

## SUMMARY AND RESULTS

PCLM methods can greatly improve the computation of pace-shape approximations. Several methods are compared. The most precise seems to be calculation of shape measures directly from PCLM output with  $A_x=0.5$ . The two methods that attempt to estimate  $A_x$  and work on original age intervals are almost equally precise, but slightly (substantially for Gini coefficient) worse than direct-PCLM method.

### Methods of approximation

**A)** Data is smoothed by PCLM and then grouped back to the original interval length. Re-grouping the data enables to calculate  $A_x$ , which is in turn used for calculation of pace-shape measures together with smoothed-re-grouped data.

**B)** Data is smoothed by PCLM, but NOT grouped back to the original interval length.  $A_x$  cannot be calculated and it is assumed to equal 0.5. Pace-shape measures are thus calculated directly from PCLM output.

**C)** Similar as A), however PCLM is used only to calculate  $A_x$ . Pace-shape measures are calculated from original data with  $A_x$  obtained via PCLM method.

**D)** PCLM is not used, and  $A_x = 0.5$ .

### Other details of the model

The relative differences between theoretical and empirical lifetable measures were calculated according to the equation:

$$D = 100 \frac{M_{Theoretical} - M_{Empirical}}{M_{Theoretical}},$$

where M is substituted by Keyfitz entropy, coefficient of variation or Gini coefficient.

The empirical life expectancy, Keyfitz entropy, coefficient of variation, and Gini coefficient are calculated with the assumption of infinite population size, that is, the standardized lifetable  $l_x$  is substituted with survivorship  $s_x$  of Gompertz distribution.

## 1. Keyfitz entropy

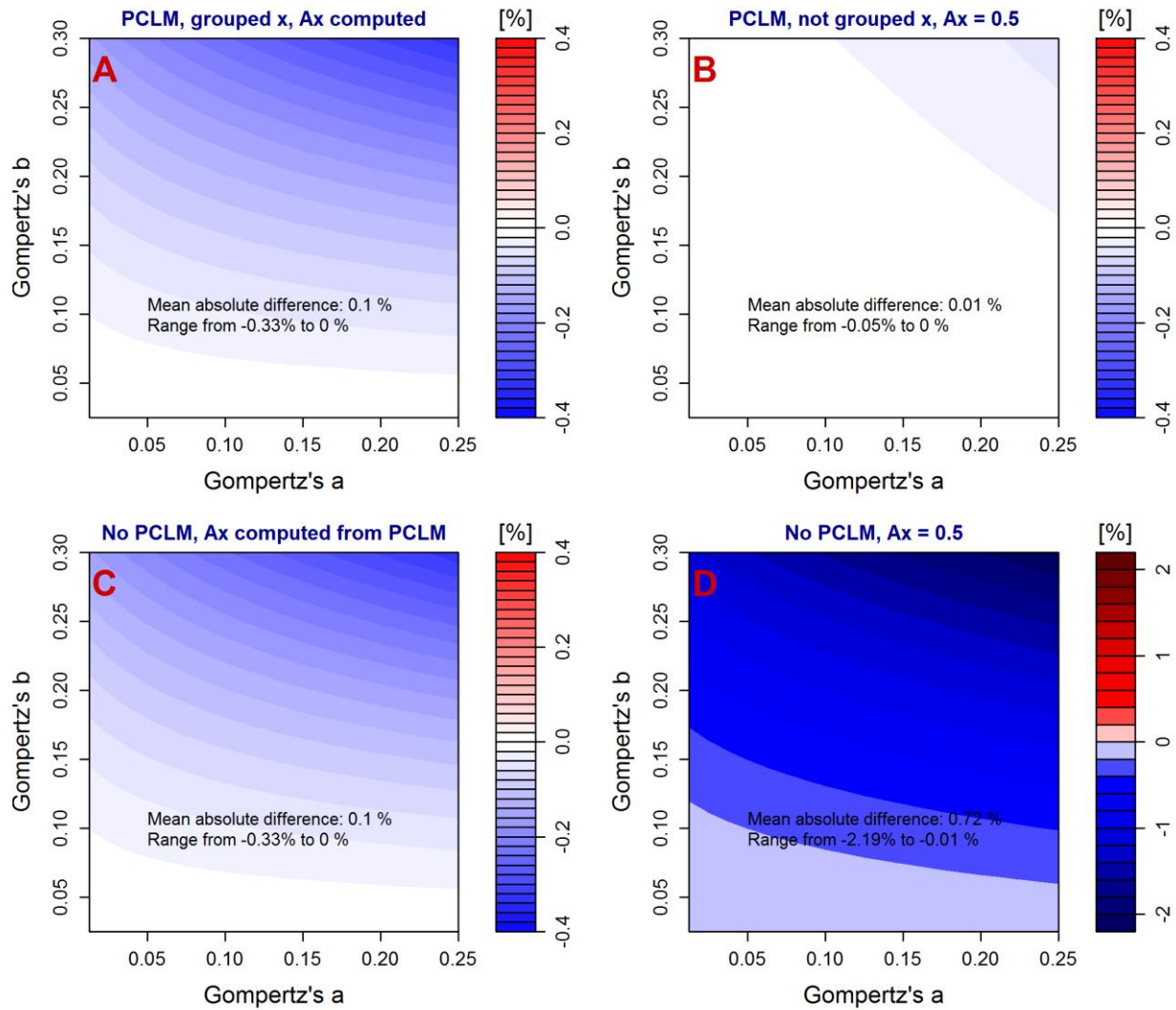
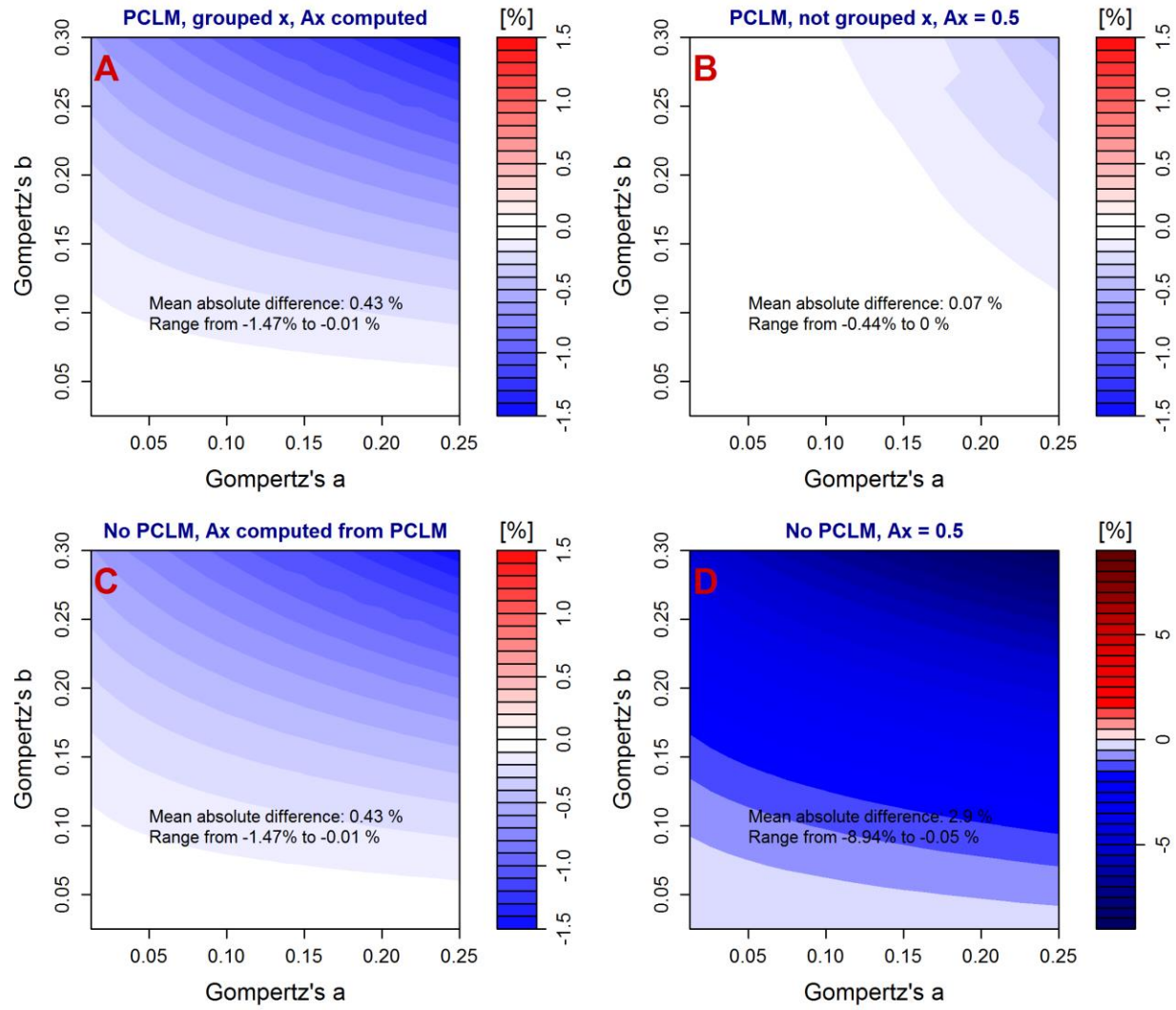
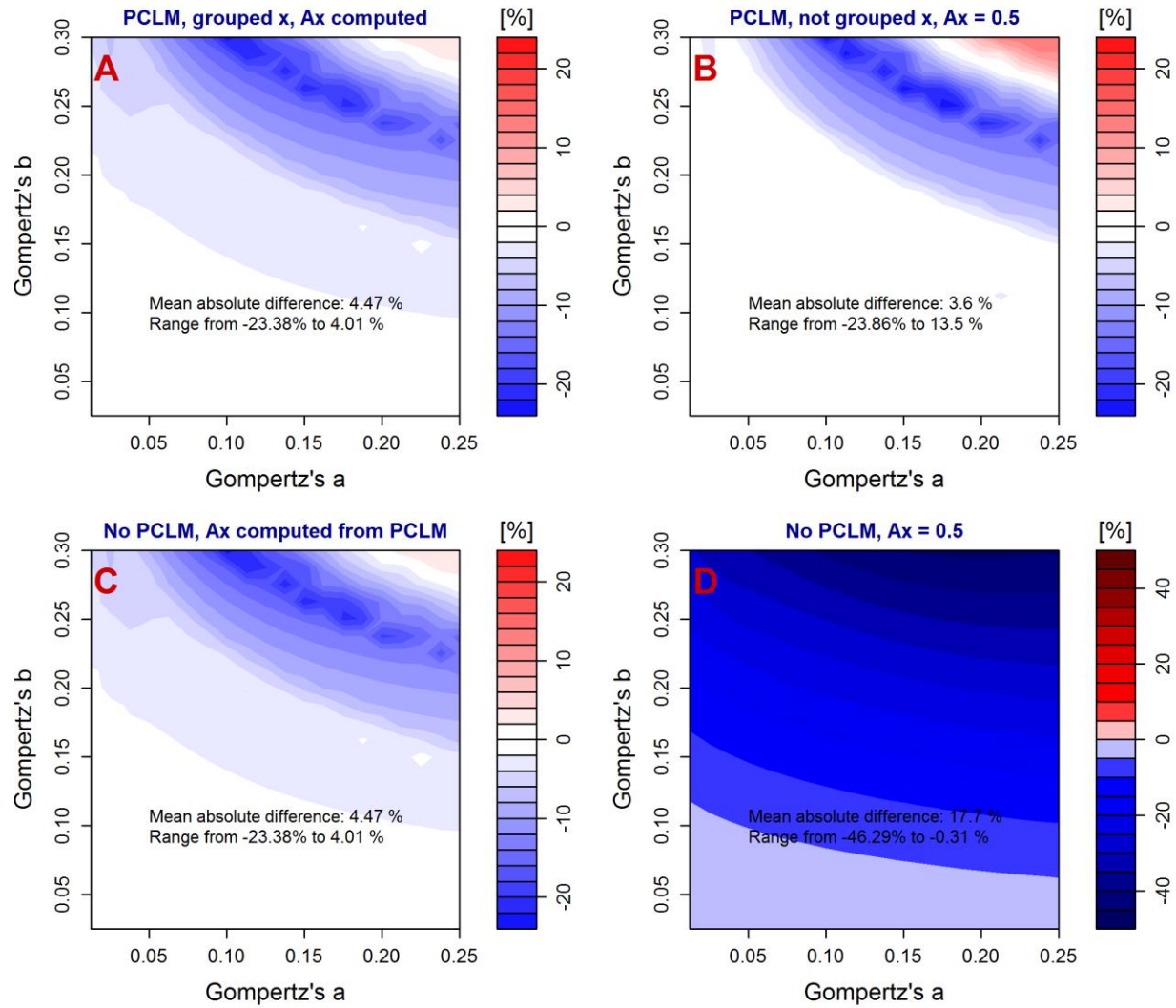


Figure 1. Relative differences between theoretical and empirical Keyfitz entropy for age interval of length 1. Different panels represents different methods of approximation.

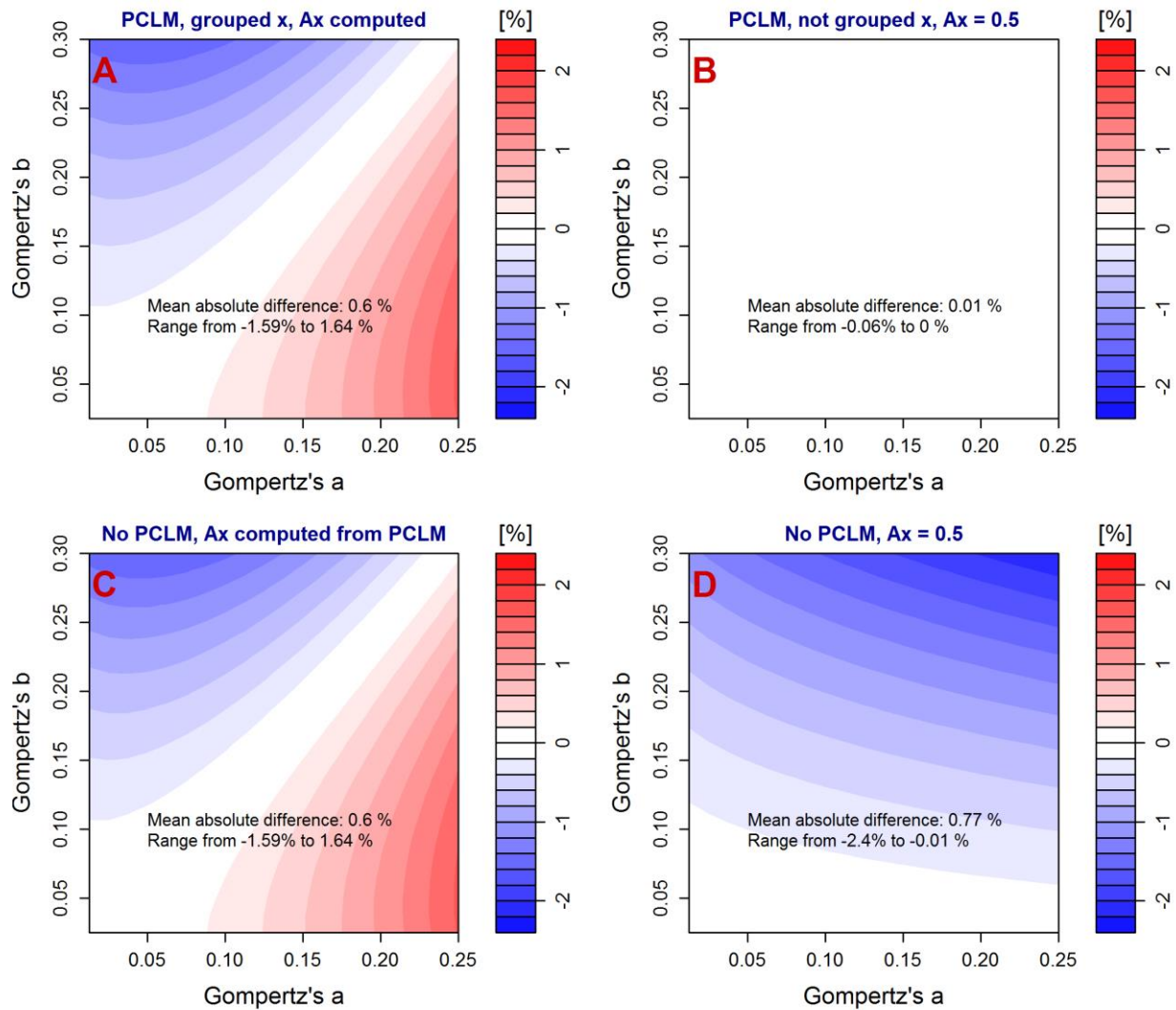


**Figure 2. Relative differences between theoretical and empirical Keyfitz entropy for age interval of length 2. Different panels represents different methods of approximation**

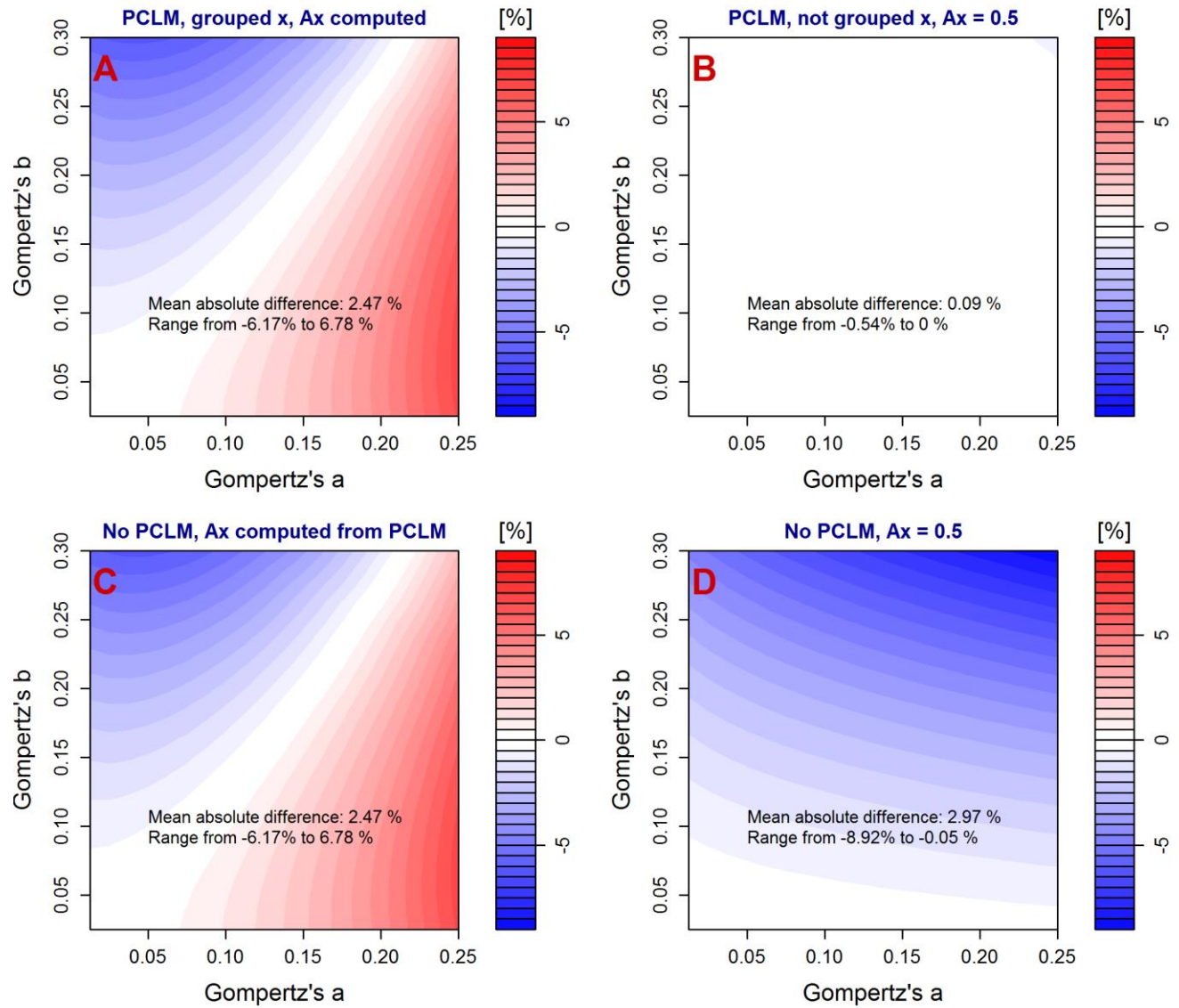


**Figure 3. Relative differences between theoretical and empirical Keyfitz entropy for age interval of length 5.** Different panels represents different methods of approximation. The red deviation in upper-right corner of A-C is the result of smoothing very few age classes.

## 2. Gini coefficient

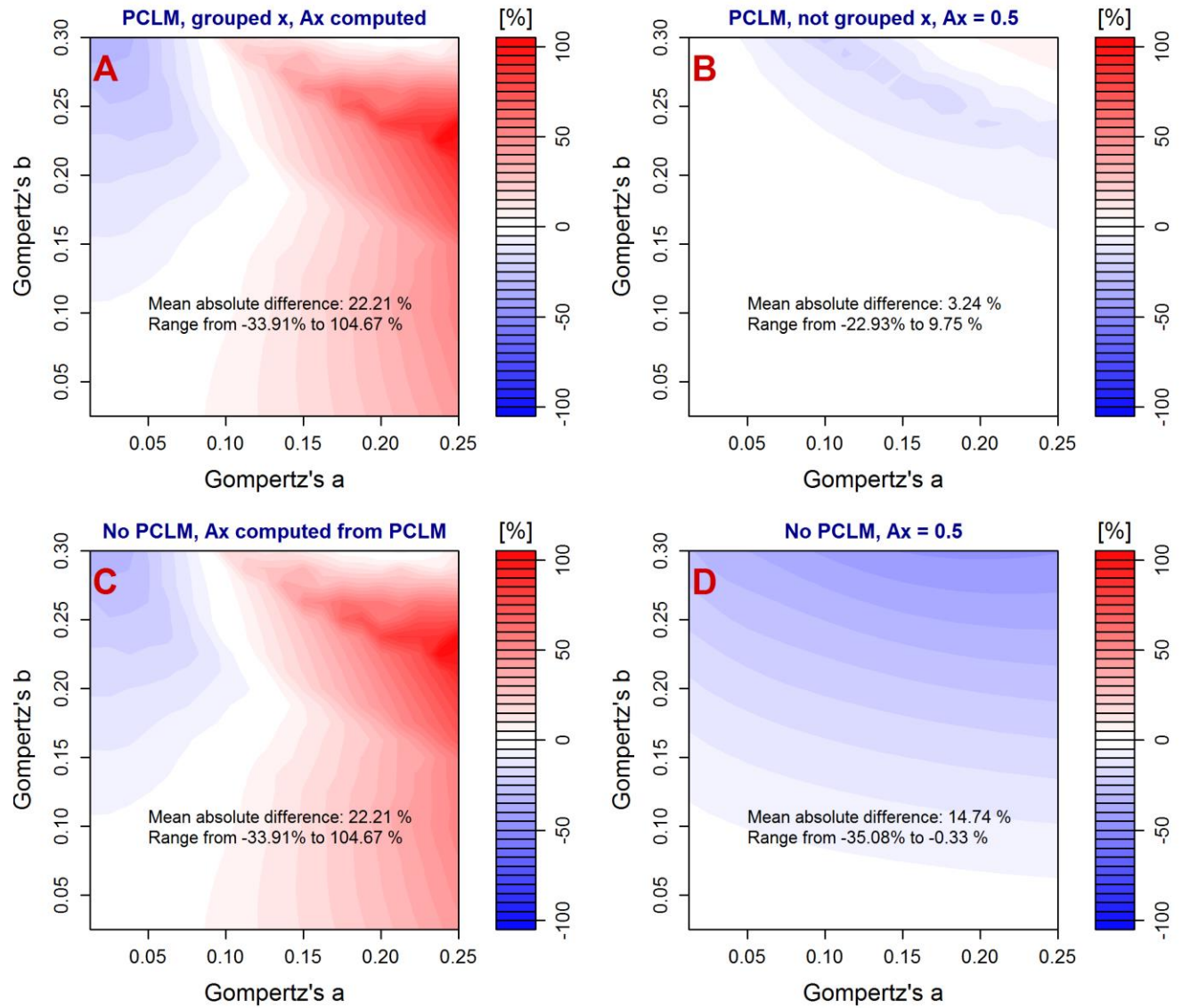


**Figure 4. Relative differences between theoretical and empirical Gini coefficient** age interval of length 1. Different panels represents different methods of approximation.



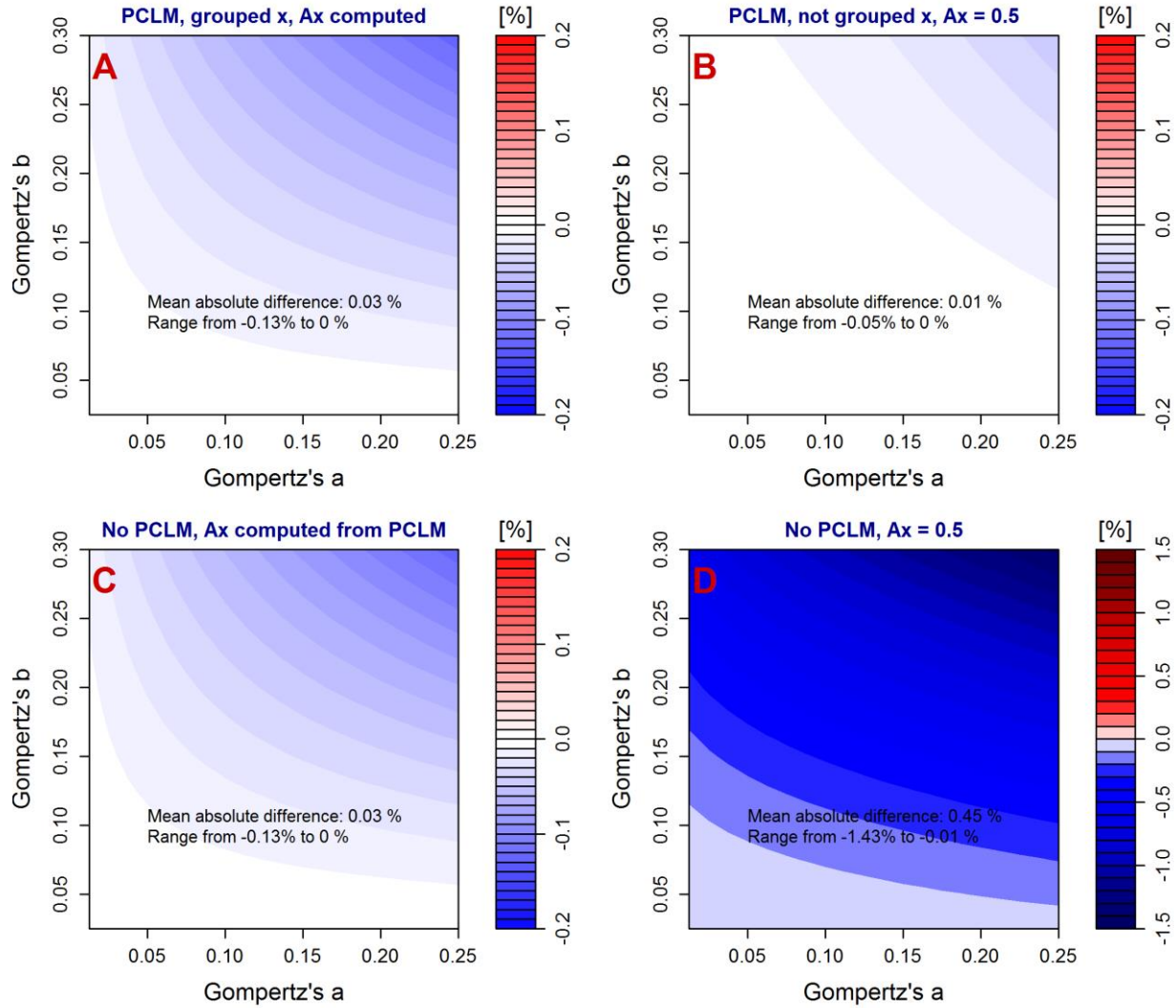
**Figure 5. Relative differences between theoretical and empirical Gini coefficient for age interval of length 2.** Different panels represents different methods of approximation.





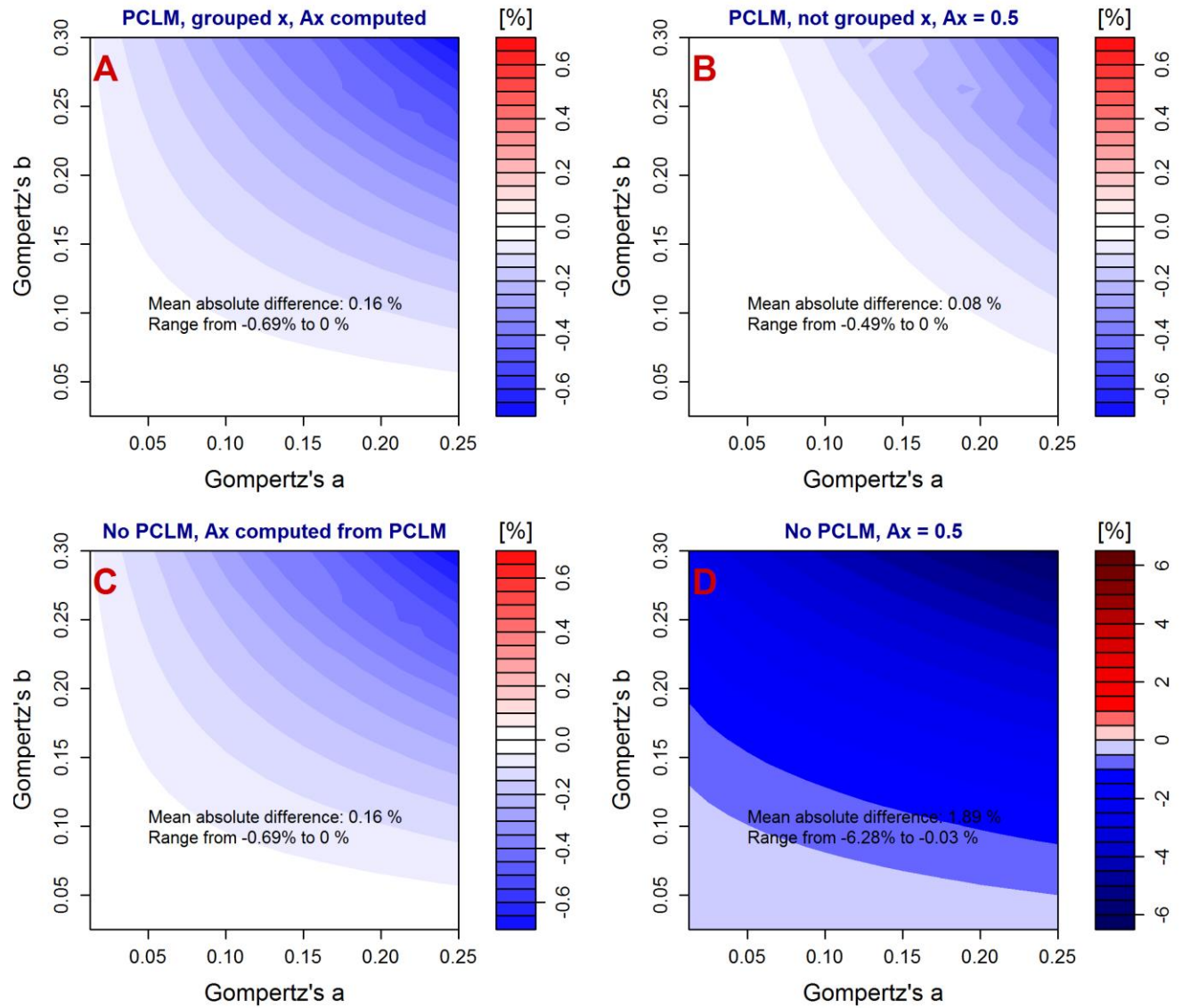
**Figure 6. Relative differences between theoretical and empirical Gini coefficient for age interval of length 5.** Different panels represents different methods of approximation. The irregular pattern in C is the result of smoothing very few age classes.

### 3. Coefficient of variation

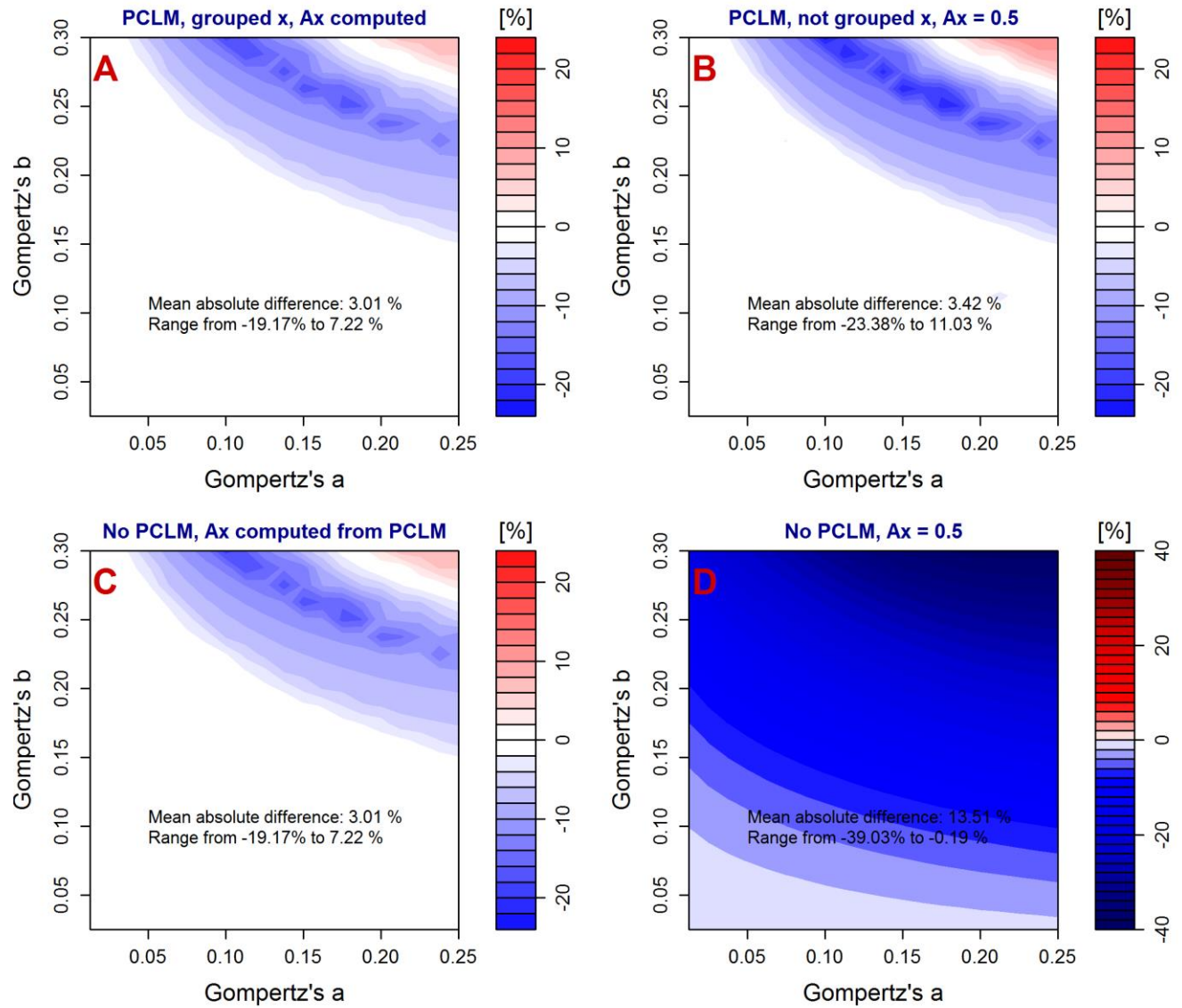


**Figure 7. Relative differences between theoretical and empirical CV for age interval of length 1.** Different panels represents different methods of approximation.



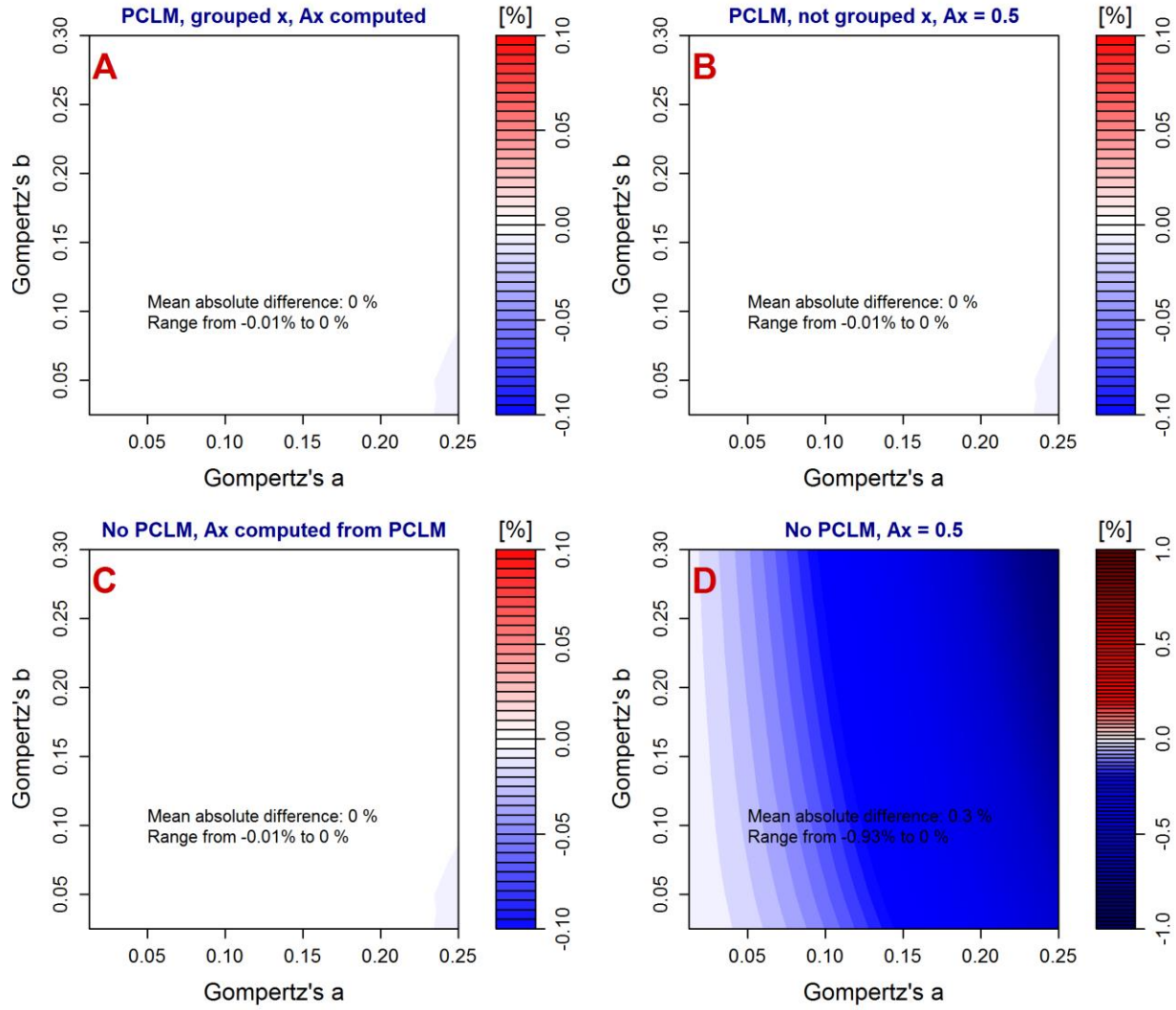


**Figure 8. Relative differences between theoretical and empirical CV for age interval of length 2.** Different panels represents different methods of approximation.

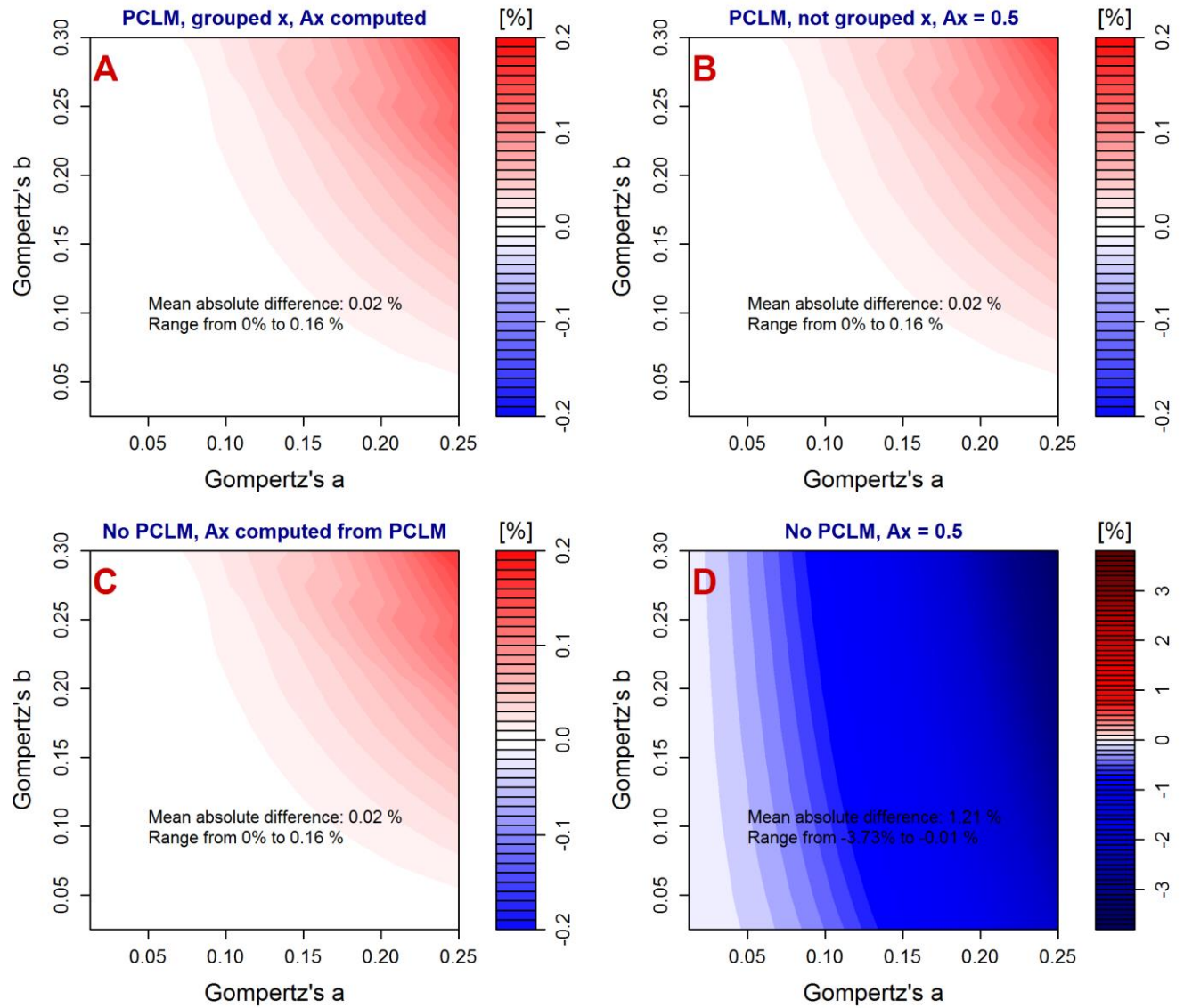


**Figure 9. Relative differences between theoretical and empirical CV for age interval of length 5.**  
Different panels represents different methods of approximation. The red deviation in upper-right corner of A-C is the result of smoothing very few age classes.

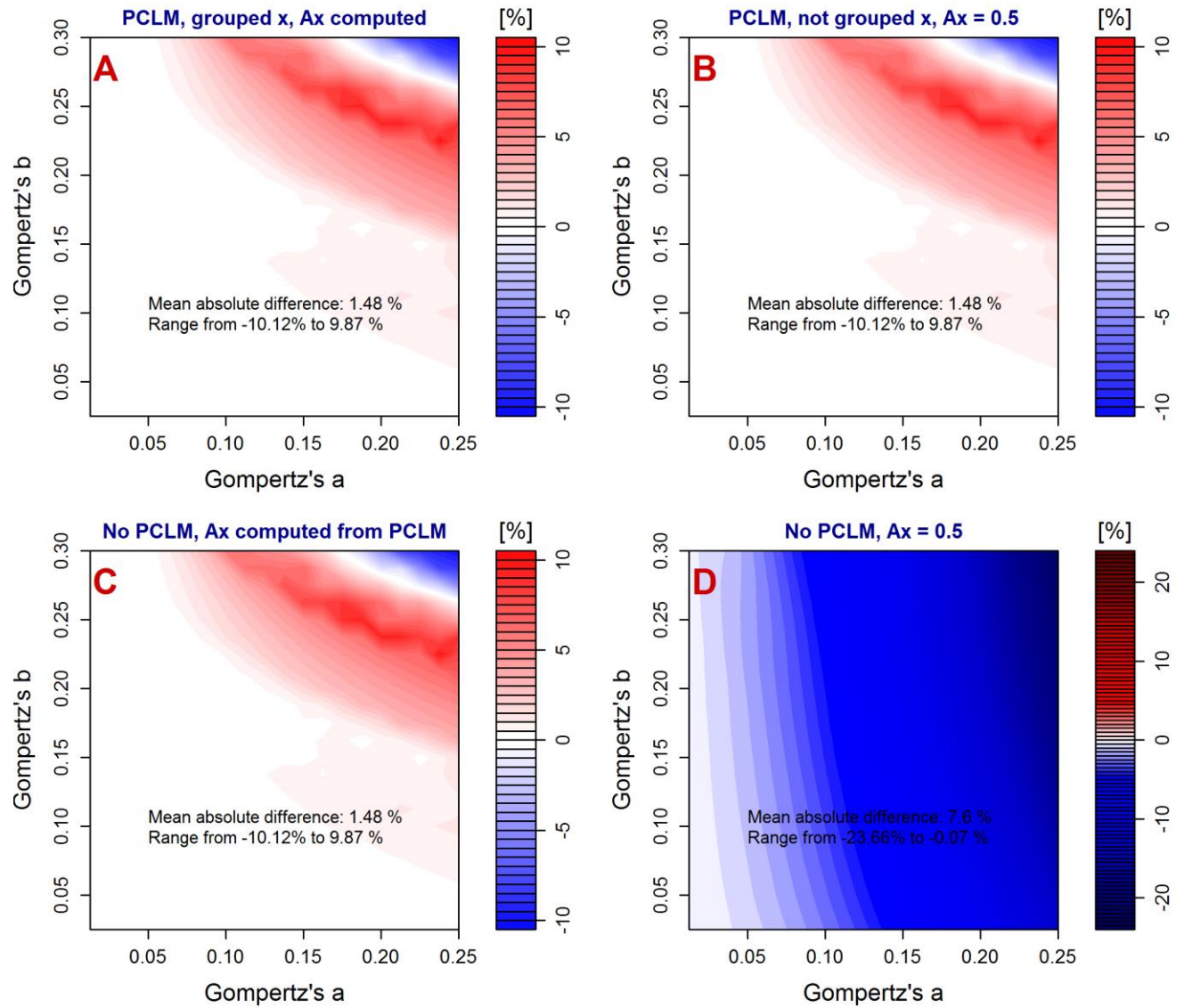
#### 4. Life expectancy



**Figure 10.** Relative differences between theoretical and empirical  $e_0$  for **age interval of length 1**. Different panels represent different methods of approximation.



**Figure 11. Relative differences between theoretical and empirical  $e_0$  for age interval length 2.** Different panels represents different methods of approximation.



**Figure 12. Relative differences between theoretical and empirical  $e_0$  for age interval of length 5.**

Different panels represent different methods of approximation. The blue deviation in upper-right corner of A-C is the result of smoothing very few age classes.