A Seminar Report
On

Time-Frequency Analysis Of Accelerogram

Submitted in partial fulfillment of the award of

Master of Engineering

Degree

By

Satyajit Maitra

MTech II Year

(Seismic Vulnerability and Risk Assessment/2019-21)

Registration Number: 19553006

Department Of Earthquake Engineering,

Indian Institute of Technology, Roorkee, Uttrakhand, Pin: 267667, India

Submitted To :

Dr. M. L. Sharma

Professor, Department of Earthquake Engineering

Dean Finance and Planning, IIT Roorkee

Ex-Head, Department of Earthquake Engineering

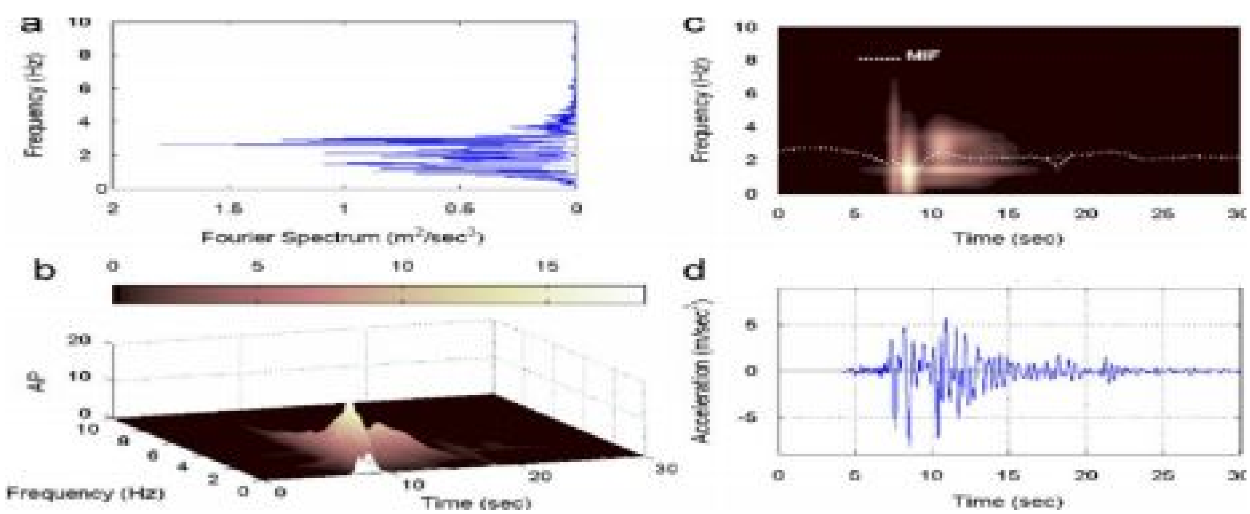
ABSTRACT

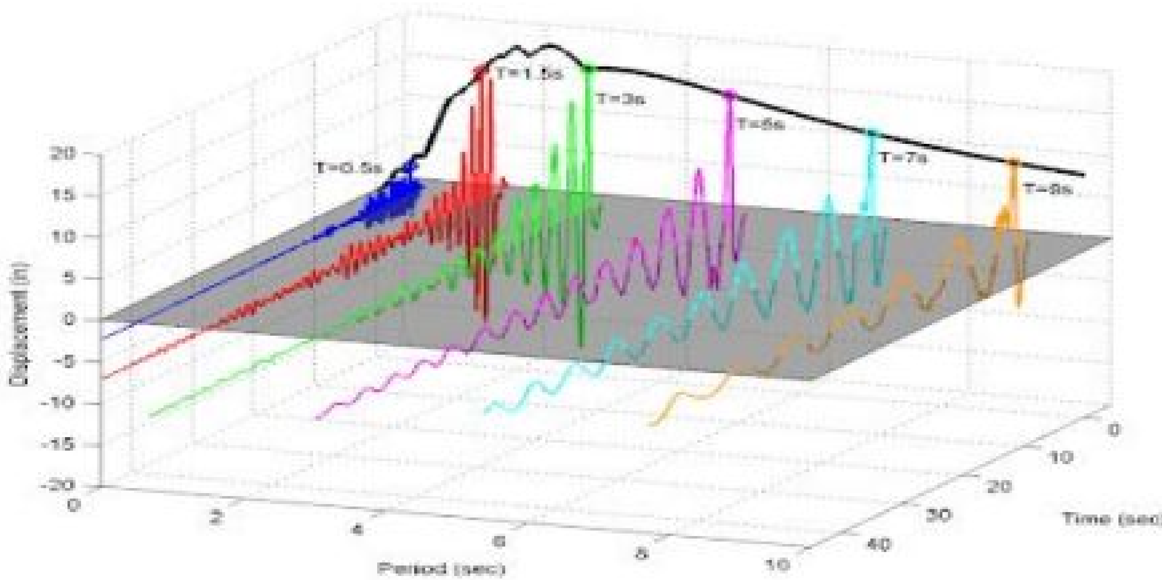
In signal processing, time-frequency analysis consists of studying a signal in time and frequency domains simultaneously. Rather than viewing a 1-dimensional signal (a function, real or complex-valued, whose domain is the real line) and some transform (another function whose domain is the real line, obtained from the original via some transform), time-frequency analysis studies a two-dimensional signal – a function whose domain is the two-dimensional real plane, obtained from the signal via a time-frequency transform. These high-level representations such as time-frequency maps convey a wealth of useful information, but they involve a large number of parameters that make statistical investigations of many signals difficult at present. In this paper, we will describe a method that performs a drastic reduction in the complexity of time-frequency representations through modeling of the maps by elementary functions, Artificial Intelligence, and Machine learning. The method is validated on artificial signals and subsequently applied to signals recorded at original stations. We will show different methods of doing Time-frequency analysis using techniques like FFT(Fast Fourier Transform), wavelet methods, and how applying Artificial neural networks, deep learning can significantly reduce the complexity of time-frequency analysis with more return in result. We will try to validate the advanced technological improvement in this field to show the potential and promise of technology in this area.

INTRODUCTION

In signal processing, time-frequency analysis consists of studying a signal in time and frequency domains simultaneously. Rather than viewing a 1-dimensional signal (a function, real or complex-valued, whose domain is the real line) and some transform (another function whose domain is the real line, obtained from the original via some transform), time-frequency analysis studies a two-dimensional signal – a function whose domain is the two-dimensional real plane, obtained from the signal via a time-frequency transform. The mathematical motivation for this study is that functions and their transform representation are often tightly connected, and they can be understood better by studying them jointly, as a two-dimensional object, rather than separately.

The practical motivation for time-frequency analysis is that classical Fourier analysis assumes that signals are infinite in time or periodic, while many signals in practice are of short duration, and change substantially over their duration. For example, accelerograph instruments do not produce infinite duration signals, but instead begin with an attack, then gradually decay. This is poorly represented by traditional methods, which motivates time-frequency analysis. For say Response Spectrum Which hangs with Earthquake Engineers most of the time and is a great source of Information about ground motion parameters but not the only solution upon which you can depend.





In this paper we will do our Experiment with simulated data from two different methods and we kept application of the methods on real data for the final semester project.

So let's get started with the project.

Instruments:

During an earthquake, vibrations caused by the breakage of rock along a fault zone radiate outward from the point of rupture. The instrument used to record and measure these vibrations is called a seismograph.

And for recording strong vibrations we are going to use Accelerometer.

Seismograph: A seismograph, or seismometer, is an instrument used to detect and record earthquakes. Generally, it consists of a mass attached to a fixed base. During an earthquake, the base moves and the mass does not. The motion of the base with respect to the mass is commonly transformed into an electrical voltage. The electrical voltage is recorded on paper, magnetic tape, or another recording medium. This record is proportional to the motion of the seismometer mass relative to the earth, but it can be mathematically converted to a record of the absolute motion of the ground. Seismograph generally refers to the seismometer and its recording device as a single unit.

Seismographs are used to determine:

- Magnitude: the size of the earthquake
- **Depth**: how deep the earthquake was
- **Location**: where the earthquake occurred

Some seismometers can measure motions with frequencies from 500 Hz to 0.00118 Hz (1/500 = 0.002 seconds per cycle, to 1/0.00118 = 850 seconds per cycle).Seismograph range:(5 X 10^6 sample/day) which equals to 5 MB per day . So for 200 stations it is roughly 1 GB per day.

Accelerograph: Another important class of seismometers was developed for recording large amplitude vibrations that are common within a few tens of kilometers of large earthquakes - these are called strong-motion seismometers. Strong-motion instruments were designed to record the high accelerations that are particularly important for designing buildings and other structures. An accelerograph can be referred to as a strong-motion instrument or seismograph, or simply an earthquake accelerometer. They are usually constructed as a self-contained box, which previously included a paper or film recorder (an analogue instrument) but now they often record directly on digital media and then the data is transmitted via the Internet.

Strong motion sensors measure large amplitude seismic signals and are usually accelerometers. Strong motion accelerometers can measure up to 3.5 g with a system noise level less than 1µg/√Hz. Weak motion sensors can measure very low amplitude seismic signals with a noise level of less than 1ng/√Hz.

Range of Accelerometer: 1 Hz to 6 kHz

A Comparison Between Accelerogram and Seismogram:

What is the difference between a seismograph and an accelerograph? Which one is used for structural monitoring and which is for earthquake detection?

A seismograph is a generic term used to describe a recording device that detects ground motion due to earthquakes. Typically this will comprise a recorder and a seismometer, which is a sensor that detects the velocity of the ground. Seismometers are usually very sensitive and will easily detect a typical quarry blast at a range of 100km. Seismometers should not be confused with geophones, which also detect ground velocity but are typically much less sensitive and are used for close range blast monitoring and surveying.

An accelerograph is a recorder that uses an accelerometer, which as you can tell from the name detects the acceleration of the ground. Accelerometers are much less sensitive than seismometers, but have a much greater range, detecting ±2g or more of ground acceleration (things start flying off the ground at 1g, when gravity is overcome). By comparison a seismometer will clip at full scale if you tap it too hard with your finger.

So, seismometers are good for detecting very small levels of ground motion (from very small or very distant events), and accelerometers are good at recording strong ground motion that is potentially damaging at the recording location. We will often install an earthquake recording station using both types of sensor to get the best of both worlds.

SMAs, or strong motion accelerographs, are usually all that is required for monitoring the response of a structure during an earthquake, whether this a building, bridge, dam, power station, or any other critical infrastructure that could be affected by a large earthquake. Signals that are too weak to be clearly visible on an accelerograph will generally not be of any concern to the structural integrity of the asset.

The SRC has used various types of SMAs over the years. Those with MEMS accelerometers use tiny sensors like those found in cars to trigger airbags on impact, but are much more sensitive. They are less sensitive than dedicated earthquake accelerometers, but are still sufficient to record acceleration from significant earthquakes as the lower sensitivity limit is still well below damage thresholds. For monitoring the natural frequency and modal response of structures to earthquakes, more sensitive earthquake accelerometers are required.

Let's Understand How Data Is Being Recorded?

Almost all observatory-grade seismic recorders use 24-bit or 32-bit analogue to digital converters (ADCs), although the useful range of currently available 32-bit ADCs is limited to the lower 24 bits – the other 8 bits are digital noise.

24-bits equates to 16,777,216 counts of recording range. If the average (RMS) noise level is just one count out of this range, then the dynamic range of recording is defined as:
$20\log10(16777216/1) = 144.5\text{dB}$

The USGS ANSS guidelines(1) require that the dynamic range of a data acquisition unit should be ≥24-bits, based on the RMS noise compared to the RMS of the zero to peak signal of a sine wave, which would be: $20\log10(8388608/1/\sqrt{2}) = 135.5\text{dB}$ Whichever way that dynamic range is defined, there is still only 1 count of noise, and the full scale range is still ±8,388,608 counts.

We will often see very large dynamic range numbers quoted for digitisers, even when based on the ANSS method. This is possible when the RMS value of the noise is less than one count. It is possible to have less than one integer count of noise because the RMS value is an average over a number of samples. A small reduction in the fraction of a count has a huge impact on the dynamic range number, but in practice it means very little. For example:
$20\log10(8388608/0.5/\sqrt{2}) = 141.5\text{dB}$

20log10(8388608/0.1/√2) = 155.5dB Apart from the digitiser noise levels we need to consider sensor noise levels. A typical ADC input range is 40 Volts peak-to-peak, so a single count in a 24-bit range is equivalent to 2.3µV. One of the first things to look at is the electronic noise level of the sensor is over the bandwidth of interest. Noise increases with frequency, so another way sensors and recorders can be quoted with high dynamic range figures is by looking at a very low frequency band or recording data at a low sample rate. In all sensors testing a 6-channel or 12-channel Kelunji EchoPro seismic recorder with 24-bit ADCs was used. Data was recorded at 100 samples per second (sps), giving a bandwidth of DC to 40Hz (after FIR filtering). The ANSS requirement for digitisers recording frequencies up to 30Hz is 123.4dB, and the EchoPro has a dynamic range of 131.5dB at 100sps and 123.3dB at 500sps.

There are measuring instruments available now that are used to lower the noise level and used in prediction of Earthquake directly developed in Rice University,Huston,Texas.

Application of Time-frequency in seismic Engineering:

- One major benefit of applying a time-frequency transform to a signal is discovering the pattern of frequency changes.
- Once the pattern is identified we can classify the signal pattern. For Example a pattern of changing frequency might indicate the entry or exit of seismic vibration in localized context.
- Another important use of time-frequency analysis is to reduce random noise in noise-corrupted signals. Like noise generated from a recording machine.
- You also can use time-frequency analysis to determine if a signal has distinct time-frequency components and isolate those components for further analysis.
- Seismological signal processing, such as detection of soil liquefaction.
- Seismic prospecting for oil and gas has undergone a digital revolution during the past decade. Most stages of the exploration process have been affected: the acquisition of data, the reduction of these data in preparation for signal processing, the design of digital

filters to detect **primary echoes (reflections) from buried interfaces**, and the development of technology to extract from these detected signals information on the geometry and physical properties of the subsurface. The seismic reflection is generally weak, and it must be strengthened by the use of signal summing (stacking) procedures.

- Indicate presence of Natural oil and gas.
- Very Much beneficial for Earthquake prediction.

Data Generation:

For generating Artificial Earthquake acceleration data I have used many approaches but choose to work with one model.

1. Kanai–Tajimi Model

Over the past few decades, primarily two approaches have been adopted in evaluation of the strong earthquake ground motions for a seismic design of the engineered structures.One is to use the recorded strong motions. In some cases the recorded motions are modified to better represent local soil conditions. The second is to generate strong ground motions by combining an appropriate seismological model with knowledge of seismicity and geology of the site. While recording cannot be possible for most of the area using some property like w^{-2}.(w is the frequency of signal).,used to generate small earthquakes. For large earthquakes we use extended sources with finite dimensions.

Some methods like ground motion using a stochastic point source model. The equation is shared below for reference.

$$a(t) = \sqrt{2}\sum_{j=1}^{N_\omega}\sqrt{2S_{aa}(t,\omega_j)\Delta\omega}\cos(\omega_j t + \phi_j);\quad \omega_j = j\Delta\omega;\quad \Delta\omega = \frac{\omega_u}{N_\omega};\quad j = 1,2,\ldots,N_\omega$$

Here the phase is random with $(0,2\pi)$. This model is asymptomatic Gaussian as N_w becomes large due to the central limit theorem. The evolutionary power spectrum takes the following form

$$S_{aa}(t,\omega) = \frac{1}{2\pi}|W(t,\omega)|^2|A(\omega)|^2; \quad |A(\omega)| = C A_S(\omega) A_D(\omega) A_A(\omega)$$

Here the scaling factors are as follows,

$$C = \frac{R(\theta,\varphi)FV}{4\pi\rho C_S^3}; \quad A_S(\omega) = \frac{M_0\omega^2}{1+(\omega/\omega_c)^2}$$

$$A_D(\omega) = \frac{1}{1+(\omega/\omega_{max})^n}\frac{1}{R}\exp(-\frac{\omega R}{2QC_S}); \quad A_A(\omega) = \frac{\sqrt{1+4h_g^2(\frac{\omega}{\omega_g})^2}}{\sqrt{(1-(\frac{\omega}{\omega_g})^2)^2+4h_g^2(\frac{\omega}{\omega_g})^2}}$$

Here the local soil amplification factor is taken in accordance with the (Kanai,1957; Tajimi 1960;) model.

The moduleting function in Eq becomes:

$$|W(t,\omega)| = \frac{e^{-(c_1\omega+c_2)t} - e^{-(c_3\omega+c_4)t}}{e^{-(c_1\omega+c_2)t^*} - e^{-(c_3\omega+c_4)t^*}}; \quad t^* = \frac{\ln(c_1\omega+c_2) - \ln(c_3\omega+c_4)}{(c_1\omega+c_2)-(c_3\omega+c_4)}$$

where

$$c_k = [(a_{k1}\Delta - a_{k2})M + a_{k3} - a_{k4}\Delta] \times a_{k5}; \quad a_{11} = a_{31} = 6.0; a_{12} = a_{32} = 1600.0;$$
$$a_{13} = 14000.0; a_{14} = a_{34} = 54.0; a_{15} = a_{35} = 10^{-6}; a_{21} = a_{41} = 4.0; a_{22} = a_{42} = 1000.0;$$
$$a_{23} = 9500.0; a_{24} = a_{44} = 36.0; a_{25} = a_{45} = 10^{-4}; a_{33} = 15000.0; a_{43} = 9510.0$$

Now in case of ground motion from an extended fault (large ground motion) using **simulation method** which is formulated from Empirical Green’s function method suggested by

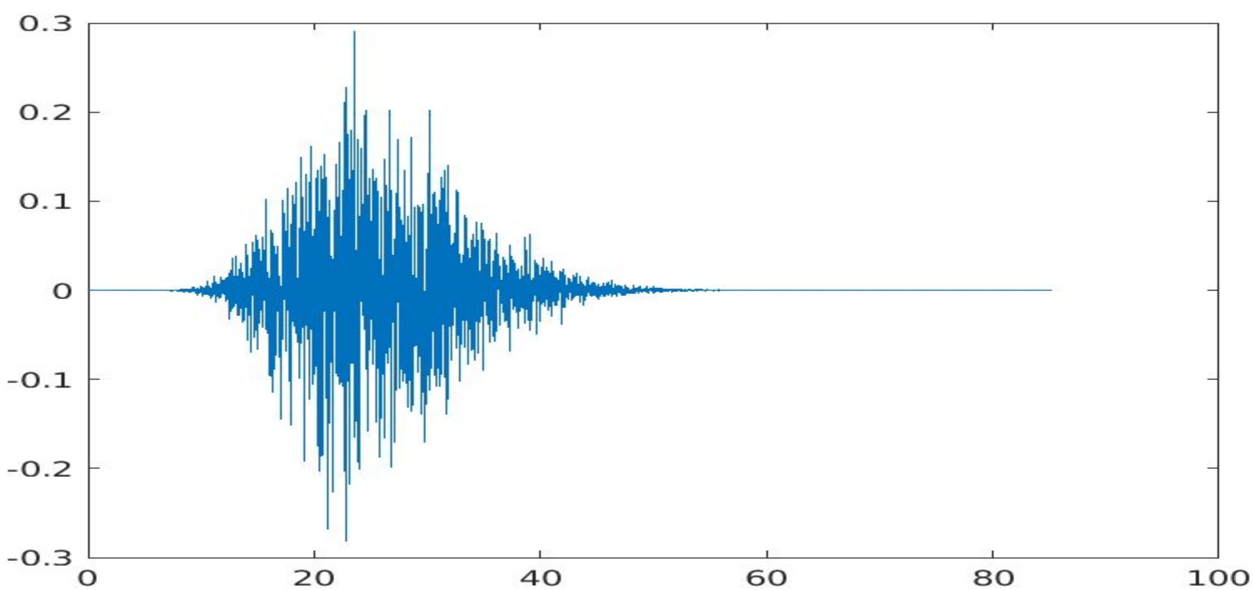
Hartzell(1978). This method is proposed by Irikura (1983) based on the concept of Elastodynamics. The far field displacement u(x,t) in homogeneous,isotropic,and layered medium can be expressed as follows

$$\boldsymbol{u}(\boldsymbol{x},t)=\sum_{m=1}^{N_L}\sum_{n=1}^{N_W}\int_{\xi_m}^{\xi_m+\Delta L}\int_{\eta_n}^{\eta_n+\Delta W}\dot{D}(\xi_m,\eta_n,t-\tau_{mn})*\boldsymbol{G}(\boldsymbol{x},\xi_m,\eta_n,t-t_{mn})d\xi d\eta$$

and the Fourier transform is

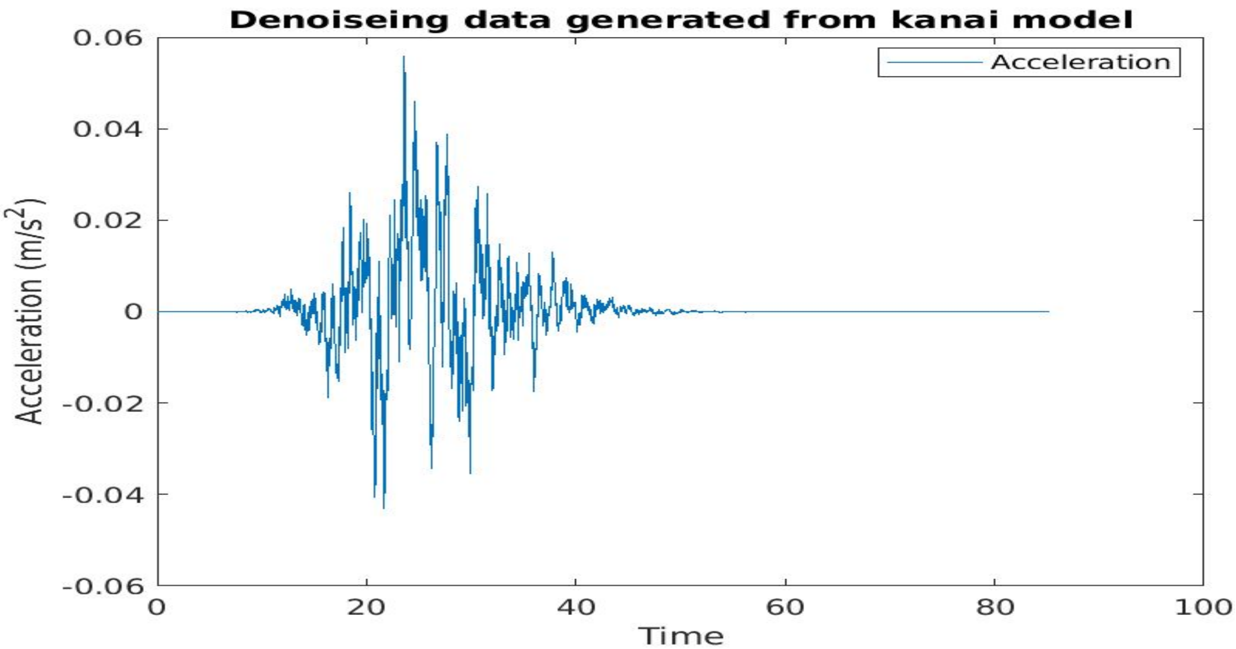
$$\boldsymbol{u}(\boldsymbol{x},\omega)=\sum_{m=1}^{N_L}\sum_{n=1}^{N_W}\int_{\xi_m}^{\xi_m+\Delta L}\int_{\eta_n}^{\eta_n+\Delta W}\dot{D}(\xi_m,\eta_n,\omega)\boldsymbol{G}(\boldsymbol{x},\xi_m,\eta_n,\omega)e^{-i\omega(\tau_{mn}+t_{mn})}d\xi d\eta$$

With kanai-Tajimi model we got an earthquake response like this

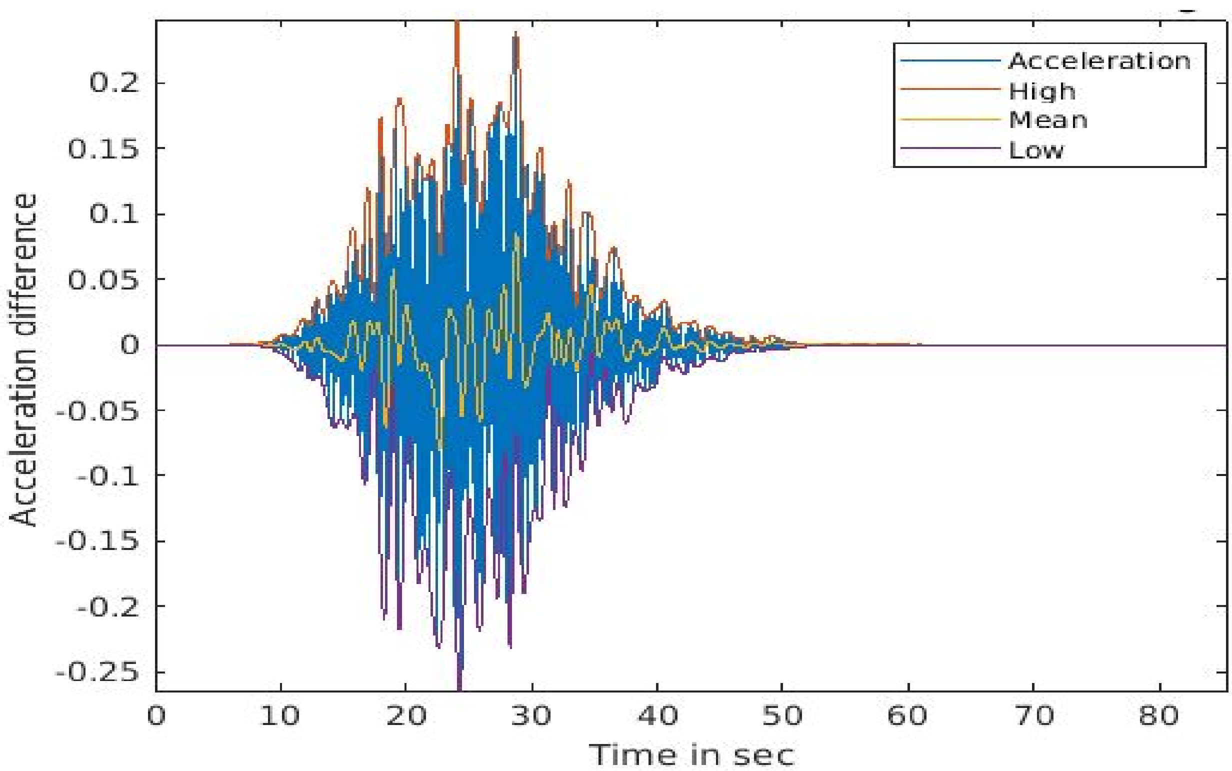


Here the x-axis represents time and Y-axis represents the acceleration value.

If we do some weighted sample of the data we got this

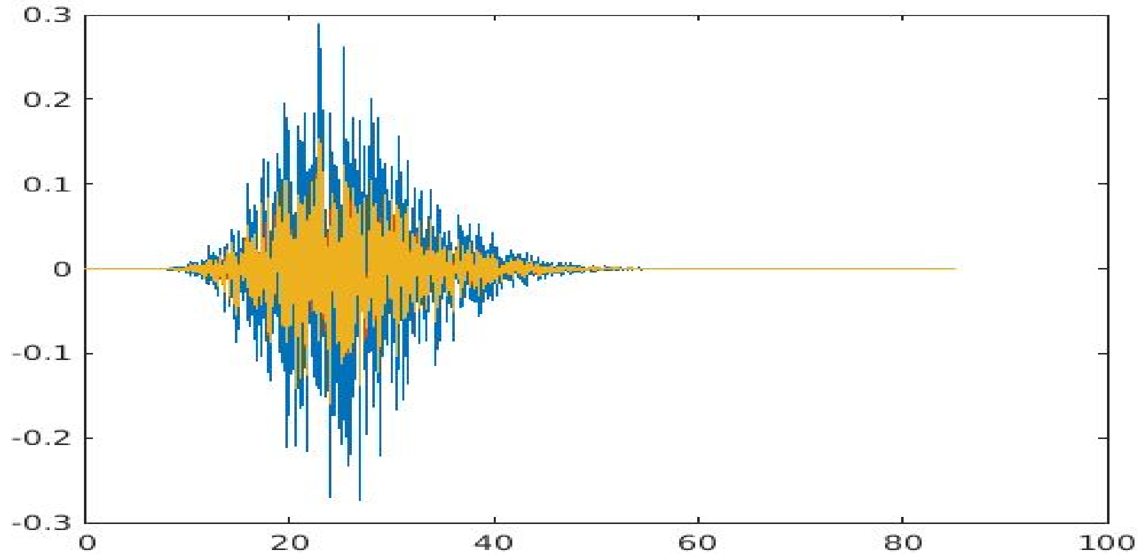


Clearly it eliminates the peak rear signals by assuming noise and gives us the low values which are frequently generated but as Earthquake is a rear event this diagram has very little importance to us.

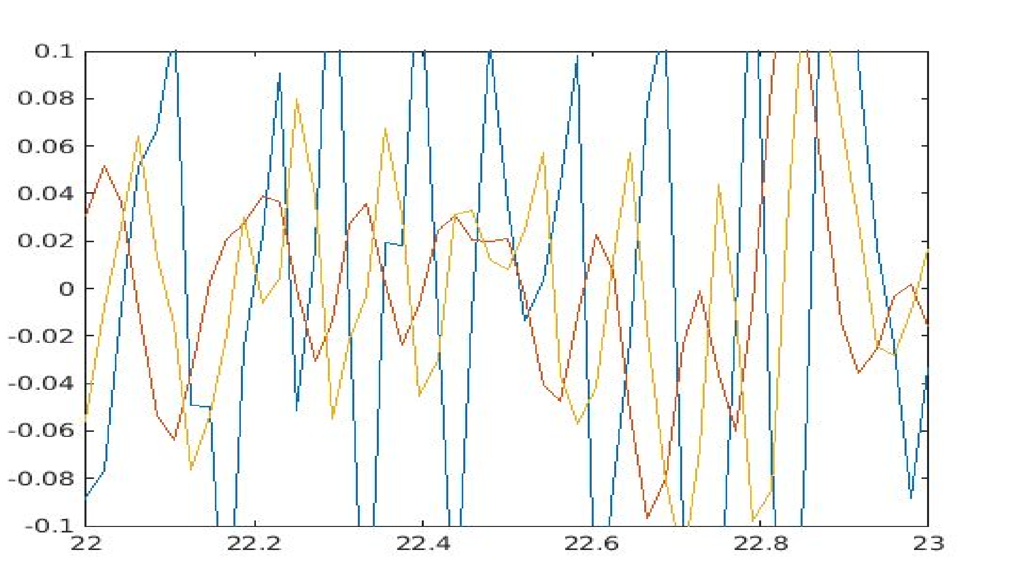


The graph above shows the high low and mean value which capture all the required information about the Earthquake.

So by applying some different smoothing techniques we got this



On zooming into we got this

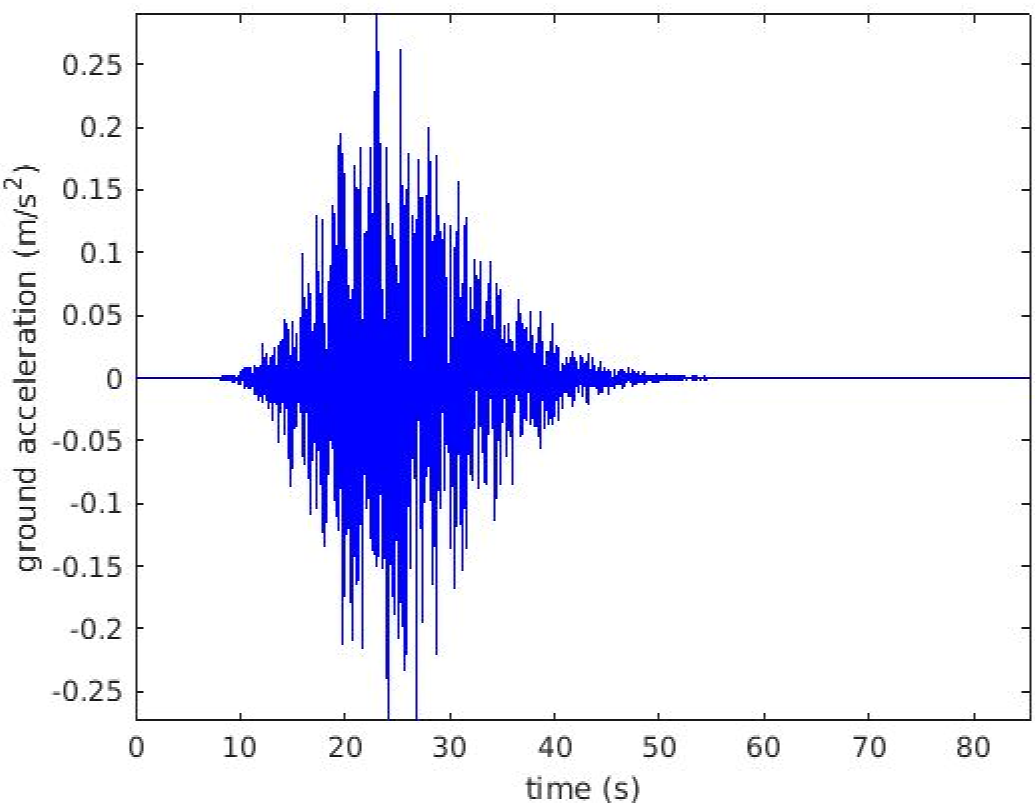


Here blue is the original signal, yellow is the exponential and orange is the binomial smoothing . So based on the evidence we can say for our data exponential smoothing fits the original data points way better than any other smoothing methods.

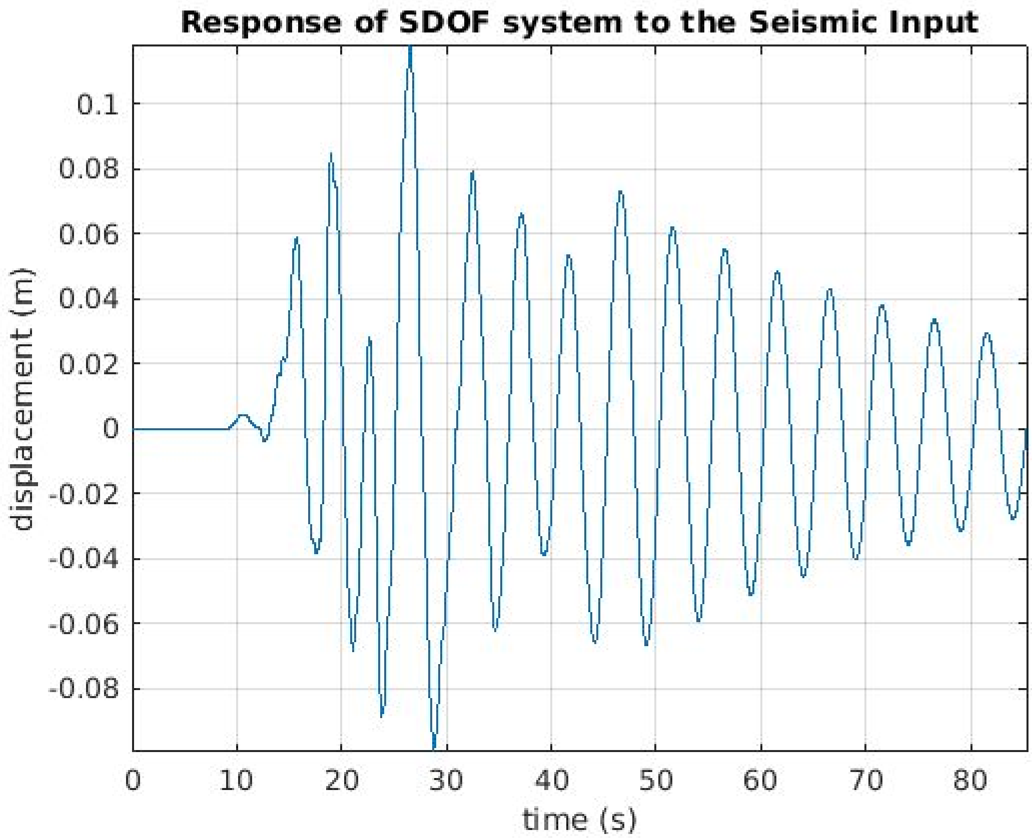
A note on modifying the response Kanai-Tajimi model, The Hu spectral model, a modified Kanai-Tajimi spectral model for the stationary stochastic process of earthquake ground motion, is analyzed. According to the Hu spectral model, the spectral density function of the Kanai-Tajimi spectrum is modified only during the low frequency range and in good accordance with the Kanai-Tajimi spectrum during the moderate and high frequencies. It is proved that the earthquake-induced ground acceleration process with the Hu spectrum is essentially the result of which a filtered Gaussian white noise process on the rock is filtered by the overlying soil represented by a linear single-freedom-degree system,namely it is a twice filtered white noise process. This shows that the Hu spectral model is not only concise in the mathematical expression but also distinct in physical meaning and reasonable in practice. Furthermore, the low-frequency control factor which determines the low frequency contents in Hu spectral model is investigated and evaluated from the observation data of earthquake ground motion.

Response Spectrum:

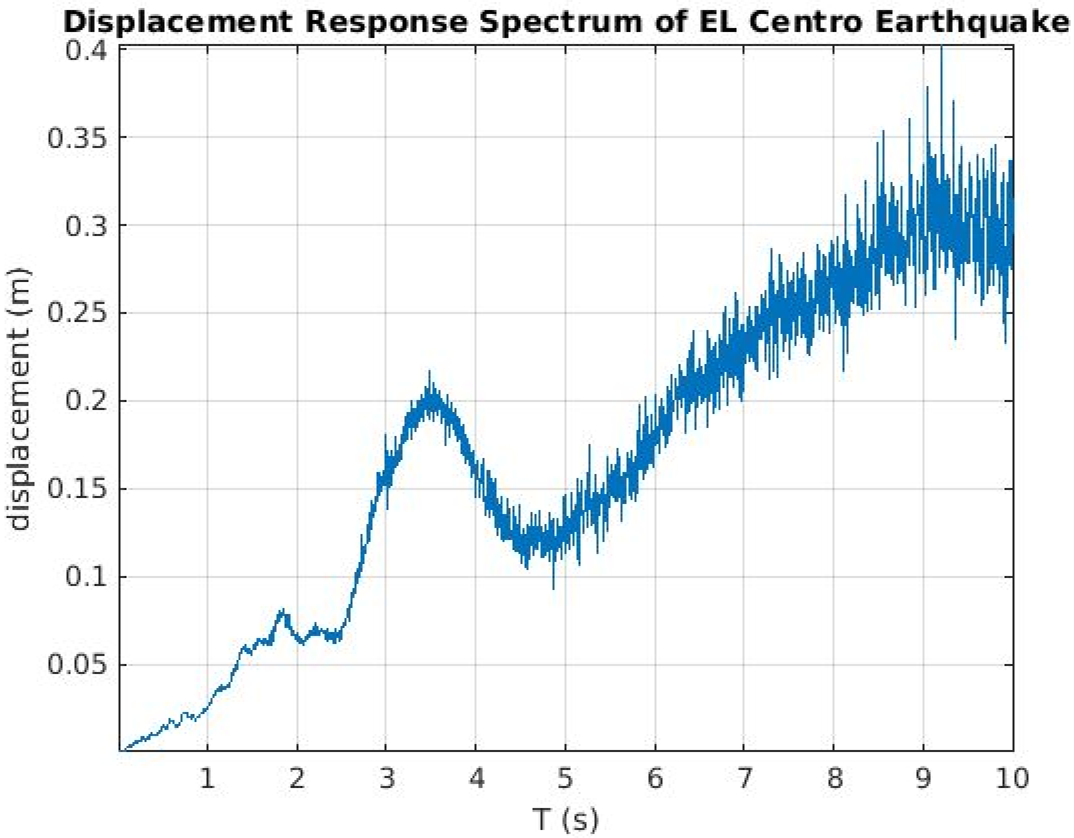
For just giving a brief description I am sharing a response spectrum plotted using Matlab.
The Earthquake in time domain



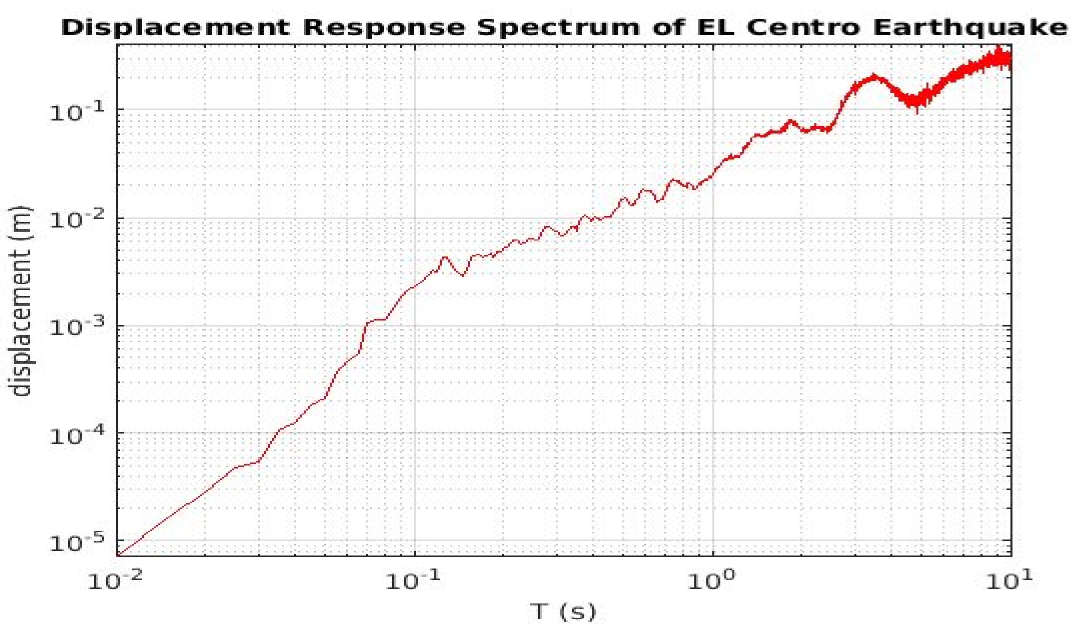
The response of a standard system wrt the Earthquake is

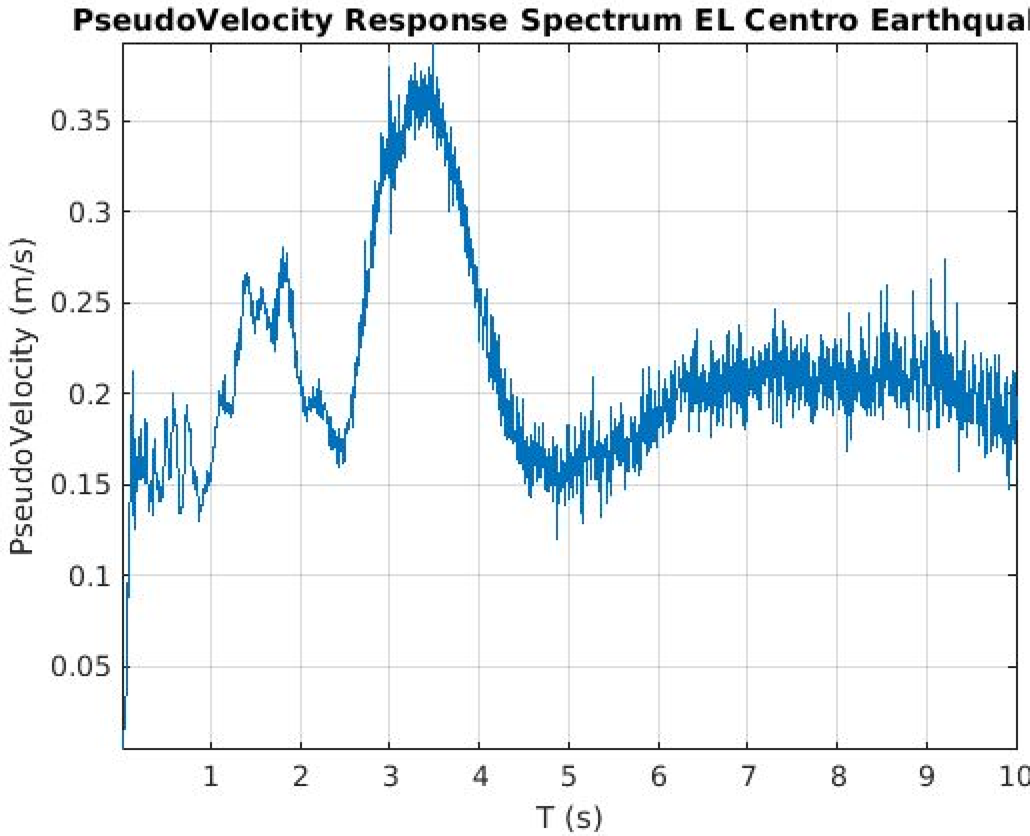


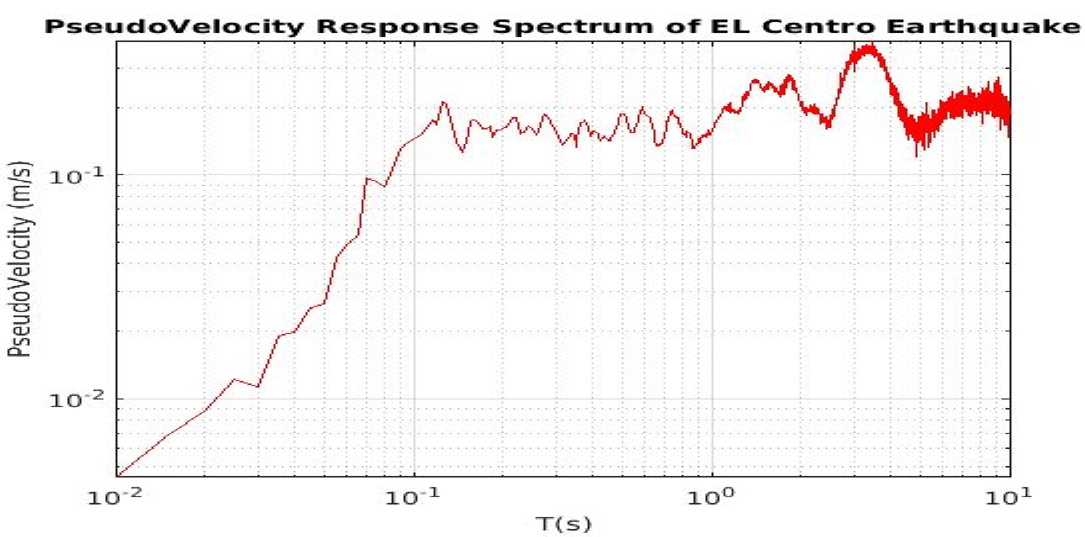
The Displacement response of SDOF system is

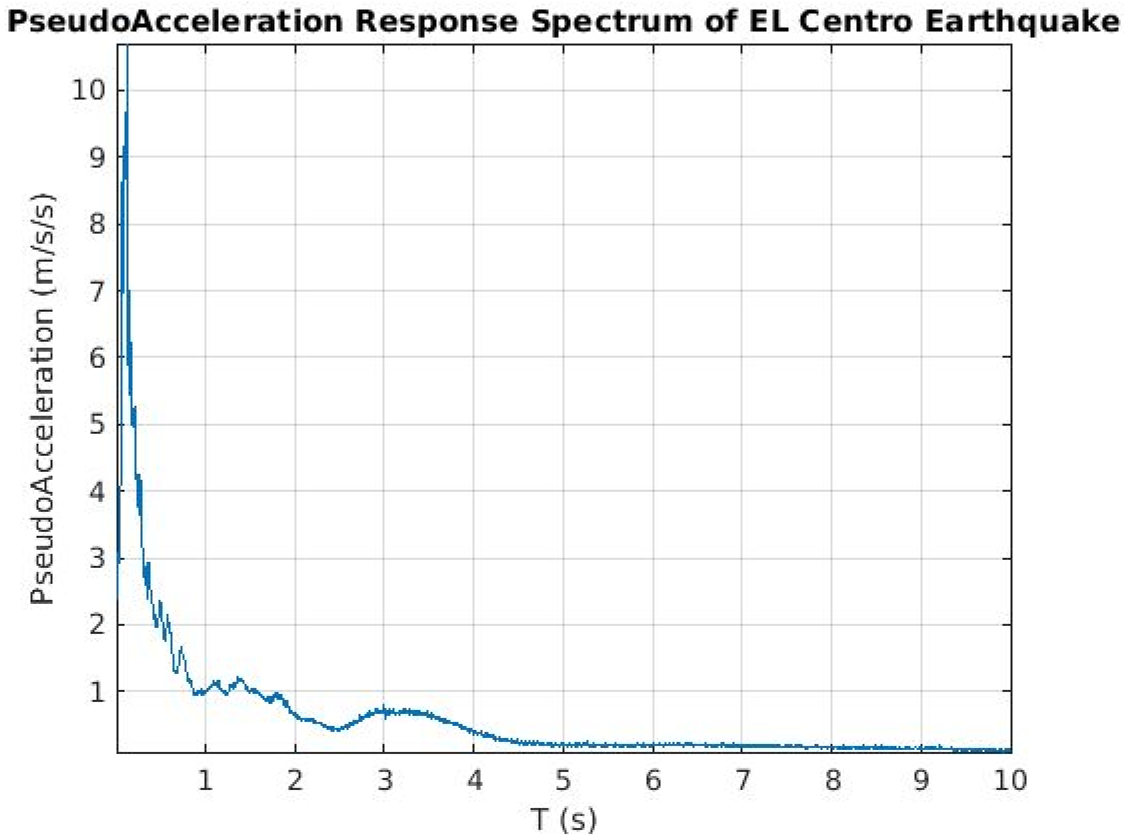


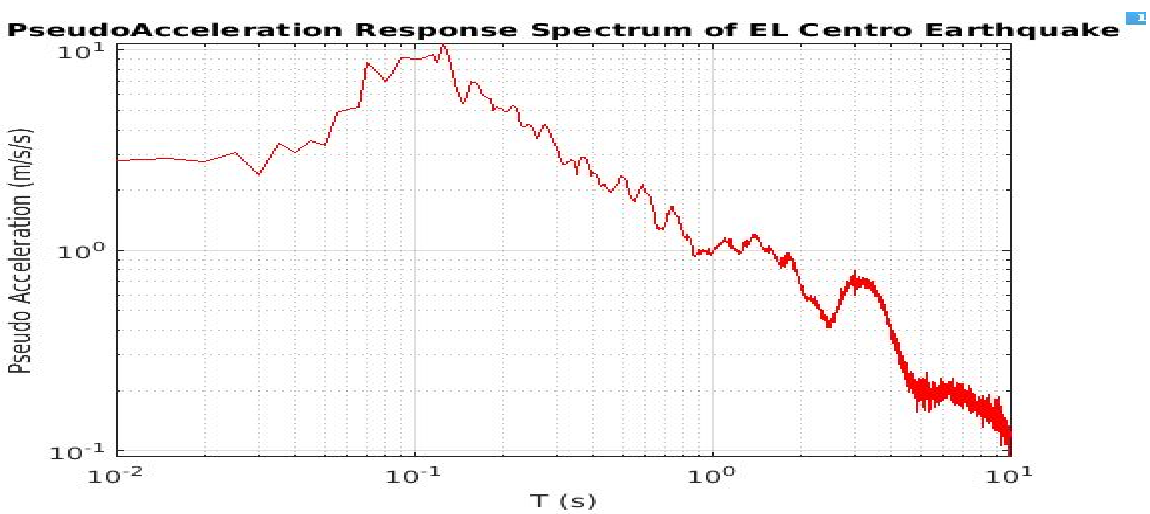
In log-log scale we got




PseudoVelocity Response Spectrum EL Centro Earthquake
PseudoVelocity (m/s)
T (s)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
1
2
3
4
5
6
7
8
9
10

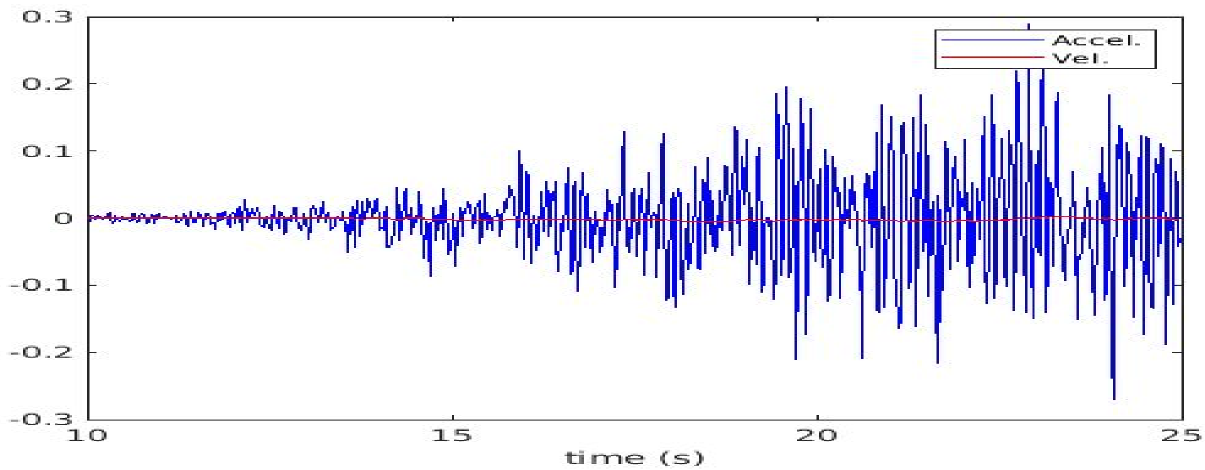

PseudoVelocity Response Spectrum of EL Centro Earthquake
PseudoVelocity (m/s)
T(s)
10^{-1}
10^{-2}
10^{-2}
10^{-1}
10^{0}
10^{1}





Analysis Of The Earthquake:

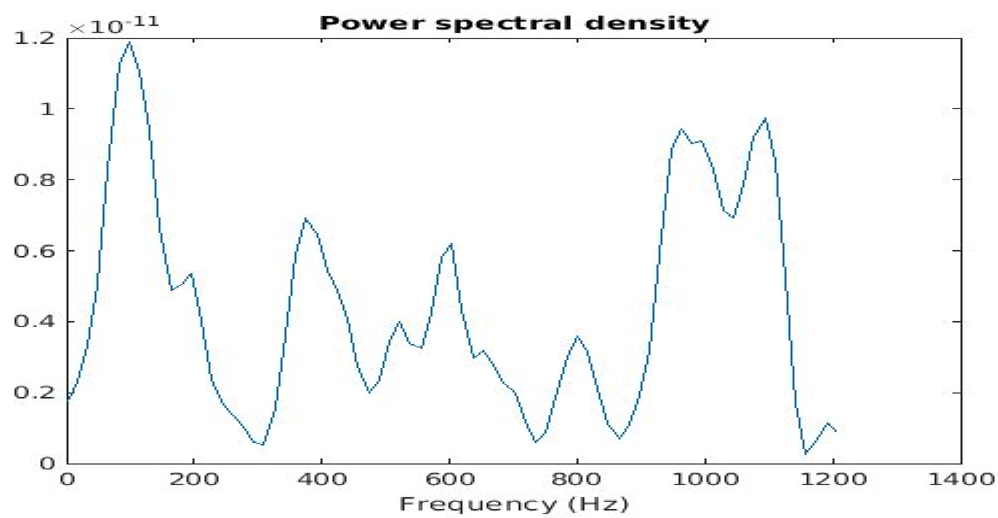
We can see the Excitation periods Acceleration and velocity closely

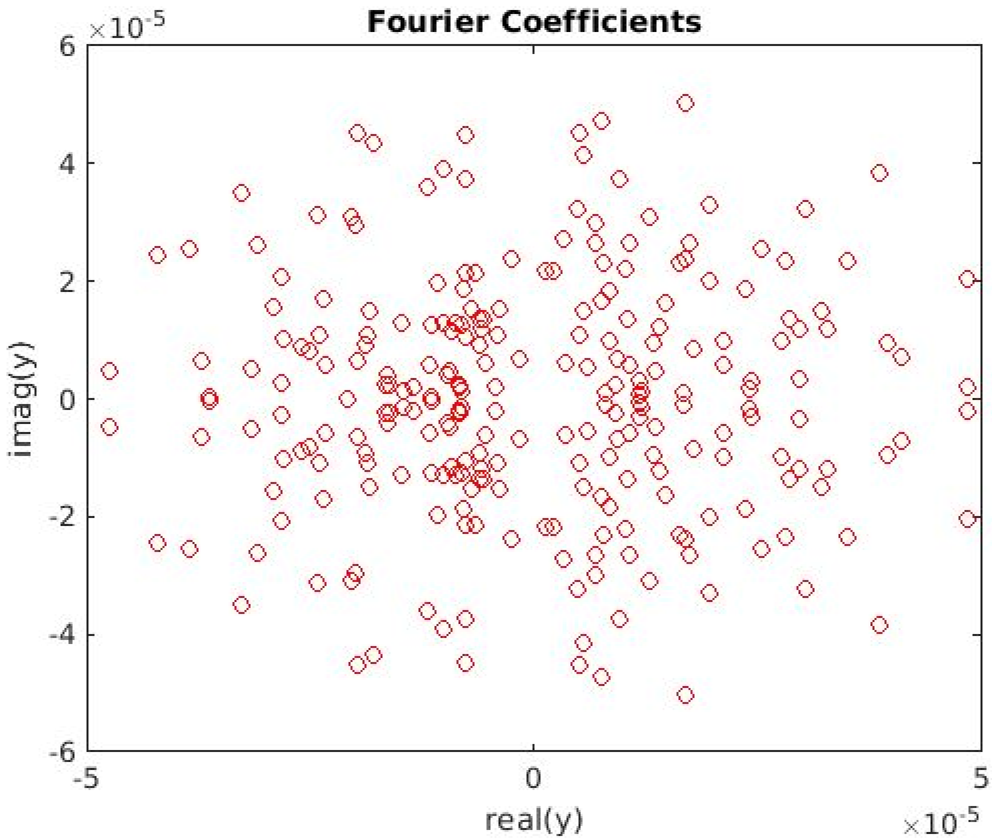


Different Methods Of Time Frequency Distributions:

Now we will discuss the different methods applied for time frequency distribution of our generated data.

1. **Fourier Transform: In Matlab we use the fft** function to transform the signal into fourier domain fft has very less time complexity O(nlogn). Brief of fourier transform is not part of this paper.here I am sharing the results we got from our sample Earthquake





2. **STFT METHOD:** The Short-time Fourier transform (STFT), is a Fourier-related transform used to determine the sinusoidal frequency and phase content of local sections of a signal as it changes over time.[1] In practice, the procedure for computing STFTs is to divide a longer time signal into shorter segments of equal length and then compute the Fourier transform separately on each shorter segment. This reveals the Fourier spectrum on each shorter segment. One then usually plots the changing spectra as a function of time, known as a spectrogram or waterfall plot.

In general with this type of data we can have continuous or discrete time STFT. Simply, in the continuous-time case, the function to be transformed is multiplied by a window function which is nonzero for only a short period of time. The Fourier transform (a one-dimensional function) of the resulting signal is taken as the window is slid along the time axis, resulting in a two-dimensional representation of the signal. Mathematically, this is written as:

$$\mathbf{STFT}\{x(t)\}(\tau,\omega) \equiv X(\tau,\omega) = \int_{-\infty}^{\infty} x(t)w(t-\tau)e^{-i\omega t}\,dt$$

where $w(\tau)$ is the window function, commonly a Hann window or Gaussian window centered around zero, and x(t) is the signal to be transformed (note the difference between the window function w and the frequency ω). $X(\tau,\omega)$ is essentially the Fourier transform of $x(t)w(t-\tau)$ a complex function representing the phase and magnitude of the signal over time and frequency. Often phase unwrapping is employed along either or both the time axis, τ, and frequency axis, ω, to suppress any jump discontinuity of the phase result of the STFT. The time index τ is normally considered to be "slow" time and usually not expressed in as high resolution as time t.

In the discrete time case, the data to be transformed could be broken up into chunks or frames (which usually overlap each other, to reduce artifacts at the boundary). Each chunk is Fourier transformed, and the complex result is added to a matrix, which records magnitude and phase for each point in time and frequency. This can be expressed as:

$$\mathbf{STFT}\{x[n]\}(m,\omega) \equiv X(m,\omega) = \sum_{n=-\infty}^{\infty} x[n]w[n-m]e^{-j\omega n}$$

likewise, with signal x[n] and window w[n]. In this case, m is discrete and ω is continuous, but in most typical applications the STFT is performed on a computer using the fast Fourier transform, so both variables are discrete and quantized.

The magnitude squared of the STFT yields the spectrogram representation of the Power Spectral Density of the function:

$$\mathrm{spectrogram}\{x(t)\}(\tau,\omega) \equiv |X(\tau,\omega)|^2$$

We can also have invertible STFT, that is, the original signal can be recovered from the transform by the Inverse STFT. The most widely accepted way of inverting the STFT is by using the overlap-add (OLA) method, which also allows for modifications to the STFT complex spectrum. This makes for a versatile signal processing method,[3] referred to as the overlap and add with modifications method.

Limitations: One of the pitfalls of the STFT is that it has a fixed resolution. The width of the windowing function relates to how the signal is represented—it determines whether there is good frequency resolution (frequency components close together can be separated) or good time resolution (the time at which frequencies change). A wide window gives better frequency resolution but poor time resolution. A narrower window gives good time resolution but poor frequency resolution. These are called narrowband and wideband transforms, respectively.

The product of the standard deviation in time and frequency is limited. The boundary of the uncertainty principle (best simultaneous resolution of both) is reached with a Gaussian window function, as the Gaussian minimizes the Fourier uncertainty principle.

So the only consideration required is window function which can be changed to get various results.

Mathematical Property Of STFT:

$$\begin{aligned} X_m(\omega) &= \sum_{n=-\infty}^{\infty} x(n)w(n-mR)e^{-j\omega n} \\ &= \mathrm{DTFT}_{\omega}(x \cdot \mathrm{SHIFT}_{mR}(w)), \end{aligned}$$

This is STFT in it's simple form.

If the window $w(n)$ has the *Constant OverLap-Add (COLA) property* at hop-size R, *i.e.*, if

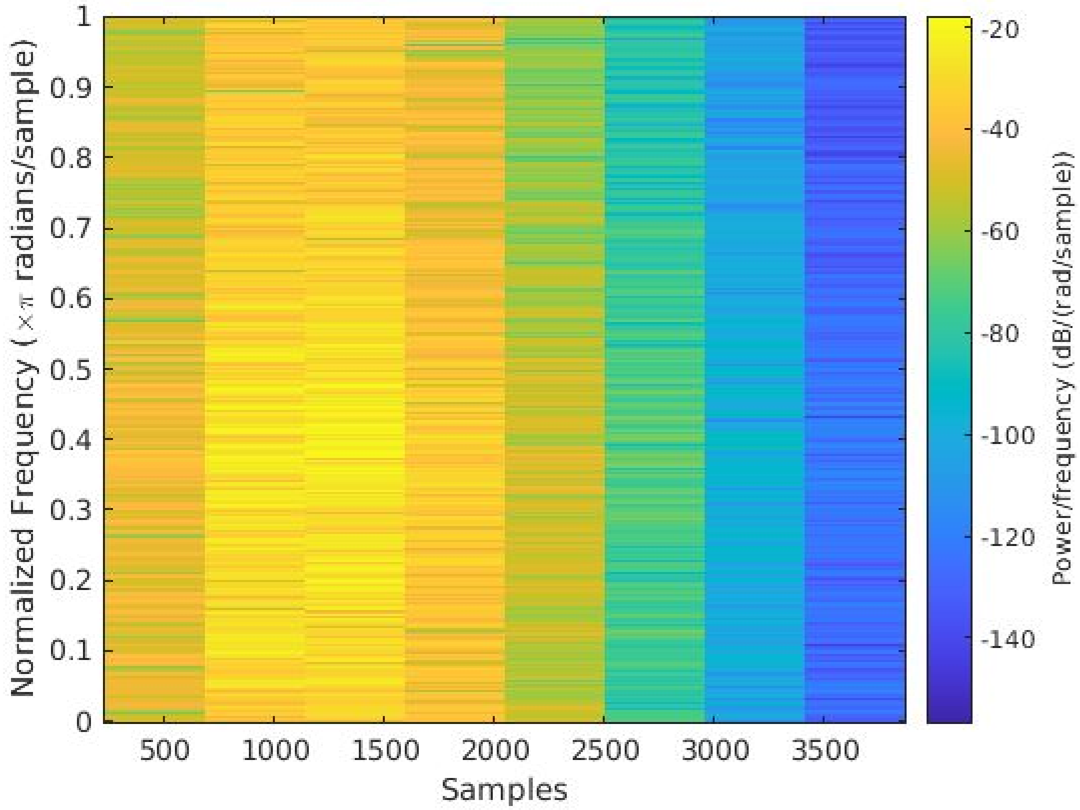
$$\boxed{\sum_{m=-\infty}^{\infty} w(n-mR) = 1, \ \forall n \in \mathbb{Z}} \quad (w \in \mathrm{COLA}(R))$$

$$
\begin{aligned}
\sum_{m=-\infty}^{\infty} X_m(\omega) &\triangleq \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(n) w(n-mR) e^{-j\omega n} \\
&= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \underbrace{\sum_{m=-\infty}^{\infty} w(n-mR)}_{1 \text{ if } w \in \text{COLA}(R)} \\
&= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \\
&\triangleq \text{DTFT}_\omega(x) = X(\omega).
\end{aligned}
$$

The above equation validates the fact that in constant overlapping both the summation will be the same. But in reality STFT is computed as a succession of FFT with windowed data.

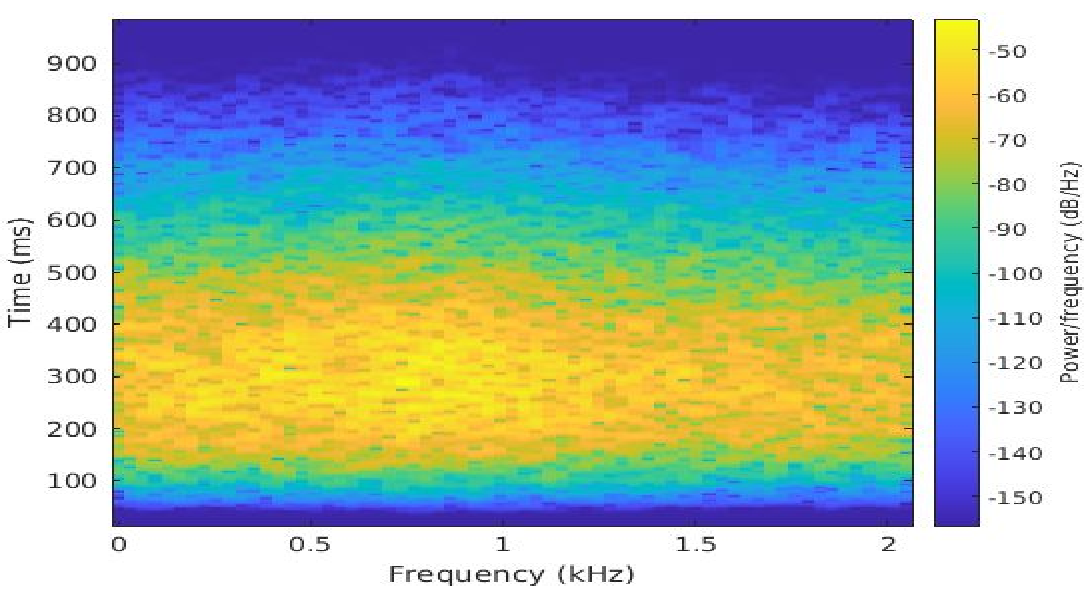
$$
\begin{aligned}
X_m(\omega) &= \sum_{n=-\infty}^{\infty} x(n+mR) w(n) e^{-j\omega(n+mR)} \\
&= e^{-j\omega mR} \sum_{n=-\infty}^{\infty} x(n+mR) w(n) e^{-j\omega n} \\
&= e^{-j\omega mR} \text{DTFT}_\omega(\text{SHIFT}_{-mR}(x) \cdot w).
\end{aligned}
$$

Using a custom STFT function we got TF distribution like this.

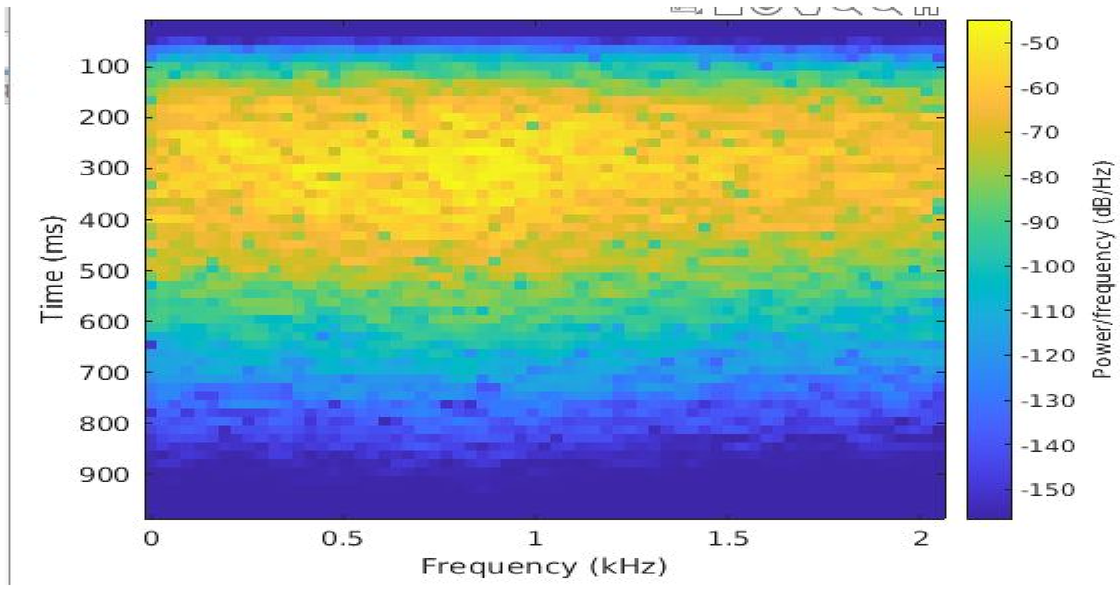


So it is clear that the power is high in the beginning range but this view is certainly not expected for any conclusion.

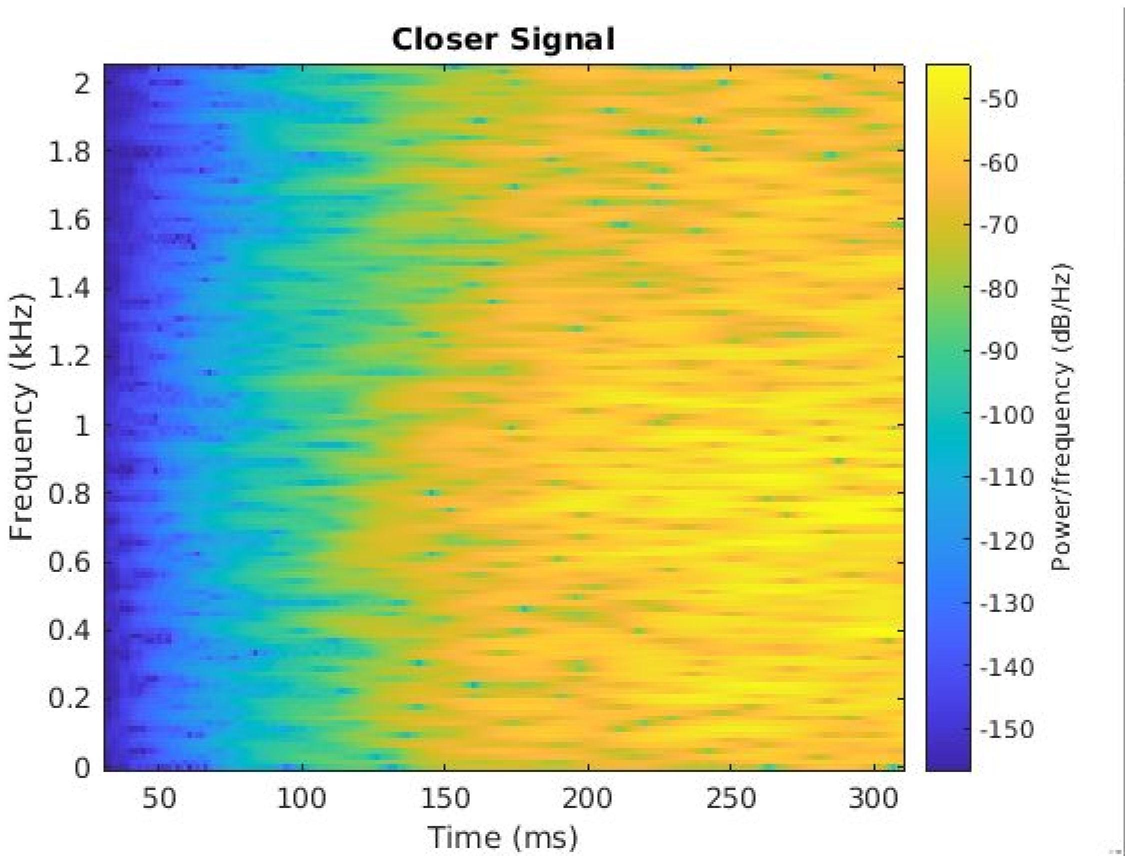
With frequency in x-axis we got a spectrogram like this @hamming window length of 128



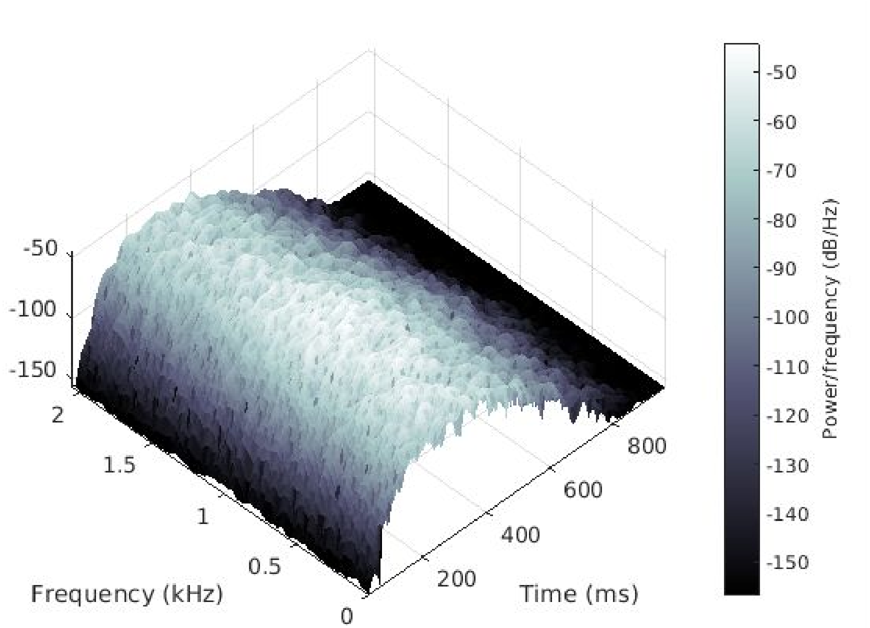
Now if we Replace the Hamming window with a Blackman window. Decrease the overlap to 60 samples. Plot the time axis so that its values increase from top to bottom.

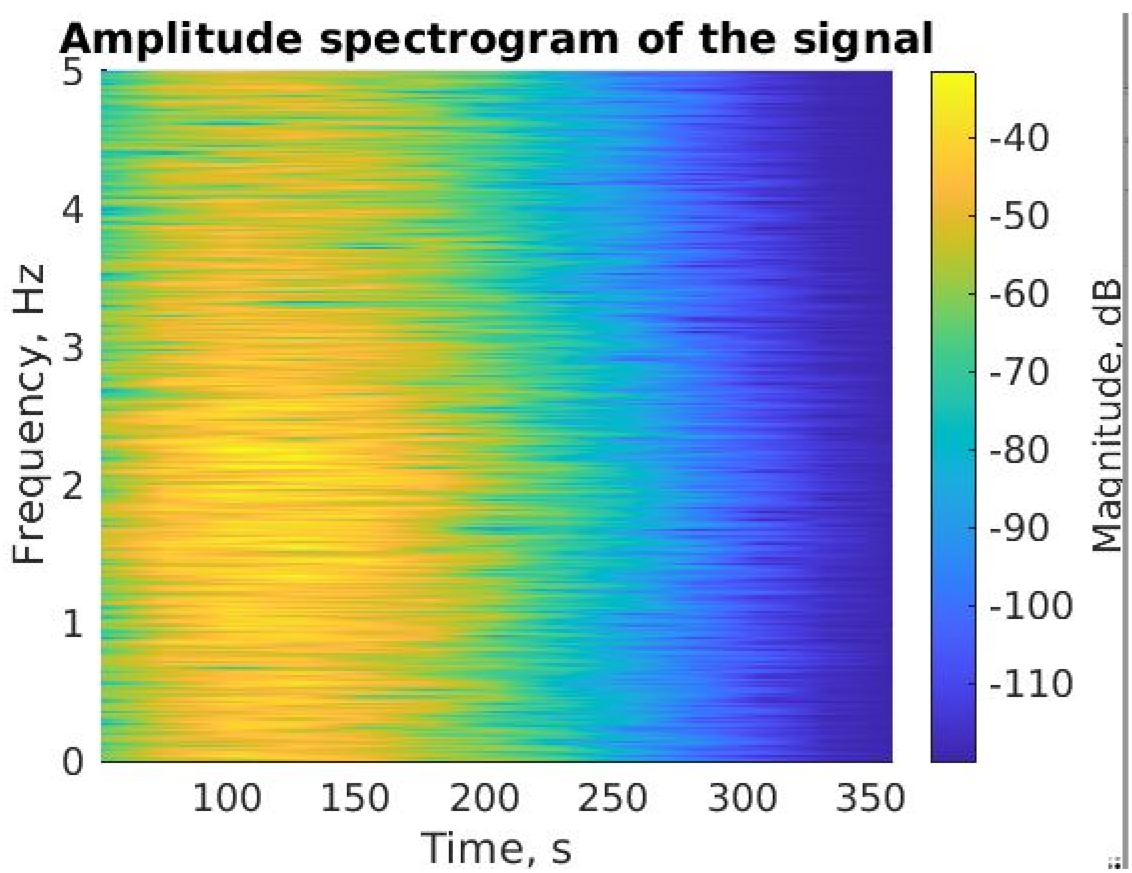


Now let’s plot the power spectral density of the signal at closer proximity, we got

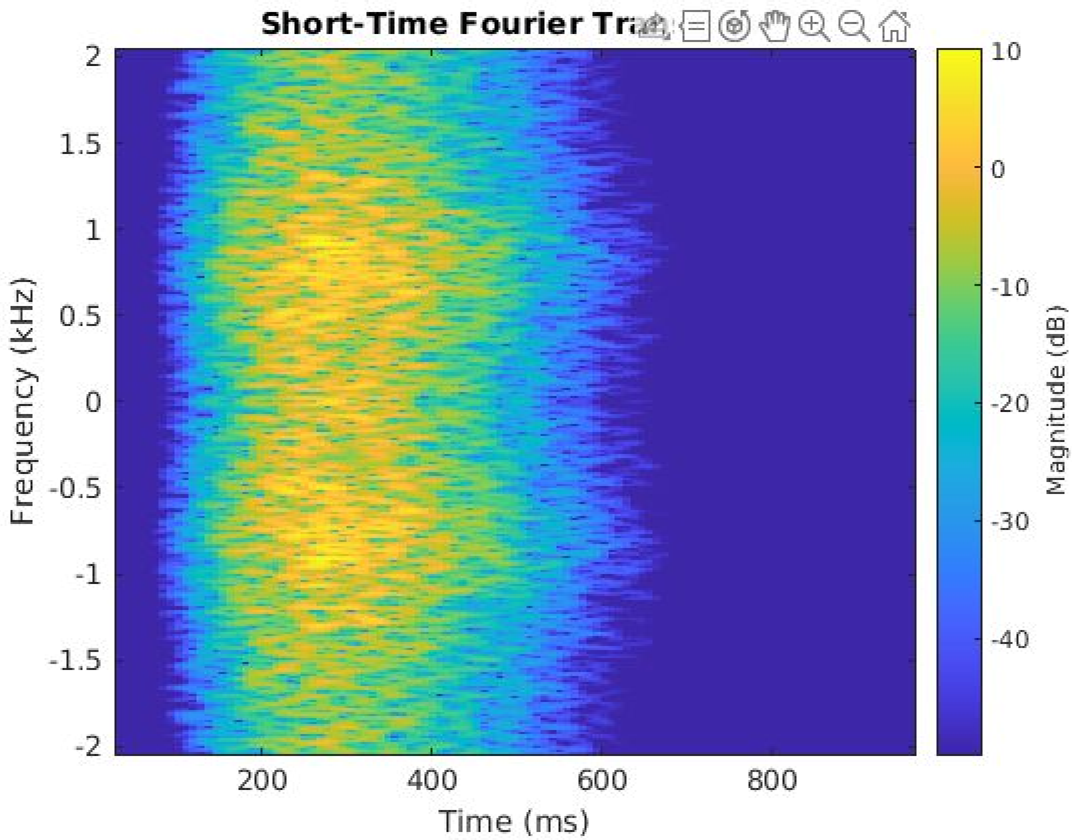


The 3-D visualization of the Spectrogram



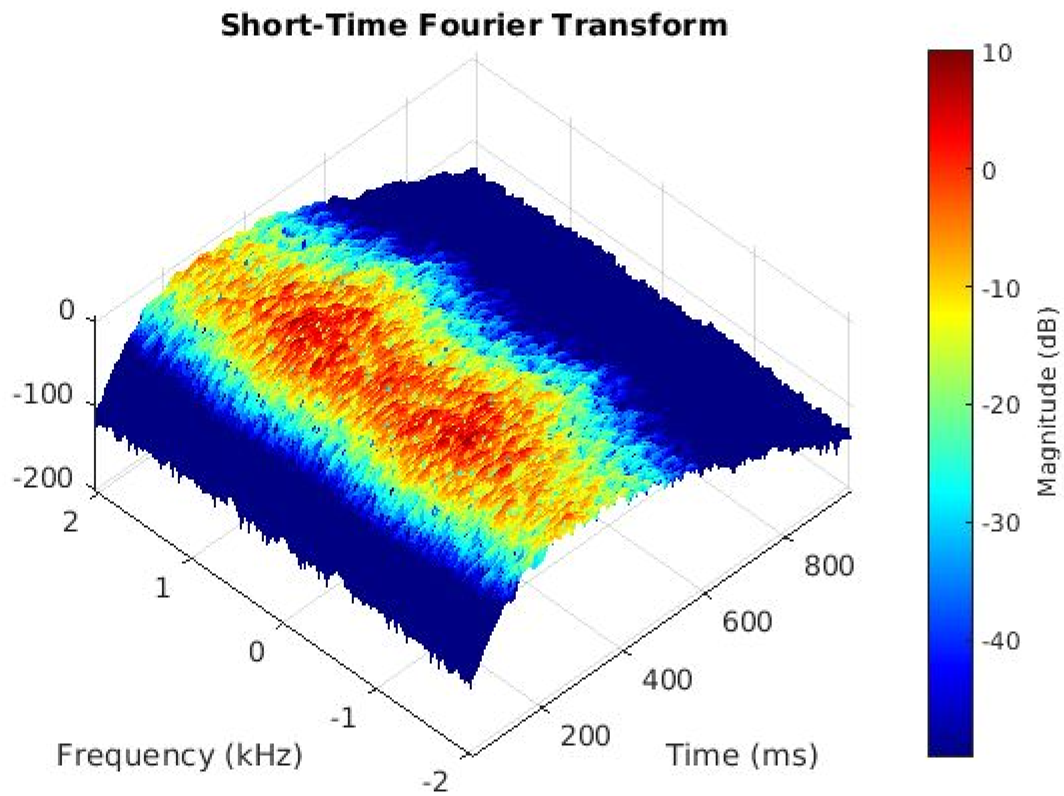


So clearly we can see the earthquake is an event of the rear time frame. We can see the magnitude of the earthquake in decibel is upto 40 db .



Using kaiser window of length 256 and parameter B as 5 we got this.

This same model in waterfall view will give us more meaningful visuals.



1. **Wavelet Method:** A wavelet is a wave-like oscillation with an amplitude that begins at zero, increases, and then decreases back to zero. It can typically be visualized as a "brief oscillation" like one recorded by a seismograph or heart monitor. Generally, wavelets are intentionally crafted to have specific properties that make them useful for signal processing. For example, a wavelet could be created to have a frequency of Middle C and a short duration of roughly a 32nd note. If this wavelet were to be convolved with a signal created from the recording of a melody, then the resulting signal would be useful for determining when the Middle C note was being played in the song. Mathematically, the wavelet will correlate with the signal if the unknown signal contains information of similar frequency. This concept of correlation is at the core of many practical applications of wavelet theory.As a mathematical tool, wavelets can be used to extract information from many different kinds of data, including – but not limited to – audio signals and images. Sets of wavelets are generally needed to analyze data fully. A set of

"complementary" wavelets will decompose data without gaps or overlap so that the decomposition process is mathematically reversible. Thus, sets of complementary wavelets are useful in wavelet based compression/decompression algorithms where it is desirable to recover the original information with minimal loss.In formal terms, this representation is a wavelet series representation of a square-integrable function with respect to either a complete, orthonormal set of basis functions, or an overcomplete set or frame of a vector space, for the Hilbert space of square integrable functions. This is accomplished through coherent states.

Wavelet Theory: Wavelet theory is applicable to several subjects. All wavelet transforms may be considered forms of time-frequency representation for continuous-time (analog) signals and so are related to harmonic analysis. Discrete wavelet transform (continuous in time) of a discrete-time (sampled) signal by using discrete-time filterbanks of dyadic (octave band) configuration is a wavelet approximation to that signal. The coefficients of such a filter bank are called the wavelet and scaling coefficients in wavelets nomenclature. These filterbanks may contain either finite impulse response (FIR) or infinite impulse response (IIR) filters. The wavelets forming a continuous wavelet transform (CWT) are subject to the uncertainty principle of Fourier analysis respective sampling theory: Given a signal with some event in it, one cannot simultaneously assign an exact time and frequency response scale to that event. The product of the uncertainties of time and frequency response scale has a lower bound. Thus, in the scaleogram of a continuous wavelet transform of this signal, such an event marks an entire region in the time-scale plane, instead of just one point. Also, discrete wavelet bases may be considered in the context of other forms of the uncertainty principle.

Wavelet transforms are broadly divided into three classes: continuous, discrete and multiresolution-based.

Continuous wavelet transforms: In continuous wavelet transforms, a given signal of finite energy is projected on a continuous family of frequency bands (or similar subspaces of the Lp function space L2(R)). For instance the signal may be represented on every frequency band of the form [f, 2f] for all positive frequencies f > 0. Then, the original signal can be reconstructed by a suitable integration over all the resulting frequency components.

The frequency bands or subspaces (sub-bands) are scaled versions of a subspace at scale 1. This subspace in turn is in most situations generated by the shifts of one generating function ψ in L2(R), the mother wavelet. For the example of the scale one frequency band [1, 2] this function is

$$\psi(t) = 2\ \mathrm{sinc}(2t) - \ \mathrm{sinc}(t) = \frac{\sin(2\pi t) - \sin(\pi t)}{\pi t}$$

with the (normalized) sinc function. That, Meyer's wavelet. A list of all the wavelet transform is given below

- Continuous wavelet transform (CWT)
- Discrete wavelet transform (DWT)
- Multiresolution analysis (MRA)
- Lifting scheme
- Binomial QMF (BQMF)
- Fast wavelet transform (FWT)
- Complex wavelet transform
- Non or undecimated wavelet transform, the downsampling is omitted
- Newland transform, an orthonormal basis of wavelets is formed from appropriately constructed top-hat filters in frequency space
- Wavelet packet decomposition (WPD), detail coefficients are decomposed and a variable tree can be formed
- Stationary wavelet transform (SWT), no downsampling and the filters at each level are different
- e-decimated discrete wavelet transform, depends on if the even or odd coefficients are selected in the downsampling
- Second generation wavelet transform (SGWT), filters and wavelets are not created in the frequency domain
- Dual-tree complex wavelet transform (DTCWT), two trees are used for decomposion to produce the real and complex coefficients

- WITS: Where Is The Starlet, a collection of a hundredth of wavelet names in -let and associated multiscale, directional, geometric, representations, from activelets to x-lets through bandelets, chirplets, contourlets, curvelets, noiselets, wedgelets

Discrete wavelet transforms: It is computationally impossible to analyze a signal using all wavelet coefficients, so one may wonder if it is sufficient to pick a discrete subset of the upper half plane to be able to reconstruct a signal from the corresponding wavelet coefficients. One such system is the affine system for some real parameters $a > 1$, $b > 0$. The corresponding discrete subset of the half plane consists of all the points (a^m, $na^m b$) with m, n in Z. The corresponding *child wavelets* are now given as

$$\psi_{m,n}(t) = \frac{1}{\sqrt{a^m}} \psi\left(\frac{t - nb}{a^m}\right).$$

A sufficient condition for the reconstruction of any signal x of finite energy by the formula

$$x(t) = \sum_{m \in \mathbb{Z}} \sum_{n \in \mathbb{Z}} \langle x,\, \psi_{m,n} \rangle \cdot \psi_{m,n}(t)$$

How wavelet transform works is completely a different fun story, and should be explained after short time Fourier Transform (STFT). The WT was developed as an alternative to the STFT. The STFT will be explained in great detail in the second part of this tutorial. It suffices at this time to say that the WT was developed to overcome some resolution related problems of the STFT, as explained in Part II.

To make a real long story short, we pass the time-domain signal from various highpass and low pass filters, which filters out either high frequency or low frequency portions of the signal. This procedure is repeated, every time some portion of the signal corresponding to some frequencies being removed from the signal.

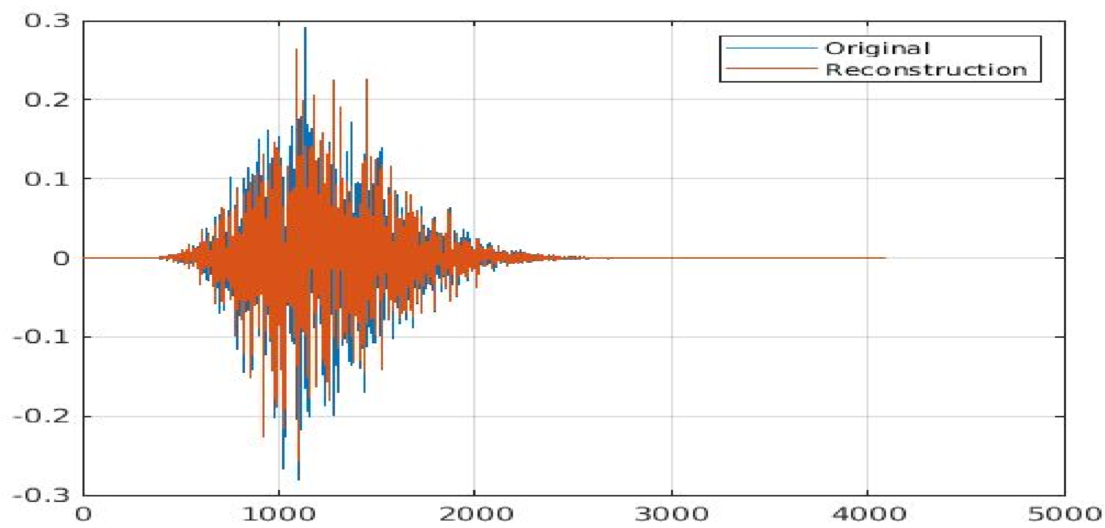
Here is how this works: Suppose we have a signal which has frequencies up to 1000 Hz. In the first stage we split up the signal in to two parts by passing the signal from a highpass and a lowpass filter (filters should satisfy some certain conditions, so-called admissibility condition) which results in two different versions of the same signal: portion of the signal corresponding to 0-500 Hz (low pass portion), and 500-1000 Hz (high pass portion).

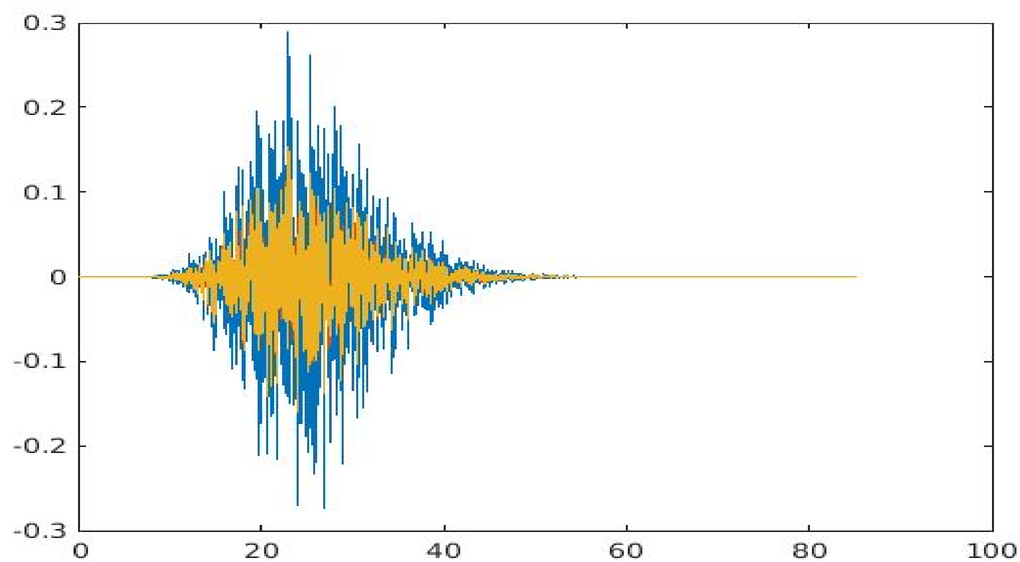
Then, we take either portion (usually low pass portion) or both, and do the same thing again. This operation is called decomposition .

Assuming that we have taken the lowpass portion, we now have 3 sets of data, each corresponding to the same signal at frequencies 0-250 Hz, 250-500 Hz, 500-1000 Hz.

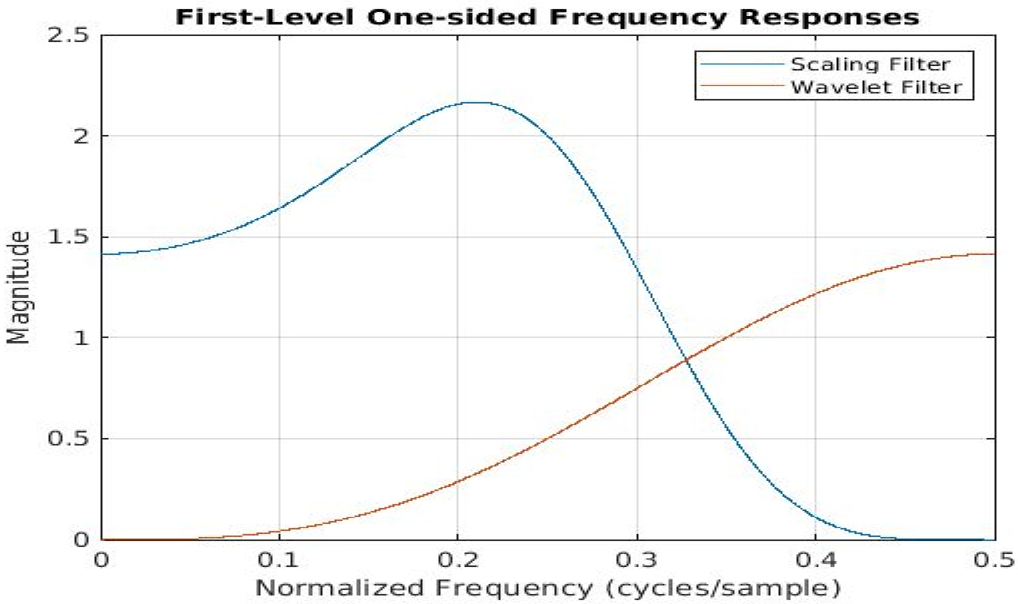
Then we take the lowpass portion again and pass it through low and high pass filters; we now have 4 sets of signals corresponding to 0-125 Hz, 125-250 Hz,250-500 Hz, and 500-1000 Hz. We continue like this until we have decomposed the signal to a pre-defined certain level. Then we have a bunch of signals, which actually represent the same signal, but all corresponding to different frequency bands. We know which signal corresponds to which frequency band, and if we put all of them together and plot them on a 3-D graph, we will have time in one axis, frequency in the second and amplitude in the third axis. This will show us which frequencies exist at which time (there is an issue, called "uncertainty principle", which states that, we cannot exactly know what frequency exists at what time instance, but we can only know what frequency bands exist at what time intervals.

Now using single level 1-D wavelet transform it is almost possible to find a more accurate and smooth curve of the Earthquake in comparison to exponential method

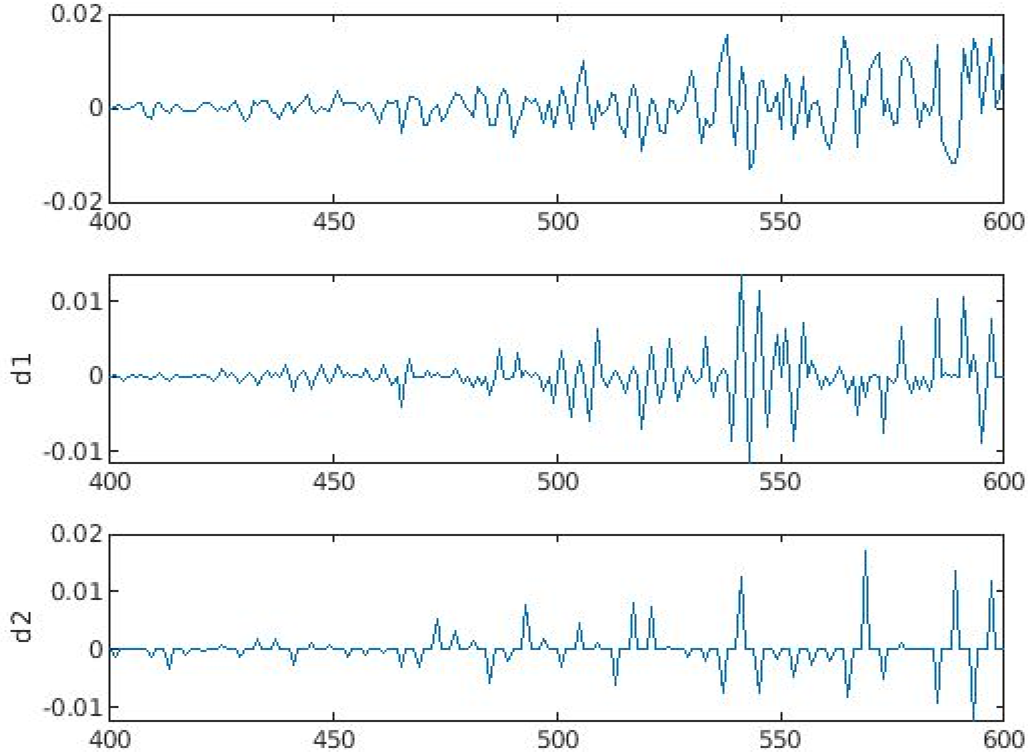




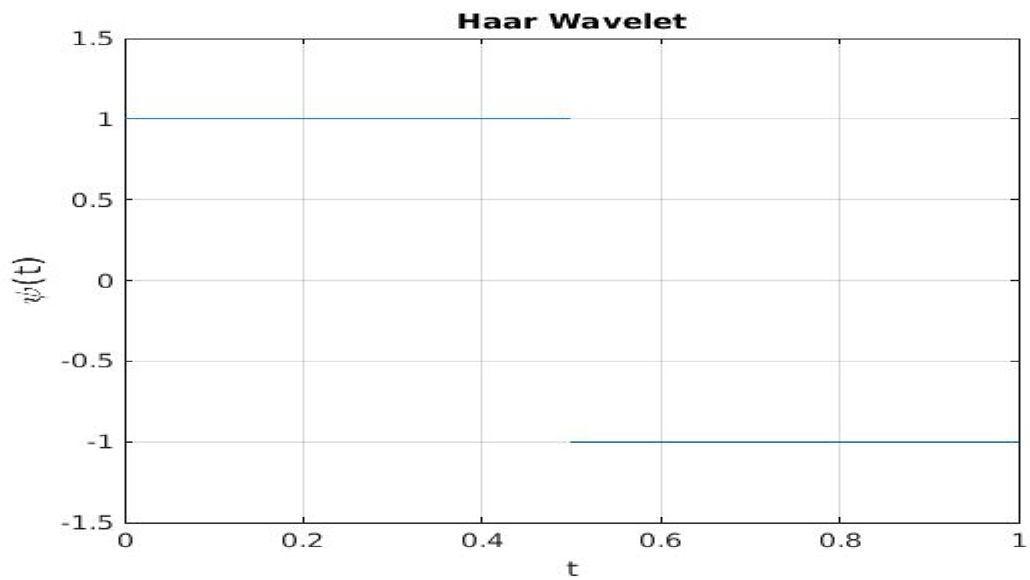
Now if we just change the wavelet and scaling factor we got this wavelet

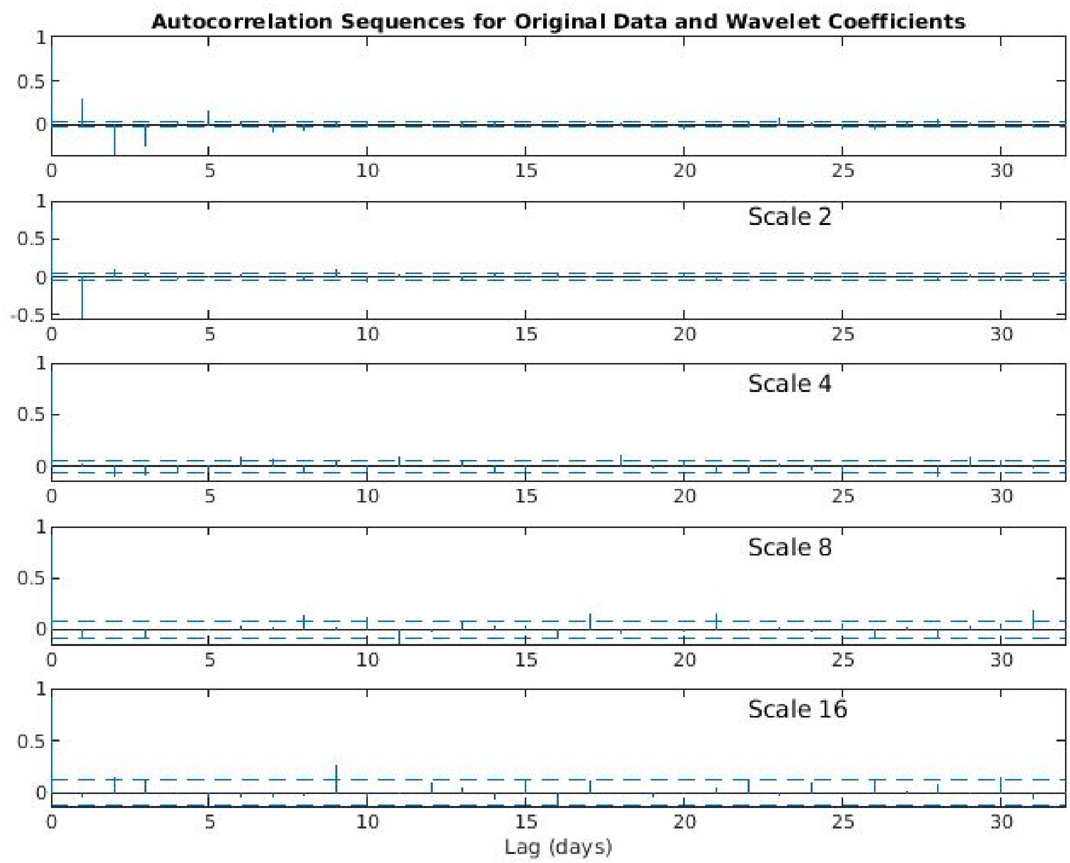


We can also show how analysis by wavelets can detect a discontinuity in one of a signal's derivatives.

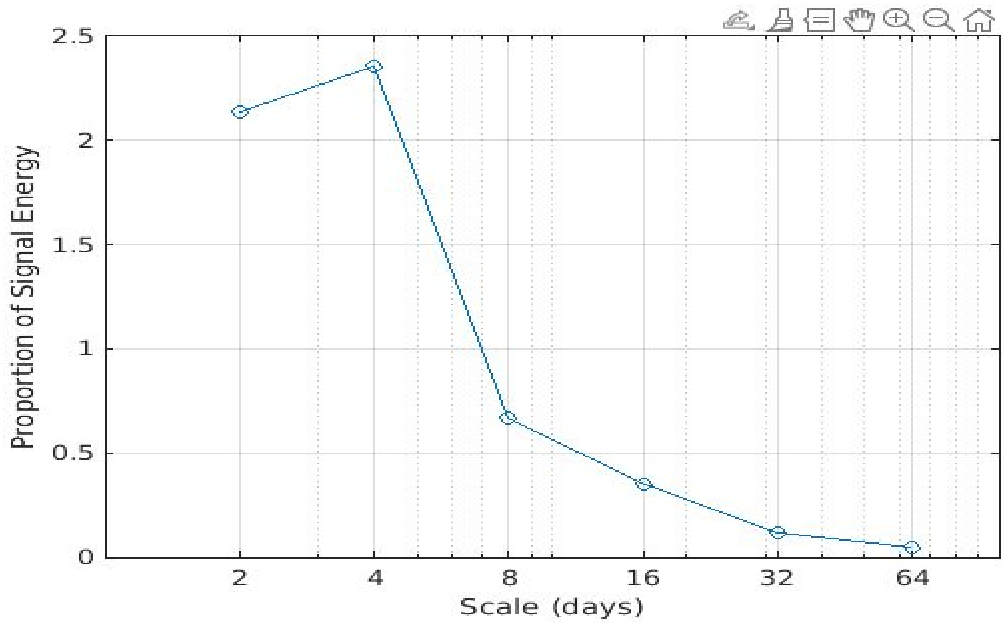


We can use Haar wavelet to find autocorrelation in signal but first see how haar looks like and then correlation

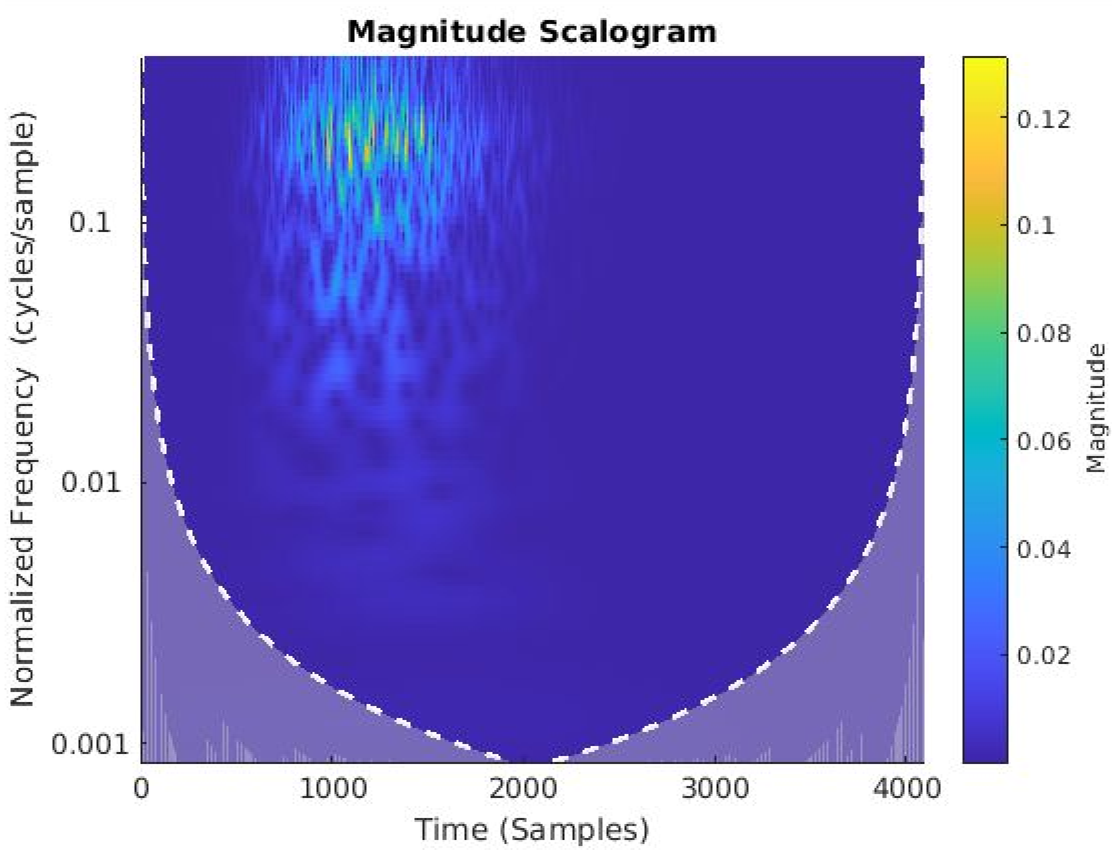




Next see how haar preserves the energy

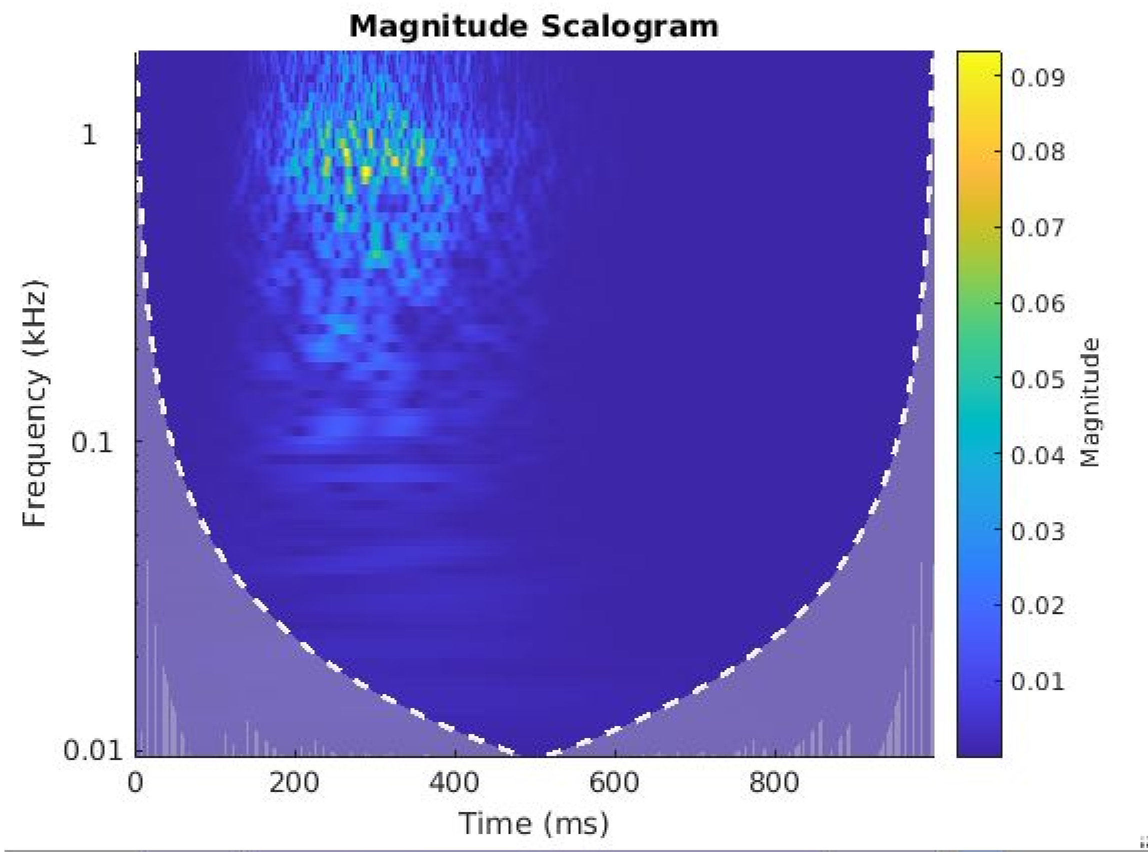


Using Scalogram we get the Time frequency details that are much understandable than STFT

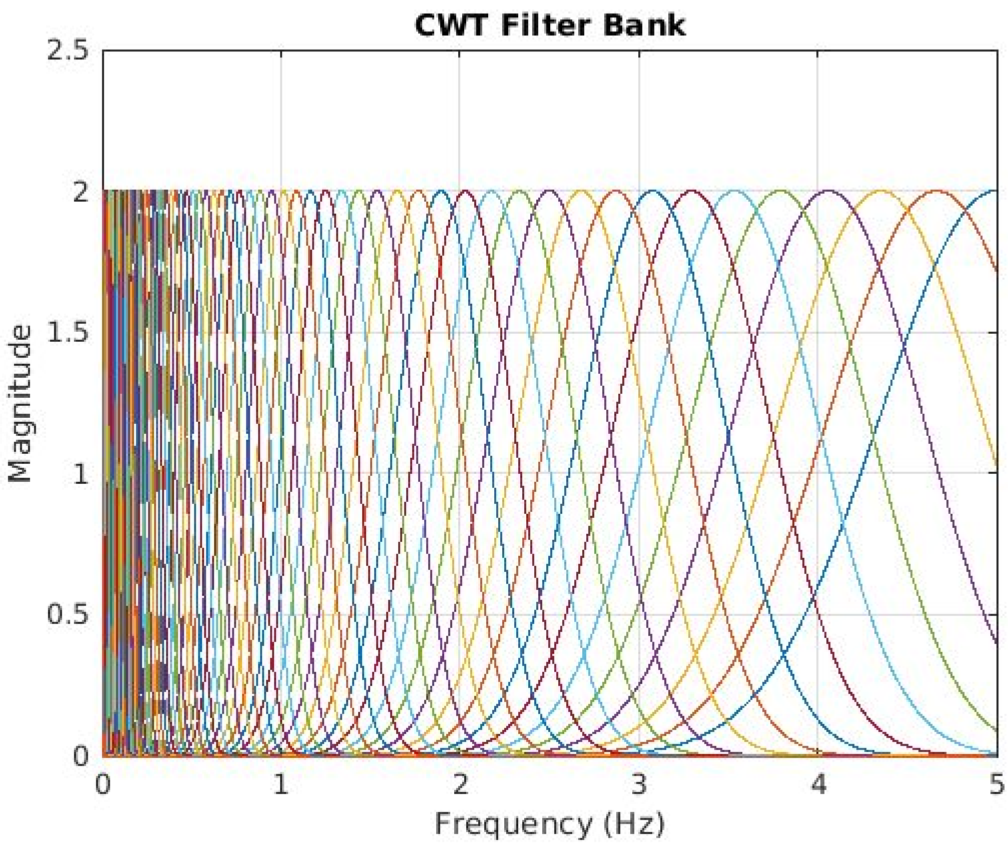


This is the default view that contains multiple excitation at around 1100 time value to 2000 timevalue.

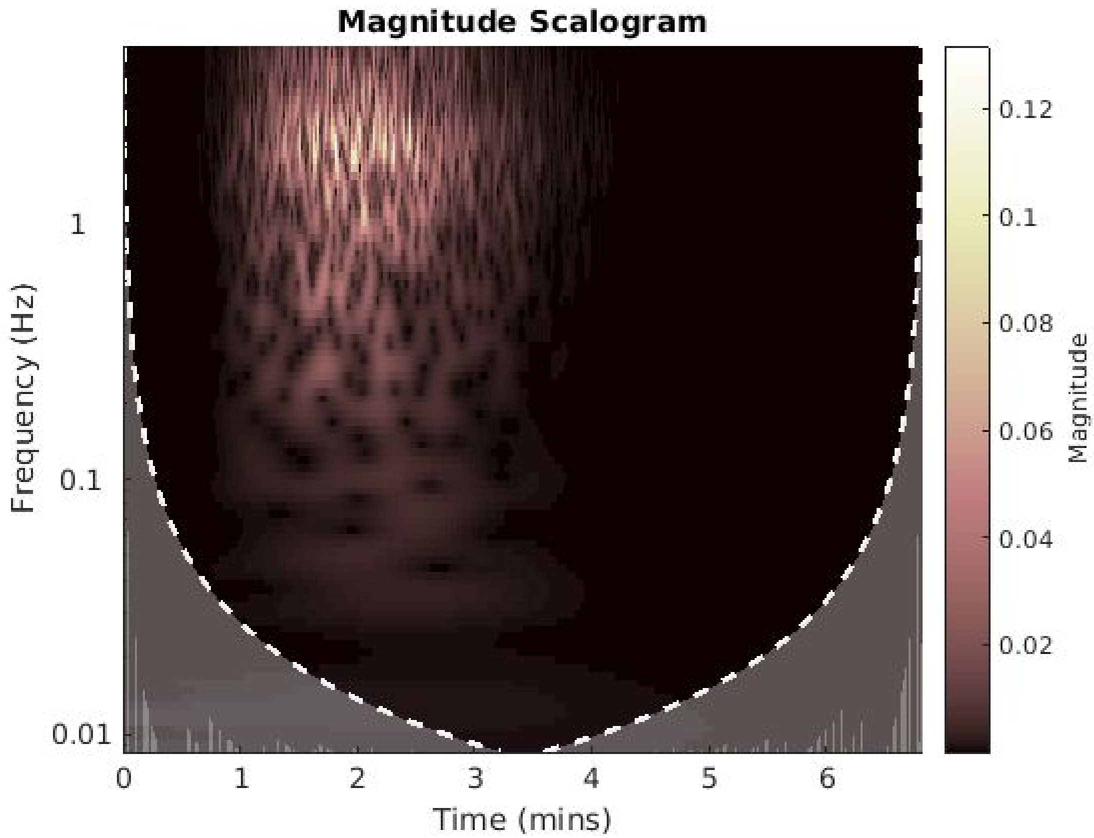
Now with values used in experiment we got this



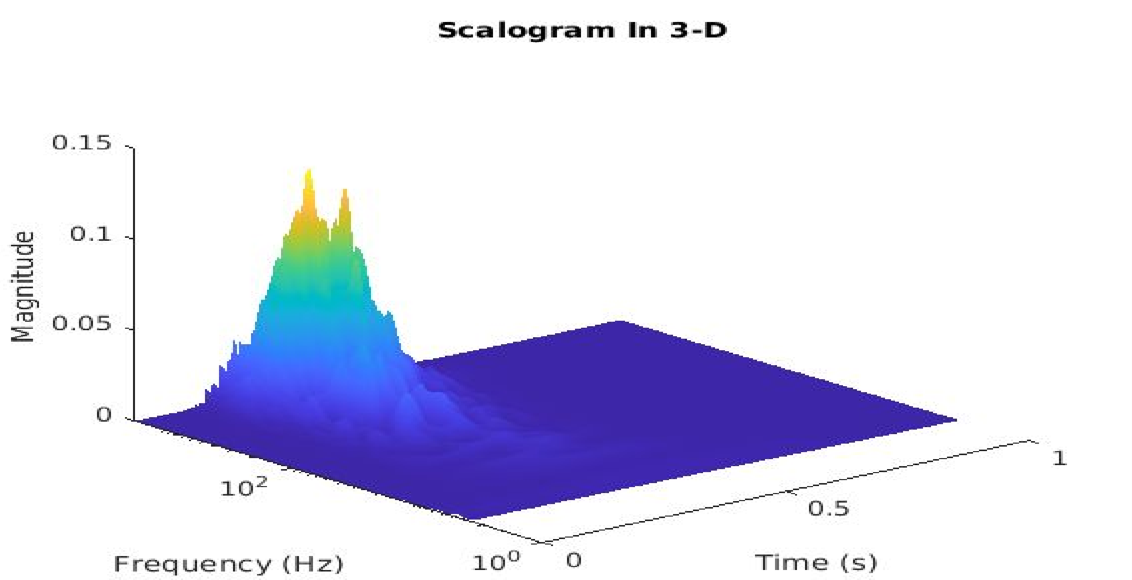
The default filter bank will look like this



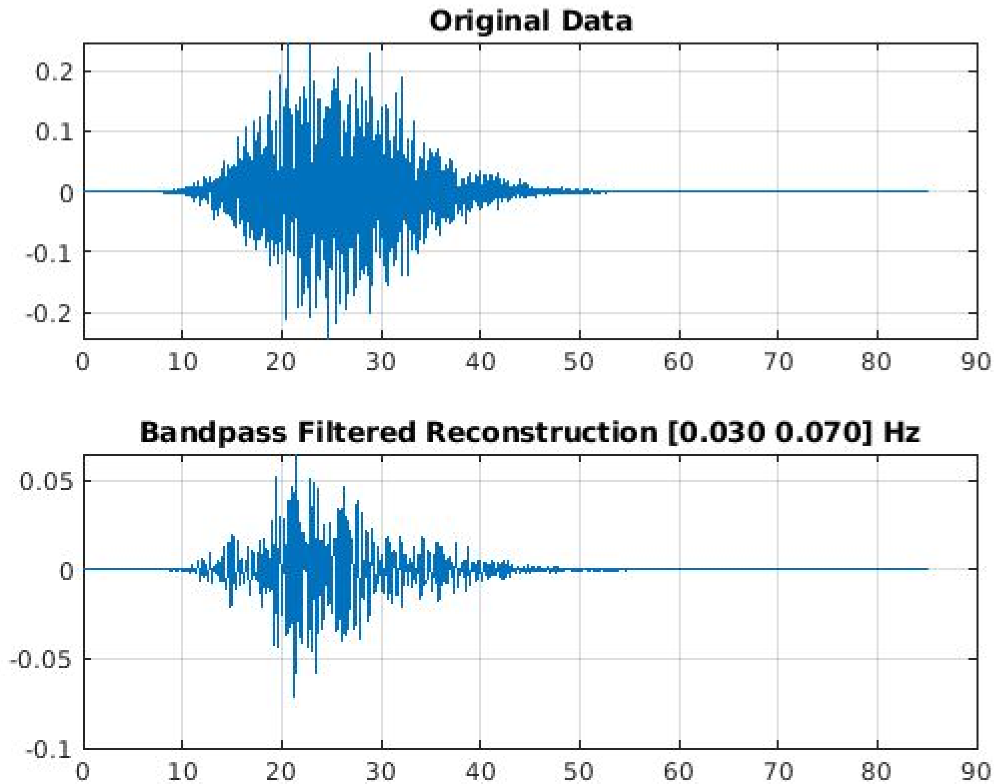
A better representation of the scalogram



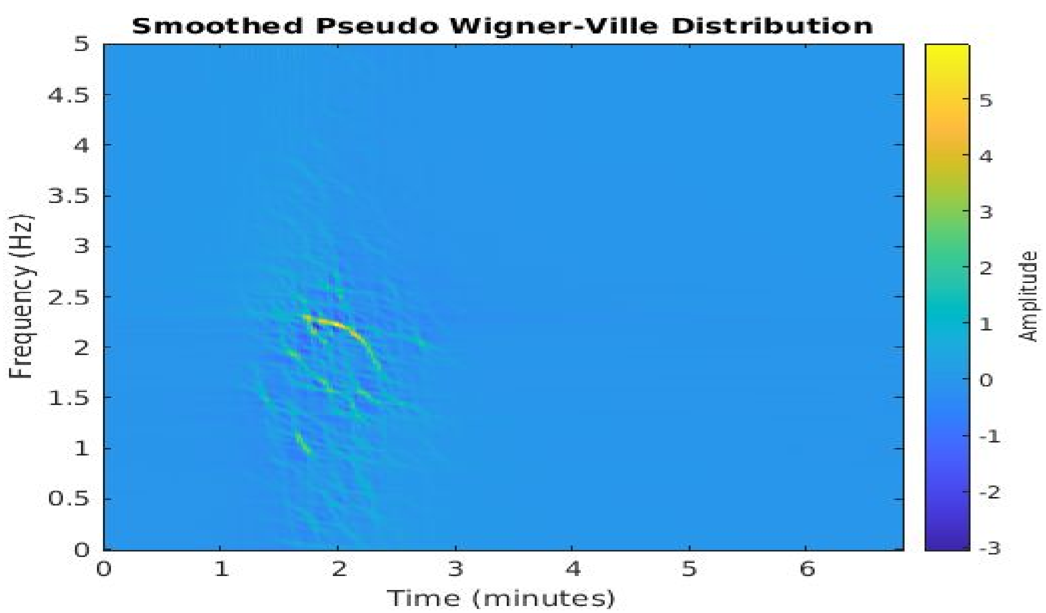
In 3-D we got this



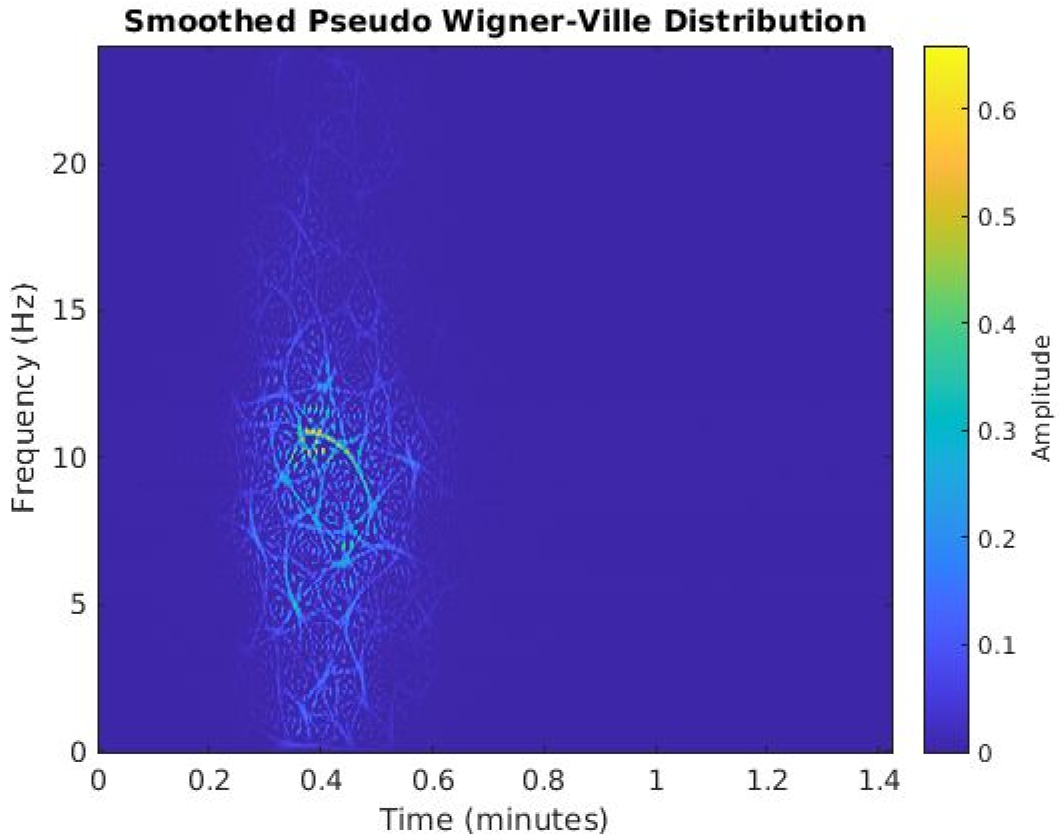
The filtered data will look like this



3. **Wigner Ville Distribution:** the results from wigner ville are shown here

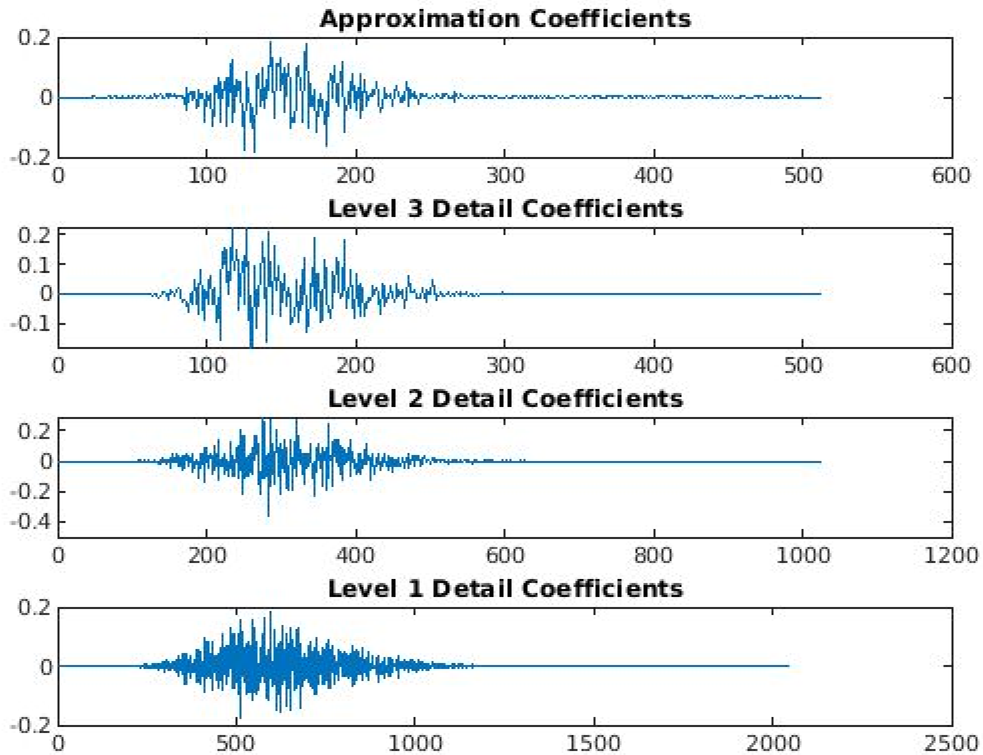


By further sharpening the result we got this

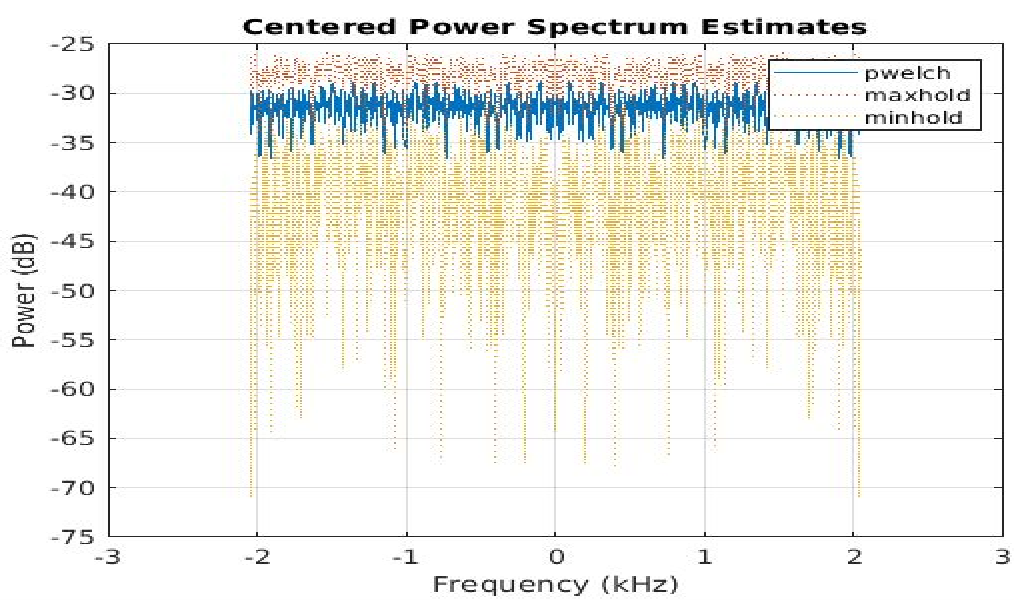


4. **Local Wave Decomposion:** If we do local value decomposition and Perform a 3-level wavelet decomposition of the signal using the order 2 Daubechies wavelet. Extract the coarse scale approximation coefficients and the detail coefficients from the decomposition.

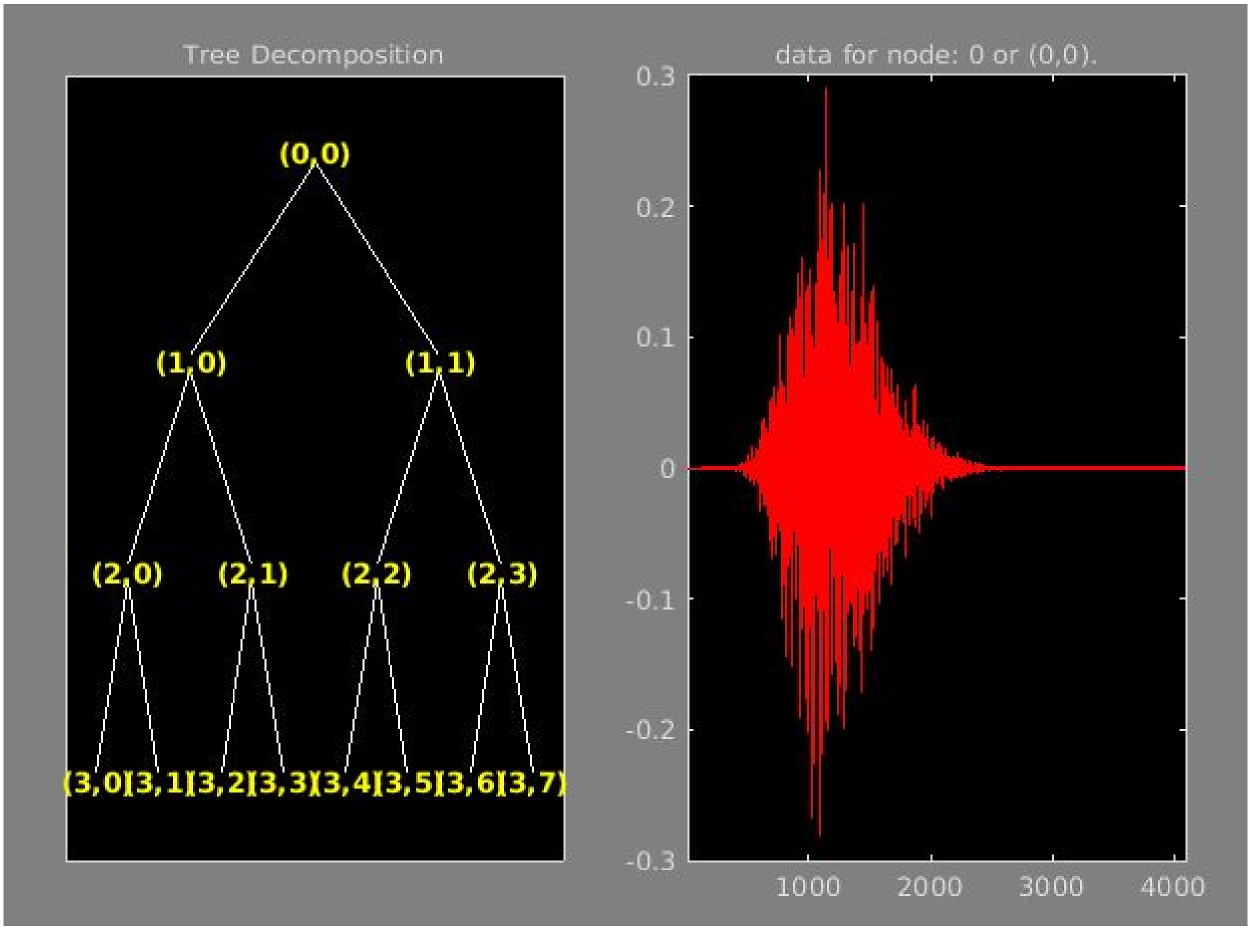
We get this



We can also compute the PSD estimate and the maximum-hold and minimum-hold spectra of the signal. Plot the results.

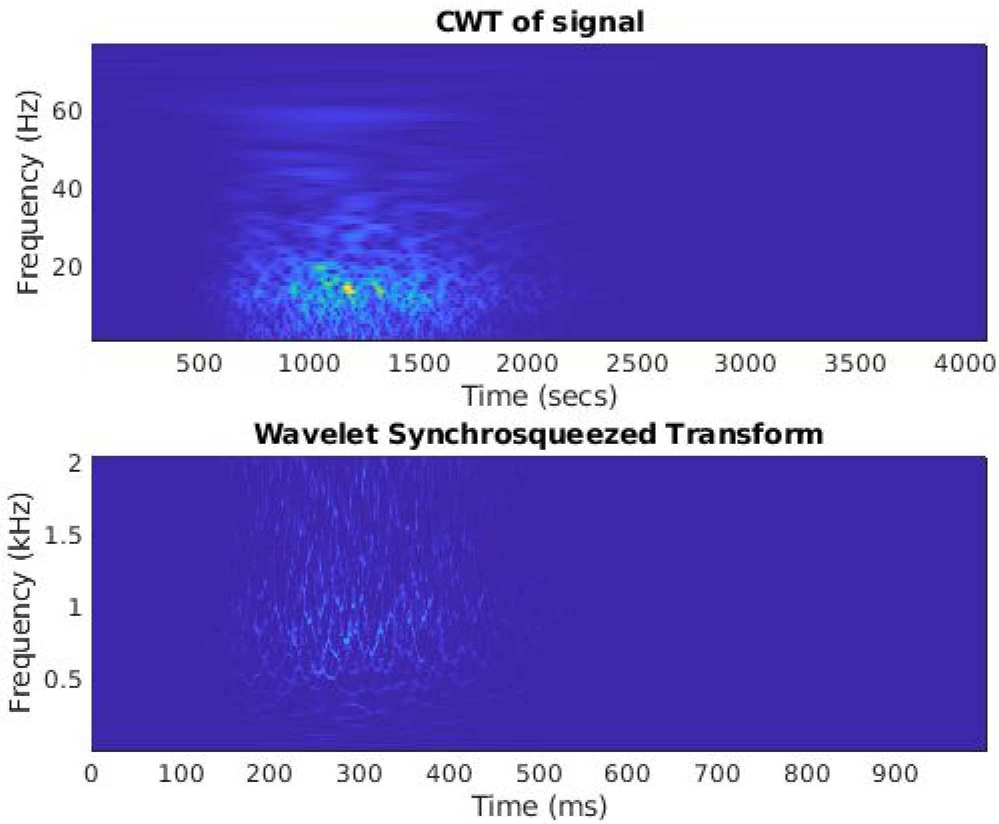


We can also plot the wavelet packet trees

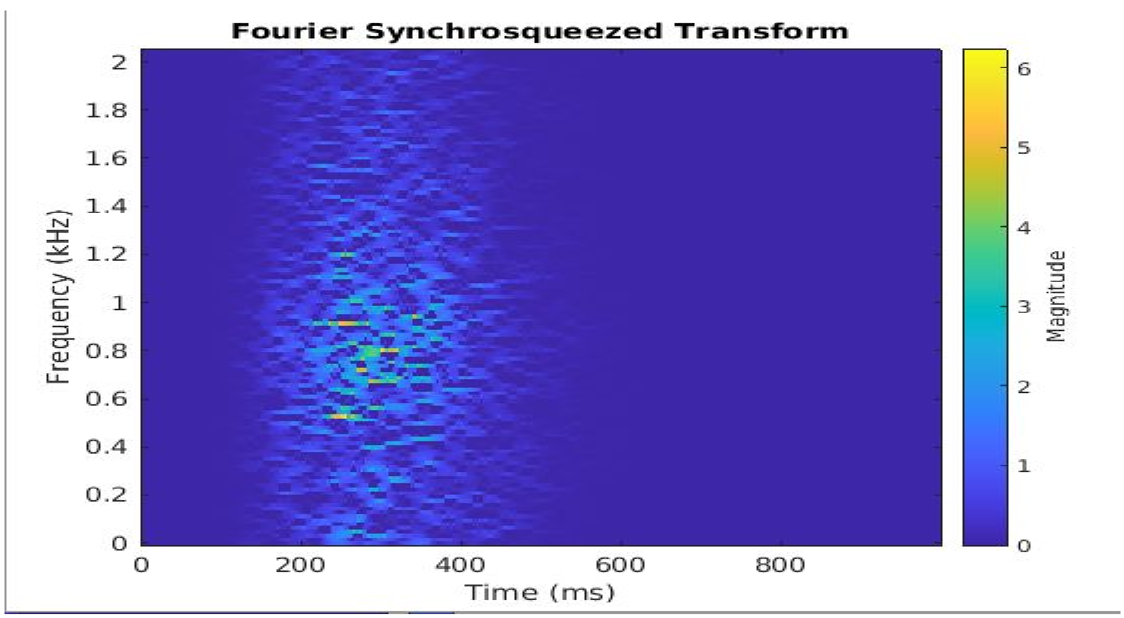


Synchrosqueezing: The wavelet synchrosqueezed transform is a time-frequency analysis method that is useful for analyzing multicomponent signals with oscillating modes. Examples of signals with oscillating modes include speech waveforms, machine vibrations, and physiologic signals. Many of these real-world signals with oscillating modes can be written as a sum of amplitude-modulated and frequency-modulated components.

Let's show you the difference between cwt and synchrosqeezing



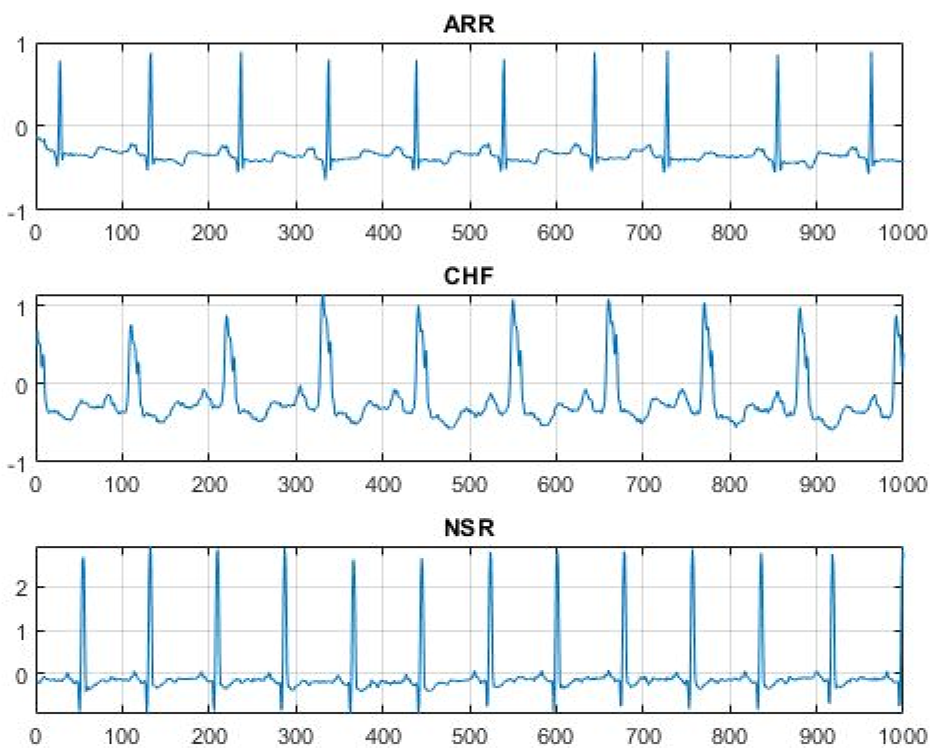
This is another result



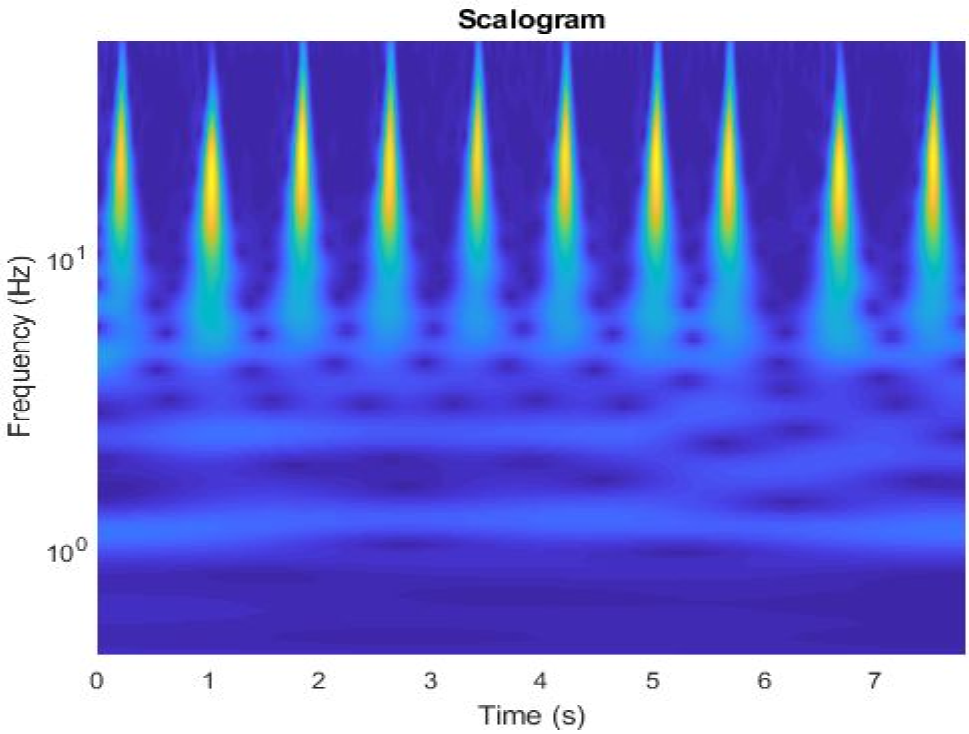
MACHINE LEARNING AND DEEP LEARNING:

We can easily classify the type of earthquake using deep learning and wavelet transform to do that we need labeled data so it is not the scope of this paper for now but sharing similar example on ECG data This example shows how to classify human electrocardiogram (ECG) signals using the continuous wavelet transform (CWT) and a deep convolutional neural network (CNN). And same concept will be applied to decode Seismic result.

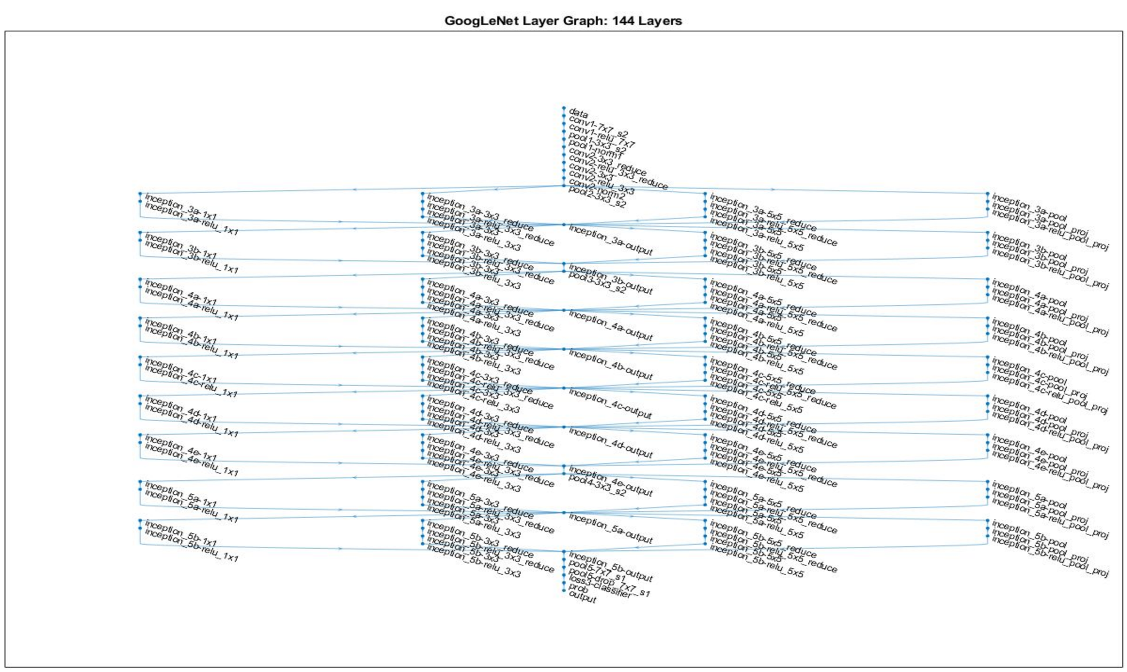
The data used in this example are publicly available from PhysioNet.Plot a representative of each ECG category.



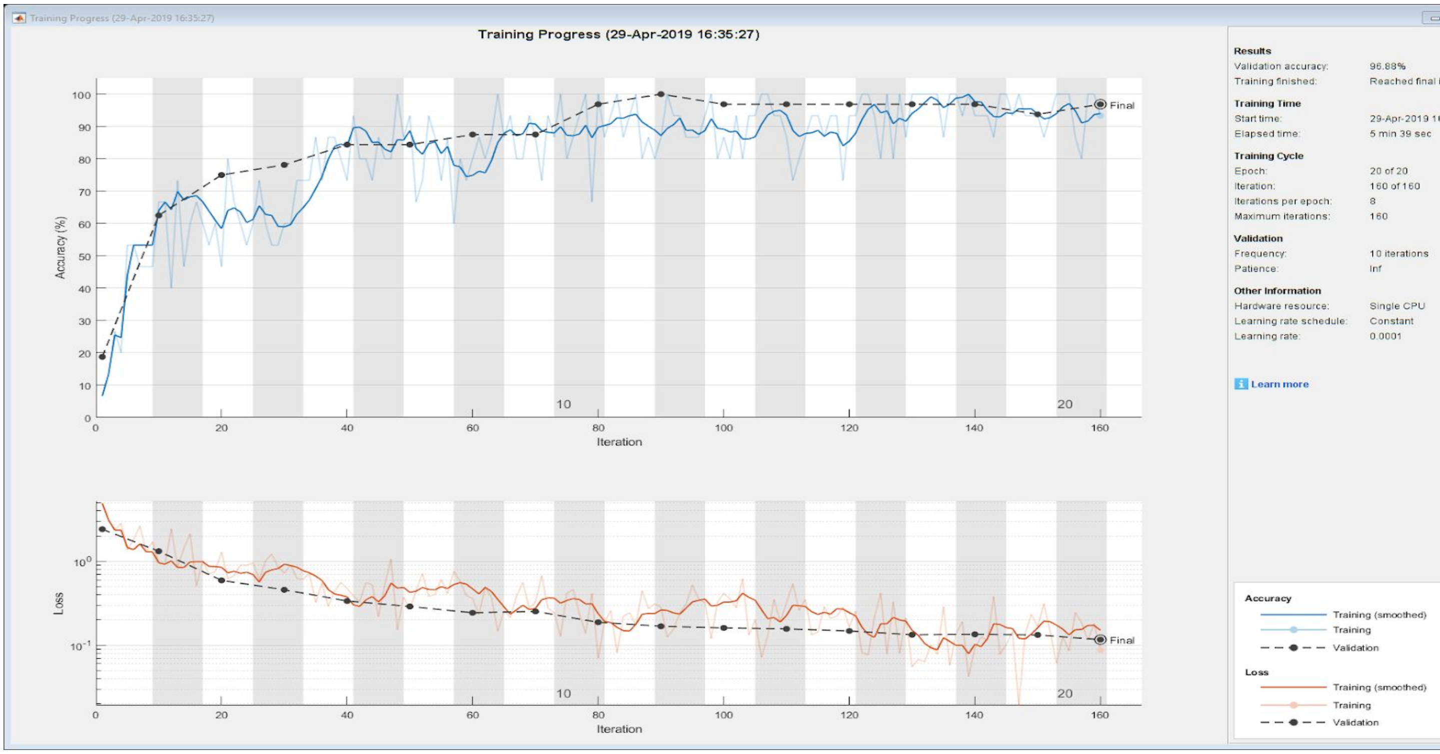
Now the scalogram



The Archetecture used is GoogleNet



Training progress ignore time frame (not done by me)

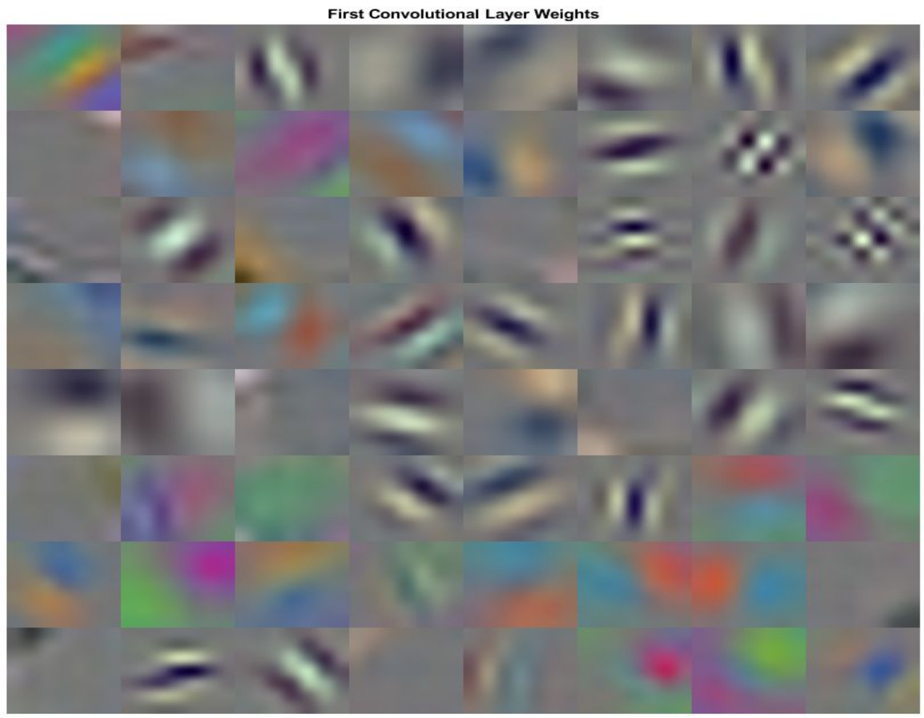


The result looks like this

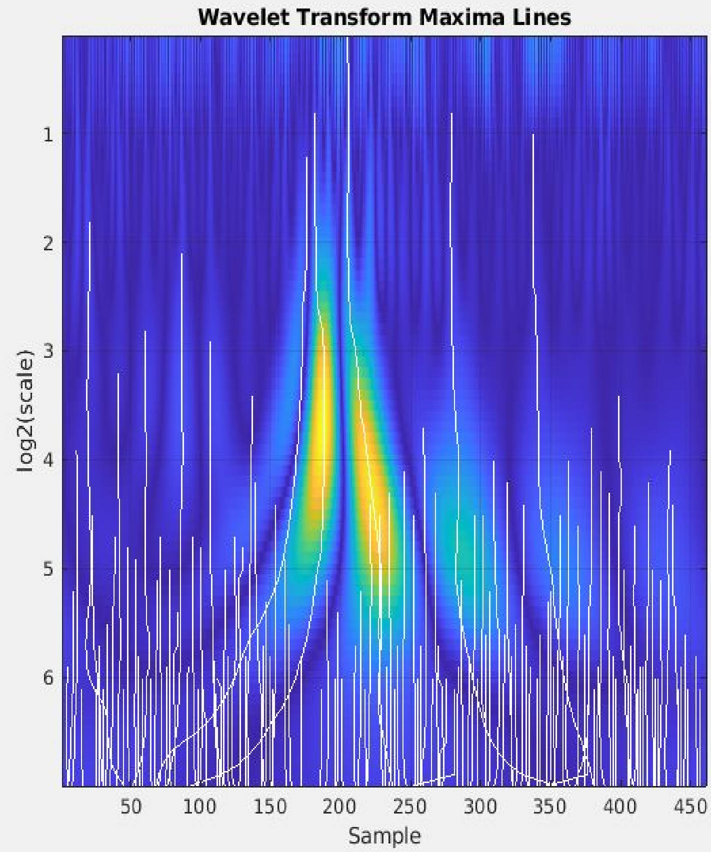
```
Initializing input data normalization.

|======================================================================================

==========================================|

|  Epoch  |  Iteration  |  Time Elapsed  |  Mini-batch  |  Validation  |

Mini-batch  |  Validation  |  Base Learning  |

|         |             |   (hh:mm:ss)   |   Accuracy   |   Accuracy   |

Loss      |     Loss     |      Rate        |

|======================================================================================

==========================================|

|       1 |           1 |       00:00:03 |        6.67% |       18.75% |

4.9207 |       2.4141 |      1.0000e-04 |

|       2 |          10 |       00:00:23 |       66.67% |       62.50% |

0.9589 |       1.3191 |      1.0000e-04 |

|       3 |          20 |       00:00:43 |       46.67% |       75.00% |

1.2973 |       0.5928 |      1.0000e-04 |
```

```
|       4 |          30 |       00:01:04 |       60.00% |       78.13% |
0.7219 |       0.4576 |      1.0000e-04 |
|       5 |          40 |       00:01:25 |       73.33% |       84.38% |
0.4750 |       0.3367 |      1.0000e-04 |
|       7 |          50 |       00:01:46 |       93.33% |       84.38% |
0.2714 |       0.2892 |      1.0000e-04 |
|       8 |          60 |       00:02:07 |       80.00% |       87.50% |
0.3617 |       0.2433 |      1.0000e-04 |
|       9 |          70 |       00:02:29 |       86.67% |       87.50% |
0.3246 |       0.2526 |      1.0000e-04 |
|      10 |          80 |       00:02:50 |      100.00% |       96.88% |
0.0701 |       0.1876 |      1.0000e-04 |
|      12 |          90 |       00:03:11 |       86.67% |      100.00% |
0.2836 |       0.1681 |      1.0000e-04 |
|      13 |         100 |       00:03:32 |       86.67% |       96.88% |
0.4160 |       0.1607 |      1.0000e-04 |
|      14 |         110 |       00:03:53 |       86.67% |       96.88% |
0.3237 |       0.1565 |      1.0000e-04 |
|      15 |         120 |       00:04:14 |       93.33% |       96.88% |
0.1646 |       0.1476 |      1.0000e-04 |
|      17 |         130 |       00:04:35 |      100.00% |       96.88% |
0.0551 |       0.1330 |      1.0000e-04 |
|      18 |         140 |       00:04:57 |       93.33% |       96.88% |
0.0927 |       0.1347 |      1.0000e-04 |
|      19 |         150 |       00:05:18 |       93.33% |       93.75% |
0.1666 |       0.1325 |      1.0000e-04 |
|      20 |         160 |       00:05:39 |       93.33% |       96.88% |
0.0873 |       0.1164 |      1.0000e-04 |
|=============================================================================
========================================|
```

Sharing the weights of first convolution layer

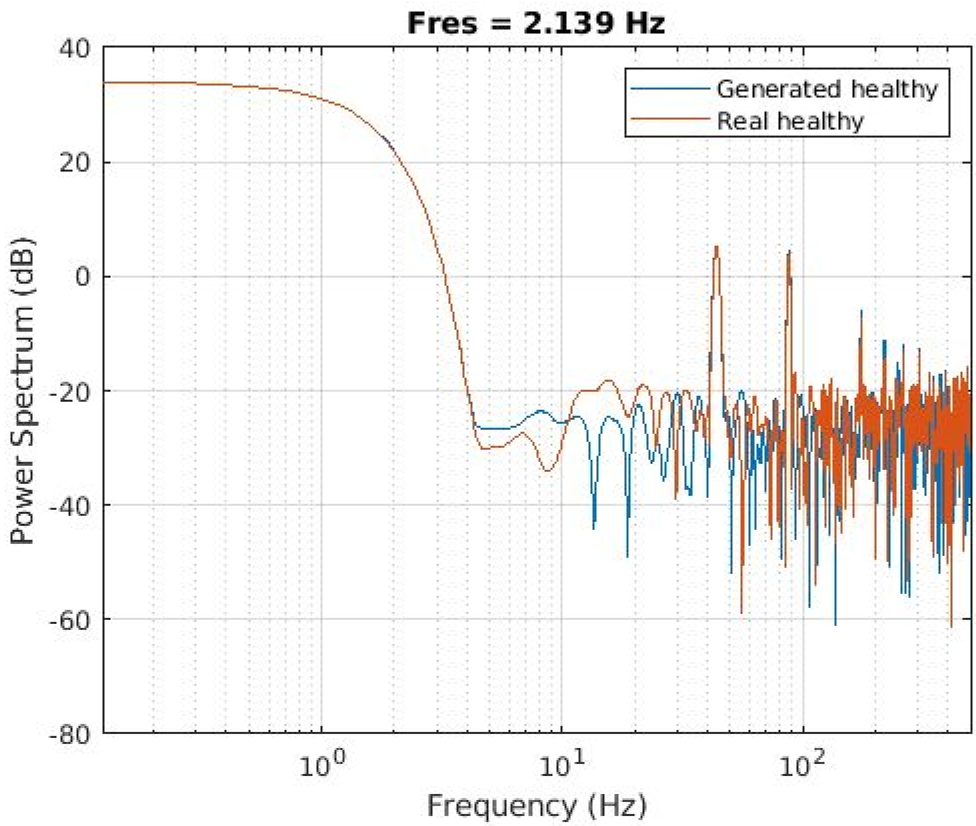


We can also detect crack from data recorded from accelerometer sharing one such result



	Sample	Holder Expone
1	5	-2.15
2	9	-1.82
3	11	-0.39
4	15	-0.69
5	21	-0.30
6	23	-1.16
7	26	-2.23
8	28	-2.11
9	31	-1.88
10	33	-1.78
11	39	-0.76
12	41	-0.59
13	45	-2.26
14	48	-1.32
15	54	-2.10
16	56	-3.38
17	59	-2.36
18	61	-0.27
19	64	-2.20
20	69	-1.52
21	71	-1.28
22	76	-2.82
23	78	-1.69
24	84	-1.13
25	87	-0.17
26	92	-2.17
27	94	-1.62
28	98	-3.13
29	100	-1.48
30	102	-2.67

We can also generate signal accurately for simulation without much knowledge of domain



Future scope :

Add deeplearning codes

Add local wave decomposition codes

Add conclusion

Add engineering Application codes