

Chapter 5.

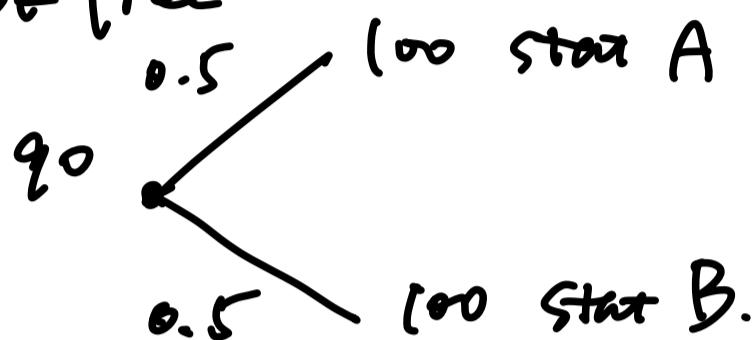
Markowitz. 1952. Portfolio Selection - CAPM + 贝塔系数
一个比较好的估计.

Covariance.

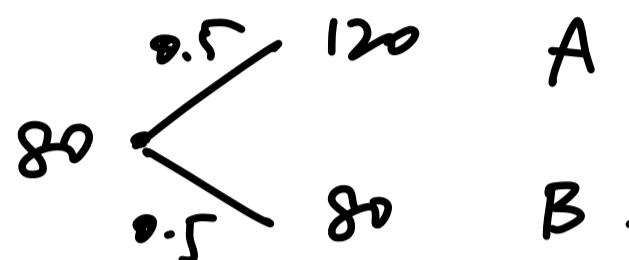
Mean + Variance 来刻画的 Portfolios. Mean Variance Analysis

Markowitz 说：最后的组合必须一起看，不能单独看每一个资产。

risk free:



risky:



ex-ante rate of return. 事前回报率 = expected rate of return. 预期

$$E(r) = 0.5 \times \left(\frac{120}{80} - 1 \right) + 0.5 \times \left(\frac{80}{80} - 1 \right) = 25\%.$$

$$E(r_f) = 100/90 - 1 = 11.1\%.$$

ex-post rate of return:

$$r_a = 50\%$$

$$r_{fa} = r_{fb} = \frac{100}{90} - 1 = 11.1\%.$$

$$r_b = 0\%$$

对市场的利率 r 是在 period 0 以来的回报率 - 事前回报率.

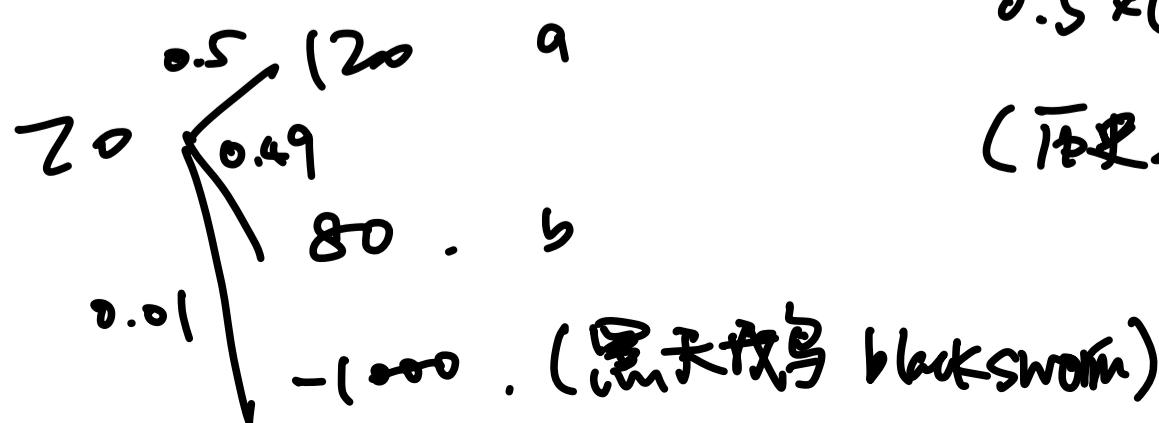
risk premium: 风险溢价。(只包含在预期回报率中).

$$E(r) - r_f = 25\% - 11\% = 14\%.$$

historical ex-post 历史数据来展现预期

Variance(r_f) = 0 ? 是对期望

幸存者偏差:



$$0.5 \times \left(\frac{120}{70} - 1 \right) + 0.49 \left(\frac{120}{80} - 1 \right) = 43\%$$

$$0.5 \times \left(\frac{120}{70} - 1 \right) + 0.49 \left(\frac{120}{80} - 1 \right) + 0.01 \times \left(\frac{-100}{80} - 1 \right) = 27\%.$$

(历史数据没有黑天鹅)

出现).

-100. (黑天鹅 blackswan)

Portfolio: 资产组合.

(w_1, w_2, \dots, w_n) . n -number of assets.

w_i : share of assets on asset i .

$$\sum_{i=1}^n w_i = 1. \quad w_i > 0 \text{ long } \frac{r_p}{\sigma_p}, \quad w_i < 0 \text{ short } \frac{r_p}{\sigma_p}.$$

如何图6. 变化 σ^2 :

固定 r_f - 线.

$$\bar{r}_p = E[(1-w)r_f + w \cdot \tilde{r}_s] \\ = (1-w)r_f + w \cdot \bar{r}_s.$$

$r_f + \tilde{r}_s$.

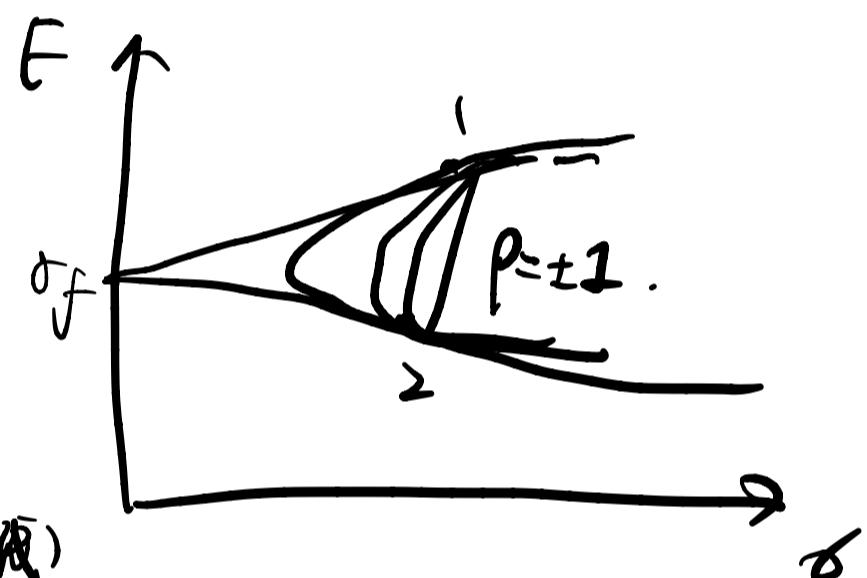
$$(1-w, w). \quad \sigma_p^2 = E[(1-w)r_f + w \tilde{r}_s - (1-w)r_f - w \bar{r}_s]^2 = w^2 \sigma_s^2.$$

$$\Rightarrow \bar{r}_p = r_f + \frac{\bar{r}_s - r_f}{\sigma_s} \cdot \sigma_p \text{ 直线方程}$$

类似图6: 固定风险资产:

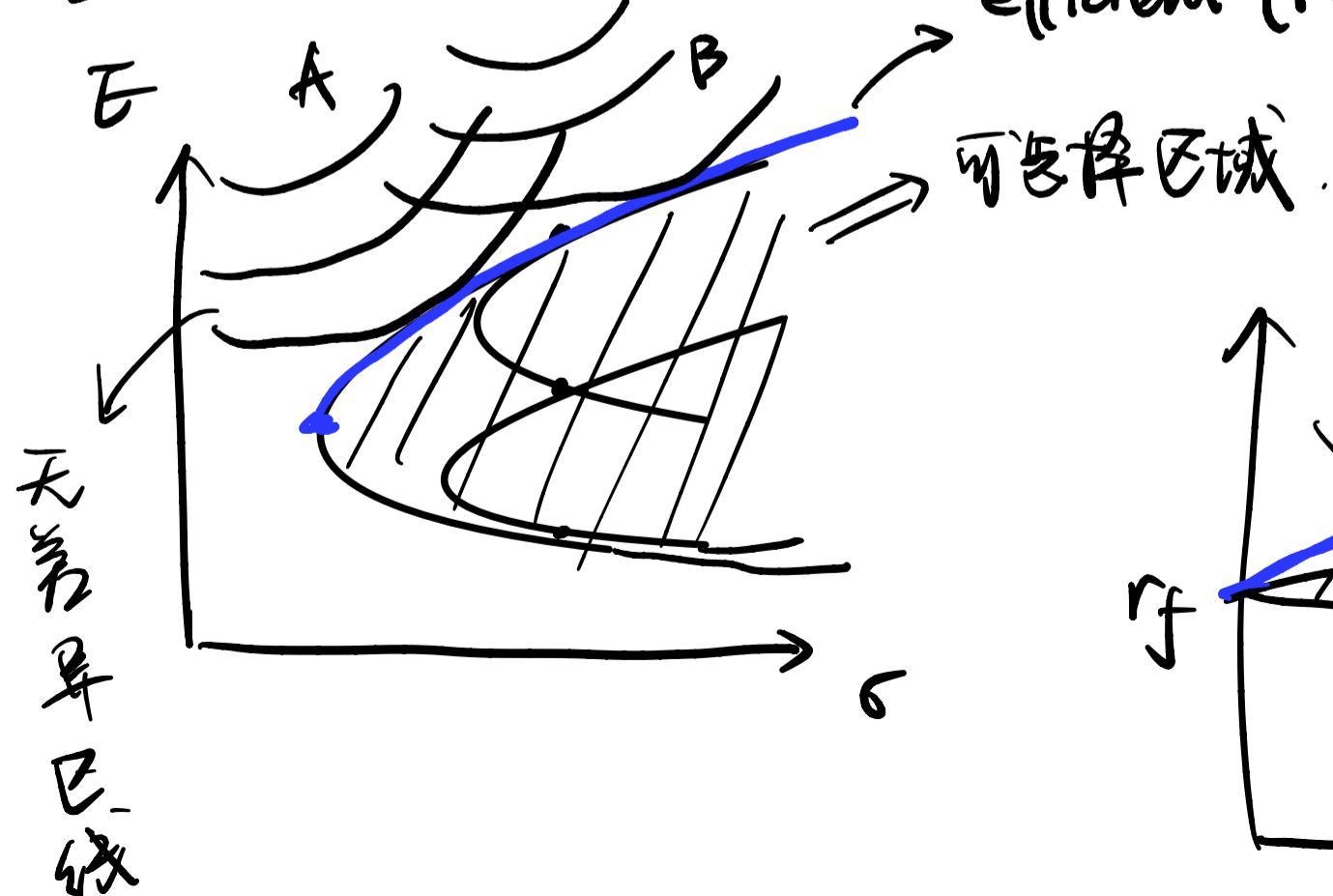
$$\tilde{r}_1 + \tilde{r}_2. \quad \bar{r}_p = w \bar{r}_1 + (1-w) \bar{r}_2$$

$$(w, 1-w). \quad \sigma_p^2 = w^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 \\ + 2(1-w)w \sigma_1 \sigma_2 \text{ (协方差)}$$



高平滑度取决于两类资产的相关程度.

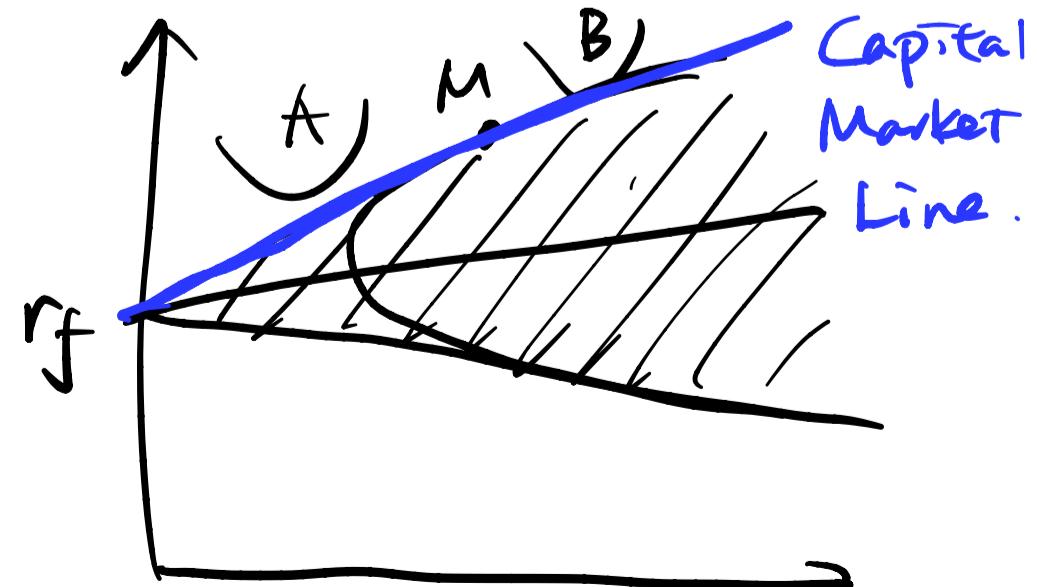
2 → n. 数字上算出.



efficient frontier 有效前沿.
(也是双曲的).

可行解区域.

加入 r_f , 变成直线



Capital
Market
Line.

M: Market Portfolios: 无论是风险厌恶，抑或风险喜好，均应该用 M 来选择 Portfolio.

Mutual Fund Separation Theorem:

Step 1: 球形组合 M.

Step 2: 球形组合 和无风险资产进行分配.

Homework:

	A	B
E	0.1	0.15
σ	0.15	0.2
w		
1-w		

$$E_A = 0.1 \quad \sigma_A = 0.15 \\ E_B = 0.15 \quad \sigma_B = 0.2$$

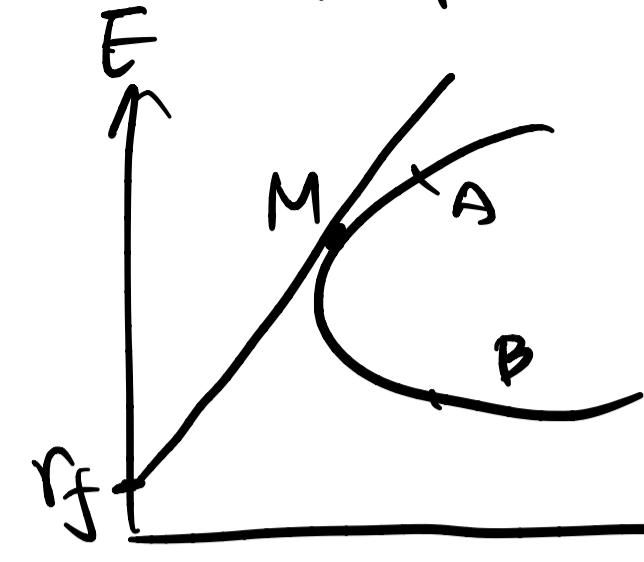
$$\sigma_p^2 = E[(w\gamma_A + (1-w)\gamma_B) - (wE_A + (1-w)E_B)]^2 \\ = w^2 \sigma_A^2 + (1-w)^2 \sigma_B^2 + 2w(1-w) \text{Cov}.$$

$$\frac{\partial \sigma_p^2}{\partial w} \Rightarrow w = \frac{\sigma_B^2 - \text{Cov}}{\sigma_A^2 + \sigma_B^2 - 2\text{Cov}} \quad 1-w = \frac{\sigma_A^2 - \text{Cov}}{\sigma_A^2 + \sigma_B^2 - 2\text{Cov}}$$

ρ	σ_{AB}	w^*	σ_p
-1	-0.03	0.57	0
0	0	0.64	0.12
0.5	0.015	0.77	0.14

$\rho \uparrow w^* \uparrow$. 原因是：相关系数越大两个资产会倾向于增大波动率更小的资产的权重.

(b). 求解 AB 双边线：



AB 双边线：

$$E_p = w E_A + (1-w) E_B$$

$$\sigma_p^2 = w^2 \sigma_A^2 + (1-w)^2 \sigma_B^2 + 2w(1-w) \sigma_{12}.$$

但是这里参数方+

突破点是这样：让 M_{\min} shape ratio 大 (shape ratio！)

$$S_p^2 = \frac{(E_p - r_f)^2}{\sigma_p^2}$$

$$\frac{\partial S_p^2}{\partial w} = \frac{1}{\sigma_p^4} \left[2(E_p - r_f) \cdot \frac{\partial E_p}{\partial w} \cdot \sigma_p^2 - (E_p - r_f) \cdot 2 \frac{\partial \sigma_p^2}{\partial w} \cdot \sigma_p \right] = 0$$

$$\Rightarrow 2 \frac{\partial E_p}{\partial w} \cdot \sigma_p^2 = (E_p - r_f) \cdot \frac{\partial \sigma_p^2}{\partial w}.$$

$$2(E_A - E_B) (w^2 \sigma_A^2 + (1-w)^2 \sigma_B^2 + 2w(1-w) \sigma_{12}) = (w E_A + (1-w) E_B - r_f) \cdot \\ 2w \sigma_A^2 - 2(1-w) \sigma_B^2 \\ + (2-2w) \cdot \sigma_{12}$$

$$(2(E_A - E_B) \sigma_A^2 + \sigma_B^2 - 2 \sigma_{12}) \cdot w^2$$

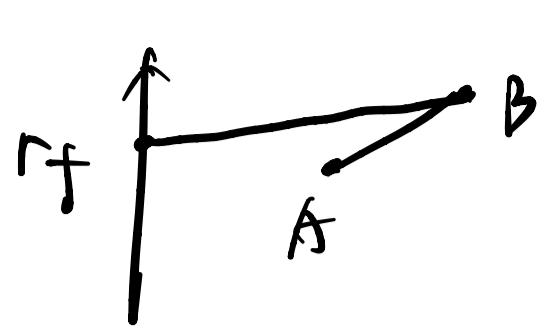
$$+ (-2(E_A - E_B) \cdot \sigma_B^2 - 2(E_A - E_B) \cdot \sigma_{12} - E_A + E_B - 2\sigma_A^2 - 2\sigma_B^2 + 2\sigma_{12}) w$$

$$+ 2(E_A - E_B) \sigma_B^2 - E_B + r_f$$

$$+ 2\sigma_B^2 - 2\sigma_{12} = 0.$$

MRP

ρ_{12}	Cov	$A \neq f$	$B \neq f$	$A \neq f$	$B \neq f$
-1	-0.03	0.57	0.43	0.57	0.43
0	0	0.64	0.36	0.47	0.53
0.5	0.25	0.77	0.23	0.25	0.75



高风险高收益，给B更多