Analysis of the reporting styles using generalized ordered probit models: an introduction to *hopit* package

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Abstract

The *hopit* package provides R functions to fit and analyze ordered response data in the context of reporting styles. In this vignette we describe the formulation and fit of *hopit* models as well as functions used to analyse reporting styles.

1. Introduction

hopit is an open source software library written in the R (R-Core-Team 2018) and C++ (Bates and Eddelbuettel 2013; Eddelbuettel and François 2011) programming languages. The hopit package provides versatile methods to fit and analyze ordered response data in the context of self reporting styles.

The ordered response data classifies a measure of interest into ordered categories collected during a survey. For example, if the dependent variable were a happiness rating, then a respondent typically answers a question like: "Taking all things together, would you say you are ...;" and then selects from response options along the lines of:"very happy", "pretty happy", "not too happy", "very unhappy" (Liao, Fu, and Yi 2005). Similarly if interviewees are asked to evaluate their health in general (e.g. "Would you say your health is ...?") they may choose among several categories, such as "very good", "good", "fair", "bad", and "very bad" (King et al. 2004; Jürges 2007; Rebelo and Pereira 2014). In political sciences a respondent may be asked for an opinion about recent legislation (e.g. "Rate your feelings about the proposed legislation") and asked to choose among categories like: "strongly oppose", "mildly oppose", "indifferent", "mildly support", "strongly support" (Greene and Hensher 2010). It is easy to imagine other multi-level ordinal variables that might be used during a survey and to which the methodology described below could be applied to.

Practically, it is assumed that when responding to a survey question about their general happiness, health, feeling, attitude or other status, participants assess their true value of this unobserved continuous variable, and project it to a provided discrete scale. The thresholds that each individual uses to categorize their true status into a specific response option may be affected by the choice of a reference group, earlier life experiences, and cross-cultural differences in using scales, and thus, may differ across individuals depending on their gender, age, cultural background, education, and personality traits, among other factors.

From the reporting-styles modeling perspective, one of the main tasks is to compute this continuous estimate of individuals' underlying, latent measure based on several specific characteristics of the considered response (e.g. health variables or happiness variables) and accounting also for variations in reporting across socio-demographic and cultural groups. More specifically, to build the latent, underlying measure a generalized hierarchical ordered threshold model is fitted, which regresses the reported status/attitude/feeling on two sets of independent variables (Boes and Winkelmann 2006; Greene et al. 2014). When a dependent reported ordered variable is self-rated health status then the first set of variables - health variables - assesses individuals' specific aspects of health,

and might include chronic conditions, mobility level, difficulties with a range of daily activities, performance on grip strength test, anthropometric measures, lifestyle behaviors, etc. Using the second set of independent variables (threshold variables), the model also adjusts for the differences across socio-demographic and cultural groups like cultural background, gender, age, education, etc. (King et al. 2004; Jürges 2007; but see Rebelo and Pereira 2014).

Once the model is fitted, its estimates (latent measure and threshold coefficients) can be used to calculate the differences in reporting styles among groups of people having different contextual characteristics realized by calculation of differences between expected and reported ordinal response measures (Jürges 2007).

2. Generalized (hierarchical) ordered threshold model

Ordered threshold models are used with ordered categorical dependent variables. The generalized ordered threshold models (Ierza 1985; Boes and Winkelmann 2006; Greene et al. 2014) are an extension to the ordered threshold models (McKelvey and Zavoina 1975). In the latter models, the thresholds are constant, whereas generalized models allows thresholds to be dependent on covariates. Greene and Hensher (2010) and Greene et al. (2014) pointed out that also thresholds must be ordered so that a model has a sense. This motivated Greene and coauthors to call this models HOPIT, which stands for hierarchical ordered probit models.

In the self-rated health example, the response variable is self-rated health and latent measure h_i can depend on different health conditions and diseases (health variables X). Variables X are modeled with parallel regression assumption. According to the assumption, coefficients, which describe the relationship between lowest and all higher response categories, are the same as those coefficients, which describe the relationship between another (e.g. adjacent) lowest and the remaining higher response categories. In the considered case h_i is modeled as a linear function of X and their coefficients β :

$$h_i = \sum_{k=1}^K \beta_k X_{i,k} = X'\beta \tag{1}$$

where index $i \in 1...N$ is number of cases (e.g. respondents), X is in the form of model matrix, and K is number of columns in X. As described above, the categorization (response mechanism) of the latent measure h_i is modeled in terms of thresholds $\alpha_{i,j}$ assuming that thresholds of lower order are never greater than thresholds of higher orders (hierarchical assumption):

$$\begin{cases} y_{i} = 1 \iff \alpha_{i,0} \leq h_{i} < \alpha_{i,1} \\ y_{i} = 2 \iff \alpha_{i,1} \leq h_{i} < \alpha_{i,2} \\ \dots \\ y_{i} = j \iff \alpha_{i,j-1} \leq h_{i} < \alpha_{i,j} \\ \dots \\ y_{i} = J \iff \alpha_{i,J-1} \leq h_{i} < \alpha_{i,J} \end{cases}$$

$$(2)$$

The thresholds (cut points, α) are modeled by threshold variables Y, their coefficients γ , and intercepts λ . It is assumed that they model contextual characteristics of the respondent (e.g. country, gender, age, etc.). Threshold variables are modeled without parallel regression assumption, thus

each threshold is modeled by a variable independently (Boes and Winkelmann 2006; Greene et al. 2014).

Different parametrizations of thresholds exist (Greene et al. 2014; Rebelo and Pereira 2014; Jürges 2007). In the package, King et al. (2004) and Jürges (2007) parametrization is used, which assumes that:

$$\alpha_{i, j} = \begin{cases} -\infty & for \ j = 0\\ \lambda_{1} + \sum_{m=1}^{M} \gamma_{1,m} Y_{i,m} & for \ j = 1\\ \alpha_{i, j-1} + exp(\lambda_{j} + \sum_{m=1}^{M} \gamma_{j,m} Y_{i,m}) & for \ J - 1 \ge j \ge 2\\ \infty & for \ j = J \end{cases}$$
(3)

The condition $y_i = j \Leftrightarrow \alpha_{j-1,i} \leq h_i < \alpha_{j,i}$ can be easily expressed in terms of the probability, which leads to:

$$P(y_i = j) = P(\alpha_{j-1,i} \le h_i < \alpha_{j,i}), \tag{4}$$

hence

$$P(y_i = j) = \Phi(\alpha_{i, j} - h_i) - \Phi(\alpha_{i, j-1} - h_i), \tag{5}$$

where Φ is a distribution function (cdf, cumulative density function). For example, for probit regression it is standard normal cdf $\Phi(x) = \frac{1}{2} + \frac{1}{2} * erf\left(\frac{x}{\sqrt{2}}\right)$ whereas for logit regression it takes the form $\Phi(x) = \frac{1}{1+e^{-x}}$. In reporting styles analyses the typical choice is the probit model. It simply assumes that h_i is affected by a random noise ϵ_i having standard normal distribution $\epsilon_i \sim \mathcal{N}(0,1)$.

Using all definitions presented above the log likelihood function can be constructed

$$\ln L = \sum_{i=1}^{N} \sum_{j=1}^{J} z_{i, j} \ln \left[\Phi(\alpha_{i, j} - h_i) - \Phi(\alpha_{i, j-1} - h_i) \right], \tag{6}$$

where $z_{i,j}$ is an indicator function defined as:

$$z_{i, j} = \begin{cases} 0 & \text{for } y_i = j \\ 1 & \text{for } y_i \neq j \end{cases}$$
 (7)

3. Analysis of reporting styles

The model estimates are used to determine reporting behavior, i.e., how the continuous latent measure is projected onto the categorical response. Practically, this is done by comparing actual categorical ordered responses with theoretical ones that are adjusted for heterogeneity in reporting behaviors and are more comparable across individuals.

One of the first steps of the analysis is standardization of the latent measure to obtain latent index H_i .

$$H_i = 1 - \frac{h_i - \min_i h_i}{\max_i h_i - \min_i h_i} \tag{8}$$

In the self-rated health example H_i is a proxy for true underlying health of an individual, and varies from 0 representing the (model-based) worst health state to 1 representing the (model-based) best health in the sample.

The predicted latent measure h_i obtained from the model is also used to standardize latent variable coefficients. In the self-rated health example the standardized coefficients are called disability weights D_k (Jürges 2007) and are calculated for each health variable to provide information about the impact of a specific health measure on the latent index H_i . The disability weight for a health variable is equal to the ratio of corresponding health coefficient and the difference between the lowest and highest values of predicted latent health. In other words, disability weight reduces H_i by some given amount or percentage (i.e. of every individual is reduced by the same amount if heart attack or other heart problems are present)(Jürges 2007).

$$D_k = \frac{\beta_k}{\max_i h_i - \min_i h_i} \tag{9}$$

While the latent index H_i is intend to reflect underlying health, happiness or other status across individuals, the standardized coefficients D_k (e.g. disability weights), are computed for an average individual in the study population. The relation between H_i and D_k follows the equation:

$$H_i = C - \sum_{k=1}^K D_k X_{i,k}, \quad \text{where } C = \frac{\max_i h_i}{\max_i h_i - \min_i h_i}$$
 (10)

Reporting styles analysis is based on the reclassification of individuals into new response categories. There are two methods of reclassification: (1) Jürges (2007) percentile method (see also Rebelo and Pereira 2014) and (2) reclassification based on estimated thresholds.

In the first method, the classification is based on calculated latent index H_i and is thus adjusted for inter-individual differences in reporting behavior. The Jürges' percentile method is based on original distribution of categorical response variable. First for each category j an empirical distribution function is constructed.

$$\hat{F}(j) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{y_i} \le j \tag{11}$$

Where **1** is indicator function taking 1 if the condition is true or 0 otherwise. The calculated cumulative frequencies of latent index H_i are used as percentiles (cut points), so each individual i can be reclassified into new response categories.

In the second method the reclassification is based on eq. (2), so each individual has its own, model-derived cut-points.

4. Installing and loading the package

The newest available version of the package is always available from GitHub. It can be installed using devtools package

```
library(devtools)
install_github("maciejdanko/hopit")
```

library(hopit)

In examples presented below we will use *healthsurvey*, which is artificially generated data set inspired by WAVE1 SHARE database (DOIs: 10.6103/SHARE.w1.600) see Börsch-Supan et al. (2013) for technical details.

```
# load *healthsurvey* dataset
data(healthsurvey)

# horizontal view on the dataset (omitting ID)
print(t(healthsurvey[1:6,-1]), quote=FALSE, na.print='NA', right=TRUE)
```

##		1	2	3	4	5	6
##	health	Very good	Good	Good	Good	Excellent	Good
##	diabetes	no	no	yes	no	no	no
##	obese	no	no	no	no	no	no
##	IADL_problems	no	no	no	no	no	no
##	high_cholesterol	no	yes	no	no	no	yes
##	respiratory_problems	no	no	no	no	no	yes
##	poor_mobility	no	no	no	no	no	yes
##	very_poor_grip	no	no	no	no	no	no
##	depression	no	no	no	yes	no	no
##	other_diseases	yes	yes	no	no	no	yes
##	sex	man	man	man	man	woman	man
##	ageclass	80+	70-79	50-59	60-69	80+	+08
##	education	prim-	prim-	prim-	sec+	prim-	prim-
##	country	Y	Y	Х	Y	Z	Y
##	CSW	2407.48	1198.12	885.26	772.04	1304.24	917.16
##	psu	YB	YB	XC	YA	ZB	YD
##	ssu	NA	NA	XCgis	NA	NA	NA
##	hypertension	no	yes	no	no	no	yes
##	$heart_attack_or_stroke$	no	yes	no	no	no	no

The first variable on the list (health) is categorical self-reported health status. This variable is followed by 11 determinants of health, which includes information on presence of diseases and health conditions. The sex, ageclass, education, and country are variables describing contextual characteristics of individuals. The last type of variables (csw, psu, and ssu) describes the survey design.

5. Fitting the model using the hopit() function

Generalized ordered probit model can be fitted using the hopit() function. The function takes two kinds of formulas: (1) latent.formula that models the impact of latent variables on categorical health and (2) *thresh.formula that models thresholds.

```
# first determine the order of the dependent variable
levels(healthsurvey$health)
## [1] "Excellent" "Very good" "Good"
                                           "Fair"
                                                       "Poor"
# the order is decreasing (from best health to the worst health)
# so we set: decreasing.levels = TRUE
model1<- hopit(latent.formula = health ~ hypertension + high_cholesterol +</pre>
                             heart_attack_or_stroke + poor_mobility + very_poor_grip +
                             depression + respiratory_problems +
                             IADL_problems + obese + diabetes + other_diseases,
               thresh.formula = ~ sex + ageclass,
               decreasing.levels = TRUE,
               control=list(trace=FALSE),
               data = healthsurvey)
summary(model1)
## Formula (latent variables):
## health ~ hypertension + high_cholesterol + heart_attack_or_stroke +
       poor_mobility + very_poor_grip + depression + respiratory_problems +
##
       IADL_problems + obese + diabetes + other_diseases
## Formula (threshold variables): ~sex + ageclass
## Link: probit
## Number of cases: 10000
## Response levels: Excellent, Very good, Good, Fair, Poor
##
## Robust SE were used (sandwich estimator of varcov).
##
##
                             Estimate Std. Error z value Pr(>|z|)
                                                    7.76 8.4e-15 ***
## hypertensionyes
                              0.19232
                                         0.02478
## high_cholesterolyes
                              0.09780
                                         0.02918
                                                    3.35 0.00080 ***
## heart_attack_or_strokeyes 0.34401
                                         0.03183
                                                   10.81 < 2e-16 ***
## poor mobilityyes
                                                   20.44 < 2e-16 ***
                              0.72832
                                         0.03564
                                                    4.04 5.3e-05 ***
## very_poor_gripyes
                              0.49720
                                         0.12299
## depressionyes
                                                   10.59 < 2e-16 ***
                              0.25323
                                         0.02390
                                                   11.02 < 2e-16 ***
## respiratory_problemsyes
                              0.36777
                                         0.03337
## IADL_problemsyes
                                                  16.93 < 2e-16 ***
                              0.61579
                                         0.03637
                                                   5.76 8.3e-09 ***
## obeseyes
                              0.18991
                                         0.03295
## diabetesyes
                              0.33726
                                         0.04010
                                                    8.41 < 2e-16 ***
## other_diseasesyes
                                         0.02370
                                                   14.15 < 2e-16 ***
                              0.33533
## (L).1|2
                                                   -2.90 0.00379 **
                             -0.09248
                                         0.03194
## (L).2|3
                             -0.26826
                                         0.03236
                                                   -8.29 < 2e-16 ***
## (L).3|4
                                                    2.59 0.00968 **
                              0.07514
                                         0.02905
## (L).4|5
                                                  -3.90 9.8e-05 ***
                             -0.20346
                                         0.05222
## (G).sexwoman.1|2
                              0.02373
                                         0.03015
                                                    0.79 0.43112
## (G).sexwoman.2|3
                              0.01366
                                         0.03460
                                                    0.39 0.69304
## (G).sexwoman.3|4
                              0.03661
                                         0.02869
                                                   1.28 0.20192
```

```
## (G).sexwoman.4|5
                                         0.05039
                                                     2.35 0.01872 *
                              0.11848
## (G).ageclass60-69.1|2
                             -0.01835
                                         0.03383
                                                    -0.54 0.58763
## (G).ageclass60-69.2|3
                                                     1.31 0.18962
                              0.05336
                                         0.04068
## (G).ageclass60-69.3|4
                              0.06003
                                         0.03616
                                                     1.66 0.09693 .
## (G).ageclass60-69.4|5
                                                     2.59 0.00949 **
                              0.16842
                                         0.06492
## (G).ageclass70-79.1|2
                                                    -7.32
                             -0.32157
                                         0.04391
                                                           2.4e-13 ***
## (G).ageclass70-79.2|3
                              0.17131
                                         0.04774
                                                     3.59
                                                          0.00033 ***
## (G).ageclass70-79.3|4
                              0.19360
                                         0.03777
                                                     5.13
                                                           3.0e-07 ***
## (G).ageclass70-79.4|5
                              0.23234
                                         0.06653
                                                     3.49 0.00048 ***
## (G).ageclass80+.1|2
                             -0.33134
                                         0.07274
                                                    -4.56 5.2e-06 ***
## (G).ageclass80+.2|3
                                         0.07590
                                                     1.97 0.04848 *
                              0.14976
## (G).ageclass80+.3|4
                                                     3.55 0.00038 ***
                              0.17851
                                         0.05025
## (G).ageclass80+.4|5
                              0.22378
                                         0.07674
                                                     2.92 0.00354 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Theta: 1
## Log-likelihood: -12945.98
## Deviance: 25891.96
## AIC: 25953.96
can be accessed by coef() function
```

model1 contains 11 dichotomous health variables and two threshold variables. The fitted coefficient

```
# extract parameters in a form of list
cm1 <- coef(model1, aslist = TRUE)</pre>
# names of returned coefficients
names(cm1)
```

```
## [1] "latent.params" "thresh.lambda" "thresh.gamma"
                                                        "logTheta"
```

```
# extracting latent health coefficients
cm1$latent.params
```

```
##
             hypertensionyes
                                     high_cholesterolyes
##
                    0.1923166
                                               0.0978032
##
  heart_attack_or_strokeyes
                                        poor_mobilityyes
##
                    0.3440052
                                               0.7283236
##
           very_poor_gripyes
                                           depressionyes
##
                    0.4972017
                                               0.2532285
                                        IADL_problemsyes
     respiratory_problemsyes
##
##
                    0.3677676
                                               0.6157910
##
                     obeseves
                                             diabetesyes
##
                    0.1899097
                                               0.3372606
##
           other_diseasesyes
##
                    0.3353300
```

model1 can be further extended by adding country of origin to the threshold formula to control for cultural differences.

```
model2<- hopit(latent.formula = health ~ hypertension + high_cholesterol +</pre>
                      heart_attack_or_stroke + poor_mobility +
                      very poor grip + depression + respiratory problems +
                      IADL_problems + obese + diabetes + other_diseases,
               thresh.formula = ~ sex + ageclass + country,
               decreasing.levels = TRUE,
               control=list(trace=FALSE),
               data = healthsurvey)
```

```
The fit of both models can be compared using AIC() function:
AIC(model2, model1)
##
     model2
              model1
## 25154.19 25953.96
or using Likelihood Ratio Test (LRT) as models are nested:
anova (model2, model1)
## Full model:
## -- Formula (latent variables):
## health ~ hypertension + high_cholesterol + heart_attack_or_stroke +
       poor_mobility + very_poor_grip + depression + respiratory_problems +
##
       IADL_problems + obese + diabetes + other_diseases
## -- Formula (threshold variables): ~sex + ageclass + country
## -- Theta: FALSE
##
## Nested model:
## -- Formula (latent variables):
## health ~ hypertension + high_cholesterol + heart_attack_or_stroke +
       poor_mobility + very_poor_grip + depression + respiratory_problems +
##
       IADL_problems + obese + diabetes + other_diseases
## -- Formula (threshold variables): ~sex + ageclass
## -- Theta: FALSE
##
## Likelihood ratio test:
     Chi^2 df Pr(>Chi^2)
## 815.78 8
                <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Both latent.formula and thresh.formula allow the user to specify interactions, like interaction between
gender (sex) and age (ageclass):
model3<- hopit(latent.formula = health ~ hypertension + high_cholesterol +</pre>
                      heart_attack_or_stroke + poor_mobility +
                      very_poor_grip + depression + respiratory_problems +
                      IADL problems + obese + diabetes + other diseases,
               thresh.formula = ~ sex * ageclass + country,
```

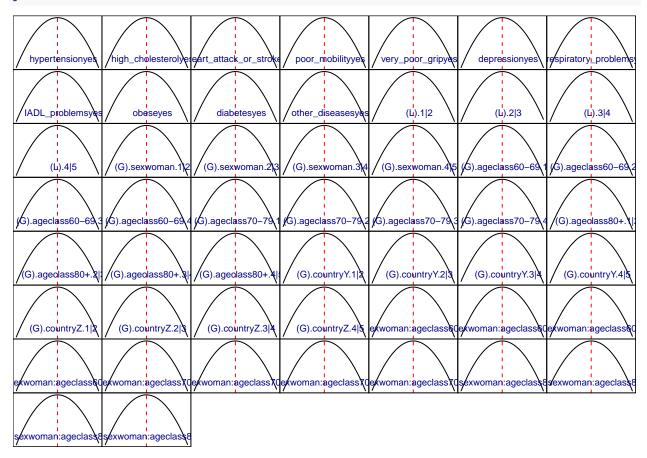
```
decreasing.levels = TRUE,
               control=list(trace=FALSE),
               data = healthsurvey)
print(anova(model3,model2), short=TRUE)
##
## Likelihood ratio test:
     Chi^2 df Pr(>Chi^2)
## 26.498 12 0.00912 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
The interactions between latent and threshold variables can be modeled using crossinter.formula:
model4<- hopit(latent.formula = health ~ hypertension + high_cholesterol +</pre>
                       heart_attack_or_stroke + poor_mobility +
                       very_poor_grip + depression + respiratory_problems +
                       IADL_problems + obese + diabetes + other_diseases,
               thresh.formula = ~ sex * ageclass + country,
               crossinter.formula = ~ sex : depression,
               decreasing.levels = TRUE,
               control=list(trace=FALSE),
               data = healthsurvey)
print(anova(model3,model4), short=TRUE)
##
## Likelihood ratio test:
## Chi^2 df Pr(>Chi^2)
## 0.043 1
                  0.835
The hopit() function has also an option to include survey design using the survey package. The
example below fit a model using simple two level cluster sampling design.
design <- svydesign(ids = ~ country + psu, weights = healthsurvey$csw,</pre>
                    data = healthsurvey)
```

Generally, ignoring survey design could lead to biased results. In the example presented here, it has low importance, which is seen by comparing coefficients of latent variable for both models:

##		No	survey	design	Has	survey	design
##	hypertensionyes		0.18	3475332		0.18	3777955
##	high_cholesterolyes		0.08	3972562		0.09	9368829
##	${\tt heart_attack_or_strokeyes}$		0.34	1659838		0.34	1676962
##	poor_mobilityyes		0.70	346456		0.70	0603471
##	very_poor_gripyes		0.53	1424418		0.54	1768793
##	depressionyes		0.24	1998274		0.24	1922297
##	respiratory_problemsyes		0.37	7863461		0.37	7984848
##	<pre>IADL_problemsyes</pre>		0.59	9262343		0.60	999389
##	obeseyes		0.19	9041874		0.18	3874130
##	diabetesyes		0.32	2839067		0.32	2328477
##	other_diseasesyes		0.32	2936970		0.32	2876106

The fit accuracy of the model can be assessed using profile() function, which calculate and plot profile of the log likelihood function around fitted coefficient values.

profile(model3)



All parameters seem to be at arg.max (at optimum).

6. Analyses of the reporting styles using hopit package

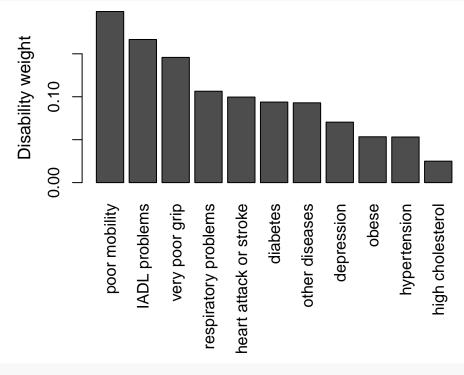
Let's look at latent health variables of model3.

model3\$coef.ls\$latent.params

```
##
                                     high_cholesterolyes
             hypertensionyes
                   0.18807939
##
                                               0.08845807
##
  heart_attack_or_strokeyes
                                        poor_mobilityyes
##
                   0.35272888
                                               0.70549450
##
           very_poor_gripyes
                                           depressionyes
##
                   0.51631806
                                               0.24936181
##
     respiratory_problemsyes
                                        IADL_problemsyes
##
                   0.37683709
                                               0.59016970
##
                     obeseyes
                                             diabetesyes
##
                   0.18900807
                                               0.33214901
##
           other_diseasesyes
##
                   0.32904701
```

We can standardize them using Jürges' approach (Jürges 2007) to obtain disability weights. The standardization can be done using standardizeCoef() function.

```
# A function that modifies coefficient names.
txtfun <- function(x) gsub('_',' ',substr(x,1,nchar(x)-3))
# Calcualte and plot disability weights
sc <- standardizeCoef(model3, plotf = TRUE, namesf = txtfun)</pre>
```



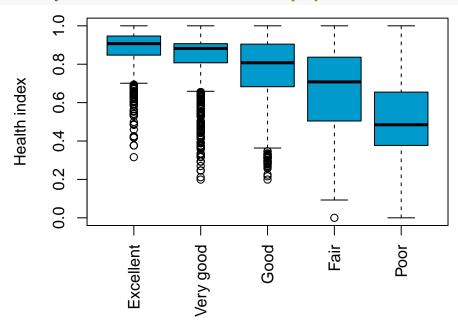
[,1]

sc

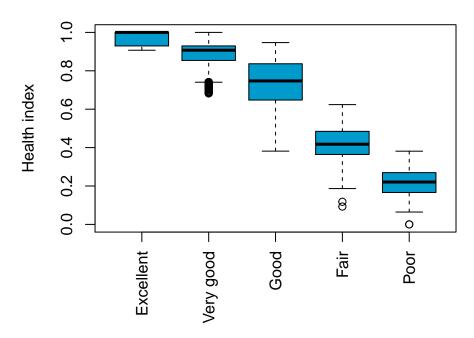
```
## poor mobility
                           0.19924639
## IADL problems
                           0.16667625
## very poor grip
                           0.14581901
## respiratory problems
                           0.10642667
## heart attack or stroke 0.09961801
## diabetes
                           0.09380582
## other diseases
                           0.09292975
## depression
                           0.07042499
## obese
                           0.05337983
## hypertension
                           0.05311755
## high cholesterol
                           0.02498241
```

The *namesf* argument is a function or a character vector that is used to rename the coefficient names. Here, it removes last 3 letters ("yes"), which is a reference level for each variable and exchanges "_" with spaces in variable names.

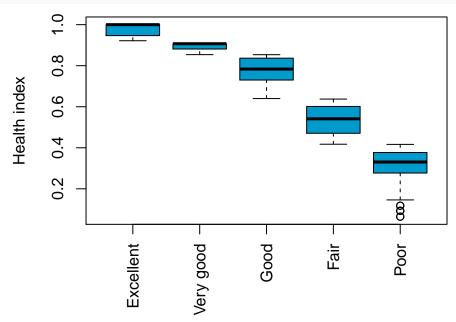
The latent index is simply calculated using *latentindex()* function.



The boxplot above shows reported health status vs. health index. It is also possible to plot expected categorical health status on Y axis calculated according to the eq. (2).

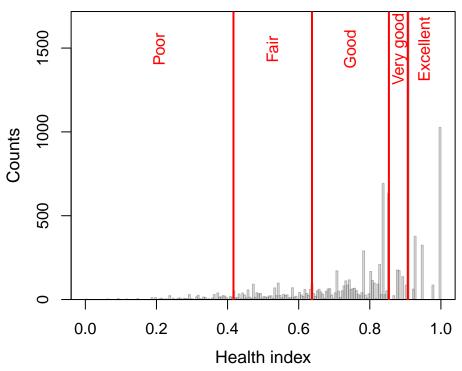


or according to Jürges (2007) method:

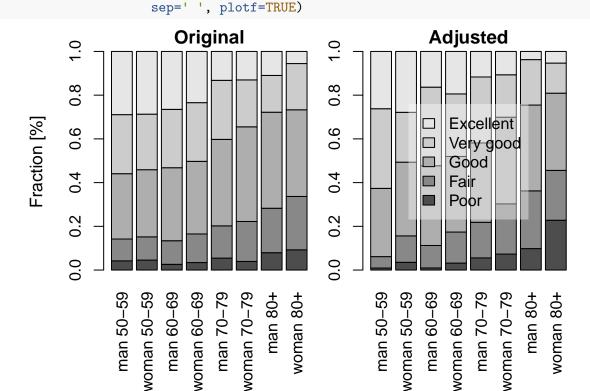


The central part of reporting styles analyses is to determine the cut-points used to calculate adjusted health status for each individual. The calculation and plotting of cut-points is realized by getCutPoints() function.

```
z <- getCutPoints(model=model3)</pre>
```



```
# Health index cut-points
z$cutpoints
       4.41%
                 17.68%
                           52.34%
##
                                      77.63%
## 0.4165472 0.6373989 0.8536904 0.9070702
# Adjusted health levels for individuals: Jürges method
rev(table(z$adjused.levels))
##
                                                   Poor
## Excellent Very good
                             Good
                                        Fair
                   2804
                             3506
                                                    444
##
        1876
                                        1368
# Original health levels for individuals
table(model3$y_i)
##
## Excellent Very good
                             Good
                                        Fair
                                                   Poor
##
        2237
                   2529
                             3466
                                        1327
                                                    441
# Adjusted health levels for individuals: Estimated model thresholds
table(model3$Ey_i)
##
## Excellent Very good
                             Good
                                        Fair
                                                   Poor
         734
                   4438
                             3959
                                         805
##
                                                     64
The analysis of health levels is done by qetLevels() function
# Health levels for combination of age and gender, and pooled country of origin.
hl <- getLevels(model=model3, formula=~ sex + ageclass, data = healthsurvey,</pre>
```



The differences between original and adjusted frequencies can be calculated directly using getLevels output:

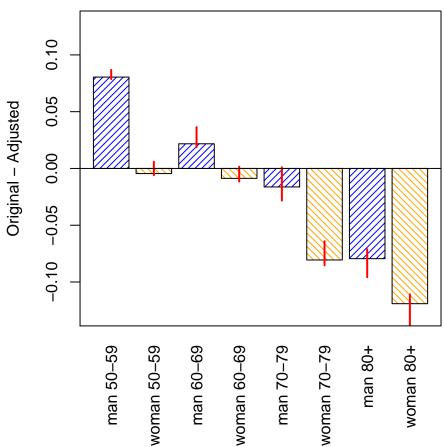
```
round(100*(hl$original - hl$adjusted),2)
##
##
                     Poor
                             Fair
                                    Good Very good Excellent
##
     man 50-59
                     3.28
                             4.77
                                   -1.32
                                              -9.43
                                                           2.70
##
     woman 50-59
                           -1.47
                                   -2.99
                                               2.60
                                                           0.85
                     1.01
##
     man 60-69
                     1.66
                            0.51
                                   -3.07
                                              -9.26
                                                          10.15
##
     woman 60-69
                     0.25
                           -1.12
                                   -1.37
                                              -1.74
                                                           3.99
     man 70-79
                           -1.53
                                              -3.16
                                                           1.53
##
                    -0.10
                                    3.26
##
     woman 70-79
                    -3.37
                           -4.69
                                    3.66
                                               2.06
                                                           2.34
                           -6.07
                                              -3.97
                                                           7.24
##
     man 80+
                    -1.87
                                    4.67
##
     woman 80+
                   -13.55
                             1.64
                                    4.31
                                               7.39
                                                           0.21
```

7. Bootstrapping Confidence Intervals

The package offers functions to calculate confidence intervals for any measure derived from the model. As an example, we show calculation of confidence intervals of the difference between original and adjusted frequencies of combined "Poor" + "Fair" health categories.

```
# the function to be bootstraped
diff_BadHealth <- function(model, data) {
  hl <- getLevels(model=model, formula=~ sex + ageclass, data = data,</pre>
```

```
sep=' ', plotf=FALSE)
  hl$original[,1] + hl$original[,2] - hl$adjusted[,1]- hl$adjusted[,2]
}
# estimate the difference
est.org <- diff_BadHealth(model = model3, data = healthsurvey)</pre>
# perform the bootstrap
B <- boot_hopit(model = model3, data = healthsurvey,</pre>
                 func = diff_BadHealth, nboot = 100)
# calculate lower and upper bounds using percentile method
est.CI <- boot_hopit_CI(B)</pre>
# plot the difference and its (assymetrical) confidence intervals
pmar \leftarrow par('mar'); par(mar = c(9.5, pmar[2:4]))
m <- max(abs(est.CI))</pre>
pos <- barplot(est.org, names.arg = names(est.org), las = 3, ylab = 'Original - Adjusted',
                ylim=c(-m, m), density = 20, angle = c(45, -45), col = c('blue', 'orange'))
for (k \text{ in seq\_along(pos)}) \text{ lines}(c(pos[k,1],pos[k,1]), est.CI[,k], lwd = 2, col = 2)
abline(h = 0); box(); par(mar = pmar)
```



The results show that men tend to over-report bad health at ages (50,60] and (50,70], whereas women at ages [70,80) and both sexes at ages (80, 120] under-report bad health.

8. References

Bates, Douglas, and Dirk Eddelbuettel. 2013. "Fast and Elegant Numerical Linear Algebra Using the RcppEigen Package." *Journal of Statistical Software* 52 (5): 1–24. http://www.jstatsoft.org/v52/i05/.

Boes, Stefan, and Rainer Winkelmann. 2006. "Ordered Response Models." Allgemeines Statistisches Archiv 90 (1): 167–81. https://doi.org/10.1007/s10182-006-0228-y.

Börsch-Supan, Axel, Martina Brandt, Christian Hunkler, Thorsten Kneip, Julie Korbmacher, Frederic Malter, Barbara Schaan, Stephanie Stuck, and Sabrina Zuber. 2013. "Data Resource Profile: The Survey of Health, Ageing and Retirement in Europe (Share)." *International Journal of Epidemiology* 42 (4): 992–1001. https://doi.org/10.1093/ije/dyt088.

Eddelbuettel, Dirk, and Romain François. 2011. "Rcpp: Seamless R and C++ Integration." *Journal of Statistical Software* 40 (8): 1–18. https://doi.org/10.18637/jss.v040.i08.

Greene, William, Mark N. Harris, Bruce Hollingsworth, and Timothy A. Weterings. 2014. "Heterogeneity in Ordered Choice Models: A Review with Applications to Self-Assessed Health." *Journal of Economic Surveys* 28 (1): 109–33. https://doi.org/10.1111/joes.12002.

Greene, William, and David Hensher. 2010. *Modeling Ordered Choices*. Cambridge University Press. https://EconPapers.repec.org/RePEc:cup:cbooks:9780521142373.

Ierza, Joseph V. 1985. "Ordinal Probit: A Generalization." Communications in Statistics - Theory and Methods 14 (1). Taylor & Francis: 1–11. https://doi.org/10.1080/03610928508828893.

Jürges, Hendrik. 2007. "True Health Vs Response Styles: Exploring Cross-Country Differences in Self-Reported Health." *Health Economics* 16 (2): 163–78. https://doi.org/10.1002/hec.1134.

King, Gary, Christopher J. L. Murray, Joshua A. Salomon, and Ajay Tandon. 2004. "Enhancing the Validity and Cross-Cultural Comparability of Measurement in Survey Research." *American Political Science Review* 98 (1). Cambridge University Press: 191–207. https://doi.org/10.1017/S000305540400108X.

Liao, Pei-Shan, Yang-Chih Fu, and Chin-Chun Yi. 2005. "Perceived Quality of Life in Taiwan and Hong Kong: An Intra-Culture Comparison." *Journal of Happiness Studies* 6 (1): 43–67. https://doi.org/10.1007/s10902-004-1753-6.

McKelvey, R. D., and W. Zavoina. 1975. "A Statistical Model for the Analysis of Ordinal Level Dependent Variables." *Journal of Mathematical Sociology* 4 (1): 103–20.

R-Core-Team. 2018. "R: A Language and Environment for Statistical Computing [Internet]. Vienna, Austria. R Version 3.4.4 (2018-03-15)." https://www.r-project.org.

Rebelo, Luis Pina, and Nuno Sousa Pereira. 2014. "Assessing Health Endowment, Access and Choice Determinants: Impact on Retired Europeans' (in)activity and Quality of Life." Social Indicators Research 119 (3). Germany: Springer: 1411–46. https://doi.org/10.1007/s11205-013-0542-1.