Chapter 12

P.O $\Longleftrightarrow$ G.E (Welfare Theorem 1.2).

Complete Market. $C_{ks}(e_s)$ $\forall k, k'$, C. correlated.

Representative Consumer 代表消费者 (HARA).

$$P_j = E\left[\delta \frac{u'(\tilde{c}_1)}{u'(\tilde{c}_0)} \tilde{x}_j\right] \Leftrightarrow 1 = E\left[\delta \frac{u'(\tilde{c}_1)}{u'(\tilde{c}_0)} (1+\tilde{r}_j)\right].$$

决定价格的是总禀赋

$$P_j = E[\quad \tilde{x}_j] \Leftrightarrow 1 = E[\tilde{m}(1+\tilde{r}_j)] \Rightarrow 1 = E[\tilde{m}](1+r_f).$$

C-CAPM里 $\tilde{m}$ 就是 $\delta \frac{u'(\tilde{c}_1)}{u'(c_0)}$ 跨期贴现因子.

Asset Pricing: $\tilde{c}_1, c_0 \longrightarrow E[\tilde{r}_j]$?

Consumption Theory: $\tilde{r}_j \to c_0, \tilde{c}_1$.

$$E[\tilde{r}_j] = r_f + (E(\tilde{r}_j) - r_f)$$

$$\tilde{g} \triangleq \frac{\tilde{c}_1}{c_0} - 1, \quad \tilde{c}_1 = c_0(1+\tilde{g}).$$

$$Var(\tilde{g}) = E[\tilde{g}-\bar{g}]^2 = E[\tilde{g}]^2 + E[\bar{g}]^2 - 2E[\tilde{g}]\bar{g}$$
$$= E[\tilde{g}^2] - \bar{g}^2$$

一般来说, $\bar{g}^2$ 是小数 $\approx E[\tilde{g}^2]$.

$$\tilde{m} = \delta \frac{u'(c_0(1+\tilde{g}))}{u'(c_0)} \Rightarrow \text{Taylor Expansion at } c_0:$$

$$= \frac{\delta}{u'(c_0)}\left[u'(c_0) + u''(c_0)c_0\tilde{g} + \frac{1}{2}u'''(c_0)c_0^2\tilde{g}^2\right].$$

$$= \delta\left[1 - R_R\tilde{g} + \frac{1}{2}R_R \cdot P_R\tilde{g}^2\right].$$ R_R: 相对风险. P_R: 相对审慎.

$$E(\tilde{m}) \approx E[\delta(1-R_R\tilde{g}+\tfrac{1}{2}R_RP_R\tilde{g}^2)] = \delta[1-R_RE(\tilde{g})+\tfrac{1}{2}R_RP_RE(\tilde{g}^2)]$$

$$= \delta(1-R_R\bar{g}+\tfrac{1}{2}R_RP_R\sigma_g^2)$$

$$\therefore r_f = \frac{1}{E(\tilde{m})}-1 \approx \frac{1}{\delta(1-R_R\bar{g}+\frac{1}{2}R_RP_R\sigma_g^2)}-1 = \frac{1-\delta}{\delta}+R_R\bar{g}-\tfrac{1}{2}R_RP_R\sigma_g^2$$

$\therefore \frac{1-\delta}{\delta}$：不耐心程度，越不耐心 ⇒ 越不愿意的储蓄

投资：做储蓄来平衡未来消费 ⇒ 未来才有利可图.

$R_R\tilde{g}$：消费平均增长率↑ 未来的消费多 ⇒ 现在储蓄意愿越低.

为了平衡这种欲望 需要更大的无风险利率.

R_R：越是风险厌恶↑ 则越会平滑消费 ⇒ 越会增加当期消费.

⇒ 越是需要更高的无风险利率来促进储蓄.

Economic Growth.

$-\frac{1}{2}R_RP_R\sigma_g^2$：Precautionary Saving. 预防性储蓄动机.

⇒ 希望多储蓄 ⇒ 不需要 r_f 来驱动.

C-CAPM 真实的无风险利率.

真实利率 + 通胀 = 名义利率

Equilibrium. Non-storable. vs. saving.

原因是宏观和微观 通过跨代微观的个人可以储蓄，但宏观上不能有储蓄，一旦有宏观储蓄，则价格去调整 ⇒ 无风险利率会越来越低 最后使得平衡.

$u(c)=\frac{c^{1-\gamma}-1}{1-\gamma}$ $R_R=\gamma$. $P_R=\gamma+1$ $\delta=0.98$

中国来说：$r_{f_{CN}} = 2\% + 2\times 9.7\% - \frac{1}{2}\times 2\times 3\times 0.05\% = 21.25\%$.

$\bar{g} = 9.7\%$. 相对风险

$r_f = \rho + \gamma \bar{g}$，大头来自于经济增长。

Risk Free Rate Puzzle 无风险利率之谜

风险溢价之谜

$$E(\tilde{r}_j) - r_f = -\frac{\delta(1+r_f)}{u'(c_0)} \mathrm{Cov}(u'(\tilde{c}_1), \tilde{r}_j)$$

CRRA: $\tilde{m} = \delta (c_0(1+\tilde{g}))^{-\gamma} / c_0^{-\gamma} = \delta (1+\tilde{g})^{-\gamma}$

$$E(\tilde{r}_j) - r_f = -\delta(1+r_f) \cdot \mathrm{Cov}((1+\tilde{g})^{-\gamma}, \tilde{r}_j)$$

$$\approx -\delta(1+r_f) \cdot \mathrm{Cov}(1-\gamma\tilde{g}, \tilde{r}_j)$$

$$= \delta\gamma(1+r_f) \cdot \mathrm{Cov}(\tilde{g}, \tilde{r}_j) \qquad (1+r_f) = \frac{1}{\delta} + \gamma\bar{g}$$

$$= \gamma(1+\delta\gamma\bar{g}) \cdot \mathrm{Cov}(\tilde{g}, \tilde{r}_j)$$

Mehra & Prescott 1889–1978年

$$6.2\% = \gamma(1+0.999 \times 1.8\% \times \gamma) \times 0.3\%$$

⇒ 解出 $\gamma = 16$，这个是一个非常高的概率。

相对风险系数估计

$\frac{1}{2}$ win 50%

$\frac{1}{2}$ loss 50%

⇒ 付出 x 的钱后，可以不参与赌局。

x	21%	41%	46%	47%
γ	2	5	10	15

γ 应该低于5，高于10已经非常高了。

如果 $\gamma = 16$，则美国 $r_{f,us}$:

$$r_{f,us} = \rho + \gamma\bar{g} = 0.1\% + 16 \times 1.8\% = 28.9\%$$

Equity Premium Puzzle 风险溢价谜题

Homework 12

12.2).

	φ_1	φ_2	φ_3
	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
	state 1	state 2	state3
A	2	0	1
B	0	1	1
r_f	1	1	1

r_f: 确定付给消费者1单位.

$$u_k(c_{k,0}, \tilde{c}_{k,1}) = \ln c_{k,0} + 0.9 \times E(\ln \tilde{c}_{k,1}). \quad k=1,2,3.$$

ⓐ. 对于消费者K:

一期消费 c_{k0}, 二期消费 c_{k1}, c_{k2}, c_{k3}

$$\max \quad \ln c_{k,0} + 0.9 \times \left(\frac{1}{3}\ln c_{k1} + \frac{1}{3}\ln c_{k2} + \frac{1}{3}\ln c_{k3}\right).$$

$$\text{s.t.} \quad c_{k0} + \varphi_1 c_{k1} + \varphi_2 c_{k2} + \varphi_3 c_{k3} = e_k. \quad k=1,2,3.$$

$e_1 = 1 + 2\varphi_1 + \varphi_3$

$e_2 = 2\varphi_1 + \varphi_2 + \varphi_3$

$e_3 = 1 + \varphi_2 + \varphi_3$.

ⓑ. 解上述拉格朗日方程:

① $L_1 \ \ln c_{10} + \left(0.9 \times \frac{1}{3}\ln c_{11} + \frac{1}{3}\ln c_{12} + \frac{1}{3} c_{13}\right) - \lambda_1 (c_{10} + \varphi_1 c_{11} + \varphi_2 c_{12} + \varphi_3 c_{13} - 1 - 2\varphi_1 - \varphi_3)$

$\frac{\partial L_1}{\partial c_{10}} = 0 \Rightarrow \frac{1}{c_{10}} - \lambda_1 = 0$. $\quad \frac{\partial L_1}{\partial c_{12}} = 0 \Rightarrow \frac{0.3}{c_{12}} - \lambda_1 \varphi_2 = 0$.

$\frac{\partial L_1}{\partial c_{11}} = 0 \Rightarrow \frac{0.3}{c_{11}} - \lambda_1 \varphi_1 = 0$. $\quad \frac{\partial L_1}{\partial c_{13}} = 0 \Rightarrow \frac{0.3}{c_{13}} - \lambda_1 \varphi_3 c_{13} = 0$

$\Rightarrow \frac{1}{\lambda_1}+\frac{0.3}{\lambda_1}+\frac{0.3}{\lambda_1}+\frac{0.3}{\lambda_1}=1+2\varphi_1+\varphi_3 \Leftrightarrow \frac{1.9}{\lambda_1}=1+2\varphi_1+\varphi_3$

同理：$\frac{1.9}{\lambda_2}=2\varphi_1+\varphi_2+\varphi_3 \quad \frac{1.9}{\lambda_3}=1+\varphi_2+\varphi_3$.

市场出清：

禀赋：$e_1=1+2\varphi_1+\varphi_3$ $\quad \varphi_1=4$

$e_2=2\varphi_1+\varphi_2+2\varphi_3$ $\quad \varphi_2=2$

$e_3=1+\varphi_2+\varphi_3$. $\quad \varphi_3=4$,

$\therefore \sum_{k=1}^{3} c_{k0}=2 \quad \sum_{k=1}^{3} c_{k1}=4 \quad \sum_{k=1}^{3} c_{k2}=2 \quad \sum_{k=1}^{3} c_{k3}=4$

$\Rightarrow \varphi_1=0.15, \quad \varphi_2=0.3, \quad \varphi_3=0.15$.

$\Rightarrow P_A=2\varphi_1+\varphi_3=0.45$.

$P_B=\varphi_2+\varphi_3=0.45$.

$P_{无风险}=\varphi_1+\varphi_2+\varphi_3=0.6$.

③ $r_f=0.6$

r_f与贴现率不一致：

收益率与效用不一样

0.9与0.6确定不一样.