Lec 3 Interest Rate and Bond Analysis.

Recap: Time, Uncertainty.

Asset Pricing: $X \to P[E(F)]$. 定价

均衡定价: $D \to S \to P$

demand supply price.

N.A. 无套利定价分析: some $P_s' \to$ other P_s'.

本讲就主要 focus 在计算利率.

1 principle 本金.

lender ⇄ borrower

principle 1+0.1 interest(利息). interest rate(利率).

risky rate − risky free rate = credit spread

Simple Interest: $A = (1+nr)A_0$ 利息不产生利息.

principle. years. rate.

Compound Interest: $A = (1+r)^n \cdot A_0$.

连续复利: $A(1+\frac{r}{m})^{nm} \to \lim_{m\to\infty} A(1+\frac{r}{m})^{nm}$

$= \lim_{m\to\infty} A(1+\frac{r}{m})^{\frac{m}{r}\cdot nr} = Ae^{nr}$.

FV: future value. PV: present value.

$FV = PV(1+r)^n$. $PV = FV/(1+r)^n$.

$FV = PV e^{nr}$ $PV = FV e^{-nr}$.

项目分析：

0	1	2	3
−100	30	60	40

cash flow. $r=10\%$.

需要将未来的钱折算到现在（也可以折算到未来时）

Net Present Value (NPV).

$$NPV = -100 + \frac{30}{(1+r)} + \frac{60}{(1+r)^2} + \frac{40}{(1+r)^3} = 6.9.$$

所以决定现金的回报与 r, cash flow 都有关系.

$r\uparrow$. $NPV\downarrow$：因为回报是未来给的未来折算现值会随着 $r\uparrow$ 而贬值.

Internal Rate of Return. IRR

s.t. $NPV=0$. 解 IRR：只需要数值解.

$\Longrightarrow$ IRR = 13.7%.

IRR > r（市场收益率）✓ IRR < r ✗

Ex. 京东白条：6 months. 6299 → 1081.14. 0.5%起.

$$0 = 6299 + \frac{1081.14}{1+IRR} + \cdots + \frac{1081.14}{(1+IRR)^6} \Rightarrow IRR \approx 10\%.$$

Reinvestment Risk.

在中期获得的现金流再投资，在实业中不行，但债券行.

债券拿的钱取决于再投资风险.

3.4 Bond Analysis

Government Bond 国债.

	Face Value (面值)	Maturity (期限)	Coupon	Current Price
A	100	1	0	95
B	100	2	5	97.

对于 zero coupon bond 零息债券.

问 A好还是B好?

A: -95 100

B: -97 5 5+100.

yield and maturity (IRR).

coupon rate = $\frac{\text{coupon}}{\text{face value}}$ 和收益率无关.

对 A: $0 = -95 + \frac{100}{(1+y_A)} \Rightarrow y_A = 5.26\%$. $y_B > y_A$

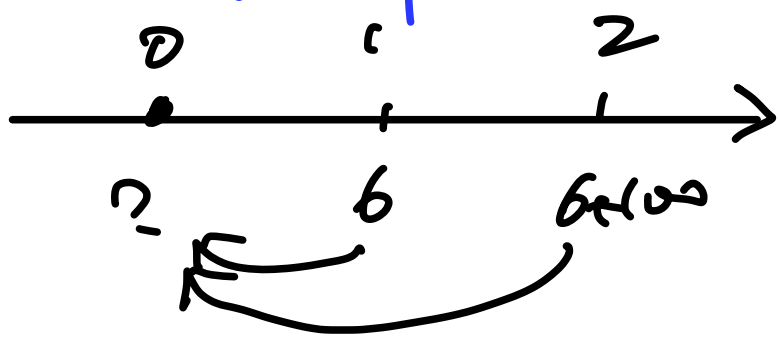
对 B: $0 = -97 + \frac{5}{(1+y_B)} + \frac{105}{(1+y_B)^2} \Rightarrow y_B = 6.65\%$

但 y_A 和 y_B 不可比, 因为有再投资风险, 以及投资期限不一样. 以及受到市场利率 r 有关系.

CurrentPrice (Cashflow(r)) 函数关系

3.7. 即期利率 远期利率:

引入 C 债券:



C: 100, 2, 6, ?

CurrentPrice ↑

析现时. 安计算好利息:

① 一年息零息债券. 可以直接计算.

② spot rate. zero rate.

yield → spot rate → pricing bond.

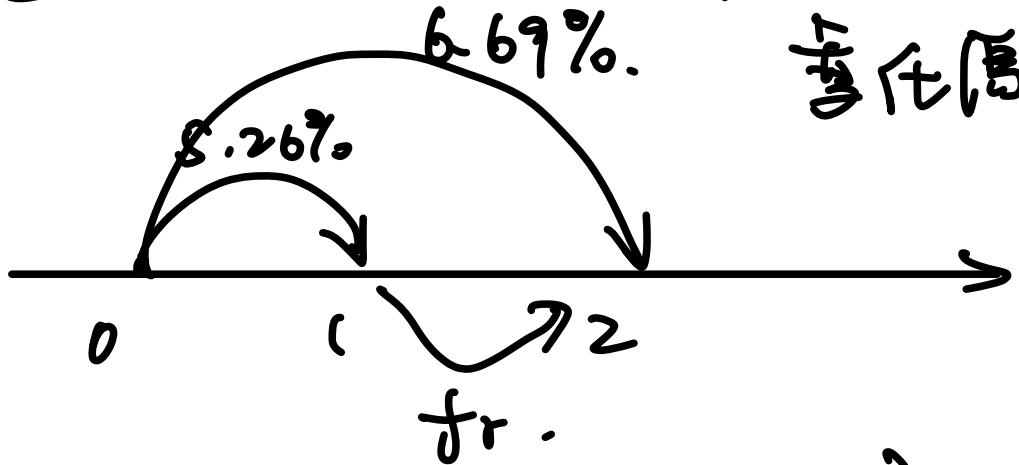
↑ bootstrap methods.

$r_1 = \frac{100}{95} - 1 = 5.26\%$ (A直接计算).

$97 = \frac{5}{1+r_1} + \frac{105}{(1+r_2)^2} \Rightarrow r_2 = 6.69\%$.

同理如法炮制 $r_3, r_4, \cdots$. (bootstrap Method).

$P_C = \frac{6}{1+r_1} + \frac{106}{(1+r_2)^2} = 98.83$ 所以当 P_C 贵, 则卖.



当代偶同是市场预期在产生变化.

f_r: 远期利率.

$f_r = 100 \times (1+6.69\%)^2 = 100 \times (1+5.26\%)(1+f_r)$.

$\Rightarrow f_r = 8.13\%$. forward rate.

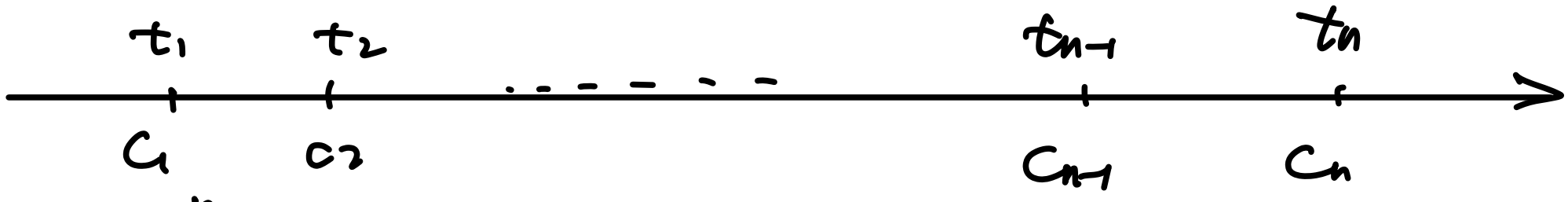
f_r 反应了市场预期利率.

Duration: 久期. 债券平均到期期限:

债券价格对利率的敏感性:

① $r\uparrow$. duration $\downarrow$. ② $r\downarrow$ duration $\uparrow$.

这样才能赚钱.



$$P = \sum_{i=1}^{n} c_i e^{-y t_i}$$

$\therefore D = \frac{1}{P} \cdot \sum_{i=1}^{n} t_i \cdot c_i e^{-y t_i}$, $dP = -P \cdot D \cdot dy$.

$\therefore$ 对于一只久期10年债券. dy 变化1%. 则 P 下降10%.

连续复利: $\frac{\Delta P}{P} = -\frac{D}{1+\frac{y}{m}} \cdot \Delta y$.

HW3.

3.1 $0 = -1200 + 500 + \frac{500}{1+IRR} + \frac{500}{(1+IRR)^2}$

$\Rightarrow IRR = 27.5\%$.

3.2.

a. $\frac{100}{90} - 1 = r_1 \Rightarrow r_1 = 11.1\%$.

$95 = \frac{5}{1+r_1} + \frac{105}{(1+r_2)^2} \Rightarrow r_2 = 7.71\%$.

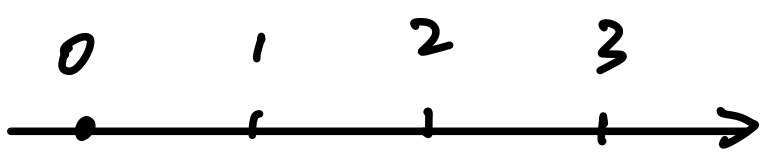
$96 = \frac{6}{1+r_1} + \frac{6}{(1+r_2)^2} + \frac{106}{(1+r_3)^3} \Rightarrow r_3 = 7.46\%$.

b. $90 = \frac{100}{r_1'} \Rightarrow r_1' = 11.1\%$

$95 = \frac{5}{1+r_2'} + \frac{105}{(1+r_2')^2} \Rightarrow r_2' = 7.8\%$

$96 = \frac{6}{1+r_3'} + \frac{6}{(1+r_3')^2} + \frac{106}{(1+r_3')^3} \Rightarrow r_3' = 7.54\%$

c.



对于 f_{12}: $(1+r_1)(1+f_{12}) = (1+r_2)^2$. $f_{12} = 4.42\%$.

f_{23}: $(1+r_1)(1+f_{12})(1+f_{23}) = (1+r_3)^3$: $f_{23} = 6.95\%$.

d. 对于 f_{13}: $(1+r_1)\cdot(1+f_{13})^2 = (1+r_3)^3$

$\Rightarrow f_{13} = 5.68\%$

e. 假设市值为 CP.

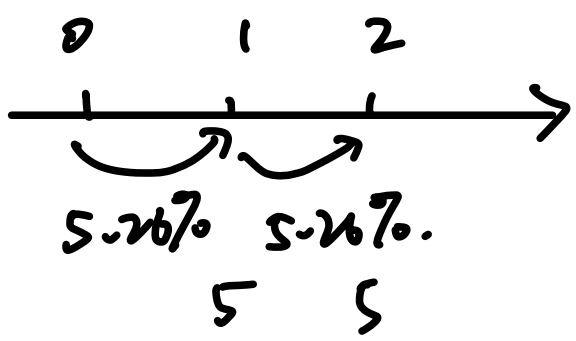
$CP = \frac{10}{1+r_3} + \frac{10}{(1+r_3)^2} + \frac{110}{(1+r_3)^3} \Rightarrow CP = 106.62$.

3.3

3.4.3 收益率赌博：借短买长，

连续2年以1年期即期利率借入资金来购买2年期债券。

假设未来2年都是5.26%



$\boxed{97\text{价格}}$

总共：$5\times(1+5.26\%)+105=110.26$(元).

投资者借入：97元.

第2年偿还：$97\times(1+5.26\%)^2=108.21$(元).

如果第2年还是5.26%而是10%，则：

偿还：$97\times(1+5.26\%)(1+10\%)=112.31$(元).

$105+5(1+10\%)-97\times(1+5.26\%)(1+10\%)=1.81$(元)、