PHYS460 Homework 1

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Preface: This homework solution may have some steps that seem odd to include. I am honestly trying to convince myself that this stuff works, and isn't wibbly wobbly timey wimey voodoo still lol. Language used in this homework may be offensive to some audiences.

Secondly, I am unfamiliar with some of the symbolism in Tex. Any feedback regarding formatting and or usage of symbols (Ex: ψ vs Ψ) would be greatly appreciated so I can improve these documents.

Associated code can be found on my github repository:
https://github.com/Machinza/Homework/upload/main/PHYS460 Quantum Mechanics/Homework 1

1 Problem 1:

1.1 Part Aa: Given Data

$$N = 30, P(j) = \frac{N(j)}{N}$$

$$P(2) = \frac{1}{30}$$

$$P(3) = \frac{2}{30}$$

$$P(4) = \frac{1}{30}$$

$$P(5) = \frac{2}{30}$$

$$P(6) = \frac{8}{30}$$

$$P(7) = \frac{6}{30}$$

$$P(8) = \frac{4}{30}$$

$$P(9) = \frac{2}{30}$$

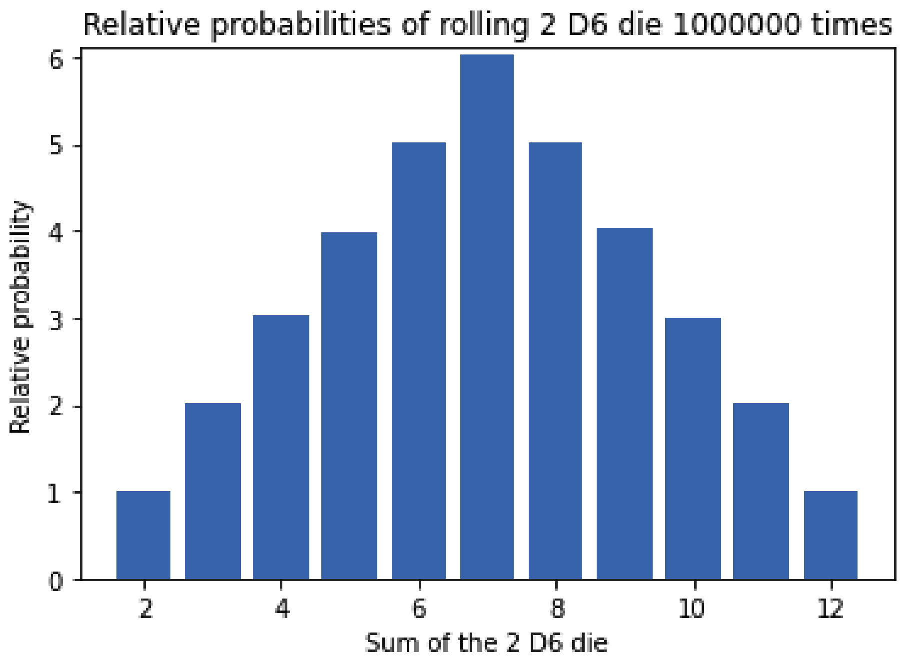
$$P(10) = \frac{1}{30}$$

$$P(11) = \frac{1}{30}$$

$$P(12) = \frac{2}{30}$$

1.2 Part A: "Spherical Cow"

See attached code, I wrote a program that will create graphs for N amount of rolls (tries variable) and relativize them.



From this result, we can see that a sum of 7 is the most likely. The probabilities relative to sum 7 are below. To find our probabilities, we can sum our relative probabilities to find N.

$$N = 6 + 2(5) + 2(4) + 2(3) + 2(2) + 2(1) = 36$$

$$P(2, 12) = \frac{1}{36}$$

$$P(3, 11) = \frac{2}{36}$$

$$P(4, 10) = \frac{3}{36}$$

$$P(5,9) = \frac{4}{36}$$
$$P(6,8) = \frac{5}{36}$$
$$P(7) = \frac{6}{36}$$

We can verify this works by summing all the $P(j)$.

$$2\left(\frac{1}{36}\right) + 2\left(\frac{2}{36}\right) + 2\left(\frac{3}{36}\right) + 2\left(\frac{4}{36}\right) + 2\left(\frac{5}{36}\right) + \frac{6}{36} = 1$$

1.3 Part B:

This was already kind of answered in the previous question, but for the experimental case the most probable value is 6 and for the spherical cow scenario it's 7.

1.4 Part C:

1.4.1 Experimental case:

$$\langle x \rangle = \sum_{j=2}^{12} \frac{N(j)}{N} = \frac{1(2) + 2(3) + 1(4) + 2(5) + 8(6) + 6(7) + 4(8) + 2(9) + 1(10) + 1(11) + 2(12)}{30} \approx 6.9$$

Even at 30 trials we have a decent approximation, however this is probably by chance. I ran my code multiple times with 30 trials, and the averages were wildly different each time.

1.4.2 Spherical Cow:

$$\langle x \rangle = \sum_{j=2}^{12} \frac{N(j)}{N} = \frac{1(2+12) + 2(3+11) + 3(4+10) + 4(5+9) + 5(6+8) + 6(7)}{36} = 7$$

As expected, as we approach infinite trials our expectation value approaches 7

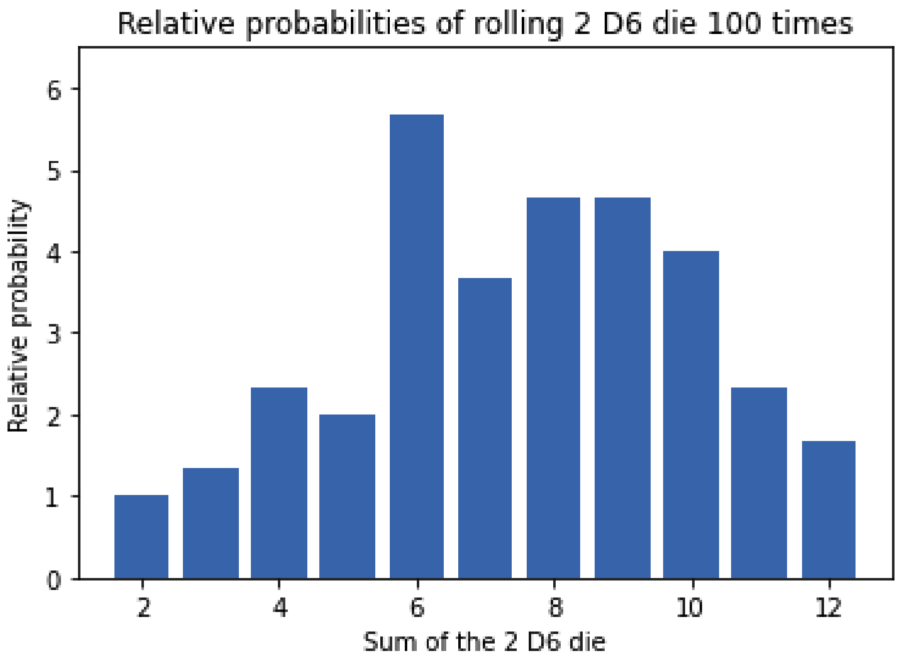
1.5 Part D:

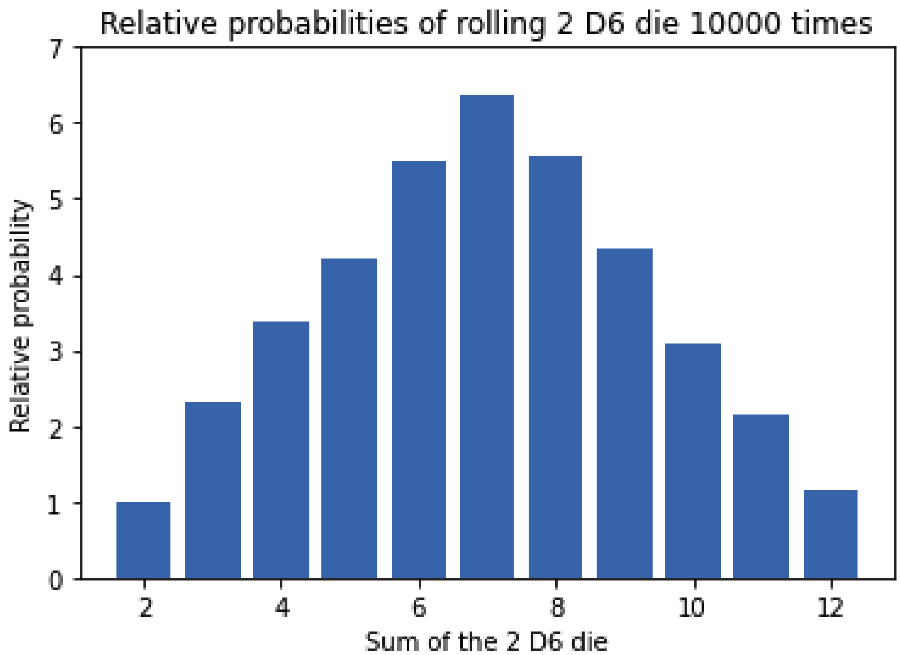
Yeah... I'm lazy, sue me. I wrote a section in the attached code to do this math for me.

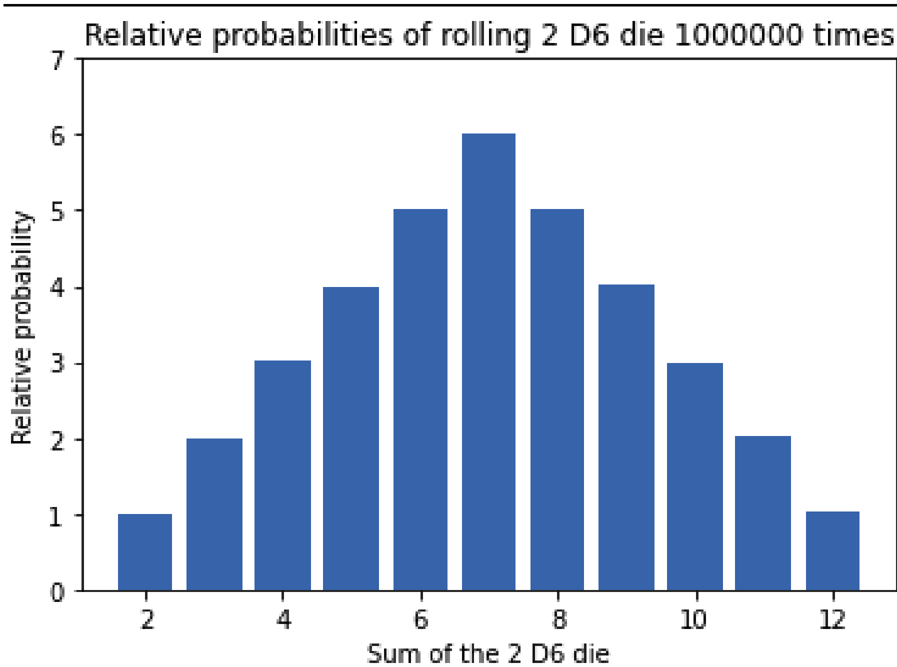
$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$
$$\sigma_{ideal} = 2.415229$$
$$\sigma_{exp} = 2.371356$$

1.6 Part E:

Simply, its because we don't have enough trials. The ideal case assumes an infinite amount of trials, and thus random chances variances vanish and the trend dominates. The "noise" for lack of a better term with small trial quantities overwhelms the overall ideal trend.




Relative probabilities of rolling 2 D6 die 10000 times
Relative probability
Sum of the 2 D6 die
0
1
2
3
4
5
6
7
2
4
6
8
10
12


Relative probabilities of rolling 2 D6 die 1000000 times
Relative probability
Sum of the 2 D6 die
0
1
2
3
4
5
6
7
2
4
6
8
10
12

2 Problem 2:

$$\psi(x,0) = \begin{cases} A(x/a) & 0 \leq x \leq a \\ A(b-x)(b-a)^{-1} & a \leq x \leq b \\ 0 & x < 0 \text{and} x > b \end{cases} \tag{1}$$

- Hey, so there's this neat formula. Might prove useful or something idk.

$$|\psi(x,t)|^2 = 1$$

- So like, what if we said the system of equations squared and absolute valued was equal to 1. That'd be crazy to do right?! Let's do it anyways.

$$1 = \int_0^a |A|^2 \left(\frac{x}{a}\right)^2 dx + \int_a^b |A|^2 \left[(b-x)(b-a)^{-1}\right]^2 dx$$

$$1 = |A|^2 \left(a^{-2} \int_0^a x^2 dx + (a^2 - 2ab)^{-1} \int_a^b -2xb + x^2 dx\right)$$

- Well, that first integral is easy enough

$$1 = |A|^2 \left(\frac{a}{3} + (a^2 - 2ab)^{-1} \left[-x^2 b + \frac{x^3}{2}\right|_a^b\right)$$

$$1 = |A|^2 \left(\frac{a}{3} + (a^2 - 2ab)^{-1} \left[\left(-b^3 + \frac{b^3}{2}\right) - \left(-ab^2 + \frac{a^3}{2}\right)\right]\right)$$

$$1 = |A|^2 \left(\frac{a}{3} \frac{-b^3 + \frac{b^3}{2} + ab^2 - \frac{a^3}{2}}{(a^2 - 2ab)}\right)$$

2.1 Part B:

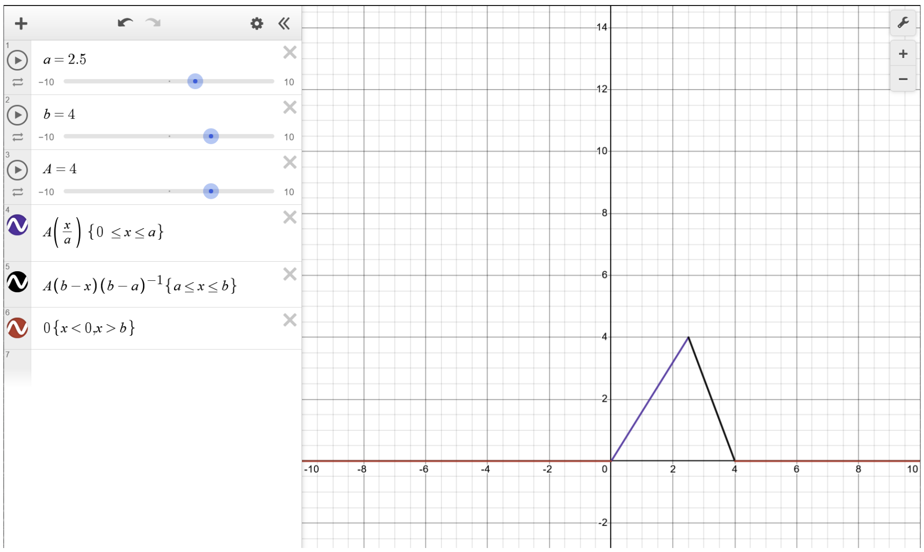


Figure 1: Plotted in Desmos

2.2 Part C:

Our piecewise function in the above plot has a maximum at $\psi(a)$, therefore it is most likely to be found at that location.

2.3 Part D:

$$P = \int_a^b |\Psi|^2 dx = \frac{|A|^2}{a^2}\int_0^a x^d x \implies \frac{|A|^2}{b^2}\int_0^a x^d x = \frac{|A|^2}{a^2}\left[\frac{x^3}{3}\right|_0^a = \frac{|A|^2}{3}$$

Requires result from Part A to finish. Going to office hours.

2.4 Part E:

Ditto as finishing Part D

3 Problem 3:

$$\Psi(x,t) = Ae^{-\kappa|x|-i\omega t} \text{ where } A, \kappa, \omega \text{ are positive real constants.}$$

My first assumption here is that since we have an imaginary term in the exponential is that we need to normalize using the complex conjugate.

$$|\Psi(x,t)|^2 = \psi\psi^* = 1$$

$$1 = \int |\Psi|^2 dx$$
$$1 = \int \left(Ae^{-\kappa|x|-i\omega t}\right)\left(Ae^{-\kappa|x|+i\omega t}\right) dx$$
$$1 = |A|^2 \int_0^\infty \left(e^{-\kappa|x|-i\omega t+(-\kappa|x|+i\omega t)}\right) dx$$
$$1 = |A|^2 \int_0^\infty e^{-2\kappa x} dx$$
$$1 = |A|^2 \left[\frac{e^{-2\kappa x}}{2\kappa}\right|_0^\infty$$
$$1 = |A|^2 \left[0 - \frac{1}{2\kappa}\right]$$
$$1 = -\frac{|A|^2}{2\kappa}$$
$$A = \sqrt{2\kappa}$$

3.1 Part B:

$$\langle x \rangle = \int x|\Psi|^2 dx = |A|^2 \int_{-\infty}^{\infty} xe^{-2\kappa x} dx$$

Used https://www.integral-calculator.com/ to calculate the integral.

$$\langle x \rangle = \left[-\frac{(2kx+1)\,\mathrm{e}^{-2kx}}{4k^2}\right|_{-\infty}^{\infty} \implies \langle x \rangle = 0$$

$$\left\langle x^2 \right\rangle = \langle x \rangle = \int x^2|\Psi|^2 dx = |A|^2 \int_0^\infty x^2 e^{-2\kappa x} dx$$

Used https://www.integral-calculator.com/ to calculate the integral.

$$\left\langle x^2 \right\rangle = \left[-\frac{(2\kappa x \cdot (\kappa x + 1) + 1)\,\mathrm{e}^{-2\kappa x}}{4\kappa^3}\right|_0^\infty$$