1
Introduction

> Under normal conditions the research scientist is not an innovator but a solver of puzzles, and the puzzles upon which he concentrates are just those which he believes can be both stated and solved within the existing scientific tradition.
>
> – *Thomas Kuhn, The Essential Tension, 1977.*

Quantum theory has been puzzling physicists and philosophers since its birth in the early 20th century. However, starting in the 1980s, rather than asking why quantum theory is so weird, many people started to ask the question:

What can we do with quantum weirdness?

In this book we not only embrace this perspective shift, but challenge the quantum icons even more. We contend that one should not only change the kinds of questions we ask about quantum theory, but also:

change the very language we use to discuss it!

Before meeting this challenge head-on, we will tell a short tale to demonstrate how the quantum world defies conventional intuitions ...

1.1 The Penguins and the Polar Bear

Quantum theory is about very special kinds of physical systems – often very small systems – and the ways in which their behaviour differs from what we observe in everyday life. Typical examples of physical systems obeying quantum theory are microscopic particles such as photons and electrons. We will ignore these for the moment, and begin by considering a more 'feathered' quantum system. This is Dave:

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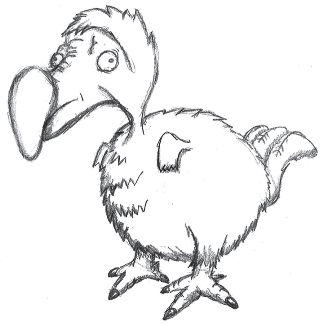

He's a dodo. Not your typical run-of-the-mill dodo, but a *quantum dodo*. We will assume that Dave behaves in the same manner as the smallest non-trivial quantum system, a two-level system, which these days gets referred to as a quantum bit, or *qubit*. Let's compare Dave's state to the state of his classical counterpart, the *bit*. Bits form the building blocks of classical computers, whereas (we will see that) qubits form the building blocks of quantum computers. A bit:

1. admits two states, which we tend to label 0 and 1,
2. can be subjected to any function, and
3. can be freely read.

Here, 'can be subjected to any function' means that we can apply any function on a bit to change its state. For example, we can apply the 'NOT' function to a bit, which interchanges the states 0 and 1, or the 'constant 0' function which sends any state to 0. What we mean by 'can be freely read' is that we can read the state of any bit in a computer's memory without any kind of obstruction and without changing that state.

The fact that we even mention all of this may sound a bit odd...until we compare this to the quantum analogue. A qubit:

1. admits an entire sphere of states,
2. can only be subjected to rotations of the sphere, and
3. can only be accessed by special processes called *quantum measurements*, which only provide limited access, and are moreover extremely invasive.

The set of states a system can occupy is called the *state space* of that system. For classical bits, this state space contains just two states, whereas a qubit can be in infinitely-many states, which we can visualise as a sphere. In the context of quantum theory, this state space is called the *Bloch sphere*. For the sake of explanation, any sphere will do, so we'll just take the Earth. There's plenty of space on Earth for two states of a bit, so put 0 on the North Pole and 1 on the South Pole:

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The particular choice of North Pole/South Pole is not important, but it is important that they are *antipodal* points on the sphere.

Since we can only apply rotations to the sphere of qubit-states, we cannot map both 0 and 1 to 0 (as we could with classical bits), simply because there is no rotation that does that. On the other hand, there are lots of ways to interchange 0 and 1, since there are many (different!) rotations that will turn a sphere upside-down.

So what are quantum measurements? Just like when we read a normal bit, measuring a qubit will produce one of two answers (e.g. 0 or 1, hence the name qubit). However, this act of 'measuring' is not quite as innocent as simply reading a bit to get its value. To get a feel for this, we return to Dave. Since qubits can live anywhere in the world, Dave – like one particularly famous (classical) dodo – lives in Oxford:

Now, suppose we wish to ascertain where in the world certain animals live, subject to the following assumptions:

1. we are only allowed to ask whether an animal lives at a specific location on Earth or its antipodal location;
2. all animals can talk and will always answer 'correctly'; and
3. predatory animals will refrain from eating the questioner.

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If we ask a polar bear whether she lives at the North Pole or the South Pole, then she'll say 'the North Pole'. If we ask again, she'll say 'the North Pole' again, because that's just where polar bears are from. Similarly, if we ask a penguin, he'll keep saying 'the South Pole', as long as we keep asking.

On the other hand, what will Dave say if we ask him whether he lives at the North Pole or the South Pole? Now, Dave doesn't really understand the question, but since dodos are a bit thick, he'll give an answer anyway. However, assumption 2 was that all animals will answer correctly. Consequently, as soon as Dave says 'the North Pole', his statement is correct: he actually is at the North Pole!

Now, if we ask him again, he'll say 'the North Pole' again, and he'll keep answering thus until he's eaten by a polar bear (Fig. 1.1). Alternatively, if he had initially said 'the South Pole', he would immediately have been at the South Pole.

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Figure 1.1 A polar bear attempting a 'demolition measurement' on Dave.

So, no matter what answer Dave gives, his state has changed. The fact that he was originally in Oxford is permanently lost. This phenomenon, known as the *collapse* of the quantum state, happens for almost all questions (i.e. measurements) we might perform. Crucially, this collapse is almost always *non-deterministic*. We almost never know until we measure Dave whether he'll be at the North Pole or the South Pole. We say 'almost', because there is one exception: if we ask whether Dave is in Oxford or the Antipodes Islands, he'll say 'Oxford' and stay put.

While quantum theory cannot predict with certainty the fate of Dave, what it does provide are the *probabilities* for Dave to either collapse to the North Pole or to the South Pole. In this case, quantum theory will tell us that Dave is more likely to go to the North Pole and get eaten by a polar bear than to go to the South Pole and chill with some penguins. The dodo is extinct for a reason after all ...

1.2 So What's New?

Almost a century has passed since Dave's unfortunate travels to the North Pole. In particular, the past two decades have seen a humongous surge in new kinds of research surrounding quantum theory, ranging from re-considering basic concepts (Fig. 1.2) to envisioning radically new technologies. A paradigmatic example is *quantum teleportation*, whereby the non-local features of quantum theory are exploited to send a quantum state across (sometimes) great distances, using nothing but a little bit (actually two little bits ...) of classical communication. Quantum teleportation exposes a delicate interaction between quantum theory and the structure of spacetime at the most fundamental level. At the same time, it is also a template for an important quantum computational model (measurement-based quantum computing), as well as a component in many quantum communication protocols.

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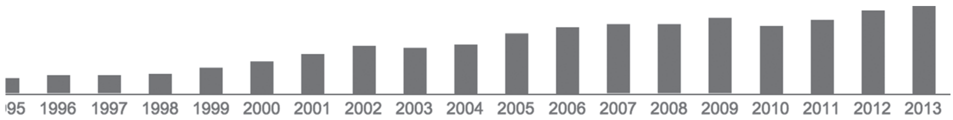

Figure 1.2 The paper by Einstein, Podolsky, and Rosen, which was the first to identify quantum non-locality, has enjoyed a huge surge in citations over the past two decades according to Google Scholar, now making it Albert Einstein's most cited paper. And considering the competition, that's saying something.

Quantum theory as we now know it – that is to say, its formulation in terms of *Hilbert spaces* – first saw daylight in 1932 with John von Neumann's book *Mathematische Grundlagen der Quantenmechanik*. On the other hand, quantum teleportation was only discovered in 1992. Hence the question:

Why did it take 60 years for quantum teleportation to be discovered?

A first explanation is that within the tradition of physics research during those 60 years, the question of whether something like quantum teleportation would be possible was simply never asked. It only became apparent when researchers stepped outside the existing scientific tradition and asked a seemingly bizarre question:

What are the information processing features of quantum theory?

However, one could go a step further and ask why it was even necessary to first pose such a question for teleportation to be discovered. Why wasn't it plainly obvious that quantum theory allowed for quantum teleportation, in the same way that it is plainly obvious that hammers are capable of hitting nails? Our answer to this question is that the traditional language of Hilbert spaces just isn't very good at exposing many of the features of quantum theory, and in particular, those features such as teleportation that involve the interaction of multiple systems across time and space. Thus, we pose a new question:

What is the most appropriate language to reason about quantum theory?

The answer to this question is what this book is all about. The reader will learn about many important new quantum features that rose to prominence within the emerging fields of quantum computation, quantum information, and quantum technologies, and how these developments went hand-in-hand with a revival of research into the foundations of quantum theory. All of this will be done by using a novel presentation of quantum theory in a purely diagrammatic manner. This not only consists of developing a two-dimensional notation for describing and reasoning about quantum processes, but also of a unique methodology that treats quantum processes, and most importantly *compositions* of processes, as first-class citizens.

1.2.1 A New Attitude to Quantum Theory: 'Features'

Since its inception, many prominent thinkers were deeply unsettled by quantum theory. A great deal of effort and ingenious mathematics in the early twentieth century went

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into demonstrating the *bugs* in quantum theory, starting with the now famous EPR paper by Einstein, Podolsky, and Rosen (EPR) in 1935, which claimed that the quantum state provided an 'incomplete description' of physical reality. Roughly speaking, they claimed that something must be missing in order to make sense of quantum theory in a manner compatible with our conventional intuitions. However, John Bell showed in 1964 that any attempt to 'complete' quantum theory to EPR's standards was doomed to failure and thereby binned our conventional intuitions as far as quantum theory is concerned. Bell showed that quantum theory contains at its heart a fundamental, irreducible non-locality (Fig. 1.3).

While relativity theory led Einstein to a beautiful and elegant description of the universe in-the-large, quantum theory seemed to muddy the waters. And this more or less characterises how most scientists perceived quantum theory. There were essentially two ways of dealing with this discomfort with 'quantum weirdness'. One way is to simply ignore any conceptual considerations. This has been the main attitude within the particle physics community, who exemplify the motto 'shut up and calculate'. Alternatively, one can be obsessively concerned with the conceptual problems surrounding

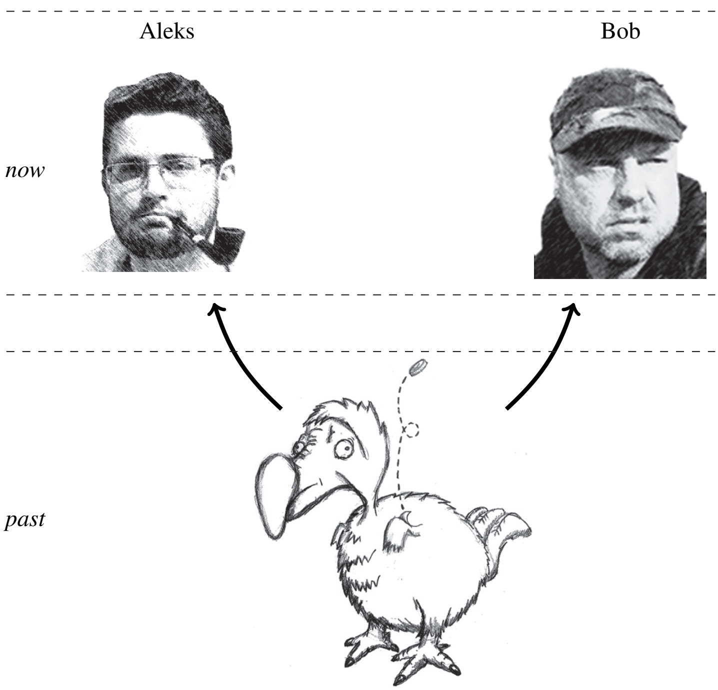

Figure 1.3 Non-locality of quantum theory means that quantum features cannot be explained by means of a classical probabilistic model. In other words, there are situations (unlike the one above) where distantly located observers can experience statistical correlations when they make quantum measurements that cannot be explained by a common cause.

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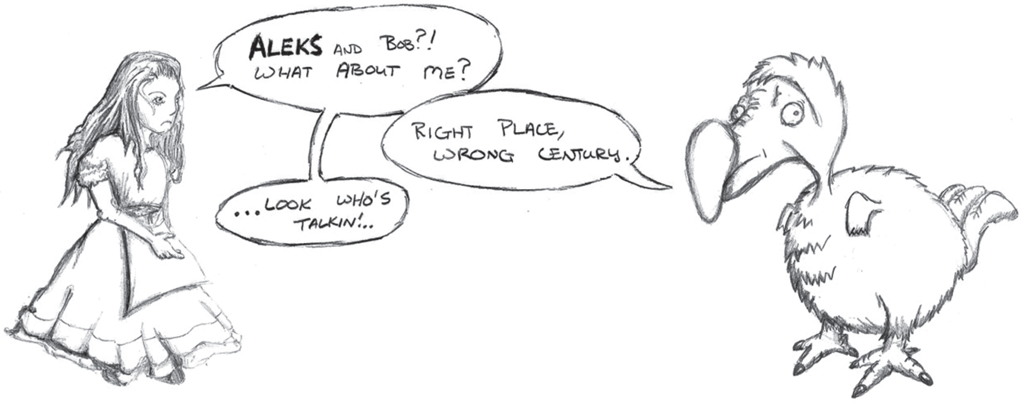

Figure 1.4 Alice queries Dave about Aleks having been partnered with Bob.

quantum theory, sacrificing most of one's life (not to mention sanity) trying to 'fix' them.

Then, starting in the early 1980s, there was an important attitude change, which could be summed up in a simple question:

What if the purported bugs of quantum theory are actually features?

In other words, people began to realise that there was much to be gained by embracing quantum theory as it is and trying to figure out how one can actually exploit 'quantum weirdness'. One may even hope that by doing so, we will become more acquainted with quantumness, get more comfortable with its quirkiness, and maybe the resulting less conventional intuitions might even start to make a lot of sense.

And indeed, quantum non-locality, once perceived by Einstein as some unwanted 'spooky action at a distance', suddenly became a key resource. In fact, decades before software developers started using the motto above to excuse their lazy debugging practices ('It's not a bug, it's a feature!'), Richard Feynman had already pointed out that there was at least one thing that quantum systems were really good at: simulating quantum systems! As it turns out, this problem is pretty difficult using a normal, classical computer. Over the next few decades, scientists discovered lots of weird and wonderful things that quantum systems can do: send secure messages, teleport physical systems, and efficiently factor large numbers.

The new focus on quantum features gave birth to several new fields: quantum computing, which studies how quantum systems can be used to compute; quantum information theory, which studies the implications of incorporating quantum phenomena into gathering and sharing information; and quantum technologies, which concerns the actual business of building devices that exploit quantum effects to make our lives better.

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0	0	1	1	0	1	0	0	1	1	1	0	1	0
0	1	0	1	0	1	0	0	1	0	0	1	0	0
1	0	1	0	0	1	0	1	0	0	0	1	0	0
0	1	1	1	0	0	1	1	1	0	0	0	1	1
1	0	0	0	1	0	0	0	0	1	1	0	1	0
0	1	1	0	0	1	1	1	0	1	0	0	1	0
0	1	1	1	0	1	0	0	1	0	1	0	1	0
1	1	0	1	0	0	0	0	1	0	1	0	1	0
0	1	0	1	0	1	0	0	1	0	1	0	1	0
0	1	0	0	1	1	0	1	0	0	1	1	0	1
1	0	0	1	1	0	1	0	1	0	0	1	1	0
0	1	0	0	1	0	0	1	1	0	0	0	1	1
1	1	0	0	0	1	1	0	1	0	1	1	1	0
0	1	0	1	1	1	0	1	1	0	1	0	0	0

vs.

Figure 1.5 Contrasting a low-level and a high-level representation of the digital data that one may find within a computational device.

1.2.2 A New Form of Mathematics: 'Diagrams'

It should be emphasised that discovering these new quantum features wasn't trivial and involved some very smart people. Our bold claim is that when one adopts the appropriate language for quantum theory, these features jump right off the page. Conversely, the traditional, Hilbert space–based language of quantum theory forms a major obstruction to discovering such features. To give some idea of why this is the case, we will make use of some simple metaphors.

Imagine that you were trying to determine what was happening in a video just by looking at its digital encoding (Fig. 1.5). Obviously this is a more or less impossible task. While digital data, i.e. strings of 0s and 1s, is the workhorse of digital technology, and while it is possible to understand 'in principle' how they encode all of the media stored on your hard drive, asking a person to decode a particular string of binary by hand is more suitable for punishing greedy bankers and corrupt politicians than solving interesting problems.

Of course, even skilled computer programmers wouldn't be expected to interact directly with binary data. Somewhere along the way to modern computer programming came the advent of assembly language, which gives a (somewhat) human-readable translation for individual instructions sent to a computer processor. While this made it more practical to write programs to drive computers, it still takes a lot of head-scratching to figure out what any particular piece of assembly code does. Using *low-level languages* such as assembly language creates an artificial barrier between programs and the concepts that they represent and places practical limits on the complexity of problems those programs can solve. For this reason, virtually every programmer today uses *high-level languages* in their day-to-day work (Fig. 1.6).

Similarly, 'detecting new quantum features' in terms of the traditional (i.e. low-level) language for quantum theory, namely 'strings of complex numbers' (rather than 'strings of 0s and 1s'), isn't that easy either. This could explain why it took six highly esteemed researchers to discover quantum teleportation, some 60 years since the actual birth of the quantum theoretical formalism. By contrast, the diagrammatic language we use in this book is a *high-level* language for exploring quantum features (Fig. 1.7). We will soon see

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```
.LC0:
    .string "QUANTUM!"
    .text
    .globl  main
    .type   main, @function
main:
.LFB0:
    .cfi_startproc
    pushq   %rbp
    .cfi_def_cfa_offset 16
    .cfi_offset 6, -16
    movq    %rsp, %rbp
    .cfi_def_cfa_register 6
    subq    $16, %rsp
    movl    $0, -4(%rbp)
    jmp .L2
.L3:
    movl    $.LC0, %edi
    movl    $0, %eax
    call    printf
    addl    $1, -4(%rbp)
.L2:
    cmpl    $4, -4(%rbp)
    jle .L3
    leave
    .cfi_def_cfa 7, 8
    ret
    .cfi_endproc
```

vs.

```
5.times do
  print "QUANTUM!"
end
```

Figure 1.6 Contrasting a low-level and a high-level language for computer programs. The programs on the left and right perform the same task, but one is written in the low-level x86 assembly language and one in the high-level language Ruby.

$$\frac{1}{4}\begin{pmatrix} -1+i & 1+i & 1+i & -1+i & 1+i & 1-i & 1-i & 1+i \\ 1+i & 1-i & 1-i & 1+i & -1+i & 1+i & 1+i & -1+i \\ 1+i & 1-i & 1-i & 1+i & 1-i & -1-i & -1-i & 1-i \\ 1-i & -1-i & -1-i & 1-i & 1+i & 1-i & 1-i & 1+i \\ 1+i & 1-i & 1-i & 1+i & 1-i & -1-i & -1-i & 1-i \\ 1-i & -1-i & -1-i & 1-i & 1+i & 1-i & 1-i & 1+i \\ -1+i & 1+i & 1+i & -1+i & 1+i & 1-i & 1-i & 1+i \\ 1+i & 1-i & 1-i & 1+i & -1+i & 1+i & 1+i & -1+i \end{pmatrix}$$

vs.

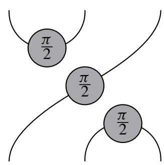

Figure 1.7 Contrasting a low-level and a high-level language for quantum processes, just like we contrasted the low-level and a high-level representation for digital data in Fig. 1.5 and a low-level and a high-level programming language in Fig. 1.6.

that by embracing the diagrammatic language for quantum theory, features like quantum teleportation are pretty much staring you in the face!

Although it goes beyond the scope of this book, it is worth mentioning that the diagrammatic language we use has found applications in other areas as well, such as modelling meaning in natural language (Fig. 1.8), doing proofs in formal logic, control theory, and modelling electrical circuits.

Diagrams are also becoming increasingly important in some fancy research areas of pure mathematics, such as knot theory, representation theory, and algebraic topology. By using diagrams we eliminate a huge amount of redundant syntactic garbage in representing

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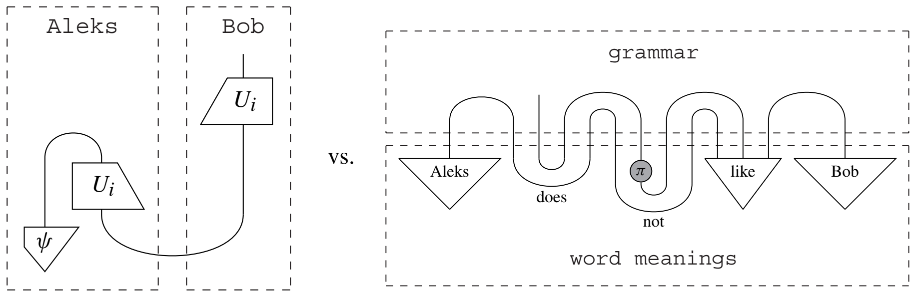

Figure 1.8 Comparing diagrammatic representations of quantum processes to those of 'the flow of meaning' in natural language. While these are two very different contexts, Aleks and Bob feel well at home in both due to their diagrammatic similarity. In the diagram representing natural language, the upper half represents the grammatical structure, while the bottom half represents meaning of individual words, and the overall wiring exposes how the meanings of these words interact in order to produce the meaning of the entire sentence.

$$(g_1 \otimes g_2) \circ (f_1 \otimes f_2) = (g_1 \circ f_1) \otimes (g_2 \circ f_2) \qquad \text{vs.}$$

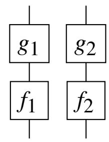

Figure 1.9 Two distinct syntactic descriptions corresponding to the same diagram. In terms of the symbolic language that is used on the left, two syntactically non-equal expressions might mean the same thing. On the other hand, in terms of the graphical language that is used on the right, there is only one representation. This example is explained in great detail in Section 3.2.4.

mathematical objects (Fig. 1.9), freeing us to concentrate on the important features of the mathematical objects themselves.

There are clear indications that diagrammatic reasoning will become increasingly important in the sciences in general, and this book represents the first attempt to comprehensively introduce a big subject like quantum theory entirely in this new language. By reading this book – or even more, taking a course based on this book – you, like the monkeys launched into space in the 1960s, are the 'early adopters' (a.k.a. 'test subjects') in a totally new enterprise.

1.2.3 A New Foundation for Physics: 'Process Theories'

By taking diagrammatic language as a formal backbone for describing quantum theory (or any other physical theory, for that matter) one also subscribes to a new perspective on physical theories.

First, traditional physical theories take the notion of a 'state of a system' as the primary focus, whereas in diagrammatic theories, it is natural to treat arbitrary processes on equal footing with states. States are then treated just as a special kind of process, a

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'preparation' process. In other words, there is a shift from focussing on 'what is' to 'what happens', which is clearly a lot more fun. This is very much in line with the concerns of computer science, where the majority of time and energy goes into reasoning about processes (i.e. programs), and states (i.e. data) only exist to be used and communicated by programs. It is also becoming clear that one should focus not just on single programs but on collections of *interacting* programs to understand the complex, distributed computer systems that are becoming increasingly prevalent in the modern world.

Another example where studying interaction is crucial to understanding a system comes from biology. While one can (in principle) deduce the coat of an animal from its genetic code, this does not explain why that animal has such a coat. On the other hand, if we look at where an animal lives or how it attracts a mate, for example, this can immediately become clear. Similarly, rather than concentrating on systems in isolation, our approach to physics looks at the overall structure of many systems and processes and how they compose. We call such a structure consisting of all the 'allowed processes' and how these interact a *process theory*.

Schrödinger realised early on that the most startlingly non-classical features of quantum theory came from looking not at a single system, but rather at how multiple systems behave together. Rather than acting as a collection of individuals, quantum systems establish complex relationships, and it is these relationships that suddenly enable amazing new things:

> When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.

Schrödinger says that the most important trait of quantum mechanics only becomes apparent when we study the interactions of two systems. So, one might expect any approach to quantum theory to start from a point of view that emphasises compositionality from page 1. But oddly, if you pick up a random textbook on quantum theory, this is not the point of view you will see. It was not until the late 1990s – largely prompted by the discoveries we discussed in Section 1.2.1 – that this idea made it back into the mainstream.

The concept of a process theory does of course put composition at the forefront, and it suggests a natural way of reasoning about processes. One should pare down the nitty-gritty aspects of how processes are mathematically defined and seek out high-level principles that govern their interactions. These principles taken together constitute what can be called the *logic of interaction* for a process theory. Von Neumann also thought that quantum theory should be understood in terms of logical principles. Three years after he published *Mathematische Grundlagen*, von Neumann wrote:

> I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space no more. [sic]

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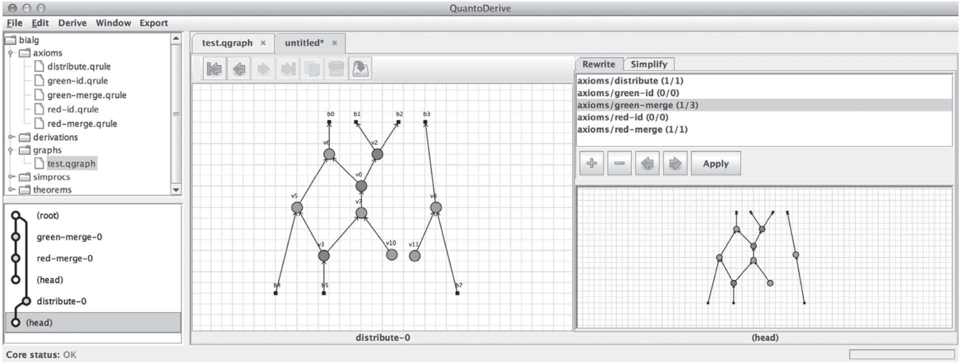

Figure 1.10 Quantomatic: a diagrammatic proof assistant.

He went on to say that it is not the Hilbert-space structure of quantum theory that is physically relevant, but rather the quantum analogue of 'logical propositions', namely those properties verifiable by means of quantum measurements. However, this new kind of logic, called *quantum logic*, ultimately failed to replace Hilbert space as a conceptual basis for quantum theory. Its biggest stumbling block was its complete focus on systems in isolation and its inability to obtain any conceptual account of composed systems. More pragmatically, the passage from Hilbert space to quantum logic seems to make it more difficult to establish new facts or discover new features of quantum theory, typically requiring extreme cleverness on the part of its practitioners to establish even basic facts.

In contrast, this new kind of interaction logic via process theories has very quickly become a practical tool for high-level reasoning about quantum systems and beyond, not in the least due to its intuitive diagrammatic language. It has even formed the basis of a diagrammatic *proof assistant* – i.e. an interactive software tool that constructs proofs (semi-)automatically – called Quantomatic (Fig. 1.10).

So, what should we call this new way of reasoning about quantum theory entirely with diagrams, focussing crucially on its processes and the logic of their interactions? Since the term 'quantum logic' is already trademarked, we'll have to use something a bit more descriptive ...

1.2.4 A New Paradigm: 'Quantum Picturalism'

In Section 1.2.1 we said that the dissatisfaction with quantum theory obstructed people from realising what the actual features of quantum theory were and how those features could be put to good use. Then, by asking the right 'positive' question, many new features were discovered. We went on to argue that in an adequate mathematical language, the features of quantum theory should be plainly obvious. Taking things one step further, one could wonder if this adequate mathematical language isn't just easier to work with, but is also closer to what the world is made up of!

The Holy Grail of theoretical physics is to come up with a theory of quantum gravity. It is likely that to develop a consistent theory of quantum gravity, some of the core assumptions

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of quantum theory will need to be relaxed. As the standard, Hilbert-space presentation of the theory comes as a packaged deal, it is therefore necessary to seek an alternative presentation that lets us tease out the important features from the incidental ones. Hence, figuring out the presentation that matches what is actually out there in the world is of crucial importance.

Until recently, most if not all attempts to do so suffered from the same obsessions we mentioned before. That is, they take as their starting points some failure of quantum theory:

- *C*-algebras*: the non-commutativity of 'quantum observables'
- *quantum logic*: the non-distributivity of 'quantum propositions'
- *quantum measure theory*: the non-additivity of 'quantum measures'

(Sorry about all the jargon.) It doesn't matter what all of these exactly mean, but the key thing to observe is that they all emphasise something that quantum theory fails to be. What can you do with that? How useful is it to know that a fish is not a dodo? Not much, since a screwdriver is also not a dodo.

Instead of highlighting properties that quantum theory fails to satisfy, one should instead seek out the unique new possibilities highlighted by quantum theory. We contend that the most interesting features of quantum theory are diagrammatic ones, which brings us to the first 'definition' in this book.

Definition 1.1 *Quantum picturalism* refers to the use of diagrams to capture and reason about all of the essential features of interacting quantum processes, in a manner that these diagrammatic equations become the very foundation of quantum theory.

Now, let's see quantum picturalism in action. Consider the following failure of quantum theory: 'A state of two quantum systems fails in general to separate into distinct states of systems A and B'. Flipping this around, we can 'witness' non-separability by noting that there exist *maximally entangled states* for two systems, which can be succinctly characterised by a single (very useful!) graphical equation (see Fig. 1.11).

A second 'positive' feature we explore in this book is the *existence of complementary measurements*. Here the corresponding 'negative' statement is that most quantum measurements fail to be *compatible* (i.e. they cannot both be performed simultaneously). In fact, the aforementioned conditions of non-commutativity and non-distributivity both aimed to capture the incompatibility of measurements. By contrast, *maximum incompatibility*

Figure 1.11 The two quantum features, 'existence of maximally entangled states' and 'existence of complementary measurements', in terms of graphical primitives. Chapters 4 and 9 are devoted to these two features.

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captured by the second equation in Fig. 1.11 represents an actual quantum behaviour that we can observe in experiments and exploit, e.g. in the design of quantum security protocols.

One can now even ask whether such features are enough to completely characterise quantum theory. Can we craft a new formalism for quantum theory for which the defining axioms represent essential physical features through elegant diagrammatic properties? Though this story is by no means complete, the answer seems to be 'yes'. We hope that during the course of this book you will enjoy discovering this fact as much as we did.

1.3 Historical Notes and References

At the end of each chapter, including this one, we will present a brief overview of the history of its main content. This particular section, due to the very nature of an introductory chapter, will cover a lot of ground, and many things touched on here will be discussed with greater care at the end of later chapters.

First, let us say a word about dodos. Dodos became extinct in 1680. The world's only preserved dodo remains are resting at Oxford University's Museum of Natural History. Thus the appearance of Dave the Dodo is an homage both to the 'Oxford Dodo' and to another dodo famously appearing in a fellow Oxonian and local logician's hallucinogenic trip *Alice in Wonderland* (Carroll, 1942). It is fortuitous that our hero is entombed less than 100 metres from both the place where the first experimental demonstration of a quantum algorithm took place (Jones et al., 1998) and the offices where this book was written.

Actual quantum systems, rather than imaginary quantum dodos, were first identified as such by Max Planck (1900), which initiated some 30 years of constructing the formalism of quantum theory as we know it now, ultimately yielding von Neumann's formulation of quantum theory based on Hilbert space and linear maps (von Neumann, 1932). Since then, pretty much any standard textbook of quantum theory still very much resembles the original, with the exception of this one of course!

The paper most associated with Einstein's discomfort with quantum theory is the EPR paper (Einstein et al., 1935). Einstein himself never subscribed to the precise wording of the EPR paper and republished a single-authored paper (Einstein, 1936). As we mentioned earlier, the EPR paper is now considered to be the first paper pointing in the direction of quantum non-locality, by highlighting a conflict between quantum theory and the assumption of 'local realism'. John Bell (1964) strengthened this claim by proving a general theorem about local realistic models and showed that if quantum theory is correct, then it must violate local realism. The modern conception of local realism is that the probabilistic correlations between events observed at distinct locations can be explained by a causal, classical probabilistic model (see e.g. Pearl, 2000). Even today, seeking refinements and generalisations of Bell's theorem is a topic of active research (see e.g. Wood and Spekkens, 2012).

Importantly, Bell's version was directly experimentally verifiable, and the violation of local realism has been experimentally verified many times, starting with Aspect et al. (1981, 1982). Hence it has been experimentally established that the world is indeed 'non-local',

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exactly as predicted by quantum theory. It should be mentioned that there were a handful of objections to that initial experiment, in the form of 'loopholes' in that particular demonstration of non-locality. All of these have meanwhile been closed by other experiments (Weihs et al., 1998; Rowe et al., 2001; Hensen et al., 2015). On top of that, the huge variety of experiments that have been done confirming some form or another of non-locality for quantum theory is pretty compelling, to say the least (see e.g. Rauch et al., 1975; Zeilinger, 1999; Pan et al., 2000; Gröblacher et al., 2007).

Another development prompted by the EPR paper was the quest for interpretations of quantum theory. Given EPR's claim of incompleteness of quantum theory, one family of interpretations was all about completing quantum theory, although due to Bell's theorem, any such an attempt is bound to be non-local. The most famous one of these is the hidden variable interpretation due to David Bohm (1952a,b). Another one loved by Hollywood is the many-worlds interpretation due to Hugh Everett III (1957). The official default interpretation of quantum theory is the Copenhagen interpretation due to Niels Bohr and Werner Heisenberg, which in the eyes of many is a non-interpretation, given that at first sight it provides nothing more than a recipe to compute probabilities. A detailed survey and extensive discussion of the interpretations of quantum theory can be found in Bub (1999).

The shut-up-and-calculate slogan is often associated with Richard Feynman, who did in many ways embody this way of working, but in fact it was coined by David Mermin (May 2004), who very much did not skirt around foundational questions in quantum theory. He used the term not to refer to this common practice in particle physics, but rather to give his view on the Copenhagen interpretation (Mermin, April 1989).

While on the topic of Feynman, it is worth mentioning that he was the first to realise that there was something quantum systems are really good at: simulating themselves (Feynman, 1982). Thus, his notion of a quantum simulator contained the first seeds of the idea of quantum computation. The discovery of less self-referential applications for quantum features in information processing began a few years later with the advent of quantum key distribution (Bennett and Brassard, 1984). A year later, at the University of Oxford, David Deutsch (1985) gave a formulation for a universal quantum computer, the quantum analogue to Turing's universal machine (Turing, 1937). This led to the discovery of quantum algorithms that substantially outperformed any classical algorithm (Deutsch and Jozsa, 1992; Shor, 1994; Grover, 1996; Simon, 1997). The term 'qubit' was coined by Schumacher (1995).

Quantum teleportation was proposed by Bennett et al. (1993), and its first experimental realisation was by Bouwmeester et al. (1997). The question of why it took 60 years for quantum teleportation to be discovered was asked by one of the authors in a seminar at the Perimeter Institute of Theoretical Physics and was immediately answered by Gilles Brassard, co-inventor of quantum teleportation and a pioneer of the quantum information endeavour as a whole, who happened to be in the audience. He said that no one before had considered the information-processing features of quantum theory and had therefore simply not thought to ask the question. This exchange is reported in Coecke (2005).

The diagrams used in this book are an extension of those used by Roger Penrose (1971), who introduced them as an alternative for ordinary tensor notation (see Section 3.6.1).

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However, many similar diagrammatic languages were invented prior to this or reinvented later.

In programming language theory, flow charts were among the first abstract presentations of programs and algorithms. These flow charts, introduced in Gilbreth and Gilbreth (1922) under the name 'process charts', are widely used in many other disciplines too. In quantum information the use of diagrammatic representations started with quantum circuits, a notation borrowed from circuits made up of Boolean logic gates, to which a number of new properly quantum gates were added (see e.g. Nielsen and Chuang, 2010).

A diagrammatic notation specifically tailored towards the processes responsible for quantum weirdness was first introduced in Coecke (2003, 2014a) and, independently, also in Kauffman (2005). These diagrams were provided with an axiomatic underpinning in Abramsky and Coecke (2004), and independently in Baez (2006), paving the way to a diagrammatic approach for quantum theory as a whole. The main pillars supporting the story outlined in this book are the diagrammatic representation of mixed states and completely positive maps by Selinger (2007), the diagrammatic representation of classical data as 'spiders' in (Coecke and Pavlovic, 2007; Coecke et al., 2010a), the diagrammatic representation of phases, complementarity, and the introduction of strong complementarity by Coecke and Duncan (2008, 2011), again in terms of spiders, and the causality postulate introduced by Chiribella et al. (2010). 'Quantum picturalism' was coined in Coecke (2009).

Important topics in the area of quantum computing and related areas that were diagrammatically explored are quantum circuits (Coecke and Duncan, 2008), (topological) measurement-based quantum computing (Coecke and Duncan, 2008; Duncan and Perdrix, 2010; Horsman, 2011), quantum error correction (Duncan and Lucas, 2013), quantum key exchange (Coecke and Perdrix, 2010; Coecke et al., 2011a), non-locality (Coecke et al., 2011b, 2012), and quantum algorithms (Vicary, 2013; Zeng and Vicary, 2014).

Structural theorems about quantum theory emerging from the diagrammatic approach include a number of completeness theorems (Selinger, 2011a; Duncan and Perdrix, 2013; Backens, 2014a; Kissinger, 2014b) and some representation theorems (Kissinger, 2012a; Coecke et al., 2013c).

For applications of the kinds of diagrams that we consider here to other scientific disciplines, there are, for example, Coecke et al. (2010c) and Sadrzadeh et al. (2013) for applications to natural language, Mellies (2012) for logic in computer science, Pavlovic (2013) for computability, Hinze and Marsden (2016) for programming, Baez and Fong (2015) for applications to electrical circuits, Bonchi et al. (2014a) and Baez and Erbele (n.d.) for applications in control theory, Hedges et al. (2016) for applications in economic game theory, Baez and Lauda (2011) for a prehistory, Baez and Stay (2011) for a Rosetta Stone, and Coecke (2013) for an alternative Gospel.

A discussion of von Neumann's discontent with Hilbert space is in Redei (1996), from which we also took the second quote in Section 1.2.3. Attempted modifications/generalisations/axiomatisations of quantum theory, all to a great extent inspired by quantum logic (Birkhoff and von Neumann, 1936), were pioneered by Mackey (1963), Jauch (1968), Foulis and Randall (1972), Piron (1976), and Ludwig (1985). Coecke et al. (2000) provides

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a survey of these approaches. A tutorial on how property lattices yield Hilbert spaces is in Stubbe and van Steirteghem (2007). A survey of Foulis and Randall's 'test space' formalism (or 'manuals' formalism) is in Wilce (2000). Ludwig's approach has recently become very prominent again under the name 'generalised probabilistic theories' (Barrett, 2007). Also, several researchers have tried to combine the earlier axiomatic approaches with diagrams and/or compositional structure (Harding, 2009; Heunen and Jacobs, 2010; Jacobs, 2010; Vicary, 2011; Abramsky and Heunen, 2012; Coecke et al., 2013a,b; Tull, 2016).

Giving processes a privileged role in quantum theory was already present in the work of Whitehead (1957) (cf. the quotation at the beginning of Chapter 6) and in Bohr (1961) and became more prominent in Bohm (1986). Process ontologies trace back to the pre-Socratics, most notably to Heraclitus of Ephesus in the sixth century BC (cf. the quotation at the beginning of Chapter 2). Diagrams, and hence a privileged role for processes and composition thereof, are used as a canvas for drafting theories of physics in Chiribella et al. (2010), Coecke (2011), and Hardy (2011, 2013b).

The second quote in Section 1.2.3 on the importance of the role of composition in quantum theory is taken from Schrödinger (1935). The first proper 'interaction logic' was *Geometry of Interaction* due to Jean-Yves Girard (1989), which was recast in a form more resembling the language used in this book in Abramsky and Jagadeesan (1994), and even more so in Duncan (2006). That quantum picturalism can be seen as a logic of interaction is argued in Coecke (2016), on the basis that the roots of logic are language and that the use of logic is artificial reasoning. The diagrammatic proof assistant Quantomatic is described in (Kissinger and Zamdzhiev, 2015) and at the time of this writing is still being actively developed. It is available from the project Website quantomatic.github.io.

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