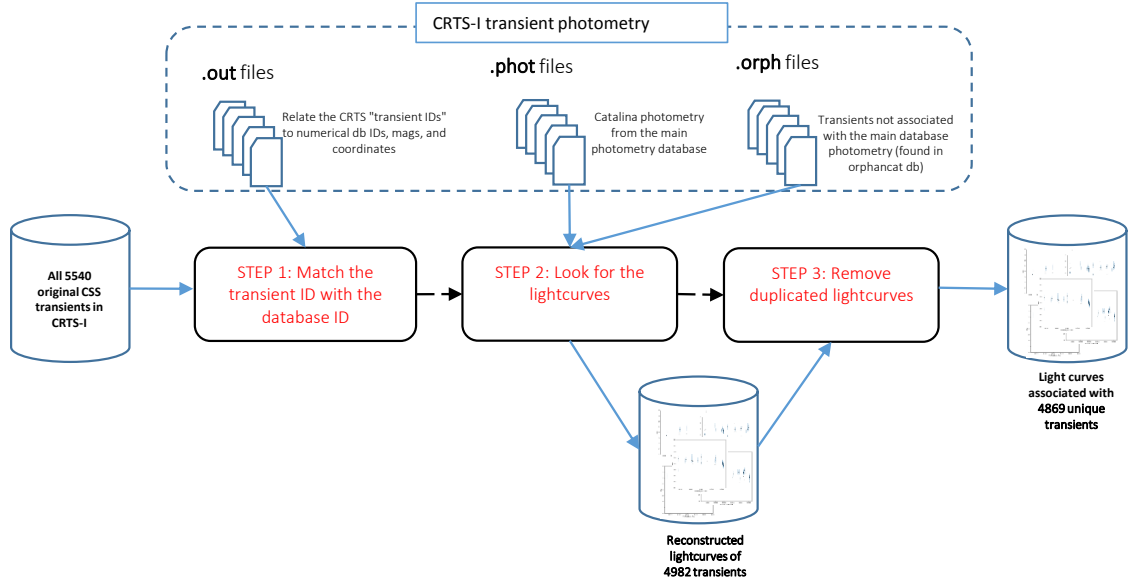


Figure 1. ATRANCCATA Dataset Set Up: Lightcurve compilation for transient classes.

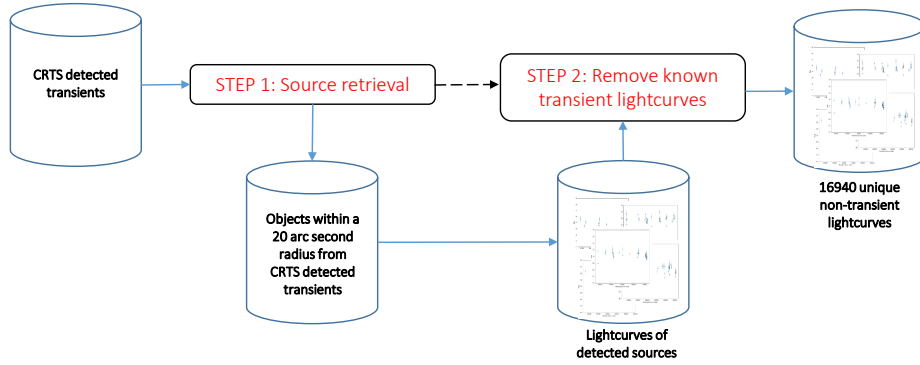


Figure 2. ATRANCCATA Dataset Set Up: Lightcurve compilation for non-transients.

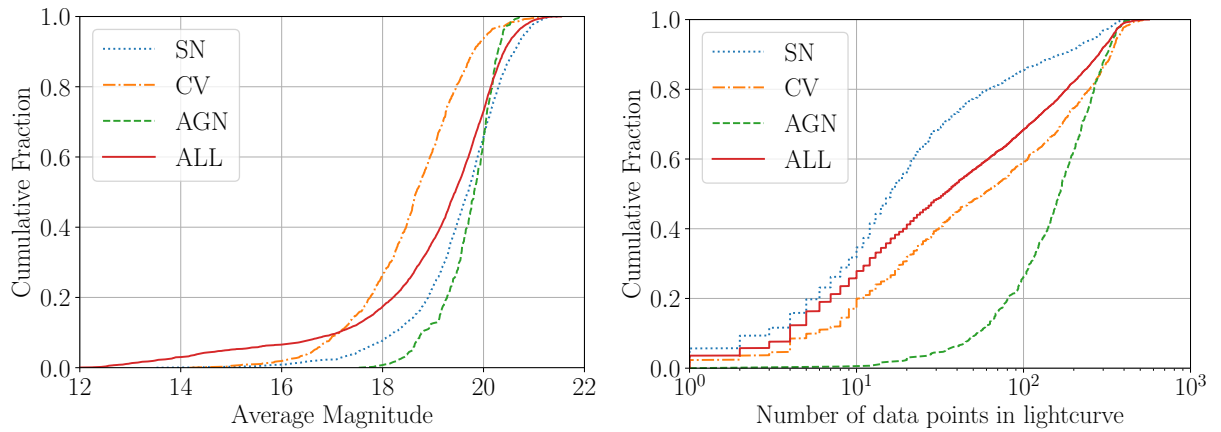


Figure 3. Cumulative number of lightcurves (expressed as a fraction) as a function of average magnitude (left) and number of data points in the lightcurve (right). This includes information for the three most representative classes (SN, CV, AGN) and the whole database (ALL).

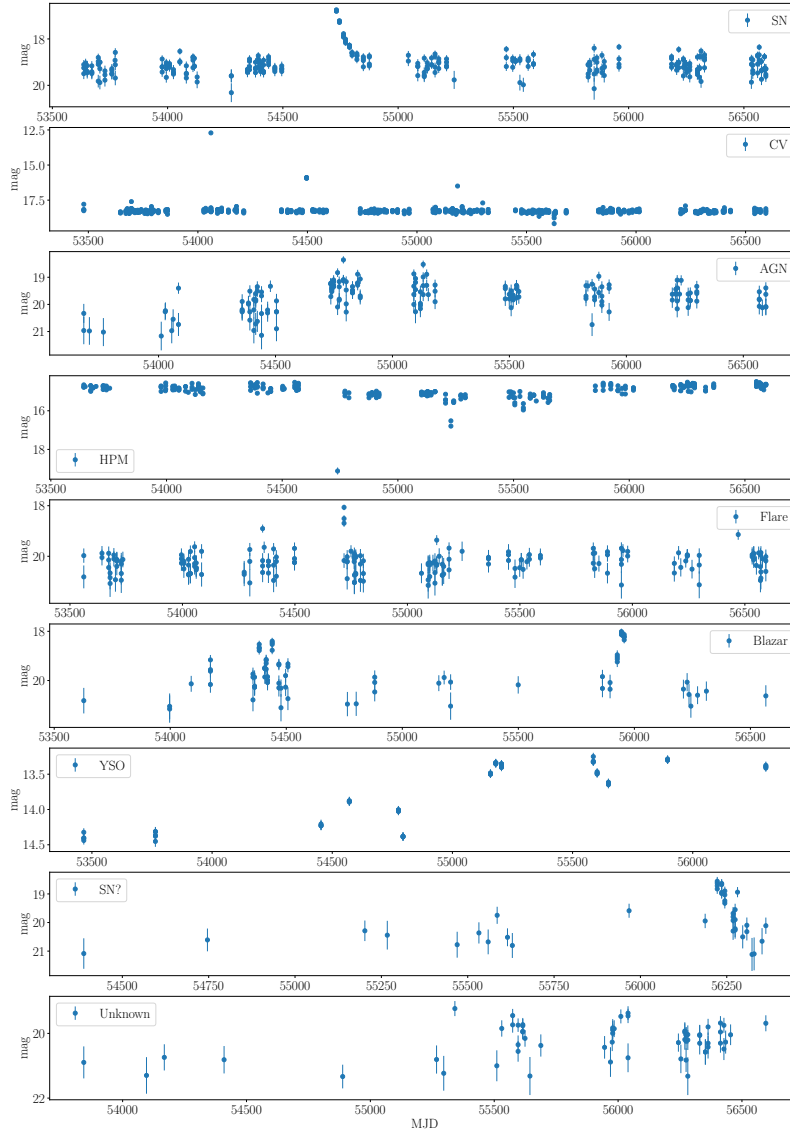


Figure 4. Randomly selected lightcurves for the most represented transient classes as compiled in ATRANCCATA. The class of each sample is within the legend box.

2.1 Classification Tasks

We study two classification tasks on the ATRANCCATA dataset:

- **Binary Classification.** Using a balanced number of events from both classes in order to investigate the capability of distinguishing between Transient and non-transient sources.
- **8-Class Classification.** Using the unbalanced number of objects across classes to perform a classification into the following categories: AGN, Blazar, CV, Flare, HPM, Other, SN and Non-Transient.

We evaluate both tasks using the metrics of a detection problem. For each class in the testing set, we report the maximum F1-Score that is defined as the harmonic mean of precision and recall. We construct Precision-Recall (PR) curves by setting different thresholds on the output probabilities of belonging to each class.

3 REPOSITORY DESCRIPTION

The repository contains the lightcurves and a Jupyter notebook to reproduce some of the Figures and Tables in this paper. The repository can be found in <https://github.com/MachineLearningUniandes/ATRANCCATA>. To date the repository has two main folders:

- **data/lightcurves:** contains three text files in CSV format the transient lightcurves (`transient_lightcurves.csv`), the labels for the transients (`transient_labels.csv`) and the lightcurves for non-transient objects (`nontransient_lightcurves.csv`). The first two files can be linked by unique transient IDs and provided in the CRTS database.
- **nb-explore:** includes a jupyter notebook (`explore_light_curves.ipynb`) with examples on how to read and plot transient and non-transient lightcurves, extract the statistics in Table 1 and prepare the summary

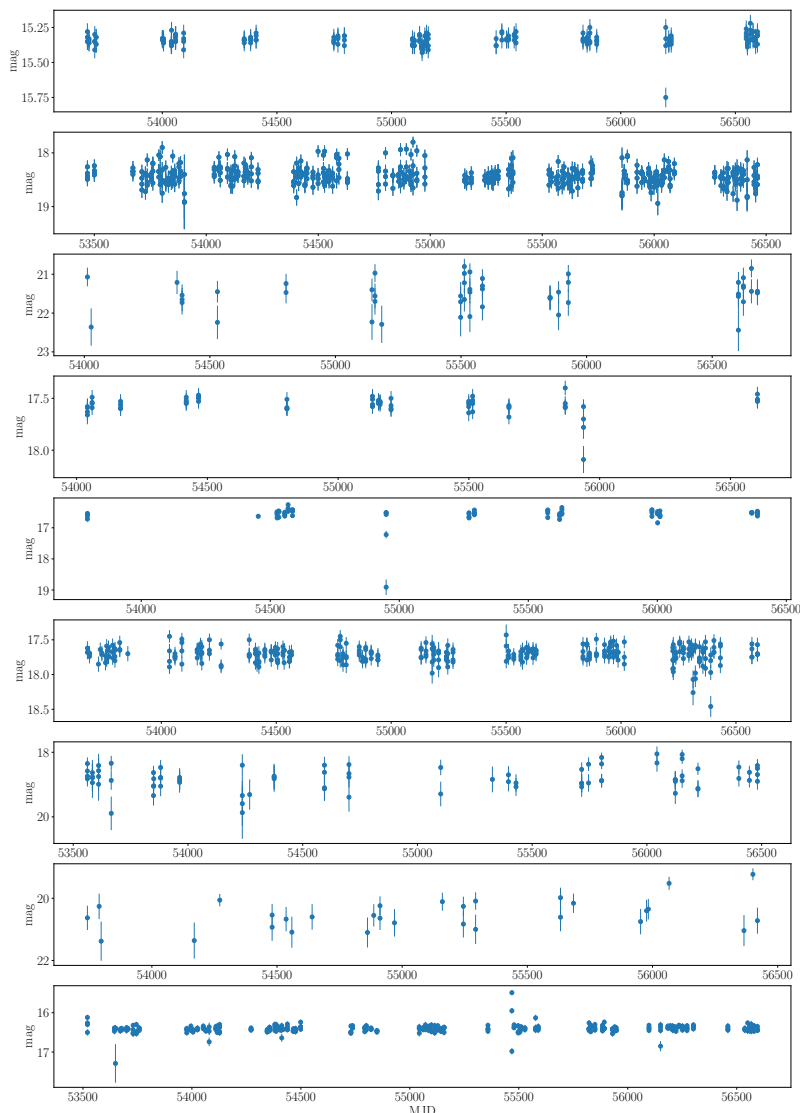


Figure 5. Randomly selected lightcurves for non-transient sources retrieved for ATRANCCATA.

statistics in Figure 3. Additional python files (`features.py`, `helpers.py` and `inputs.py`) allow to read and perform simple operations on the CSV data files.

4 MACHINE LEARNING METHODS

In order to provide baseline algorithms on the ATRANCCATA dataset that can be used as a reference for future work, we apply ML methods to perform different classification tasks. In Figure 6 we show an overview of our method for transient classification. The main steps include data processing, feature extraction and classification.

4.1 Preprocessing and Feature Extraction

We do not input directly the annotated lightcurves to the ML algorithms. We perform a preprocessing stage as follows. First, we discard lightcurves with less than 10 data points

observations as they may not contain enough information to be classified correctly.

Given that the number of lightcurves per class is imbalanced, in order to have the same number of instances for each class, we implement an oversampling step by artificially generating multiple mock lightcurves from an observed one. We generate a slightly different lightcurve from the observed lightcurve and then sample the observed magnitude from a Gaussian distribution centered on the observational apparent magnitude with the magnitude’s error as the standard deviation.

Finally, we compute a standard set of features for each lightcurve. These features are scalars derived from statistical and model-specific fitting techniques. The first features (moment-based, magnitude-based and percentile-based) were formally introduced in Richards et al. (2011), and have been used in other studies (Lochner et al. 2016; D’Isanto et al. 2016). We extend that list to include another set (polynomial fitting-based features). At the end of this

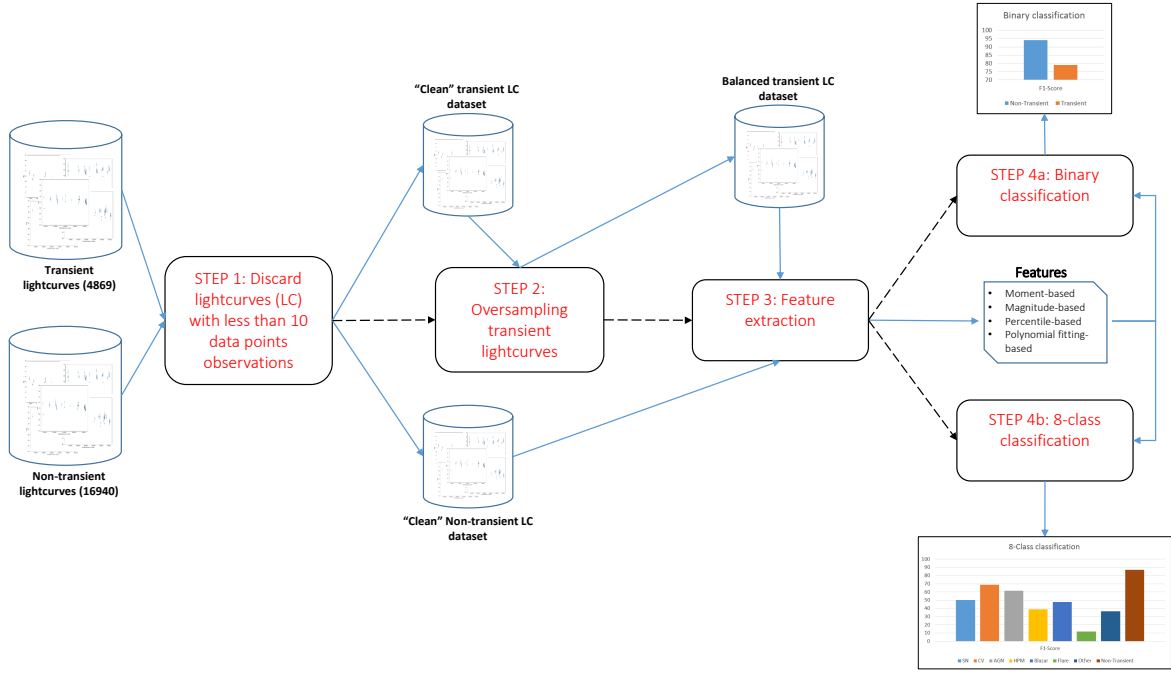


Figure 6. Overview of the Machine Learning process on the ATRANCCATA dataset for the binary and 8-class classification tasks. We take the raw lightcurves as input, preprocess the data (step 1) and balance the classes for the training phase (step 2). The final classification (step 4) follows the feature extraction stage (step 3).

process we normalize the features to have zero mean and unit variance.

These groups of features are:

(i) Moment-based features, which use the magnitude for each lightcurve.

- **beyond1std:** Percentage of observations which are over or under one standard deviation from the weighted average. Each weight is calculated as the inverse of the corresponding observation's photometric error.
- **kurtosis:** The fourth moment of the data distribution.
- **skew:** Skewness. Third moment of the data distribution.
- **sk:** Small sample kurtosis.
- **std::** The standard deviation.
- **stetson_j:** The Welch-Stetson J variability index (Stetson 1996). A robust standard deviation.
- **stetson_k:** The Welch-Stetson K variability index (Stetson 1996). A robust kurtosis measure.

(ii) Features based on the magnitudes.

- **amp:** The difference between the maximum and minimum magnitudes.
- **max_slope:** Maximum absolute slope between two consecutive observations.
- **mad:** The median of the difference between magnitudes and the median magnitude.
- **mbrp:** The percentage of points within 10% of the median magnitude.
- **pst:** Percentage of all pairs of consecutive magnitude measurements that have positive slope.
- **pst_last30:** Percentage of the last 30 pairs of consec-

utive magnitudes that have a positive slope, minus percentage of the last 30 pairs of consecutive magnitudes with a negative slope.

(iii) Percentile-based features, which use the sorted flux distribution for each source. The flux is computed as $F = 10^{0.4\text{mag}}$. We define $F_{n,m}$ as the difference between the m -th and n -th flux percentiles.

- **p_amp:** Largest percentage difference between the absolute maximum magnitude and the median.
- **pdfp:** Ratio between $F_{5,95}$ and the median flux.
- **fpr20:** Ratio $F_{40,60}/F_{5,95}$
- **fpr35:** Ratio $F_{32.5,67.5}/F_{5,95}$
- **fpr50:** Ratio $F_{25,75}/F_{5,95}$
- **fpr65:** Ratio $F_{17.5,82.5}/F_{5,95}$
- **fpr80:** Ratio $F_{10,90}/F_{5,95}$

(iv) Polynomial Fitting-based features, which are the coefficients of multi-level terms in a polynomial curve fitting. This is a new set of features proposed in this paper. **Polyn_Tm** indicates the coefficient of the term of order m in a fit to a polynomial of order n .

- Poly1_T1.
- Poly2_T1.
- Poly2_T2.
- Poly3_T1.
- Poly3_T2.
- Poly3_T3.
- Poly4_T1.
- Poly4_T2.
- Poly4_T3.
- Poly4_T4.

4.2 ML algorithms

We conduct experiments with three widely used families of supervised classification algorithms (Bloom et al. 2012; D’Isanto et al. 2016): Neural Networks (NNs), Random Forests (RFs) and Support Vector Machines (SVMs).

These algorithms are popular in published studies and are efficient for low dimensional feature datasets as is our case. We use sklearn (Pedregosa et al. 2011) Python’s implementation of these algorithms. Details on the inner workings of these machine learning models can be found in Hastie et al. (2016). The hyperparameters explored for each algorithm are the following.

- Neural Networks:
 - Learning Rate: Either constant vs. adaptive.
 - Hidden Layer Sizes: Single Layer with 100 nodes vs. Two layers with 100 nodes each.
 - L2 Penalty (α): 10^{-1} , 10^{-2} , 10^{-3} , 10^{-4} .
 - Activation Function: Logistic vs. Relu.
- Random Forest:
 - Number of Estimators: 200 or 700.
 - Number of features considered: Square Root or the Logarithm base 2 of the total number of features.
- Support Vector Machines:
 - Kernel: Radial Basis Function (RBF).
 - Kernel Coefficient (γ): 10^{-1} , 10^{-2} , 10^{-3} , 10^{-4} , 10^{-5} .
 - Error Penalty (C): 1 vs 10 vs 100 vs 1000.

4.3 Validation

We split the input lightcurves into training and testing. The testing set contains 4869 unique transient lightcurves. We use 2-fold cross-validation during training as evaluation protocol. Moreover, we use grid search during training to test multiple hyperparameter configurations for each one of the proposed methods.

4.4 Results

4.4.1 Binary Classification

The best algorithm in this task is RFs with an average F1-Score of 87.69%. SVMs are the second best-performing model with a F1-Score of 85.36%. Changing the number of features does not significantly affect the score. NNs are ranked third, although their scores are very similar to those of SVMs. The highest achieved score for NNs is 85.03%.

Table 3 shows the confusion matrix of the best performing algorithm and Table 2 summarizes the metrics for transient and non-transient class. These results suggest that in an imbalanced set up, non-transient sources are better classified, while transients are more difficult, showing a difference of about 14 points in F1-Score compared to the non-transient class. This difference in performance could be attributed to the intra-class variation within the transient class because of the different types of transient sources.

Figure 7 displays the most important features for the RFs classifier. The top five inputs for classification are `stetson_j`, `std`, `mad`, `poly1_t1` and `poly2_t1`. The first feature

	Precision	Recall	F1-Score	No. instances
Non-Transient	94.13	94.13	94.13	3798
Transient	79.10	79.10	79.10	1067

Table 2. Precision, Recall and F1-Score for the Binary Classification Task.

	Non-Transient	Transient
Non-Transient	3575	223
Transient	223	844

Table 3. Confusion Matrix for the best performing model in the Binary task. Rows represent prediction and columns the ground truth.

achieved the highest importance of 21%, compared to the following with values in the range 6% - 8%.

4.4.2 Eight-Class Classification

For this task, RFs are again the best classifier. The best F1-Score is 66.05%. NNs are the second best model. Their highest F1-Score is 60.19%, while SVMs are the worst-performing model only achieving an average F1-Score of 57.30%. Table 4 summarizes the results for individual classes and Table 5 presents the confusion matrix for the RFs.

The two classes with highest F1-Score are non-transient (87.12%) and CV (68.77%). The recall decreases for the non-transient class in comparison to the binary experiment, meaning that the algorithm misclassified some instances that belong to non-transient class among transient classes. However, transient sources are not commonly confused with non-transient ones. The worst performing classes are Flare, Other and HPM, with F1-Scores in the range 11% - 40%. It is worth noting that the less frequent classes present a lower performance, such as Flare and HPM. Even though the most frequent classes are more easily identified, the "other" type class has a low F1-score due to the different nature of sources that were assigned to this category.

SN is the class with which most other class instances are incorrectly classified. Moreover, Flares have about 50% of the test samples classified as non-transients, AGNs have about 20% of their samples classified as Other, and Blazars and Other had most of its samples classified as AGN. Additionally, most incorrectly classified AGNs (~20.5%) are identified as Other, and most Blazar instances are incorrectly categorized as either SN or AGN.

Figure 8 displays the feature importance ranking. This list ranks first `stetson_j` with an 8% importance, followed by `amp`, `sk`, `std`, `mad`, with values around 6%.

5 CONCLUSIONS

The scope of forthcoming large astronomical synoptic surveys motivates the development and exploration of automatized ways to detect transient sources. In turn, this need prompts the compilation of publicly available databases to

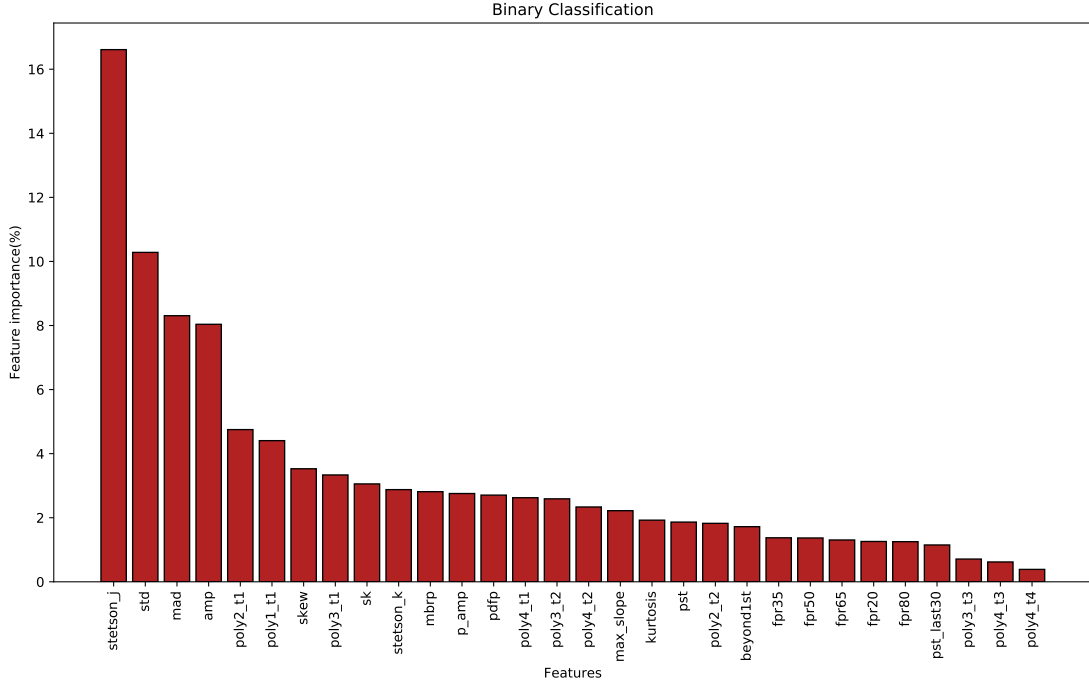


Figure 7. Feature importance rank for the best Random Forest classifier for the Binary classification task. Feature importance is represented with percentages.

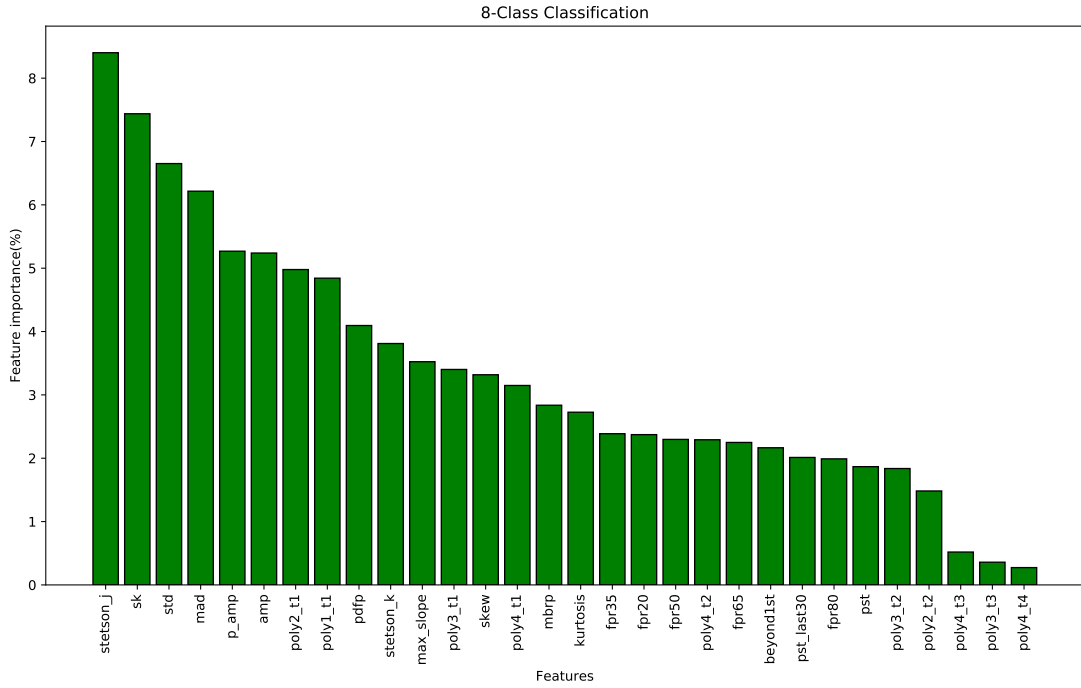


Figure 8. Feature importance rank for the best Random Forest classifier for the best 8-Class classification task. Feature importance is represented with percentages.

	Precision	Recall	F1-Score	No. instances
SN	48.82	51.39	50.07	323
CV	66.96	70.69	68.77	215
AGN	48.14	85.84	61.69	106
HPM	25.19	86.84	39.05	76
Blazar	46.77	49.15	47.93	59
Flare	7.00	41.17	11.96	51
Other	31.11	44.01	36.46	234
Non-Transient	96.06	79.69	87.12	3798
avg/total	46.25	63.59	50.38	4862

Table 4. Precision, Recall and F1-Score for the 8-Class Classification Task.

	SN	CV	AGN	HPM	Blazar	Flare	Other	Non-Tr.
SN	166	25	0	0	7	5	40	97
CV	17	152	0	1	5	3	12	37
AGN	1	2	91	0	10	1	35	49
HPM	5	0	0	66	0	0	5	186
Blazar	8	13	4	0	29	0	6	2
Flare	16	5	0	0	3	21	4	251
Other	53	12	7	1	3	3	103	149
Non-Tr.	57	6	4	8	2	18	29	3027

Table 5. Confusion Matrix for the best performing model in the 8-class task. The classes follow the abbreviations in Table 4. Rows represent the predictions, and columns the ground truth.

train and test new algorithms. In this paper we presented the results of such a compilation based on data from the Catalina Real-Time Transient Survey. The data-set compiles 4869 transient and 16940 non-transient lightcurves. The dataset is publicly available at <https://github.com/MachineLearningUnianides/ATRANCCATA>.

We illustrated how to use this database by extracting characteristic features to use them as input to train three different machine learning algorithms (Random Forests, Neural Networks and Support Vector Machines) for classification tasks. The features extracted from lightcurves were either statistical descriptors of the observations, or polynomial curve fitting coefficients applied to the lightcurves. Overall, the best classifier for all tasks was the Random Forest. In this model the most important feature was always `stetson_j`, i.e. a robust estimate for the standard deviation.

In a second paper we will present another reference dataset for astronomical transient event recognition based on images of the CRTS. The corresponding tests will use state-of-the art deep learning techniques for transient classification.

ACKNOWLEDGEMENTS

We thank Andrew Drake for sharing with us the CRTS Transient dataset used in this project. We acknowledge funding from Universidad de los Andes in the call for project finalization. We also thank contributors and collaborators of the SciKit-Learn, Jupiter Notebooks and Pandas' Python libraries.

CRTS and CSDR2 are supported by the U.S. National Science Foundation under grant NSF grants AST-1313422, AST-1413600, and AST-1518308. The CSS survey is funded by the National Aeronautics and Space Administration under Grant No. NNG05GF22G issued through the Science Mission Directorate Near-Earth Objects Observations Program.

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