1 Problem 1.

2 Problem 2.

Since X is pof, it is orbit-finite. Since given relation is equivariant, each orbit of X is either entirely contained within one equivalence class or each element of the orbit is in a different equivalence class.

If the first case holds for every orbit, then every equivalence class is a union of orbits, therefore:

- There are finitely many equivalence classes.
- Each equivalence class is equivariant.

If the second case holds for at least one orbit, than:

- There are infinitely many equivalence classes (because an orbit has one or infinitely many elements and if it had only one, the first case would hold).
- Not every equivalence class is equivariant, because elements of the same orbit are by definition equivariantly indistinguishable.

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3 Problem 3.

Note that such function f exists if and only if every orbit (orbit generating element) of X is in a relation with an oribt of Y that does not introduce new atoms (ones that were not used in the X's orbit generating element).

Proof: If for every relation of given orbit of X (O) it would introduce new atoms, than f would have to create new atoms, and since it has to be equivariant we could only take all atoms or none of them, meaning $f(O)$ doesn't exist or has infinitely many possible values, therefore f is not a function.

With that we iterate over all orbits of $X \times Y$ ignoring the ones that are not in R (R is an equivariant subset of $X \times Y$) and if the given orbit (let's call the X part O_x and the Y part O_y) satisfies the condition that O_y does not introduce new atoms, we mark O_x as an orbit for which f exists.

Lastly, we check for every orbit of X if f exists for it. Described algorithm will terminate because we operate on pof sets, which are orbit-finite.

T.H.M.W.