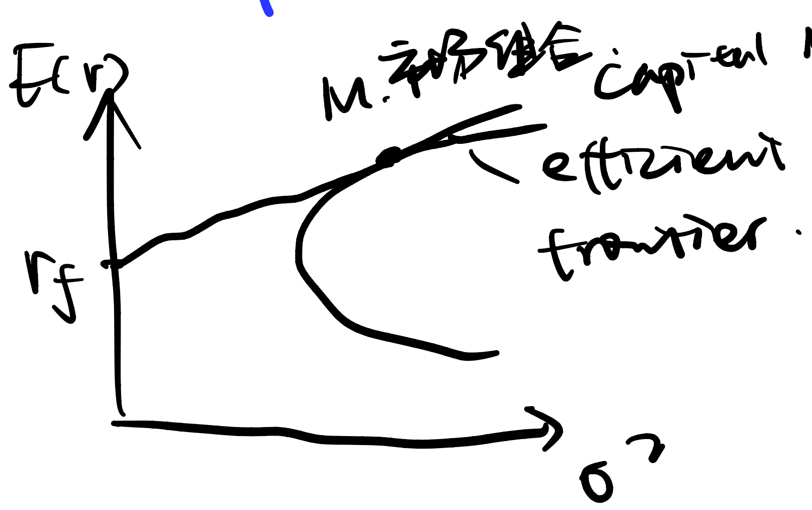
Chapter 6



资本资产定价模型: Capital Asset Pricing Model.

M 即市场.

CAPM Assumption:

(1) Market: No transaction costs. Taxes. Perfect Competition. Infinitely divisible. Equilibrium.

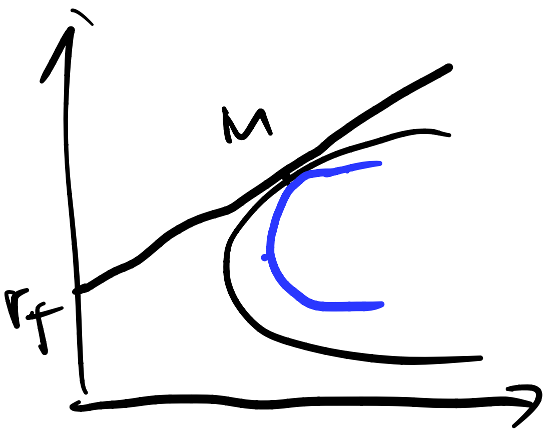
(2). Investor: Mean-Variance Preference. No limits on short long. Common belief.

效用函数: $u(r) = E(r) - A\sigma^2(r)$ $A > 0$.

$r_p = \underset{1-w \quad w}{(M,\ r_i)} \Rightarrow u(r_p) = u(w r_i + (1-w)\cdot r_M)$. $\forall i$. 对于其扰动 w. 都不如 r_M.

Sharpe Ratio

$$SR_i = \frac{\bar{r}_i - r_f}{\sigma_i}$$ (市场组合 r_M 有最高 Sharpe Ratio).



Proof: CML: $E(r) = r_f + \frac{E(r_M) - r_f}{\sigma_M}\cdot\sigma$.

$\underset{w,\ 1-w}{(i, M)} \triangleq r_w = w r_i + (1-w)\cdot r_M$.

$E(r_w) = w\left[E(r_i) - E(r_M)\right] + E(r_M)$.

$\sigma^2(r_w) = \sigma^2(w r_i + (1-w) r_M)$

$= w^2\sigma^2(r_i) + (1-w)^2\sigma(r_M) + 2w(1-w)\cdot\sigma_{iM}$.

$$\frac{dE(r)}{d\sigma(r_M)} = \frac{dE(r)}{dw} \Big/ \frac{d\sigma(r_M)}{dw} = E(r_i) - E(r_M) \Big/ \frac{2(\sigma_{iM} - \sigma_M^2)}{2\sigma_M} = \frac{E(r_M) - r_f}{\sigma_M}.$$

$$\Rightarrow \frac{\sigma_M[E(r_i)-E(r_M)]}{\sigma_{iM}-\sigma_M^2} = \frac{E(r_M)-r_f}{\sigma_M}$$

$$\beta_i \triangleq \frac{\sigma_{iM}}{\sigma_M^2}$$

$$\sigma_M^2[E(r_i)-r_f] = \sigma_{iM}[E(r_M)-r_f]$$

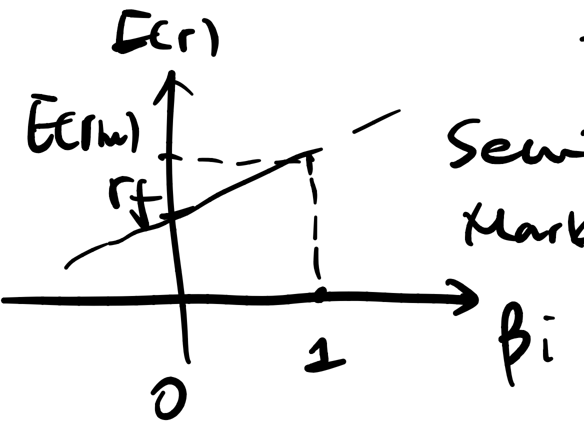
$$\Rightarrow E(r_i)-r_f = \frac{\sigma_{iM}}{\sigma_M^2}[E(r_M)-r_f]$$

CAPM.

CAPM 依然是线性方程.

无风险 β_f 是 0: $\frac{\sigma_{fM}}{\sigma_M^2} = \frac{\sigma_{fM}}{\sigma_{MM}^2} = 0$.

无风险与市场组合无关



Expected rate of return = Time Value of money + risk premium.

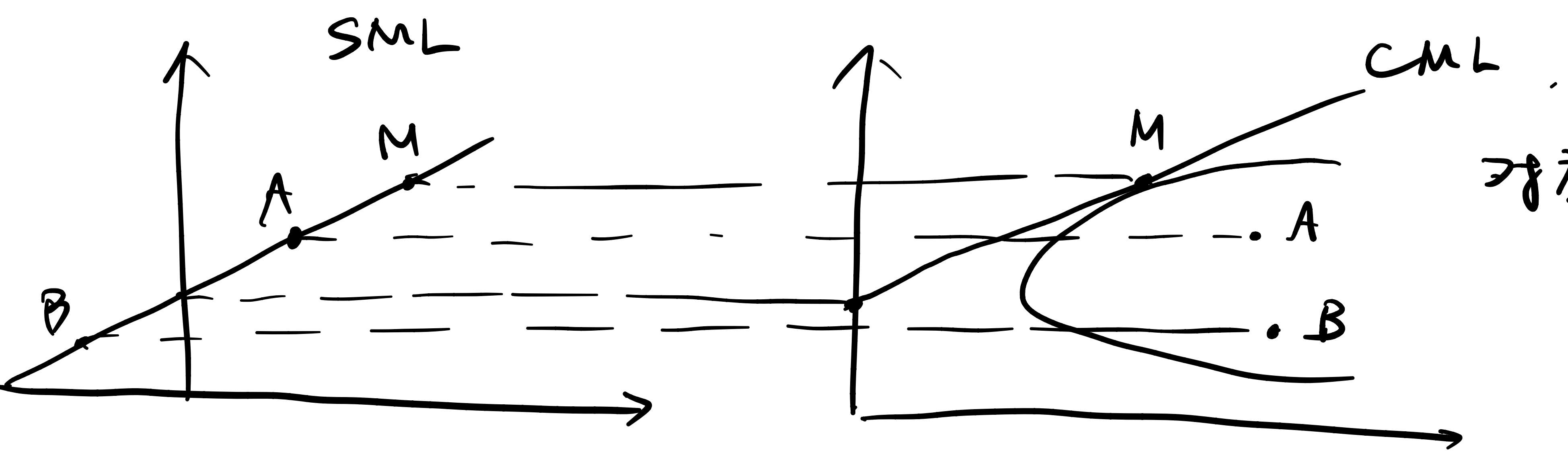
risk premium = measurement of risk × price of risk

$$E(r) = r_f + \frac{E(r_M)-r_f}{\sigma_M}\cdot\sigma.$$

为何 CML 和 SML 都成立? 成立对象不一样

SML 对所有资产成立. 才能说是线性方程.

CML 只对最高 Sharpe ratio 资产才成立.



对应关系.

HW 6.

6.1). $(r_M,\ r_f) \Rightarrow r_i$. 假设对任意资产投资: r_I.

$\quad\ \ \mu \quad\ 1-\mu.$

by: $\quad (r_i,\ r_I) \Rightarrow r_p.$

$\quad\quad\ w \quad\ 1-w$

$\Rightarrow.\ E(r_p) = w[E(r_i) - E(r_I)] + E(r_I).$

$\Rightarrow\ \sigma(r_p) = w^2\sigma_i^2 + (1-w)^2\sigma_I^2 + 2w(1-w)\cdot Cov.$

$\Rightarrow\ u(r_p) = E(r_p) - A\cdot\sigma^2(r_p)$

$\Rightarrow\ \frac{du}{dw} = E(r_i) - E(r_I) - A\left(2w\sigma_i^2 + 2(1-w)\cdot\sigma_I^2 + (2-2w)\,Cov\right) = 0$

$\Rightarrow.\ (2A\sigma_i^2 - 2A\sigma_I^2 - 2Cov)\ w = E(r_i) - E(r_I) - 2A\sigma_I^2 + \cancel{A}Cov.$

$\Rightarrow.\ \frac{du}{dw}\Big|_{w=0} = 0.\ \Rightarrow.\quad E(r_i) - E(r_I) = -2A\sigma_I^2 + 2ACov.$

$\Rightarrow.\ r_i = r_f.\ Cov(r_f, r_i) = 0.\quad r_f - r_I = 2A\sigma_I^2.$

$\Rightarrow.\ r_i - r_I = \frac{r_I - r_f}{\sigma_I^2}(Cov - \sigma_I^2) \Rightarrow r_i - r_f = \frac{cov}{\sigma_I^2}(r_I - r_f).$

$Cov = Cov(r_i, r_I)$

$\quad\ = Cov(r_i,\ \mu r_M + (1-\mu)\cdot r_f).$

$\quad\ = .Cov(r_i,\ \mu r_M) = \mu\sigma_{iM}.$

$\sigma_I^2 = Var(\mu r_M + (1-\mu)\cdot r_f) = \mu^2\sigma_M^2$

$\Rightarrow.\quad r_i - r_f = \frac{\mu\sigma_{iM}}{\mu^2\sigma_M^2}\cdot(\mu r_M - r_f) \Rightarrow r_i - r_f = \frac{\sigma_{iM}}{\sigma_M^2}(r_M - r_f).$

6.2)

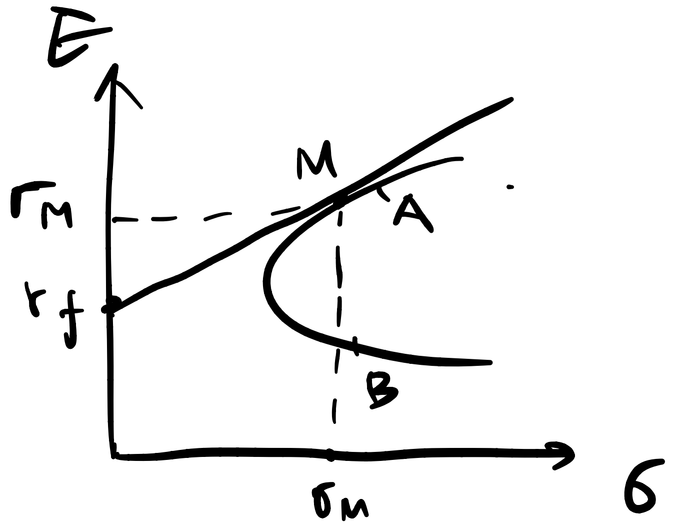
$\rho_{AB} = 0.5$

$r_f = 5\%$

r_A: $E(r_A) = 15\%$ $\sigma_A = 10\%$

r_B: $E(r_B) = 13\%$ $\sigma_B = 8\%$

① 求双曲线组合：



$$E(r_p) = w\, r_A + (1-w)\cdot r_B$$

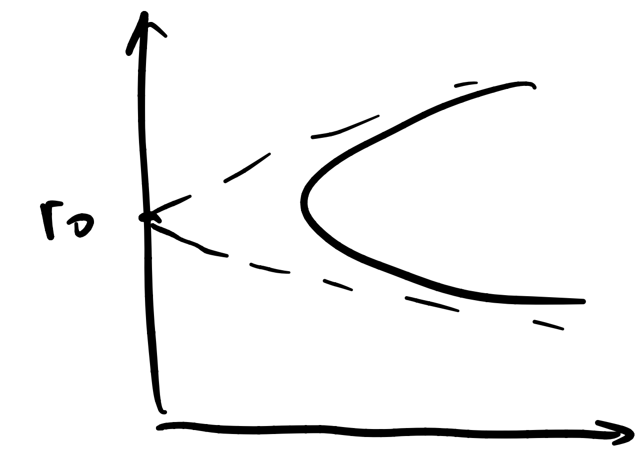
$$\sigma^2(r_p) = w^2 r_A^2 + (1-w)^2 r_B^2 + 2w(1-w)\cdot \rho_{AB}$$

$$\bar{r} - r_f = \frac{\sigma}{\sigma_M}\cdot(\bar{r}_M - r_f)$$

∴ 只有：$\dfrac{\bar{r}_M - r_f}{\sigma_M}$ ⇒ 最大：

$$\Rightarrow w_A = \frac{R_A\cdot\sigma_B^2 - R_B\rho_{AB}}{R_A\sigma_B^2 + R_B\sigma_A^2 - (R_A+R_B)\rho_{AB}} = 0.815$$

6.3)



① $r_f \geq r_0$ 时，r_0不会有人持有，则 $r_f\uparrow$，r_f始终会大于r_0。