# Notes on 'Hungarian Algorithm for assignment problem'

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Lecture link -https://www.youtube.com/watch?v=BUGIhEecipE

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## 1 Problem statement

Given a set of **n jobs** and a set of **n people** who can do those jobs with varying cost , assign each job to a person such that each job is done by only 1 person and the net cost is **MINIMUM**.

## 2 Representation of the problem

Start the solution by arranging the jobs in a n × n matrix such that the rows represent the jobs and the columns represent the people. Each cell of the matrix represents $C_{ij}$ which is the cost of job i being done by the jth person . We have to minimize Sum of $(C_{ij} \times X_{ij})$ for all values of i and j where $X_{ij}$ represent whether job i is being done by person j subjected to the constraints - Sum of $X_{ij}$ for all j is 1 and sum of all $X_{ij}$ is 1 for all i is 1 i.e. 1 job per person and each person gets a job.

## 3 Important observations

- Reducing all entries of a row by same amount will not affect the final assignment since the same person will charge the best amount even then .
- Similarly reducing all entries of a column by same value does not affect assignment.

Following the above observations we can reduce the columns and rows in our matrix by the least value in the respective column or row so as to get a 0 in every row and every column. This way we can reduce our problem to assigning jobs such that the **total sum of cost is 0**.

## 4 The algorithm

1. Arrange the cost as described in the representation section.
2. Reduce all rows by the least value of that row to get atleast one 0 in every row.
3. Reduce all columns by the least value of that column to get atleast one 0 in every column.
4. If a row has exactly one 0 then mark it and cross out all 0 in the corresponding column. Do this for all rows. Then do similar procedure for all column till all zeroes have either been marked or crossed. If the number of marked zeroes equal n then skip to step 11.
5. Tick all the unassigned rows. For every ticked row if there is a zero then cross the corresponding column of that zero.
6. For every crossed column if there is a zero such that it's row is unticked then tick it. Repeat steps 5 and 6 till no more ticking is possible.
7. Cross out unticked rows.
8. Find the smalled value of the matrix such that neither it's row nor column are marked. Call it theta.
9. For all entries of the matrix , if there is no line the subtract theta . If there is one line passing through then do nothing. If there are 2 lines passing through then add theta.
10. Mark zeroes similar to how they were done in step 4 i.e if there is a single zero then mark it and cross all other zero in the corresponding column. Do this for all rows and columns. If number of marked zeroes is n the continue else go back to step 5.
11. Report all the marked zeroes . Assigning these cells will give us the least cost.