Chapter 8.

C- CAPM.

Preference: M-V.

Decision: Portfolio Optimization

Equilibrium: Partial (Asset Market).

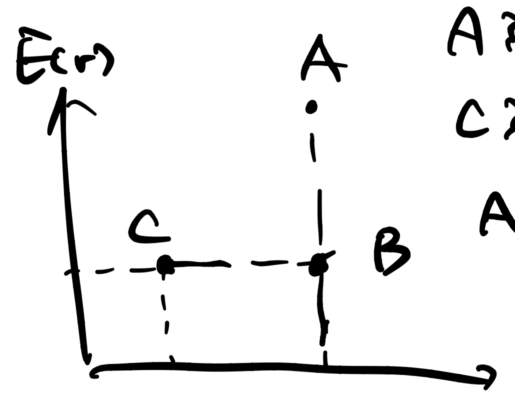
Asset pricing: CAPM (SML).

Preference: Rational: $\begin{cases} \text{Compete 可比较. } \forall x,y\in X,\ x\succsim y \text{ or } y\succsim x. \\ \text{transitivity. } x\succsim y,\ y\succsim z \Rightarrow x\succsim z. \end{cases}$

Continuous $x^n \succsim y^n$, $\lim_{n\to\infty} x^n \geq \lim_{n\to\infty} y^n \Rightarrow u(\cdot)$ s.t. $x\succsim y \Leftrightarrow u(x) \succsim u(y)$

(序列).

$u(r) = E(r) - A\sigma^2(r)$ 在偏好上存在问题.



$A \succsim B$

$C \succsim B$

$A ? C$.

C-CAPM: (Lec 9).

Expected Utility. 多在 u(·) 函数下的偏好.

Decision under uncertainty (Lec 10).

General (whole economy) (Lec 11)

C-CAPM (11, 12).

均值方差分析里丢失了很多信息.

① $\tilde{x}$, moment $E(\tilde{x}-c)^k$, $k>0$. 矩. 随变量所有信息包含在

mean和Variance 都一样. 但它们的分布完全不一样.

St. Petersburg Paradox 圣彼得堡悖论. 抛硬币.

①. Expected Payoff: $\frac{1}{2}\times 1 + \frac{1}{4}\times 2 + \frac{1}{8}\times 4 + \dots = +\infty$. 门票多少?

$$E(x) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \cdot 2^{n-1} = +\infty.$$

Bernoulli: 边际效用下降

Expected Utility: $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \cdot \ln(2^{n-1}) = \ln 2 \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n (n-1) = \ln 2$.

认为效用只有 ln2.

期望效用框架:

Step1: Modelling Choice set under uncertainty
Step2: Modelling Preference under uncertainty.
Step3: Find Expected Uncertainty Function.
Without Uncertainty: consumption bundle 消费束.

Apple Pear 比较 choice set $x \in \mathbb{R}^2$. $y \in \mathbb{R}^2$.

(1, 2) x
(2, 1) y

Under Uncertainty:
Definition: 8.3: Simple Lottery. $L=(p_1, \dots, p_N)$ $p_i \geq 0$. $\sum_{i=1}^{N} p_i = 1$.

Example: $L_1 = (0.5, 0.5)$. A 是拿到 1苹果 2个梨.
A B. B 是拿到 2苹果 1个梨.
$L_2 = (0.25, 0.75)$

Compound Lottery: $(L_1, L_2 : 0.5, 0.5)$.

Apple: $0.5 \times 0.5 + 0.5 \times 0.25 = 0.375$
Pear: $0.5 \times 0.5 + 0.5 \times 0.75 = 0.625$
还是 simple lottery.

$\mathcal{L}$: space of simple lottery.

Complete + ~~transitivity~~ + continuous ⇒ ∃ $u(\cdot)$ 来代表偏好.
$A, B, C \in \mathcal{L}$. $\forall \alpha \in (0,1)$. Independent Axiom.
$A \succsim B \Leftrightarrow \alpha A + (1-\alpha) C \succsim \alpha B + (1-\alpha) C$.
Prop. 8.2. $\succsim$ 偏好 定义在 $\mathcal{L}$. rational + continuous + IA.
$U(L) = \sum_{n=1}^{N} p_n u(x_n)$. x_n 是每个小的 → 期望效用函数.

Allais Paradox 阿莱悖论.

2.5 million (u_{25}) 0.5 million (u_5) brocken legs. (u_0).

$L_1 = (0 , 1 , 0)$

$L_1' = (0.1 , 0.89 , 0.01)$

$L_2 = (0 , 0.11 , 0.89)$

$L_2' = (0.1 , 0 , 0.9)$.

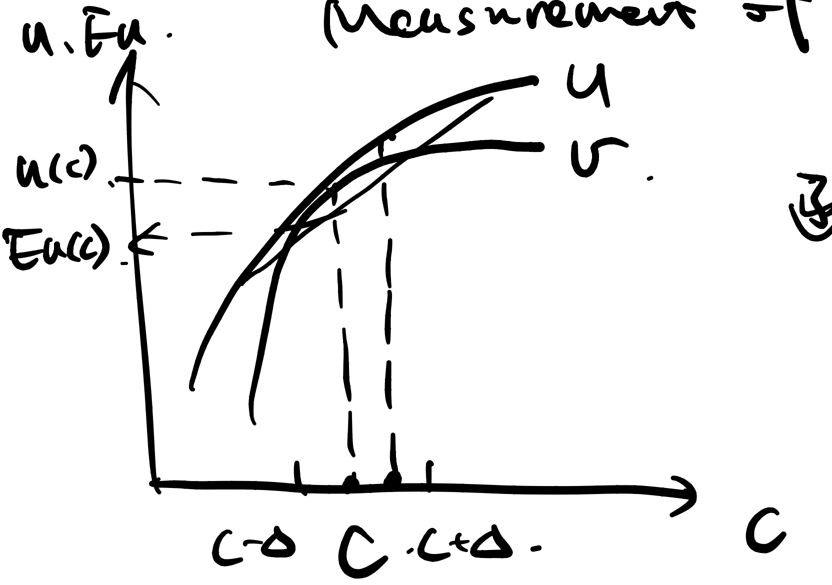
$L_1 \succsim L_1'$. $L_2 \precsim L_2'$. 是普遍选择.

$L_1 \succsim L_1' : u_5 > 0.1 u_{25} + 0.89 u_5 + 0.01 u_0$. $(+0.89u_0 - 0.89u_5)$.

$\Rightarrow \underbrace{0.11 u_5 + 0.89 u_0}_{u(L_2)} > \underbrace{0.1 u_{25} + 0.9 u_0}_{u(L_2')} \Rightarrow$ 意味着矛盾.

说明：在处理极端情况下并不合理.

Measurement of Risk Aversion.



风险厌恶评估.

函数弯曲程度代表着厌恶程度.

Certainty equivalence (ce).

为了愿意付出多少来消灭风险:

risk premium = c − ce

$\Rightarrow$ 希望去找到一个参数来描述风险的喜好:

· Coefficient of absolute Risk Aversion (ARA).

$u(y) = \pi^* u(y+h) + (1-\pi^*) u(y-h) \Rightarrow$ 总有一个 π^* 能被找到.

π^* 则刻画了风险厌恶的程度 ~~且 π^*~~. 则用 Taylor Expansion:

$u(y) = \pi^* [u(y) + h u'(y) + \frac{h^2}{2} u''(y)] + (1-\pi^*) [u(y) - h u'(y) + \frac{h^2}{2} u''(y)]$

$\Rightarrow 0 = (2\pi^* - 1) h u'(y) + \frac{h^2}{2} u''(y)$

$\Rightarrow \pi^* = \frac{1}{2} + \frac{h}{4} \left[- \frac{u''(y)}{u'(y)} \right]$.

$R_A(y) \triangleq - \frac{u''(y)}{u'(y)}$ (ARA) 绝对风险规避系数.

一相对风险规避系数推导:

$u(y) = \pi^* u(y+\theta y) + (1-\pi^*) u(y-\theta y)$.

$\Rightarrow \pi^* = \frac{1}{2} + \frac{\theta}{4}\left[-\frac{y u''(y)}{u'(y)}\right]$ $\quad R_R(y) \triangleq -\frac{y u''(y)}{u'(y)}$ (RRA).

CARA: $u(c) = -e^{\alpha c}$. $R_A(y) = \alpha$.

CRRA: $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ $\quad R_R(y) = \gamma$.

$\gamma = 1$: $u(c) = \ln(c)$.

Linear: $u(c) = \alpha c$. (风险中性, 不惧怕风险).

- Quadratic $\Longrightarrow u(r) = E(r) - A\sigma^2(r)$.
- CARA + lognormal return

Homework 8.

8) ⓐ $u(A) = 100$. $u(B) = 10$. $u(C) = 50$.

加买的效用: $0.5 \times u(A) + 0.5 \times u(B) = 55$.

直接拿的效用: $1 \times u(C) = 50$.

ⓑ. $f(x) = a + bx$)

加买: $0.5 \times (a\,u(A) + b) + 0.5 \times (a\,u(B) + b)$

$= 50a + 0.5b + 5a + 0.5b = 55a + b$.

直接拿: $1 \cdot (a\,u(C) + b) = 50a + b$. $a \geq 0$. $b \geq 0$. 不影响.

ⓒ $g(x) = \ln x$.

加买: $0.5 \times \ln u(A) + 0.5 \times \ln u(B) = 3.454$

直接拿: $1 \cdot \ln u(C) = 3.912$. $\therefore$ 选直接拿.

8-2).

房产: 150万.

现金: 50万.

用 compound lottery 的方法求解:

	房产150万.	房产50万	现金50万.	现金50-x万.	现金150-x
无保险	0.99	0.01	1	0,	0 A
有保险	0.99	0.01	0	0.99	0.01 .B

笨方法: $u(A) = 0.99 \times u(200) + 0.01 \times u(100)$.

$\gamma = 1$. $u(B) = 0.99 \times u(200-x) + 0.01 \times (200-x) = u(200-x)$.

$\Rightarrow \ln(200-x) = 0.99 \times \ln 200 + 0.01 \times \ln 100$.

$\Rightarrow \quad x = 1.38$ (万元).

$\gamma = 2$: $u(w) = \frac{\frac{1}{w}-1}{-1} = 1 - \frac{1}{w}$.

$\Rightarrow 0.99 \times \left(1 - \frac{1}{200}\right) + 0.01 \times \left(1 - \frac{1}{100}\right) = 1 - \frac{1}{200-x}$.

$\Rightarrow x = 1.98$ (万元).

$\gamma = 4$: $u(w) = \frac{1}{w^2} - 1/-3 = \frac{1}{3} - \frac{1}{3w^3}$.

$\Rightarrow 0.99 \times \left(1 - \frac{1}{200^3}\right) + 0.01 \times \left(1 - \frac{1}{100^3}\right) = \left(1 - \frac{1}{(200-x)^3}\right)$

$\Rightarrow \quad x = 4.46$ (万元).