2

Guide to Reading This Textbook

Nothing endures but change.

– Heraclitus of Ephesus, 535–475 BC

2.1 Who Are You and What Do You Want?

While there is already a plethora of textbooks on quantum theory and its features, this one is unique because it is based on quantum picturalism.

Prerequisites. There are hardly any prerequisites to this textbook. We do not expect our readers to have a background in physics or computer science or to have any profound background in mathematics. In principle, some basic secondary-school mathematics should be sufficient.

For example, linear algebra (and of course quantum theory) is presented from scratch in a diagrammatic manner. However, this does not mean that the first half of this book will be a boring read for the specialist, since these presentations from scratch are radically different from the usual ones.

Target audience. Given its low entrance fee, as well as its unique form and content, this textbook should appeal to a broad audience, ranging from students to experts from a wide range of disciplines including physicists, computer scientists, mathematicians, logicians, philosophers of science, and researchers from other areas with a multidisciplinary interest, such as biologists, engineers, cognitive scientists, and educational scientists.

One particular target audience for this book consists of students and researchers in quantum computation and quantum information, as we will apply the tools from quantum picturalism directly to these areas. A practising quantum computing researcher may discover a new set of tools to attack open problems where traditional methods have failed, and a student may find some subjects explained in a manner that is much easier to grasp.

Another target audience consists of students and experts with an interest in foundations and/or philosophy of physics, who can read in this book about a process-oriented approach to physics that takes composition of systems as a first-class citizen, rather than a derived notion. In particular, this is the first book that uses diagrammatic language to capture the idea of a process theory and puts such process theories forward as a new foundation for quantum theory, in which all standard quantum theoretical notions can be expressed.

Yet another target audience consists of logicians and computer scientists, who may want to learn about diagrams as a new kind of logical paradigm, which emphasises ‘composition’ over ‘proposition’. For them, learning quantum theory may just be an added bonus to learning about this new paradigm for theory development.

But since this textbook covers the essential ingredients of a standard textbook on quantum computation and quantum foundations, it can just as well be used as a first introduction to those fields. Even though our notation is different from the one used in other textbooks, we cover the core curriculum of a first course in quantum computation and make a continual effort to relate the concepts and notations we introduce to those used more commonly in the literature.

Similarly, this textbook can be used as a first introduction to diagrammatic reasoning, being pretty much the first textbook that does that too.

2.2 The Menu

While we cannot yet offer dodo steaks, with recent advances in science, it may not be too long before pigeons give birth to new Daves and Davettes.

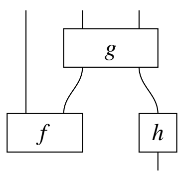
2.2.1 How Diagrams Evolve in This Book

In this book two stories more or less evolve in parallel:

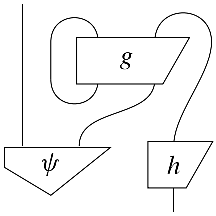
- the development of the diagrammatic language and
- the presentation of quantum theory as a process theory.

Quantum picturalism is indeed all about how these two are closely intertwined. We begin with a very general diagrammatic language and gradually add features to increase its expressiveness. Thus, we start with a language that is general enough to describe many different kinds of processes and gradually home in on quantum processes. Along the way, we present quantum features as and when the language is rich enough to discuss them. As a result, we will encounter features such as quantum teleportation way before more concrete notions such as qubits. All together there are five major jumps in the expressiveness of diagrams on the way to capturing full-blown quantum theory.

1. We first introduce a very basic diagrammatic language in Chapter 3 consisting of nothing but *boxes* and *wires*. This gives us a natural way to express compositions of processes in any process theory:

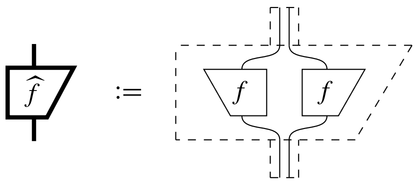


2. *String diagrams*, defined in Chapter 4, single out special kinds of process theories for which wires can be cup- and cap-shaped, and each box can be both horizontally and vertically reflected:

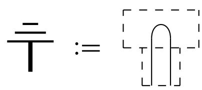


They already expose several quantum-like features, such as non-separability, unitarity, and the impossibility to clone arbitrary states. Following our discussion at the beginning of Section 1.2 on why it took 60 years for teleportation to be discovered, the language of string diagrams makes it plainly obvious that something like quantum teleportation is possible.

3. Next, we allow for two kinds of boxes and wires in diagrams, 'thin' and 'thick' ones, which allows us to distinguish quantum systems from classical ones. Thick boxes and wires, defined in Chapter 6, arise by *doubling* their thin counterparts:

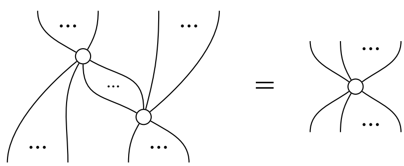


This doubling guarantees that numbers produced by the theory are positive, so these can be interpreted as probabilities. It also allows us to define *discarding*:



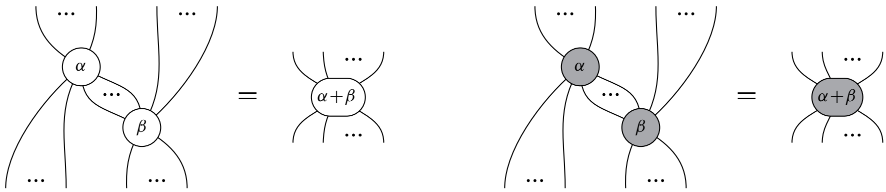
a special process that plays a crucial role in our presentation of quantum theory and the causality postulate, which imposes compliance with the theory of relativity.

4. *Spiders*, defined in Chapter 8, are a funky generalisation of wires. Whereas wires have just one input and one output, spiders represent a connection between any number of inputs/outputs. They are governed by a 'spider fusion' rule, which states that when two spiders are connected, they fuse together into a single spider.

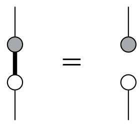


Among other things, spiders are used to capture the unique behaviour of classical data, in that it can be copied and deleted. They also allow us to represent the interaction of classical and quantum systems directly in our diagrams, via measurement and encoding operations:

5. For the final jump, we allow for diversification of spiders in Chapter 9. Spiders can now be decorated by *different colours* and *phases*:



This extra data allows us to define all of the processes we need in a purely diagrammatic way (without invoking e.g. matrices) and gives elegant expressions of extremely important quantum features, such as complementarity:



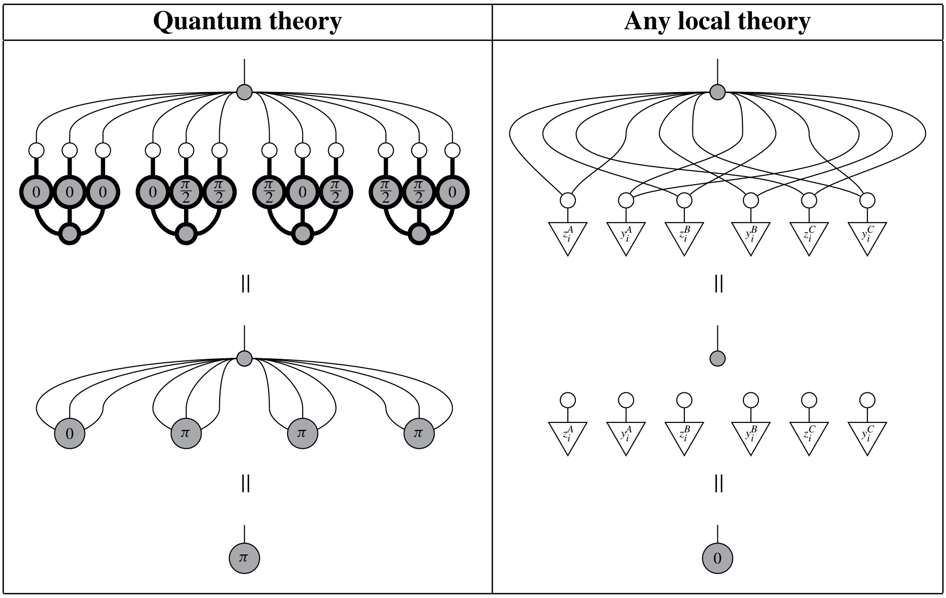
The diagrammatic language now becomes rich enough to unambiguously write down any process from m qubits to n qubits; that is, it becomes *universal* for qubits. The rules for manipulating these diagrams are called the *ZX-calculus*. In fact, this calculus is not just universal, but also complete for important fragments of qubit quantum theory. This means that all equations that can be derived using matrices can also be derived diagrammatically.

Once we have the full diagrammatic language available, we give a succinct and elegant picture of quantum theory in Chapter 10, 'Quantum Theory: The Full Picture'. Here's a preview of what this looks like in the form of a ...

2.2.2 Hollywood-Style Trailer

The coolest and at the same time least understood quantum feature is most certainly quantum non-locality. Near the end of this book, we provide a detailed account (and proof) of the existence of non-locality. Here, we'll give the reader a glimpse into the mysteries that will be revealed.

While the account of non-locality in standard textbooks involves pages mixing up words and formulas, for us it simply boils down to two diagrammatic computations, one about quantum theory and one about local theories, which yield contradictory results:

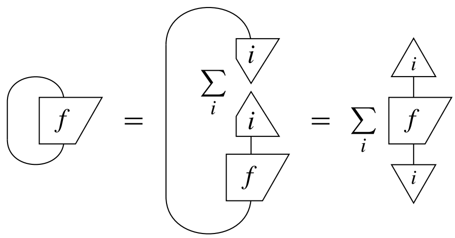


We will learn that the diagram on the left models four measurements performed on a GHZ-state, followed by computing the *parity* of the measurement outcomes. In other words, our theory is telling us whether to expect an even or an odd number of clicks (or beeps, flashes, etc.) coming from our measurement devices. We will see that the reduction to π on the left says that, according to quantum theory, the parity will be odd. Compare this with the derivation on the right, which assumes that there are some pre-established correlations that determine the measurement outcomes. This is always the case with a local theory, since all correlations of distant events can be traced back to some common cause. The reduction to 0 on the right says that any local theory predicts an even number of clicks, and hence there is a contradiction between quantum theory and locality.

This example is taken from Chapter 11, ‘Quantum Foundations’, in which we also present a toy theory that looks very much like qubit quantum theory, but fails to be non-local. There are two more themed chapters: Chapter 12, ‘Quantum Computation’, where we address standard topics such as the circuit model of quantum computation and quantum algorithms, as well as less standard (but increasingly important) topics such as measurement-based quantum computation. In Chapter 6, ‘Quantum Resources’, we present a general framework to study resources in quantum theory, with quantum entanglement as a particularly important example. We also show how qualitatively distinct types of quantum entanglement can be thought of as very differently behaving spiders.

2.2.3 Some Intermediate Symbolic Pollution

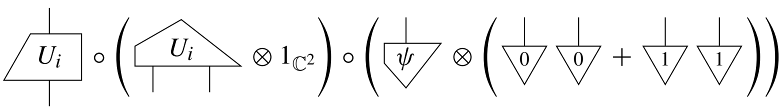
We haven't mentioned Chapter 5 yet. This chapter doesn't contribute to the development of quantum picturalism, but makes the connection with the usual quantum theoretical formalism. The main question addressed is the following. Given a process theory with string diagrams, when do wires represent Hilbert spaces and boxes represent linear maps? As an answer to this question, we adjoin some symbols to string diagrams, resulting in a hybrid diagram–symbol formalism in which one encounters computations like this one:



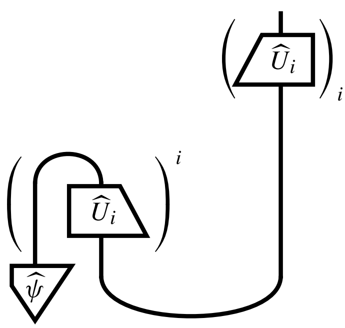
Such a hybrid formalism is in itself useful, and is widely used in areas of mathematics such as knot theory. More importantly, it introduces the usual formalism to those who don't know it already, and for those who are familiar with it, it makes clear how the diagrams relate to it. Finally, and perhaps most importantly, it allows us to state what exactly one can compute with string diagrams.

Something this allows us to do, which cannot be found in standard textbooks, is to make a smooth transition from linear maps to quantum processes and ultimately to processes that model quantum non-determinism and the classical-quantum interaction in a purely graphical way.

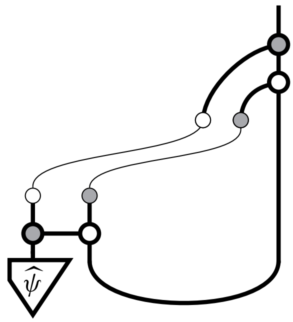
Along the way, we, like Dante, will proceed from linear algebraic Hell:



through Purgatory:



and ultimately to a purely diagrammatic Paradise:



2.2.4 *Summaries, Historical Notes, References, Epigraphs*

Each chapter (with the exception of Chapter 10, which is already a summary) contains a short section entitled 'Summary: What to Remember' listing the essential material to be taken from it.

At the end of each chapter there is also a short section entitled 'Historical Notes and References' in which we sketch the historical development of the material covered in that chapter, list some key references, and suggest some further reading.

The epigraphs appearing at the beginning of each chapter are all relevant to the contents of that chapter. For some it will be immediately clear from the text. Determining the relevance of the others is left as an exercise for the reader.

2.2.5 *Starred Headings and Advanced Material Sections*

Any sections, theorems, remarks, examples, or exercises that have a star (*) as superscript, e.g. 'Remark* x.y.z', are to be considered as optional. Typically they require some knowledge that only a fraction of our readers would either know about or be interested in, and hence they are only intended for that particular fraction of readers. For instance, a starred remark may require knowledge of some advanced concepts from linear algebra, quantum theory, or programming. Notably, each chapter has a section entitled 'Advanced Material', which contains material that particularly advanced students or specialists may find interesting. These in particular include clarification of the connection between diagrams and monoidal categories. Some attention is also given to currently ongoing research in quantum picturalism, how it is related to recent developments in pure mathematics, and the surprising connection with natural language meaning.

2.3 FAQ

Over the years, we have noticed that people ask a few questions all of the time. We also anticipate a couple of new questions arising about this book in particular. We'll try to address both of these here.

Q1: *Why does it take X pages to get to some basic stuff such as Y?*

A: There are a few reasons for this:

- As the title suggests, this is a first course not only in quantum theory, but also in diagrammatic reasoning. Thus, we introduce features at the exact points where we have a rich enough language to talk about them. This doesn't always happen in the order you might expect.
- We assume no preliminaries and construct as much as possible from first principles, anticipating a very broad spectrum of potential readers. This means introducing many things, such as linear algebra, that will be old hat for many readers. However, we develop these basic concepts in such a drastically different, diagrammatic way that we think there is something for everyone in each chapter.
- It turns out that diagrams take up a lot of space. Sorry, trees.

Q2: *Where's the beef (i.e. the numbers)?*

A: A traditionally held belief is that the predictive content of a physical theory lies in its ability to produce numbers, such as probabilities. Many find it difficult to reconcile this idea with a diagram, which seems like a discrete, logical object. However, we will see in Section 3.4.1 that numbers arise naturally as special kinds of diagrams. That being said, the most interesting features we will highlight in this book are qualitative, not quantitative. As previewed in the Hollywood-style trailer, we will use diagrams continuously to see that quantum theory exhibits behaviours that are simply not possible in classical physics.

Q3: *What about infinite-dimensional Hilbert spaces?*

A: The lesson of quantum computing and quantum information processing is that we can access many new, revolutionary features of quantum theory by restricting to finite (and often just two!) dimensions. In fact, the long-held belief that 'real physics' only happens in the infinite-dimensional realm, with all of its associated difficulties, could have contributed to the blindness to new quantum features that were not discovered until the past couple of decades. Of course, we are not claiming that infinite dimensions should therefore be ignored, but it does present a unique set of difficulties for quantum picturalism. Most notably, the main diagrammatic workhorses in this book (cups, caps, and spiders) simply don't exist as bounded operators between infinite-dimensional Hilbert spaces. So, for many years, we thought one had to choose between the power (and complexity) of infinite dimensions or the elegance of diagrammatic reasoning with cups, caps, and spiders. However, certain constructions in this book that seem to rely on caps and cups can be done in ways to avoid them altogether (Coecke and Heunen, 2011), and recent results of Gogioso and Genovese (2016) suggest it is possible for us to have our caps and eat them too! Namely, they showed using techniques from non-standard analysis that, while cups, caps, and spiders don't actually <u>exist</u> in infinite dimensions, reasoning with them is still sound, as long as they don't appear in the final answer that comes out.

While this new way of working with infinite-dimensional systems is still in its infancy, it shows promise for the advent of a true infinite-dimensional quantum picturalism.

Q4: *What about Schrödinger's equation?*

A: You will notice that we are careful to use the term 'quantum theory', as opposed to 'quantum mechanics', which we take to mean the core of quantum mechanics that ignores things like positions, momenta, and continuous time-evolution. For our purposes, it suffices to just consider the overall change of systems between some time t_1 and some time t_2, without expounding on the details of what exactly happens in between these times. As in the case of finite dimensions, the huge advances in quantum information/computation have shown us that, even working at this level, we can access many fascinating features of the theory. That being said, there has recently been some exciting new research to accommodate dynamics in quantum picturalism by Gogioso (2015b,c), and it seems likely that many of the features we discuss in this book (e.g. strong complementarity) play a major role.

Q5: *Has quantum picturalism ever produced anything new?*

A: What people posing this question usually mean is: Has quantum picturalism helped solve problems that were already out there and that one couldn't solve with other existing methods? The answer is yes, and some examples of that are Duncan and Perdrix (2010), Coecke et al. (2011b), Horsman (2011), and Boixo and Heunen (2012). However, in science, coming up with new, interesting questions is sometimes more important than answering old ones. Issues such as completeness of calculi is a question that to our knowledge has never been asked before in physics, and quantum picturalism has meanwhile produced a string of results in that area (Backens, 2014a,b; Schröder de Witt and Zamdzhiev, 2014; Hadzihasanovic, 2015). It would be difficult to see how any answer could arise when sticking to the traditional Hilbert space formalism. Another new question is whether we can automate reasoning about physics, for which the Quantomatic software discussed in Chapter 14 has been produced (Kissinger and Zamdzhiev, 2015). Finally, there is the communality of structure and use of quantum picturalism methods elsewhere, like in developing a compositional distributional model of natural language meaning (Clark et al., 2014; Coecke, 2016), which has actually outperformed other existing methods when applied to empirical data (Grefenstette and Sadrzadeh, 2011; Kartsaklis and Sadrzadeh, 2013).