

**Interview**  
for  
**Statistical Analyst Position GGHS**

Maciej J. Dańko

MPIDR January 2017

## Selected major research projects of 2016 (chronologically)

- A. Does quantity of nutrients mediate sex specific fitness costs in *Callosobruchus maculatus*? [Małek, D.](#); [Dańko, M. J.](#); [Czarnoleski, M.](#) In preparation
- B. Identifying the Pattern of Human Mortality at Its Front End. [Missov, T. I.](#); [Németh, L.](#); [Ribeiro, F.](#); [Dańko, M. J.](#) Submitted\* to Science
- C. Density shapes patterns of survival and reproduction in hydromedusa *Eleutheria dichotoma*. [Dańko, A.](#); [Schaible, R.](#); [Pijanowska, J.](#); [Dańko M. J.](#) Submitted\* to Mechanisms of Aging and Development
- D. Latitudinal and age-specific patterns of larval mortality in the damselfly *Lestes sponsa*: Senescence before maturity? [Dańko, M.J.](#); [Dańko, A.](#); [Golab, M.J.](#); [Stoks R.](#); [Sniegula, S.](#) Submitted to Experimental Gerontology
- E. Life history traits are shaped by the interaction of extrinsic mortality and density-dependence. [Dańko, M.J.](#); [Burger, O.F.](#); [Kozłowski, J.](#) Submitted to PLOS Computational Biology
- F. Age-related changes of physiological performance and survivorship of bank voles selected for high aerobic capacity. [Rudolf, A.M.](#); [Dańko, M.J.](#); [Sadowska, E.T.](#); [Dheyongera, G.](#); [Koteja, P.](#) Submitted to Experimental Gerontology
- G. How much can we trust life tables? Sensitivity of mortality measures to right-censoring treatment. [Missov, T. I.](#); [Németh, L.](#); [Dańko, M. J.](#) Palgrave Communications, 2:15049

And more.....

# Selected major research projects of 2016 (chronologically)

## Statistical methods used

- A. Does quantity of nutrients mediate sex specific fitness costs in *Callosobruchus maculatus*?  
(Frailty parametric proportional hazard models, frailty Cox proportional models, Mortality smoothing, ...)
- B. Identifying the Pattern of Human Mortality at Its Front End.  
(Fitting Gamma-Gompertz-Makeham model via ML, Parametric bootstrap, AIC/Hierarchical LRT models selection,...)
- C. Density shapes patterns of survival and reproduction in hydromedusa *Eleutheria dichotoma*.  
(Poisson regression, Mortality Smoothing, Fitting Gamma-Gompertz Model via ML,...)
- D. Latitudinal and age-specific patterns of larval mortality in the damselfly *Lestes sponsa*: Senescence before maturity?  
(Weighted logrank tests, Fitting family of Gompertz models, AIC/Hierarchical LRT model selection,...)
- E. Life history traits are shaped by the interaction of extrinsic mortality and density-dependence.  
(Matrix projection models, optimal resource allocations models, Gompertz-Makeham, AFT/PH models,...)
- F. Age-related changes of physiological performance and survivorship of bank voles selected for high aerobic capacity. (Weighted logrank tests, Mortality smoothing, Mixture effects models,...)
- G. How much can we trust life tables? Sensitivity of mortality measures to right-censoring treatment.  
(Fitting Gamma-Gompertz-Makeham model by ML, Parametric bootstrap, Simulation of censoring, LT measures,...)

Other projects: Lifetables analysis of large databases, PCLM used to approximate LT measures, bootstrap to calculate confidence intervals of LT measures, weighted linear models, ...

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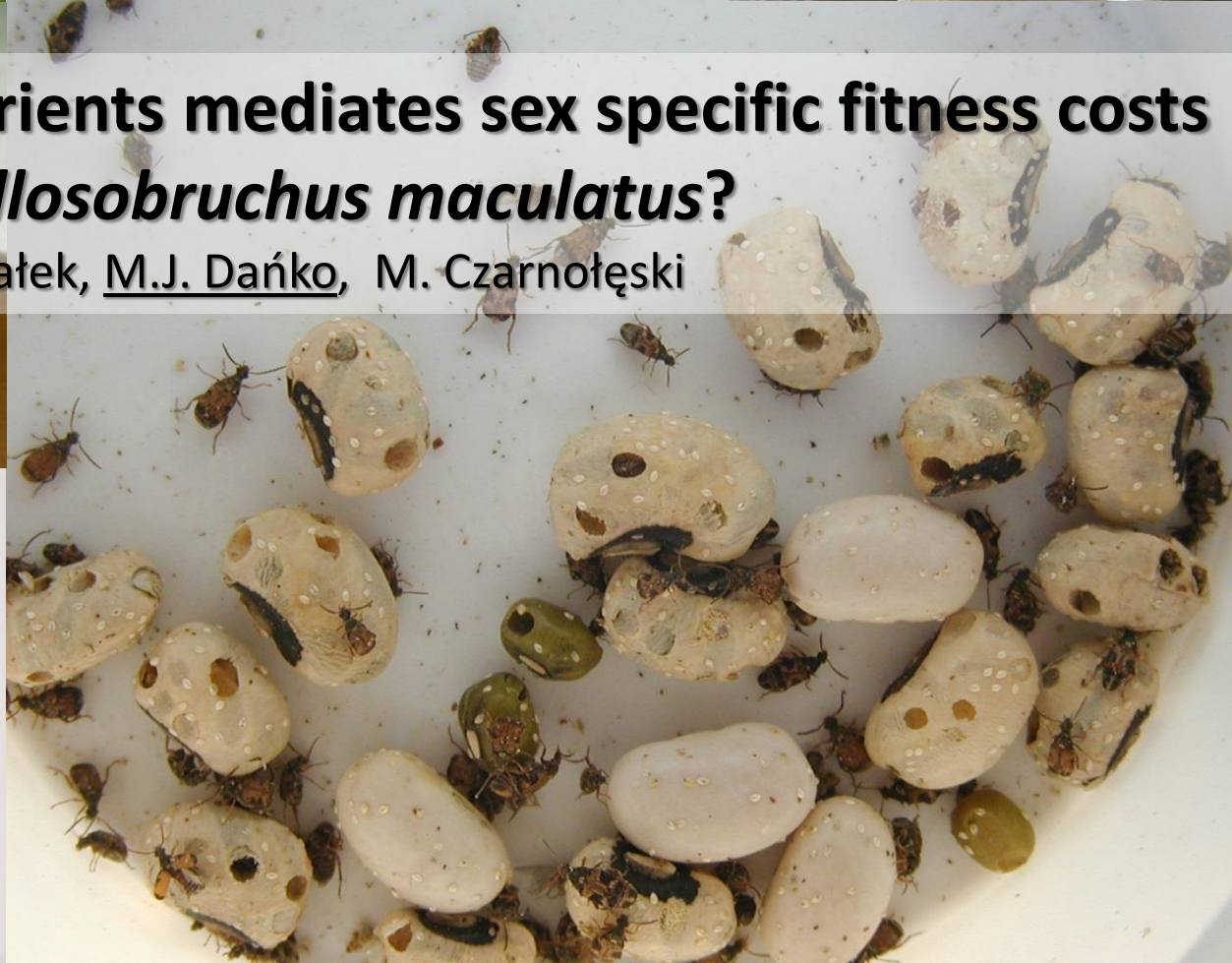
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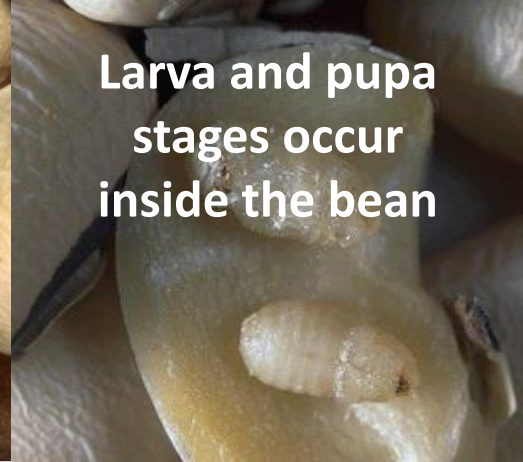


# Does quantity of nutrients mediate sex specific fitness costs in *Callosobruchus maculatus*?

D. Małek, M.J. Dańko, M. Czarnołęski







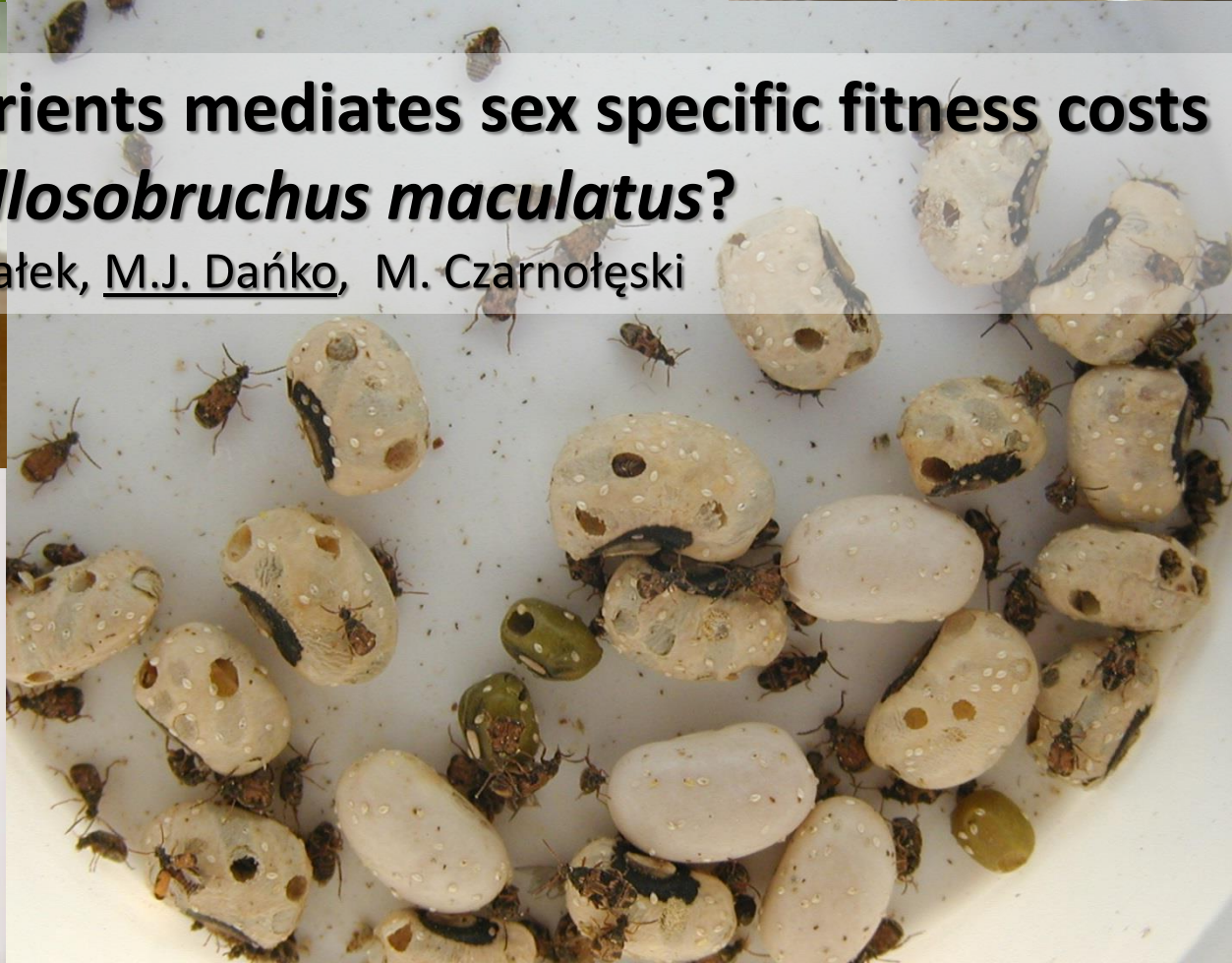
Larva and pupa  
stages occur  
inside the bean

Laying eggs on beans



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Nuptial gifts in other animals



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- Reproductive costs in both sexes can also influence one another, for example by a phenomenon known as nuptial gifts.
- An endogenous gift should be especially costly to a donor because adult individuals of *Callosobruchus maculatus* in laboratory conditions do not ingest food or water, resulting in a very limited energy budget.



Nuptial gifts in other animals



# Goals of the project (relevant to survival analysis)

**Treatment:** reproduction allowed, **control:** reproduction not allowed.

**Two sexes**

**Covariates:** bean size, adult size and gift size (reproducing animals)

**Random effects:** mother id

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**Treatment:** reproduction allowed, **control:** reproduction not allowed.

**Two sexes**

**Covariates:** bean size, adult size and gift size (reproducing animals)

**Random effects:** mother id

- 1) Investigate the effect of **sex**, **bean size**, **presence of reproduction**, and **adult size** on survival.
- 2) Investigate the role of **nuptial gifts** in reproducing males and females for their survival



# Frailty models

(Random effects models in survival analysis)

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- Frailty models:
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  - Shared frailty models are random effects model (a group of individuals share the same hidden frailty)
- Shared frailty models:
  - Covered in books, but research is still ongoing
  - Only partially covered in statistical packages



# Proportional hazard shared frailty models

Conditional hazard of subject  $i$   
that belongs to the group  $g$

$$h_{g,i}(t | u_g) = u_g h_0(t) \exp(X_{g,i}^T \beta)$$

Frailty term in group  $g$ .  
Realization of a random  
variable with certain pdf  
(e.g. gamma distribution)

Baseline hazard  
(e.g. Weibull)

Vector of regression coefficients applied  
to a vector of observed covariates for  
subject  $i$  in group  $g$



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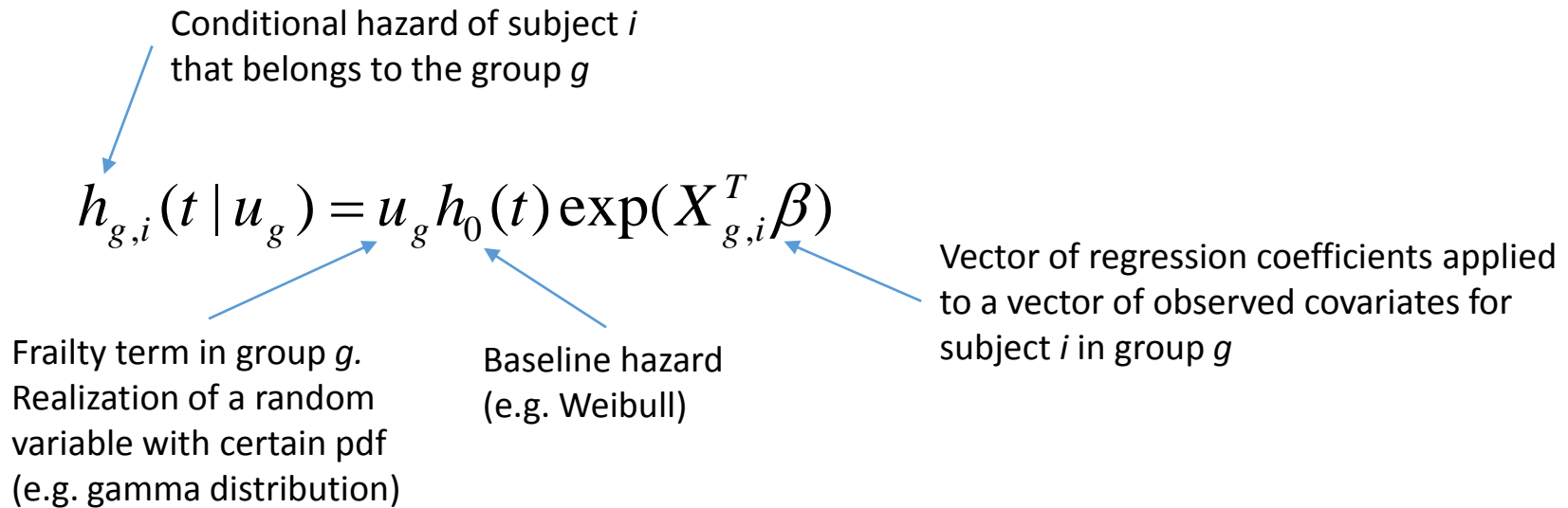
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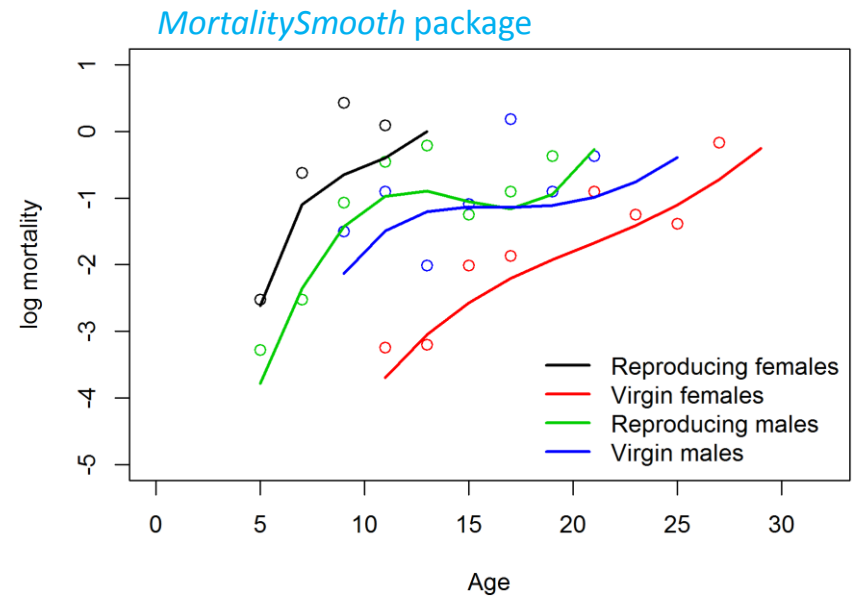
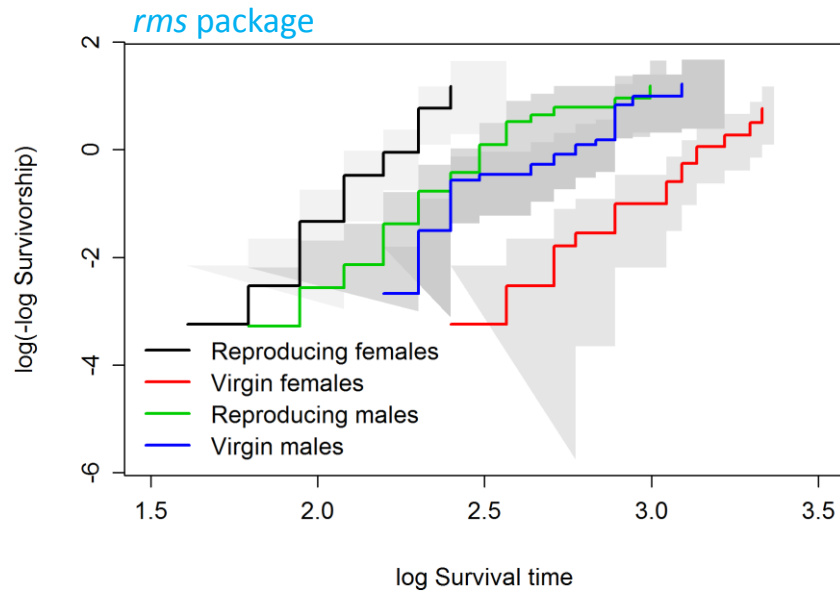


The estimation procedure is based on expectation-maximization (EM) algorithm, which is a sequence of:

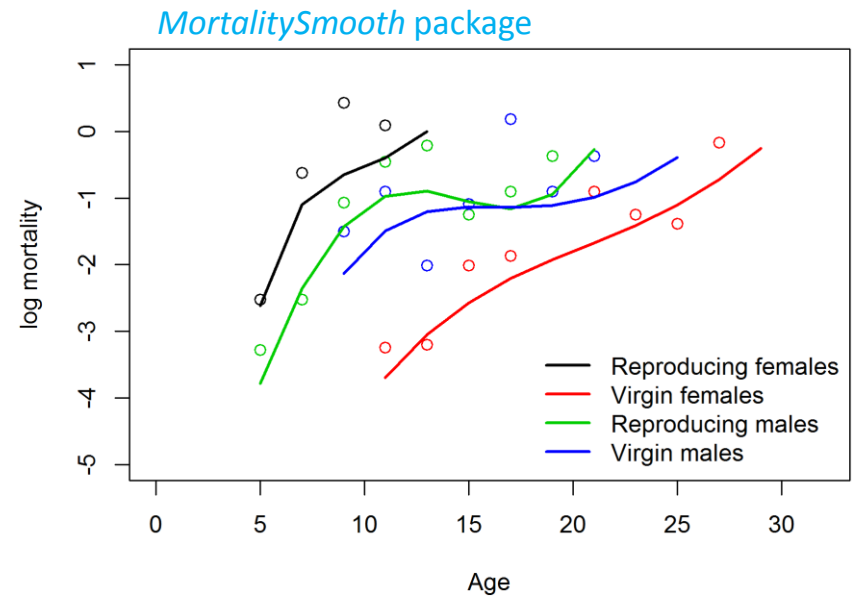
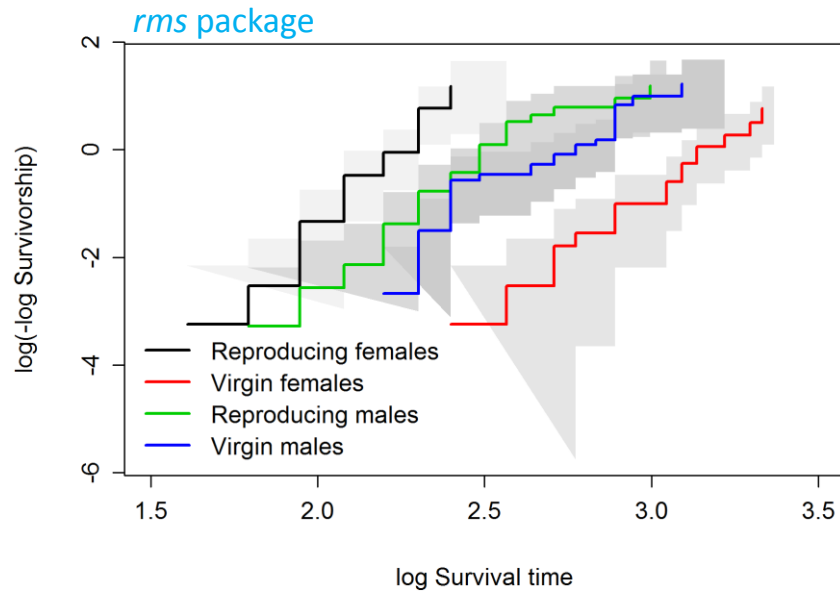
- A) Expectation → Prediction of  $u$  given the estimates of parameters of theoretical frailty distribution, baseline hazard, and regression coefficients.
- B) Maximization → Use predicted  $u$  to find all parameter estimates by maximization marginal log-likelihood function.

The sequence is repeated until the convergence

# Testing proportional hazard assumption (categorical variables)



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Clearly visible interaction between sex and treatment!

# Testing proportional hazard assumption (continuous variables)

- It is hard to perform similar non-parametric analysis on continuous variables
- We will use a semi-parametric approach based on Cox proportional hazard frailty model.

**Model:**  $\text{Surv} \sim \text{Sex} + \text{Treatment} + \text{Bean size} + \text{Adult body mass} + \text{Sex} : \text{Treatment}$



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*survival* package\*

	rho	chisq	p
Sex (Males)	-0.0052	0.0035	0.9526
Treatment (Virgin)	0.0677	0.4514	0.5017
Bean size	0.0176	0.0453	0.8315
Adult body mass	0.0139	0.0267	0.8703
Interaction (sex:treatment)	-0.0305	0.1037	0.7474
GLOBAL		0.9675	0.9651

\* P. Grambsch and T. Therneau (1994), Proportional hazards tests and diagnostics based on weighted residuals. *Biometrika*, **81**, 515-26.

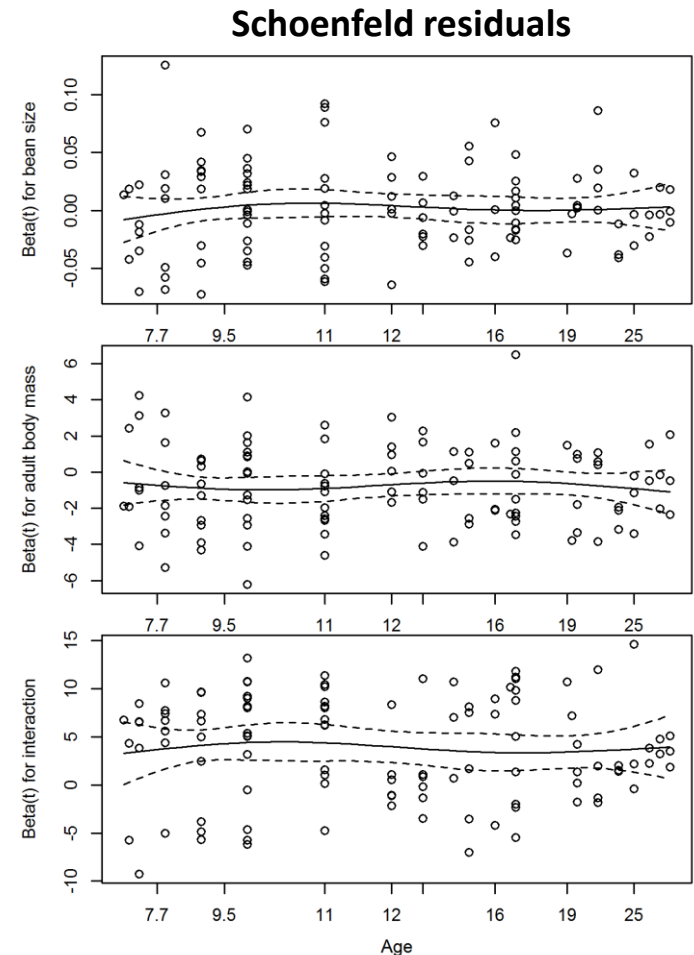
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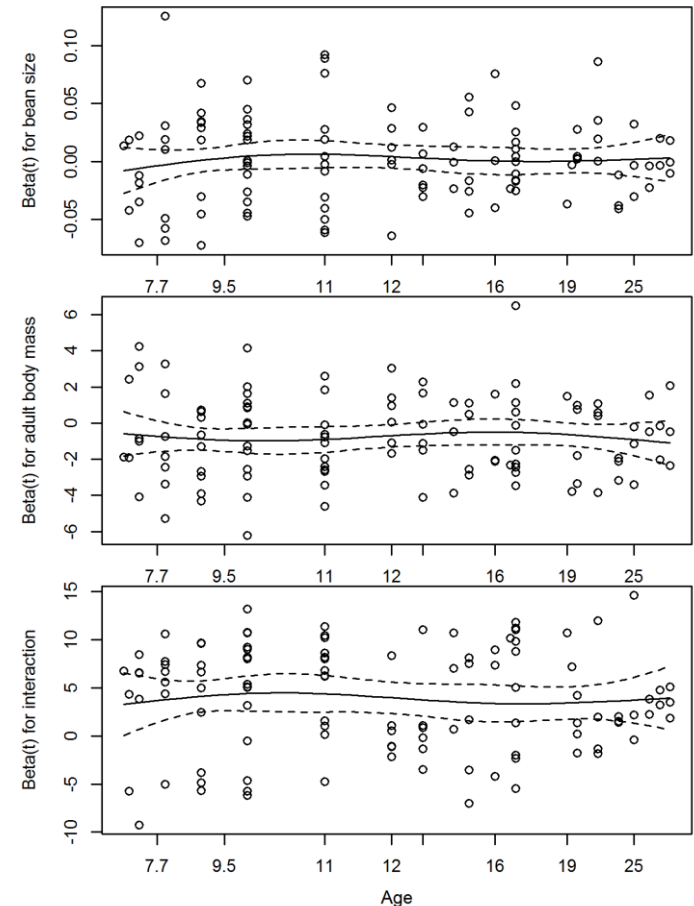
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**Conclusion:** PH assumptions holds (at least for Cox model).

Schoenfeld residuals



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# First guess about baseline hazard

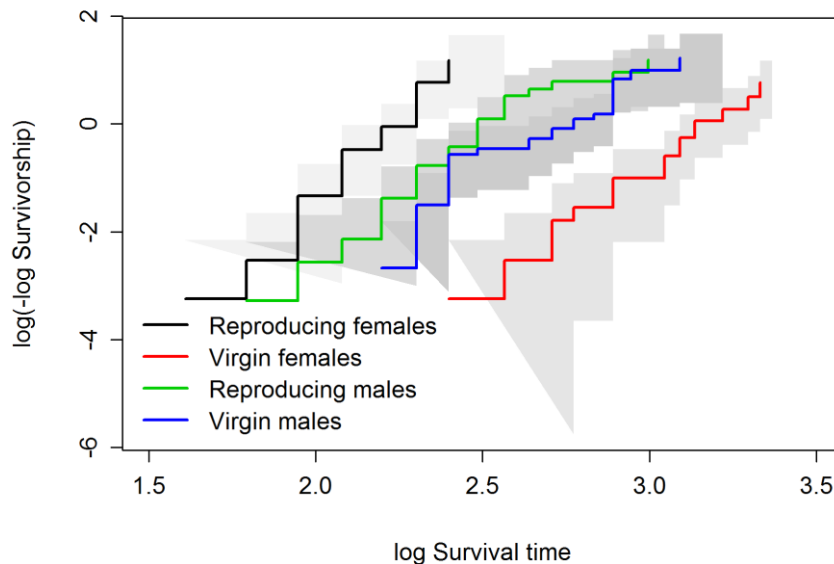
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Curves seems to be linear in early ages

# Selection of baseline hazard and frailty distribution

We can use GoF tests, however heterogeneity can affect observed mortality patterns.

**A mixture of different Weibull distributions is not necessarily a Weibull distribution**

**Tests are not decisive, but only informative**



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	p-val	
	KS Bootstrap*	Generalized Gamma**
Reproducing females	0.6054	0.2486
Virgin females	0.8352	0.5075
Reproducing males	0.2071	0.0219
Virgin males	0.1509	0.1079

- Parametric bootstrap of null hypothesis of Kolmogorov-Smirnoff statistics

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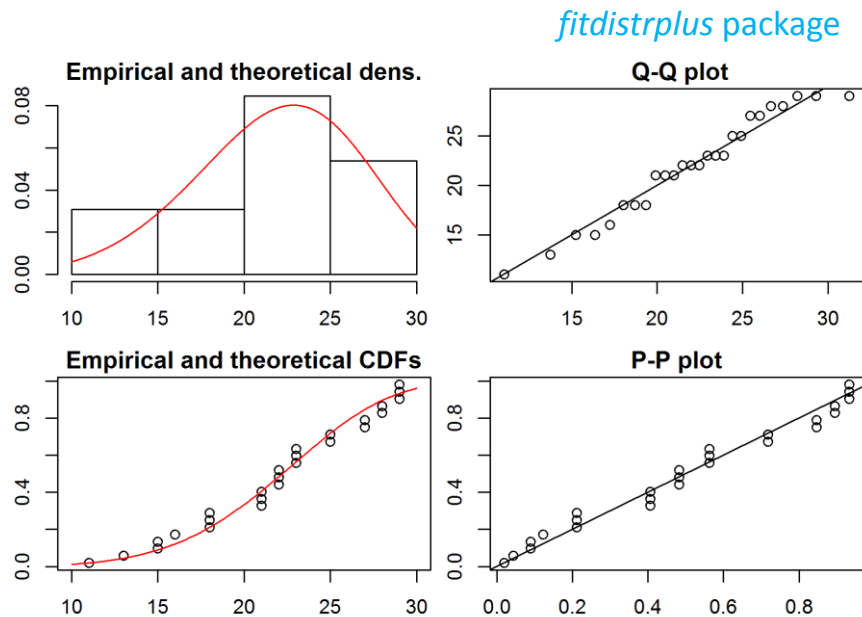


FIG. Empirical vs. Theoretical Weibull distribution for **virgin females**.

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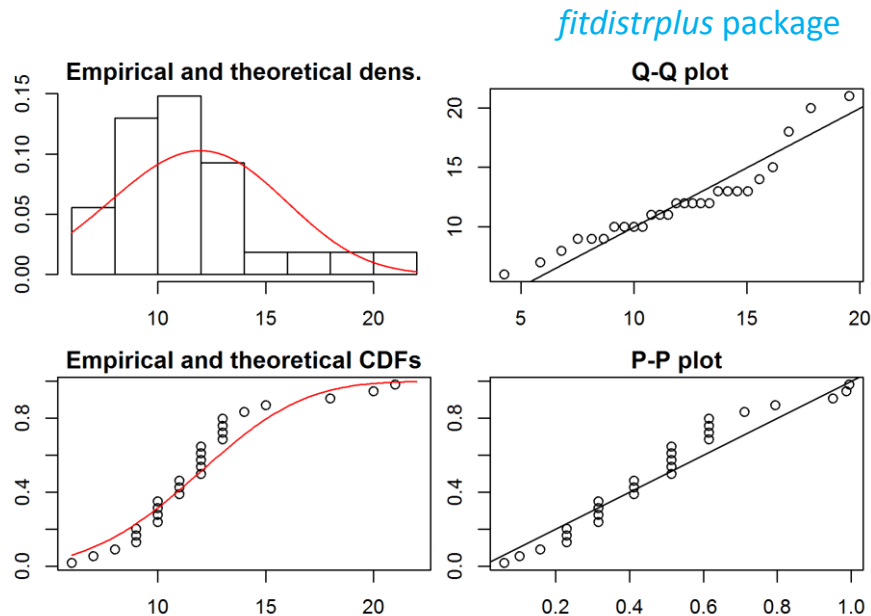


FIG. Empirical vs. Theoretical Weibull distribution for **reproducing males**.

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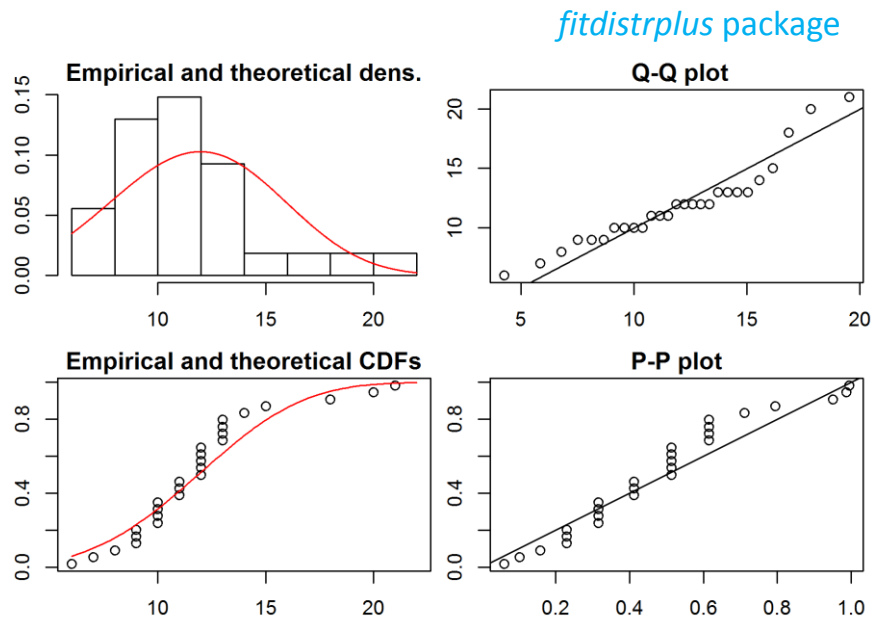


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## Model selection based on AIC

**Model:**  $\text{Surv} \sim \text{Sex} + \text{Treatment} + \text{Bean size} + \text{Adult body mass} + \text{Sex} : \text{Treatment}$

Baseline hazard	Frailty distribution	
	none	gamma
Gompertz	616.19	613.87
Weibull	584.98	583.68
Exponential	795.7	797.77

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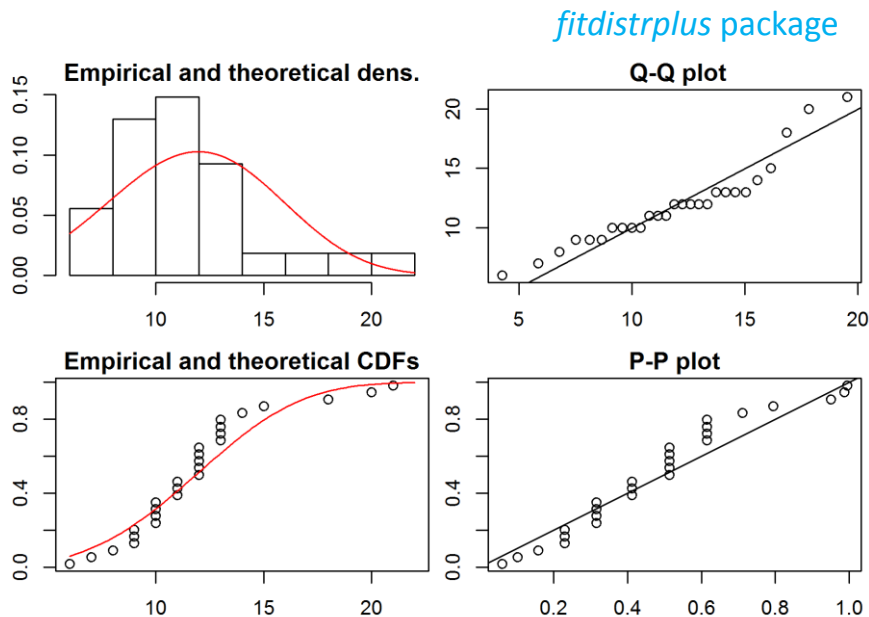


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# Basic model and model selection

Improved maximization part of EM algorithm  
of *parfm* package

	ESTIMATE	SE	p-val
Theta (frailty par.)	0.1572	0.1290	
Rho (Weibull shape par.)	4.7296	0.3877	
Lambda (Weibull scale par.)	0.0026	0.0036	
Sex (Males)	-2.8240	0.4835	0.0000
Treatment (Virgin)	-4.5469	0.4850	0.0000
Bean size	0.0011	0.0031	0.7263
Addult body mass	-0.6894	0.1998	0.0006
Interaction (sex:treatment)	3.8895	0.5384	0.0000

Basic model with included Sex:Treatment interaction.  
The p-values are calculated from Wald test.



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LRT added to *parfm* package

Added interaction	chisq	df	pval
Bean size : adult body mass	2.22	1	0.1362
Sex : adult body mass	0.27	1	0.6003
Sex : bean size	1.51	1	0.2195
Treatment : adult body mass	1.91	1	0.1673
Treatment : bean size	1.75	1	0.1858
Sex : treatment : bean size *	2.36	4	0.6697
Sex : treatment : adult body mass *	2.46	4	0.6519
Sex : bean size : adult body mass	2.25	2	0.3243
Treatment : bean size : adult body mass	1.76	2	0.4156

\* "Negative variance" problem

The model cannot be further improved by "sequential" LRT

# Basic model and model selection

Improved maximization part of EM algorithm  
of *parfm* package

	ESTIMATE	SE	p-val
Theta (frailty par.)	0.1572	0.1290	
Rho (Weibull shape par.)	4.7296	0.3877	
Lambda (Weibull scale par.)	0.0026	0.0036	
Sex (Males)	-2.8240	0.4835	0.0000
Treatment (Virgin)	-4.5469	0.4850	0.0000
Bean size	0.0011	0.0031	0.7263
Adult body mass	-0.6894	0.1998	0.0006
Interaction (sex:treatment)	3.8895	0.5384	0.0000

Basic model with included Sex:Treatment interaction.  
The p-values are calculated from Wald test.

We will perform model selection by sequentially adding  
single 2-way or 3-way interaction and performing  
Likelihood ratio test.

The estimates of the basic model will be used as starting  
values for optimization method of more complicated  
models.

**Variance inflation factors (VIF) measure how much  
the variance of the estimated regression coefficients  
are inflated as compared to when the predictor  
variables are not linearly related.**

LRT added to *parfm* package

Added interaction	chisq	df	pval
Bean size : adult body mass	2.22	1	0.1362
Sex : adult body mass	0.27	1	0.6003
Sex : bean size	1.51	1	0.2195
Treatment : adult body mass	1.91	1	0.1673
Treatment : bean size	1.75	1	0.1858
Sex : treatment : bean size	2.36	4	0.6697
Sex : treatment : adult body mass *	2.46	4	0.6519
Sex : bean size : adult body mass	2.25	2	0.3243
Treatment : bean size : adult body mass	1.76	2	0.4156

\* "Negative variance" problem

The model cannot be further improved by "sequential" LRT

*vif()* of *rms* package adopted to work with *parfm* package

## Testing collinearity by Variance Inflation Factor (VIF)

Rule of thumb:  $VIF < 10$

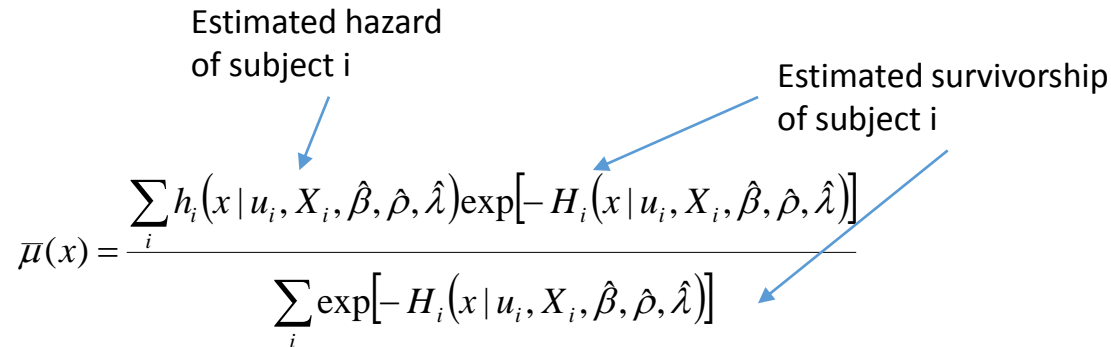
	VIF
Theta (frailty par.)	1.30
Rho (Weibull shape par.)	3.14
Sex (Males)	5.40
Treatment (Virgin)	5.62
Bean size	1.12
Adult body mass	2.69
Interaction (sex:treatment)	4.75

# Predictions of the model – marginal hazard

My new extension to *parfm* package

Estimated hazard  
of subject i

Estimated survivorship  
of subject i

$$\bar{\mu}(x) = \frac{\sum_i h_i(x | u_i, X_i, \hat{\beta}, \hat{\rho}, \hat{\lambda}) \exp[-H_i(x | u_i, X_i, \hat{\beta}, \hat{\rho}, \hat{\lambda})]}{\sum_i \exp[-H_i(x | u_i, X_i, \hat{\beta}, \hat{\rho}, \hat{\lambda})]}$$
The diagram illustrates the formula for the marginal hazard, \bar{\mu}(x). It features three blue arrows pointing to specific parts of the equation. The first arrow, labeled 'Estimated hazard of subject i', points to the hazard function h\_i(x | u\_i, X\_i, \hat{\beta}, \hat{\rho}, \hat{\lambda}) in the numerator. The second arrow, labeled 'Estimated survivorship of subject i', points to the exponential term \exp[-H\_i(x | u\_i, X\_i, \hat{\beta}, \hat{\rho}, \hat{\lambda})] in the numerator. The third arrow points to the denominator, which is the sum of the exponential terms across all subjects i.

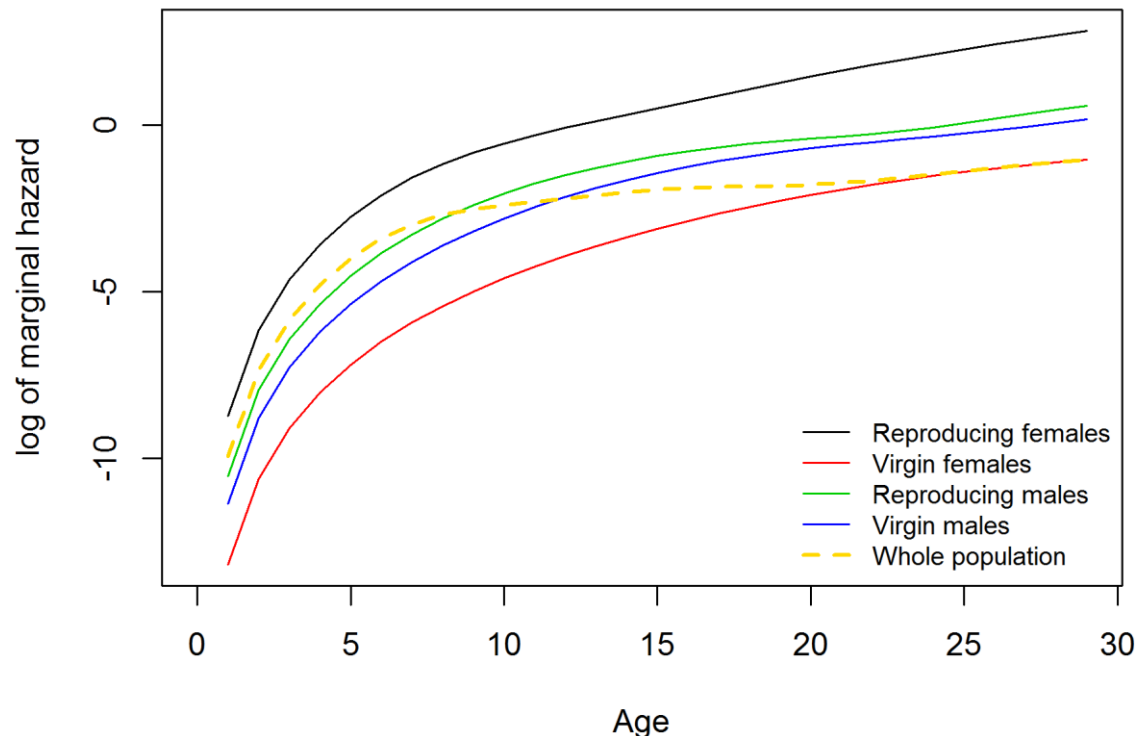
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Estimated survivorship of subject  $i$

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Marginal hazard for each sex and treatment calculated with respect to individual differences in frailty, adult body mass, and bean size

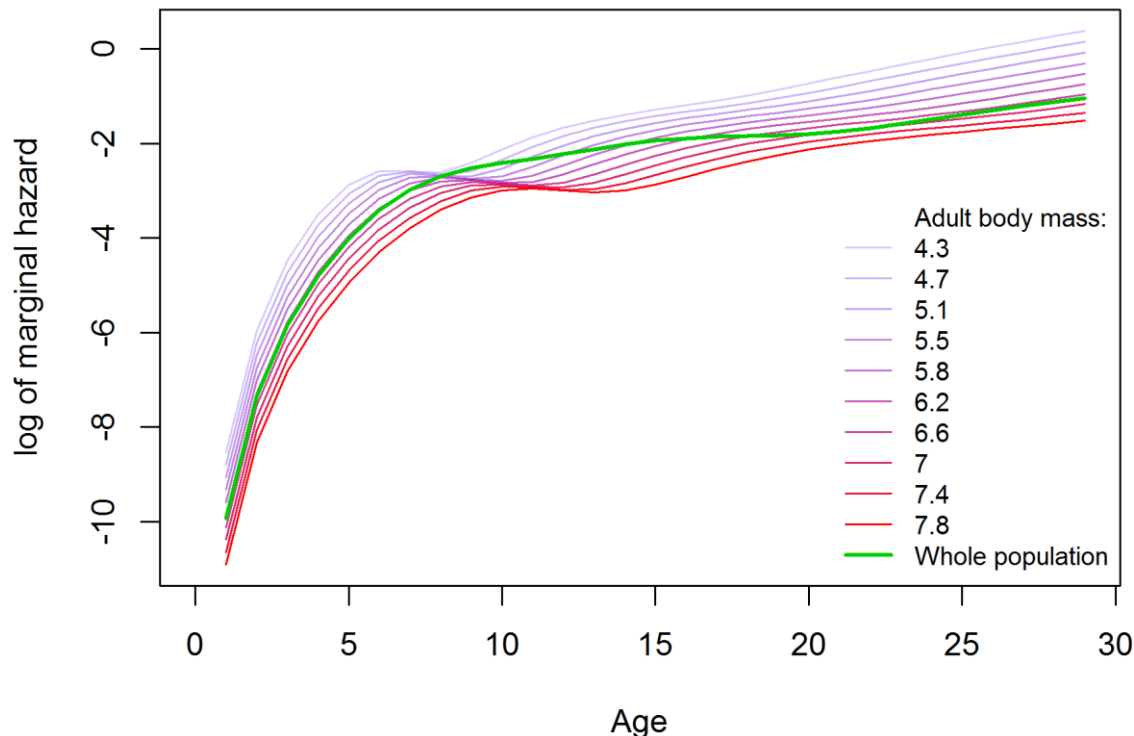
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My new extension to *parfm* package

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Marginal hazard as a function of adult body mass calculated with respect to individual differences in frailty, sex, treatment, and bean size

# Analyzing model fit by Martingale residuals

My new extension to *parfm* package

Frailty of subject  $i$

Follow-up (event) time of subject  $i$

$$\hat{M}_i = \delta_i - u_i \hat{H}_0(t_i) \exp(\hat{\beta}_1 X_{1,i} + \dots + \hat{\beta}_n X_{n,i})$$

Event indicator  
(0=censoring, 1=death)

Estimated cumulative  
baseline hazard

Estimated coefficients applied to observed  
covariate for subject  $i$



# Analyzing model fit by Martingale residuals

My new extension to *parfm* package

Frailty of subject  $i$

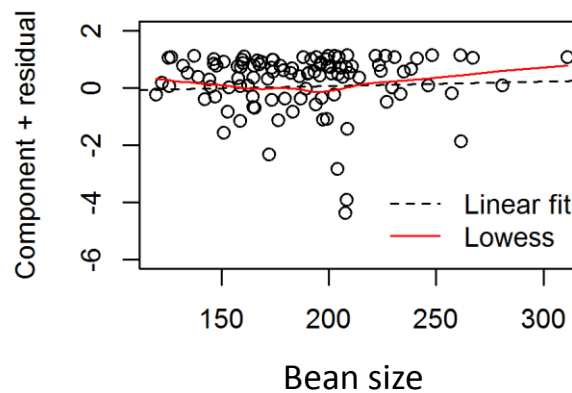
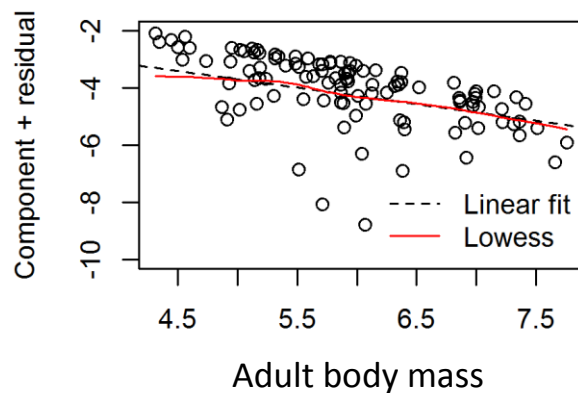
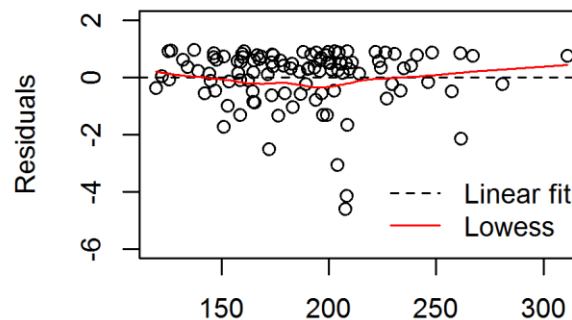
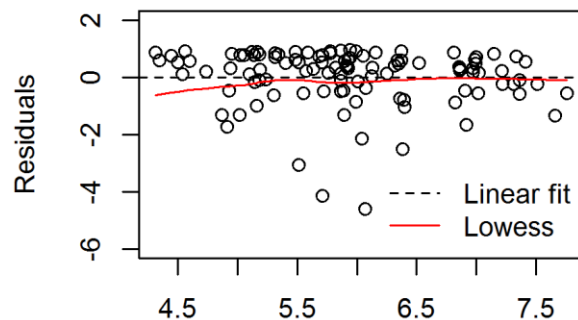
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# Modeling the effect of nuptial gift size

Only reproducing animals are relevant

Model will include new set of independent variables:

- Sex
- Gift size
- Bean size
- Adult body mass

**Basic model:**  $\text{Surv} \sim \text{Sex} + \text{Gift size} + \text{Bean size} + \text{Adult body mass}$

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It is hard to guess about interactions as we cannot easily plot them.

We cannot include all → Building too complicated models can:

- Greatly increase VIF
- Can lead to lack of convergence in EM algorithm
- Make computations extremely long

From the same reasons: exhaustive model selection (e.g. AIC) would be ineffective.

# Gift size

## Model selection via Hierarchical Likelihood Ratio Test (hLRT)

My new extension to *parfm* package

Model without interaction, but all main effects are present

Basic model:  $\text{Surv} \sim \text{Sex} + \text{Gift size} + \text{Bean size} + \text{Adult body mass}$

### #1 Sequential LRT test of each of single interaction terms

	logLikelihood	chisq	df	pval
Sex : gift size	-119.77	7.9216	1	0.0049
Sex : bean size	-123.69	0.0886	1	0.7660
Sex : Adult body mass	-123.55	0.3616	1	0.5476
Gift size : bean size	-123.68	0.0932	1	0.7602
Gift size : Adult body mass	-120.56	6.3355	1	0.0118
Bean size : Adult body mass	-123.43	0.6088	1	0.4353

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New model = Basic model + Sex : gift size

#2 Selection of significant interaction that has the highest log likelihood, then proceed as in #1

# Gift size

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	logLikelihood	chisq	df	pval
Sex : bean size	-119.43	0.6826	1	0.4087
Sex : Adult body mass	-119.56	0.4282	1	0.5129
Gift size : bean size	-119.66	0.2215	1	0.6379
Gift size : Adult body mass	-119.46	0.6210	1	0.4307
Bean size : Adult body mass	-118.30	2.9423	1	0.0863



# Gift size

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No further improvement is possible.

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### No further improvement is possible.

	ESTIMATE	SE	p-val
Theta (frailty par.)	0.2558	0.2314	
Rho (Weibull shape par.)	5.4213	0.7059	
Lambda (Weibull scale par.)	0.0009	0.0021	
Sex (Males)	-4.8499	1.0594	0.0000
Bean size	-4.2520	2.4488	0.0825
Adult body mass	-0.0045	0.0051	0.3764
Gift size	-0.4326	0.2837	0.1272
Interaction (sex : gift size)	8.5111	3.1087	0.0062

# Gift size

## Model selection via Hierarchical Likelihood Ratio Test (hLRT)

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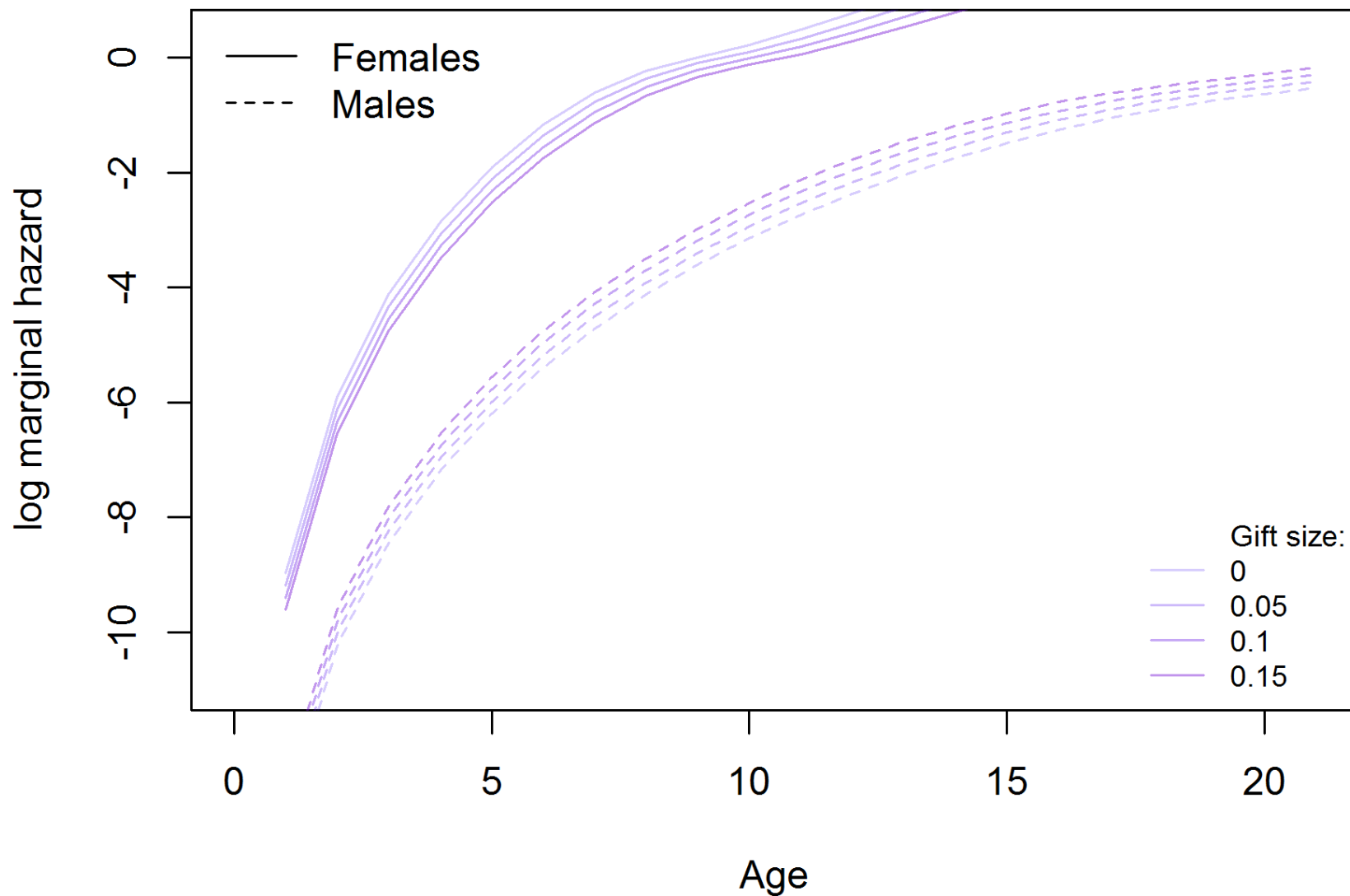
### No further improvement is possible.

Non-significant terms (like bean size or adult mass) could be dropped from the model (LRT) to decrease VIF (some VIFs slightly exceeded 10, not shown).

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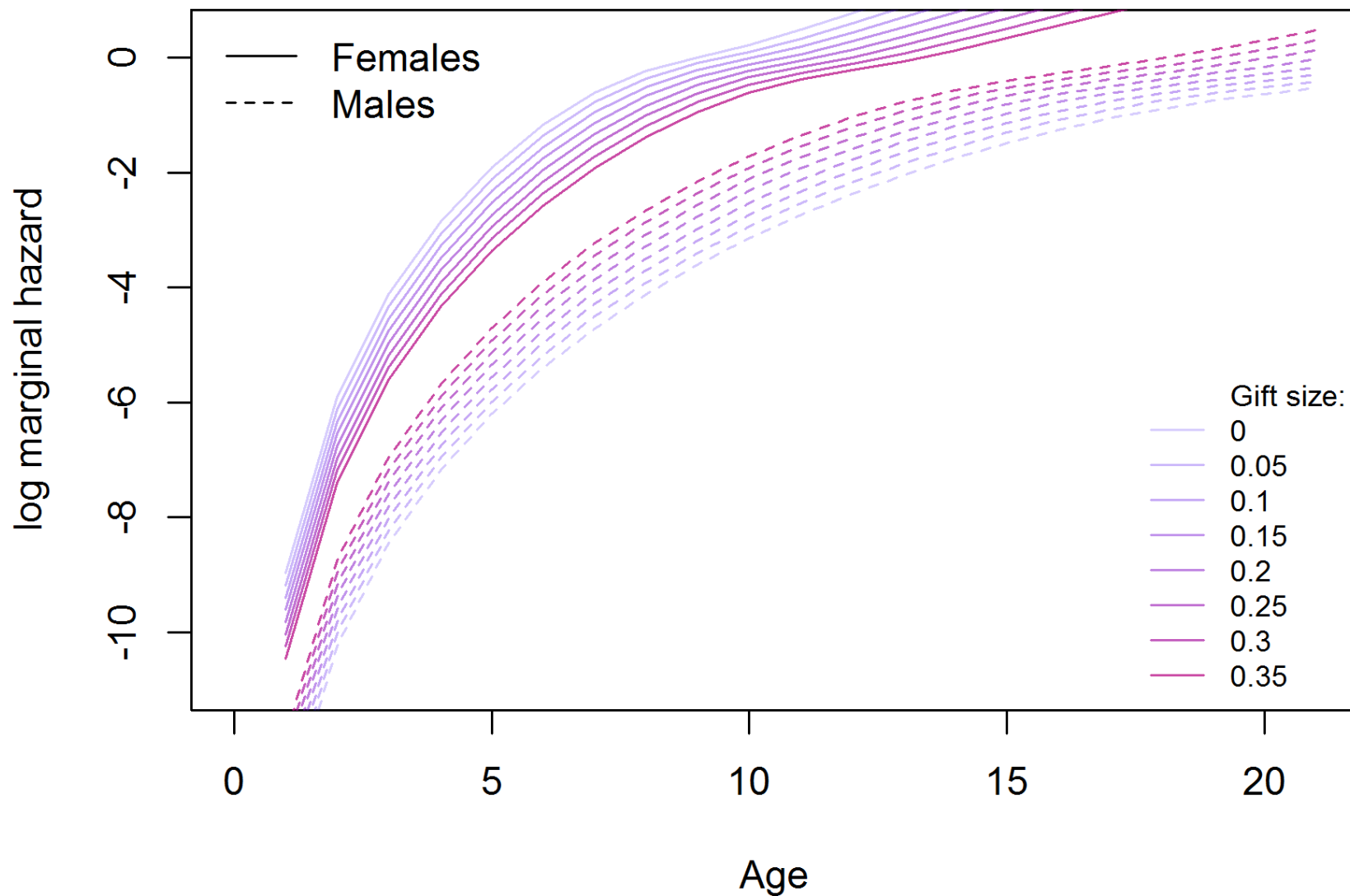
# Effect of gift size on marginal hazard rate in males and females

My new extension to *parfm* package



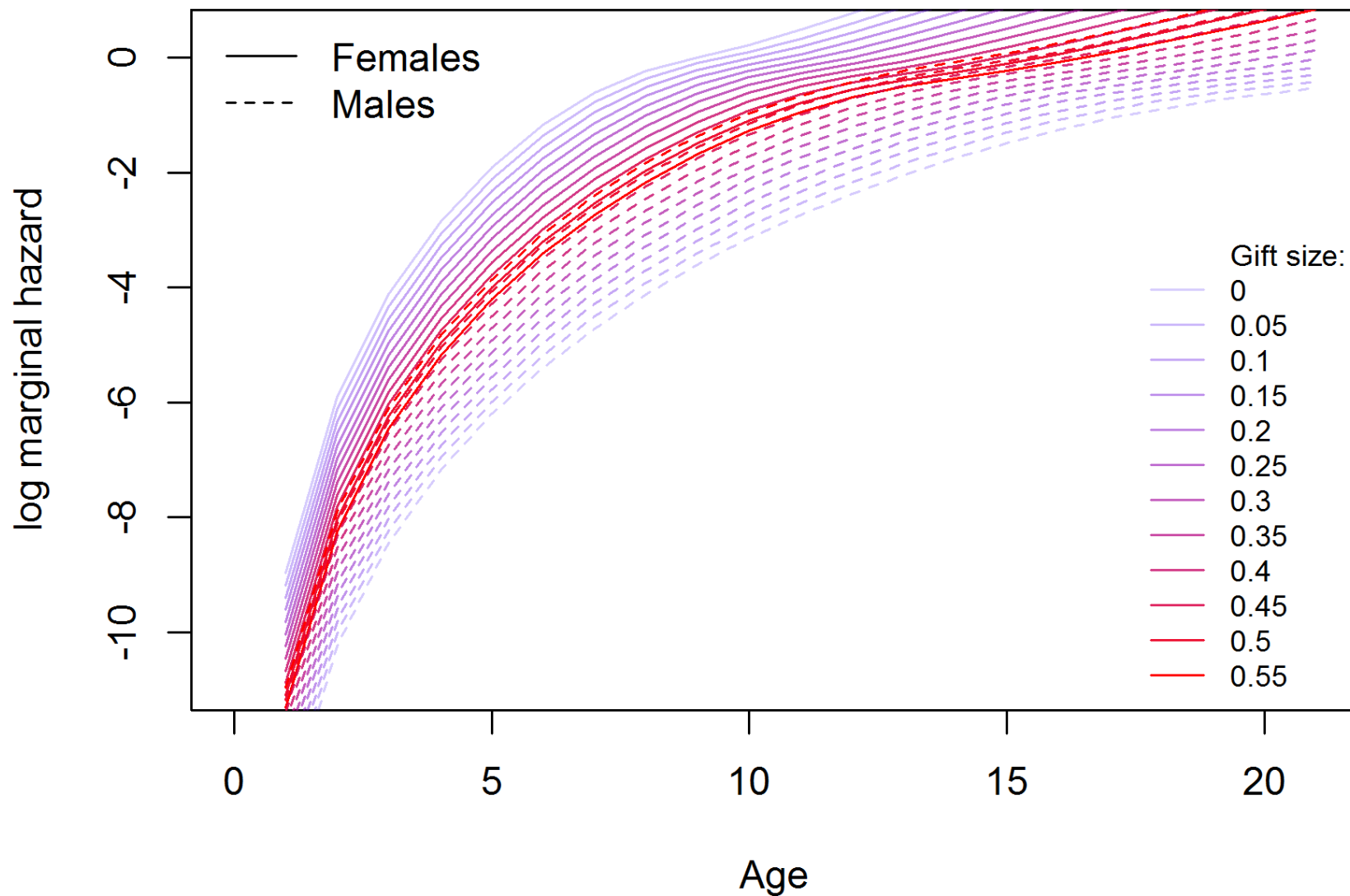
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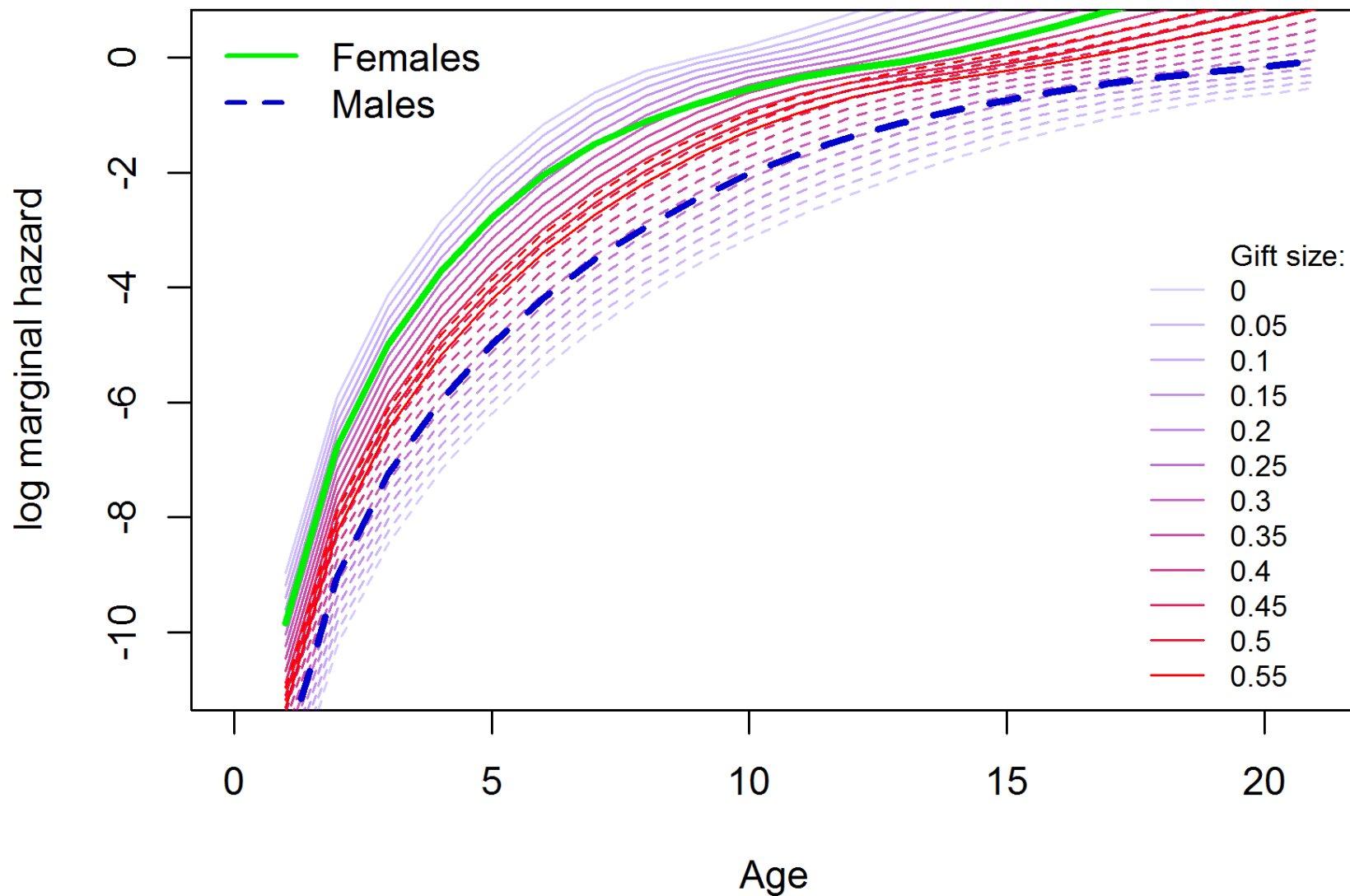
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# Summary

- The survival costs of reproduction are more pronounced for females than for males:
  - non-reproducing females live longer than non-reproducing males
  - Reproducing females live shorter than reproducing males
- Nuptial gifts increases substantially survival of the females at the costs of decreased survival of males.
- Bigger body mass is related to better survival in both males and females
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Is frailty term significant?

#1 Chisq=3.31, pval=0.0689

#2 Chisq=2.14, pval=0.1435