Project 2 Monte Carlo

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Problem description

We are interested in estimating the following (called an option, with discounted payoff at time 1) with price given by the formula

$$I = e^{-r} E\left(A_n - K\right)_+,$$

where

$$A_n = \frac{1}{n}\sum_{i=1}^{n} S(i/n),$$

and

$$S(t) = S(0)\exp\left(\mu^* t + \sigma B(t)\right), \quad 0 \le t \le T$$

where $B(t)(0 \le t \le T)$ is Brownian motion.

TODO: extend interpretation by my own comments.

European and Asian option

In the case n = 1, this is called a European call option; otherwise, it is called an Asian call option.

Used methods

1. Crude Monte Carlo estimator
2. Stratified estimator

Monte Carlo description

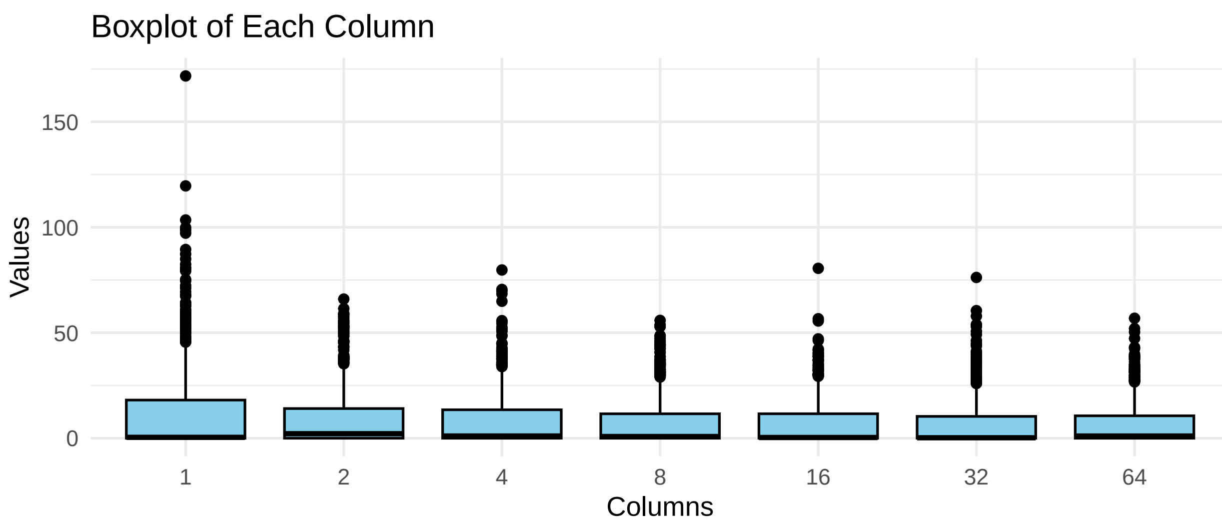
TO DO Add brownian motion description.

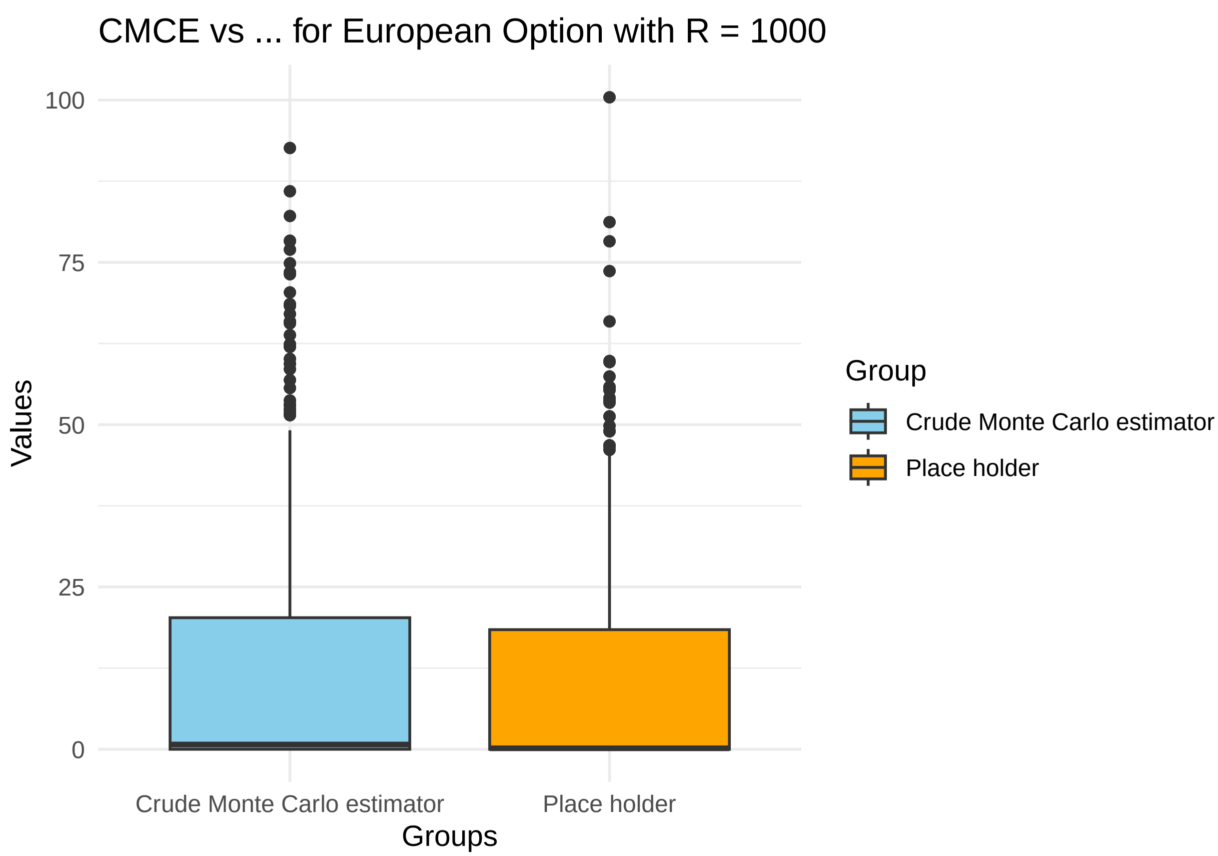
First we need to generate Brownian Motion n points, equally spaced sample on $[0, 1]$. We will use the fact that $\mathbf{B} = (B(1/n), B(2/n), \dots, B(1))$ is a multivariate normal random variable $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$ with the covariance matrix

$$\mathbf{\Sigma}(i, j) = \frac{1}{n}\min(i, j).$$

Table 1: MC estimators values for R=1000

1	2	4	8	16	32	64
11.682	8.946	8.156	7.182	6.808	6.686	6.651





Theoretical calculation for European option using Black-Scholes formula

We can compare the simulation results for European option with theoretical calulation.

$$E(S(1)-K)_+ = S(0)\Phi\left(d_1\right) - Ke^{-r}\Phi\left(d_2\right),$$

where

$$d_1 = \frac{1}{\sigma}\left[\log\left(\frac{S(0)}{K}\right) + r + \frac{\sigma^2}{2}\right],$$

and

$$d_2 = d_1 - \sigma.$$

Using our setup parameters we obtain value 11.7343652. It is alaign with the MC simulation result. Even using $R = 1000$ replication we have quite bias simulation.