Basic Maths

FTNA Past Paper Questions and Answers By Topic

Contents

Numbers

Fractions

Decimals and Percentages

Units

Approximations

Geometry

Real Numbers

Ratio, Profit and Loss

Coordinate Geometry

Perimeters and Areas

Exponents and Radicals

Algebra

Quadratic equations

Logarithms

Congruence

Similarity

Geometrical Transformations

Pythagoras theorem

Trigonometry

Sets

Numbers

2020

1. (a) Write each of the numbers 18, 24 and 36 as a product of prime factors and hence find their greatest common factor.

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Soln

Given:

18, 24, 36

$18 = 3 \times 3 \times 2$

$24 = 3 \times 2 \times 2 \times 2$

$36 = 3 \times 3 \times 2 \times 2$

GCF = Product of numbers which appear in all three numbers (common prime factor)

$= 3 \times 2$

$= 6$

$\therefore$ GCF (Greatest common factor) is 6

2018

1. (a) A block is cut into equal units of 10 g, 20 g and 35 g. Use prime factorization method to find the smallest possible mass of the block from which the pieces can be cut.

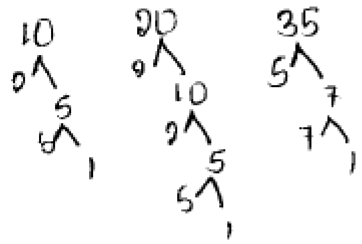
1. (a) A block is cut into equal units of 10 g, 20 g and 35 g. Use prime factorization method to find the smallest possible mass of the block from which the pieces can be cut.

Soln

10 = 1, 2, 5 and 10

20 = 1, 2, 4, 5, 10 and 20

35 = 1, 5 and 35



= 5

The smallest possible number is 5

2017

1. (a) Find the LCM and GCF of 13, 52 and 104.

(b) Round off the number 568,356 to the nearest thousands and ten thousands.

1. (a) Find the LCM and GCF of 13, 52 and 104.

Solution.

2	13	52	104
2	13	26	52
2	13	13	26
13	13	13	13
	1	1	1

LCM $= 2 \times 2 \times 2 \times 13$

$= \underline{104}$

GCF $= \underline{13}$

∴ The LCM = 104 and GCF = 13.

(b) Round off the number 568,356 to the nearest thousands and ten thousands.

Solution.

568356

- 6 — ones
- 5 — tens
- 3 — hundreds
- 8 — thousands
- 6 — ten thousands
- 5 — hundred thousands

Round off to thousands

56**8**356 = $\underline{568000}$

Round off to ten thousands.

5**6**8356 = $\underline{570{,}000}$

∴ Round off the number to thousands is 568,000 and ten thousands is 570,000.

2015

17. Find the product of the G.C.F and L.C.M of 4, 8 and 12.

Solution.

2	4	8	12
2	2	4	6
2	1	2	3
3	1	1	3
	1	1	1

L.C.M $= 2 \times 2 \times 2 \times 3$
$= 24.$
G.C.F $= 2 \times 2 = 4$

$\therefore$ LCM $= 24$ and GCF $= 4$

Product $= \underset{\text{LCM} \times \text{GCF}}{24 \times 4}$
$= 96.$

$\therefore$ The product of GCF and LCM of 4, 8 and 12 $= 96$

Fractions

2020

2. (a) Find the value of the expression $\frac{5}{2} - \left(3\frac{3}{5} \div 1\frac{1}{5} - \frac{4}{5}\right)$.

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soln

Given $\frac{5}{2} - \left[3\,^3/_5 \div 1\,^1/_5 - \,^4/_5\right] =$

$\frac{5}{2} - \left[\frac{18}{5} \div \frac{6}{5} - \,^4/_5\right] =$

$\frac{5}{2} - \left[\frac{18}{5} \times \frac{5}{6} - \,^4/_5\right] =$

$\frac{5}{2} - \left[\frac{3}{1} - \,^4/_5\right] =$

$\frac{5}{2} - \left[\frac{15-4}{5}\right] =$

$\frac{5}{2} - \frac{11}{5}$

$\frac{25-22}{10} = \frac{3}{10}$

$\therefore \frac{5}{2} - \left(3\,^3/_5 \div 1\,^1/_5 - \,^4/_5\right) = \,^3/_{10}$

2018

2\. (a) Find out which of the two fractions, $\frac{5}{7}$ or $\frac{6}{9}$ is greater.

2. (a) Find out which of the two fractions, $\frac{5}{7}$ or $\frac{6}{9}$ is greater.

Solution

- LCM of denominator

7 and 9 = 63

$\frac{5}{7} \times 63 = 5 \times 9$

$= 45$

$\frac{6}{9} \times 63 = 6 \times 7$

$= 42$

$\therefore$ 5/7 is greater than 6/9

2017

2\. (a) Determine the improper fraction of $\frac{3}{5} \times 4\frac{1}{5} \div \frac{18}{25}$.

(b) Convert $\frac{1}{3}$ into a repeating decimal.

Solution

$\frac{3}{5} \times 4\frac{1}{5} \div \frac{18}{25}$

$\frac{3}{5} \times \frac{21}{5} \div \frac{18}{25}$

BODMAS

$\frac{3}{5} \times \frac{21}{5} \times \frac{25}{18} = \frac{3}{\cancel{5}} \times \frac{21}{\cancel{5}} \times \frac{\cancel{25}}{18}$

$= \frac{3 \times 21}{18}$

$= \frac{21}{6} = \frac{7}{2}$

$\therefore = \frac{7}{2}$

(b) Convert $\frac{1}{3}$ into a repeating decimal.

Solution

To convert 1/3 into a repeating decimal.

Divide 1/3

$$
\begin{array}{r}
0.33333\ldots \\
3\overline{)10} \\
-9 \\
\hline
10 \\
-9 \\
\hline
10 \\
-9 \\
\hline
10 \\
-9 \\
\hline
10 \\
-9 \\
\hline
1
\end{array}
$$

0.33333..... into repeating decimal = $0.\dot{3}$

2015

12. In a certain animal farm 10% of the animals are horses, $\frac{1}{4}$ are goats, 0.15 are sheep and $\frac{1}{2}$ are cattle. Arrange these numbers in ascending order.

$\frac{1}{4} \times 100\% = 25\%$, $0.15 \times 100\% = 15\%$, $\frac{1}{2} \times 100\% = 50\%$, 10%

Goats, Sheep, Cattle, Horses

Ascending order = 10%, 0.15, $\frac{1}{4}$, $\frac{1}{2}$

$\therefore$ Ascending order = 10%, 0.15, $\frac{1}{4}$, $\frac{1}{2}$

2015

13. Write 750 grams as a fraction of 5 kilograms.

1 kilogram = 1000 grams
5 kilograms = ?
? = $\frac{5\cancel{kg} \times 1000g}{1\cancel{kg}}$ $x = 5{,}000$ grams

$= \frac{75\cancel{0}\ \cancel{grams}}{500\cancel{0}\ \cancel{grams}}$

$= \frac{3}{20}$ Ans

$= \frac{3}{20}$ Answer

Decimals and Percentages

2020

(b) (i) In a sales promotion, the price of a shirt costing shs. 15,000 is reduced by 15%. What is the new price of the shirt?

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Data. Soln

Percentage reduced = 15%
The price of a shirt = 15000
The amount reduced = ?

$\therefore$ Amount reduced $= \frac{15}{100} \times 15000$

Amount reduced = 2250

$\therefore$ The new price = 15000 − 2250
= 12750

$\therefore$ The new price of the shirt is shs. 12750

2020

(ii) Change $0.5\dot{6}$ into a fraction in its simplest form.

Soln

Given $0.5\dot{6}$

change it into fraction.

Let $x = 0.5\dot{6}$

Multiply by 10 in both sides

$10x = 5.66$

$10x - x = 5.66 - 0.56$

$9x = 5.1$

$9x = 5.1$

$\frac{9x}{9} = \frac{5.1}{9}$

$x = \frac{5.1 \times 10}{9 \times 10}$

$x = \frac{51}{90} = \frac{17}{30}$

$\therefore 0.5\dot{6} = \frac{17}{30}$

2018

(b) If 0.125 of all students in a mixed class are girls, what percentage of the students are boys?

(b) If 0.125 of all students in a mixed class are girls, what percentage of the students are boys?

Solution.

The number of students = 100%.

% of girls + % of boys = 100%

0.125 of students = girls.

0.125 to percentage

$0.125 \times 100\% = 12.5\%$

Recall:

% of girls + % of boys = 100%

$12.5\% + x = 100\%$

$x = 100\% - 12.5\%$

$x = 87.5\%$

$\therefore$ Percentage of the students who are boys is 87.5%

Units

2020

3. (a) A lorry carries 7.2 tonnes of sand from the mining area to the industrial site. On the way 230 kg of sand either fall off or blow away. What mass of sand will remain by the end of the journey? Give the answer in tonnes.

Soln.

Given

Total tonnes of sand = 7.2 tonnes.

We change into kg.

1 tonne = 1000 kg.

7.2 t = ?

$\frac{7.2\,t \times 1000\,kg}{1\,t}$

= 7200 kg.

Mass remaining = 7200 kg − 230 kg = 6970 kg.

Mass remaining was 6970 kg.

We change into tonnes.

1 t = 1000 kg.

? = 6970 kg

$\frac{6970\,kg \times 1\,t}{1000\,kg}$

= 6.97 tonnes.

∴ Mass remaining at the end of the journey was 6.97 tonnes.

2018

3. (a) Subtract:

m	dm	cm	mm
10	9	31	2
- 8	9	38	9

3. (a) Subtract:

m	dm	cm	mm
0 9 $\cancel{10}$	10 + 8 $\cancel{9}$	30 $\cancel{31}$ + 10	2 + 10
- 8	9	38	9

Solution

m	dm	cm	mm
9	18	40	12
− 8	9	38	9
1	9	2	3

∴ = 1m 9dm 2cm 3mm.

(b) Find the simple interest on sh. 10,000,000 invested for 5 years at the rate of 6% per annum.

(b) Find the simple interest on sh. 10,000,000 invested for 5 years at the rate of 6% per annum.

Solution

Given:

P = 10,000,000

t = 5 years

R = 6% p.a

From $\boxed{I = \frac{P \times R \times T}{100}}$

$I = \frac{10,000,0\cancel{00} \times 6 \times 5}{1\cancel{00}}$

$I = 100,000 \times 30$

$I = 3000,000$

$\therefore$ Interest was sh. 3000,000

2015

3. (a) Change 15 km into centimeters.

(b) Find the time in which sh. 200,000 will earn sh. 48,000 at the rate of 4% interest per annum.

3. (a) Change 15 km into centimeters.

K	H	D	M	d	c	m
1	0	0	0	0	0	0
	1	0	0	0	0	0
		1	0	0	0	0
			1	0	0	0
				1	0	0
					1	0

1km = 100,000cm

15km = ?

$\frac{15\cancel{km} \times 100,000cm}{1\cancel{km}} = 1500,000cm$

$\therefore = 1,500,000cm$

(b) Find the time in which sh. 200,000 will earn sh. 48,000 at the rate of 4% inte per annum.

Solution:

Principal = 200,000/=

Interest = 48,000/=

Rate = 4%

$\times 100 \quad I = \frac{PRT}{100} \times 100$

$\frac{100I}{PR} = \frac{PRT}{PR}$

$\boxed{\therefore T = \frac{100I}{PR}}$

$\text{Time} = \frac{100I}{PR}$

$\text{Time} = \frac{100 \times 48,000}{200,000 \times 4}$

$\text{Time} = \frac{\cancel{4800000}\ 6}{\cancel{800000}\ 1}$

$\text{Time} = 6/1 = 6$

$\therefore$ Time = 6 years

Approximations

2020

(b) Write the number 0.009765;
(i) correct to three decimal places
(ii) correct to three significant figures
State the place value of 9 in the given number.

(b) Write the number 0.009765;
(i) correct to three decimal places
(ii) correct to three significant figures
State the place value of 9 in the given number.

Soln

given
0.009765

i) to Three decimal places

$0.009765 \approx 0.010$ (7 is greater than 5)

$\therefore$ $0.009765 \approx 0.010$

ii) to three significant figure

$0.009765 \approx 0.00976$ (Number before 5 is even (6))

$\therefore$ $0.009765 \approx 0.00976$

Place value of 9 in 0.009765
One Tenth hundredth thousandth

$\therefore$ Place value of 9 in 0.009765 is a ~~hundredth~~ thousandth

2017

(b) Evaluate $0.864 \div 0.0246$ giving your answer correct to 2 significant figures.

Soln

$0.864 \div 0.0246 =$

$\frac{0.864}{0.00246} \times \frac{100\cancel{000}}{10\cancel{0000}} =$

$= \frac{86400}{246}$

$= 351.21$

$= 35000$

2015

2. The number of students who sat for the Primary School Leaving Examination (PSLE) in 2013 was $44,918. Express this number in standard notation.

Solution

844,938

$\Rightarrow$ 8 4 4 9 3 8

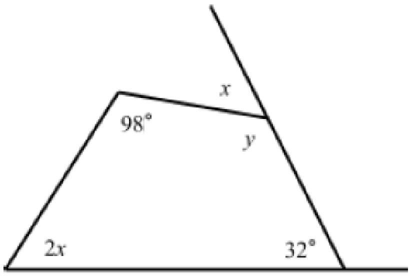
$\Rightarrow 8.44938 \times 10^5$

$\therefore$ Answer $=$ $\underline{8.44938 \times 10^5}$

Geometry

2020

4. (a) Find the values of x and y in the following figure.



Soln.

Given: Quadrilateral $= 360°$.

$2x + 98° + y + 32° = 360°$

$2x + y + 130° = 360°$

$2x + y = 360° - 130°$

$2x + y = 230°$

$2x + y = 230°$ ---- 1st Equation

Straight line $= 180°$

$x + y = 180°$.

$x + y = 180$ ---- 2nd Equation.

$\begin{cases} 2x + y = 230° \\ x + y = 180° \end{cases}$

$-\begin{cases} 2x + y = 230° \\ x + y = 180° \end{cases}$

$x = 230° - 180°$

$x = 50°$.

But: If $= x = 50°, y = ?$

Take eqn ii)

$x + y = 180°$.

$50° + y = 180°$

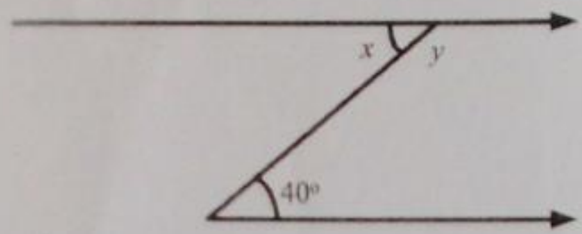
$y = 180° - 50°$

$y = 130°$

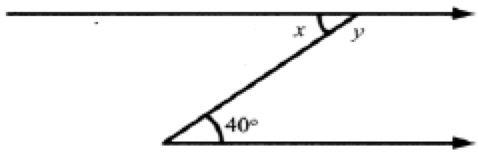
$\therefore$ Value of $\underline{x = 50°}$ and $\underline{y = 130°}$

2018

4. (a) Calculate the size of angle x and y in the following figure:



4. (a) Calculate the size of angle x and y in the following figure:



Solution:

$x = 40°$ < Alternate Interior Angles >

$x + y = 180°$ < Sum of angles in a straight line >

But, $x = 40°$

So,

$40° + y = 180°$

$y = 180° - 40°$

$y = 140°$

∴ The size of angle x and y are $40°$ and $140°$ respectively

2015

3. If A and B are complementary angles such that angle A is $18°$ less than angle B, determine the angles.

soln:

$A + B = 90°$

But $A = B - 18°$

$B - 18° + B = 90°$

$2B - 18° = 90°$

$2B = 90° + 18°$

$\frac{2B}{2} = \frac{108°}{2}$

$B = 54°$

from:

$A = B - 18°$

$A = 54° - 18°$

$A = 36°$

∴ $A = 36°$ while $B = 54°$

Real Numbers

Ratio, Profit and Loss

2020

(b) An article was sold for shs 160,000 at a profit of 25%. Find the buying price of the article.

soln

Given

Percentage profit = 25%.

Selling price = 160,000/=

Buying price = ?

From,

$$\text{Percentage Profit} = \frac{S.P - B.P}{B.P} \times 100\%$$

$$25\% = \frac{160,000 - B.P \times 100}{B.P}$$

$$25 = \frac{16000000 - 100B.P}{B.P}$$

$$25B.P = 16,000,000 - 100B.P$$

$$100B.P + 25B.P = 16,000,000$$

$$\frac{125B.P}{125} = \frac{16,000,000}{125}$$

$$B.P = 128,000/=$$

Buying price = Shs. 128,000/=

2015

20. Kapona bought a computer for 250,000/= and sold it after one years at a loss of 5 percent. Calculate the amount of the loss.

solution

Buying price = 250,000/=

Loss = 5%

Amount loss = ?

$$\text{Percentage loss} = \frac{\text{loss}}{\text{Buying price}} \times 100\%$$

$$5\% = \frac{\text{Loss}}{250,000} \times 100\%$$

$$\frac{5\%}{1} = \frac{L \times 100\%}{250,000}$$

$$\frac{100L}{100} = \frac{250,000 \times 5}{100}$$

$$L = 12500$$

$\therefore$ Loss is 12,500/=

Coordinate Geometry

2020

6. (a) (i) Find the equation of a line passing through the point P(-1,4) and has a gradient of 10.

Soln.

$P(x_1,y_1) = P(-1,4)$; Gradient $(M) = 10$.

From, $y - y_1 = M(x - x_1)$

$y - 4 = 10(x - {}^-1)$

$y - 4 = 10(x + 1)$

$y - 4 = 10x + 10$

$y - 4 = 10x + 10$

$y = 10x + 10 + 4$

$y = 10x + 14$.

$\therefore$ The equation of the line is $y = 10x + 14$.

(ii) If the line of the equation you obtained in part (a) (i) passes through the points $(a,0)$ and $(0,b)$, what will be the values of a and b?

Soln.

$(a,0)$ $(0,b)$

$y = 10x + 14$ eqn.

i, $(a,0)$ (x intercept) $y=0$

$y = 10x + 14$.

$0 = 10a + 14$

$-14 = 10a$

$\frac{-14}{10} = \frac{10a}{10}$

$a = {}^-1.4$.

ii, $(0,b)$ (y intercept) $x=0$.

$y = 10x + 14$.

$b = 10(0) + 14$.

$b = 0 + 14$

$b = 14$

$\therefore$ The value of a is ${}^-1.4$ and b is 14.

2017

6. (a) If the slope of the straight line through the points (7, 4) and (-2, *k*) is 1, find the value of *k*.

(a) If the slope of the straight line through the points (7, 4) and (-2, *k*) is 1, find the value of *k*.

Solution

$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad (7, 4)$ $(-2, k)$

$1 = \frac{k - 4}{{}^-2 - 7}$

$\frac{1}{1} = \frac{k - 4}{-9}$

$k - 4 = {}^-9$

$k = {}^-9 + 4$

$k = {}^-5$

$\therefore k = {}^-5$

2015

18. If the straight line AB that is passing through the points $A(2, 6)$ and $B(t, 3)$ has gradient -1, find the value of t.

$$M = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-1 = \frac{3-6}{t-2}$$

$$-t + 2 = -3$$

$$t = 2 + 3$$

$$t = 5$$

$$\therefore t = 5$$

Perimeters and Areas

2020

(b) Suppose a metal wire is bent to form a semi-circle with a radius of 14 cm. Find;

(i) the total length of the metal wire.

(ii) the area bounded by the metal wire.

Soln.

14cm 14cm.

i) Total length = Perimeter/Circumference

Circumference of Semi Circle $= \frac{\pi d}{2} + d$

diameter (d) = radius $\times 2$

$= 14 \times 2$

$= 28$ cm

$\frac{\pi d}{2} + d$.

$\frac{\frac{22}{7} \times 28}{2} + 28$

$\frac{22 \times 4}{2} + 28$

$44 + 28$

72 cm.

Total length = 72 cm

ii) Area of Semicircle Circle.

$= \frac{\pi r^2}{2}$

$\frac{\frac{22}{7} \times 14 \times 14}{2}$

$\frac{22 \times 2 \times 14}{2}$

$22 \times 2 \times 7$

44×7

308 cm^2

Area bounded $= 308 \text{ cm}^2$

2018

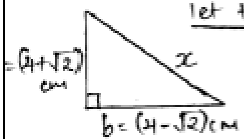
(b) Find the perimeter of a right angled triangle whose base is $(4-\sqrt{2})$ cm and height is $(4+\sqrt{2})$ cm.

(b) Find the perimeter of a right angled triangle whose base is $(4-\sqrt{2})$ cm and height is $(4+\sqrt{2})$ cm.

Solution:

From the right angled triangle:

let the hypotenuse be x



Perimeter:

Perimeter of triangle $= L+L+L$

$P = b+h+x$

Apply Pythagoras theorem:

$c^2 = a^2+b^2$

$x^2 = (4+\sqrt{2})^2 + (4-\sqrt{2})^2$

$x^2 = (4+\sqrt{2})(4+\sqrt{2}) + (4-\sqrt{2})(4-\sqrt{2})$

$x^2 = 16+4\sqrt{2}+4\sqrt{2}+2+16-4\sqrt{2}-4\sqrt{2}+2$

$x^2 = 16+16+2+2+8\sqrt{2}-8\sqrt{2}$

$x^2 = 36+(8-8)\sqrt{2}$

$\sqrt{x^2} = \sqrt{36}$

$x = \pm 6$cm

From, $P = b+h+x$

$= (4-\sqrt{2})+(4+\sqrt{2})+6$

$= 4-\sqrt{2}+4+\sqrt{2}+6$

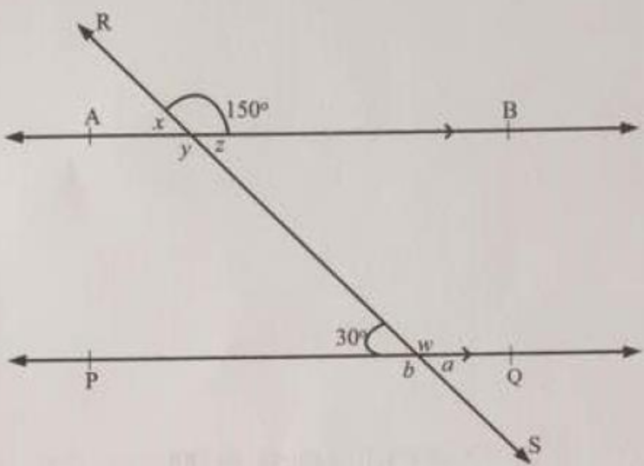
$= 4+4+6+(1-1)\sqrt{2}$

$P = 14$cm

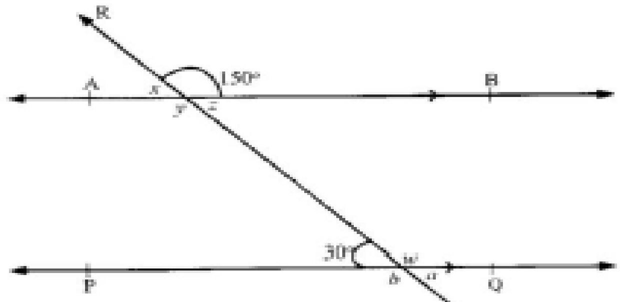
$\therefore$ Perimeter of a right angled triangle is 14cm

2017

4\. (a) In the following figure, $\overline{AB}$ is parallel to $\overline{PQ}$ and $\overline{RS}$ is a transversal. Find the angles labeled a, b, w, x, y and z.



(b) Find the perimeter of a square, if its area is $25\ cm^2$.



$x + 150° = 180°$ (By straight line rule)

$x = 180° - 150°$

$\underline{x = 30°}$

$\underline{w = 150°}$ (corresponding angles)

$w = b$ (vertical opposite angles)

$\underline{b = 150°}$

$30° = a$ (vertical opposite angles)

$\underline{a = 30°}$

$x = z$ (vertical opposite angles)

$\underline{z = 30°}$

$150° = y$ (vertical opposite angles)

$\underline{y = 150°}$

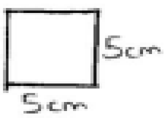
(b) Find the perimeter of a square, if its area is $25\ cm^2$.

$A = S \times S$

$\sqrt{25cm^2} = \sqrt{S^2}$

$5cm = S$

One side $= 5cm$



$P = S \times 4$

$P = 5cm \times 4$

$\underline{P = 20cm}$

2015

8. The length of one side of a square is $(3x + 4)$ cm. If the side lengths of this square are doubled, find the equation for the perimeter after changing the length of the square.

<u>soln</u>

perimeter $= 4 \times l = 4l$

given $l = 3x + 4$ cm

after being doubled

New length $= 2(3x + 4)$ cm

New length $= 6x + 8$

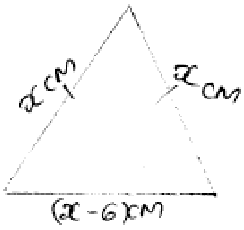
perimeter $= 4(6x + 8)$

$= (24x + 32)$ cm

2015

19. If a triangle has two equal sides of length x cm each and the third side measures 6 cm more than the length of these congruent sides, write down an equation that represents the perimeter of this triangle.

soln



Perimeter of Δ = $\text{side}_1 + \text{side}_2 + \text{side}_3$

$= x\text{cm} + x\text{cm} + (x+6)\text{cm}$

$= 2x\text{cm} + x + 6$

$= (3x + 6)\text{cm}$

$\therefore$ Perimeter of a triangle $= (3x+6)$cm

2015

21. The area of a rectangular room is 1125 cm^2 . If its length is five times its width, find its perimeter.

solution

$5 \times w = 5w$

Area $= L \times W$

$1125 = 5w \times w$

$1125 = 5w^2$

$\frac{1125}{5} = \frac{5w^2}{5}$

$\sqrt{225} = \sqrt{w^2}$

$15 = w$

Width = 15cm

Length = 5×15

= 75cm

Perimeter $= 2(L + w)$

$P = 2(75+15)$

$P = 2(90)$

$P = 180$

$\therefore$ Perimeter is 180cm.

Exponents and Radicals

2020

7. (a) If $(3^{x+3})(5^{2-y}) = \left(\frac{1}{3}\right)^5 \left(\frac{1}{5}\right)$, find the values of x and y.

(b) (i) Find the value of 0.0000234×120 in standard notation, correct to three significant figures.

(ii) Rationalize the denominator of the expression $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

7. (a) If $(3^{x+3})(5^{2-y}) = \left(\frac{1}{3}\right)^5 \left(\frac{1}{5}\right)$, find the values of x and y.

Soln:

$(3^{x+3})(5^{2-y}) = (3^{-1})^5(5^{-1})$

$(3^{x+3})(5^{2-y}) = (3^{-5})(5^{-1})$

$3^{x+3} \times 5^{2-y} = 3^{-5} \times 5^{-1}$

$3^{x+3} = 3^{-5}$ $\qquad$ $2-y = -1$

$x+3 = -5$ $\qquad$ $-y = -1-2$

$x = -5-3$ $\qquad$ $-y = -3$

$x = -8$ $\qquad$ $\frac{-y}{-1} = \frac{-3}{-1}$

$y = 3$

$\therefore$ The values of x is -8 and y is 3

(b) (i) Find the value of 0.0000234×120 in standard notation, correct to three significant figures.

Soln:

0.0000234×120

$2.34 \times 10^{-5} \times 1.2 \times 10^{2}$

$2.34 \times 1.2 \times 10^{-5} \times 10^{2}$

2.808×10^{-3}

$\approx 2.81 \times 10^{-3}$

$\therefore \underline{0.0000234 \times 120 = 2.81 \times 10^{-3}}$

(ii) Rationalize the denominator of the expression $\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

Soln:

$$\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$\frac{\sqrt{2}(\sqrt{3}-\sqrt{2})}{\sqrt{3}(\sqrt{3}-\sqrt{2})+\sqrt{2}(\sqrt{3}-\sqrt{2})}$$

$$\frac{\sqrt{6}-2}{3-\sqrt{6}+\sqrt{6}-2}$$

$$\frac{\sqrt{6}-2}{3-2} = \frac{\sqrt{6}-2}{1} = \sqrt{6}-2$$

$$\therefore \frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \underline{\sqrt{6}-2}$$

2018

7. (a) Use laws of exponents to simplify $\frac{(2r^3)^2}{(2r)^3}$.

7. (a) Use laws of exponents to simplify $\frac{(2r^3)^2}{(2r)^3}$.

Solution.

from: $(ab)^2 = a^2 \times b^2$

$$\frac{(2r^3)^2}{(2r)^3}$$

$$= \frac{2^2 \times r^{3\times 2}}{2^3 \times r^3}$$

$$= \frac{4^1 \times r^6}{8_2 \times r^3}$$

$$= \frac{r^{6-3}}{2} = \frac{r^3}{2}$$

$$\therefore \underline{\frac{(2r^3)^2}{(2r)^3} = \frac{r^3}{2}}$$

2015

9. Find the value $\frac{a}{b}$, given that $3^a \times 5^b = 675$.

SOLUTION.

Find prime factors of 675

3	675
3	225
3	75
5	25
5	5
	1

$= 3^3 \cdot 5^2$

$3^a \times 5^b = 675$

$\therefore 3^a \times 5^b = 3^3 \times 5^2$

The value of $a = 3$

The value of $b = 2$

The value of $a/b = 3/2$

$\therefore$ The value of $a/b = 3/2$

Algebra

2020

5. (a) The sum of two numbers is 127. If the difference between the numbers is 7, find the numbers.

Soln.

Given:-

sum $= 127$

difference $= 7$

let larger no. be x

smaller no. be y

$x + y = 127$ - - - (i)

$x - y = 7$ - - - (ii)

combine:-

$$- \begin{cases} x + y = 127 \\ x - y = 7 \end{cases}$$

$2y = 120$

$\frac{2y}{2} = \frac{120}{2}$

$y = 60$.

From - $x - y = 7$

but $y = 60$

$x - 60 = 7$

$x = 7 + 60$

$x = 67$

$\therefore$ The two numbers are 67 and 60.

2018

5. (a) Solve $\begin{cases} 2x + y = 20 \\ x = 35 - 3y \end{cases}$ by the elimination method.

5. (a) Solve $\begin{cases} 2x + y = 20 \\ x = 35 - 3y \end{cases}$ by the elimination method.

$\begin{cases} 2x + y = 20 \\ x + 3y = 35 \end{cases}$ $\left(\begin{array}{l} x = 35 - 3y \\ = x + 3y = 35 \end{array} \right)$

$\begin{cases} (2x + y = 20)1 \\ (x + 3y = 35)2 \end{cases}$

$- \begin{cases} 2x + y = 20 \\ 2x + 6y = 70 \end{cases}$

$2x - 2x + y - 6y = 20 - 70$

$-5y = -50$

$y = 10.$

$\begin{cases} (2x + y = 20)3 \\ (x + 3y = 35)1 \end{cases}$

$\begin{cases} 6x + 3y = 60 \\ x + 3y = 35 \end{cases}$

$6x - x + 3y - 3y = 60 - 35$

$5x = 25$

$x = 5.$

$\therefore x = \underline{5}$

$y = \underline{10.}$

2017

5. (a) Find the value of x in the equation $9 \times 3^{4x} = 27^{(x-1)}$.

(b) Factorize the expression $6x^2 - 11x + 4$ by splitting the middle term.

5. (a) Find the value of x in the equation $9 \times 3^{4x} = 27^{(x-1)}$.

Soln.

$$9 \times 3^{4x} = 27^{(x-1)}$$

$$3^2 \times 3^{4x} = 27^{(x-1)}$$

$$3^2 \times 3^{4x} = 3^{3(x-1)}$$

$$3^2 \times 3^{4x} = 3^{3x-3}$$

$$3^{(2+4x)} = 3^{(3x-3)}$$

$$2+4x = 3x-3$$

$$2+3 = 3x-4x$$

$$5 = -x.$$

$$\frac{5}{-1} = \frac{-x}{-1}$$

$$x = -5$$

$\therefore\ x = -5.$

(b) Factorize the expression $6x^2 - 11x + 4$ by splitting the middle term.

Soln.

Required to factorize $6x^2 - 11x + 4$.

$6x^2 - 11x + 4$, $6 \times 4 = 24$

Factors of $24 = 1, 2, 3, 4, 6, 8, 12, 24$.

Appropriate pair is 3 and 8.

$$6x^2 - 3x - 8x + 4$$

$$(6x^2 - 3x)(-8x + 4)$$

$$3x(2x-1) - 4(2x-1)$$

$$(3x-4)(2x-1).$$

$\therefore 6x^2 - 11x + 4 = (3x-4)(2x-1).$

2015

1. Calculate the value of $x + y + 2z - 12$, when $x = 5$, $y = 8$ and $z = 9$.

Solution

$= x + y + 2z - 12$

where $x = 5, y = 8, z = 9$ (substitute)

$= 5 + 8 + 2 \times 9 - 12$

$= 5 + 8 + 18 - 12$

$= 13 + 18 - 12$

$= 31 - 12$

$= 19$

$\therefore x + y + 2z - 12 = 19$

2015

4. Find the value of x in the equation $\frac{6}{x+1} = 12$.

$\frac{6}{x+1} = 12$

$\frac{6}{x+1} \times \frac{12}{1}$

$12(x+1) = 6(1)$

$12x + 12 = 6$.

$12x = 6 - 12$.

$\frac{12x}{12} = \frac{-6}{12}$

$x = -\frac{1}{2}$.

2015

5. Simplify the expression $9(a - 3b) + 5(4b + a) - b$.

Solution:

$= 9(a - 3b) + 5(4b + a) - b$

$= 9a - 27b + 20b + 5a - b$

$= 14a - 7b - b$

$= 14a - 8b$

$\therefore \Rightarrow 14a - 8b$.

2015

6. When 6 is subtracted from a certain number, the result is greater than 29. Write down an inequality that represents the possible values of this number.

Soln

Let the number be x

$x - 6 > 29$

$x - 6 + 6 > 29 + 6$

$x > 35$

$\therefore \; x : x > 35$

2015

7. Without using mathematical tables, evaluate: $\frac{(0.136)^2-(0.148)^2}{0.136-0.148}$.

solution

$= \frac{(0{\cdot}136)^2 - (0{\cdot}148)^2}{0{\cdot}136 + 0{\cdot}148}$

$= \frac{(0{\cdot}136 + 0{\cdot}148)(0{\cdot}136 - 0{\cdot}148)}{(0{\cdot}136 + 0{\cdot}148)}$

$= 0{\cdot}136 - 0{\cdot}148$

$= {}^{-}0{\cdot}012$

$\therefore \frac{(0{\cdot}136)^2 - (0{\cdot}148)^2}{0{\cdot}136 + 0{\cdot}148} = {}^{-}0{\cdot}012$ (Answer

2015

10. The football ground at Merita secondary school is $12\frac{1}{2}$ times as long as the length of the basketball ground. If the football ground is 100 meters long, find the length of the basketball ground.

Solution

let the Length of basketball be x

Football ground is 100meters long

$100m = 12\frac{1}{2}x$

$100m \times = \frac{25x}{2}$

$\frac{\overset{8}{\cancel{200}}m}{\cancel{25}} = \frac{\cancel{25}x}{\cancel{25}}$

$8m = x$

But x is the Length of basketball ground.

$\therefore$ The length of basketball ground is 8m

2015

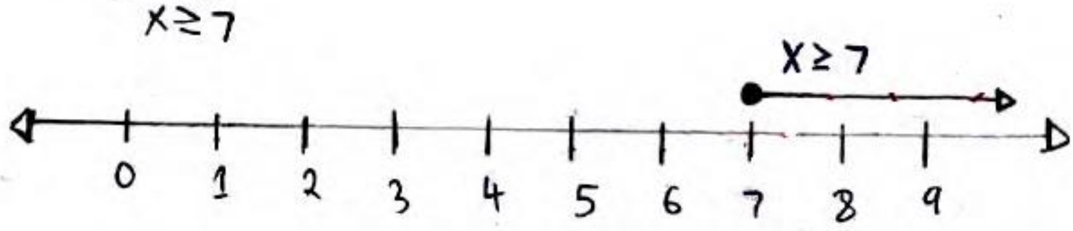
11. Represent the solution set of the inequality $3x + 4 \geq 25$ on a number line.

$3x + 4 \geq 25$

$3x + 4 - 4 \geq 25 - 4$

$\frac{\cancel{3}x}{\cancel{3}} \geq \frac{21}{3}$

$x \geq 7$



2015

14. If John is x years old and Mary is 3 years older than John, write down an equation for the sum of their ages

Solution

John is X years old

But Mary is $X+3$ years old

$X + (X+3)$

$\therefore$ The equation will be $x+(x+3)$ or $2x+3$

2015

15. Determine the value of x that satisfies the equation $\frac{x-y^2}{x+2} = 7$ given that $y = 2$.

Given $y = 2$

$\frac{x - y^2}{x+2} = 7$

$\frac{x - (2)^2}{x+2} = 7$

$= \frac{x-4}{x+2} = 7$

$x - 4 = 7(x+2)$

$x - 4 = 7x + 14$

$x - 7x = 14 + 4$

$-6x = 18$

$\frac{-6x}{-6} = \frac{18}{-6}$

$x = -3$

$\therefore$ The value of x is -3

Quadratic equations

2020

(b) Solve the equation $x^2 - 10x + 13 = 0$ by completing the square. Leave the answer in surd form.

soln.

Given:-

$x^2 - 10x + 13 = 0$

$x^2 - 10x + 13 = 0$

$x^2 - 10x = -13$

$b = -10$

$x^2 - 10x + \left(-10 \times \tfrac{1}{2}\right)^2 = -13 + \left(-10 \times \tfrac{1}{2}\right)^2$

$x^2 - 10x + (-5)^2 = -13 + (-5)^2$

$x^2 - 10x + (-5)^2 = -13 + 25$

$(x-5)^2 = 12$

Find square root.

$\pm\sqrt{(x-5)^2} = \pm\sqrt{12}$

$x - 5 = \pm\sqrt{12}$

$x = 5 \pm \sqrt{12}$

$x = 5 \pm \sqrt{12}$

$\therefore x = 5 \pm 2\sqrt{3}$

2018

(b) Solve the equation $4(p+1)(1-p) = 3$.

(b) Solve the equation $4(p+1)(1-p) = 3$.

$4(p+1)(1-p) = 3$

$4(1+p)(1-p) = 3$

$4(1^2 - p^2) = 3$

$4(1 - p^2) = 3$

$4 - 4p^2 = 3$

$4p^2 - 4 + 3 = 0$

$4p^2 - 1 = 0$

$(2p)^2 - (1)^2 = 0$.

$(2p-1)(2p+1) = 0$

$2p - 1 = 0$

$2p + 1 = 0$.

$2p - 1 = 2p = 1$

$p = \tfrac{1}{2}$

$2p + 1 = 0$

$2p = -1$

$p = -\tfrac{1}{2}$

$\therefore p = \tfrac{1}{2}$ or $-\tfrac{1}{2}$

Logarithms

2018

(b) If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 7 = 0.8451$, find $\log 42$.

(b) If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 7 = 0.8451$, find $\log 42$.

Solution

$\log 42$

$= \log (7 \times 6)$

$= \log (7 \times 2 \times 3)$

Product rule

$- \log (7 \times 2 \times 3)$

$= \log 7 + \log 2 + \log 3$

Recall:

$\log 2 = 0.3010$

$\log 3 = 0.4771$

$\log 7 = 0.8451$

Then

$= \log 7 + \log 2 + \log 3$

$= 0.8451 + 0.3010 + 0.4771$

$= 0.8451 + 0.7781$

$= 1.6232$

$\underline{\log 42 = 1.6232}$

2017

7\. (a) Rationalize the denominator of $\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}}$.

(b) Without using mathematical tables, find the value of $3\log_{10} 5 + 5\log_{10} 2 - 2\log_{10} 2$

(a) Rationalize the denominator of $\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}}$.

Solution

$$\frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}}$$

$$= \frac{\sqrt{2}}{\sqrt{10}-\sqrt{2}} \times \frac{\sqrt{10}+\sqrt{2}}{\sqrt{10}+\sqrt{2}}$$

$$= \frac{\sqrt{20}+\sqrt{4}}{\sqrt{100}+\cancel{\sqrt{20}}-\cancel{\sqrt{20}}-\sqrt{4}}$$

$$= \frac{\sqrt{20}+2}{10-2}$$

$$= \frac{\sqrt{20}+2}{8}$$

(b) Without using mathematical tables, find the value of $3\log_{10} 5 + 5\log_{10} 2 - 2\log_{10} 2$.

Soln

$3\log_{10}5 + 5\log_{10}2 - 2\log_{10}2$

$= \log_{10}5^3 + \log_{10}2^5 - \log_{10}2^2$

$= \log_{10}125 + \log_{10}32 - \log_{10}4$

$= \log_{10}125 + \log_{10}32/4$

$= \log_{10}125 + \log_{10}8$

$= \log_{10}125 \times 8$

$= \log_{10}1000$

$= \log_{10}10^3$

$= 3\log_{10}10.$

$= 3$

2015

16. Write $4\log 2 - \frac{1}{2}\log 64$ as a single logarithmic expression.

solution

$4\log 2 - \log (64)^{\frac{1}{2}}$

$\log 2^4 - \log\sqrt{64}$

$\log 16 - \log 8$

$\log 16/8$

$\log 2$

$\therefore$ The answer is $\log 2$.

2015

22. Evaluate $\frac{1.34 \times 5.804}{\sqrt{0.4391}}$ using logarithmic tables.

Solution

Number	Standard form	Logarithm
1·34	$1{\cdot}34 \times 10^0$	0·1271
5·804	$5{\cdot}804 \times 10^0$	0·7637 +
	NUMERATOR:	0·8908
$(0{\cdot}4391)^{1/2}$	$4{\cdot}391 \times 10^{-1}$	$\frac{1}{2}(\bar{1}{\cdot}6426) = \bar{1}{\cdot}8213$
	DENOMINATOR:	$\bar{1}{\cdot}8213$

$= 0{\cdot}8908 - \bar{1}{\cdot}8213$

$= 1{\cdot}0695$

$= 1{\cdot}173 \times 10^1$

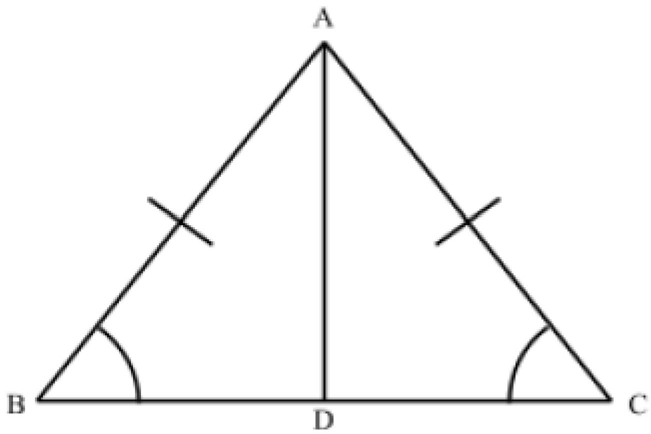
$\therefore$ The answer is $\underline{1{\cdot}173 \times 10^1}$ or $\underline{11{\cdot}73}$

$\therefore \frac{1{\cdot}34 \times 5{\cdot}804}{\sqrt{0{\cdot}4391}} = 11{\cdot}73$

Congruence

2020

8. (a) In the following figure, $\overline{AB} = \overline{AC}$, prove that ABC and ACD are also equal.



B D C

Soln.

Given $\overline{AB} = \overline{AC}$

Required to prove that $A\hat{B}D$ and $A\hat{C}D$ are equal.

$\overline{AB} = \overline{AC}$ (given)

$\overline{AD}$ is common.

$D\hat{A}C = D\hat{A}B$ (included angles)

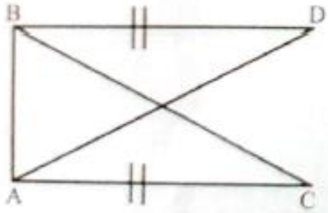
$\Delta ABD \equiv \Delta ACD$ (SAS rule).

Since $\Delta ABD \equiv \Delta ACD$ then $A\hat{B}D = A\hat{C}D$

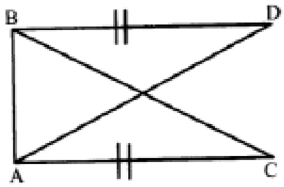
$\therefore$ $\underline{A\hat{B}D = A\hat{C}D \text{ Hence proven}}$

2018

(b) The figure below shows that $\overline{AC} = \overline{BD}$. Prove that $A\hat{C}B = A\hat{D}B$.



(b) The figure below shows that $\overline{AC} = BD$. Prove that $ACB = ADB$.



Solution

Consider ΔABD and ΔBAC

$\overline{BD} = \overline{AC}$ (Given) ... S

$\overline{BA}$ is common ... S

$B\hat{A}C = A\hat{B}D = 90^\circ$ ... A

$\therefore \Delta ABD \equiv \Delta BAC$ (by SAS)

Hence; All corresponding sides and angles are equal

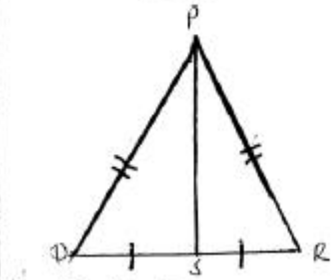
$A\hat{C}B = A\hat{D}B$ (by definition of congruence)

2017

8. (a) PQR is an isosceles triangle whereby $\overline{PQ} = \overline{PR}$ and $\overline{QS} = \overline{SR}$. If S is a point between Q and R prove that $\Delta PQS \equiv \Delta PRS$.

8. (a) PQR is an isosceles triangle whereby $\overline{PQ} = \overline{PR}$ and $\overline{QS} = \overline{SR}$. If S is a point between Q and R prove that $\Delta PQS \equiv \Delta PRS$.

Soln



Given: $\overline{PQ} = \overline{PR}$, $\overline{QS} = \overline{SR}$

Required to prove that $\Delta PQS \equiv \Delta PRS$

Argument	Reason
$\overline{PQ} = \overline{PR}$	Given
$\overline{QS} = \overline{SR}$	Given
$\overline{PS} = \overline{PS}$	Common

$\Delta PQS \equiv \Delta PRS$ by (SSS - Theorem)

$\therefore$ Hence Proved

16. Similarity

2020

(b) If the rectangular metal sheets $ABCD$ and $WXYZ$ are similar, calculate the length of $\overline{XY}$ when $\overline{AB} = 2\ cm$, $\overline{BC} = 4\ cm$ and $\overline{WX} = 2.5\ cm$.

Sol'n

$\frac{XY}{AB} = \frac{BC}{WX}$

$\frac{XY}{2cm} = \frac{4cm}{2.5cm}$

$2.5cm\ XY = 4cm \times 2cm$

$\frac{2.5\cancel{cm}\ XY}{\cancel{cm}} = \frac{8\cancel{cm}^2}{\cancel{cm}}$

$\frac{2.5XY}{2.5cm} = \frac{8cm^2}{2.5cm}$

$XY = \frac{8cm \times 10}{25 \times 10}$

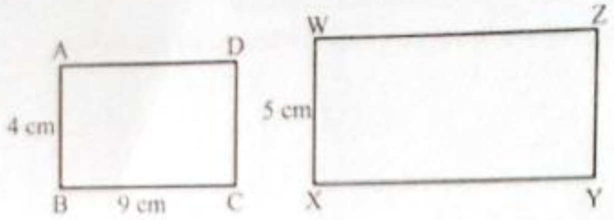
$XY = \frac{80cm}{25}$

$XY = 3\ cm$

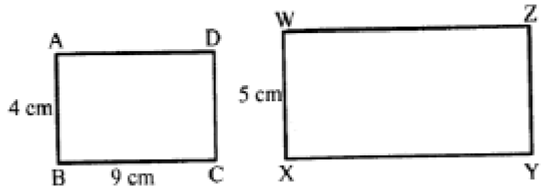
$\therefore$ The length of $\overline{XY}$ is $3cm$

2018

8. (a) Rectangle $ABCD$ is similar to rectangle $WXYZ$. If $\overline{BC} = 9cm$, $\overline{AB} = 4\ cm$ and $\overline{WX} = 5\ cm$; Calculate the length of $\overline{XY}$.



8. (a) Rectangle $ABCD$ is similar to rectangle $WXYZ$. If $\overline{BC} = 9cm$, $\overline{AB} = 4\ cm$ and $\overline{WX} = 5\ cm$; Calculate the length of $\overline{XY}$.



Solution

Since rectangle ABCD ~ rectangle WXYZ

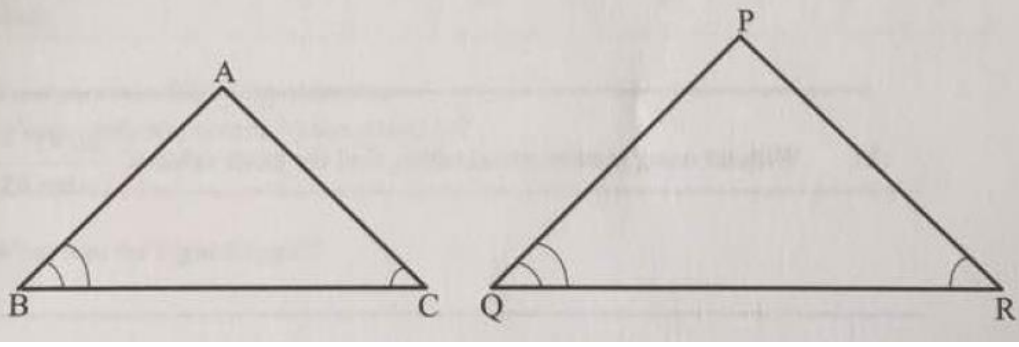
$$\frac{5}{4} = \frac{\overline{XY}}{9}$$

$$\frac{\overline{XY} \times 4}{4} = \frac{9 \times 5}{4} = \frac{45}{4}$$

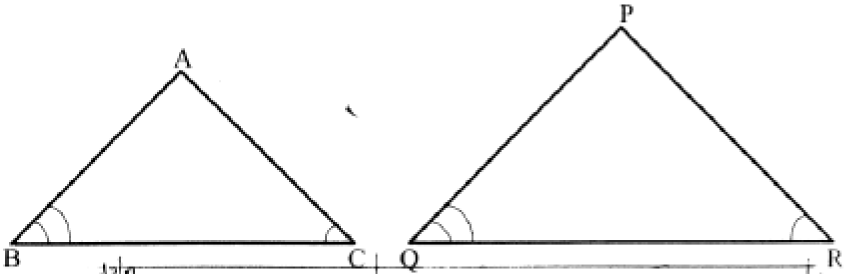
$\therefore\ \overline{XY} = 11.25\text{ cm}$

2017

(b) In the following figure, $\Delta ABC \sim \Delta PQR$, $\overline{AC} = 4.8$ cm, $\overline{AB} = 4$ cm and $\overline{PQ} = 9\ cm$. Find $\overline{PR}$.



(b) In the following figure, $\Delta ABC \sim \Delta PQR$, $\overline{AC} = 4.8$ cm, $\overline{AB} = 4$ cm and $\overline{PQ} = 9\ cm$. Find $\overline{PR}$.



soln

From similarity

If $\Delta ABC \sim \Delta PQR$

Then $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

$\frac{AB}{PQ} = \frac{AC}{PR}$

Given: $AC = 4.8cm$ Required: PR

$AB = 4cm$

$PQ = 9cm$

$\frac{AB}{PQ} = \frac{AC}{PR}$

$PR = \frac{AC \times PQ}{AB}$

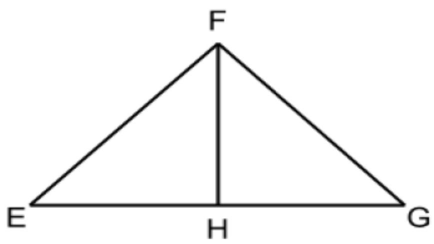
$= \frac{4.8 \times 9}{4} = 1.2 \times 9$

$= 10.8cm$

$\therefore \overline{PR} = 10.8cm$

2015

25. In the figure below $\overline{EF} = \overline{FG}$ and $\overline{EH} = \overline{HG}$. Show that triangles EFH and GFH are similar.



soln

Given: ΔEFH and ΔGFH and $\overline{EF} = \overline{FG}$, $\overline{EH} = \overline{HG}$

Required to prove: triangle $EFH \sim$ triangle GFH

Proof: $\overline{EF} = \overline{FG}$ ------ given (s)

$\overline{EH} = \overline{HG}$ ---- given (s)

$\overline{FH}$ ------- Common (s

$\therefore \Delta EFH \sim \Delta GFH$ by (SSS) similarity theorem

Geometrical Transformations

2020

(b) Find the image of the point P(4,1) when it is;
(i) reflected in the x-axis.
(ii) reflected in the line $y = x$.
(iii) translated by the point T(3,5).

Soln.

i, reflected in x axis.

$M_{x\,axis}(x,y) = (x,-y)$.

$M_{x\,axis}(4,1) = (4,-1)$ ans.

ii, Reflected in $y=x$.

$M_{y=x}(x,y) = (y,x)$

$M_{y=x}(4,1) = (1,4)$ ans.

$T'(x',y') = (a,b) + (x,y)$.

$T'(x',y') = (3,5) + (4,1)$

$T'(x',y') = (7,6)$

$\therefore$ The image is $(7,6)$.

2018

(b) By using a sketch, find the image of the point $A(5, 2)$ under a reflection in the line $y = 0$, followed by another reflection in the line $y = x$.

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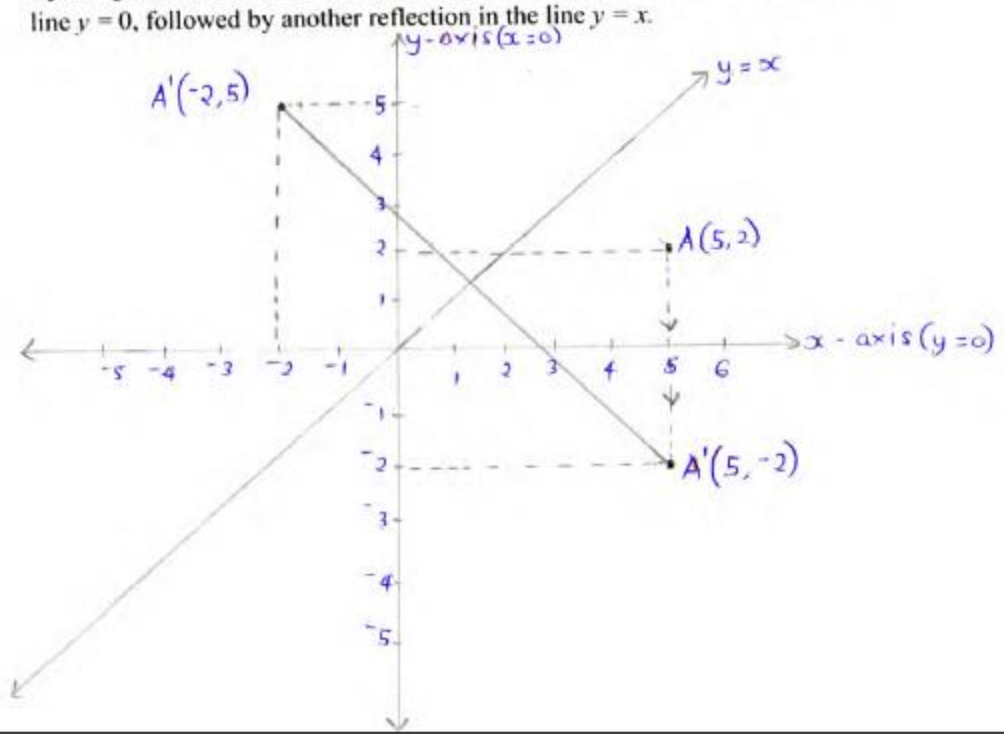


2015

6. (a) Find the equation of the straight line passing through the points $(3, 5)$ and $(7, 9)$. (Express your answer in the form $y = mx + c$).

(b) The vertices of a triangle are A (2, 2), B (3, 4) and C (4, 3). If the triangle is reflected in the y-axis, write down the coordinates of the image of points A, B and C.

6. (a) Find the equation of the straight line passing through the points $(3, 5)$ and $(7, 9)$. (Express your answer in the form $y = mx + c$).

Soln

gradient (m) = $\frac{\text{change in } y}{\text{change in } x}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Given points, $(3, 5)$ and $(7, 9)$

$m = \frac{9-5}{7-3} = \frac{4}{4}$

$m = 1$

but equation of the line

$= m = \frac{y - y_1}{x - x_1}$ using point $(3, 5)$

$\frac{1}{1} = \frac{y-5}{x-3}$

$y - 5 = x - 3$

$y = x - 3 + 5$

$y = x + 2$

$\therefore$ The equation of the line $= y = x + 2$

(b) The vertices of a triangle are A (2, 2), B (3, 4) and C (4, 3). If the triangle is reflected in the y-axis, write down the coordinates of the image of points A, B and C.

Soln

$A(x, y) = A'(-x, y)$

$B(x, y) = B'(-x, y)$

$C(x, y) = C'(-x, y)$

$\therefore A' = (-2, 2)$, $B' = (-3, 4)$ and $C' = (-4, 3)$

NOTE:- Reflection on the y-axis, $x = 0$.

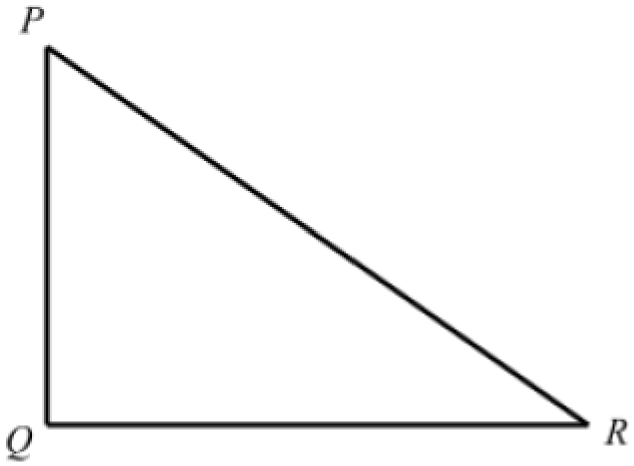
$\therefore$ Coordinates of the image of points A, B, C

$= A'(-2, 2)$, $B'(-3, 4)$ and $C'(-4, 3)$.

Pythagoras theorem

2020

9. (a) Figure PQR represents a triangular floor such that $\overline{PQ} = \overline{QR} = 2$ cm and angle PQR is $90°$. Find $\overline{PR}$, correct to two decimal places.

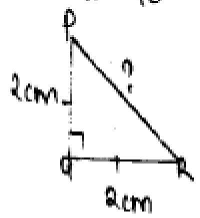


soln.

Given

$\overline{PQ} = \overline{QR} = 2cm$

$P\hat{Q}R = 90°$



By pythagoras theorem

$\overline{PR} = x$

$x^2 = 2^2 + 2^2$

$x^2 = 4 + 4$

$x^2 = 8$

$\sqrt{x^2} = \sqrt{8}$

$x = 2.828$ cm

correct to two decimal places

$2.82|8$

$$\begin{array}{r} 2.82 \\ +\ \ \ 1 \\ \hline 2.83 \end{array}$$

$\therefore$ The length of $\overline{PR}$ is

≈ 2.83cm.

2018

9. (a) A ladder on the ground leans against a vertical wall whose height is 5 metres. The ground distance between the ladder and the wall is 12 metres.

(i) Draw a diagram to represent this information.

(ii) Using the diagram in part (i), find the length of the ladder.

(a) A ladder on the ground leans against a vertical wall whose height is 5 metres. The ground distance between the ladder and the wall is 12 metres.

(i) Draw a diagram to represent this information.

(ii) Using the diagram in part (i), find the length of the ladder.

Soln.

Logically

i) Mathematically:

ii) Apply the pythagoras theorem

$c^2 = a^2 + b^2$

$c^2 = (12)^2 + (5)^2$

$c^2 = 144 + 25$

$\sqrt{c^2} = \sqrt{169}$

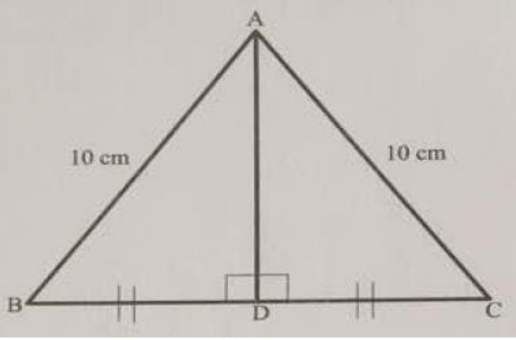
$c = 13$

c = hypoten = length of the ladder

$\therefore$ Length of the ladder is 13m

2017

9. (a) The sides of an equilateral triangle ABC are 10 cm each. Find the length marked $\overline{AD}$ in surd form.



Let a be $\overline{AD}$ soln.

b be $\overline{DC}$

c be $\overline{AC}$

$a^2 + b^2 = c^2$

$x^2 + 5^2 = 10^2$

$x^2 = 100 - 25$

$x^2 = 75$

$x = \sqrt{75} = 5\sqrt{3}$

$\therefore$ The length of $\overline{AD}$ is $5\sqrt{3}$ cm

Trigonometry

2020

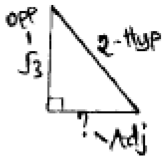
(b) Given that $\sin\theta = \frac{\sqrt{3}}{2}$ where θ is an acute angle; without using mathematical table, find;

(i) $\cos\theta$.

(ii) $\tan\theta$.

SOLN.

(i) $\cos\theta = \frac{Adj}{Hyp}$



From $\sin\theta = \frac{\sqrt{3}}{2}$ - opp, 2 - Hyp

By pythagoras theorem

$a^2+b^2=c^2$

$b^2=c^2-a^2$

$b^2=2^2-(\sqrt{3})^2$

$b^2=4-3$

$b^2=1$

$\sqrt{b^2}=\sqrt{1}$

$b=1$ (Adj)

$\cos\theta = \frac{1}{2}$

$\cos\theta = 0.5$

$\therefore \cos\theta = 0.5$

(ii) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin\theta = \sqrt{3}/2$ $\cos\theta = 1/2$

$= \sqrt{3}/2 \div 1/2$

$= \sqrt{3}/2 \times 2$

$= \sqrt{3}$

$\therefore \tan\theta = \sqrt{3}$

2018

(b) Given that $\sin A = \frac{3}{5}$ where A is an acute angle, find without using mathematical tables the values of:

(i) $\cos A$

(ii) $\tan A$

(iii) $\frac{1-\sin A}{1-\cos A}$

(b) Given that $\sin A = \frac{3}{5}$ where A is an acute angle, find without using mathematical tables the values of:

(i) $\cos A$

(ii) $\tan A$

(iii) $\frac{1-\sin A}{1-\cos A}$

Solution

SO TO CA
H A H

$\sin A = \frac{3}{5}$

$\sin = \frac{Opp}{hyp}$

Apply pythagoras theorem

$c^2 = a^2 + b^2$

$5^2 = a^2 + 3^2$

$25 - 9 = a^2$

$\sqrt{16} = \sqrt{a^2}$ $a = 4$ (Adj)

i) $\cos A = \frac{Adj}{hypte} = \frac{4}{5}$

$\therefore \cos A = 4/5$

ii) $\tan A = \frac{Opposite}{Adj} = \frac{3}{4}$

$\therefore \tan A = 3/4$

iii) $\frac{1-\sin A}{1-\cos A} = \frac{1-3/5}{1-4/5}$

$= \frac{3/5}{1/5} = \frac{2}{5} \times \frac{5}{1} = 2$

$\therefore \frac{1-\sin A}{1-\cos A} = 2$

2017

(b) Without using mathematical tables, find the exact value of $\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$.

(b) Without using mathematical tables, find the exact value of $\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$.

soln.

$\tan 45^\circ = 1, \ \tan 30^\circ = \sqrt{3}/3$

$$\frac{1 + \sqrt{3}/3}{1 - (1 \times \sqrt{3}/3)} = \frac{\frac{3+\sqrt{3}}{3}}{1 - \sqrt{3}/3}$$

$$\frac{3+\sqrt{3}}{3} \div 1 - \sqrt{3}/3 = \frac{3+\sqrt{3}}{3} \times \frac{3}{3-\sqrt{3}}$$

$$= \frac{3+\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{9 + 3\sqrt{3} + 3\sqrt{3} + 3}{9-3} = \frac{12 + 6\sqrt{3}}{6}$$

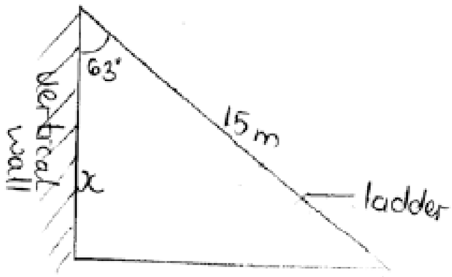
$$= 2 + 1\sqrt{3}$$

$\therefore \ 2 + \sqrt{3}$

2015

23. A ladder 15m long leans against a vertical wall such that the top of the ladder makes an angle of 63 degrees with the vertical wall. Find the height of the wall.

Solution



$$\cos 63^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{x \text{ m}}{15 \text{ m}}$$

$x = (\cos 63^\circ) \times 15$

$\cos 63^\circ = 0.4540$

$x = 0.4540 \times 15$

$x = 6.8100$

$\therefore$ The height of the wall is 6.81 meters.

Sets

2020

10. (a) In a certain village, 300 people were interviewed about their food preference. It was found that, 200 people like banana, 120 people like rice and 60 people like both banana and rice. By using formula, find the number of people who like neither banana nor rice.

Soln

$\mu = 300$ people.

$n\langle B\rangle = 200$ people.

$n\langle R\rangle = 120$ people.

$n\langle B\cap R\rangle = 60$ people.

$n\langle B\cup R\rangle = ?$

From.

$n\langle B\cup R\rangle = n\langle B\rangle + n\langle R\rangle - n\langle B\cap R\rangle$

$n\langle B\cup R\rangle = 200 + 120 - 60$.

$n\langle B\cup R\rangle = \underline{\underline{260 \text{ people}}}$.

$\mu = n\langle B\cup R\rangle + n\langle B\cup R\rangle'$

$\mu = 260 + n\langle B\cup R\rangle'$

$300 = 260 + n\langle B\cup R\rangle'$

$300 - 260 = n\langle B\cup R\rangle'$

$40 = n\langle B\cup R\rangle'$

∴ 40 people like neither Banana nor Rice.

2018

10. (a) In a class of 32 students, 18 play golf, 16 play piano and 7 play both golf and piano. Use a formula to find the number of students who play neither golf nor piano.

10. (a) In a class of 32 students, 18 play golf, 16 play piano and 7 play both golf and piano. Use a formula to find the number of students who play neither golf nor piano.

Solution.

- let the number of those who play golf be set A⟨18⟩
- let the number of those who play piano be set B⟨16⟩

n(Universal set) = 32

from:

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$n(A\cup B) = 18 + 16 - 7$

$n(A\cup B) = 34 - 7$

$n(A\cup B) = 27$

$n(\mu) = n(A\cup B) + n(A\cup B)'$

$32 = 27 + n(A\cup B)'$

$n(A\cup B)' = 32 - 27$

$n(A\cup B) = 5$

∴ Those who play neither golf nor piano is 5 pupils /students

2017

(b) The marks of 61 students are represented in the following table:

Marks in %	30	35	45	50	60	75	80	85	90
Number of students	3	5	7	10	18	9	4	3	2

From the table answer the following questions:

(i) Which mark was scored by few students?

____

(ii) What was the highest mark?

____

(iii) If 50% was the pass mark in the examination, how many students passed the examination?

____

(iv) Which mark was scored by many students?

(b) The marks of 61 students are represented in the following table:

Marks in %	30	35	45	50	60	75	80	85	90
Number of students	3	5	7	10	18	9	4	3	2

From the table answer the following questions:

(i) Which mark was scored by few students?

90% mark was scored by few students.

(ii) What was the highest mark?

90% was the highest mark.

(iii) If 50% was the pass mark in the examination, how many students passed the examination?

46 students passed the examination.

(iv) Which mark was scored by many students?

60% mark was scored by many students

2015

24. In a class of 50 students, 16 like watching television, 41 like reading story books and 7 do not like neither watching television nor reading story books. Find the number of students who like both watching television and reading story books using the formula.

Soln

All students = 50

Who do not like both = 7

$50 - 7 = 43$ students

Let the set of students who like watching television be A and those like reading story be B.

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$43 = 16 + 41 - n(A \cap B)$

$43 = 57 - n(A \cap B)$

$n(A \cap B) = 57 - 43$

$n(A \cap B) = 14$

$\therefore$ The number of students who like both watching television and reading story books is 14 student

Statistics

2020

(b) The masses of a group of students from Kilimani secondary school were recorded as shown in the following table:

Mass in kilograms	31 − 40	41 − 50	51 − 60	61 − 70	71 − 80
Frequency	2	5	3	9	1

(i) How many students are there in the group?

(ii) State the class interval that has the largest number of students.

(iii) Prepare a table showing the class boundaries and the corresponding cumulative frequencies.

(i) How many students are there in the group?

$2+3+5+9+1 = 20$ students $\therefore$ There are 20 students.

(ii) State the class interval that has the largest number of students.

Class interval of 61 - 70

(iii) Prepare a table showing the class boundaries and the corresponding cumulative frequencies.

Class interval	Frequency	Cummulative frequency	Class boundaries
31 - 40	2	2	30·5 - 40·5
41 - 50	5	7	40·5 - 50·5
51 - 60	3	10	50·5 - 60·5
61 - 70	9	19	60·5 - 70·5
71 - 80	1	1	70·5 - 80·5

2018

(b) A survey was done among students in a certain school in order to find the most popular subject. In this survey each student voted once and the results were as follows:

Subject	Mathematics	English	Biology	History	Geography	Physics
Number of Pupils	50	80	120	40	80	30

Show this information in a pie chart.

2017

10\. (a) In a primary school of 150 pupils 50 study Hisabati, 70 study Sayansi and 40 study both subjects. By using the appropriate formula, calculate the number of pupils who study neither Hisabati nor Sayansi.

Soln.

$50+80+120+40+80+30 = 400$

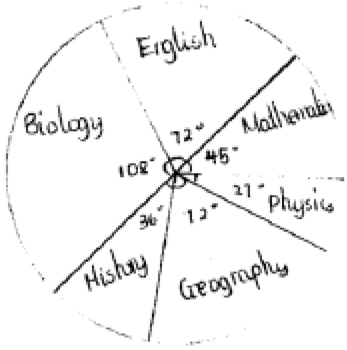
400 pupils = 360°

$\frac{50}{400} \times 360 = 45°$ | $\frac{40}{400} \times 360 = 36°$

$\frac{80}{400} \times 360 = 72°$ | $\frac{80}{400} \times 360 = 72°$

$\frac{120}{400} \times 360 = 108$ | $\frac{30}{400} \times 360 = 27°$

$45° + 72° + 108° + 36° + 72° + 27° = 360°$



PIE CHART.