In the vote on the amendment, the Republicans and the Northern Democrats were in the majority, thereby substituting the amended bill for the original. In the vote on the amended bill, the Republicans and the Southern Democrats were in the majority, and the amended bill was defeated. However, before being amended the original bill had a majority of the votes!

36.9 The Vickrey-Clarke-Groves Mechanism

Let us think about the public good problem in a very general framework. The goal is to choose some outcome (for example, whether or not to provide a streetlight) so as to maximize the sum of utilities of the agents involved. The challenge is to determine just what those individual utility functions are, since consumers may not have good incentives to report true values.

In the simplest case the choice might be a zero-one decision: if $x = 1$ the streetlight is built, if $x = 0$ it is not. In a more general case, the choice might be how much of something to provide—how many streetlights, or how bright they are, or where they are located. We will use x to represent the possible choices, whatever they may be. We suppose that there are n agents, and let $u_i(x)$ be the utility of agent i. The goal is to choose x to maximize the sum of the agents' utility, $\sum_i u_i(x)$.

This would be easy if the decision maker knew the utility functions. Unfortunately, in any realistic situation, the decision maker won't know this. And, as we've seen, the agents may well have an incentive to misrepresent their true utility functions.

Somewhat surprisingly, there is a clever way to get the agents to tell the truth and achieve an efficient outcome. This **economic mechanism** is known as the **Vickrey-Clarke-Groves mechanism**, or **VCG mechanism**.

Groves Mechanism

We will describe the VCG mechanism in two stages. First, we describe what is known as a **Groves mechanism**.

1. The center asks each agent i to report how much he is willing to pay to have x units of the public good provided. We denote this reported utility for x units of the public good by $r_i(x)$.

2. The center chooses the level of the public good x^* that maximizes the sum of the reported utilities, $R = \sum_{i=1}^{n} r_i(x)$.

3. Each agent i receives a sidepayment which is the sum of the reported utilities of everyone else, evaluated at the level of x determined in step 2. Denote this sidepayment by $R_i = \sum_{j \neq i} r_j(x^*)$.

It turns out that in this mechanism it is a **dominant strategy** for each agent to report his true utility function. To see why, consider the total payoff to agent i, which is his utility plus his sidepayment

$$u_i(x) + \sum_{j \neq i} r_j(x).$$

Note that agent i cares about his *true* utility function but his sidepayment depends on the sum of the others' *reported* utility functions.

Agent i recognizes that the decision maker will maximize the sum of utilities using his *reported* utility,

$$r_i(x) + \sum_{j \neq i} r_j(x).$$

However, agent i wants the decision maker to maximize his own true utility plus sidepayment,

$$u_i(x) + \sum_{j \neq i} r_j(x).$$

Agent i can ensure that the decision maker makes a choice that will maximize this expression by reporting his true utility; that is, by setting $r_i(x) = u_i(x)$.

The Groves mechanism essentially "internalizes the externality" among the agents. It makes each agent face the costs and benefits that his report imposes on the other agents. Each agent wants to report his true utility, since that is what he wants to be maximized.

The VCG Mechanism

The trouble with the Groves mechanism alone is it is potentially very costly: the center has to pay every agent an amount equal to the sum of the others' reported utilities. How can the magnitude of the sidepayments be reduced?

One important observation is that we can impose a "tax" on each agent as long as this tax is independent of the agent's choice. If the tax is independent of i's choice, then it can't affect his decision.[3] We will choose the tax in a way that guarantees that the net payments the center will receive are either positive or zero. Thus the center will always have at least as much money as necessary to pay for the public good.

A particularly convenient tax is to charge agent i an amount equal to the *maximum* sum of reported utilities excluding agent i. That is, we charge

[3] This is where the quasilinear assumption about utility is important.

each agent the sum of the reported utilities that would occur if he were not present. The net tax imposed on agent i is then

$$W_i - R_i = \sum_{j \neq i} r_j(x) - \max_z \sum_{j \neq i} r_j(z).$$

Note that this number is either positive or zero. Why? Because the *maximum* sum of the $n-1$ reported utilities has to be larger than any other value for that sum.

What we are computing here is the difference between what would happen with agent i present, and what would happen with him absent. Thus it measures the net cost that agent i imposes on the other agents. As long as i faces the cost that he imposes on the other agents, he will have appropriate incentives to report his true utility.

Now we can complete the description of the VCG mechanism. We use steps 1 and 2 above, but then substitute the following steps for step 3 above.

3. The center also calculates the outcome that maximizes the sum of the $n-1$ reported utilities if agent $1, 2, \ldots, n$ were not present. Let W_i be the maximum sum of reported utilities that results without agent i.

4. Each agent i pays a tax equal to $W_i - R_i$.

36.10 Examples of VCG

The discussion in the last section was admittedly abstract, so it is helpful to examine some specific cases.

Vickrey Auction

The first case we look at is the **Vickrey auction**, as described in Chapter 17. Here the outcome is simple: which person should get the item being auctioned. Let $v_1 > v_2$ be the true values of two bidders and $r_1 > r_2$ be the reported values.

If agent 1 is present, he gets a utility of v_1. If he is absent, the item is awarded to the other agent so agent 1's total payoff is $v_1 - r_2$. Agent 2 gets a payoff of zero no matter what. Each agent has an incentive to report its true value, so we end up with the optimal outcome.

Clarke-Groves Mechanism

The next example is a public goods problem along the lines of the TV-buying game described in Table 36.1. As in that example, suppose that there are two roommates who are trying to decide whether they will buy a TV. Let c_i be how much agent i will pay if the TV is purchased. Since the total cost of the TV is \$150, we must have $c_1 + c_2 = 150$.

According to the VCG mechanism, each agent reports a value for the TV, denoted by r_i. If $r_1 + r_2 > 150$ the TV will be purchased and the agents will make the payments according to the mechanism. Let $x = 1$ if the TV is purchased and $x = 0$ if it is not.

Before we look at the VCG mechanism, let us think about what would happen if we followed a naive mechanism: ask each agent to report his value and then acquire the TV if the sum of the reported values exceeds the cost of the TV.

Suppose person 1's value exceeds his cost share, so that $v_1 - c_1 > 0$. Then person 1 may as well report a million dollars; this will ensure the TV get purchased, which is what he wants to see. On the other hand if $v_1 < c_1$ person 1 may as well report a negative million dollars.

The problem is that each agent, acting independently, has no reason to take into account the other agent's values. The agents have a strong incentive to exaggerate their reported values one way or the other.

Let's see how the VCG mechanism solves this problem. The payoff to agent 1 is

$$(v_1 - c_1)x + (r_2 - c_2)x - max_y(r_2 - c_2)y.$$

The first term is his net utility from the TV: the value to him minus the cost he has to pay. The second term is the reported net utility to his roommate. The last term is the maximum utility his roommate would get if agent 1 were not present. Since agent 1 can't influence this, we can just ignore it for now.

Rearranging the first 2 terms we have agent's payoff as

$$[(v_1 + r_2) - (c_1 + c_2)]x.$$

If this is positive then he can ensure the TV is purchased if he reports $r_1 = v_1$, since then the sum of the *reported* values will exceed the total cost. If this is negative he can ensure that the TV is not purchased by reporting $r_1 = v_1$. Either way, it is optimal to report the true value. The same thing is true for agent 2. If both report the truth, the TV will be purchased only when $v_1 + v_2 > 150$, which is the optimal thing to do.

Note that agent i will have to make a payment only if he changes the social decision. In this case we say agent i is **pivotal**. The amount of the payment a pivotal agent makes is simply the cost that he imposes on the other agents.

36.11 Problems with the VCG

The VCG mechanism leads to truthtelling and leads to the optimal level of the public good. However, it is not without problems.

The first problem is that it only works with quasilinear preferences. This is because we can't have the amount that you have to pay influence your demand for the public good. It is important that there is a unique optimal level of the public good.

The second problem is that the VCG mechanism doesn't really generate a Pareto efficient outcome. The level of the public good will be optimal, but the private consumption could be greater. This is because of the tax collection. Remember that in order to have the correct incentives, the pivotal people must actually pay some taxes that reflect the harm that they do to the other people. And these taxes cannot go to anybody else involved in the decision process, since that might affect their decisions. The taxes have to disappear from the system. And that's the problem—if the taxes actually have to be paid, the private consumption will end up being lower than it could be otherwise, and therefore be Pareto inefficient.

However, the taxes only have to be paid if someone is pivotal. If there are many people involved in the decision, the probability that any one person is pivotal may not be very large; thus the tax collections might typically be expected to be rather small.

A third problem with VCG is that it is susceptible to collusion. Consider, for example, the public goods problem described above. Suppose that there are 3 roommates participating in the TV auction, but two of them collude. The colluders agree to each state \$1 million as their net benefit from the TV. This ensures that the TV will be purchased but since neither of the agents is pivotal (i.e., neither of the colluding agents changed the decision) then neither one has to pay the tax.

The final problem concerns the equity and efficiency tradeoff inherent in the VCG mechanism. Since the payment scheme must be fixed in advance, there will generally be situations where some people will be made worse off by providing the public good, even though the Pareto efficient *amount* of the public good will be provided. To say that it is Pareto efficient to provide the public good is to say that there is *some* payment scheme under which everyone is better off having the public good provided than not having it. But this doesn't mean that for an *arbitrary* payment scheme everyone will be better off. The Clarke tax ensures that if everyone *could* be better off having the good provided, then it will be provided. But that doesn't imply that everyone will actually be better off.

It would be nice if there were a scheme that determined not only whether or not to provide the public good, but also a Pareto efficient way to pay for it—that is, a payment plan that makes everyone better off. However, it does not appear that such a general plan is available.

Summary

1. Public goods are goods for which everyone must "consume" the same