A Seminar Report

On

Time-Frequency Analysis Of Accelerogram

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Master of Engineering

Degree

By

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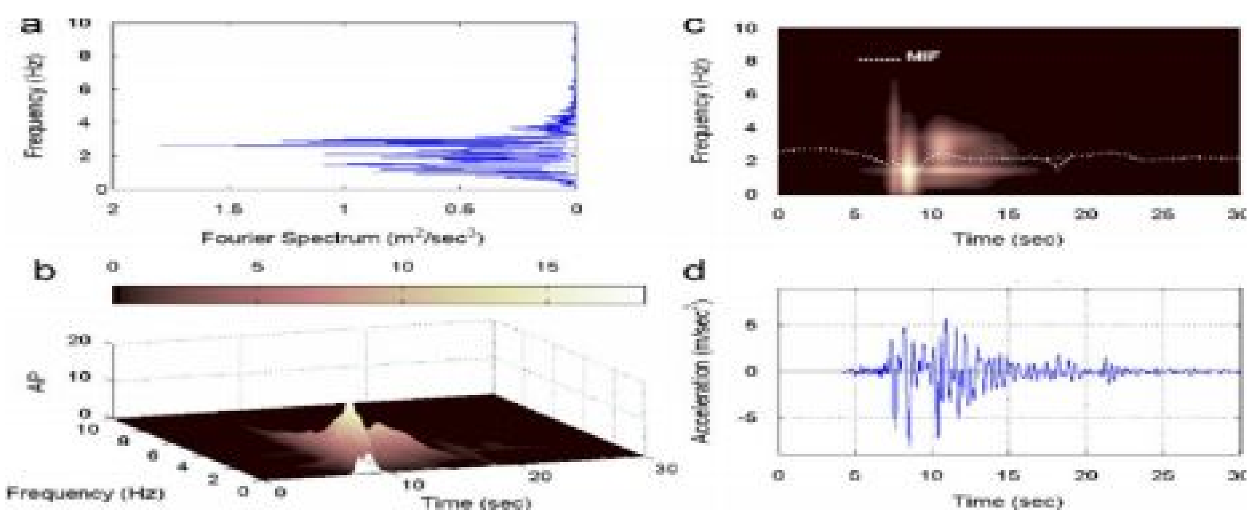
ABSTRACT

In signal processing, time-frequency analysis consists of studying a signal in time and frequency domains simultaneously. Rather than viewing a 1-dimensional signal (a function, real or complex-valued, whose domain is the real line) and some transform (another function whose domain is the real line, obtained from the original via some transform), time-frequency analysis studies a two-dimensional signal – a function whose domain is the two-dimensional real plane, obtained from the signal via a time-frequency transform. These high-level representations such as time-frequency maps convey a wealth of useful information, but they involve a **large number of parameters** that make statistical investigations of many signals difficult at present. In this paper, we will describe a method that performs a drastic reduction in the complexity of time-frequency representations through modeling of the maps by elementary functions, Artificial Intelligence, and Machine learning. The method is validated on artificial signals and subsequently applied to signals recorded at original stations. We will show different methods of doing Time-frequency analysis using techniques like FFT(Fast Fourier Transform), wavelet methods, and how applying Artificial neural networks, deep learning can significantly reduce the complexity of time-frequency analysis with more return in result. We will try to validate the advanced technological improvement in this field to show the potential and promise of technology in this area.

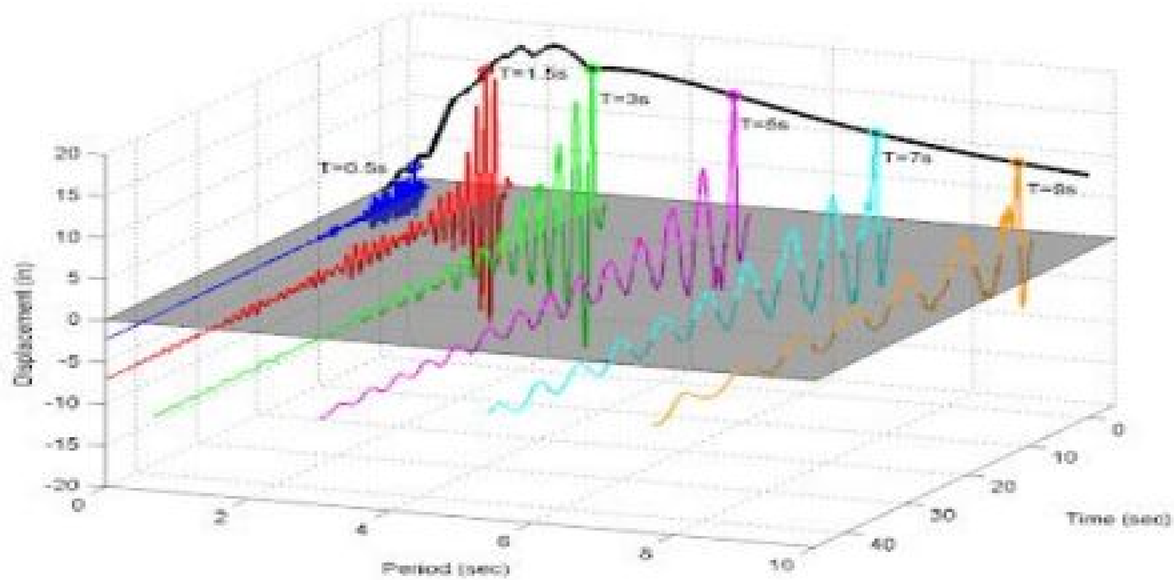
INTRODUCTION

In signal processing, time-frequency analysis consists of studying a signal in time and frequency domains simultaneously. Rather than viewing a 1-dimensional signal (a function, real or complex-valued, whose domain is the real line) and some transform (another function whose domain is the real line, obtained from the original via some transform), time-frequency analysis studies a two-dimensional signal – a function whose domain is the two-dimensional real plane, obtained from the signal via a time-frequency transform. The mathematical motivation for this study is that functions and their transform representation are often tightly connected, and they can be understood better by studying them jointly, as a two-dimensional object, rather than separately.

The practical motivation for time-frequency analysis is that classical Fourier analysis,assumes that signals are infinite in time or periodic, while many signals in practice are of short duration, and change substantially over their duration. For example, accelerograph instruments do not produce infinite duration signals, but instead begin with an attack, then gradually decay. This is poorly represented by traditional methods, which motivates time-frequency analysis. For say Response Spectrum Which hangs with Earthquake Engineers most of the time and is a great source of Information about ground motion parameters but not the only solution upon which you can depend.



A simple Time-Frequency Distribution



A Simple Plot Of Response Spectrum

In this paper we will do our Experiment with simulated data from two different methods and we kept application of the methods on real data for the final semester project.

So let's get started with the project.

Seismograph

A seismograph, or seismometer, is an instrument used to detect and record earthquakes. Generally, it consists of a mass attached to a fixed base. During an earthquake, the base moves and the mass does not. The motion of the base with respect to the mass is commonly transformed into an electrical voltage. The electrical voltage is recorded on paper, magnetic tape, or another recording medium. This record is proportional to the motion of the seismometer mass relative to the earth, but it can be mathematically converted to a record of the absolute motion of the ground. Seismograph generally refers to the seismometer and its recording device as a single unit.

Seismographs are used to determine:

- Magnitude: the size of the earthquake
- **Depth**: how deep the earthquake was
- **Location**: where the earthquake occurred

Some seismometers can measure motions with frequencies from 500 Hz to 0.00118 Hz (1/500 = 0.002 seconds per cycle, to 1/0.00118 = 850 seconds per cycle).Seismograph range:(5 X 10^6 sample/day) which equals to 5 MB per day . So for 200 stations it is roughly 1 GB per day.

Accelerograph

Another important class of seismometers was developed for recording large amplitude vibrations that are common within a few tens of kilometers of large earthquakes - these are called strong-motion seismometers. Strong-motion instruments were designed to record the high accelerations that are particularly important for designing buildings and other structures.
An accelerograph can be referred to as a strong-motion instrument or seismograph, or simply an earthquake accelerometer. They are usually constructed as a self-contained box, which previously included a paper or film recorder (an analogue instrument) but now they often record directly on digital media and then the data is transmitted via the Internet.
Strong motion sensors measure large amplitude seismic signals and are usually accelerometers. Strong motion accelerometers can measure up to 3.5 g with a system noise level less than 1μg/√Hz. Weak motion sensors can measure very low amplitude seismic signals with a noise level of less than 1ng/√Hz.
Range of Accelerometer: 1 Hz to 6 kHz

A Comparison Between Accelerogram and Seismogram

What is the difference between a seismograph and an accelerograph? Which one is used for structural monitoring and which is for earthquake detection?

A seismograph is a generic term used to describe a recording device that detects ground motion due to earthquakes. Typically this will comprise a recorder and a seismometer, which is a sensor that detects the velocity of the ground. Seismometers are usually very sensitive and will easily detect a typical quarry blast at a range of 100km. Seismometers should not be confused with geophones, which also detect ground velocity but are typically much less sensitive and are used for close range blast monitoring and surveying.
An accelerograph is a recorder that uses an accelerometer, which as you can tell from the name detects the acceleration of the ground. Accelerometers are much less sensitive than seismometers, but have a much greater range, detecting ±2g or more of ground acceleration

(things start flying off the ground at 1g, when gravity is overcome). By comparison a seismometer will clip at full scale if you tap it too hard with your finger.

So, seismometers are good for detecting very small levels of ground motion (from very small or very distant events), and accelerometers are good at recording strong ground motion that is potentially damaging at the recording location. We will often install an earthquake recording station using both types of sensor to get the best of both worlds.

SMAs, or strong motion accelerographs, are usually all that is required for monitoring the response of a structure during an earthquake, whether this a building, bridge, dam, power station, or any other critical infrastructure that could be affected by a large earthquake. Signals that are too weak to be clearly visible on an accelerograph will generally not be of any concern to the structural integrity of the asset.

The SRC has used various types of SMAs over the years. Those with MEMS accelerometers use tiny sensors like those found in cars to trigger airbags on impact, but are much more sensitive. They are less sensitive than dedicated earthquake accelerometers, but are still sufficient to record acceleration from significant earthquakes as the lower sensitivity limit is still well below damage thresholds. For monitoring the natural frequency and modal response of structures to earthquakes, more sensitive earthquake accelerometers are required.

Data Recording

Almost all observatory-grade seismic recorders use 24-bit or 32-bit analogue to digital converters (ADCs), although the useful range of currently available 32-bit ADCs is limited to the lower 24 bits – the other 8 bits are digital noise.

24-bits equates to 16,777,216 counts of recording range. If the average (RMS) noise level is just one count out of this range, then the dynamic range of recording is defined as:
$20\log10(16777216/1) = 144.5dB$

The USGS ANSS guidelines(1) require that the dynamic range of a data acquisition unit should be ≥24-bits, based on the RMS noise compared to the RMS of the zero to peak signal of a sine wave, which would be: $20\log10(8388608/1/\sqrt{2}) = 135.5dB$ Whichever way that dynamic range is defined, there is still only 1 count of noise, and the full scale range is still ±8,388,608 counts.

We will often see very large dynamic range numbers quoted for digitisers, even when based on the ANSS method. This is possible when the RMS value of the noise is less than one count. It is possible to have less than one integer count of noise because the RMS value is an average over a number of samples. A small reduction in the fraction of a count has a huge impact on the dynamic range number, but in practice it means very little. For example:
$20\log10(8388608/0.5/\sqrt{2}) = 141.5dB$

$20\log10(8388608/0.1/\sqrt{2}) = 155.5dB$ Apart from the digitiser noise levels we need to consider sensor noise levels. A typical ADC input range is 40 Volts peak-to-peak, so a single count in a 24-bit range is equivalent to 2.3µV. One of the first things to look at is the electronic noise level of the sensor is over the bandwidth of interest. Noise increases with frequency, so another way sensors and recorders can be quoted with high dynamic range figures is by looking at a very low frequency band or recording data at a low sample rate. In all sensors testing a 6-channel or 12-channel Kelunji EchoPro seismic recorder with 24-bit ADCs was used. Data was recorded at 100 samples per second (sps), giving a bandwidth of DC to 40Hz (after FIR filtering). The ANSS requirement for digitisers recording frequencies up to 30Hz is 123.4dB, and the EchoPro has a dynamic range of 131.5dB at 100sps and 123.3dB at 500sps.

There are measuring instruments available now that are used to lower the noise level and used in prediction of Earthquake directly developed in Rice University,Huston,Texas.

Application of Time-frequency in seismic Engineering:

- One major benefit of applying a time-frequency transform to a signal is discovering the pattern of frequency changes.
- Once the pattern is identified we can classify the signal pattern. For Example a pattern of changing frequency might indicate the entry or exit of seismic vibration in localized context.
- Another important use of time-frequency analysis is to reduce random noise in noise-corrupted signals. Like noise generated from a recording machine.
- You also can use time-frequency analysis to determine if a signal has distinct time-frequency components and isolate those components for further analysis.
- Seismological signal processing, such as detection of soil liquefaction.
- Seismic prospecting for oil and gas has undergone a digital revolution during the past decade. Most stages of the exploration process have been affected: the acquisition of data, the reduction of these data in preparation for signal processing, the design of digital filters to detect **primary echoes (reflections) from buried interfaces**, and the development of technology to extract from these detected signals information on the geometry and physical properties of the subsurface. The seismic reflection is generally weak, and it must be strengthened by the use of signal summing (stacking) procedures.
- Indicate presence of Natural oil and gas.
- Very Much beneficial for Earthquake prediction.

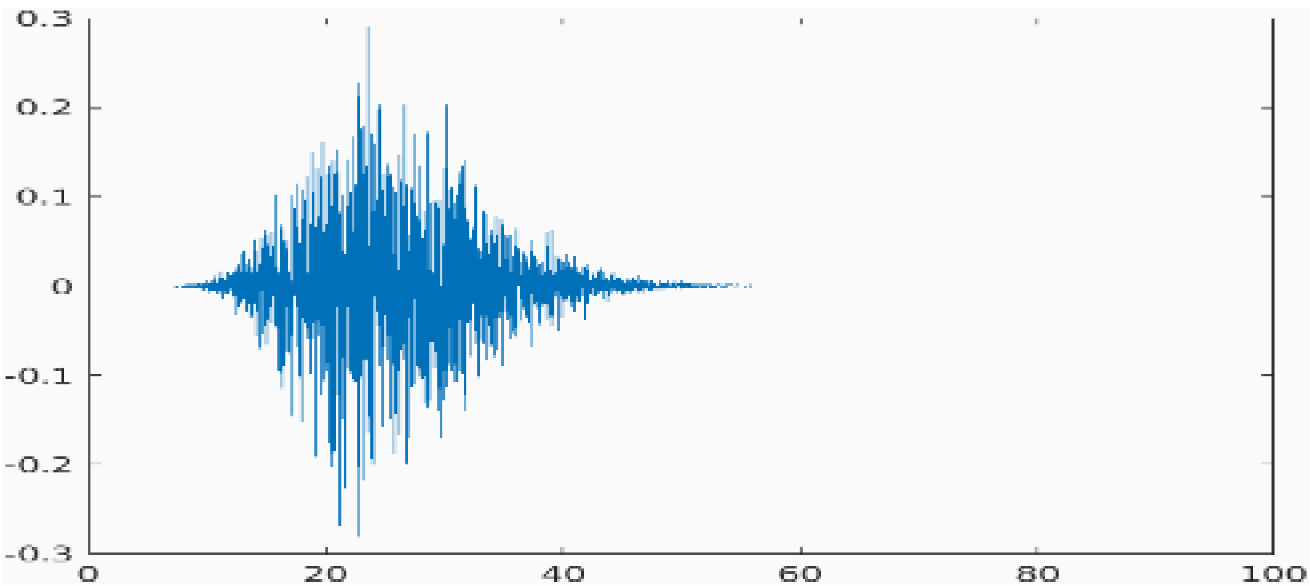
Generation Of Artificial Earthquake Data:

Traditional Method

For generating Artificial Earthquake acceleration data I have used many approaches but choose to work with Kanai–Tajimi Model.
A few words regarding data generation using different approaches.
Over the past few decades, primarily two approaches have been adopted in evaluation of the strong earthquake ground motions for a seismic design of the engineered structures.One is to use the recorded strong motions. In some cases the recorded motions are modified to better represent local soil conditions. The second is to generate strong ground motions by combining an appropriate seismological model with knowledge of seismicity and geology of the site. While

recording cannot be possible for most of the area using some property like w^{-2}.(w is the frequency of signal).,used to generate small earthquakes. For large earthquakes we use extended sources with finite dimensions.

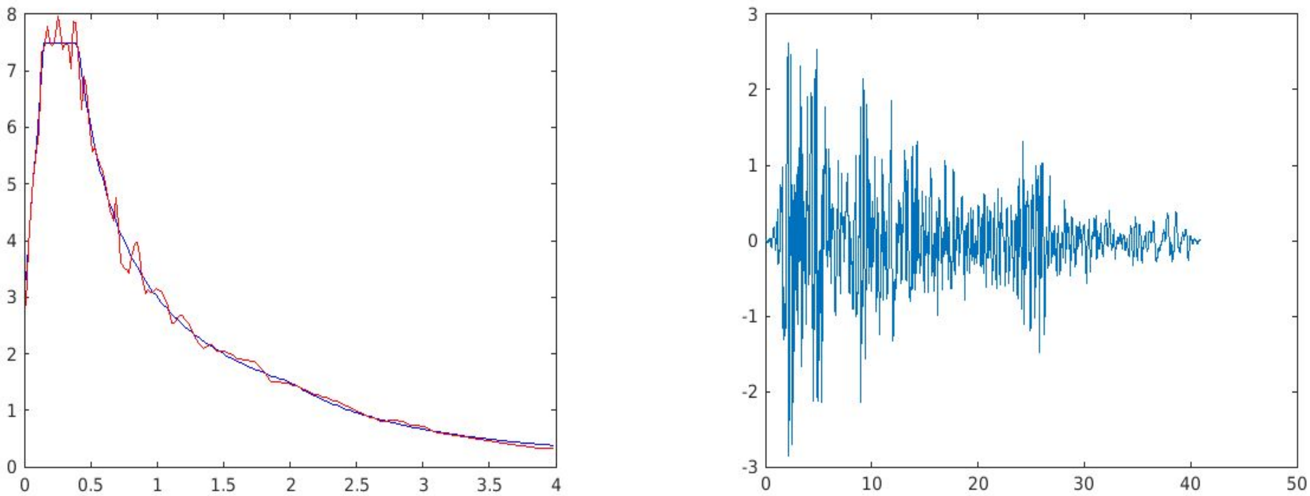
With kanai-Tajimi model we plotted the generated Earthquake Acceleration is time domain(below image).



Earthquake Generated With Kanai-Tajimi Model(X-axis:time Y-axis:Acceleration both in MKS unit)

Code Based Approach

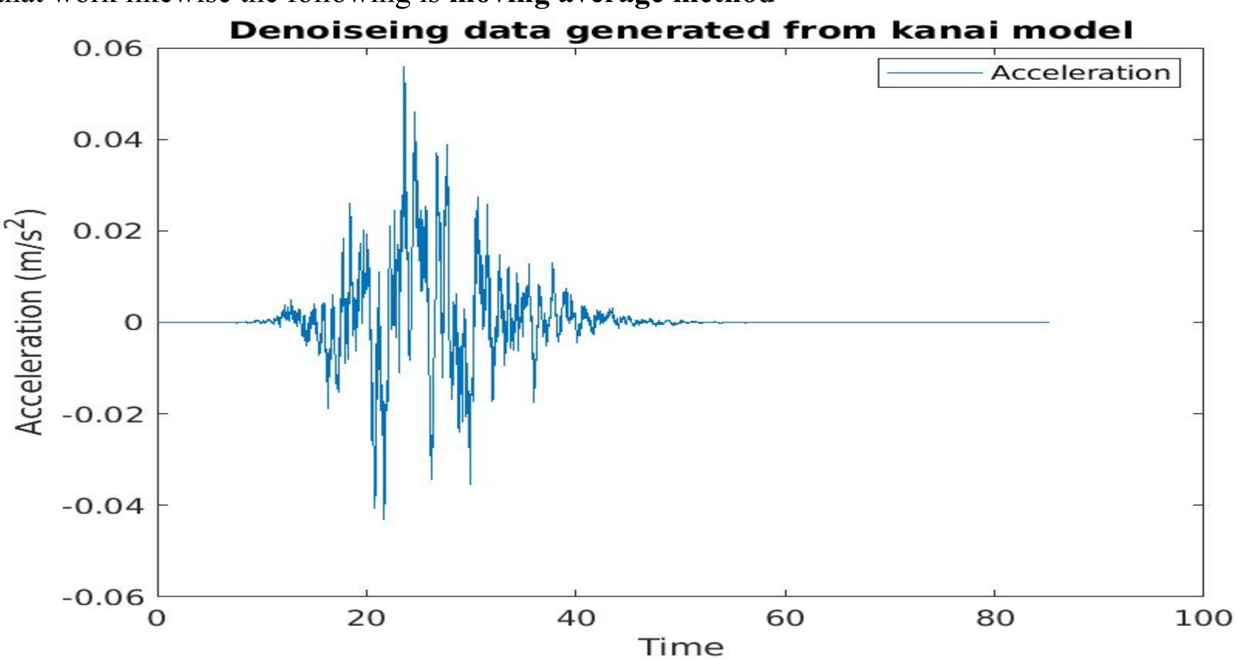
Now here comes the technology as you can see in the code there are a lot of variables which are used to generate this earthquake that involves a lot of Domain Knowledge as well as technical Expertise . We can also with the help of data generate Earthquakes.The function took soil category,type,importance factor and base peak acceleration reference then with the help of data (contains ElCentro Earthquake (1940) Gebze Earthquake (1999) Mexico City Earthquake (1985)) and based upon preference writes output of artificial sign and artificial spectrum.The working of Algorithm is not part of this paper so without further mentioning about it I am sharing one such plot that I have generated using This.



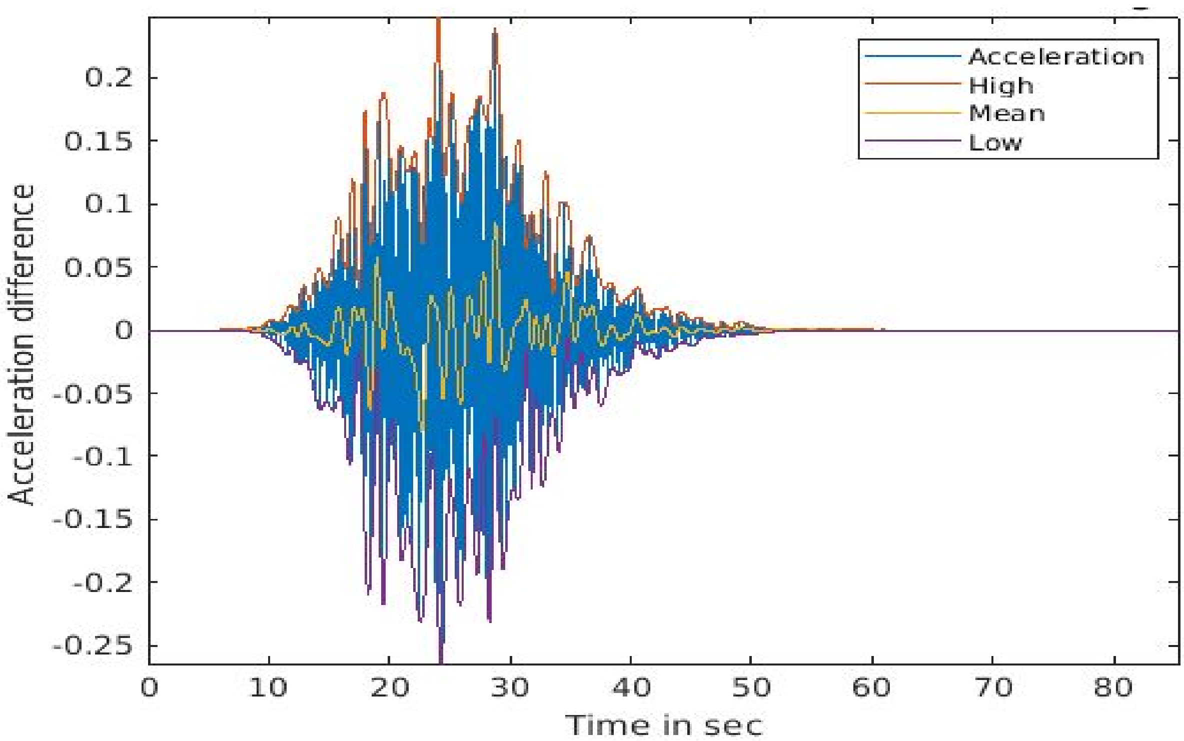
From Left to Right Response Spectrum (X axis:period Y axis:Sa) Corresponding Earthquake(X axis : period Y axis:Acceleration Record)

Denoising Methods

As the data contains a lot of noise By applying Various Denoise Methods We get a more simplified Signal to analyse Which gives our analysis more insights. But we have to use methods that work likewise the following is **moving average method**

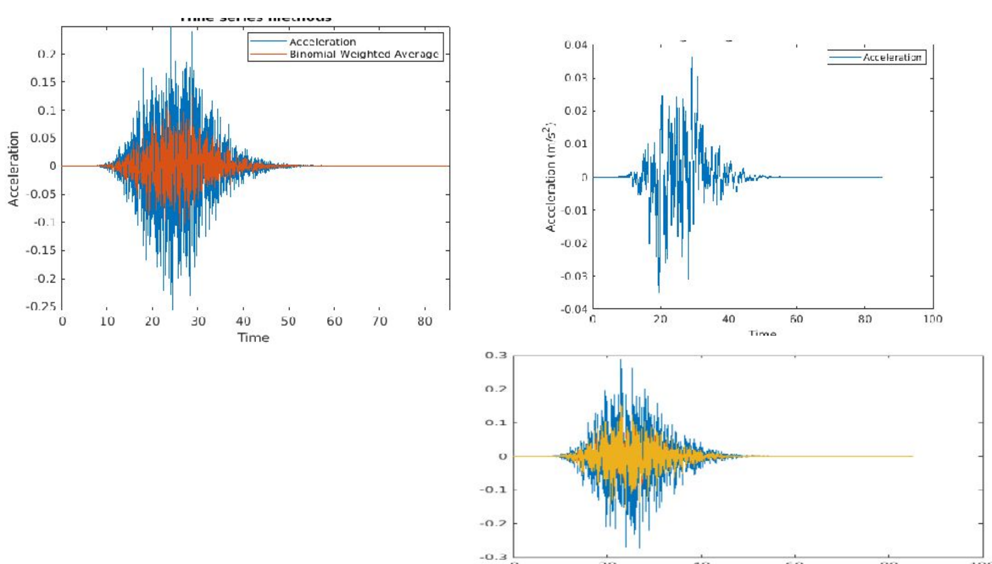


Clearly it eliminates the peak rear signals by assuming noise and gives us the low values which are frequently generated but as Earthquake is a rear event this diagram has very little importance to us.



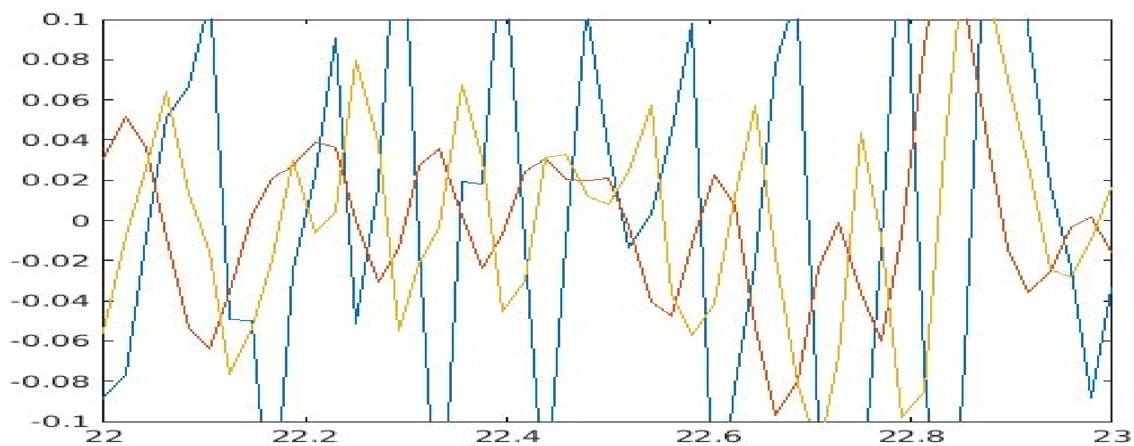
The graph above shows the high low and mean value which capture all the required information about the Earthquake.

Next, On Applying Different methods we further smoothen our generated data showing some results



From Top Left Binomial Model Average Weight Model Exponential Model

If we zoom into the plot containing Information of Exponential and Binomial methods we have this result



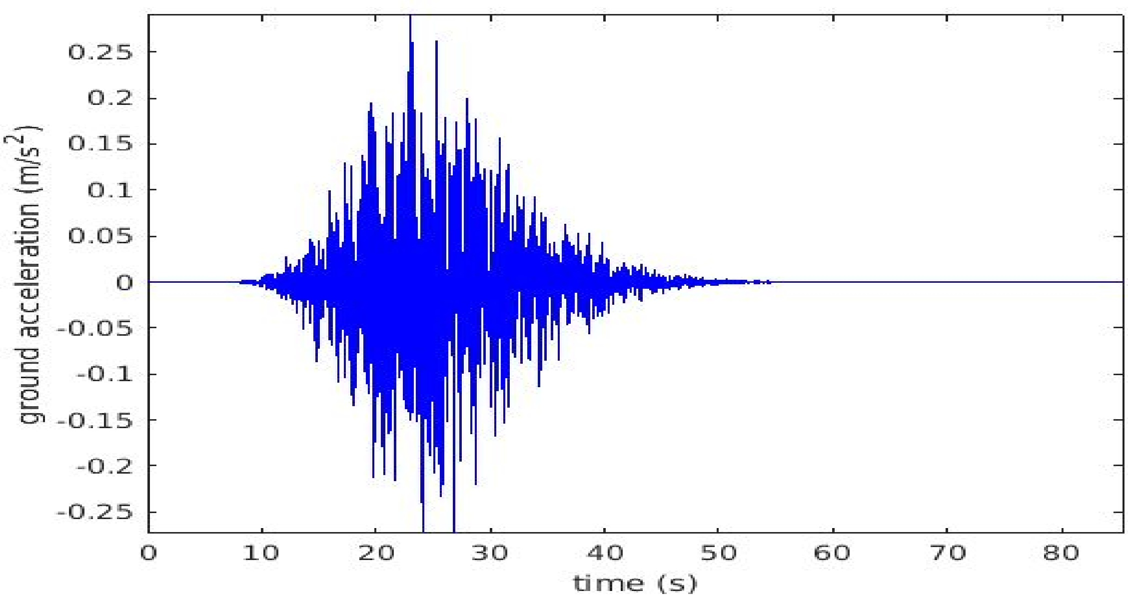
Blue Original data Yellow Exponential Method Orange Binomial Method

So based on the evidence we can say for our data exponential smoothing fits the original data points way better than any other smoothing methods.

Response Spectrum

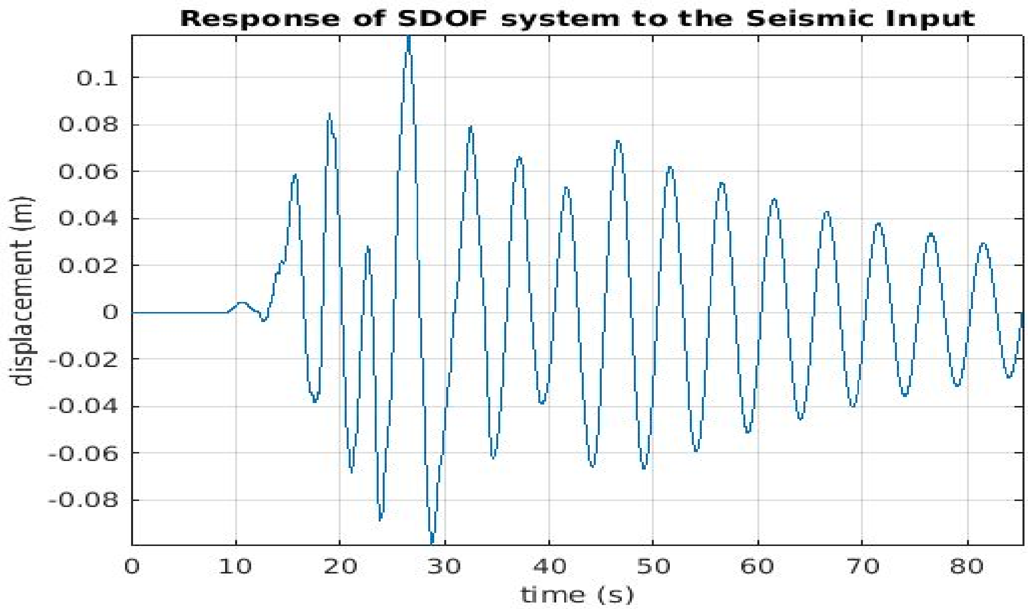
For just giving a brief description I am sharing a response spectrum plotted using Matlab.

The Earthquake in time domain



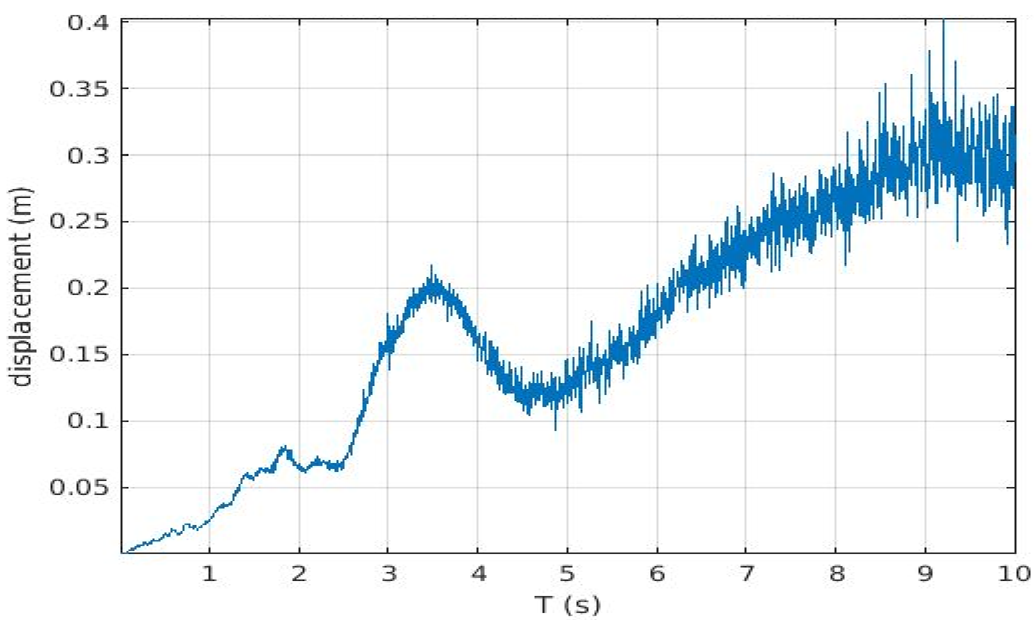
Earthquake plot in Time domain (Acceleration VS Time)

The response of a standard system wrt the Earthquake is



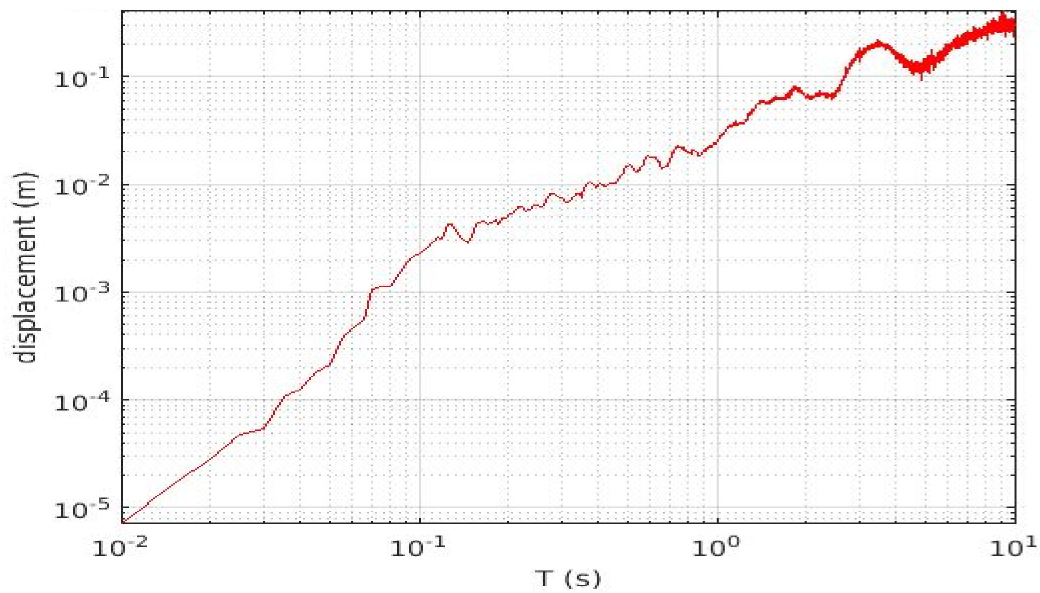
Response Of SDOF System In Earthquake

The Displacement response of SDOF system is



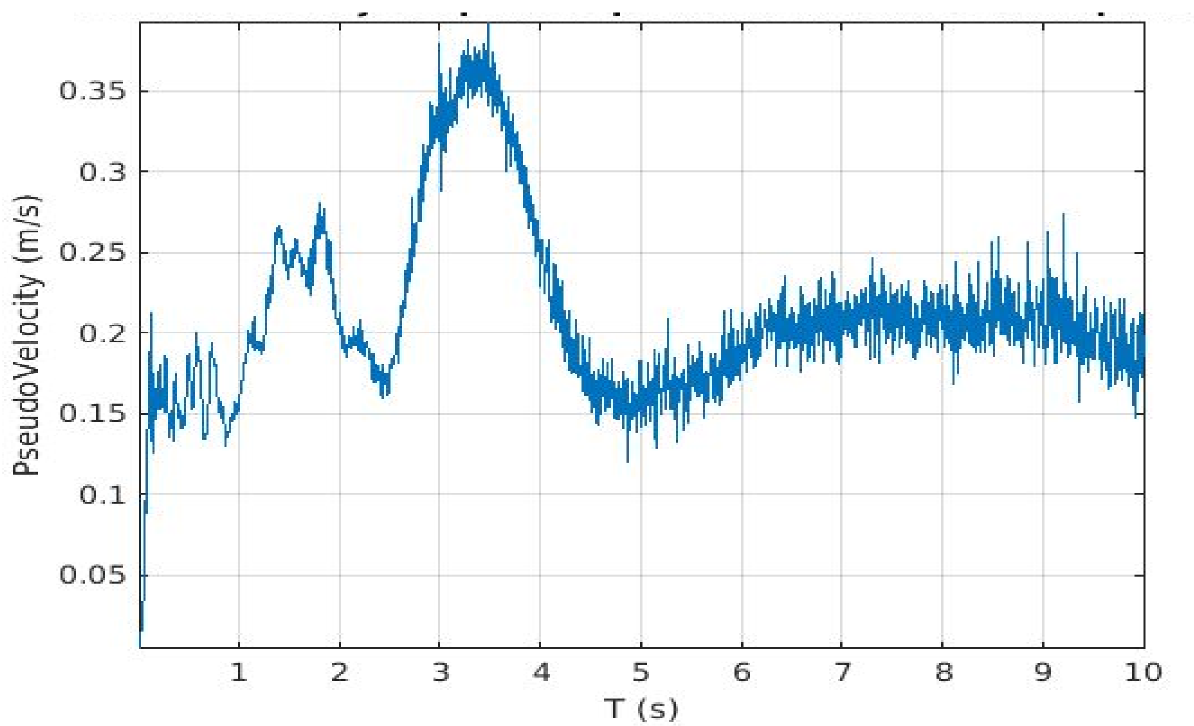
Displacement Spectrum Of The Earthquake

In log-log scale

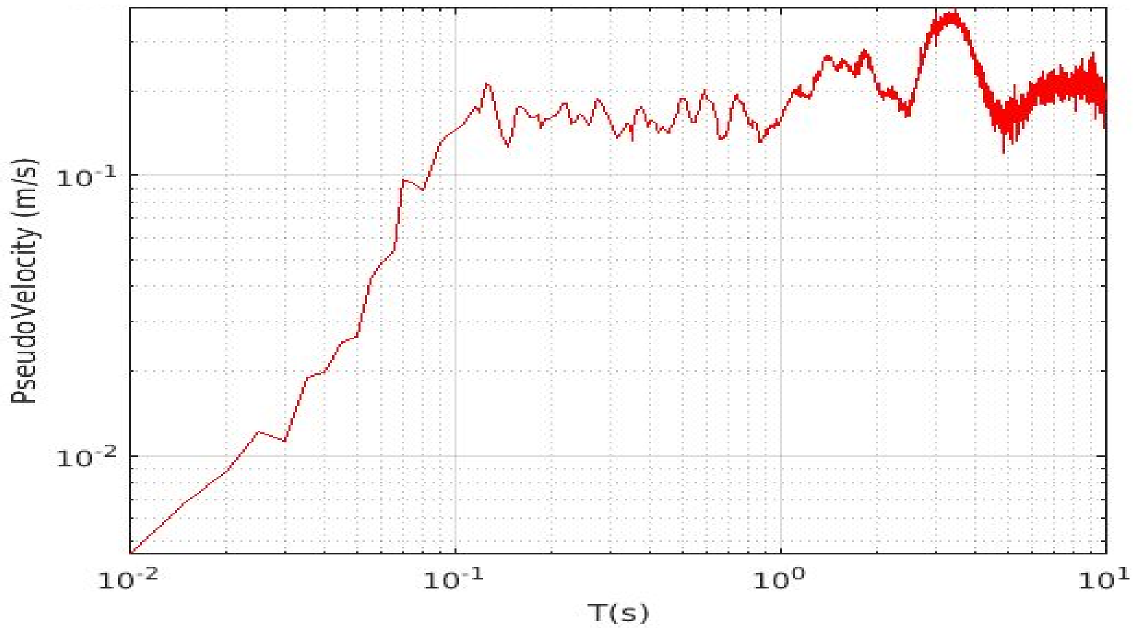


Displacement In log-log Scale

The velocity Spectrum in Normal and log scale

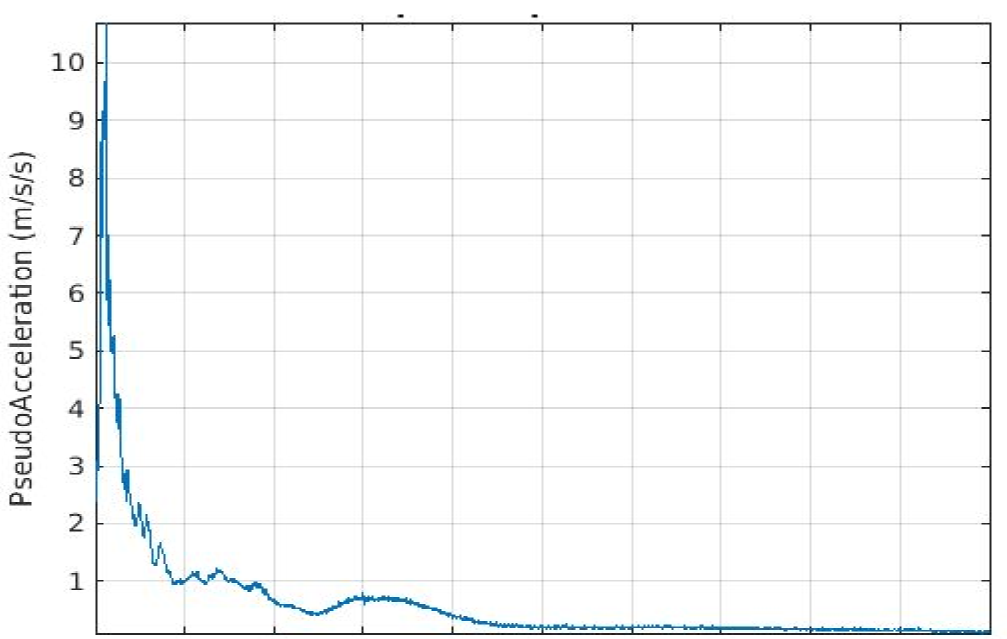


PseudoVelocity Response Spectrum Of Earthquake

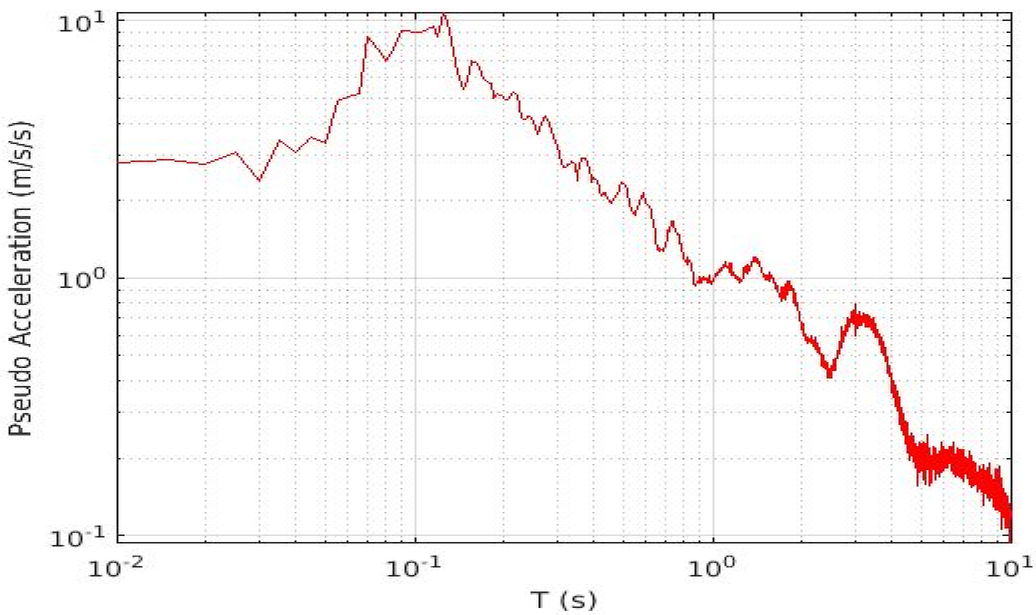


PseudoVelocity Response Spectrum Of Earthquake In Log-log

And the acceleration curve



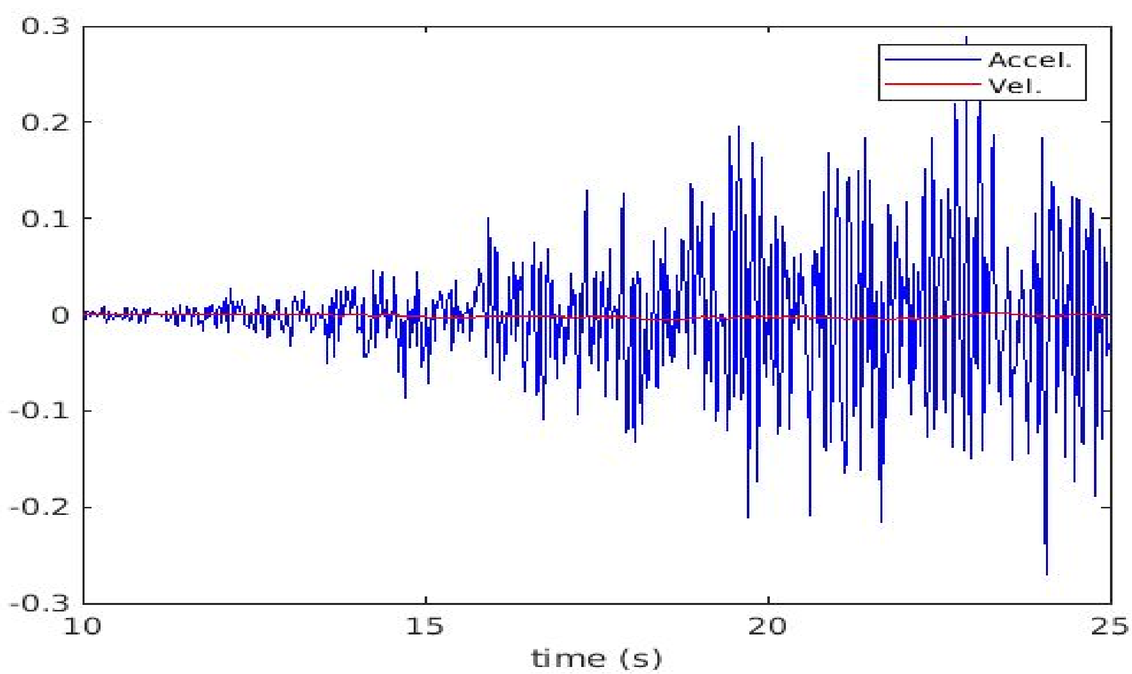
PseudoAcceleration Response Spectrum Of Earthquake



PseudoAcceleration Response Spectrum Of Earthquake in log Scale

Analysis Of The Earthquake

We can see the Excitation periods, Acceleration and velocity closely in this graph



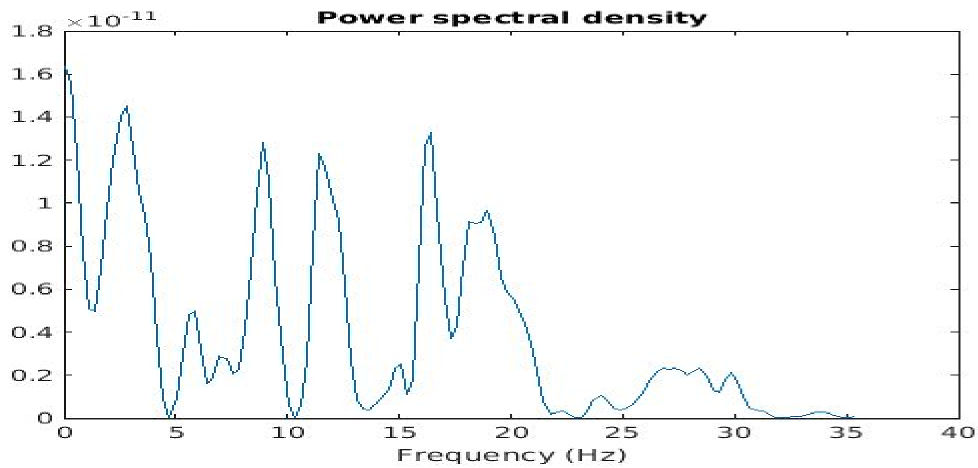
Peak Earthquake Response

Methods Of Time Frequency Distributions

Now we will discuss the different methods applied for time frequency distribution of our generated data.

- **Fourier Transform**

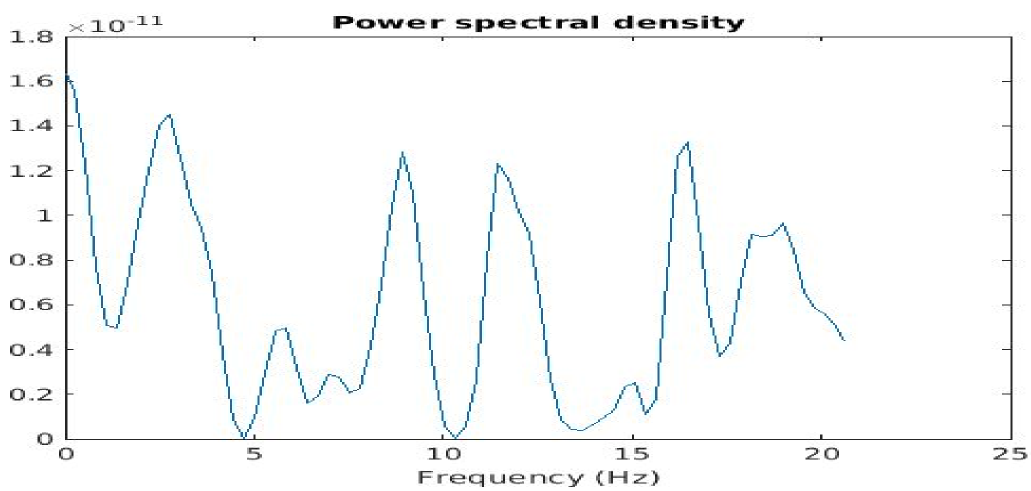
  In Matlab we use the fft function to transform the signal into fourier domain fft has very less time complexity O(nlogn). Brief of fourier transform is not part of this paper.
  Results



Fourier Transform

One point to be noted our Earthquake resides for 60 sec and we got 4096 data point at a rate of 70 sampling frequency.

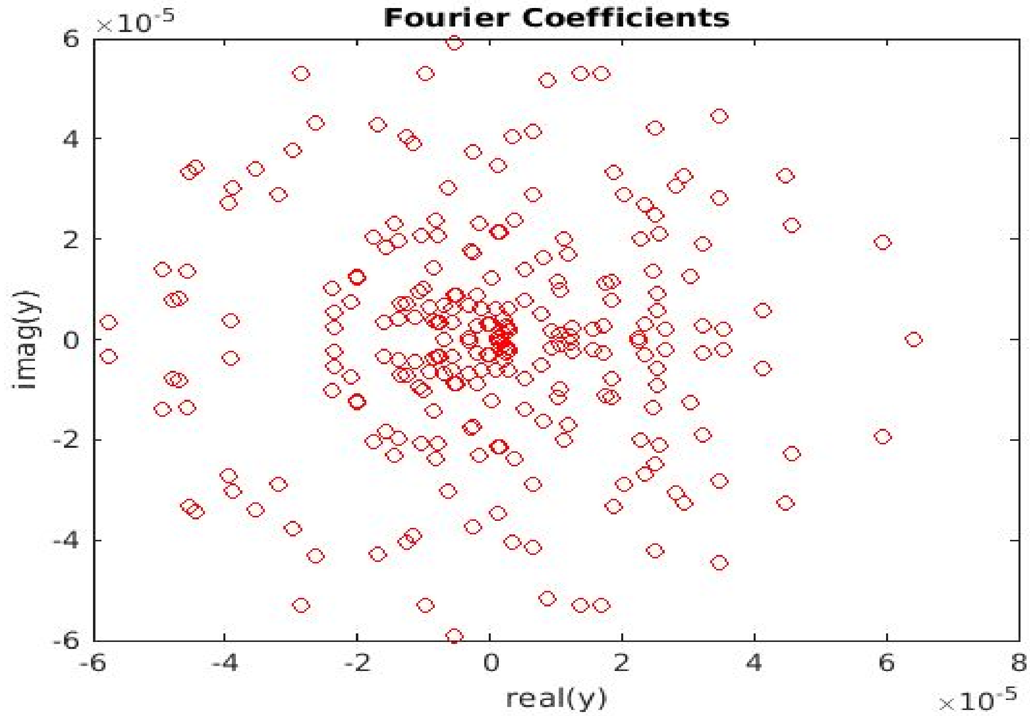
On zooming the figure we can see clearly



Fourier Transform

The dominating frequency here is 10 HZ.

The Fourier Coefficients are



Fourier Coefficients

- **STFT METHOD**
- The Short-time Fourier transform (STFT), is a Fourier-related transform used to determine the sinusoidal frequency and phase content of local sections of a signal as it changes over time.[1] In practice, the procedure for computing STFTs is to divide a longer time signal into shorter segments of equal length and then compute the Fourier transform separately on each shorter segment. This reveals the Fourier spectrum on each shorter segment. One then usually plots the changing spectra as a function of time, known as a spectrogram or waterfall plot.

In general with this type of data we can have continuous or discrete time STFT. Simply, in the continuous-time case, the function to be transformed is multiplied by a window function which is nonzero for only a short period of time. The Fourier transform (a one-dimensional function) of the resulting signal is taken as the window is slid along the time axis, resulting in a two-dimensional representation of the signal. Mathematically, this is written as:

$$\mathbf{STFT}\{x(t)\}(\tau,\omega) \equiv X(\tau,\omega) = \int_{-\infty}^{\infty} x(t)w(t-\tau)e^{-i\omega t}\,dt$$

where w is the window function, commonly a Hann window or Gaussian window centered around zero, and x(t) is the signal to be transformed (note the difference between the window function w and the frequency ω). $X(\tau,\omega)$ is essentially the Fourier transform of $x(t)w(t-\tau)$ a complex function representing the phase and magnitude of the signal over time and frequency. Often phase unwrapping is employed along either or both the time axis, τ, and frequency axis ω, to suppress any jump discontinuity of the phase result of the STFT. The time index τ is normally considered to be "slow" time and usually not expressed in as high resolution as time t.

In the discrete time case, the data to be transformed could be broken up into chunks or frames (which usually overlap each other, to reduce artifacts at the boundary). Each chunk is Fourier transformed, and the complex result is added to a matrix, which records magnitude and phase for each point in time and frequency. This can be expressed as:

$$\mathbf{STFT}\{x[n]\}(m,\omega) \equiv X(m,\omega) = \sum_{n=-\infty}^{\infty} x[n]w[n-m]e^{-j\omega n}$$

likewise, with signal x[n] and window w[n]. In this case, m is discrete and ω is continuous, but in most typical applications the STFT is performed on a computer using the fast Fourier transform, so both variables are discrete and quantized.

The magnitude squared of the STFT yields the spectrogram representation of the Power Spectral Density of the function

$$\mathrm{spectrogram}\{x(t)\}(\tau,\omega) \equiv |X(\tau,\omega)|^2$$

We can also have invertible STFT, that is, the original signal can be recovered from the transform by the Inverse STFT. The most widely accepted way of inverting the STFT is by using the overlap-add (OLA) method, which also allows for modifications to the STFT complex spectrum. This makes for a versatile signal processing method,[3] referred to as the overlap and add with modifications method.

Limitations: One of the pitfalls of the STFT is that it has a fixed resolution. The width of the windowing function relates to how the signal is represented—it determines whether there is good frequency resolution (frequency components close together can be separated) or good time resolution (the time at which frequencies change). A wide window gives better frequency resolution but poor time resolution. A narrower window gives good time resolution but poor frequency resolution. These are called narrowband and wideband transforms, respectively.
The product of the standard deviation in time and frequency is limited. The boundary of the uncertainty principle (best simultaneous resolution of both) is reached with a Gaussian window function, as the Gaussian minimizes the Fourier uncertainty principle.
So the only consideration required is window function which can be changed to get various results.

Mathematical Property Of STFT

$$\begin{aligned} X_m(\omega) &= \sum_{n=-\infty}^{\infty} x(n)w(n-mR)e^{-j\omega n} \\ &= \text{DTFT}_\omega(x \cdot \text{SHIFT}_{mR}(w)), \end{aligned}$$

This is STFT in it's simple form.

If the window $w(n)$ has the *Constant OverLap-Add (COLA) property* at hop-size R , *i.e.*, if

$$\boxed{\sum_{m=-\infty}^{\infty} w(n-mR) = 1,\ \forall n \in \mathbb{Z}} \quad (w \in \text{COLA}(R))$$

$$
\begin{aligned}
\sum_{m=-\infty}^{\infty} X_m(\omega) &\triangleq \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(n) w(n-mR) e^{-j\omega n} \\
&= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \underbrace{\sum_{m=-\infty}^{\infty} w(n-mR)}_{1 \text{ if } w \in \text{COLA}(R)} \\
&= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \\
&\triangleq \text{DTFT}_\omega(x) = X(\omega).
\end{aligned}
$$

The above equation validates the fact that in constant overlapping both the summation will be the same. But in reality STFT is computed as a succession of FFT with windowed data.

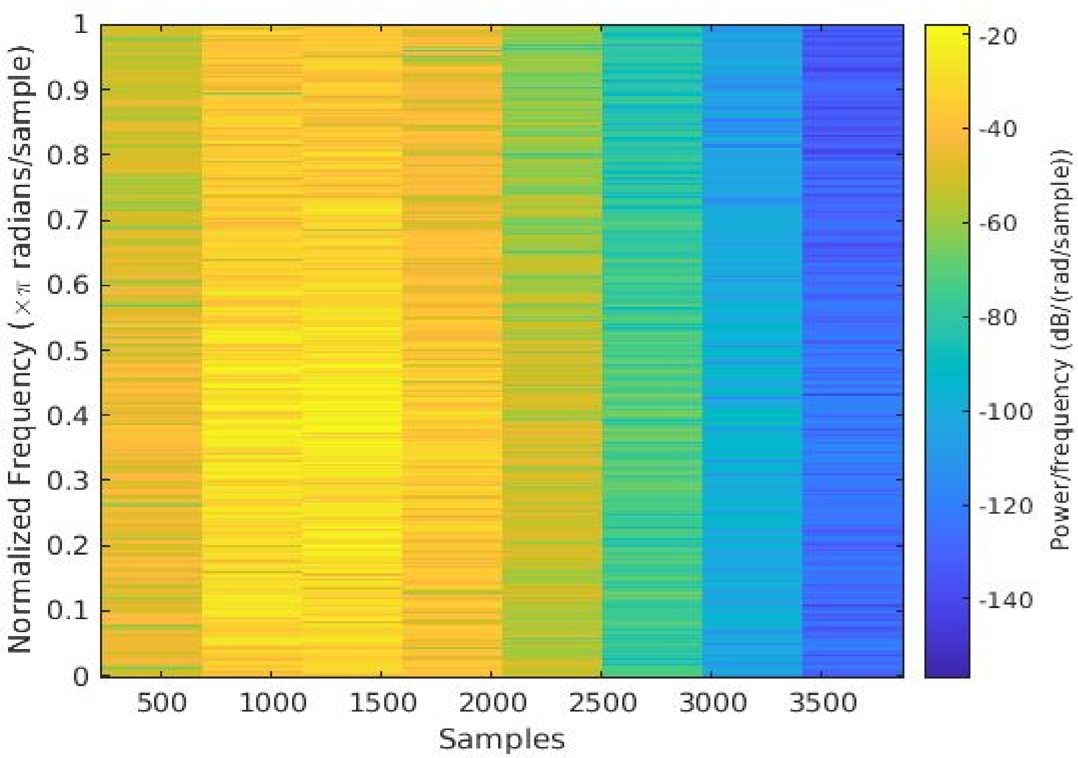
$$X_m(\omega) = \sum_{n=-\infty}^{\infty} x(n+mR)w(n)e^{-j\omega(n+mR)}$$

$$= e^{-j\omega mR} \sum_{n=-\infty}^{\infty} x(n+mR)w(n)e^{-j\omega n}$$

$$= e^{-j\omega mR}\mathrm{DTFT}_\omega(\mathrm{S{\scriptsize HIFT}}_{-mR}(x) \cdot w).$$

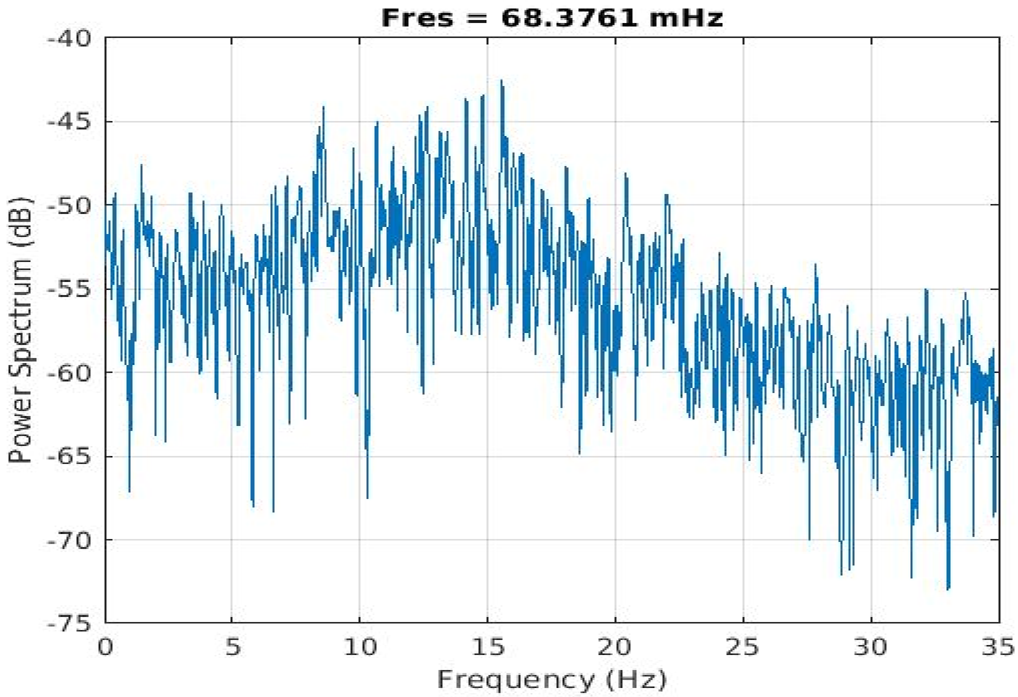
Results

Using a custom STFT function we got TF distribution



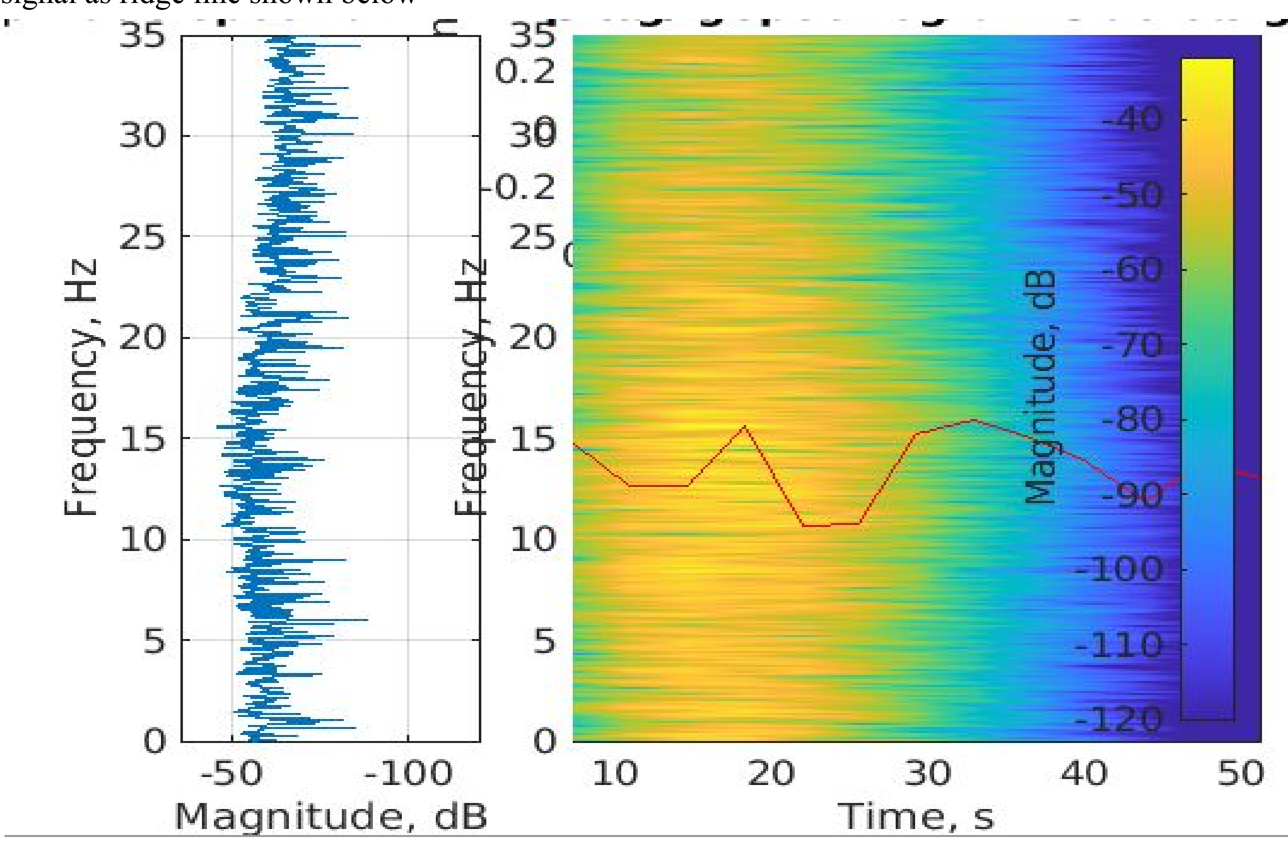
Spectrogram

The signal in frequency domain with default frequency resolution



Frequency Domain

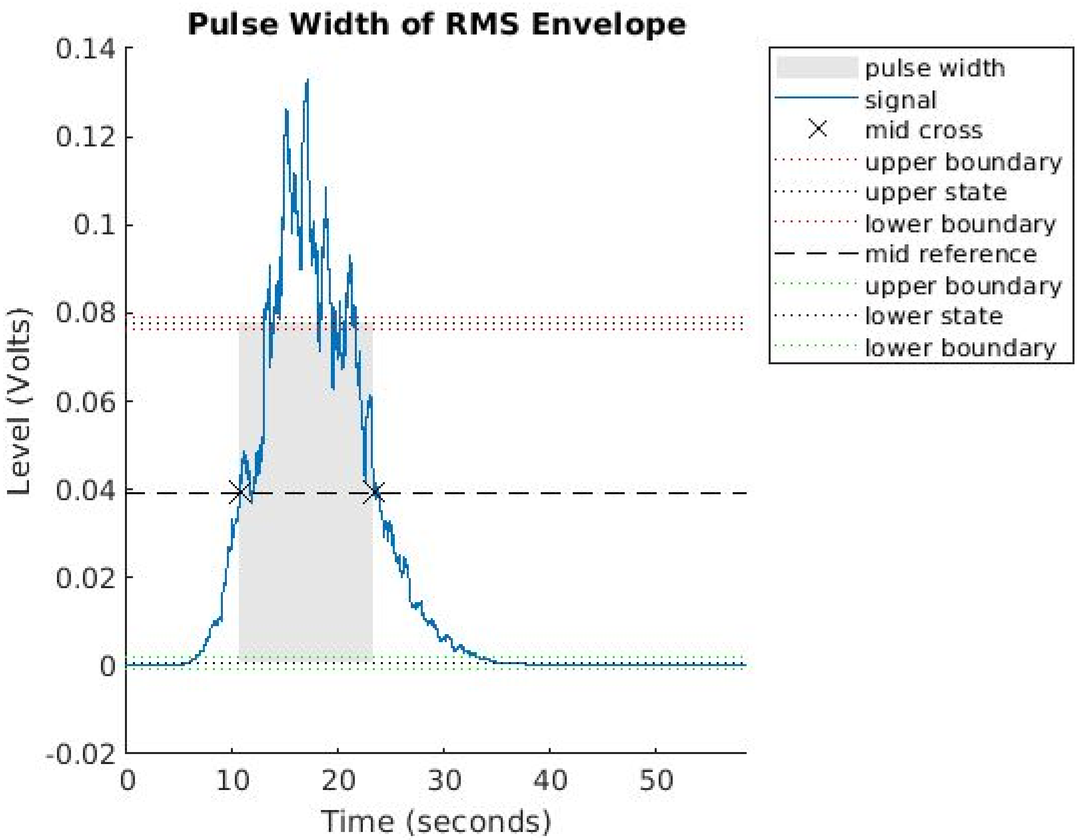
Now we can visualize the Signal in Frequency domain as well as Time Frequency domain with signal as ridge line shown below



Frequency and Tf Plot

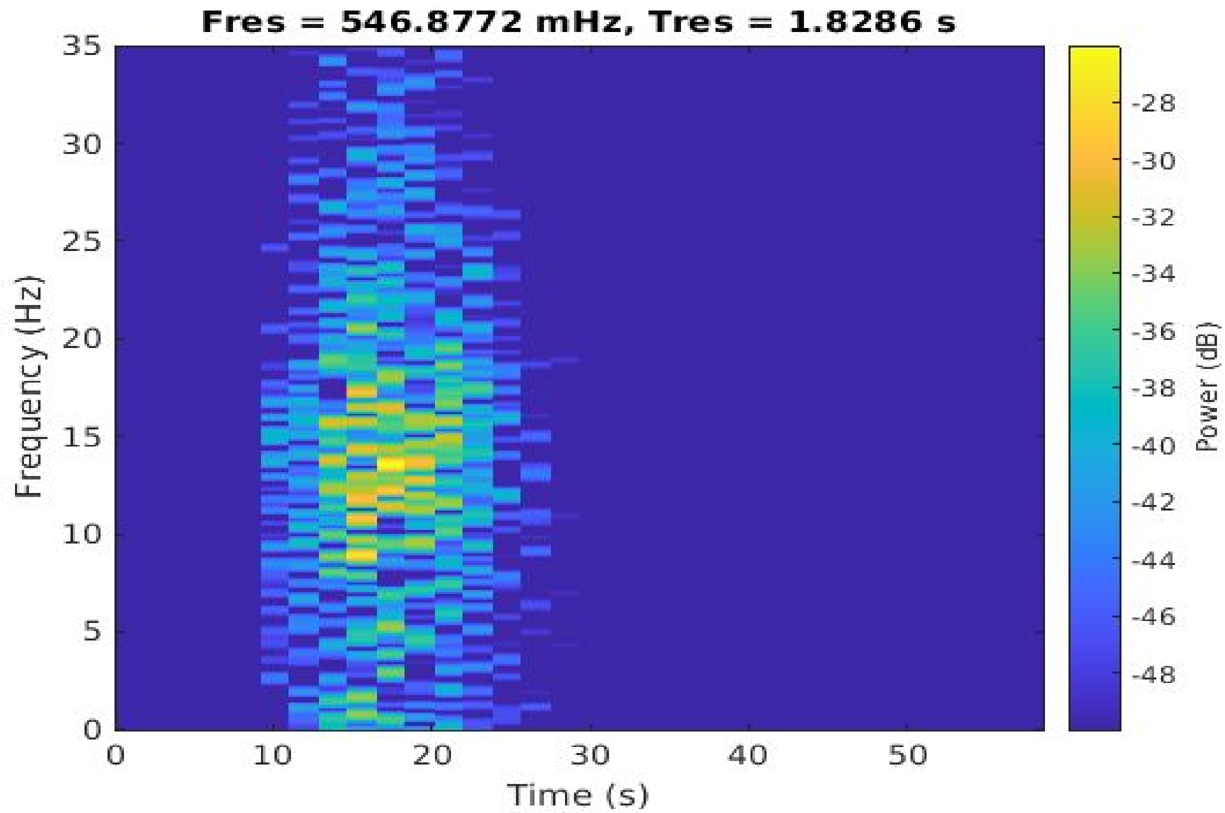
We can understand that the time when Earthquake happens ranges from 10 to 25 sec and the frequency ranges 10 to 20 Hz for it. And There are 2 Earthquake main shocks.

The RMS envelop

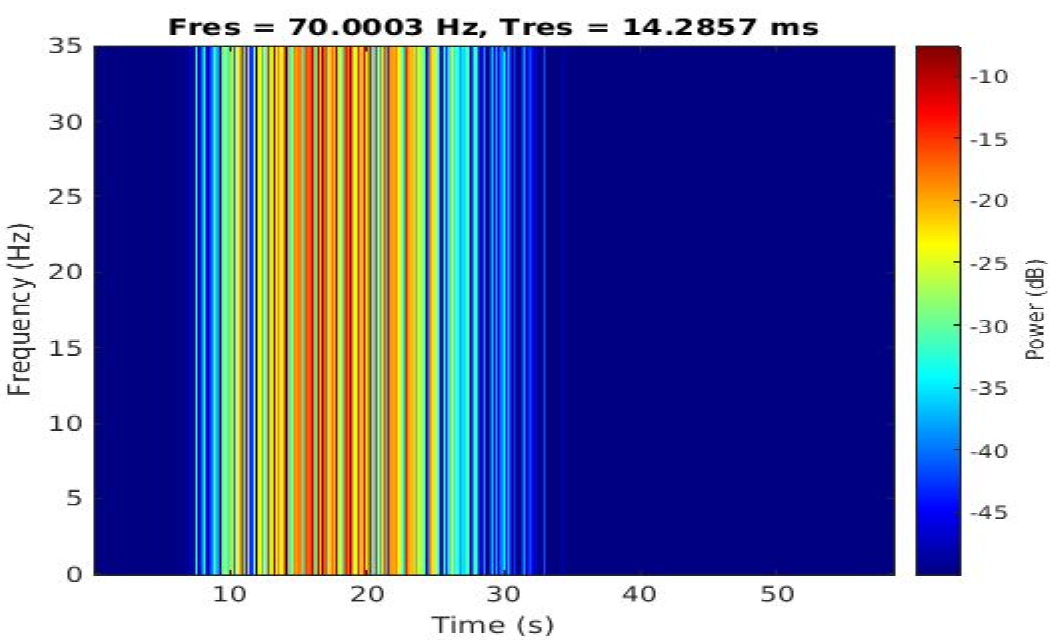


You can see the spikes that resemble the Earthquake.

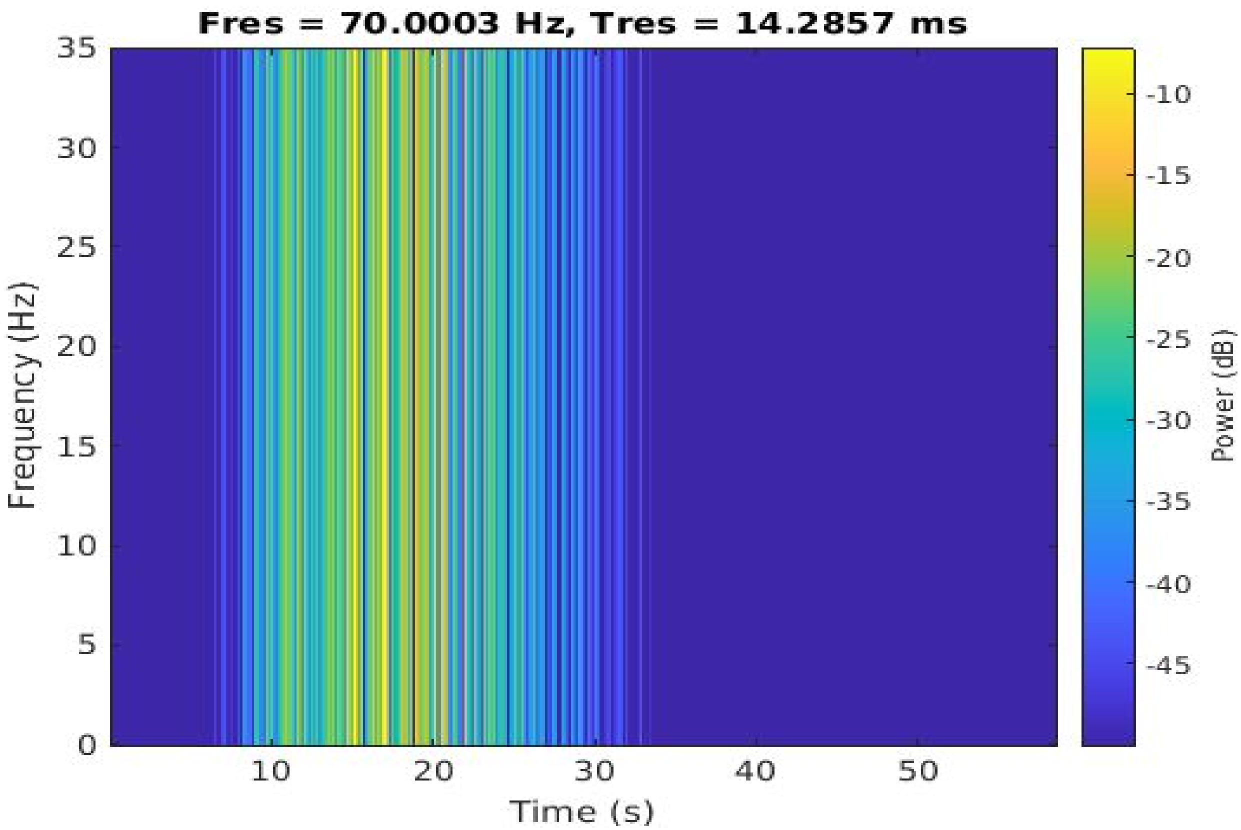
By suppressing All the values lesser than 50 db we get the spectrogram



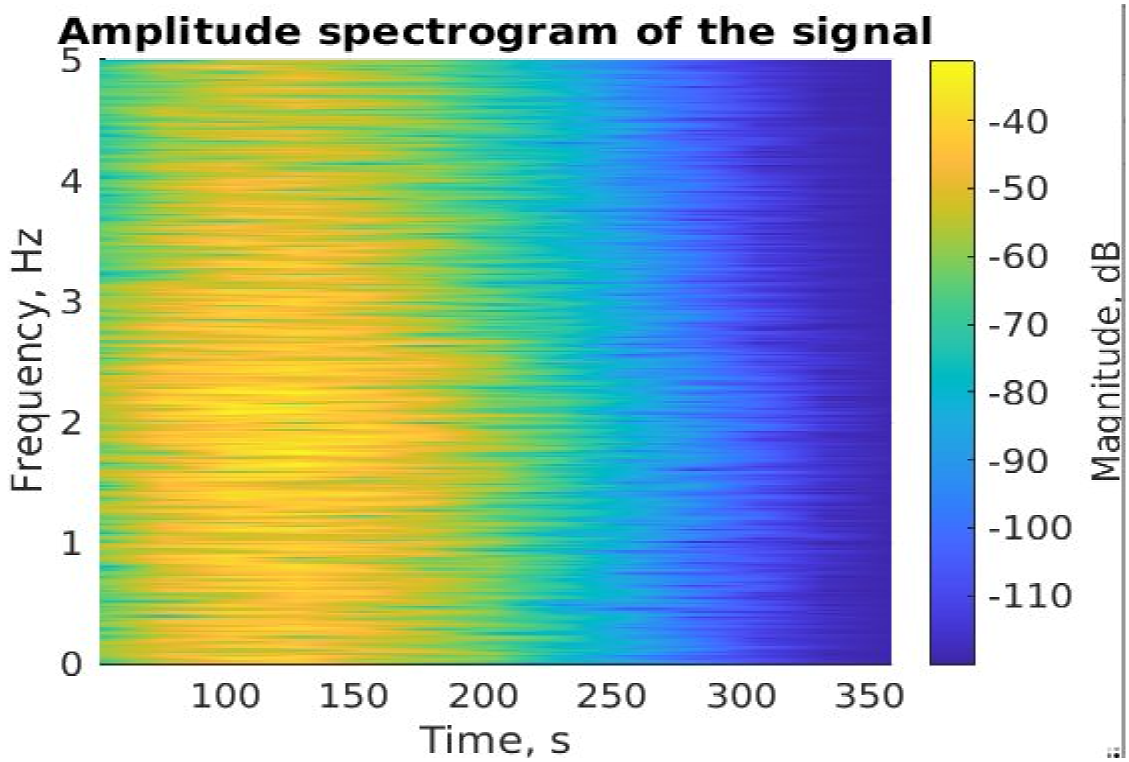
With smallest possible time window and a overlap percent of 99 we got this spectrogram



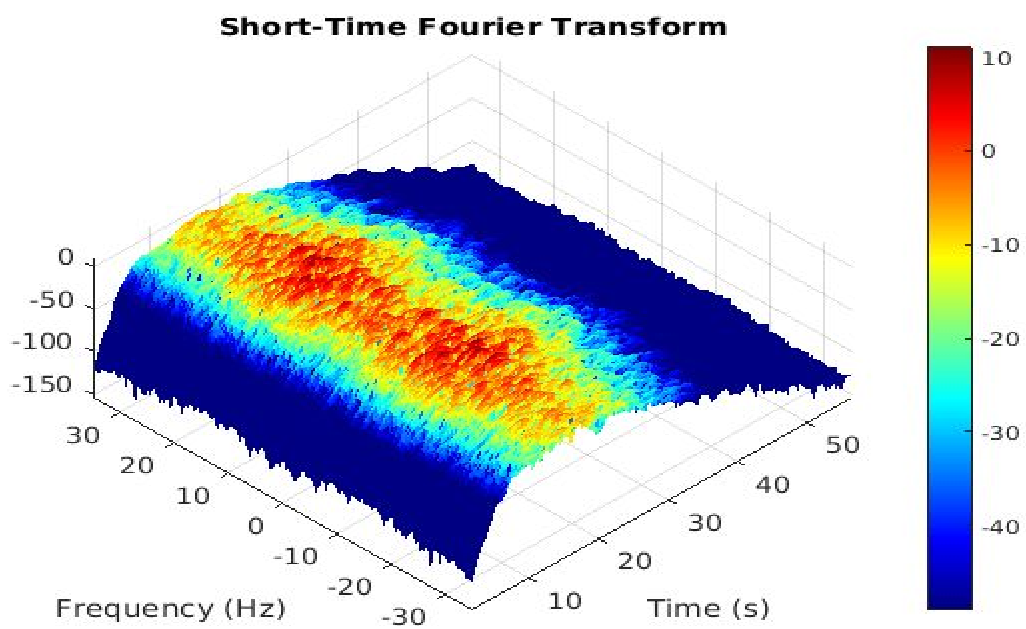
With 0 overlap we got this



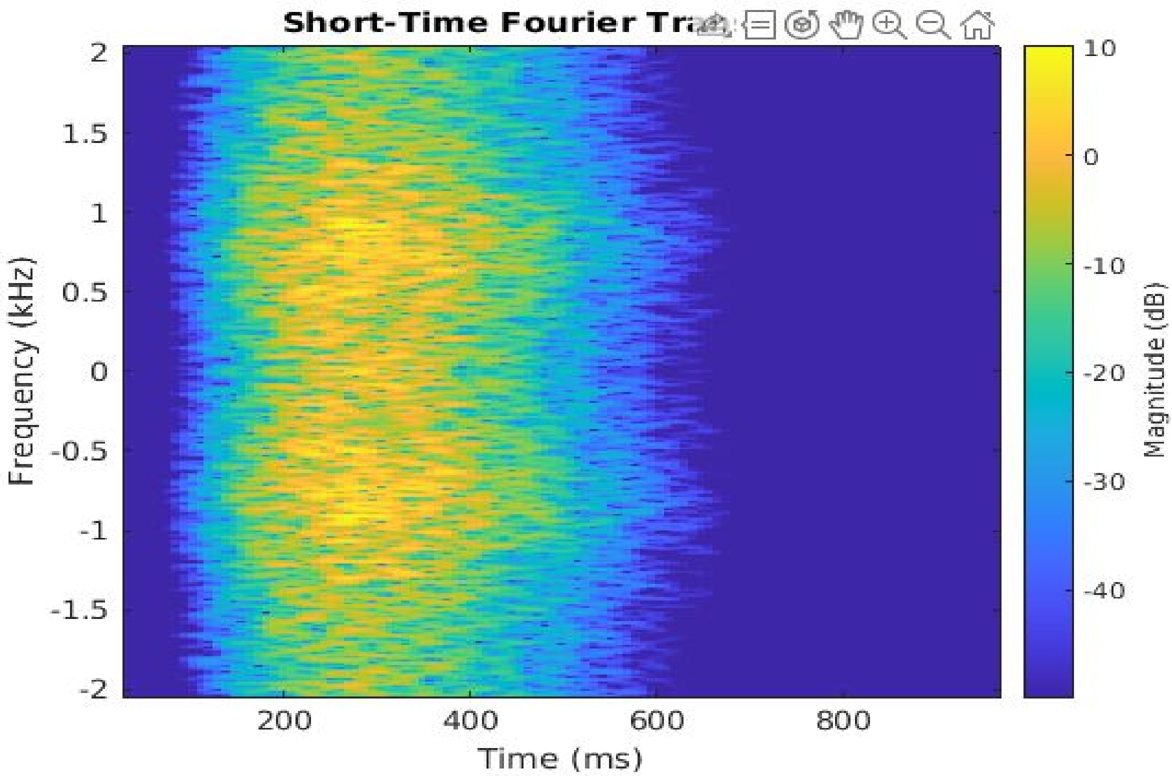
The Amplitude Spectrogram with default value and 4096 sampling rate



For better visualization we can plot the STFT in 3D space



With best representing Time-frequency parameters we have this 2-D plot with default time



- **Wavelet Method**
- A wavelet is a wave-like oscillation with an amplitude that begins at zero, increases, and then decreases back to zero. It can typically be visualized as a "brief oscillation" like one recorded by a seismograph or heart monitor. Generally, wavelets are intentionally crafted to have specific properties that make them useful for signal processing. For example, a wavelet could be created to have a frequency of Middle C and a short duration of roughly a 32nd note. If this wavelet were to be convolved with a signal created from the recording of a melody, then the resulting signal would be useful for determining when the Middle C note was being played in the song. Mathematically, the wavelet will correlate with the signal if the unknown signal contains information of similar frequency. This concept of correlation is at the core of many practical applications of wavelet theory.As a mathematical tool, wavelets can be used to extract information from many different kinds of data, including – but not limited to – audio signals and images. Sets of wavelets are generally needed to analyze data fully. A set of "complementary" wavelets will decompose data without gaps or overlap so that the decomposition process is

mathematically reversible. Thus, sets of complementary wavelets are useful in wavelet based compression/decompression algorithms where it is desirable to recover the original information with minimal loss.In formal terms, this representation is a wavelet series representation of a square-integrable function with respect to either a complete, orthonormal set of basis functions, or an overcomplete set or frame of a vector space, for the Hilbert space of square integrable functions. This is accomplished through coherent states.

Wavelet Theory

Wavelet theory is applicable to several subjects. All wavelet transforms may be considered forms of time-frequency representation for continuous-time (analog) signals and so are related to harmonic analysis. Discrete wavelet transform (continuous in time) of a discrete-time (sampled) signal by using discrete-time filterbanks of dyadic (octave band) configuration is a wavelet approximation to that signal. The coefficients of such a filter bank are called the wavelet and scaling coefficients in wavelets nomenclature. These filterbanks may contain either finite impulse response (FIR) or infinite impulse response (IIR) filters. The wavelets forming a continuous wavelet transform (CWT) are subject to the uncertainty principle of Fourier analysis respective sampling theory: Given a signal with some event in it, one cannot simultaneously assign an exact time and frequency response scale to that event. The product of the uncertainties of time and frequency response scale has a lower bound. Thus, in the scaleogram of a continuous wavelet transform of this signal, such an event marks an entire region in the time-scale plane, instead of just one point. Also, discrete wavelet bases may be considered in the context of other forms of the uncertainty principle.

Wavelet transforms are broadly divided into three classes: continuous, discrete and multiresolution-based.

Continuous wavelet transforms

In continuous wavelet transforms, a given signal of finite energy is projected on a continuous family of frequency bands (or similar subspaces of the Lp function space L2(R)). For instance the signal may be represented on every frequency band of the form [f, 2f] for all positive frequencies $f > 0$. Then, the original signal can be reconstructed by a suitable integration over all the resulting frequency components.

The frequency bands or subspaces (sub-bands) are scaled versions of a subspace at scale 1. This subspace in turn is in most situations generated by the shifts of one generating function ψ in L2(R), the mother wavelet. For the example of the scale one frequency band [1, 2] this function is

$$\psi(t) = 2\ \operatorname{sinc}(2t) - \ \operatorname{sinc}(t) = \frac{\sin(2\pi t) - \sin(\pi t)}{\pi t}$$

with the (normalized) sinc function. That, Meyer's wavelet. A list of all the wavelet transform is given below

- Continuous wavelet transform (CWT)
- Discrete wavelet transform (DWT)
- Multiresolution analysis (MRA)
- Lifting scheme
- Binomial QMF (BQMF)
- Fast wavelet transform (FWT)
- Complex wavelet transform
- Non or undecimated wavelet transform, the downsampling is omitted
- Newland transform, an orthonormal basis of wavelets is formed from appropriately constructed top-hat filters in frequency space
- Wavelet packet decomposition (WPD), detail coefficients are decomposed and a variable tree can be formed
- Stationary wavelet transform (SWT), no downsampling and the filters at each level are different
- e-decimated discrete wavelet transform, depends on if the even or odd coefficients are selected in the downsampling
- Second generation wavelet transform (SGWT), filters and wavelets are not created in the frequency domain
- Dual-tree complex wavelet transform (DTCWT), two trees are used for decomposion to produce the real and complex coefficients

- WITS: Where Is The Starlet, a collection of a hundredth of wavelet names in -let and associated multiscale, directional, geometric, representations, from activelets to x-lets through bandelets, chirplets, contourlets, curvelets, noiselets, wedgelets

Discrete wavelet transforms

It is computationally impossible to analyze a signal using all wavelet coefficients, so one may wonder if it is sufficient to pick a discrete subset of the upper half plane to be able to reconstruct a signal from the corresponding wavelet coefficients. One such system is the affine system for some real parameters $a > 1$, $b > 0$. The corresponding discrete subset of the half plane consists of all the points $(a^m, na^m b)$ with m, n in Z. The corresponding *child wavelets* are now given as

$$\psi_{m,n}(t) = \frac{1}{\sqrt{a^m}} \psi\left(\frac{t - nb}{a^m}\right).$$

A sufficient condition for the reconstruction of any signal x of finite energy by the formula

$$x(t) = \sum_{m \in \mathbb{Z}} \sum_{n \in \mathbb{Z}} \langle x,\, \psi_{m,n} \rangle \cdot \psi_{m,n}(t)$$

How wavelet transform works is completely a different fun story, and should be explained after short time Fourier Transform (STFT). The WT was developed as an alternative to the STFT. The STFT will be explained in great detail in the second part of this tutorial. It suffices at this time to say that the WT was developed to overcome some resolution related problems of the STFT, as explained in Part II.

To make a real long story short, we pass the time-domain signal from various highpass and low pass filters, which filters out either high frequency or low frequency portions of the signal. This procedure is repeated, every time some portion of the signal corresponding to some frequencies being removed from the signal.

Here is how this works: Suppose we have a signal which has frequencies up to 1000 Hz. In the first stage we split up the signal in to two parts by passing the signal from a highpass and a lowpass filter (filters should satisfy some certain conditions, so-called admissibility condition) which results in two different versions of the same signal: portion of the signal corresponding to 0-500 Hz (low pass portion), and 500-1000 Hz (high pass portion).

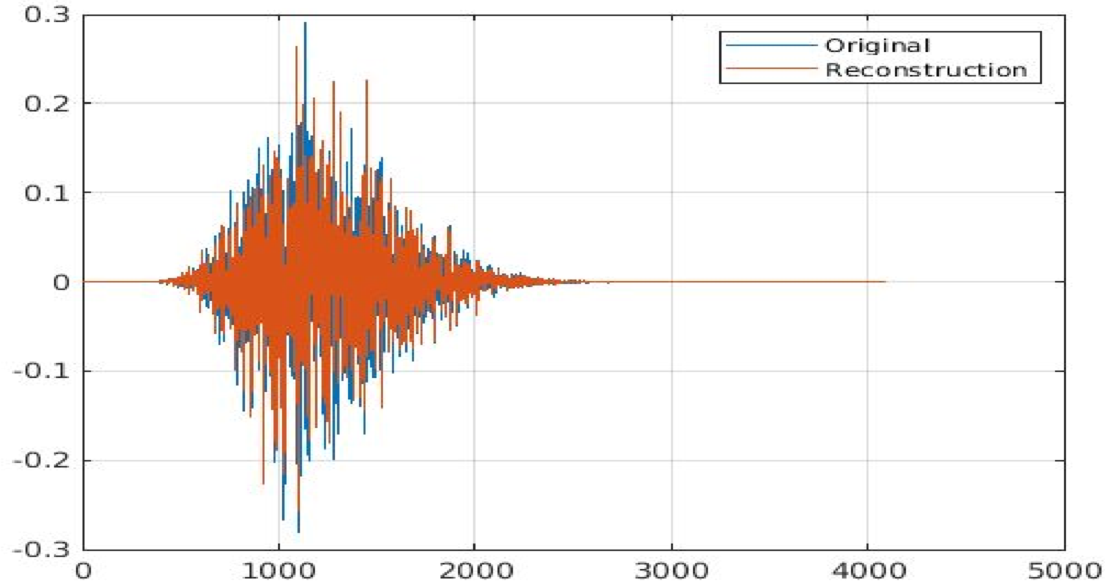
Then, we take either portion (usually low pass portion) or both, and do the same thing again. This operation is called decomposition .

Assuming that we have taken the lowpass portion, we now have 3 sets of data, each corresponding to the same signal at frequencies 0-250 Hz, 250-500 Hz, 500-1000 Hz.

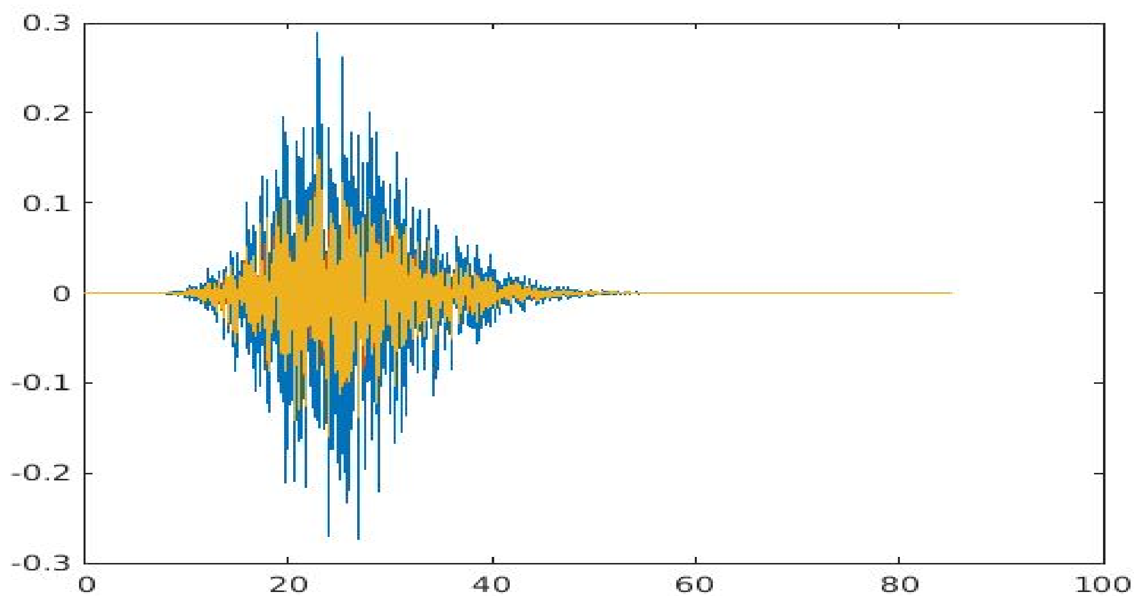
Then we take the lowpass portion again and pass it through low and high pass filters; we now have 4 sets of signals corresponding to 0-125 Hz, 125-250 Hz,250-500 Hz, and 500-1000 Hz. We continue like this until we have decomposed the signal to a pre-defined certain level. Then we have a bunch of signals, which actually represent the same signal, but all corresponding to different frequency bands. We know which signal corresponds to which frequency band, and if we put all of them together and plot them on a 3-D graph, we will have time in one axis, frequency in the second and amplitude in the third axis. This will show us which frequencies exist at which time (there is an issue, called "uncertainty principle", which states that, we cannot exactly know what frequency exists at what time instance, but we can only know what frequency bands exist at what time intervals.

Results

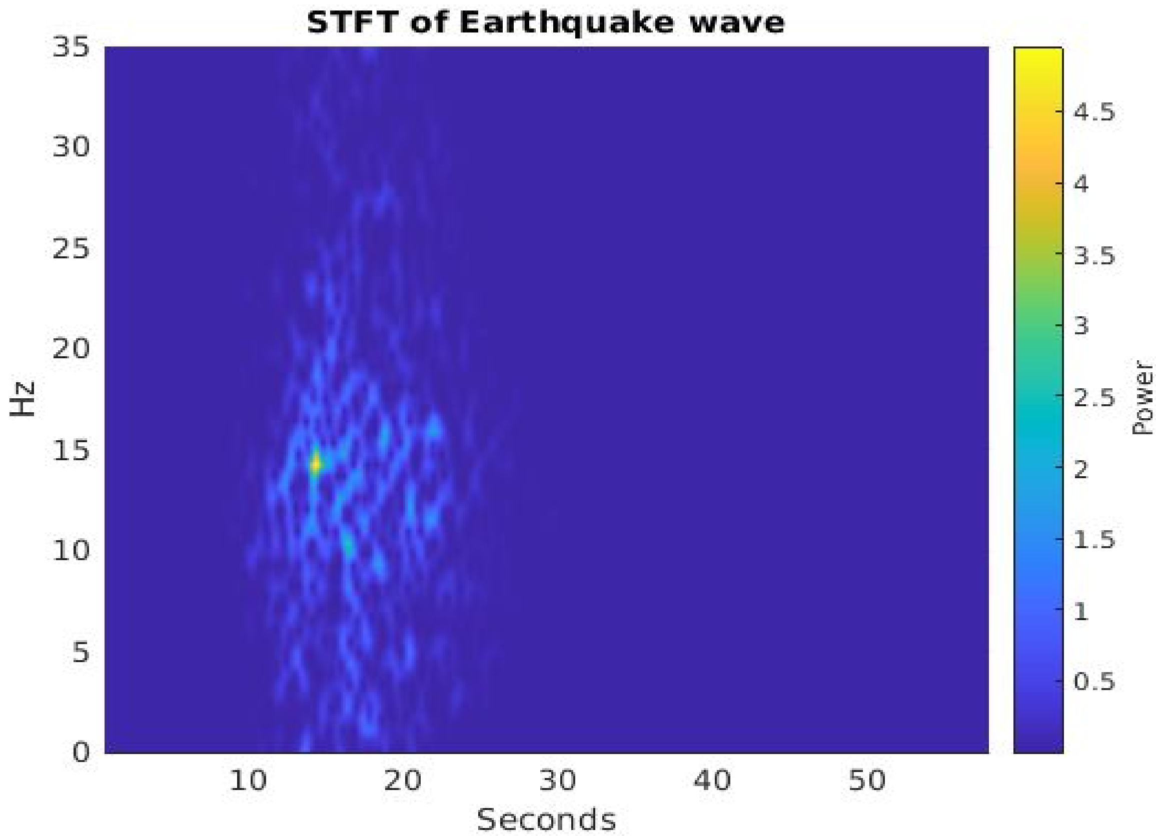
Now using single level 1-D wavelet transform it is almost possible to find a more accurate and smooth curve of the Earthquake in comparison to exponential method



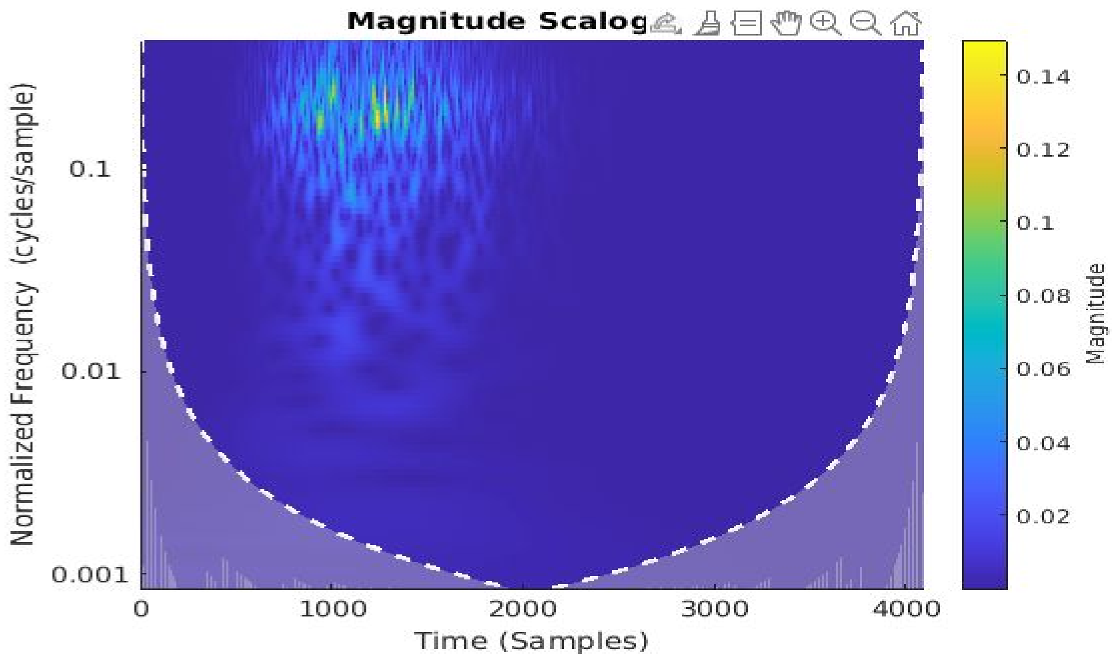
With modififing scaling and shifting parameters we got this



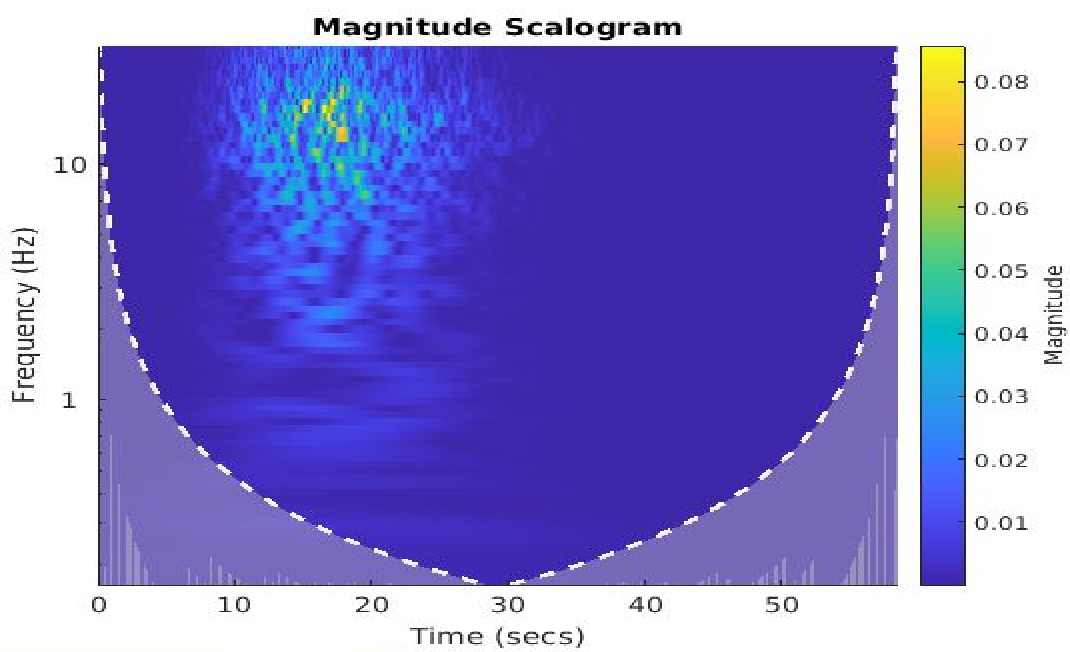
Let's visualize the problem with STFT to show importance of wavelet if we want to localize the Earthquake we got this as response spectrogram



Now let's look how a scalogram will look like with our data

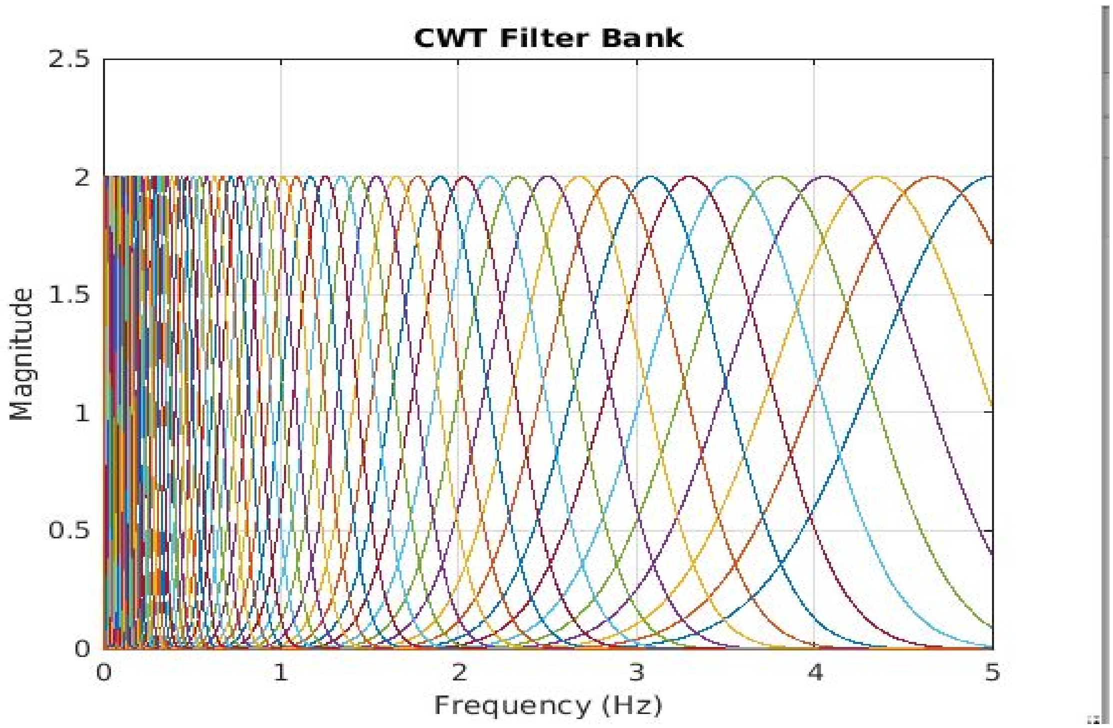


With sampling frequency 70 we can improve the view much better

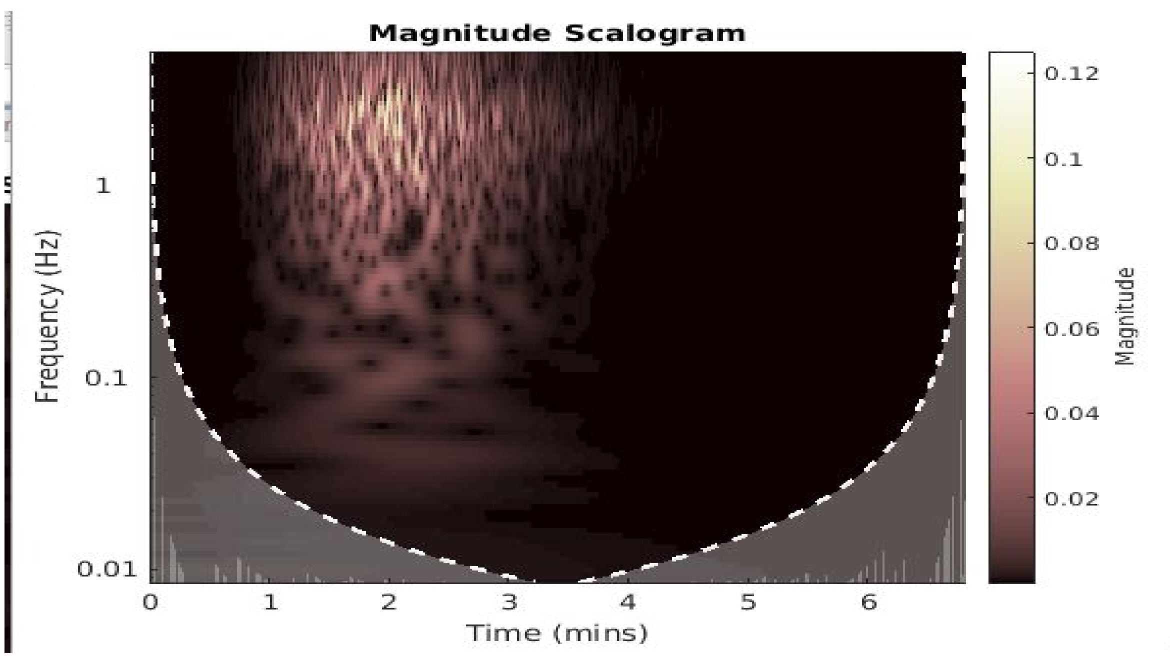


We can see now lot's of main shocks are localized in discrete time frequency plot.

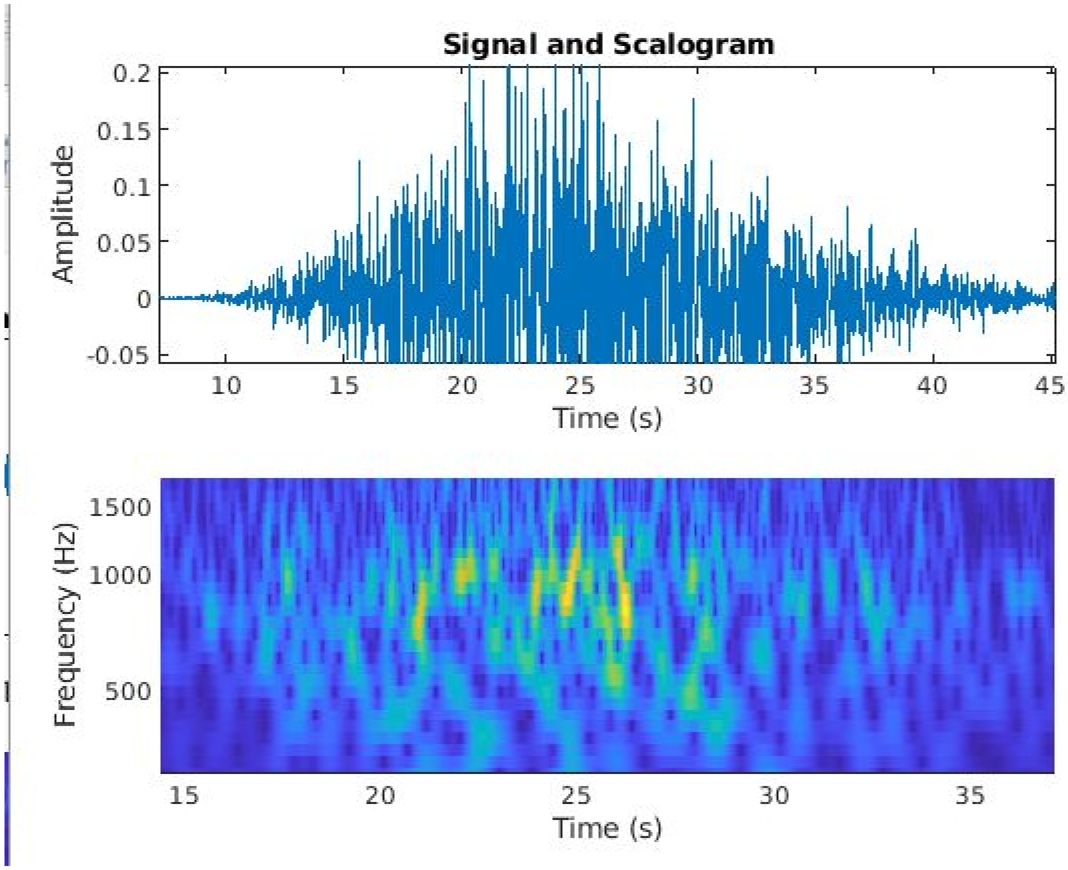
The CWT filter bank used in this transform



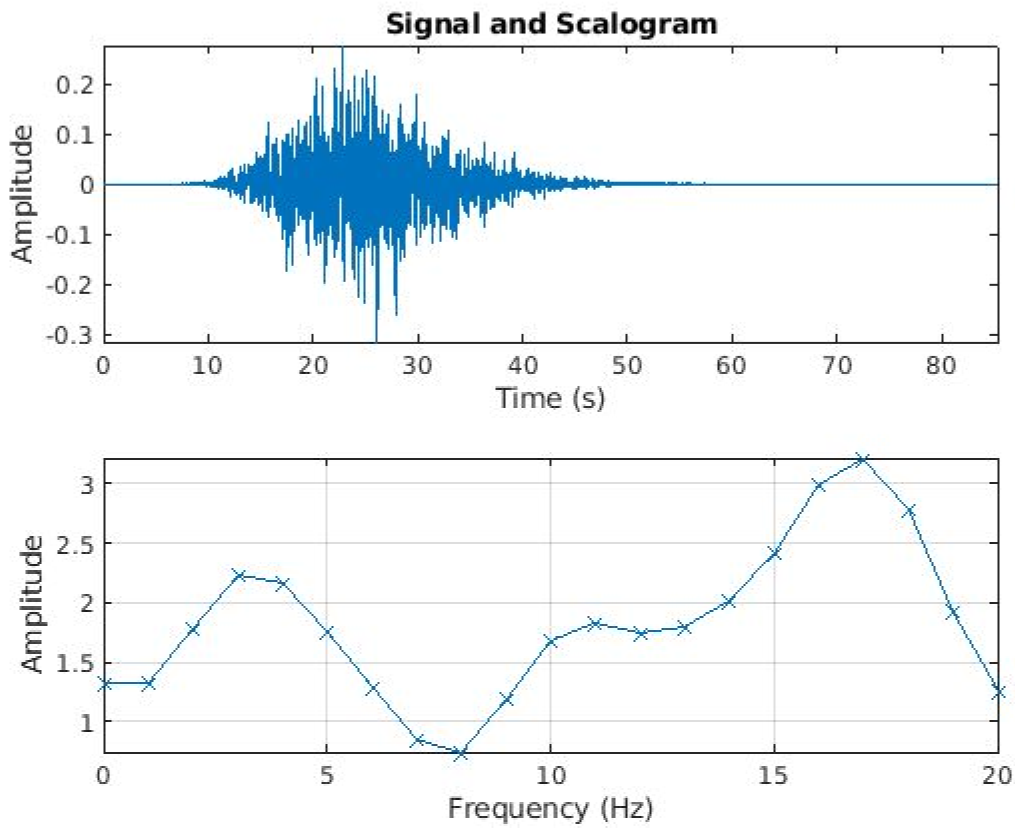
A better representation of the TF distribution plotted



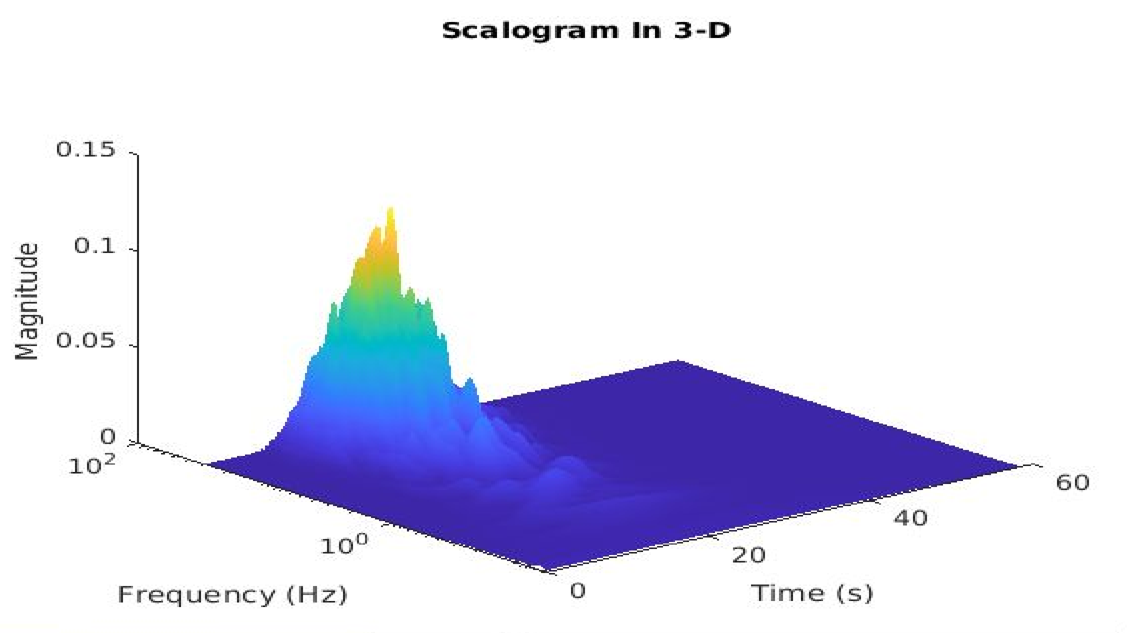
We can plot the scalogram and signal side by side to visualize the Effects



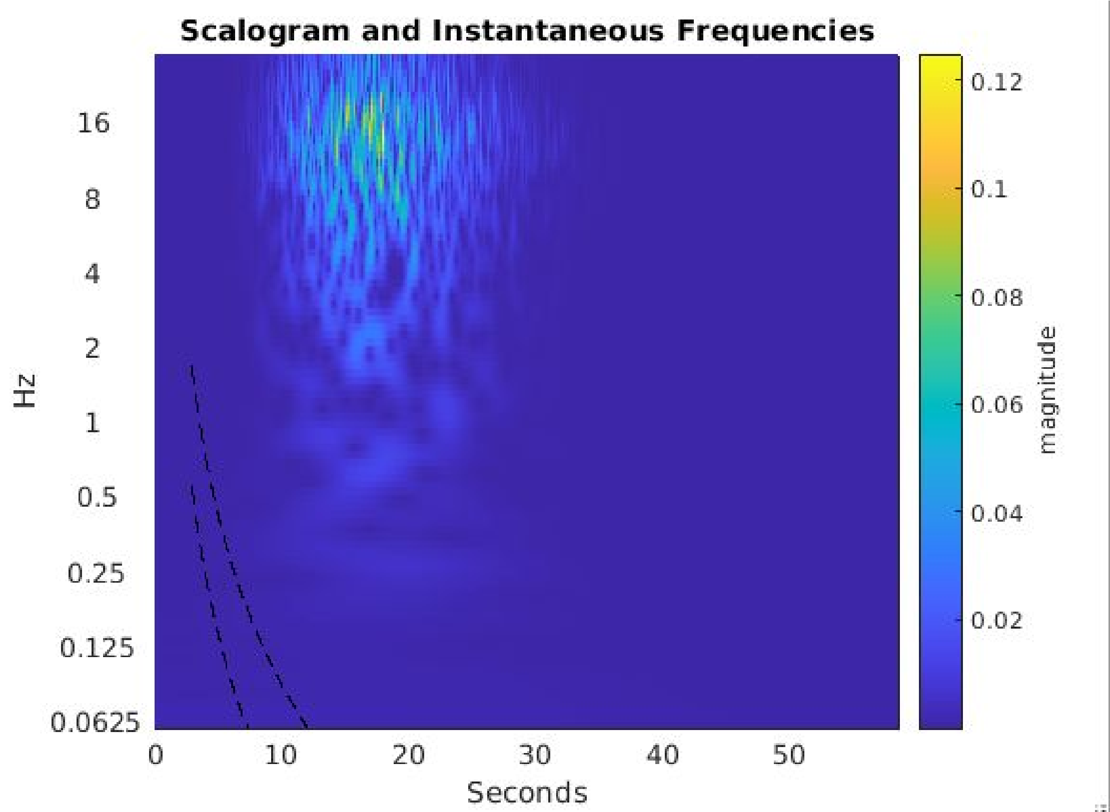
The same plot in Frequency domain resembles



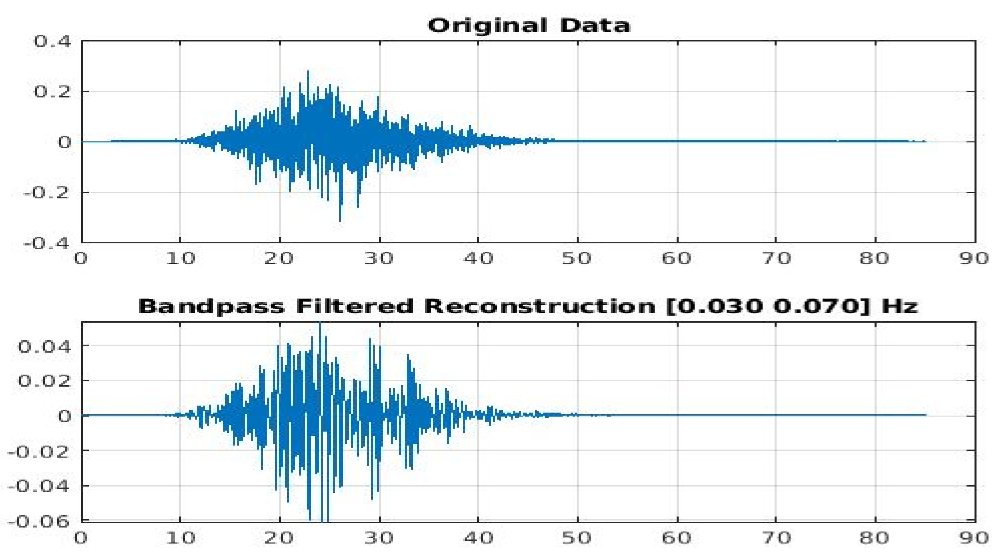
Better way to put this into 3-D diagram



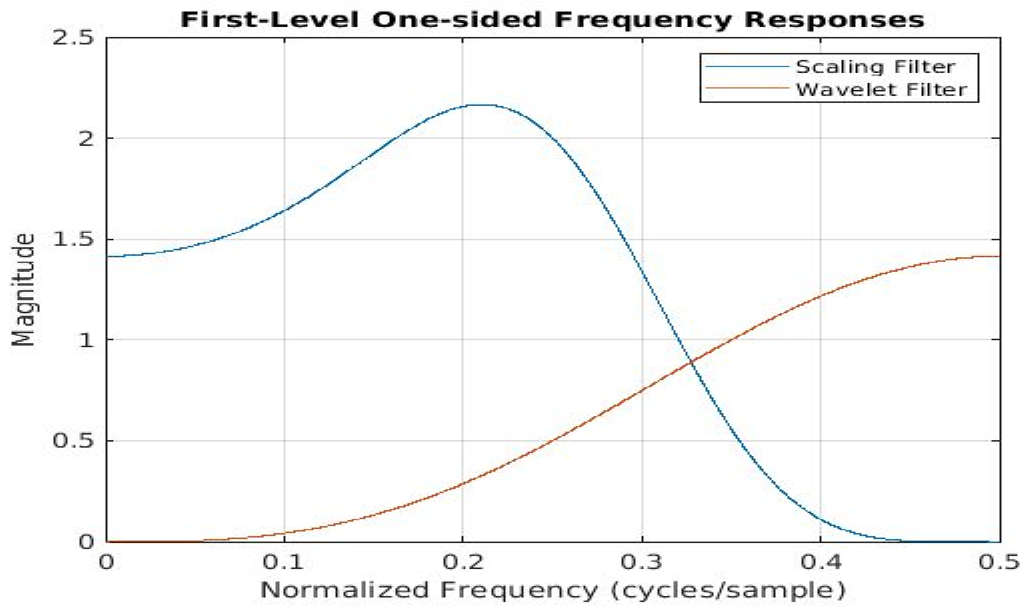
Now we can see the instantaneous frequency plot as well



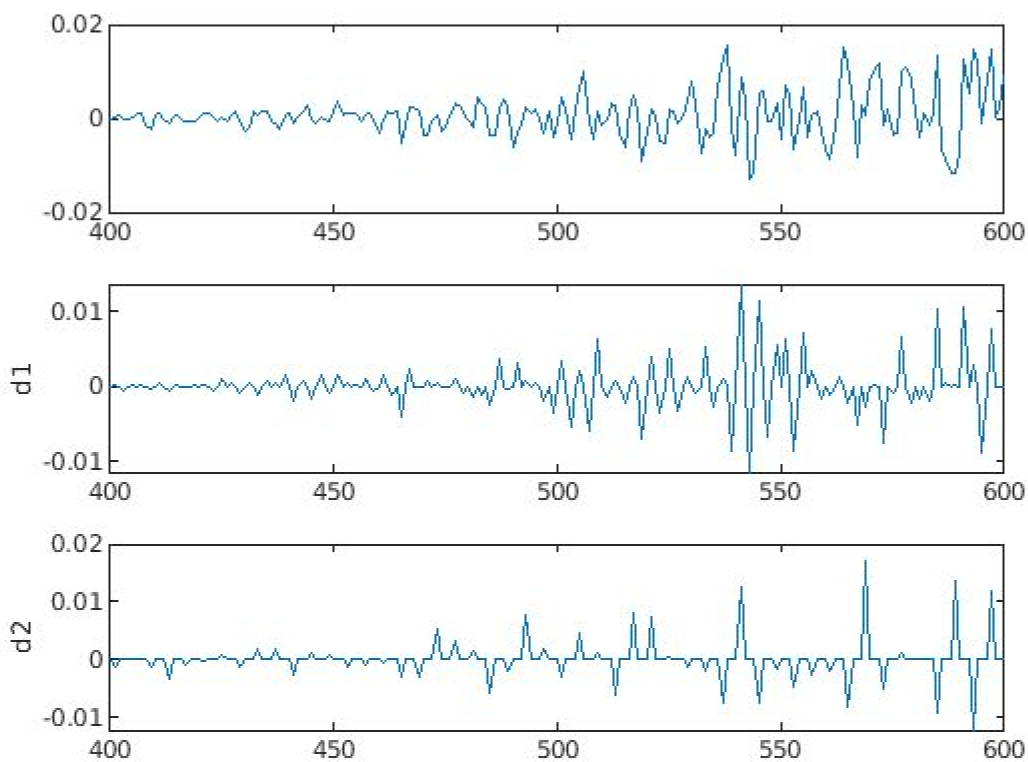
We can use wavelets to filter data and plot a smoother noise free plot for say if we pass the data from a band of [0.03 and 0.07 Hz] we get much smoother original earthquake signal.



Some facts of wavelet we used in our programme



We can also show how analysis by wavelets can detect a discontinuity in one of a signal's derivatives



- **Wigner Ville Distribution**

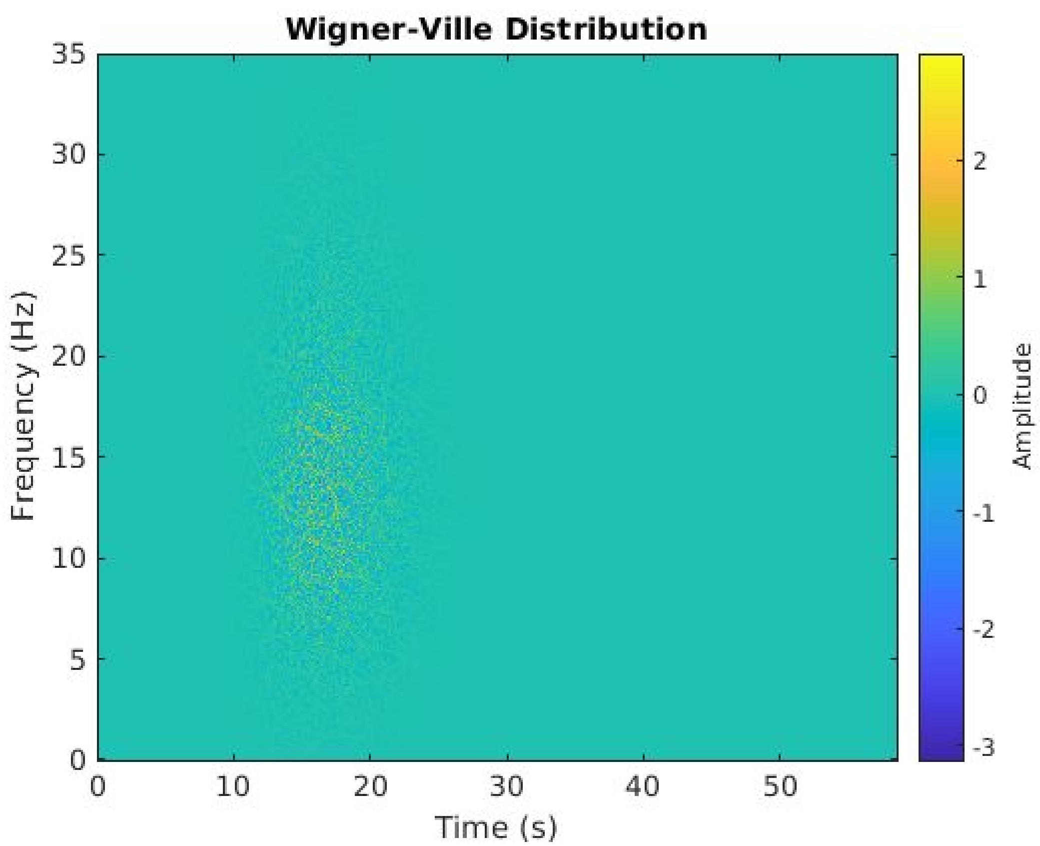
The Wigner-Ville distribution (WVD) is a quadratic energy density computed by correlating the signal with a time and frequency translated and complex-conjugated version of itself.

The Wigner-Ville distribution is always real even if the signal is complex.Time and frequency marginal densities correspond to instantaneous power and spectral energy density, respectively.

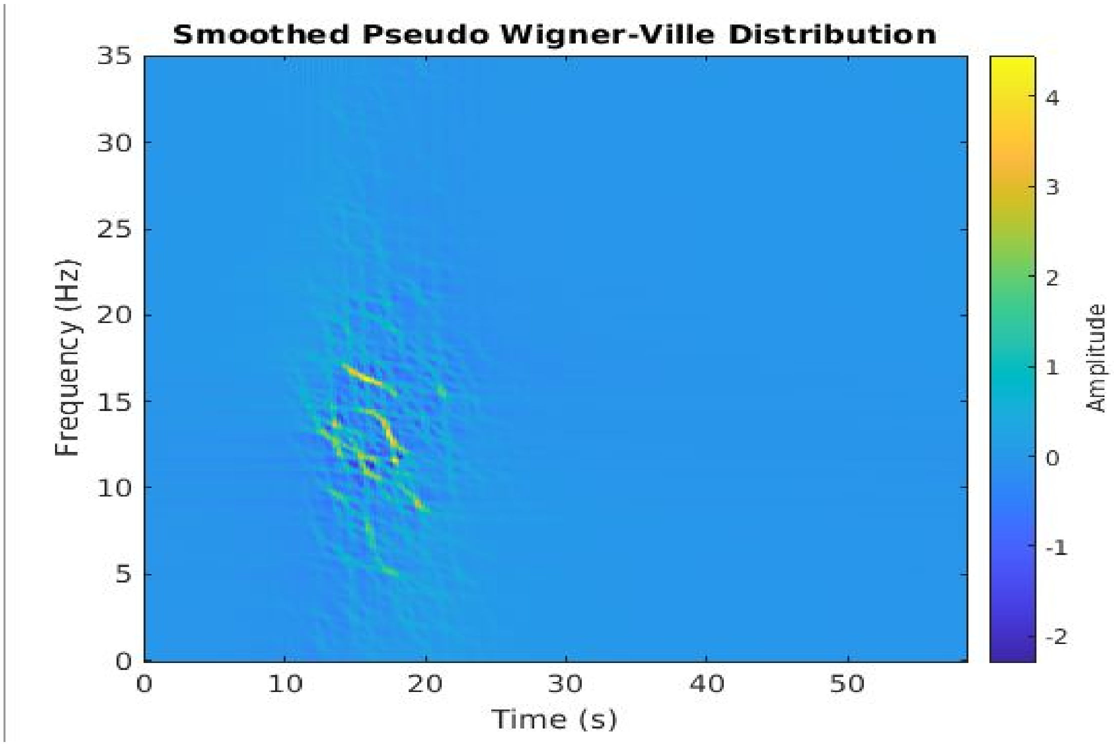
The instantaneous frequency and group delay can be evaluated using local first-order moments of the Wigner distribution.The time resolution of the WVD is equal to the number of input samples.The Wigner distribution can locally assume negative values.

Results

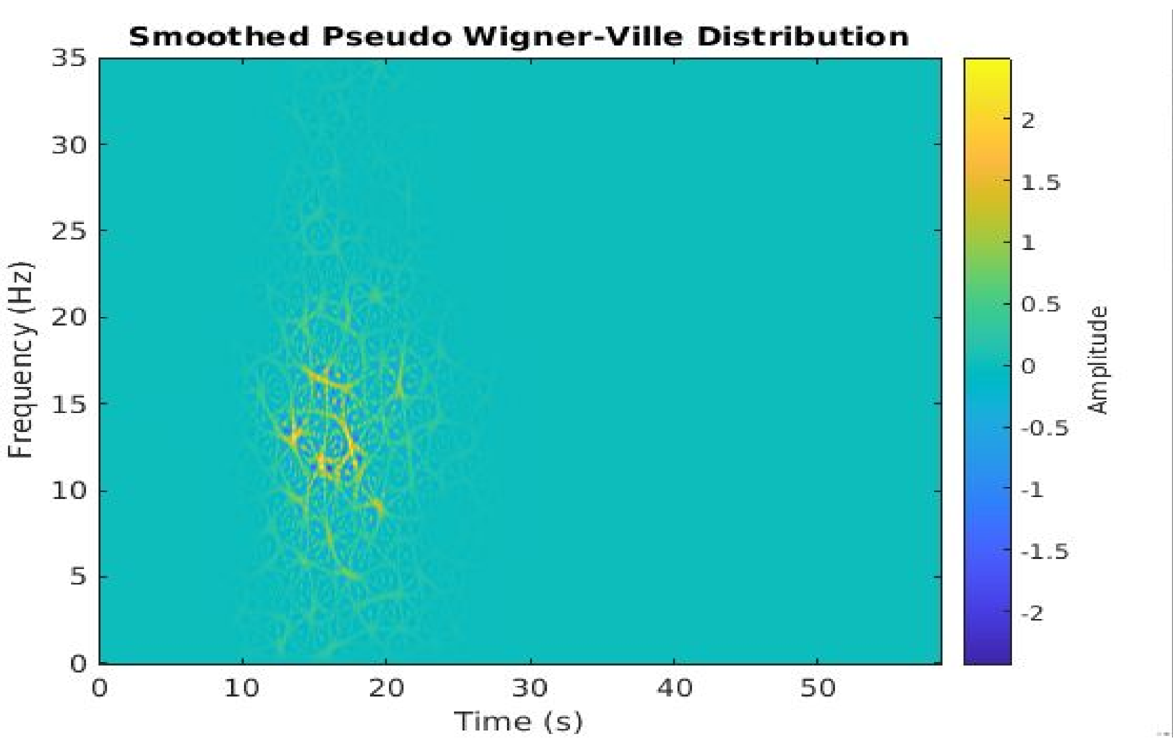
The Default representation of The Earthquake in Wigner Ville distribution is



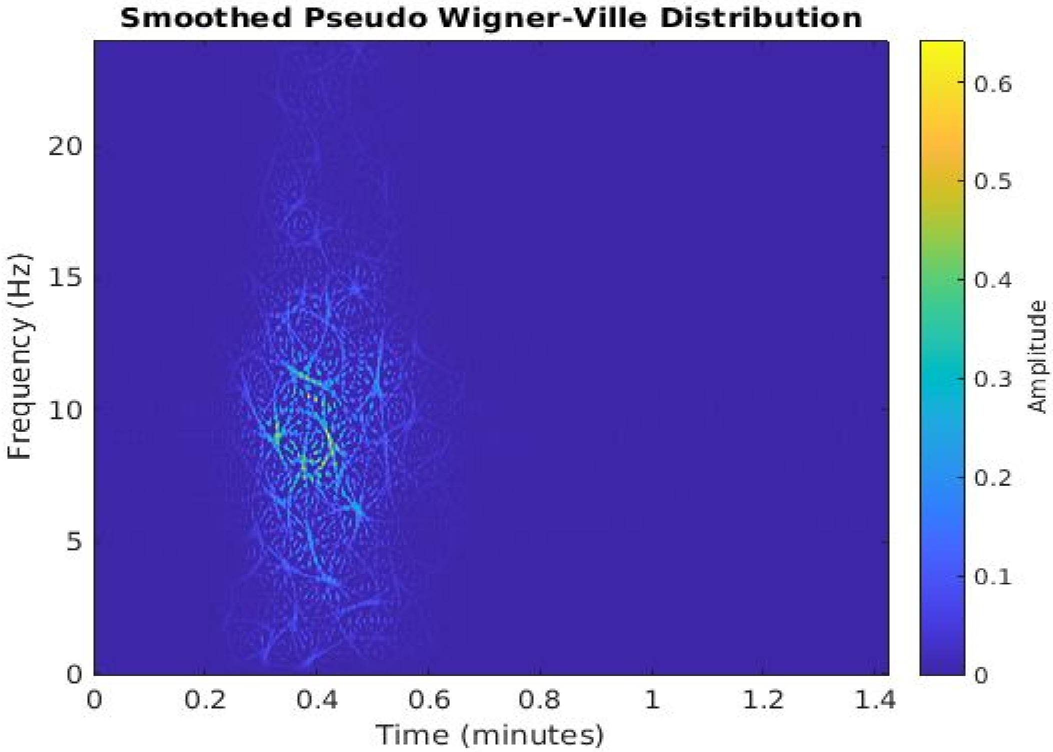
We hardy can visualize any good in this plot. But we can improve the plot .If we smooth the plot with number of frequency and time resolution we can visualize the signal in a better manner



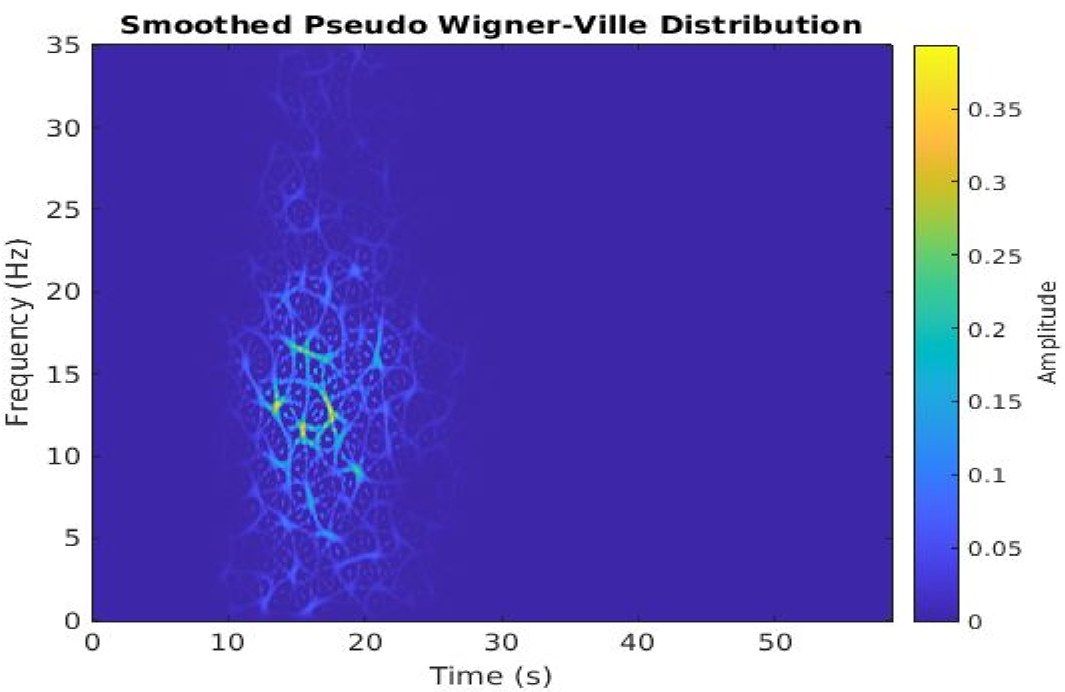
Now we can understand the signal but it can be improved upon sharpening



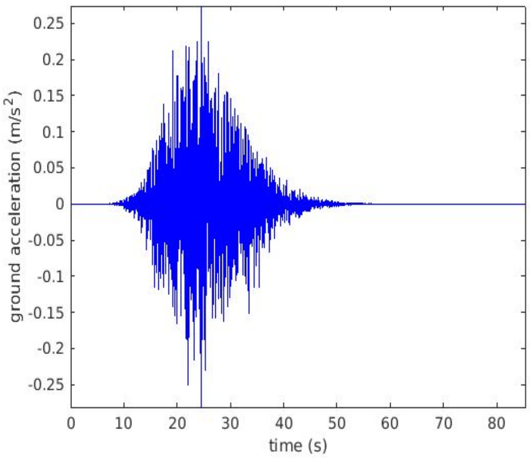
Now if we improve this by Window the distribution in time using a 601-sample Hamming window and in frequency using a 305-sample rectangular window. Use 600 frequency points for the display. Set to zero those components of the distribution with amplitude less than 0.



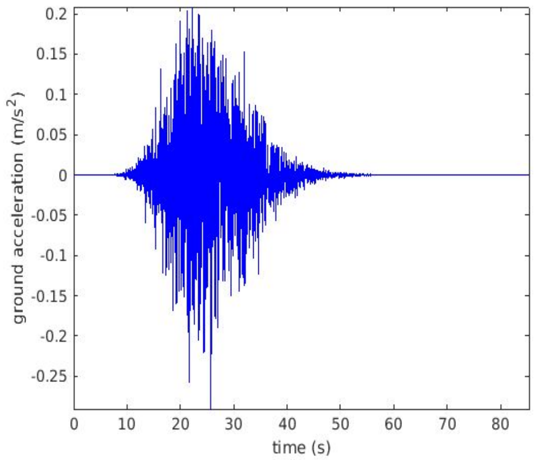
With Kaiser window we have something different than hamming



Now to visualize **Cross Wigner-Ville Distribution** let's build two Earthquake signal of same duration with excitation frequency as 10 and 20 Hz keeping all other factor same.

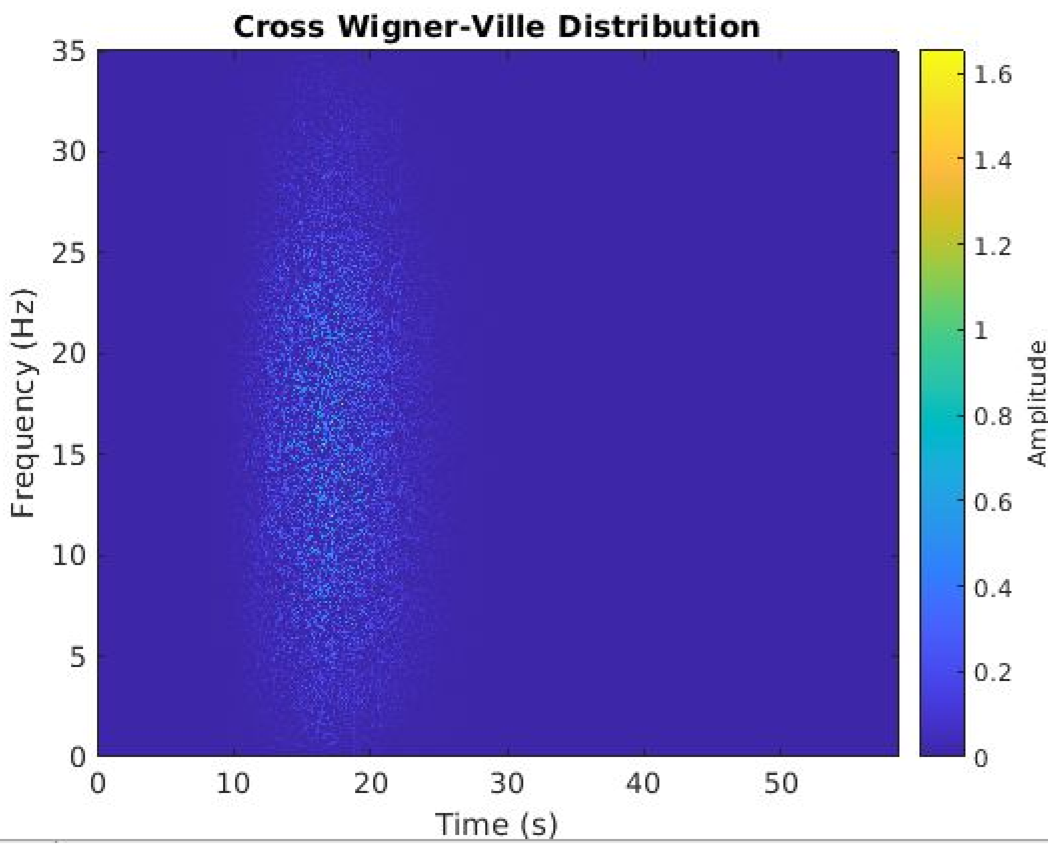


Frequency = 20



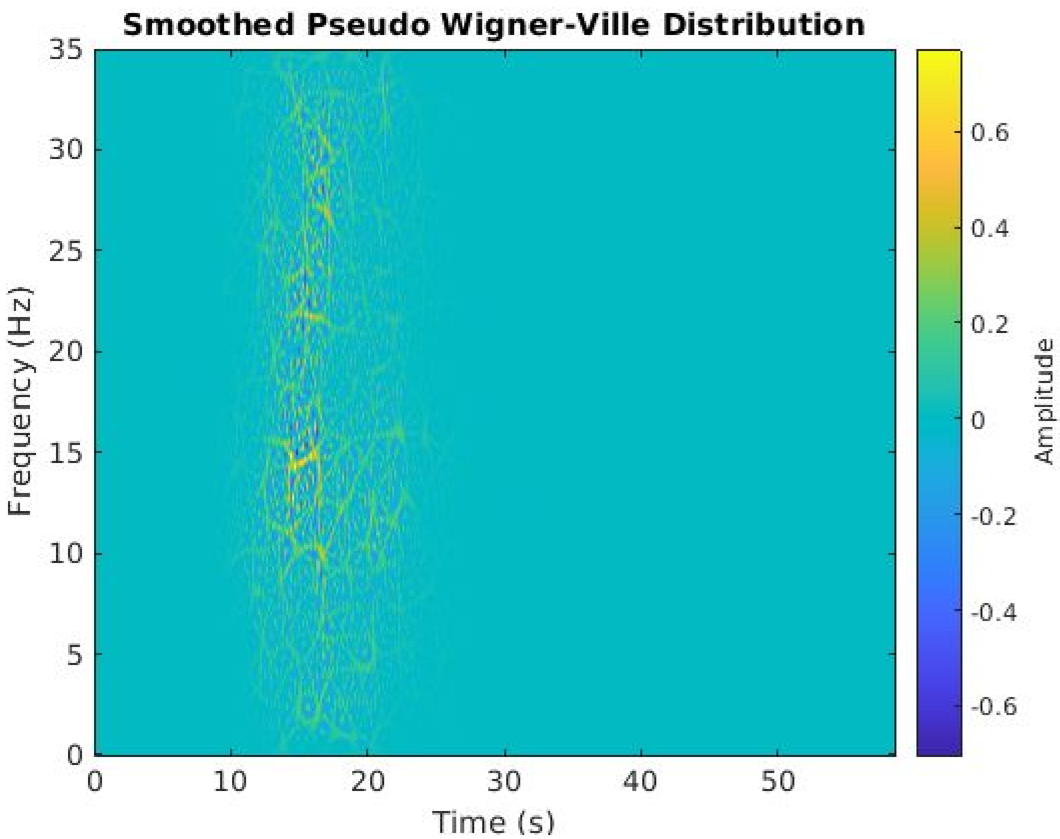
Frequency = 10

The cross wigner ville transform looks like



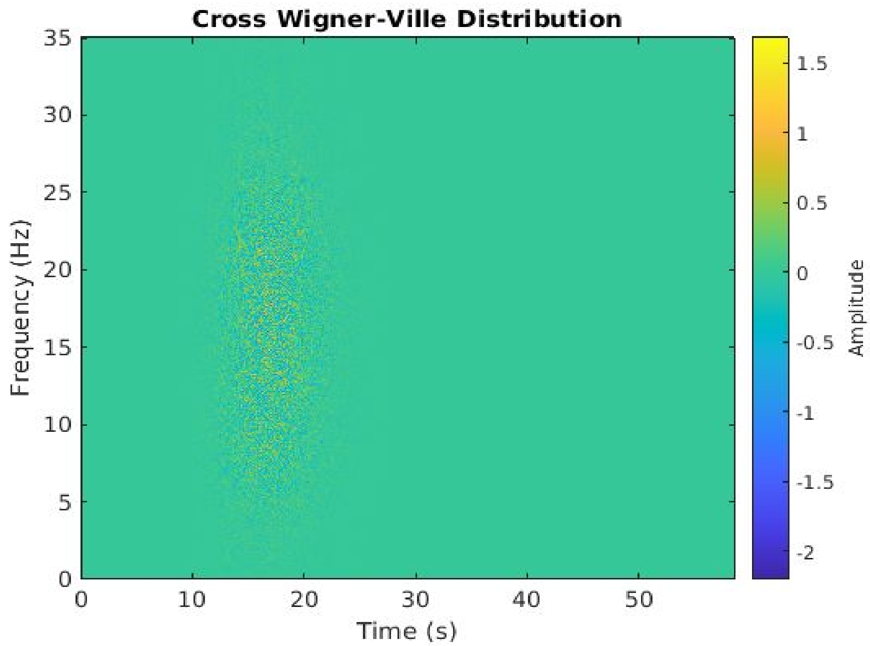
Now let's Create a two-component signal consisting of the sum of the unknown and reference signals. The smoothed pseudo Wigner-Ville distribution of the result provides an "ideal" time-frequency representation.

Compute and display the smoothed pseudo Wigner-Ville distribution.

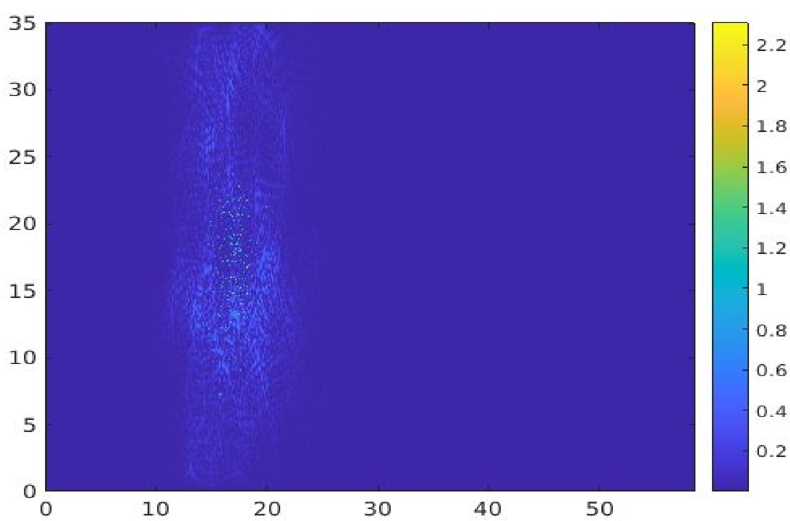


Compute the cross Wigner-Ville distribution of the unknown and reference signals. Take the absolute value of the distribution and set to zero the elements with amplitude less than 0. The cross Wigner-Ville distribution is equal to the cross-terms of the two-component signal.

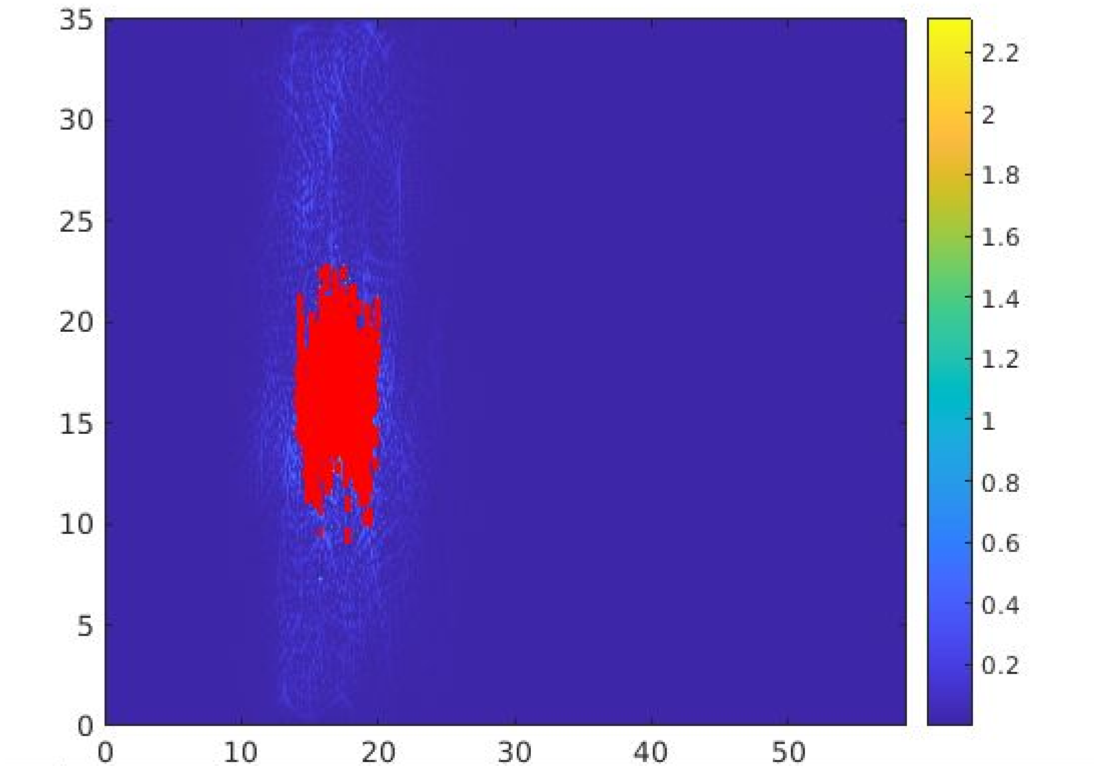
Plot the real part of the cross Wigner-Ville distribution.



Now Enhance the Wigner-Ville cross-terms by adding the ideal time-frequency representation to the cross Wigner-Ville distribution. The cross-terms of the Wigner-Ville distribution occur halfway between the reference signal and the unknown signal.



Identify and plot the high-energy ridge corresponding to the cross-terms. To isolate the ridge, find the time values where the cross-distribution has nonzero energy.

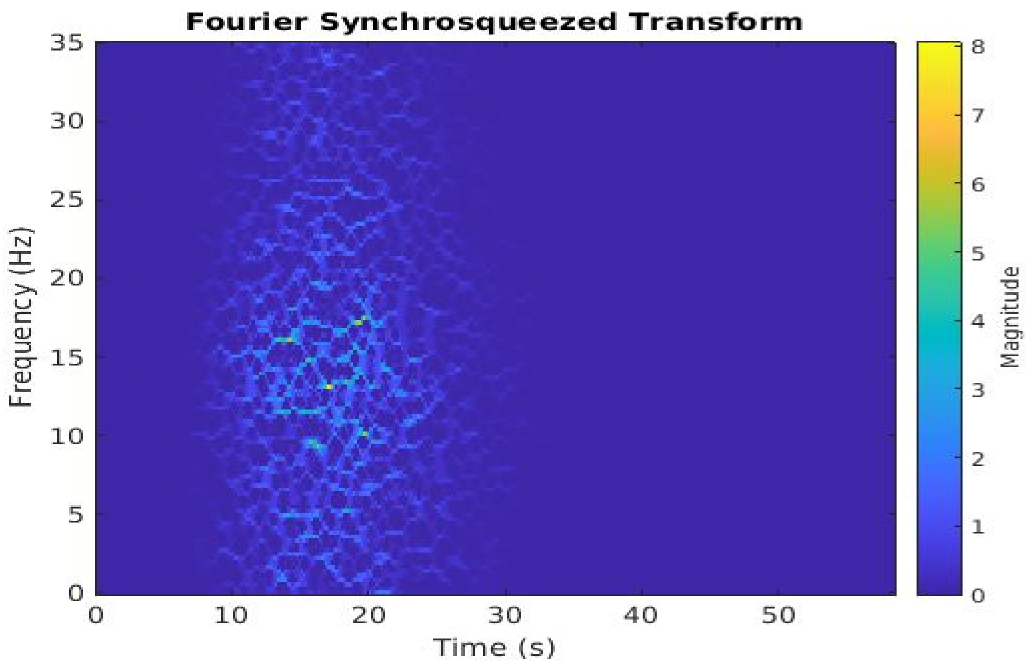


- **Reassignment and Synchrosqueezing**

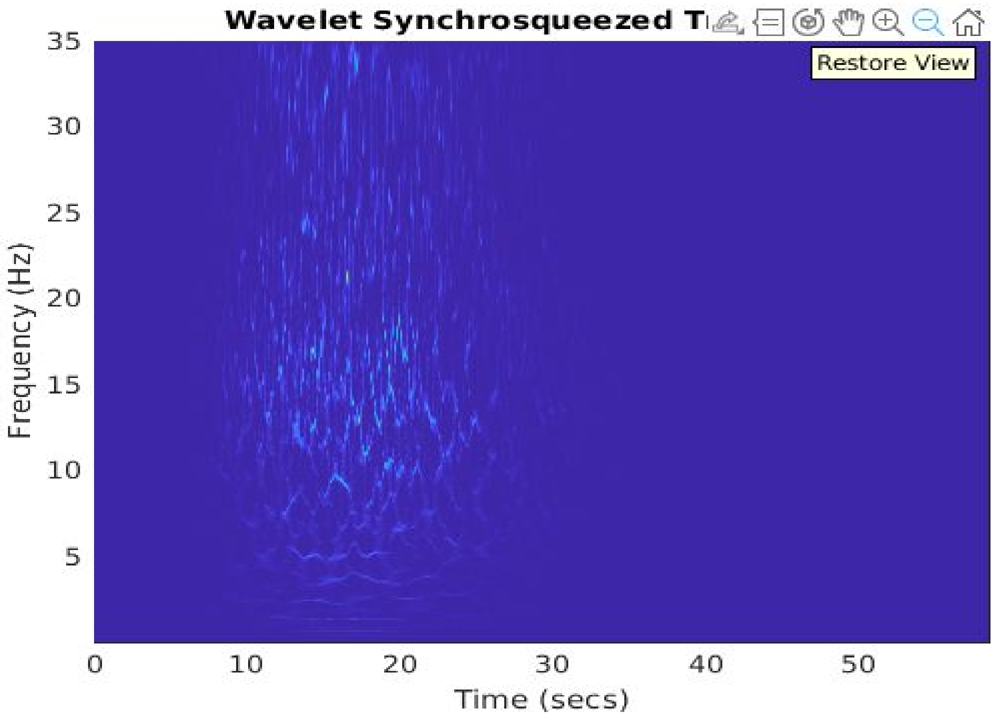
Reassignment sharpens the localization of spectral estimates and produces spectrograms that are easier to read and interpret. The technique relocates each spectral estimate to the center of energy of its bin instead of the bin's geometric center. It provides exact localization for chirps and impulses.The *Fourier synchrosqueezed transform* starts from the short-time Fourier transform and "squeezes" its values so that they concentrate around curves of instantaneous frequency in the time-frequency plane.The *wavelet synchrosqueezed transform* reassigns the signal energy in frequency.Both the Fourier synchrosqueezed transform and the wavelet synchrosqueezed transform are invertible.The reassigned and synchrosqueezing methods are especially suited to track and extract time-frequency *ridges*.It is used to analysis of seismic data to find oil and gas traps. Synchrosqueezing can also detect deep-layer weak signals that are usually smeared in seismic data

Results

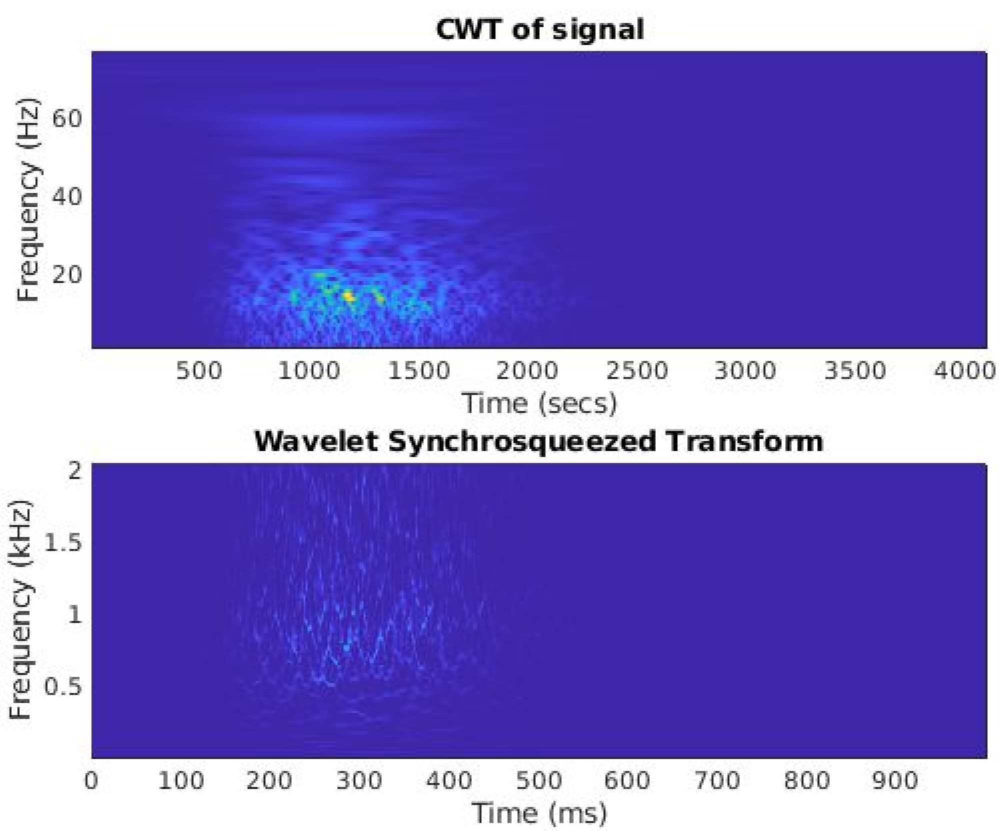
Compute the synchrosqueezed Fourier transform of the signal. Window the signal using a Kaiser window with shape factor β=20.



Compute the wavelet synchrosqueezed transform of the acceleration measurements.



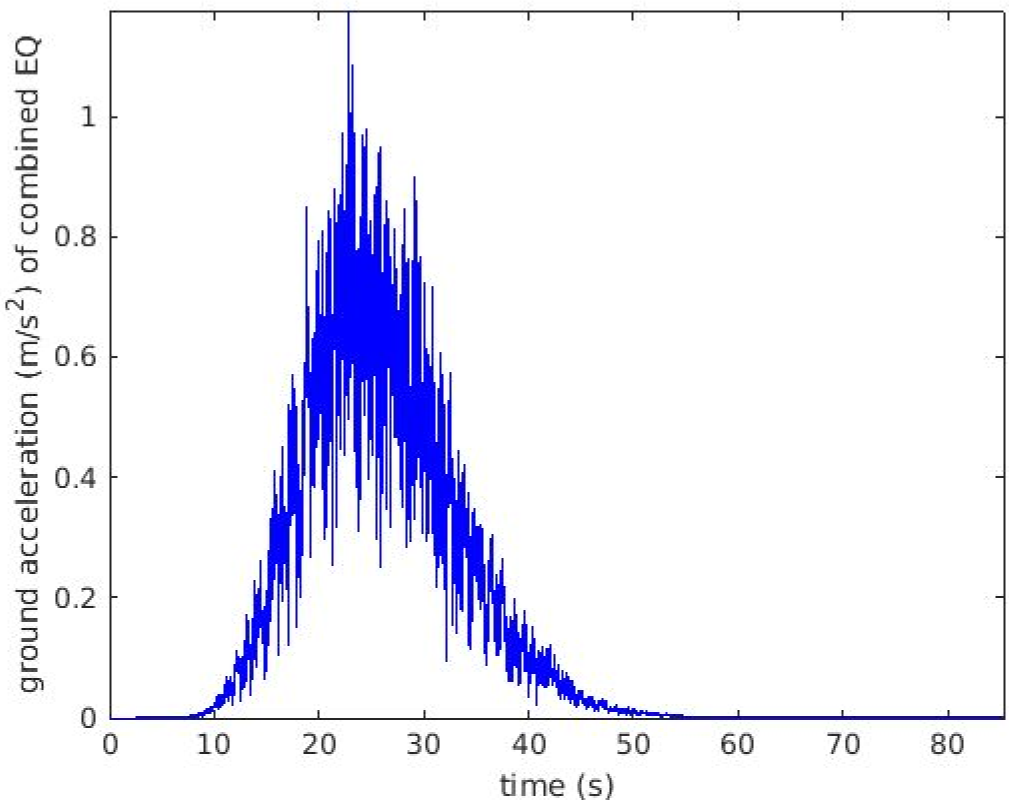
See how CWT and wavelet transforms are different for same signal



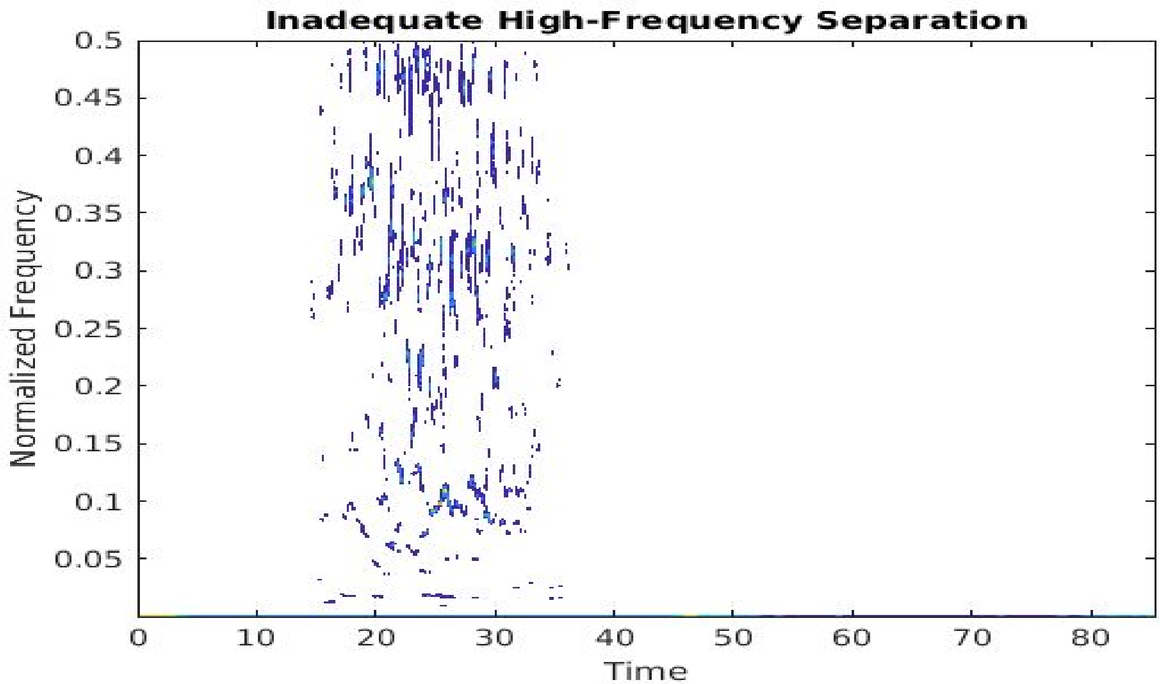
Now let’s show you how to do **Low-Frequency vs. High-Frequency Component Separation**

Build 5 Earthquakes with frequency correspond to 1Hz,5Hz,0.001Hz,15Hz,1000Hz and combined them together to build one Earthquake.

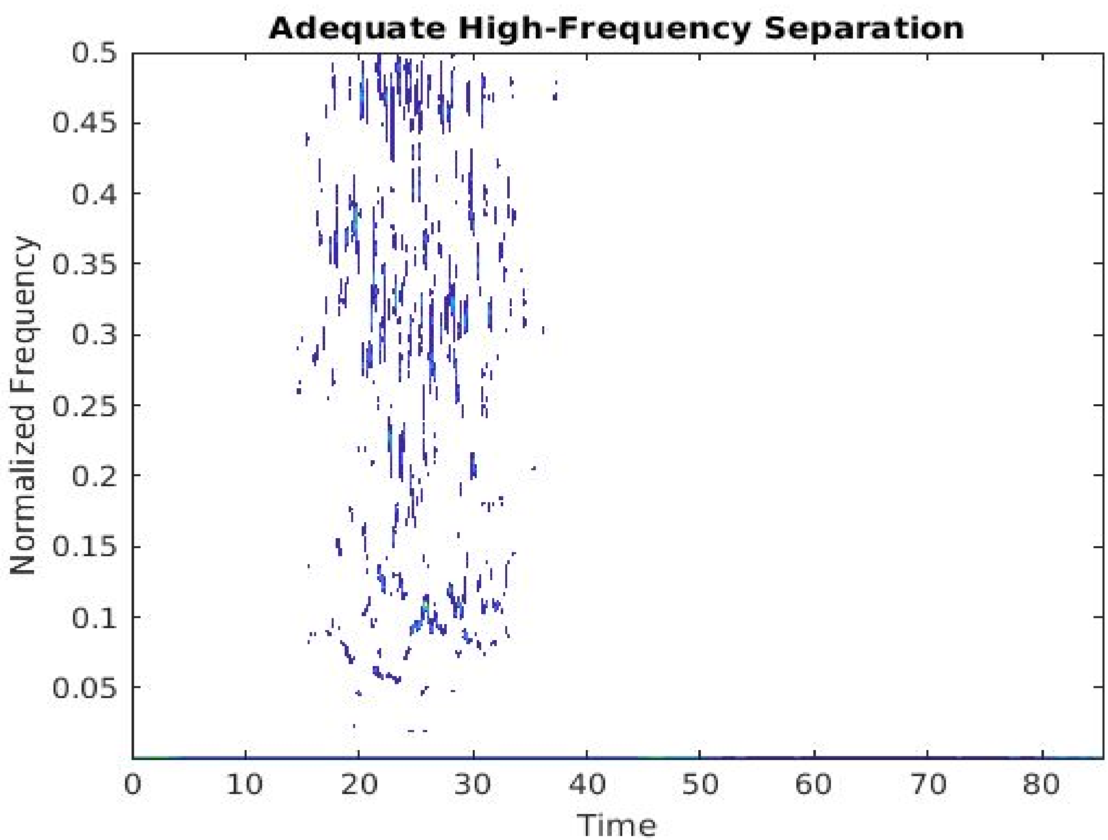
Now the Earthquake will look like this



Define the signal and plot the synchrosqueezed components



Now if we remove the 1 Hz frequency we will get a more better representation in below section

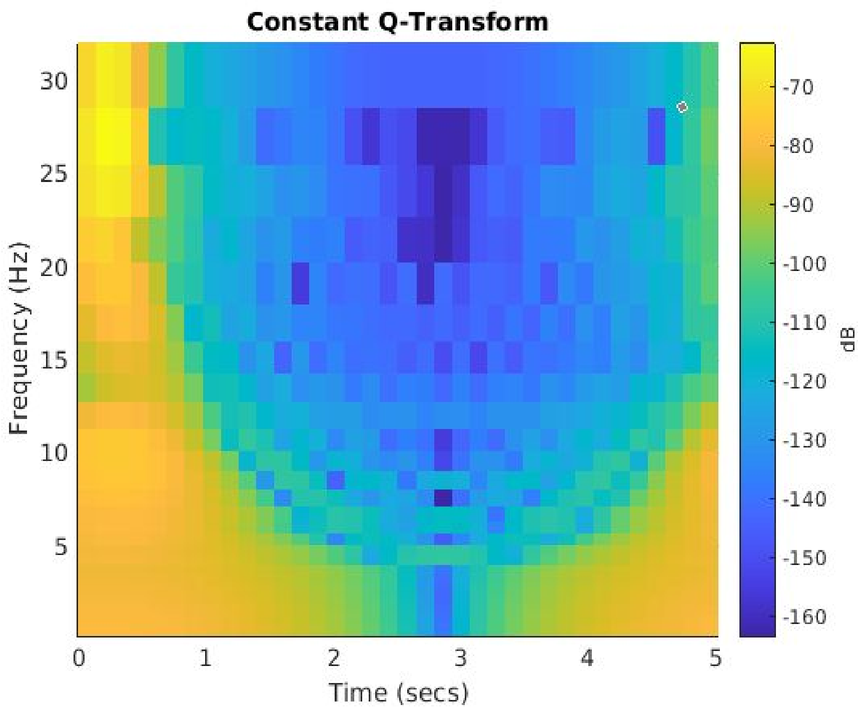


- **Constant-*Q* Gabor Transform**

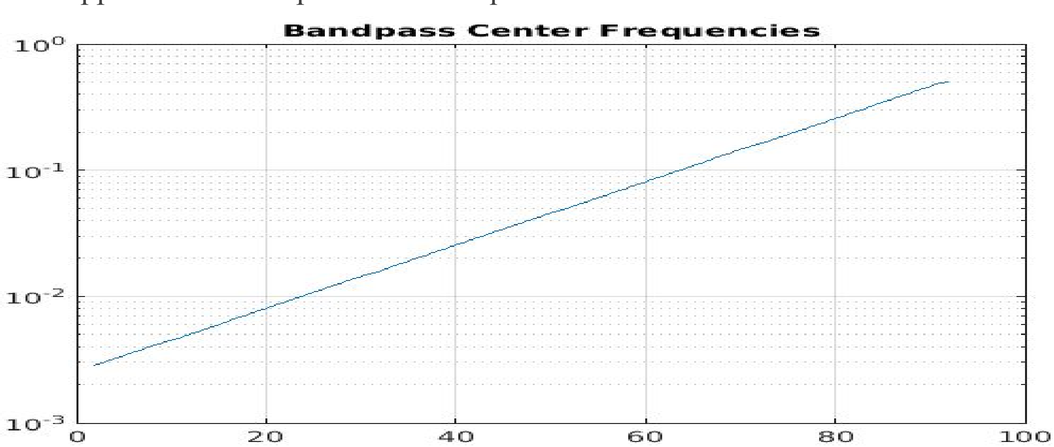
The constant-Q nonstationary Gabor transform uses windows with different center frequencies and bandwidths such that the ratio of center frequency to bandwidth, the Q factor, remains constant.The constant-*Q* Gabor transform enables the construction of stable inverses, yielding perfect signal reconstruction.In frequency space, the windows are centered at logarithmically spaced center frequencies.

Results

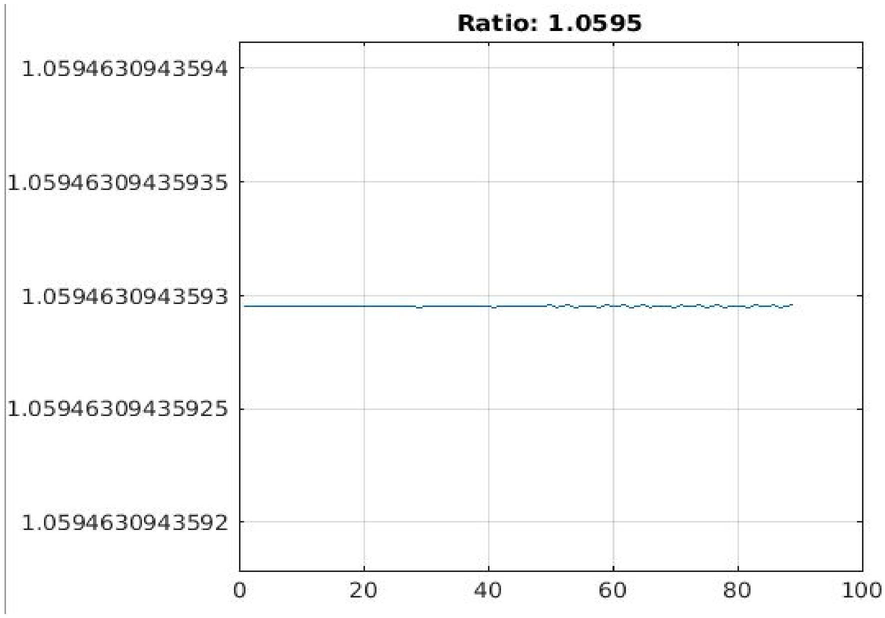
The transformed TF distribution looks like this



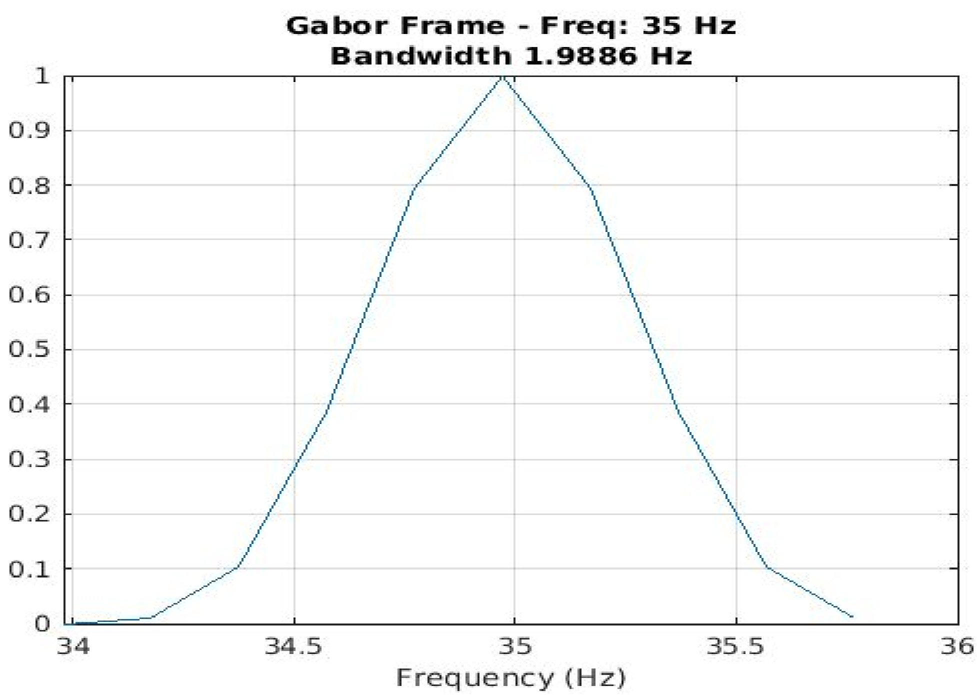
The the approximate bandpass center frequencies.



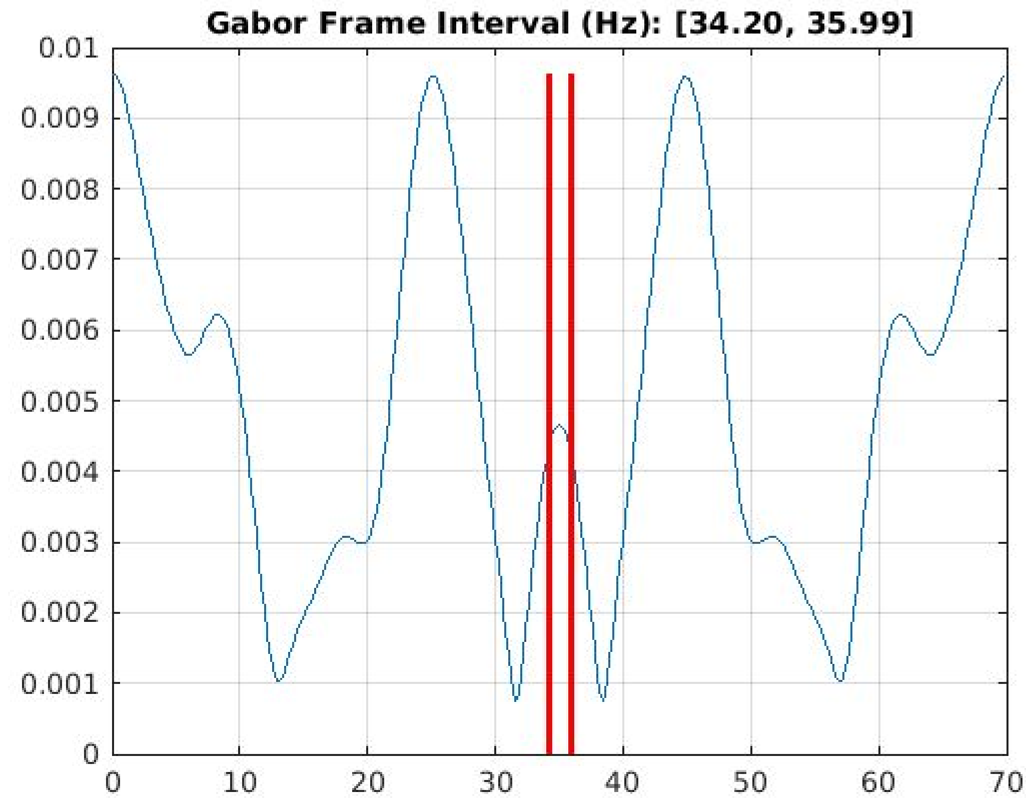
To confirm the ratios of consecutive pairs of frequencies are constant, plot the ratios. . Since the DC and Nyquist frequencies are not members of the geometric sequence of center frequencies but are included in the frequency vector, exclude them from the plot.



Now let **Visualize and Apply Constant-Q Transform Gabor Frames**



In the constant-Q transform, the Gabor frames are applied to the discrete Fourier transform of the input signal, and the inverse discrete Fourier transform is performed. The k-th Gabor frame is applied to the k-th frequency interval specified in fintervals. Take the discrete Fourier transform of the signal and plot its magnitude spectrum. Use fintervals to indicate over which Fourier coefficients are the Gabor frame associated with the Nyquist frequency are applied.

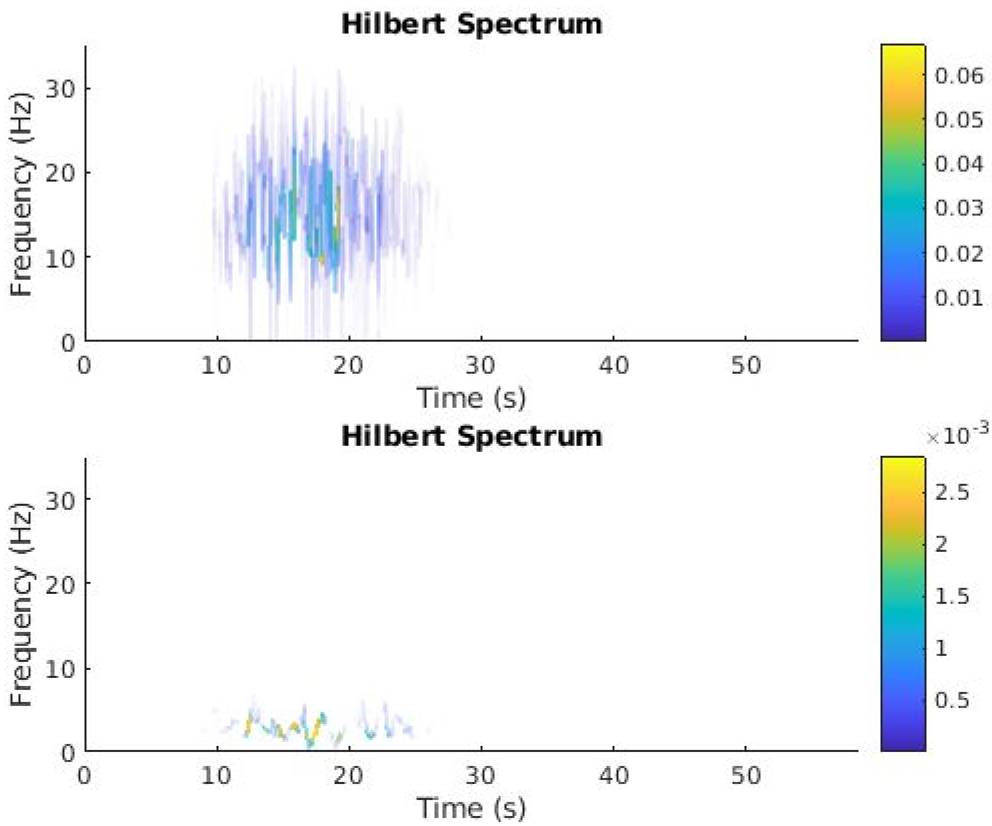


- **Empirical Mode Decomposition and Hilbert-Huang Transform**

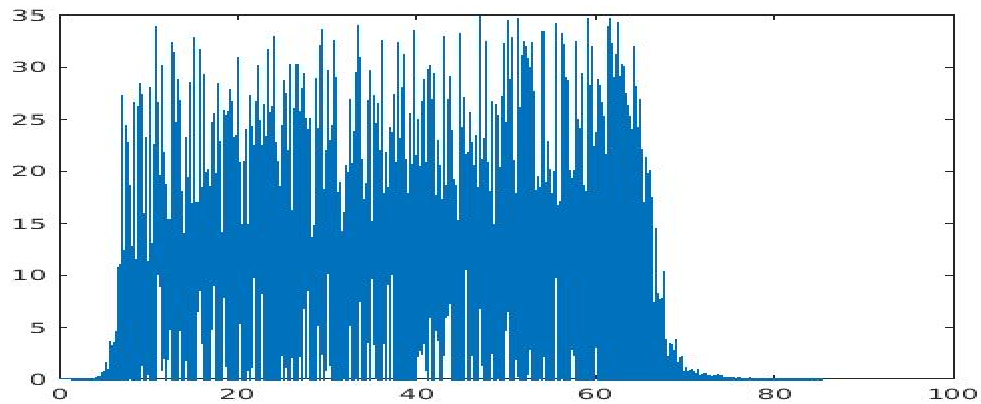
The *empirical mode decomposition* decomposes the signals into *intrinsic mode functions* which form a complete and nearly orthogonal basis for the original signal.Applied on Structural applications,Locate anomalies that appear as cracks, delamination, or stiffness loss in beams and plates.The Hilbert-Huang transform computes the instantaneous frequency of each intrinsic mode function.

Results

Compute the first five intrinsic mode functions (IMFs) of the signal. Plot the Hilbert spectrum of the first and third empirical modes. The first mode reveals increasing wear due to high-frequency impacts. The third mode shows a resonance occurring halfway through the measurement process that caused the defect.



Compute the analytic signal and differentiate its phase to measure the instantaneous frequency. The scaled derivative yields a meaningful estimate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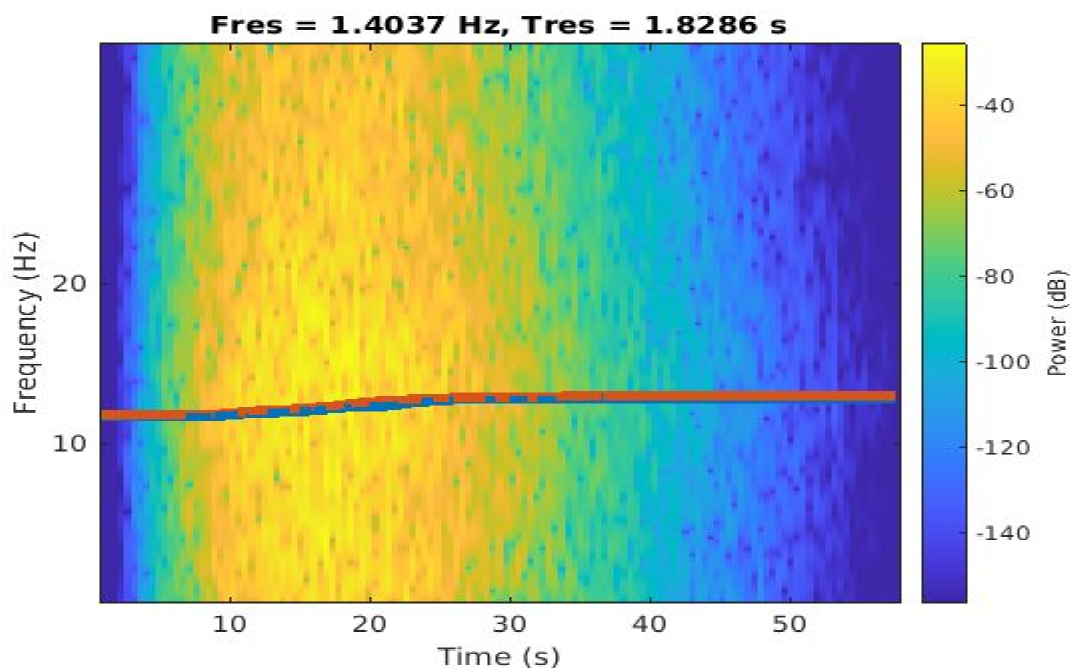
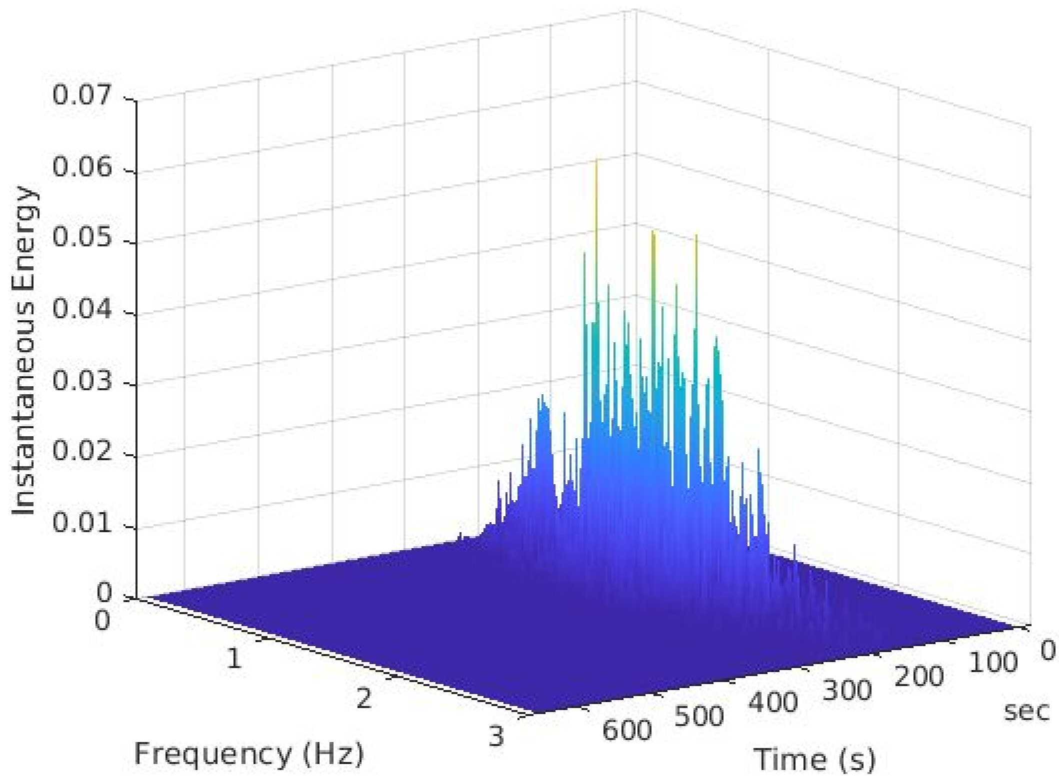
25
20
15
10
5
0
0
20
40
60
80
100


Fres = 1.4037 Hz, Tres = 1.8286 s
Frequency (Hz)
20
10
10
20
30
40
50
Time (s)
-40
-60
-80
-100
-120
-140
Power (dB)

A 3-D view of the Signal in Hilbert space



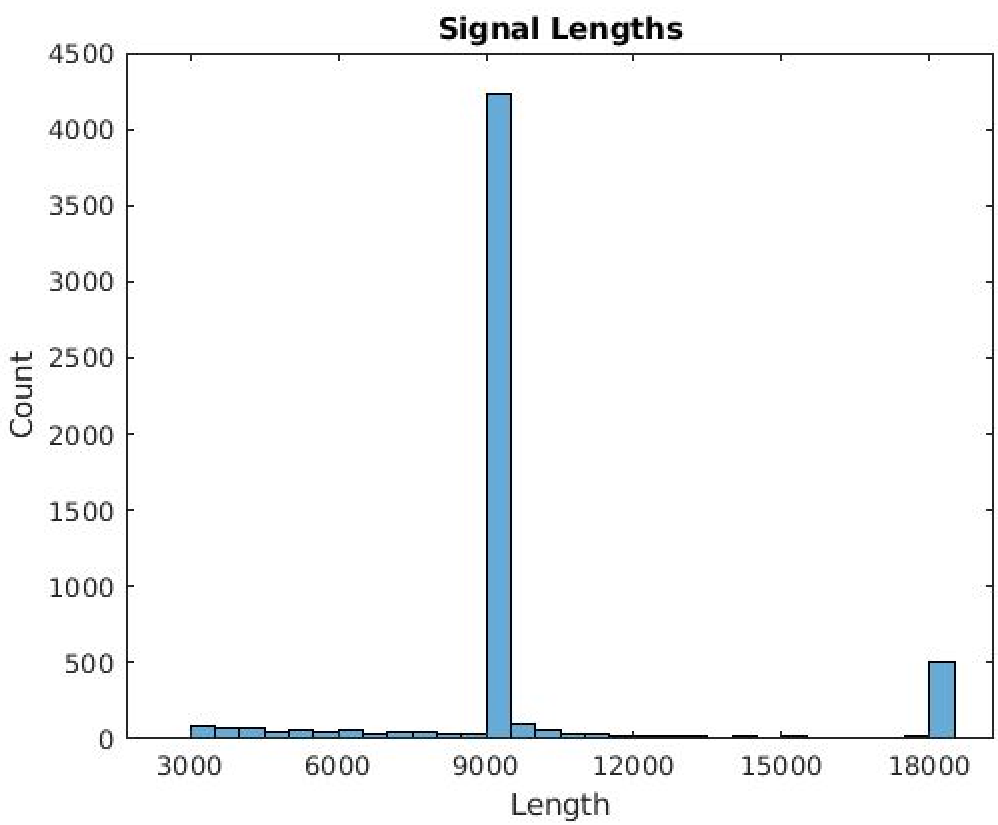
Machine Learning And Deep Learning

We can apply different techniques to extract information from Signal using Machine learning likewise,

This section refers to the method of using the deep neural long-short-term memory (LSTM) network for the problem of signal classification of Earthquake(EQ). EQ signals contain a lot of subtle information analyzed by seismologist to determine the type of Earthquake. Due to the large number of signal features that are difficult to identify, raw EQ data is usually not suitable for use in machine learning. This section presents how to transform individual EQ time series into spectral images for which two characteristics are determined, which are instantaneous frequency and spectral entropy. Feature extraction consists of converting the EQ signal into a series of spectral images using short-term Fourier transformation. Then the images were converted using Fourier transform again to two signals, which includes instantaneous frequency and spectral entropy. The data set transformed in this way was used to train the LSTM network. During the experiments, the LSTM networks were trained for both raw and spectrally transformed data. Then, the LSTM networks trained in this way were compared with each other. The obtained results prove that the transformation of input signals into images can be an effective method of improving the quality of classifiers based on deep learning.

Let me give a brief hypothetical situation and then analyze it with the help of ECG signal (due to unavailability of data at the point).

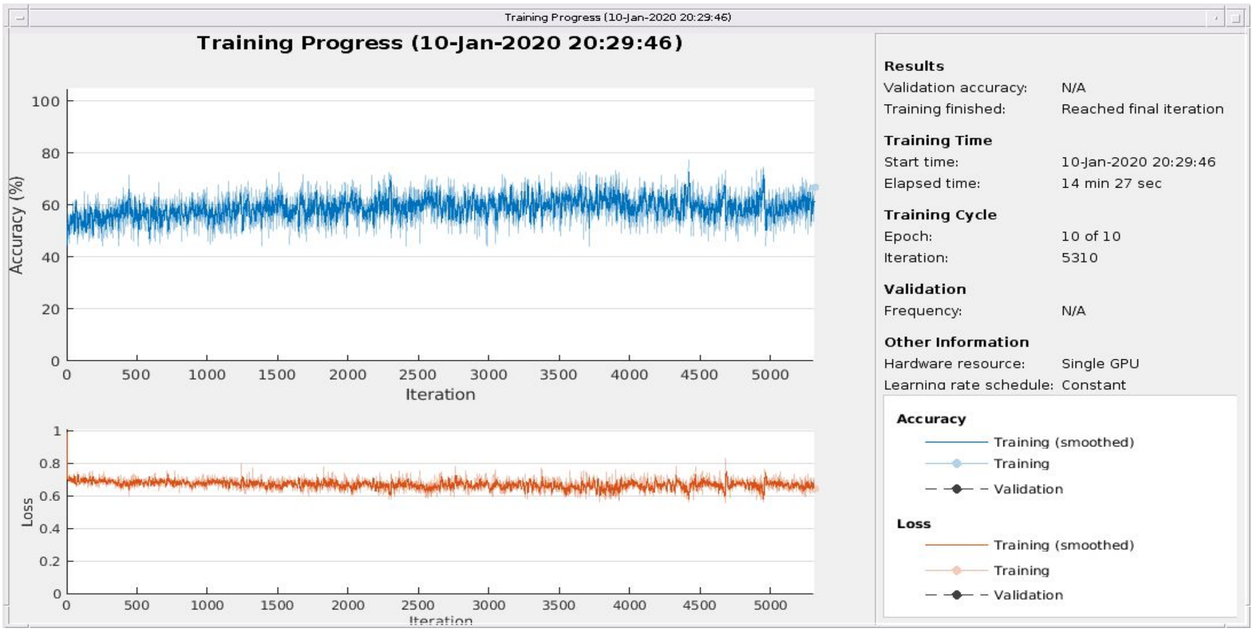
So let's assume we have a data with time series records of acceleration taken from accelerogram with some interval and labeled as is it an EQ or not . The first problem we will be facing is Uneven Label distribution that looks like this

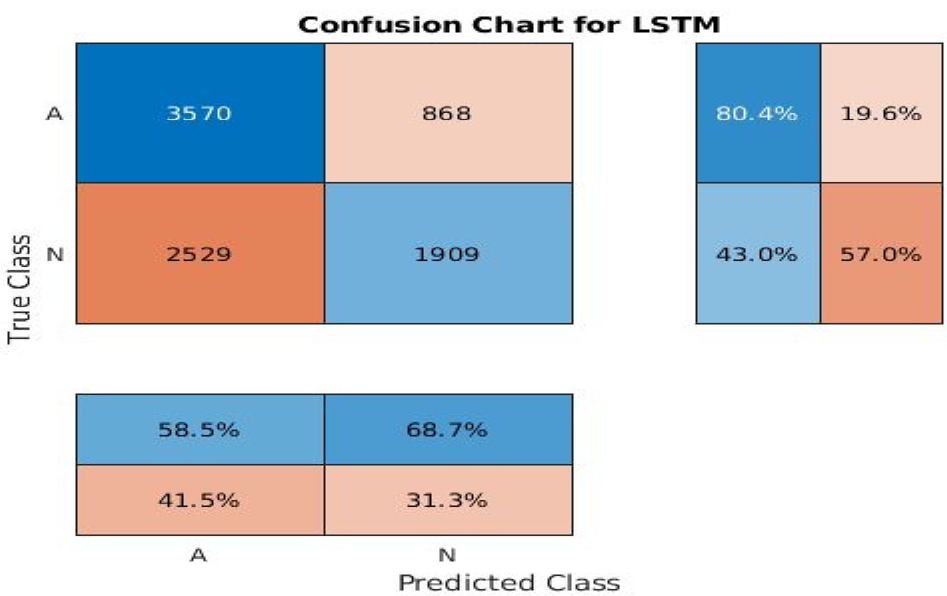


For now assume all the small peak bars as different label EQ signal and the long peak as Non EQ signal. So any Navie Machine learning or probabilistic model will result in less accuracy while predicting an Earthquake due to it's less frequency. So we can use LSTM network that remembers any significant signal property of past and utilize it to predict signal of future.

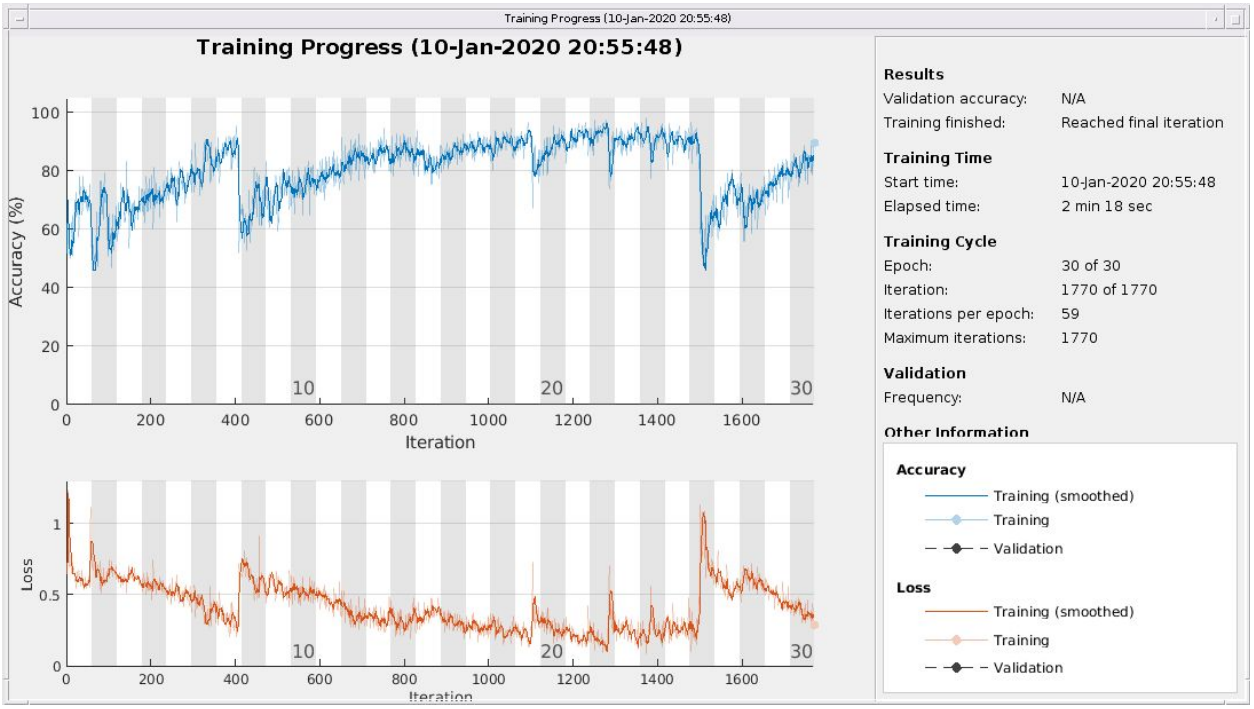
And using tf distribution as feature will help in accuracy sharing picture of same type of problem with ECG data.

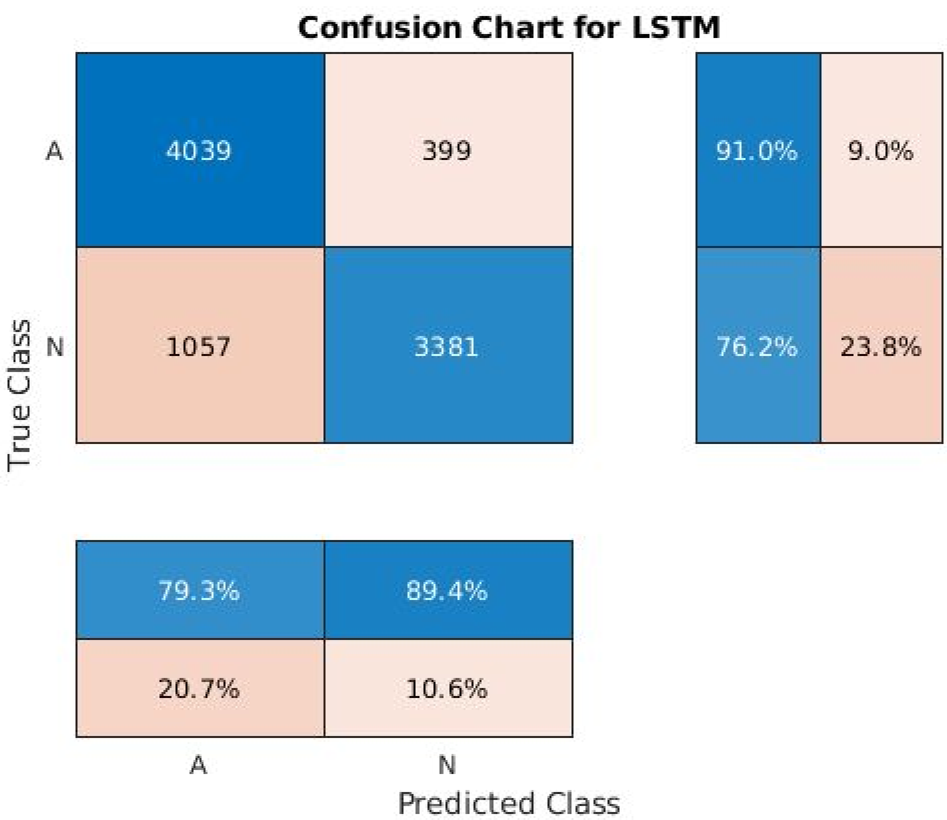
Screen of LSTM with Raw data





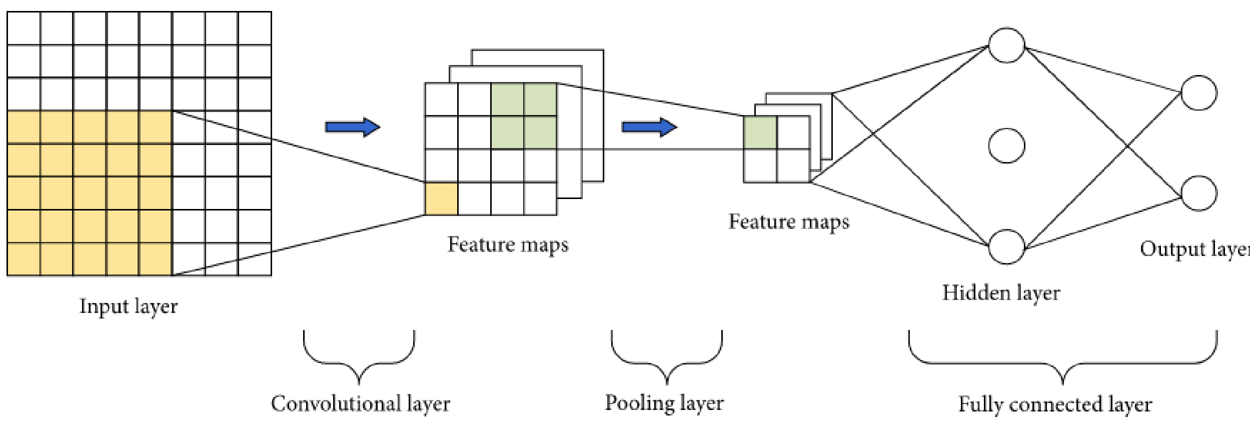
Screen with Time frequency as feature

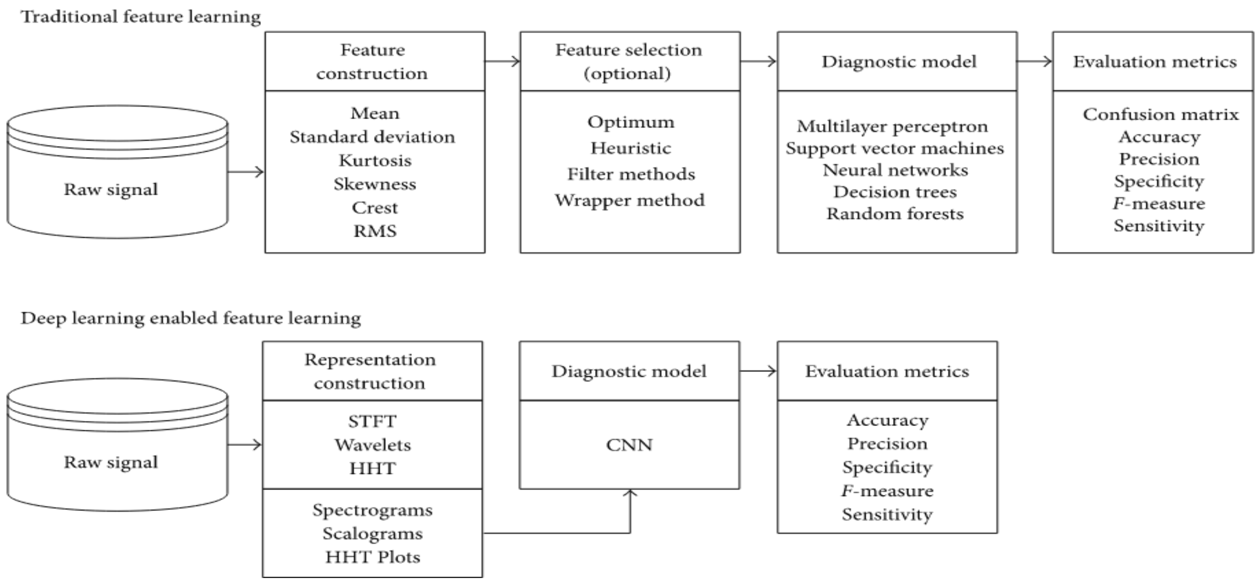




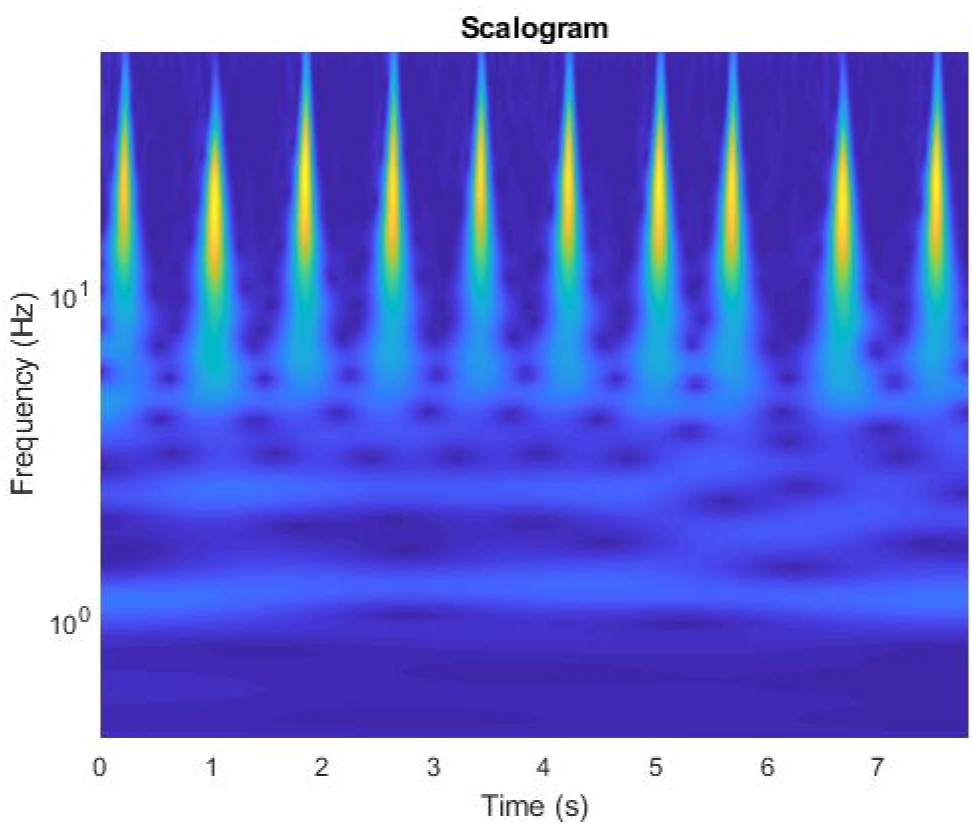
See a significant improvement and less bias in result.

Now we can also say machine learning is useful in time frequency analysis as Traditional feature extraction and selection is a labor-intensive process requiring expert knowledge of the relevant features pertinent to the system. This knowledge is sometimes a luxury and could introduce added uncertainty and bias to the results. To address this problem a deep learning enabled featureless methodology is proposed to automatically learn the features of the data. Time-frequency representations of the raw data are used to generate image representations of the raw signal, which are then fed into a deep convolutional neural network (CNN) architecture for classification and fault diagnosis. This methodology was applied to two public data sets of rolling element bearing vibration signals. Three time-frequency analysis methods (short-time Fourier transform, wavelet transform, and Hilbert-Huang transform) were explored for their representation effectiveness by **David Verstraete**. The proposed CNN architecture achieves better results with less learnable parameters than similar architectures used for fault detection, including cases with experimental noise.

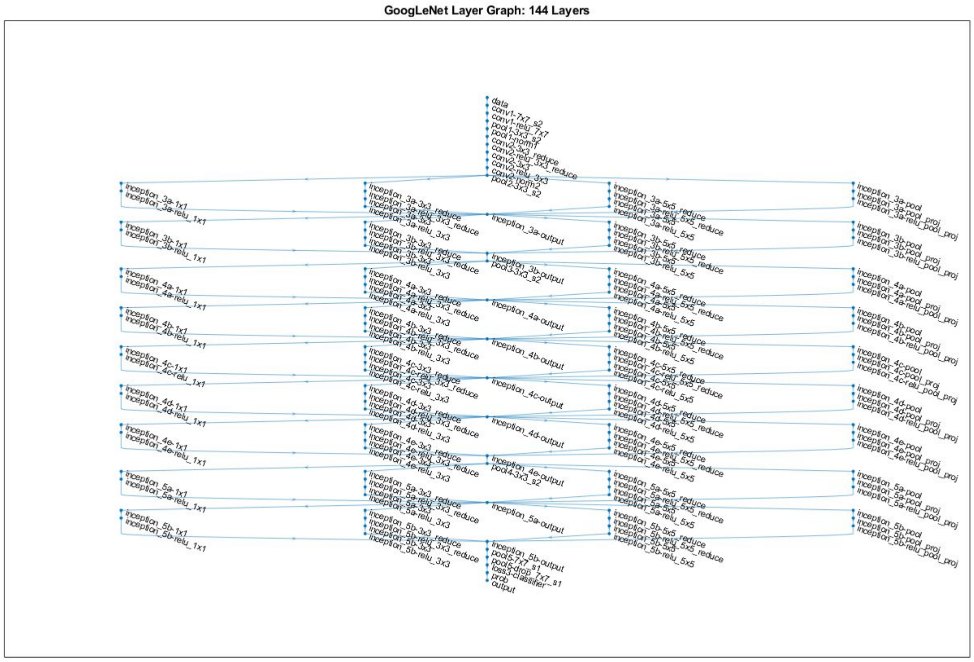




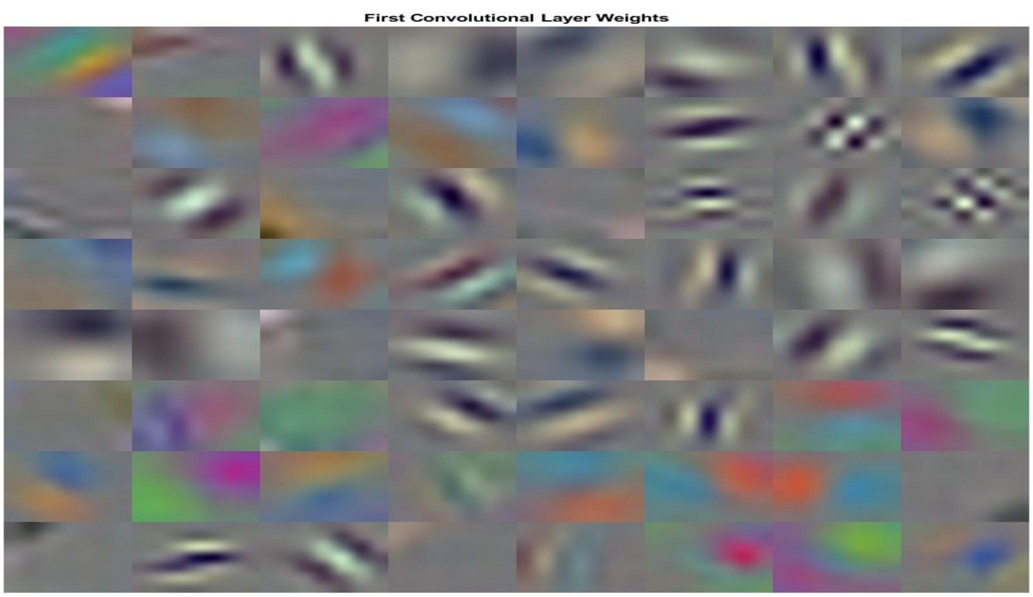
We can use wavelet to feature extract from signal and ANN to classify time series EQ signals. Such Examples are shown below with scalogram



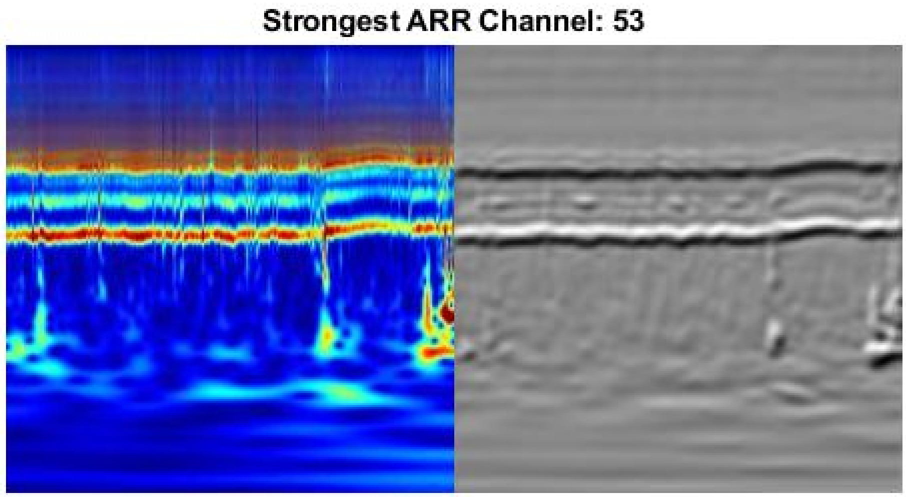
Now let use GoogleLens



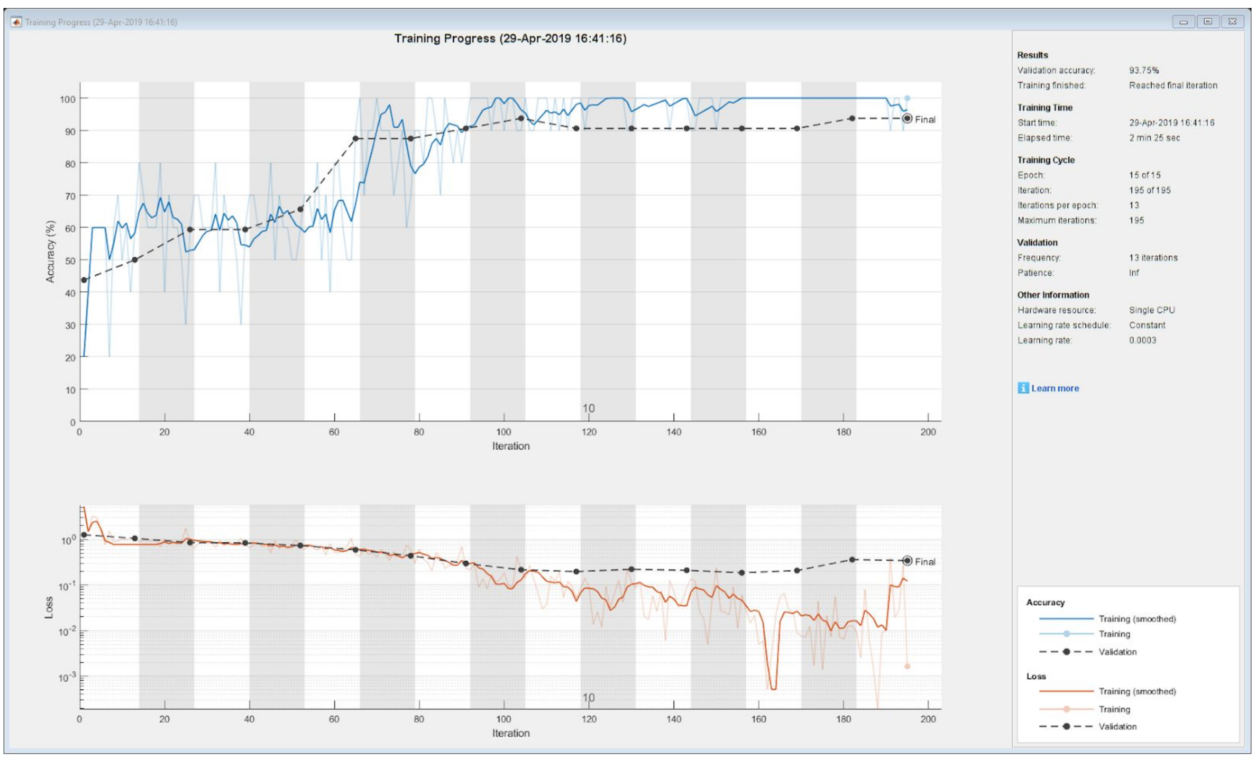
1st layer of convolution weights



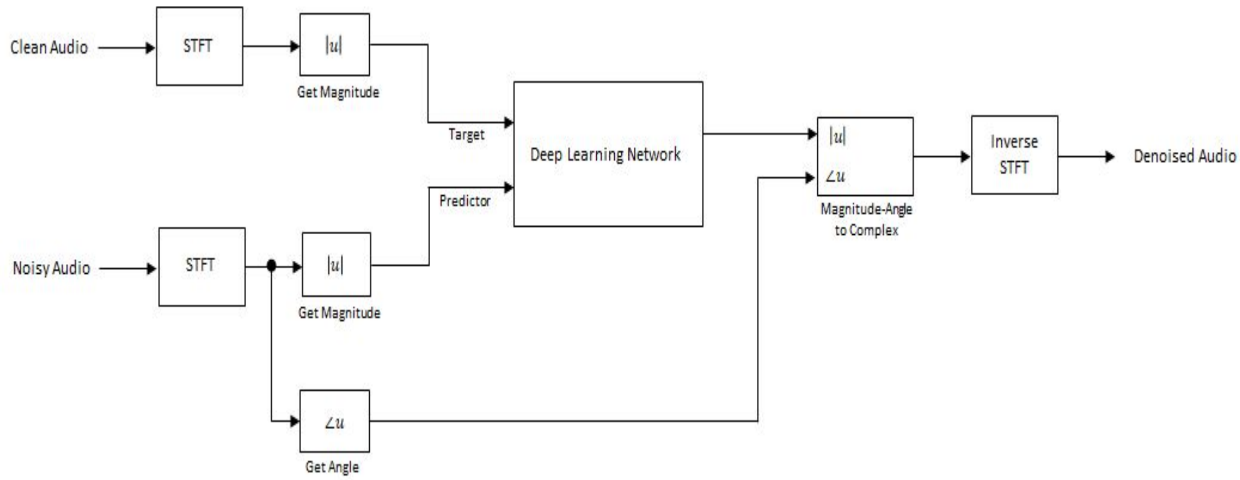
See how convolution finds information from signal



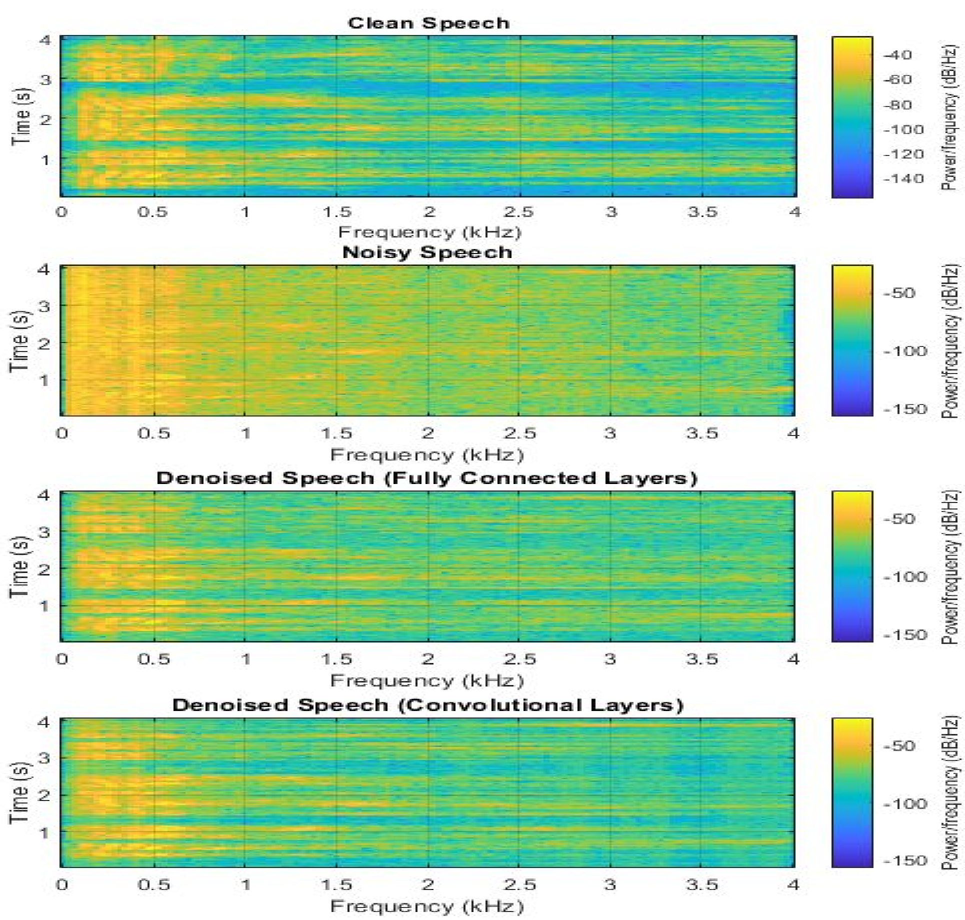
Results



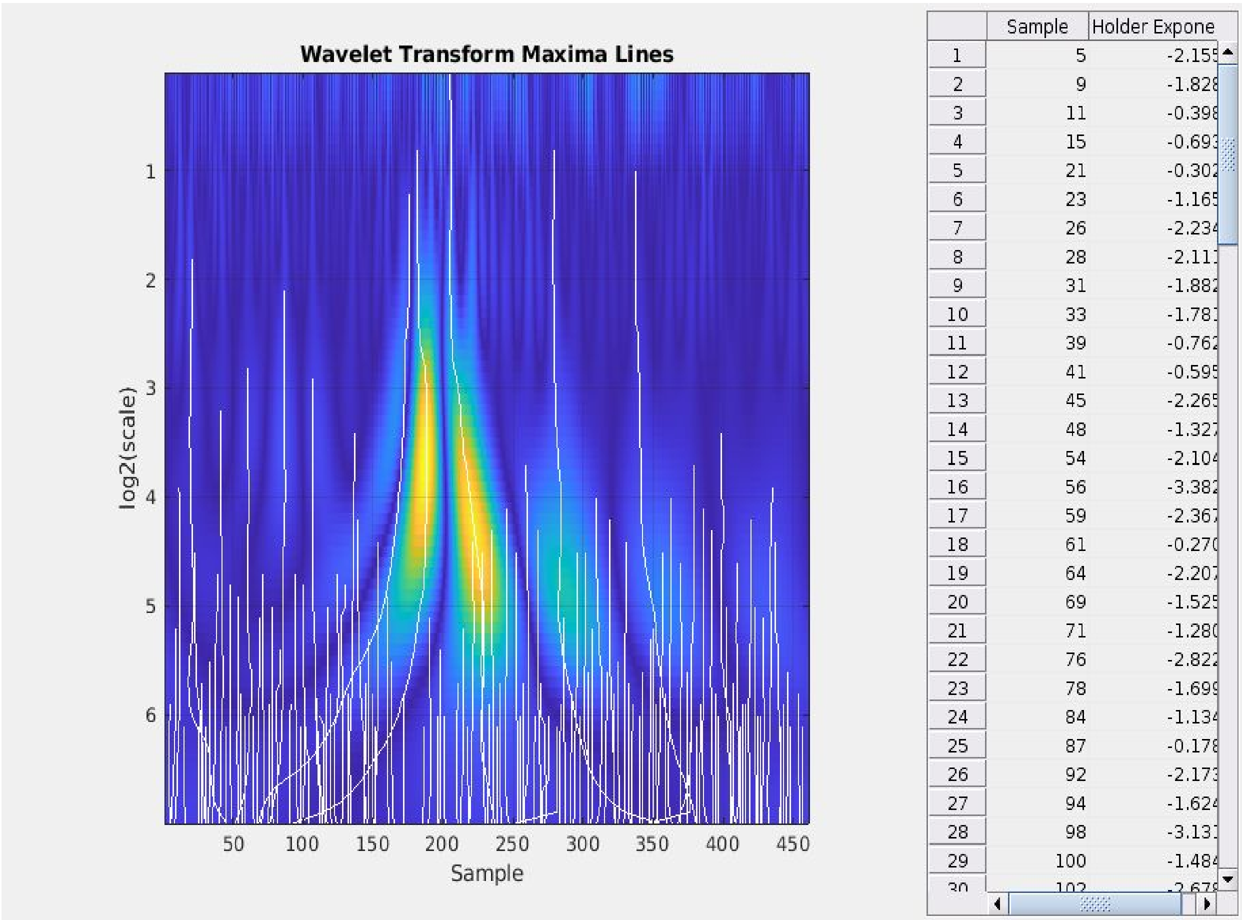
We can also demoise EQ signals using deep learning



We can see a spectrogram of how deeplearning denoise signal using convolution



We can also detect crack using accelerogram record on a site



	Sample	Holder Expone
1	5	-2.155
2	9	-1.828
3	11	-0.398
4	15	-0.693
5	21	-0.302
6	23	-1.165
7	26	-2.234
8	28	-2.111
9	31	-1.882
10	33	-1.781
11	39	-0.762
12	41	-0.595
13	45	-2.265
14	48	-1.327
15	54	-2.104
16	56	-3.382
17	59	-2.367
18	61	-0.270
19	64	-2.207
20	69	-1.525
21	71	-1.280
22	76	-2.822
23	78	-1.699
24	84	-1.134
25	87	-0.178
26	92	-2.173
27	94	-1.624
28	98	-3.131
29	100	-1.484
30	102	-2.678

- **Conclusion**

So far we have seen many methods in this paper and many modern approaches to easy researchers work in this field . By combining all this we strongly believe that we can achieve many impossible and hard task easily .

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