Basic Maths

CSEE Past Paper Questions and Answers By Topic

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Basic Maths
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1. Numbers, Approximations, Fractions, Logarithms

2018

1. (a). If $m = 0.\dot{2}\dot{7}$ and $n = 0.\dot{1}\dot{5}$, find the fraction $\frac{n}{m}$ in its simplest form.

(b). Find the GCF of 210, 357 and 252.

1.a. Given $m = 0.\dot{2}\dot{7}$ $\frac{n}{m}$

$n = 0.\dot{1}\dot{5}$

Let $m = 0.\dot{2}\dot{7}$ ---(i)

Multiply by 100 both sides.

$100m = 27.\dot{2}\dot{7}$ ---(ii)

Subtract equation (i) from (ii)

$100m - m = 27.\dot{2}\dot{7} - 0.\dot{2}\dot{7}$

$\frac{99m}{99} = \frac{27}{99}$

$m = \frac{3}{11}$

Let $n = 0.\dot{1}\dot{5}$ ---(i)

Multiply by 100 both sides.

$100n = 15.\dot{1}\dot{5}$ ---(ii)

Subtract equation (i) from (ii)

$100n - n = 15.\dot{1}\dot{5} - 0.\dot{1}\dot{5}$

$\frac{99n}{99} = \frac{15}{99}$

$n = \frac{5}{33}$

$\frac{n}{m} = \frac{5/33}{3/11}$

$= \frac{5}{33} \div \frac{3}{11}$

$= \frac{5}{\cancel{33}\,3} \times \frac{\cancel{11}\,1}{3}$

$= \frac{5}{9}$

$\therefore \frac{n}{m} = \frac{5}{9}$

(b.) GCF of 210, 357 and 252.

2	210	357	252
2	105	357	126
3	105	357	63
3	35	119	21
7	35	119	7
5	5	17	1
17	1	17	1
	1	1	1

$GCF = 3 \times 7$

$= 21.$

$\therefore$ GCF of 210, 557 and 252 = 21.

The candidates were able to convert repeating decimals into fractions, divide fractions, simplify fractions and find the GCF by using either the listing method or prime factorization method.

2017

1. (a) Round off:
   (i) 9.67 to ones,
   (ii) 0.205 to one decimal place
   (iii) 0.0197 to two decimal places.
   Hence, estimate the value of $\frac{9.67\times0.205}{0.0197}$.

   (b) Simplify the expressions:
   (i) $(3+\sqrt{2})(4-2\sqrt{2})$,
   (ii) $\sqrt{40}\times\sqrt{45}$.

   (c) Express 0.3636… in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

1. a) Solution
i) $9.67 \approx 10$ (To ones)
ii) $0.205 \approx 0.2$ (To one decimal place)
iii) $0.0197 \approx 0.02$ (To two decimal place)

Then,

$$\frac{9.67\times0.205}{0.0197} = \frac{10\times0.2}{0.02}$$

$$= 100$$

$\therefore$ Estimated value for $\frac{9.67\times0.205}{0.0197} = 100$

b) i) $(3+\sqrt{2})(4-2\sqrt{2}) = 12-6\sqrt{2}+4\sqrt{2}-2\sqrt{4}$
$= 12-2\sqrt{2}-2\times2$
$= 12-4-2\sqrt{2}$
$= 8-2\sqrt{2}$

$\therefore (3+\sqrt{2})(4-2\sqrt{2}) = 8-2\sqrt{2}$

ii) $\sqrt{40}\times\sqrt{45} = \sqrt{40\times45}$
$= \sqrt{(2\times2\times2\times5)\times3\times3\times5}$
$= \sqrt{2\times2\times2\times3\times3\times5\times5}$
$= \sqrt{2^3.3^2.5^2}$
$= 2\times3\times5\sqrt{2}$
$= 30\sqrt{2}$

$\therefore \sqrt{40}\times\sqrt{45} = 30\sqrt{2}$

1 c) $0.3636.... = 0.\dot{3}\dot{6}$

Let, 0.36 is equal to x

Then, $x = 0.3636...$

Thus, $100x = 36.3636...$

$- \quad x = 0.3636...$

$99x = 36$

$x = \frac{36}{99} = \frac{4}{11}$

$\therefore$ $0.3636...$ in form of a/b is $4/11$

2016

1. (a) From the set of numbers {1, 3, 4, 5, 6, 8, 10, 15, 17, 21, 27}; write down:
   (i) the prime numbers,
   (ii) the multiples of 3,
   (iii) the factors of 60.

   (b) Four wooden rods with length of 70 cm, 119 cm, 84 cm and 105 cm are cut into pieces of the same length. Find the greatest possible length for these pieces if no wood is left over.

2015

1. (a) If $p = 6.4 \times 10^4$ and $q = 3.2 \times 10^5$, find the values of:

   (i) $p \times q$,

   (ii) $p + q$.

   Write the answers in standard form.

   (b) Evaluate $\sqrt{\frac{0.684^3 \times 43.7}{3.26}}$ using mathematical tables and write the answer correctly to 3 significant figures.

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Question Number | SUBJECT NAME Maths | INDEX NUMBER S0205/0092 | For Examiners' use only

1

a) $p = 6.4 \times 10^4$, $q = 3.2 \times 10^5$

i) $p \times q$

$6.4 \times 10^4 \times 3.2 \times 10^5$

$20.48 \times 10^{4+5}$

20.48×10^9

2.048×10^{10}

ii) $p + q$

$(6.4 \times 10^4) + (3.2 \times 10^5)$

$64000 + 320000$

384000

$= 3.84 \times 10^5$

b) $\sqrt{\frac{0.684^3 \times 43.7}{3.26}}$

Soln

$0.684 = 6.84 \times 10^{-1}$

$43.7 = 4.37 \times 10^1$

$3.26 = 3.26 \times 10^0$

No	log
0.684	$\bar{1}.8351 \times 3$
	$\bar{1}.5053$
43.7	1.6405
Numerator	1.1458
3.26	0.5132
	$0.6326 \times \frac{1}{2}$

Antilog 0.3163

$= 2.071 \times 10^0$

$= 2.07$

2014

1. (a) Kisiki and Jembe are riding on a circular path. Kisiki completes a round in 24 minutes whereas Jembe completes a round in 36 minutes. If they both started at the same place and time and go in the same direction, after how many minutes will they meet again at the starting point?

   (b) An empty bottle weighs 115 grams. If 45 tablets each weighing $\frac{3}{5}$ gram are put in the bottle, what is the total weight?

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Question Number | SUBJECT NAME Mathematics | INDEX NUMBER S..../0016 | For Examiner's Use only

1. a) Time to meet again = Lowest common factor of Kisiki's and Jembe's time;

2	36	24
2	18	12
2	9	6
3	9	3
3	3	1
	1	1

$\therefore$ L.C.M $= 2\times2\times2\times3\times3$

$= 8\times9 = 72$

$\therefore$ After 72 minutes they will meet again.

b) Weight of bottle = 115 grams

1 tablet = $\frac{3}{5}$ gram

45 = x

$\frac{3}{5}\times\frac{45}{1} = 1\times x$

$27 = x$

$\therefore$ Total weight = Weight of bottle + weight of all tablets

$= (115 + 27)$ grams $= 142$

$\therefore$ Total weight is 142 grams.

06

2. a) i) $(\sqrt{3}+5)^2 = (\sqrt{3}+5)(\sqrt{3}+5)$

$= \sqrt{3}(\sqrt{3}+5) + 5(\sqrt{3}+5)$

$= \sqrt{3\times3} + 5\sqrt{3} + 5\sqrt{3} + 25$

$= 3 + 10\sqrt{3} + 25$

$(\sqrt{3}+5)^2 = 28 + 10\sqrt{3}$

$\therefore 28 + 10\sqrt{3}$

ii) $\frac{(\sqrt{3}+5)^2}{(7\sqrt{3}+2)} = \frac{(\sqrt{3}+5)(\sqrt{3}+5)}{7\sqrt{3}+2} = \frac{28+10\sqrt{3}}{7\sqrt{3}+2}$

01½

2013

1. (a) A shopkeeper sold 292 t-shirts at the price of sh. 6950. Estimate how much money she got.

   (b) Express $0.9\dot{6}$ in the form $\frac{a}{b}$ where a and b are integers such that $b \neq 0$.

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Question Number | SUBJECT NAME MATHEMATICS | INDEX NUMBER S0255/0059 | For Examiner's use only

1\. (a) soln.

1 t-shirt = 6950 shs

292 t-shirt = x ?

292 – To nearest hundreds → 300 6950 – To nearest thousands

x = 292 × 6950 shs.

= 300 × 7000 — 02

= 2,100000 shs.

∴ She got 2,100,000 shs. — 01

(b) soln.

Let x be $0.9\dot{6}$ —(i)

$10x = 9.6\dot{6}$ —(ii) — 00½

(ii) – (i)

$10x - x = 9.6\dot{6} - 0.9\dot{6}$

$\frac{9x}{9} = \frac{8.7 \times 10}{9 \times 10}$ — 00½

$x = \frac{87}{90}$ — 01

$= \frac{29}{30}$

$\therefore 0.9\dot{6} = \frac{29}{30}$ — 01

06

2012

1. (a) By using mathematical tables, evaluate $\frac{\sqrt[3]{0.0072}\times(81.3)^2}{\sqrt{23140}}$ to three significant figures.

   (b) Rationalize $\frac{2+\sqrt{3}}{1-\sqrt{3}}$

1a

Number	logarithm	Operation
$(7.2\times 10^{-3})^{1/3}$	$\frac{1}{3}\times \bar{3}.8573$	$\bar{1}.2858$ +
$(8.13\times 10^{1})^{2}$	2×1.9101	3.8202
		3.1060
$(2.314\times 10^{4})^{1/2}$	$\frac{1}{2}\times 4.3643$	2.1822 −
		0.9238

Antilog of 0.9238

$= 10^{0}\times 8.390$

The answer is 8.39.

1(b) Solution:-

$\frac{2+\sqrt{3}}{1-\sqrt{3}}$ Rationalize factor $1-\sqrt{3}$

$=\frac{2+\sqrt{3}\,(1+\sqrt{3})}{1-\sqrt{3}\,(1+\sqrt{3})}$

$=\frac{2+2\sqrt{3}+\sqrt{3}+3}{1+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}-3}$

$=\frac{2+3+3\sqrt{3}}{1-3}$

$=\frac{5+3\sqrt{3}}{-2}$

$\therefore \frac{2+\sqrt{3}}{1-3}=\frac{5+3\sqrt{3}}{-2}$

2011

1. (a) Express 0.05473
   (i) correct to three (3) significant figures
   (ii) correct to three (3) decimal places
   (iii) in standard form.

   (b) Evaluate $\frac{0.0084 \times 1.23 \times 3.5}{2.87 \times 0.056}$ without using mathematical tables and express the answer as a fraction in its simplest form.

2010

1. (a) Write 624.3278 correct to:
   (i) five (5) significant figures
   (ii) three (3) decimal places.

   (b) A mathematics teacher bought 40 expensive calculators at shs.16,400 each and a number of other cheaper calculators costing shs.5,900 each. She spent a total of shs. 774,000. How many of the cheaper calculators did she buy?

2009

1. (a) Estimate the value of $\frac{57.2 \times 110}{2.146 \times 46.9}$ correct to one (1) significant figure

   (b) Express $1.8\dot{6}$ as an improper fraction in its simplest form.

(6 marks)

2008

1. (a) Find the product of the L.C.M. and G.C.F of 40, 120, and 240.

   (b) Round off each of the following numbers to one decimal place.
   $L = 20.354$
   $M = 40.842$
   $N = 10.789$

   (c) Use the results obtained in 1 (b) above to find the value of X, given that $X = \frac{LM}{N}$.

(6 marks)

2007

1. (a) Given $x = 4.5 \times 10^{-7}$ and $z = 7.2 \times 10^{5}$, find y in standard form if $z = xy$. **(2 marks)**

   (b) Express $2\ \dot{1}35\dot{3}$ as a fraction. **(2 marks)**

   (c) Evaluate
   $$\frac{\frac{1}{2.5} + 0.28 + 1\frac{17}{25}}{3\frac{1}{5} \div \frac{0.32}{0.15}}$$
   Give your answer in fraction form. **(2 marks)**

2. Logarithms, Exponents and Radicals

2018

2. (a). Evaluate $\log_{10} 40{,}500$ given that $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$ and $\log_{10} 5 = 0.6990$.

(b). Find the values of x and y if $\frac{3^{x+2}}{5^{2y-5}} = 2025$.

2 a) Soln:

Given $\log_{10} 40500$

$\log_{10} 40500 = \log_{10}(2\times2\times3\times3\times3\times3\times5\times5\times5)$

$= \log_{10}(2^2 \cdot 3^4 \cdot 5^3)$

but, $\log_{10}(a\times b) = \log a + \log b$

$\log_{10}(2^2 \cdot 3^4 \cdot 5^3) = \log_{10} 2^2 + \log_{10} 3^4 + \log_{10} 5^3$

$= 2\log_{10} 2 + 4\log_{10} 3 + 3\log_{10} 5$

given $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 5 = 0.6990$

$= 2(0.3010) + 4(0.4771) + 3(0.6990)$

$= 0.6020 + 1.9084 + 2.097$

$= 4.6074$

$\therefore \log_{10} 40{,}500 = 4.6074$

2b) Soln

$\frac{3^{x+2}}{5^{2y-8}} = 2025$

$3^{x+2} \times (1/5)^{2y-8} = 2025$

$3^{x+2} \times ((5)^{-1})^{2y-8} = 3^4 \times 5^2$

2b) $3^{x+2} \times 5^{-2y+8} = 3^4 \times 5^2$

$3^{x+2} = 3^4$

$x + 2 = 4$

$x = 4 - 2$

$x = 2$

$5^{-2y+8} = 5^2$

$-2y + 8 = 2$

$-2y = 2 - 8$

$\frac{-2y}{-2} = \frac{-6}{-2}$

$y = 3$

$\therefore$ The value of x is 2 and y is 3

Candidates had adequate knowledge and skills on the laws of exponents and logarithms as they correctly evaluated 10 log 40500 and solved the equation 4 2 2 8 2 3 5 5 3 = × − + y x for x and y by comparing the exponents of the same base.

2017

2. (a) Simplify:

(i) $27^{1/4} \times 3^{1/4} \times (\sqrt{3})^{-2}$,

(ii) $\log_3 10 + \log_3 8.1$.

(b) If $n\log_5 125 = \log_2 64$, find the value of n.

2 a) i) $27^{1/4} \times 3^{1/4} \times (\sqrt{3^1})^{-2} = 3^{3\times 1/4} \times 3^{1/4} \times 3^{1/2 \times -2}$

$= 3^{3/4} \times 3^{1/4} \times 3^{-1}$

$= 3^{3/4 + 1/4 - 1}$

$= 3^{1-1}$

$= 3^0$

$= 1$

$\therefore 27^{1/4} \times 3^{1/4} \times (\sqrt{3^1})^{-2} = \underline{1}$

ii) $\log_3 10 + \log_3 8.1$

From, $\log_b a + \log_b c = \log_b ac$

Then

$\log_3 10 + \log_3 8.1 = \log_3 (10 \times 8.1)$

$= \log_3 81 = \log_3 3^4 = 4\log_3 3 = 4$

$\therefore \log_3 10 + \log_3 8.1 = 4$.

2 b) $n\log_5 125 = \log_2 64$

$n\log_5 5^3 = \log_2 2^6$

$3n\log_5 5 = 6\log_2 2$

$3n = 6$

$n = 2$

$\therefore n = 2$.

2016

2. (a) Solve for x in the equation $9^{(x-3)} \times 81^{(1-x)} = 27^{-x}$.

(b) Show that $\dfrac{\log 16 + \log 81}{\log 27 + \log 8} = \dfrac{4}{3}$.

2015

2. (a) Solve for x in the equation $4^{-2x} \times 8^2 = 4 \times 16^x$.

(b) Find the value of $\log 900$ given that $\log 3 = 0.4771$.

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

Question Number	SUBJECT NAME B/MATHS INDEX NUMBER S.0101/0123	For Examiners' use only
	$2^{-4x} \times 2^6 = 2^2 \times 2^{4x}$	
	$2^{-4x+6} = 2^{2+4x}$	
	Since the base are equal, compare the exponent	
	$-4x+6 = 2+4x$	
	$-4x-4x = 2-6$	
	$\frac{-8x}{-8} = \frac{-4}{-8}$	
	$x = \cancel{4/8}\ \frac{1}{2}$	
	$x = \frac{1}{2}$	
	$\therefore x = \frac{1}{2}$	
	(b) Soln	
	$\log 900 = \log(9 \times 100)$	
	$= \log(3^2 \times 100)$	
	$= \log 3^2 + \log 100$	
	but; $\log 3 = 0.4771$	
	$\log 100 = 2$	
	$= \log 3^2 + \log 100$	
	$= 2\log 3 + \log 100$	
	$= 2(0.4771) + 2$	
	$= 0.9542 + 2$	
	$= 2.9542$	
	$\therefore \log 900 = 2.9542$	
3	(a) Soln	
	$x/3 - 1 \geq \frac{2}{1} - \frac{x}{2}$	
	$x/3 - 1/1 \geq \frac{4-x}{2}$	
	$\frac{x-3}{3} \geq \frac{4-x}{2}$	

2014

2. (a) (i) Express $(\sqrt{3}+5)^2$ in the form $a+b\sqrt{3}$, where a and b are integers.

(ii) Express $\frac{(\sqrt{3}+5)^2}{(7\sqrt{3}+2)}$ in the form $p+q\sqrt{3}$, where p and q are rational numbers.

(b) Solve for x if $\left(\frac{1}{81}\right)^{-6x} \times 81 = \sqrt{9}$.

3 COUNCIL OF TANZANIA

Question Number	SUBJECT NAME BASIC MATHEMATICS INDEX NUMBER	For Examiners' use only
	$28 + 10\sqrt{3}$	
		01
	$\therefore = 28 + 10\sqrt{3}$	
	ii) $\frac{(\sqrt{3}+5)^2}{(7\sqrt{3}+2)}$	
	From (i) above	
	$(\sqrt{3}+5)^2 = 28+10\sqrt{3}$	
	$\frac{28+10\sqrt{3}}{2+7\sqrt{3}}$	
	We rationalize the denominator	
	$\frac{(28+\sqrt{300})}{(2+\sqrt{147})} \times \frac{2-\sqrt{147}}{2-\sqrt{147}}$	
	$= \frac{56 - 28\sqrt{147} + 2\sqrt{300} - 210}{4 - 2\sqrt{147} + 2\sqrt{147} - 147}$	01
	$= \frac{-154 - 28\sqrt{147} + 2\sqrt{300}}{-143}$	
	$= \frac{-154 - 28\times7\sqrt{3} + 2\times10\sqrt{3}}{-143}$	
	$= \frac{-154 - 196\sqrt{3} + 20\sqrt{3}}{-143}$	
	$= \frac{-154 - 176\sqrt{3}}{-143}$	
	$= \frac{-(154+176\sqrt{3})}{-143}$	
	$= \frac{154}{143} + \frac{176\sqrt{3}}{143}$	01
	$= \frac{14}{13} + \frac{16}{13}\sqrt{3}$	
	$= \frac{14}{13} + \frac{16}{13}\sqrt{3}$	01
		03

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

Question Number | SUBJECT NAME: BASIC MATHEMATICS | INDEX NUMBER: [illegible] | For Examiners' Use Only

2 b) $\left(\frac{1}{81}\right)^{-6x} \times 81 = \sqrt{9}$

$(81^{-1})^{-6x} \times 81 = 9^{1/2}$

$81^{6x} \times 81 = 3$

$(3^4)^{6x} \times 3^4 = 3$

$3^{24x} \times 3^4 = 3$ 01

$3^{24x+4} = 3^1$

$24x + 4 = 1$

$24x = 1 - 4$ 01

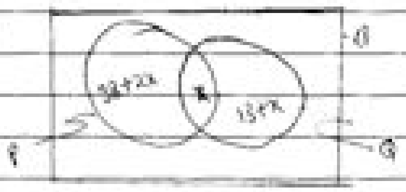
$\frac{24x}{24} = \frac{-3}{24}$

$x = -\frac{1}{8}$

$\therefore x = -\frac{1}{8}$ 01

06

03. a)



i) $n(P \cup Q) = 95$

$38 + 2x + x + 13 + x = 95$ 01

$51 + 4x = 95$

$4x = 95 - 51$

$4x = 44$

$x = 11$ 01

$\therefore x = 11$

02

2013

2. (a) If $\log y + 2\log(3x+1) = 1$, express y in terms of x.

(b) Simplify $\frac{\sqrt{7}}{\sqrt{7}+\sqrt{5}}$ by rationalizing the denominator.

2 a) $\log y + 2\log(3x+1) = 1$

$\log y + \log(3x+1)^2 = 1$

$\log_{10}\left(y(3x+1)^2\right) = 1$

$\log_{10} 10 = 1$

$\log_{10}\left(y(3x+1)^2\right) = \log_{10} 10$

$\frac{y(3x+1)^2}{(3x+1)^2} = \frac{10}{(3x+1)^2}$

$y = \frac{10}{(3x+1)(3x+1)}$

$\therefore y = \frac{10}{9x^2+6x+1}$

b) $\frac{\sqrt{7}}{\sqrt{7}+\sqrt{5}} \times \frac{(\sqrt{7}-\sqrt{5})}{(\sqrt{2}-\sqrt{5})}$

$= \frac{7-\sqrt{35}}{\sqrt{2}(\sqrt{2}-\sqrt{5})+\sqrt{5}(\sqrt{2}-\sqrt{5})} = \frac{7-\sqrt{35}}{2-\sqrt{10}+\sqrt{10}-5}$

$= \frac{7-\sqrt{35}}{2-5}$

$\frac{7-\sqrt{35}}{-3} = \frac{-7+\sqrt{35}}{3}$

2012

2. (a) Find the value of x for which $2^x \cdot 16 = \frac{1}{8^x}$

(b) Solve $\log_a(x^2+3) - \log_a x = 2\log_a 2$

2a. $2^x \times 16 = \frac{1}{8^x}$ $\quad 8 = 2^3$, $16 = 2^4$

$\therefore 2^x \times 2^4 = \frac{1}{2^{3x}}$

$2^{x+4} = 2^{-3x}$ taking the exponents

$x + 4 = -3x$

$4 = -3x - x$

$\frac{4}{-4} = \frac{-4x}{-4}$ $\quad x = ^-1$

$\therefore x = ^-1$

2b. $\log_a(x^2+3) - \log_a x = 2\log_a 2$

$\log_a\left(\frac{x^2+3}{x}\right) = \log_a 4$

$\frac{x^2+3}{x} = \frac{4}{x}$

$x^2 + 3 = 4x$

$x^2 - 4x + 3 = 0$

$x^2 - x - 3x + 3 = 0$

$x(x-1) - 3(x-1)$

$(x-3)(x-1)$ $\quad x - 3 = 0 \quad x = 3$ or $x - 1 = 0$, $x = 1$

$\therefore x = 1$ or 3

2011

2. (a) Solve the equation $\log_4 5x - \log_4(x+2) - \log_4 3 = 0$

   (b) By rationalizing the denominator, simplify the following expression.
   $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

2010

2. (a) Evaluate without using mathematical tables 2 log 5 + log 36 – log 9.

   (b) Simplify $\dfrac{27^{n+2} - 6\times 3^{3n+3}}{3^n \times 9^{n+2}}$

(6 marks)

2009

2. (a) Solve for y if $\left(\frac{1}{9}\right)^{2y}\left(\frac{1}{3}\right)^{-y} \div \frac{1}{27} = 3^{(-5y)}$

   (b) Simplify the expression $\dfrac{5}{\sqrt{11}-3} \div \dfrac{\sqrt{2}}{\sqrt{22}+3\sqrt{2}}$

2008

2. (a) By using the properties of exponents, simplify the expression
   $\dfrac{2^{18} - 2^{15} + 7}{2^{15}+1}$. (Do not use tables).

   (b) Solve for x in the logarithmic equation $2\log x = \log 4 + \log(2x-3)$.

(6 marks)

3. Algebra and Sets

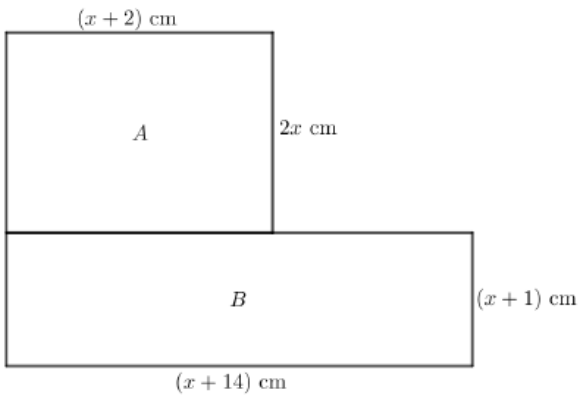
2018

3. (a). In a school of 60 teachers, some drink Fanta and some drink Coca-Cola. If 46 drink Fanta, 18 drink Coca-Cola and 14 drink both Coca-Cola and Fanta. How many teachers drink neither Fanta nor Coca-Cola? (Use Venn diagram)

(b). Use the figure below to answer the following questions:

(i). Write the expression for the total area of rectangles A and B.

(ii). If the total area of rectangles A and B is 98 square centimeters, find the value of x.



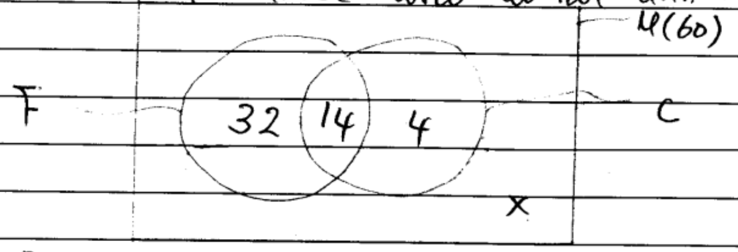
3. a)

Solution

Let: F stand for drinkers of fanta

C stand for drinkers of coca-cola

X stand for those who do not drink



$32 + 14 + 4 + x = 60$

$50 + x = 60$

$x = 60 - 50$

$x = 10$

$\therefore$ 10 teachers drink neither Fanta nor Coca-Cola

b) i)

Solution

For rectangle A:

$$\text{Area} = L \times W = (x+2)\,2x \text{ cm}^2$$
$$= 2x^2 + 4x \text{ cm}^2$$

For rectangle B:

$$\text{Area} = L \times W = (x+14)(x+1) \text{ cm}^2$$
$$= x^2 + 14x + x + 14 \text{ cm}^2$$
$$= x^2 + 15x + 14 \text{ cm}^2$$

$$\text{Total area} = (2x^2 + 4x + x^2 + 15x + 14) \text{ cm}^2$$
$$= (3x^2 + 19x + 14) \text{ cm}^2$$

$\therefore$ Total area is given by $3x^2 + 19x + 14 \text{ cm}^2$

ii)

Solution

Given: $3x^2 + 19x + 14 = 98$

$3x^2 + 19x + 14 - 98 = 0$

$3x^2 + 19x - 84 = 0$

3 From : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-19 \pm \sqrt{361 - (4 \times 3 \times -84)}}{6}$

$= \frac{-19 \pm \sqrt{361 + 1008}}{6}$

$= \frac{-19 \pm \sqrt{1369}}{6}$

$= \frac{-19 \pm 37}{6}$

$= \frac{-19 + 37}{6}$ or $\frac{-19 - 37}{6}$

$= 3$ or $-56/6$

Then : $x = 3$ since we do not have negative length

2017

3. (a) Factorize the following expressions:

(i) $16y^2 + xy - 15x^2$,

(ii) $4 - (3x - 1)^2$.

(b) At Moiva's graduation ceremony 45 people drank Pepsi-Cola, 80 drank Coca-Cola and 35 drank both Pepsi-Cola and Coca-Cola. By using a Venn diagram, found out how many people were at the ceremony if each person drank Pepsi-Cola or Coca-Cola.

03. a) i, Given; $16y^2 + xy - 15x^2$

$= (16y^2 + 16xy)(-15xy - 15x^2)$

$= 16y(y + x) - 15x(y + x)$

$= (16y - 15x)(y + x)$

$\therefore\ 16y^2 + xy - 15x^2 = (16y - 15x)(y + x).$

03 a) ii, Given; $4 - (3x - 1)^2$

$= 2^2 - (3x - 1)^2$ fiem $a^2 - b^2 = (a+b)(a-b)$

$= [2 + (3x - 1)][2 - (3x - 1)]$

$= (2 - 1 + 3x)(2 + 1 - 3x)$

$= (3x + 1)(3 - 3x)$

z

$\therefore\ 4 - (3x - 1)^2 = (1 + 3x)(3 - 3x)$

b) Let:

P = {all people who drank Pepsi-Cola}

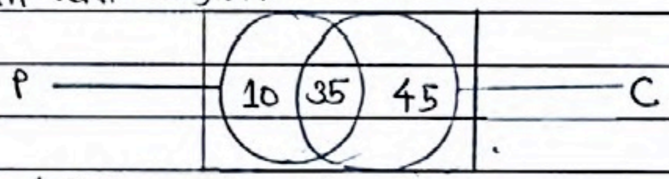
C = {all people who drank Ceca-Cola}

Then; $n(P) = 45$

$n(C)\ 80$

$n(P \cap C) = 35$

In Venn diagram.



$\therefore n(P \cup C) = 10 + 35 + 45$

$= 90$

$\therefore$ There were 90 people at the ceremony.

2016

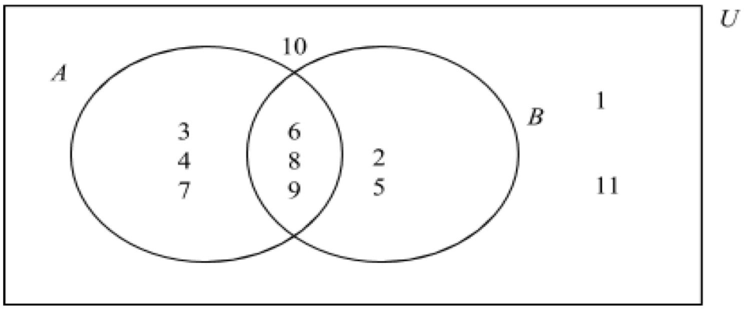
3. (a) By substituting $a = \frac{1}{x}$ and $b = \frac{1}{y}$ in the system of equations: $\begin{cases} \frac{4}{x} - \frac{6}{2y} = 1 \\ -\frac{1}{x} + \frac{3}{2y} = -1 \end{cases}$, find the solution set (x, y).

(b) Let U be a universal set and A and B be the subsets of U where:
U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {odd numbers} and B = {prime numbers}.
(i) Represent this information in a venn diagram.
(ii) Find $A \cap B'$ and $(A \cup B)'$.

2015

3. (a) Find the solution set of the inequality $\frac{x}{3} - 1 \geq 2 - \frac{x}{2}$ and indicate it on a number line.

(b) The Venn diagram below shows the universal set U and its two subsets A and B.



Write down the elements of:

(i) A',

(ii) B',

(iii) $A \cup B$,

(iv) $A' \cup B'$.

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

Question Number | SUBJECT NAME BASIC MATHEMATICS | INDEX NUMBER S0159/0044 | For Examiners' use only

2 (b) $\log 900 = 3\log 3 + \log 100$

$= 3(0.4771) + \log 10^2$

$= 1.4313 + 2\log 10$

$= 1.4313 + 2$

$\therefore \log 900 = 3.4313$

3 (a) $\frac{x}{3} - 1 \geq 2 - \frac{x}{2}$

$\frac{x}{3} + \frac{x}{2} \geq 2 + 1$

$\frac{2x + 3x}{6} \geq 3$

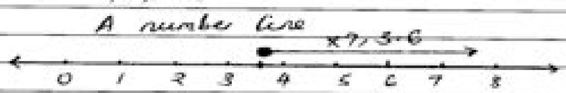
$5x \geq 6 \times 3$

$\frac{5x}{5} \geq \frac{18}{5}$

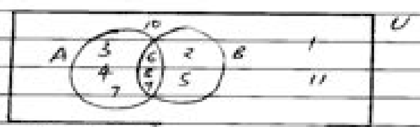
$x \geq \frac{18}{5}$

$x \geq 3.6$

A number line



(b) Given:



The element of

(i) $A' = \{1, 2, 5, 10, 11\}$

(ii) $B' = \{1, 3, 4, 7, 10, 11\}$

(iii) $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$

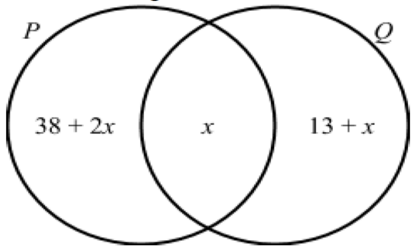
(iv) $A' \cup B' = \{2, 3, 4, 5, 7, 10, 11, 1\}$

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

Number	SUBJECT NAME: BASIC MATHEMATICS INDEX NUMBER: S0159/0044	For Examiners' use only
3	(c) Required; to show that	
	$n(A\cup B) = n(A) + n(B) - n(A\cup B)$	
	$n(A) = 6,\ n(B) = 5$	
	$n(A\cup B) = 8$	
	$n(A\cap B) = 3$.	
	$8 = 6 + 5 - 3$	
	$8 = 11 - 3$	
	$8 = 8$.	
	$\therefore n(A\cup B) = n(A) + n(B) - n(A\cup B)$	
4	(a) $\underset{\sim}{a} = 3i + 2j,\ \underset{\sim}{b} = 8i - 3j,\ \underset{\sim}{c} = 2i + 4j$	
	(i) $d = 3\underset{\sim}{a} - \underset{\sim}{b} + \frac{1}{3}c$	
	$d = 3(3i+2j) - (8i-3j) + \frac{1}{3}(2i+4j)$	
	$= 9i + 6j - 8i + 3j + \frac{2i}{3} + \frac{4j}{3}$	
	$= 9i - 8i + \frac{2i}{3} + 6j + 3j + \frac{4j}{3}$	
	$= +i + \frac{2i}{3} + 9j + \frac{4j}{3}$	
	$= \frac{3i+2i}{3} + \frac{27j+4j}{3}$	
	$\therefore d = \frac{5i}{3} + \frac{31j}{3}$	
	(ii) $\hat{d} = \frac{d}{\lvert d\rvert}$	
	$\lvert d\rvert = \sqrt{x^2+y^2}$ $x = 5/3,\ y = \frac{31}{3}$	
	$\lvert d\rvert = \sqrt{(5/3)^2 + (\frac{31}{3})^2}$	

2014

3. (a) The Venn diagram below shows the number of elements in sets P and Q.



If $n(P \cup Q) = 95$, calculate:

(i) The value of x,

(ii) $n(P \cap Q)'$.

(b) The age of Timothy is $\frac{1}{8}$ the age of his father. If the sum of their ages is 54 years, find the age of the father.

THE NATIONAL EXA... L OF TANZANIA

Question Number | SUBJECT NAME: B/mathematics | INDEX NUMBER: | For Examiners' use only

3. a) i) from $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

$n(P \cup Q) = 38 + 3x + 13 + 2x - x$

$95 = 51 + 4x$ — 01

$4x = 95 - 51$

$4x = 44$

$x = \frac{44}{4}$ — 01

$x = 11$

$\therefore$ The value of x is 11.

(ii) from $n(P \cap Q)' = n(P' \cap Q) + n(P \cap Q')$

$= 38 + 22 + 13 + 11$ — 01

$= 60 + 24$

$= 84$

$\therefore n(P \cap Q)' = 84.$ — 01

b) let the age of Timothy be x and age of his father be y.

$x = \frac{1}{8}y$

$x + y = 54$

but $x = \frac{1}{8}y$

$\frac{1}{8}y + y = 54$ — 01

$\frac{y}{8} + y = 54$

$y + 8y = 432$

$9y = 432$ — 05

$y = \frac{432}{9}$

2013

3. (a) If $m * n = m + 4n$, find x given that $3 * (x * 1) = 27$.

(b) There are 48 men at a meeting of whom 24 are teachers, 36 are parents and 16 are both teachers and parents. By using a Venn diagram, find the number of men who are neither teachers nor parents.

3. (a)

$3 * (x * 1) = 27$

$m * n = m + 4n$

$(x * 1) = x + 4(1)$

$(x * 1) = x + 4$

$3 * (x + 4) = 27$

$3 * (x+4) = 3 + 4(x+4) = 27$

$3 + 4(x+4) = 27$

$3 + 4x + 16 = 27$

$19 + 4x = 27$

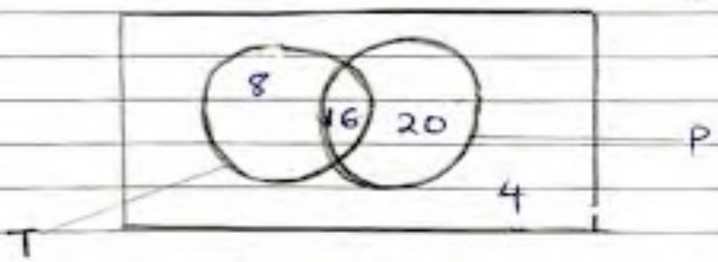
$4x = 27 - 19$

$\frac{4x}{4} = \frac{8}{4}$

$x = 2$

$\therefore x = 2$

(b) Let the set of teachers be represented by T, and the set of parents be represented by P.



$n(T \cup P) = 8 + 16 + 20$

$n(T \cup P) = 44$.

$n(T \cup P)' = n(\mathcal{U}) - n(T \cup P)$

$n(\mathcal{U}) = 48$

$n(T \cup P)' = 48 - 44 = 4$

$\therefore$ 4 men are neither teachers nor parents.

2012

3. (a) Mr. Bean lived a quarter of his life as a child, a fifth as a teenager and a third as an adult. He then spent 13 years in his old age. How old was he when he died?

(b) A and B are subsets of the universal set U. Find $n(A \cap B)$ given that $n(A) = 39$, $n(A' \cap B') = 4$, $n(B') = 24$ and $n(U) = 65$.

3 a.) $x = \frac{1}{4}x + \frac{1}{5}x + \frac{1}{3}x + \frac{13}{1}$

$x = \frac{15x + 12x + 20x + 780}{60}$

$60x = 47x + 780$

$60x - 47x = 780$

$\frac{13x}{13} = \frac{780}{13}$

$x = 60$

$\therefore$ He was 60 years old when he died.

3 b.) Soln:

$n(U) = 65 \quad n(A) = 39 \quad n(B') = 24 \quad n(A' \cap B') = 4$

$n(A \cap B)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(B) = n(U) - n(B')$

$= 65 - 24$

$n(B) = 41$

$n(A \cup B) = n(U) - n(A' \cap B')$

$= 65 - 4$

$= 61$

$n(A \cup B) = n(A) + n(B) - n(A \cup B)$

$61 = 39 + 41 - n(A \cap B)$

$n(A \cap B) = 80 - 61$

$= 19$

$\therefore n(A \cap B) = 19$

2011

3\. (a) A shopkeeper sold 500 sweets. Some costs shs. 5 and some cost shs. 8. The cash received for the more expensive sweets was shs. 100 more than for the cheaper sweets. Find the number of each kind of sweet which were sold.

(b) A survey of 240 houses showed that all of them kept a farm or a garden or both. If 180 kept gardens and 79 kept farms, how many houses kept both?

2010

3\. (a) Given that $A=\{x:0\le x\le 8\}$

$B=\{x:3\le x\le 11\}$

where x is an integer, in the same form, represent in a Venn diagram

(i) $A\cup B$

(ii) $A\cap B$

and hence find the elements in each set.

(b) In a school of 75 pupils, 42% of the pupils take Biology but not Chemistry, 32% take both subjects and 10% of them take Chemistry but not Biology. How many pupils do not take either Biology or Chemistry?

(6 marks)

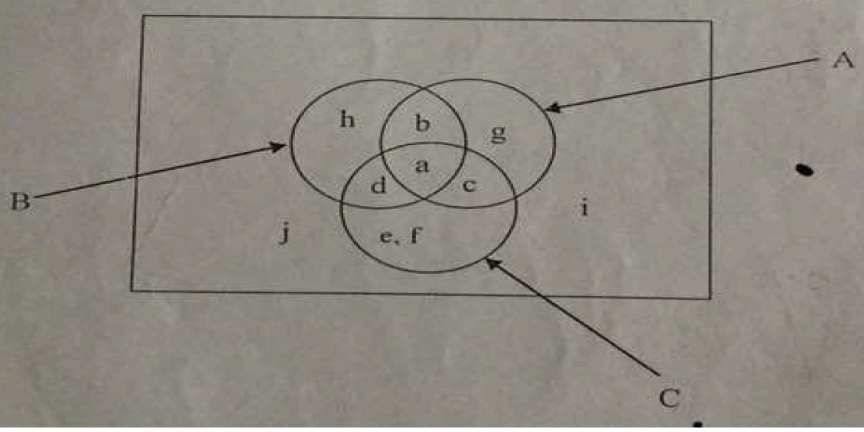
2009

(6 marks)

3\. In the figure drawn below, find the number of elements in sets:

(a) $A'\cap(B\cup C)$

(b) $(A'\cap B')\cup(B\cup C')$



(6 marks)

2008

3. (a) If $(2^{x-1})(3^{y+1}) = (3^4)(2^5)$ find

   (i) $x + y$

   (ii) $\frac{y}{x}$.

   (b) Students test results on three subjects: Mathematics, Physics and Chemistry show that 20 passed Chemistry, 5 passed all the three subjects, 12 passed Mathematics and Physics and 16 passed Mathematics and Chemistry. Each student passed at least two subjects.

   (i) Draw a well labeled Venn diagram to represent these results.

   (ii) How many students passed Physics and Chemistry?

   (iii) How many students did the test? **(6 marks)**

2007

2. (a) There are 60 people at a meeting. 35 are businesspersons, 32 are employees and 15 are both businesspersons and employees.

   (i) How many are businesspersons or employees?

   (ii) How many are neither businesspersons nor employees? **(3 marks)**

   (b) If $n(A \cap B') = 8$, $n(B \cap A') = 5$ and $n(A \cup B) = 20$.

   (i) Display the information in a Venn diagram,

   (ii) Give the values of $n(A)$ and $n(B)$. **(3 marks)**

4. Vectors and Coordinate Geometry

2018

4. (a). If $\underline{a} = 2x\underline{i} + 3\underline{j}$, $\underline{b} = (x^2 + y)\underline{i} + 4y\underline{j}$ and $\underline{v} = \frac{8}{3}\underline{i} + \frac{25}{12}\underline{j}$. Find x and y given that $\underline{v} = \frac{1}{4}\underline{a} + \frac{1}{3}\underline{b}$.

(b). Find the point of intersection of the lines $x - 2y = -5$ and $2x + 7y - 34 = 0$.

4. a)

$$\tfrac{8}{3}i + \tfrac{25}{12}j = \tfrac{1}{4}(2xi + 3j) + \tfrac{1}{3}[(x^2+y)i + 4yj]$$

multiplying by 12 throughout

$$12 \times \left(\tfrac{8}{3}i + \tfrac{25}{12}j\right) = \left\{\tfrac{1}{4}(2xi + 3j) + \tfrac{1}{3}[(x^2+y)i + 4yj]\right\} \times 12$$

$$32i + 25j = 6xi + 9j + 4x^2i + 4yi + 16yj$$

$$\underline{32i + 25j = (6xi + 4x^2i + 4yi) + (9j + 16yj)}$$

$$32i = 6xi + 4x^2i + 4yi$$

$$32i = 6x + 4x^2 + 4y$$

$$4x^2 + 6x + 4y - 32 = 0 \; \ldots \text{(i)}$$

also $25j = 9j + 16yj \; \ldots \text{(ii)}$

$$25 = 9 + 16y$$

$$25 - 9 = 16y$$

$$\frac{16}{16} = \frac{16y}{16} \qquad y = 1$$

substituting into equation (i)

$$4x^2 + 6x + 4(1) - 32 = 0$$

$$\frac{4x^2}{2} + \frac{6x}{2} + \frac{-28}{2} = \frac{0}{2}$$

$$2x^2 + 3x - 14 = 0$$

sum = 3

4 a) product = $^-28$

factors = 7, -4

$$2x^2 + 7x - 4x - 14 = 0$$
$$x(2x+7) - 2(2x+7) = 0$$
$$(x-2)(2x+7) = 0$$

$x_1 = 2$

x_2 : $2x + 7 = 0$

$$\frac{2x}{2} = \frac{^-7}{2}$$

$$x = {}^-3\tfrac{1}{2}$$

$\underline{x_1 = {}^-3\tfrac{1}{2} \quad \cancel{x_2} = 2}$

and $\underline{y = 1}$

b) $\begin{cases} x - 2y = {}^-5 & 2 \\ 2x + 7y = 34 & 1 \end{cases}$

$-\begin{cases} 2x - 4y = {}^-10 \\ 2x + 7y = 34 \end{cases}$

$$-4y - 7y = {}^-10 - 34$$

$$\frac{-11y}{-11} = \frac{-44}{-11}$$

$$y = 4$$

$$x - 2(4) = {}^-5$$
$$x - 8 = {}^-5$$
$$x = {}^-5 + 8$$
$$x = 3$$

$\therefore$ The point of intersection = $(3, 4)$

2017

4. (a) Given the three vectors $\underline{a} = 4\underline{i} + 6\underline{j}$, $\underline{b} = 4\underline{i} + 10\underline{j}$ and $\underline{c} = 2\underline{i} + 4\underline{j}$ determine the magnitude of their resultant.

(b) Camilla walks 5 km northeast, then 3 km due east and afterwards 2 km due south. Represent these displacements together with the resultant displacement graphically using the scale 1 unit = 1 km.

(c) Show that triangle ABC is right-angled where $A = (-2,-1)$, $B = (2,1)$ and $C = (1,3)$.

4 (a) $\underline{a} = 4\underline{i}+6\underline{j}$, $\underline{b} = 4\underline{i}+10\underline{j}$, $\underline{c} = 2\underline{i}+4\underline{j}$

$\underline{a}+\underline{b}+\underline{c} = 4\underline{i}+6\underline{j}+4\underline{i}+10\underline{j}+2\underline{i}+4\underline{j} = 4\underline{i}+4\underline{i}+2\underline{i}+6\underline{j}+10\underline{j}+4\underline{j}$

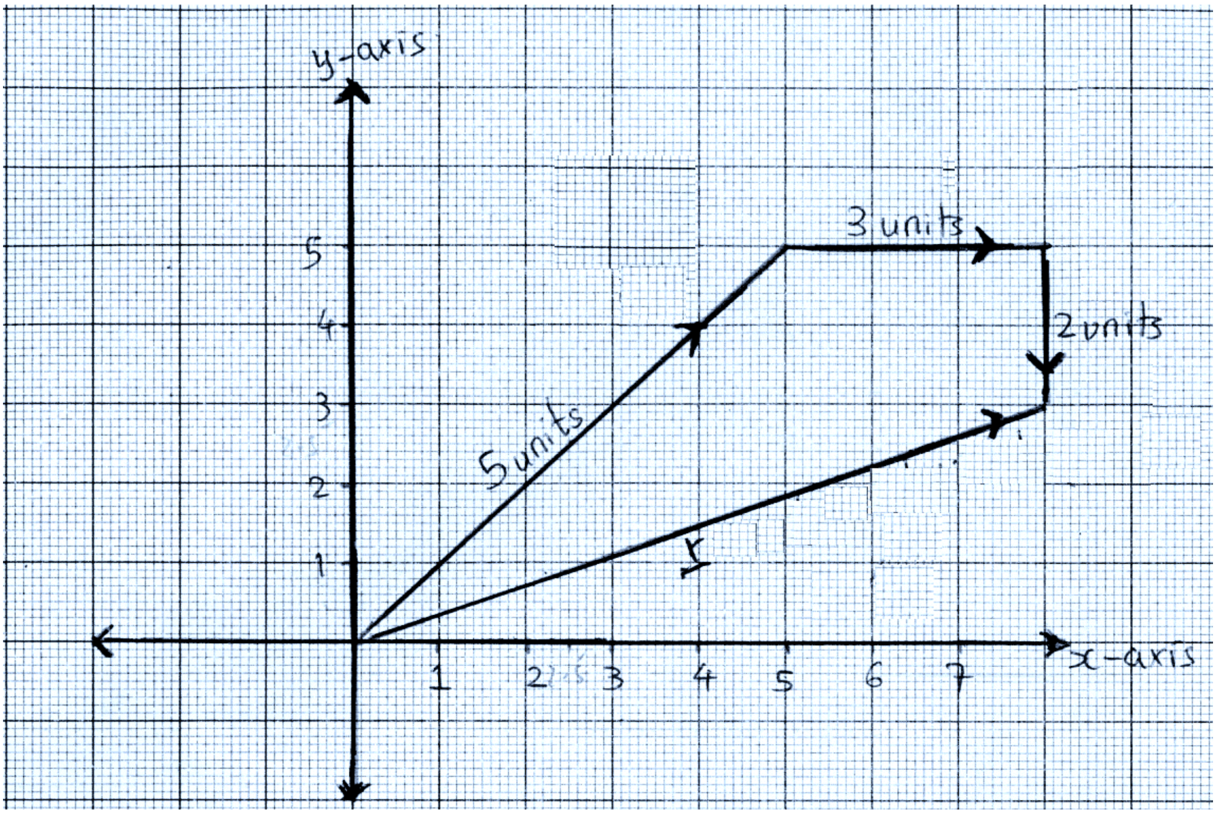
$\underline{a}+\underline{b}+\underline{c} = 10\underline{i}+20\underline{j}$

$|\underline{a}+\underline{b}+\underline{c}| = (\sqrt{10^2+20^2})$ units $= (\sqrt{100+400})$ units

$= \sqrt{500}$ units

$= \underline{10\sqrt{5}}$ units Ans

(b) IN THE GRAPH PAPER



(c) distance between A(-2,-1) and B(2,1)

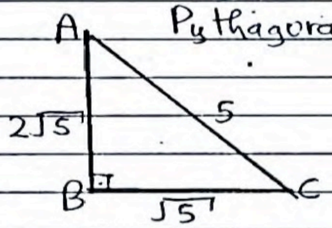
$d = \sqrt{(-2-2)^2+(-1-1)^2} = \sqrt{16+4} = \sqrt{20} = \underline{2\sqrt{5}\text{ units}}$

distance between A(-2,-1) and C(1,3)

$d = \sqrt{(-2-1)^2+(-1-3)^2} = \sqrt{9+16} = \sqrt{25} = \underline{5\text{ units}}$

4 (c) distance between B(2,1) and C(1,3)

$d = \sqrt{(2-1)^2+(1-3)^2} = \sqrt{1+4} = \underline{\sqrt{5}\text{ units}}$



Hence for a right angled triangled

Pythagoras theorem: $a^2+b^2=c^2$

$(2\sqrt{5})^2+(\sqrt{5})^2 = 5^2$

$(4\times5)+5 = 25$

$20+5 = 25$

$25 = 25$

Hence LHS = RHS, hence a right angled triangle

2016

4. (a) Given vectors $\underset{\sim}{a} = 6\mathbf{i} + 12\mathbf{j}$ and $\underset{\sim}{b} = 17\mathbf{i} + 18\mathbf{j}$:
   (i) Find the vector $\underset{\sim}{c} = 2\underset{\sim}{a} - \underset{\sim}{b}$ and its magnitude correctly to 3 significant figures.
   (ii) Represent vector $\underset{\sim}{c}$ in part (a)(i) on the x - y plane.

   (b) Find the equation of the line passing through the midpoint of the points A (-3, 2) and B (1, -4) and which is perpendicular to line AB.

2015

4. (a) Given vectors $\underline{a} = 3\mathbf{i} + 2\mathbf{j}$, $\underline{b} = 8\mathbf{i} - 3\mathbf{j}$ and $\underline{c} = 2\mathbf{i} + 4\mathbf{j}$ find:
   (i) the vector $\underline{d} = 3\underline{a} - \underline{b} + \frac{1}{2}\underline{c}$,
   (ii) a unit vector in the direction of $\underline{d}$.

   (b) Find the equation of the line passing at point (6, -2) and it is perpendicular to the line that crosses the x-axis at 3 and the y-axis at -4.

Question Number | SUBJECT NAME: BASIC MATHEMATICS | INDEX NUMBER: S2325/0011 | For Examiners' use only

4. Soln:

(a) $\underline{a} = 3i + 2j$

$\underline{b} = 8i - 3j$

$\underline{c} = 2i + 4j$.

(i) $d = 3a - b + \frac{1}{2}c$

$d = 3(3i + 2j) - (8i - 3j) + \frac{1}{2}(2i + 4j)$ — $\checkmark \frac{1}{2}$

$d = 9i + 6j - 8i + 3j + i + 2j$

$d = 2i + 11j$

$\therefore$ $d = 2i + 11j$ — $\checkmark 1$

(ii) Soln:

$\frac{d}{|d|}$ $d = 2i + 11j$ — $\checkmark \frac{1}{2}$

$|d| = \sqrt{x^2 + y^2}$

$= \sqrt{2^2 + (11)^2}$

$= \sqrt{4 + 121}$

$= \sqrt{125}$

$= 5\sqrt{5}$

$\frac{d}{|d|} = \frac{2i + 11j}{5\sqrt{5}}$

$\hat{d} = \frac{2i}{5\sqrt{5}} + \frac{11j}{5\sqrt{5}}$

$\therefore$ unit vector $= \frac{2i}{5\sqrt{5}} + \frac{11j}{5\sqrt{5}}$ — $\checkmark 1$

(b) Soln:

x-axis at 3 (3,0)

y-axis at 4 (0,4)

gradient $= \frac{\Delta y}{\Delta x}$ — 3

Question Number | SUBJECT NAME: BASIC MATHEMATICS | INDEX NUMBER: S2325/0011 | For Examiners' use only

4 (b) $= \frac{4-0}{0-3} = \frac{-4}{-3}$

$= +4/3$ — $\checkmark 1$

$= 4/3$

for perpendicular lines

$m_1 m_2 = -1$

$\frac{4}{3} m_2 = -1$

$m_2 = -1 \div 4/3$

$m_2 = -1 \times 3/4$ — $\checkmark 1$

$m_2 = -3/4$

taken $m = -3/4$ point $(6, -2)$

$m(x - x_1) = y - y_1$

$-\frac{3}{4}(x - 6) = y + 2$

$-3x + 18 = 4y + 8$.

$3x + 4y + 8 - 18 = 0$

$3x + 4y - 10 = 0$

$\therefore$ Equation of the line is $3x + 4y - 10 = 0$ — $\checkmark 1$

(05)

2014

4. (a) Find the equation of the line passing through $(6, 4)$ and perpendicular to the line whose equation is $12x + 6y = 9$.

(b) If $\underline{a} = 2\underline{i} + 3\underline{j}$, $\underline{b} = 19\underline{i} - 15\underline{j}$ and $\underline{c} = 5\underline{i} - 7\underline{j}$, find the value of x such that $x\underline{a} + y\underline{c} = \underline{b}$.

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Question Number	SUBJECT NAME: MATHEMATICS INDEX NUMBER: S.0184/0035	For Examiners' use only
4	a) $12x + 6y = 9$	
	$\frac{12x}{-6} - \frac{9}{-6} = \frac{-6y}{-6}$	
	$-2x + \frac{3}{2} = y$	
	gradient $= -2$	01
	for ⊥ lines $m_1 m_2 = -1$	
	$m_1 = -2$	
	$m_2 = -1/-2$	
	$m_2 = 1/2$	01
	$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ $x = (6, 4)$, $x\ (x, y)$	
	$\frac{1}{2} = \frac{y - 4}{x - 6}$	
	$2y - 4(2) = x - 6$	
	$2y - 8 = x - 6$	
	$2y = x - 6 + 8$	
	$\frac{2y}{2} = \frac{x}{2} + \frac{2}{2}$	
	$y = x/2 + 1$	01
	∴ The equation is $y = x/2 + 1$.	
	b) $x\underline{a} + y\underline{c} = \underline{b}$	
	$x\begin{pmatrix}2\\3\end{pmatrix} + y\begin{pmatrix}5\\-7\end{pmatrix} = \begin{pmatrix}19\\-15\end{pmatrix}$	0-1/2
	$\begin{pmatrix}2x\\3x\end{pmatrix} + \begin{pmatrix}5y\\-7y\end{pmatrix} = \begin{pmatrix}19\\-15\end{pmatrix}$	
	$2x + 5y = 19$ ---- i	
	$3x + -7y = -15$ ---- ii	01
	$\frac{2x}{2} = \frac{19 - 5y}{2}$ $x = \frac{19 - 5y}{2}$	04 1/2

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Question Number	SUBJECT NAME MATHEMATICS INDEX NUMBER S0184/0025	For Examiners' use only
4.	$3x - 7y = -15$	
	$3\left(\frac{19-5y}{2}\right) - 7y = -15$	
	$\frac{57-15y}{2} - (7y) = -15 \times 2$	
	$57 - 15y - 14y = -30$	
	$-29y = -30 - 57$	
	$\frac{-29y}{-29} = \frac{-87}{-29}$	
	$y = 3$	
	$x = \frac{19-5y}{2}$ but $y = 3$	
	$x = \frac{19-5(3)}{2} = \frac{19-15}{2} = \frac{4}{2} = 2$	
	$\therefore x = 2$	
6	a) Let one litre of milk = x	
	$x = 600$	
	$\frac{600x}{600} = \frac{208,800}{600}$	
	$x = \frac{208,800}{600}$	
	$x = 348$ litres	
	$\therefore$ Juma will sell 348 litres of milk	
	b) $l \propto T$ introduce constant k	
	$l = kT$	
	$\frac{l}{T} = \frac{T}{T}$ but $T = 2$ second $l = 0.4$cm	
	$k = l/T$	

2013

4. (a) Find the direction cosines of $\underline{C} = 9\underline{i} + 12\underline{j}$, hence show that the sum of the squares of these direction cosines is one.

(b) Find the equation of the line through the points $(4,6)$ and the midpoint of $(2,4)$ and $(10,4)$.

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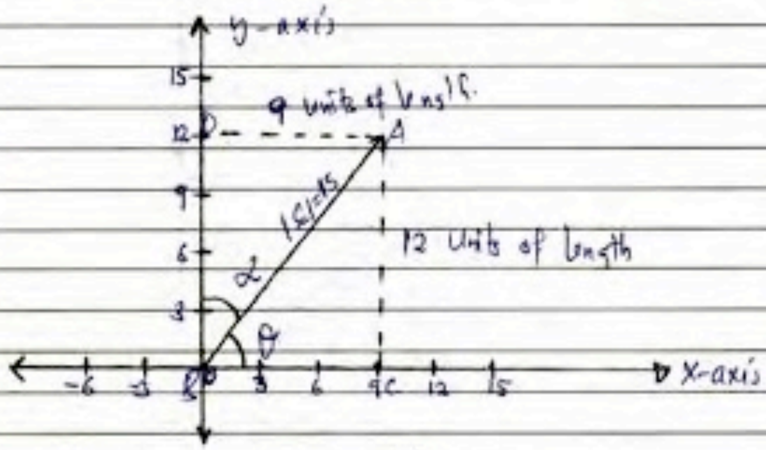
Question Number | SUBJECT NAME: BASIC MATHEMATICS | INDEX NUMBER: J·0110/0023 | For Examiners' use only

4. (a) Soln.

Given

$\underline{C} = 9\underline{i} + 12\underline{j}$

In the x-y plane.



From pythagoras theorem

$a^2 + b^2 = c^2$

$9^2 + 12^2 = (|\underline{C}|)^2$

$81 + 144 = |\underline{C}|^2$

$\sqrt{225} = \sqrt{(|\underline{C}|)^2}$

$|\underline{C}| = 15$ Units of length.

01

Direction cosine on y-axis

$\cos\alpha = 12/15$ $\left(\frac{\text{Adjacent}}{\text{hypotenuse}}\right)$

$\therefore \cos\alpha = 12/15 = 4/5$

Direction Cosine on x-axis

$\cos\theta = 9/15$ $\left(\frac{\text{Adjacent}}{\text{hypotenuse}}\right)$

Question Number | SUBJECT NAME BASIC MATHEMATICS | INDEX NUMBER S.0110/0023.

4 (a) $\therefore$ Direction Cosines are $4/5$ on y-axis and $3/5$ on x-axis.

Required to show that

$$\cos^2\alpha + \cos^2\theta = 1.$$

Consider $\cos^2\alpha + \cos^2\theta$

$$\Rightarrow \left(4/5\right)^2 + \left(3/5\right)^2$$

$$\Rightarrow 16/25 + 9/25$$

$$\Rightarrow 25/25$$

$$\Rightarrow 1$$

Since $1 = 1$.

Hence shown as required.

(b) To find Equation of line through $(4,6)$ and midpoint of $(2,4)$ and $(10,4)$

from Mid point $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{2+10}{2}, \frac{4+4}{2}\right)$$

$$= \left(12/2, 8/2\right)$$

$$= (6, 4)$$

For the Equation of the line

Let $(x_0, y_0) = (4,6)$

$$(x_1, y_1) = (6,4)$$

2012

4. Given that $\underline{a} = (3, 4)$, $\underline{b} = (1, -4)$ and $\underline{c} = (5,2)$ determine:
   (a) $\underline{d} = \underline{a} + 4\underline{b} - 2\underline{c}$;
   (b) magnitude of vector $\underline{d}$, leaving your answer in the form $m\sqrt{n}$;
   (c) the direction cosines of $\underline{d}$ and hence show that the sum of the squares of these direction cosines is one.

4 a Given $\underset{\sim}{a} = (3, 4)$ $\underset{\sim}{b} = (1, -4)$ $\underset{\sim}{c} = (5, 2)$

$\underset{\sim}{d} = \underset{\sim}{a} + 4\underset{\sim}{b} - 2\underset{\sim}{c}$

$\underset{\sim}{d} = (3, 4) + 4(1, -4) - 2(5, 2)$

$\underset{\sim}{d} = (3, 4) + (4, -16) + (-10, -4)$

$\underline{\underset{\sim}{d} = (-3, -16)}$

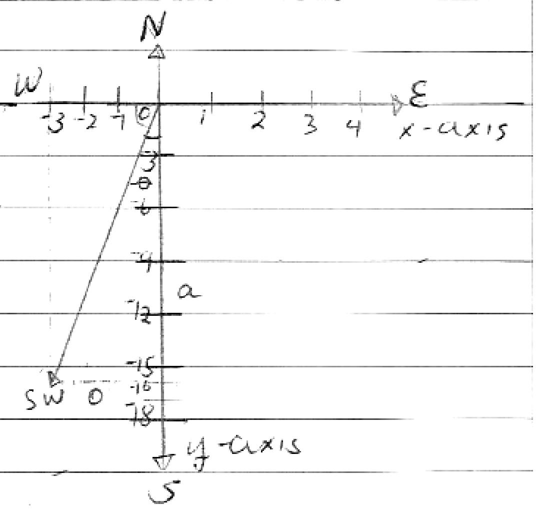
4 b $|d| = \sqrt{x^2 + y^2}$

$|d| = \sqrt{(-3)^2 + (-16)^2}$

$|d| = \sqrt{9 + 256}$

$|d| = \sqrt{265}$ units

4 c i) Direction cosine $= \dfrac{\text{Adjacent}}{\text{Hypotunese}}$



Direction cosine $= \dfrac{-16}{\sqrt{265}}$

Direction cosine $= \dfrac{-16}{16.28}$

$\underline{\text{Direction cosine}_1 = -0.9828}$

$\underline{\text{Direction cosine}_1 = S\ 10°45'W}$

Direction cosine $= \dfrac{-3}{16.28}$

$\underline{\text{Direction cosine}_2 = -0.1853} = \underline{S\ 79°15'W}$

ii) $\left(\dfrac{-16}{\sqrt{265}}\right)^2 + \left(\dfrac{-3}{\sqrt{265}}\right)^2$

$\dfrac{256}{265} + \dfrac{9}{265} = \dfrac{265}{265} = 1$ (shown)

2011

4. (a) Find the distance between point $(-3,-2)$ and the point midway between $(2,13)$ and $(4,7)$. Write your answer in the form $a\sqrt{c}$ where a and c are positive real numbers.

(b) Given the vectors $\underline{x} = 3\underline{i} + 2\underline{j}$, $\underline{y} = 5\underline{i} - 3\underline{j}$ and $\underline{z} = 4\underline{i} - 2\underline{j}$

(i) Find the resultant vector $\underline{r} = \underline{x} + \underline{y} + \underline{z}$ and its direction.

(ii) Plot the three vectors on the same axes and hence indicate the magnitude of each vector [do not perform any calculation].

2010

4. (a) (i) Without using mathematical tables, find the numerical value of

$$\frac{1}{\sin^2 45^\circ} + \frac{2}{\cos^2 45^\circ} + \frac{3}{\tan^2 45^\circ}$$

(ii) Write down the equation of the line which passes through (7, 3) and which is inclined at 45° to the positive direction of the x – axis.

(b) The position vectors of the points A, B and C are $4\underset{\sim}{i} - 3\underset{\sim}{j}$, $\underset{\sim}{i} + 3\underset{\sim}{j}$ and $-5\underset{\sim}{i} + \underset{\sim}{j}$ respectively. Find the vectors $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ hence verify that $\overline{AB} + \overline{BC} = \overline{AC}$ **(6 marks)**

2009

4. (a) Given vectors $\underline{a} = -i + 3j$, $\underline{b} = 5i - 2j$ and $\underline{c} = 3\underline{a} + 4\underline{b}$, find a unit vector in the direction of vector $\underline{c}$.

(b) The point A (5, -7) is the vertex of the right angle of a right angled-triangle whose hypotenuse lies along the line $6x - 13y = 39$. A second vertex of the triangle is B (0,-3). Find the remaining vertex C (x, y). **(6 marks)**

2008

4. (a) Solve the following Simultaneous equations.

$$\begin{cases} x = 4 - \dfrac{3y}{2} \\ -3x + \dfrac{y}{2} = 1 \end{cases}$$

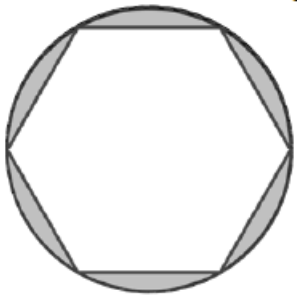
(b) If $\bar{A}$ and $\bar{B}$ are two vectors such that $\bar{A} = 2i + 5j$ and $\bar{B} = -4i + j$. Find the position vector $\overline{OM}$ where M is the midpoint of $\overline{AB}$.

(6 marks)

5. Areas, Similarity, Geometry, Perimeter

2018

5. In the following figure, a regular hexagon is inscribed in a circle. If the perimeter of the hexagon is 42 cm, find:
(a) The radius of the circle.
(b) The area of the circle and the regular polygon.
(c) The area of the shaded region.



5. a) $P = 42\,cm$ $\quad n = 6$

Perimeter $= 2nr\sin\left(\frac{180^\circ}{n}\right)$

$42 = 2 \times 6r \sin\left(\frac{180^\circ}{6}\right)$

$42 = 12r \sin 30^\circ$

$42 = 12r \times 0.5$

$\frac{42}{6} = \cancel{16}\ \frac{6r}{6}$

$r = 7cm$

$\therefore$ Radius $= 7\,cm$

b) Area of the circle $= \pi r^2$

$= 3.14 \times 7cm \times 7cm$

$= 154\,cm^2$

$\therefore$ Area of the circle $= 154\,cm^2$

Area of the polygon $= \frac{1}{2}nr^2\sin\left(\frac{360^\circ}{n}\right)$

$= \frac{1}{2} \times 6 \times 7^2 \sin\left(\frac{360^\circ}{6}\right)$

$= 3 \times 49 \sin 60^\circ$

$= 147 \times 0.8660$

$\therefore$ Area $= 127.282\,cm^2$

$\therefore$ Area of the circle $= 154\,cm^2$ and area of the polygon $= 127.282\,cm^2$

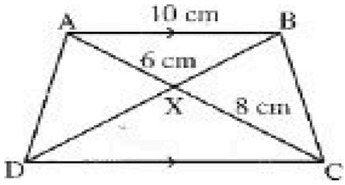
c) Area of shaded region = Area of the circle − Area of the polygon

$= 154\,cm^2 - 127.282\,cm^2 = 26.718\,cm^2$

$\therefore$ The area of shaded region $= 26.718\,cm^2$

2017

5. (a) In the figure below, AB = 10 cm, AX = 6 cm, CX = 8 cm and AB is parallel to DC.



(i) Show whether triangles AXB and CXD are similar or not.
(ii) Find the length of CD.
(iii) Find the ratio of the areas of triangles AXB and CXD.

(b) Using a ruler and compass, construct an angle of 90°.

5(a) Soln.

(i) In the triangles AXB and CXD:

$\angle DXC = \angle AXB$ (vertically opposite angles).

$\angle BAX = \angle DCX$ (alternate interior angles.)

$\angle XDC = \angle XBA$ (alternate interior angles).

$\therefore \triangle AXB \sim \triangle CXD$ (AAA).

(ii) $\frac{CX}{XA} = \frac{DC}{AB} = \frac{DX}{XB}$

$\frac{8cm}{6cm} = \frac{DC}{10cm}$

$CD = \frac{8cm \times 10cm}{6cm}$

$CD = \frac{40cm}{3}$

$CD \approx 13.3cm$ Answer.

(iii) Soln.

(Ratio of sides)^2 = Ratio of areas.

$\left(\frac{8}{6}\right)^2 = \frac{A_2}{A_1}$

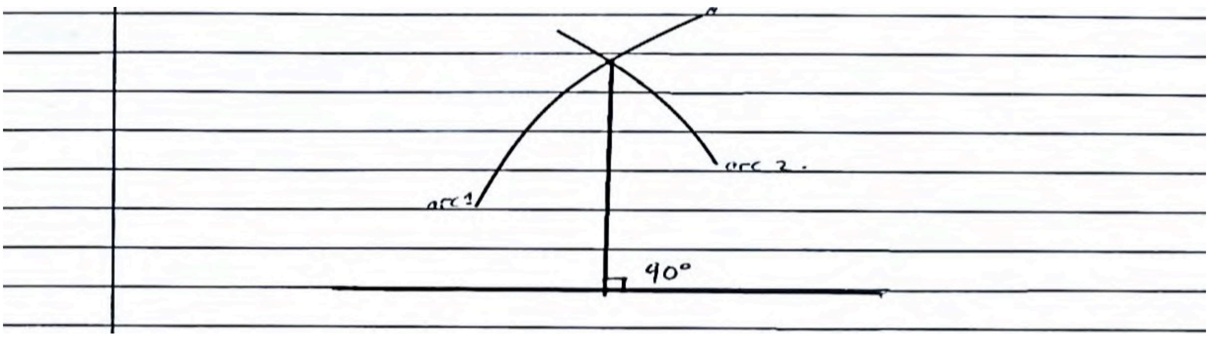
$\frac{A_2}{A_1} \Rightarrow \frac{64}{36} \Rightarrow \frac{16}{9}$

$\therefore$ Ratio of areas is 16 : 9

(b) CONSTRUCTION OF 90° BY USING RULER AND COMPASS.

steps:

(i) Draw a straight horizontal line of any length and mark its centre
(ii) start from one end put a compass and draw an arc
(iii) Repeat to draw an arc on the other side
(iv) Joint the point of intersection of arcs drawn to the centre
(v) The angles between vertical and horizontal line are 90°.



2016

5. (a) In triangle ABC; X, Y and Z are the midpoints of sides $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ respectively. If $\overline{ZX} = \overline{ZY}$ and $Z\hat{X}B = Z\hat{Y}C = 90^\circ$;

(i) Represent this information diagrammatically,

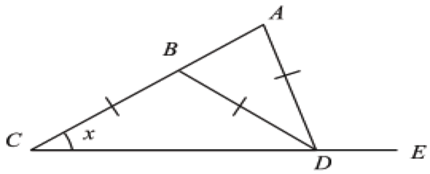
(ii) Show that $A\hat{B}Z = A\hat{C}Z$.

(b) The areas of two similar polygons are 27 and 48 square metres. If the length of one side of the smaller polygon is 4.5 cm, find the length of the corresponding side of the larger polygon.

2015

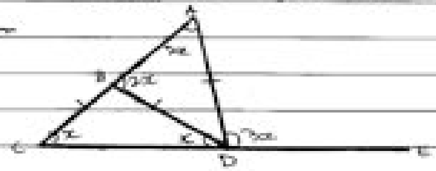
5. (a) Two triangles are similar. A side of one triangle is 10 cm long while the length of the corresponding side of the other triangle is 18 cm. If the given sides are the bases of the triangles and the area of the smaller triangle is 40 cm², find the area and the height of the larger triangle.

(b) In the figure below $\overline{CB} = \overline{BD} = \overline{DA}$ and angle $ACD = x$.



(i) Show that angle $ADE = 3x$,

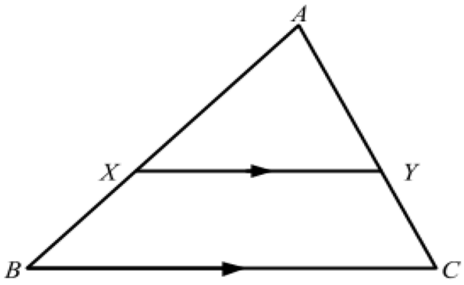
(ii) Calculate the measure of angle CDA if $x = 39°$.

Question Number	SUBJECT NAME BASIC MATHEMATICS INDEX NUMBER S0187/0056	Examiners' use only
5a)	From similarity $\frac{\text{Area 1}}{\text{Area 2}} = \left(\frac{\text{Side 1}}{\text{Side 2}}\right)^2$	2
	$\frac{40\text{ cm}^2}{\text{Area 2}} = \left(\frac{10\text{cm}}{18\text{cm}}\right)^2$, let Area 2 be x	
	$\frac{40\text{cm}^2}{x} = \frac{100\text{ cm}^2}{324\text{ cm}^2}$	
	$100\text{cm}^2\, x = 324\text{cm}^2 \times 40\text{cm}^2$	
	$x = \frac{324\text{cm}^2 \times 40\text{cm}^2}{100\text{cm}^2} = \frac{1296}{10}$	
	$x = 129.6\text{cm}^2$	
	∴ Area of larger triangle is 129.6cm^2	
	Area of a triangle $= \frac{1}{2} \times b \times h$, $A = 129.6\text{cm}^2$, $B = 18\text{cm}$, $h = ?$	
	$129.6\text{cm}^2 = \frac{1}{2} \times 18 \times h$	
	$129.6\text{cm}^2 = 9h$	
	$h = 14.4\text{cm}$	
	∴ The height of the larger triangle is 14.4cm.	2
5b)	Consider triangle CBD and ADB	
	∠BCD = ∠BDC ... Base angles of an isosceles triangle	
	The sum of these two angles equals to the measure of the	
	the opposite exterior angle namely ∠ABD	
	∠BCD + ∠BDC = ∠ABD = 2x	
	∠ABD = ∠BAD ... Base angles of an isosceles triangle	
	~~The sum~~	
		
	The sum of ∠CAD and ∠ACD = ∠ADE since angle ADE	
	is an opposite exterior angle to ∠CAD and ∠ACD	
	∠CAD + ∠ACD = ∠ADE = x + 2x = 3x.	
	∴ ∠ADE = 3x	

04½

2014

5. (a) Given that $XY = 2cm$, $BC = 3cm$ and area of $XYCB = 10cm^3$, calculate the area of triangle AXY.



(b) Determine the length of one side of a regular quadrilateral inscribed in a circle of radius $10cm$.

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02

Question Number	SUBJECT NAME Mathematics INDEX NUMBER S0265/0040	For Examiners' use only
4) a)	$2y = x - 6 + 8$	
	$2y = x + 8 - 6$	
	$\frac{2y}{2} = \frac{x + 2}{2}$	
	$y = \frac{1}{2}x + 1$	
	$\therefore$ The equation is $y = \frac{1}{2}x + 1$	01
4) b)	Soln	
	$x6 + y6 = b$	
	$x(2+3) + y(5-7) = (19-15)$	
	$2x + 3x + 5y - 7y = 19 - 15$	00
	$5x + -2y = 19 - 15$	
	$5x - 2y = 4$	
	$\frac{5x}{5} = \frac{2y + 4}{5}$	
	$\therefore x = \frac{2y + 4}{5}$	00
		03
5 a)	Soln	
	$\frac{A_1}{A_2} = \left(\frac{4}{12}\right)^2$	01
	$\frac{A_1}{A_1 + 10} = \left(\frac{2}{3}\right)^2$	
	$\frac{A_1}{A_1 + 10} = \frac{4}{9}$	01
	$4(A_1 + 10) = 9A_1$	02
	$4A_1 + 40 = 9A_1$	

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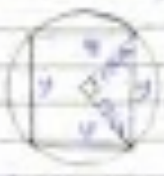
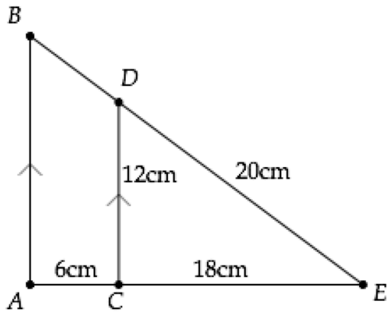
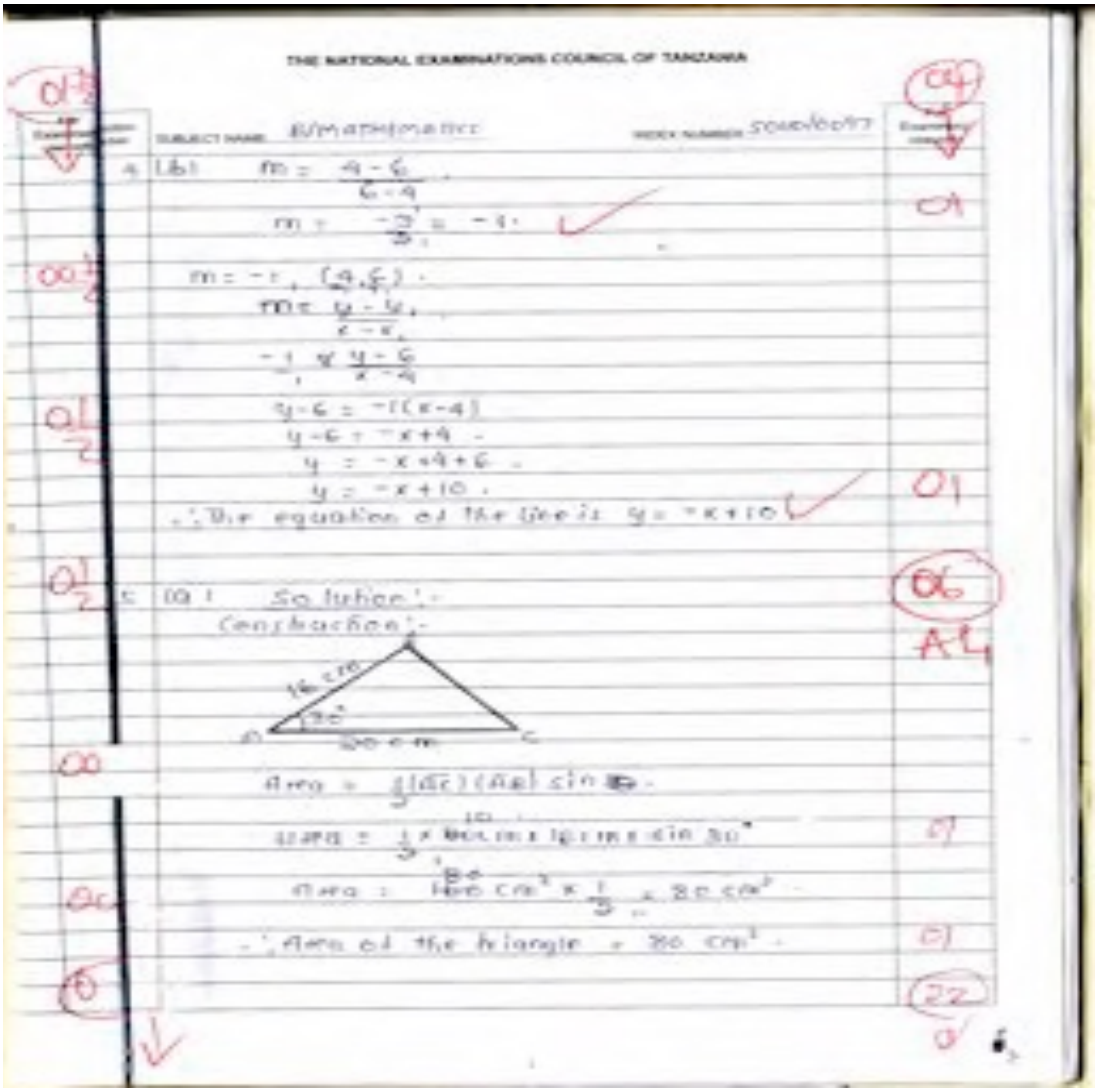
Question Number	SUBJECT NAME BASIC MATHEMATICS INDEX NUMBER [illegible]	For Examiners' use only
4	Therefore $x = 2$ and $y = 7$	
5	(a) Solution	
	$\frac{XY}{BC} = k$, but $\frac{\text{Area } AXY}{\text{Area } ABC} = k^2$	2 of 1/2
	Then $k = \frac{2}{3}$	
	Then $\frac{\text{Area } AXY}{\text{Area } ABC} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$	
	But Area ABC = Area AXY + Area XYCB	
	= Area AXY + 10 cm²	
	Therefore $\frac{\text{Area } AXY}{\text{Area } AXY + 10\text{cm}^2} = \frac{4}{9}$	
	Let Area AXY = y	
	$\frac{y}{y+10} = \frac{4}{9}$	=1
	$9y = 4y + 40$	
	$9y - 4y = 40$	
	$5y = 40$	
	$y = 8\text{cm}^2$	+1
	Therefore Area AXY = 8cm²	
	(b) Solution	
	From the figure below	
		
	$y^2 = (6\text{cm})^2 + (18\text{cm})^2 = 360\text{cm}^2$	54

2013

5. (a) The length of two sides of a triangle are $16cm$ and $20cm$. Find the area of the triangle if the included angle is $30°$.

(b) In the figure below, calculate the length BD:





THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

SUBJECT NAME B/Mathematics

4 (b) $m = \frac{4-6}{6-4}$

$m = \frac{-2}{2} = -1$

$m = -1,\ (4,6)$

$m = \frac{y-y_1}{x-x_1}$

$-1 = \frac{y-6}{x-4}$

$y-6 = -1(x-4)$

$y-6 = -x+4$

$y = -x+4+6$

$y = -x+10$

∴ The equation of the line is $y = -x+10$

5 (a) Solution:-

Construction:-

Area $= \frac{1}{2}(AC)(AB)\sin\theta$

Area $= \frac{1}{2} \times 20cm \times 16cm \times \sin 30°$

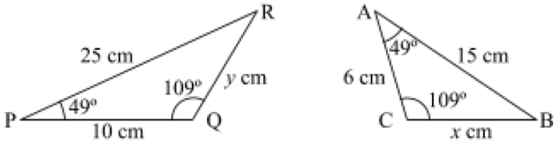
Area $= 160\,cm^2 \times \frac{1}{2} = 80\,cm^2$

∴ Area of the triangle $= 80\,cm^2$

2012

5. (a) If polygons X and Y are similar and their areas are $16cm^2$ and $49cm^2$ respectively, what is the length of a side of polygon Y if the corresponding side of polygon X is $28cm$?

(b) (i) Show whether triangles PQR and ABC are similar or not



(ii) Find the relationship between y and x in the triangles given above.

5b) i) Given: ΔPQR and ΔABC

to prove: $\Delta PQR \sim \Delta ABC$

proof: ΔPQR and ΔABC

$R\hat{P}Q = C\hat{A}B$ (given $49°$)

$R\hat{Q}P = A\hat{C}B$ (given $109°$)

$P\hat{R}Q = C\hat{B}A$ (third angle)

$\therefore \Delta PQR \sim \Delta ABC$ (AAA-theorem).

5bii) $\cancel{A}$ $\frac{PR}{AB} = \frac{PQ}{AC} = \frac{y}{x}$

$\frac{25}{15} = \frac{10}{6} = \frac{y}{x}$

$\frac{5}{3} = \frac{y}{x}$

$y = \frac{5}{3}x$

$\frac{y}{x} = \frac{5}{3}$.

2011

5. (a) In figure 1, $ABCD$ is a square. If $\overline{AR} = \overline{BR}$ prove that R is the midpoint of $\overline{DC}$.

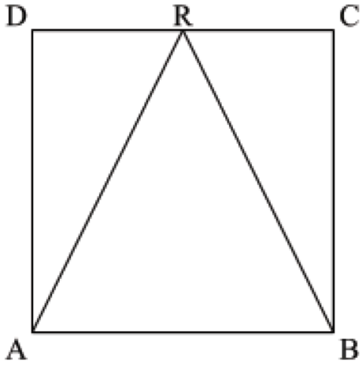


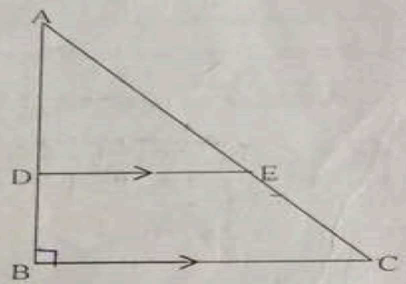
Figure 1

(b) Calculate the size of an interior angle of a regular nonagon.

2010

5. (a) A circle of radius 10 units is circumscribed by a right-angled isosceles triangle. Find the lengths of the sides of the triangle and hence its perimeter (all in 2 decimal places).

(b) In the figure below DE is parallel to BC, AD = 6 cm, BD = 3 cm DE = 4 cm. and $A\hat{B}C = 90°$

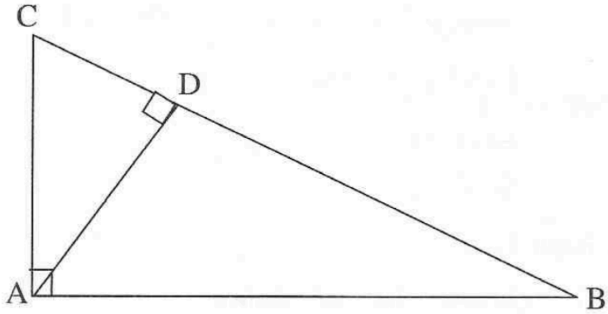


Calculate: (i) the length of BC

(ii) the ratio $\frac{AE}{AC}$ **(6 marks)**

5. (a) The volume of two similar cylinders is 125 cm^3 and 512 cm^3. If the radius of the larger cylinder is 8cm, find the radius of the smaller cylinder.

(b) In the diagram below, show that $\frac{AD}{AB} = \frac{CD}{AC}$

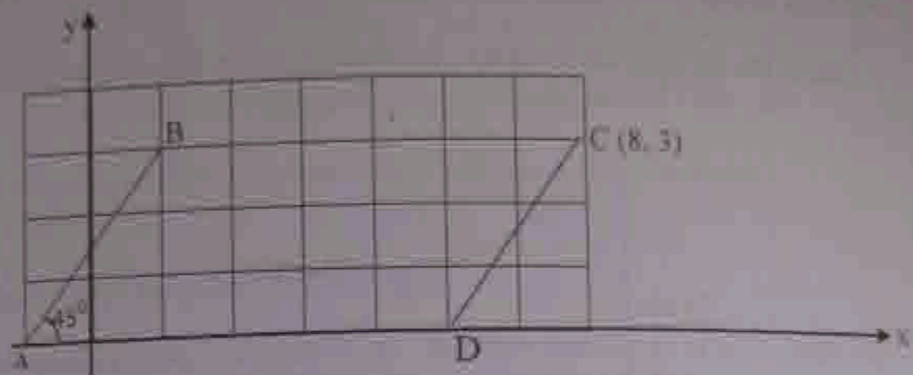


(6 marks)

2009

2008

5. (a) Find the area and the perimeter of a parallelogram ABCD given in the figure below if $\angle BAD = 45^\circ$.



(b) The ratio of the area of two similar triangles is 1:4. Find the ratio of their corresponding sides (6 marks)

2007

5. (a) Prove that the opposite angles of any quadrilateral inscribed in a circle are supplementary. **(2 marks)**

(b) In figure 1 below, 0 is the centre of the circle, $A\hat{O}B = 120°$ and $C\hat{D}B = 15°$.

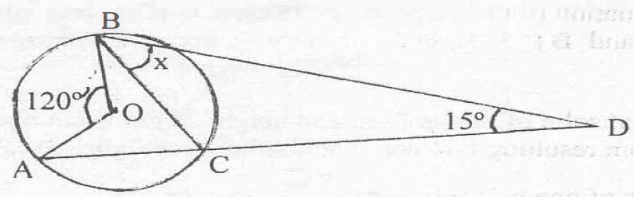


Fig. 1

Find the value of x. **(2 marks)**

(c)

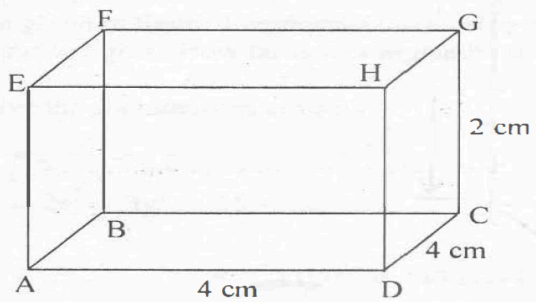


Fig. 2

For a tank given in the figure 2 above, calculate the angle between $\overline{DF}$ and the base ABCD. **(2 marks)**

6. Rates and Variations

2018

6. (a) Mukasa received Ushs 1,000,000 from his sister in Uganda. How much did he get in Tanzanian currency (Tshs) if one Ugandan shilling was equivalent to 0.65 Tanzanian shilling?

(b) The energy (E) stored in an elastic band varies as the square of the extension (x). When the elastic band is extended by 4 cm, the energy stored is 240 joules. What is the energy stored when the extension is 6 cm? What is the extension when the stored energy is 60 joules?

6 a) Ushs - 1,000,000

solution

1 Ushs = 0·65 Tshs

1,000,000 Ushs = ? x

x = $\frac{1,000,000 \text{ Ushs} \times 0{\cdot}65 \text{ Tsh}}{1 \text{ Ushs} \times ?}$

= 650,000 Tanzanian shilling

∴ Mukasa received 650,000/= in Tshs

b) solution

$E \propto x^2$

$E = kx^2$

when x = 4cm and E = 240 joules

$\Rightarrow E = kx^2$

$240 = k(4)^2$

but, $(4)^2 = 16$

$\frac{240}{16} = \frac{k\,16}{16}$

∴ $k = 15$

But also,

when (x) - extension is 6cm

find E - ?

from

$\Rightarrow E = kx^2$

$E = 15 \times 6^2$

$= 15 \times 36$

$E = 540$

∴ Energy is 540 joules

Thus

when given energy is 60 joules

to find extension = ? (x)

$k = 15$

from $E = kx^2$

60 joules $= 15 \times x^2$

$\frac{60 \text{ jouls}}{15} = \frac{15 \times x^2}{15}$

$\sqrt{x^2} = \sqrt{4}$

x = 2cm

∴ The extension is 2cm

2017

6. (a) In the preparation of fanta orange drink, a bottling filling machine can fill 1,500 bottles in 45 minutes. How many bottles will it fill in $4\frac{1}{2}$ hours?

(b) If X varies directly as Y and inversely as W, find the values of *a* and *b* in the table below.

X	8	6	*b*
Y	4	*a*	2
W	2	3	4

06. Solution.

(a) Given; 1,500 bottles are filled in 45 minutes.

$4\frac{1}{2}$ hours $= 4\times60 + \frac{1}{2}\times60$ minutes,
$= 270$ minutes,

Then; 1,500 bottles $\longrightarrow$ 45 minutes
so; $x \longrightarrow$ 270 minutes

[x is the unknown number of bottles].

$\Rightarrow x = \frac{270\times1,500}{45} = 9,000$ bottles.

$\therefore$ 9,000 bottles can be filled.

06. (b) Given; $x \propto \frac{y}{w}$.

$x_1 = 8,\ y_1 = 4,\ w_1 = 2$

$\Rightarrow x = \frac{ky}{w}$ --- (i)

$\Rightarrow k = \frac{x_1w_1}{y_1} = \frac{8\times2}{4} = 4.$

Now; $a = y_2$.

But $y = \frac{xw}{k}$ (From (i) above).

$\Rightarrow y_2, a = \frac{x_2w_2}{k} = \frac{6\times3}{4}$

$\therefore a = 4.5$

Again; $b = x_2$.

But $x_2 = \frac{ky_2}{w_2}$

$\Rightarrow x_2, b = \frac{4\times2}{4} = 2.$

$\therefore b = 2.$

$\therefore a = 4.5$ and $b = 2.$

2016

6. (a) The number of tablets given to a patient was found to be direct proportional to the weight of the patient. If a patient with 36 kg was given 9 tablets, find how many tablets would be given to a patient whose weight is 48 kg.

(b) Four people can eat 2 bags of rice each weighing 10 kg in 12 days. How many people can eat 6 bags of rice of the same weight in 18 days?

2015

6. (a) The variable v varies directly as the square of x and inversely as y. Find v when $x = 5$ and $y = 2$; given that when $v = 18$ and $x = 3$ the value of $y = 4$.

(b) The temperature (T_i) inside a house is directly proportional to the temperature (T_o) outside the house and is inversely proportional to the thickness (t) of the house wall. If $T_i = 32°C$ when $T_o = 24°C$ and $t = 9cm$, find the value of t when $T_i = 36°C$ and $T_o = 18°C$

6 (a) $V \propto x^2/y$

$V = k x^2/y$

$\frac{V \times y}{x^2} = k$

$\frac{18 \times 4}{3 \times 3} = k$

$8 = k$

from $V = k x^2/y$

$V = \frac{8 \times 5 \times 5}{2}$

$V = 4 \times 25$

$V = 100$

$\therefore V = 100$

(b) $T_i \propto T_o/t$

$T_i = k T_o/t$

$\frac{T_i \times t}{T_o} = k$

$\frac{32°C \times 9cm}{24°C} = k$

$12cm = k$

from $T_i = k T_o/t$

$t = k T_o/T_i$

$t = \frac{12cm \times 18°C}{36°C}$

$t = 6cm$

$\therefore t = 6cm$

2014

6. (a) Juma sells one litre of milk at sh 600. How many litres of milk will Juma sell to get sh 208,800?

(b) The compression l of a spring is directly proportional to the thrust, T newtons exerted on it. If a thrust of 2 newtons produces a compression of $0.4cm$, find:

(i) The compression when the thrust is 5 newtons,

(ii) The thrust when the compression is $0.7cm$.

6 a. Given

One litre of milk costs sh 600

Required to find the litres which will be sold by Juma to acquire 208, 800/=

hence,

1 litre = sh 600

x = 208 800 sh

$$x = \frac{208\,800sh \times 1 \text{ litre}}{sh\ 600}$$

01

THE NATIONAL EXAMINATIONS COUNCIL OF TANZANIA

Question Number | SUBJECT NAME B. MATHS | INDEX NUMBER S 0121/0080 | Examiners' use only

6a

$$x = \frac{2088sh \times 1 \text{ litre}}{6sh}$$

$x = 348 \times 1$ litre

$x = 348$ litres

∴ In order to get sh 208800 Juma will have to sell the total of 348 litres

b.

Given

L is directly proportional to the thrust (T) or introducing the constant K

$L \propto K$

$L = TK$

but

when $L = 0.4cm$

$T = 2N$

Required to find K

from

$L = TK$

$$\frac{0.4}{2} = \frac{2K}{2}$$

$K = 0.2$

Now required to find

1. The compression when the thrust is 5 newtons.

from

$L = TK$, where $K = 0.2$

$T = 5$ newtons

$L = 5 \times 0.2$

$L = 1$ cm

04

2013

6. (a) A bus travels $240km$ using 16 litres of diesel. How many litres of diesel are needed to drive $90km$?

(b) If y^2 varies directly to $x-1$ and inversely to $x+d$ and if $x=2$, $d=4$ for $y=1$, then find x when $y=2$ and $d=1$.

6(a)

$240\text{ km} = 16\text{ litres}$

$90\text{ km} = ?$

$\frac{90 \times 16}{240} = 6$

$\therefore$ 6 litres are needed to drive 90km.

6(b)

$y^2 \propto x-1$

$y^2 \propto \frac{1}{x+d}$

$y^2 = K\frac{x-1}{x+d}$

$(1)^2 = K\frac{(2-1)}{(2+4)}$

$1 = K\frac{1}{6}$

$\frac{1}{1} = \frac{K}{6}$ $K = 6.$

$y^2 = K\frac{x-1}{x+d}$

$2^2 = \frac{6(x-1)}{(x+1)}$

$\frac{4}{1} = \frac{6x-6}{x+1}$

$4x+4 = 6x-6$

$4x-6x = -6-4$

$\frac{-2x}{-2} = \frac{+10}{+2}$ $x = 5.$

$\therefore$ The value of $x = 5$.

06

JS

2012

6. (a) The power (P) used in an electric circuit is directly proportional to the square of the current (I). When the current is 8 Ampere (A), the power used is 640 Watts (W).

(i) write down the equation relating the power (P) and the current (I).

(ii) calculate the current (I) when the circuit uses 360 Watts.

(b) If $x * y$ is defined as $\frac{1}{2}(x+y)$, find $(5 * -2) * (3 * -4)$.

6 (a) $P \propto I^2$

$P \propto I^2$

$P = kI^2$

$640 = k64$

$k = \frac{640}{64}$

$k = 10$

Hence the equation is $P = 10I^2$

Answer $P = 10I^2$

(ii) $P = 10I^2$

$\frac{360}{10} = \frac{10I^2}{10}$

$I^2 = \frac{360}{10}$

$I^2 = 36$

$I = \sqrt{36}$

$I = 6$

Answer $I = 6$ Amperes

2011

6. The number of square tiles needed to surface the floor of a hall varies inversely as the square of the length of a side of the tile used. If 2016 tiles of side 0.4m would be needed to surface the floor of a certain hall, how many tiles of side 0.3m would be required?

2010

6. (a) Juma bought motor vehicle spare parts from Japan worth 5,900,000 Japanese Yen. When he arrived in Tanzania he was charged custom duty of 25% on the spare parts. If the exchange rates were as follows:
1 US dollar = 118 Japanese Yen
1 US dollar = 76 Tanzania Shillings
Calculate the duty he paid in Tanzania shillings.

(b) The distance of the horizon d km varies as the square root of the height h m of the observer above sea level. An observer at a height of 100m above sea level sees the horizon at a distance of 35.7 km.

Find (i) the distance of the horizon from an observer 70m above sea level.

(ii) an equation connecting d and h. **(6 marks)**

2009

6. (a) The surface area of a sphere, V mm^2 varies directly as the square of its diameter d mm. If the surface area is to be doubled, what ratio must the diameter be altered?

(b) If $a\sqrt{\left(\frac{x^2-n}{m}\right)}=\frac{a^2}{b}$ write x as a subject of the formula

(6 marks)

2008

6. The value V of a diamond is proportional to the square of its weight W. It is known that a diamond weighing 10 grams is worth shs. 200,000/=.

(a) Write down an expression which relates V and W.
(b) Find the value of a diamond weighing 30 grams.
(c) Find the weight of a diamond worth shs. 5,000,000/=.

(6 marks)

7. Fractions, Ratio, Profit and Loss

2018

7. (a) Three relatives shared Tshs 140,000 so that the first one got twice as much as the second, and the second got twice as much as the third. How much money did the first relative get?

(b) Kitwana paid Tshs 900,000 for a desktop computer and sold it the following year for Tshs 720,000. Find:

(i) The loss made,

(ii) The percentage loss.

7 a) Let the three relatives be A, B and C

Given $A = 2B$

$B = 2C$

$A:B = 2:1$

$B:C = 2:1$

$A:B = (2:1)\,2$

$B:C = (2:1)\,1$

$A:B = 4:2$

$B:C = 2:1$

$A:B:C = 4:2:1$

Thus A + B + C in ratio = $4+2+1$

Total ratio = 7

Required share of A.

$A = \frac{4}{7} \times 140{,}000$ Tsh

$= 4 \times 20{,}000$ Tsh

$= 80{,}000$ Tsh

∴ The first got Tsh 80,000.

b) i) Loss made = Buying prize − Selling prize

$= 900{,}000 - 720{,}000$

$= 180{,}000$

∴ The loss made is 180,000 Tsh

ii) Percentage loss

$\%\text{loss} = \frac{\text{loss made}}{\text{Buying prize}} \times 100$

$= \frac{180000}{900{,}000} \times 100$

$= \frac{1}{5} \times 100 = 20\%$

∴ The percentage loss is 20%.

2017

7. A computer is advertised in a shop as having a list price of sh. 2,500,000 plus value added tax (VAT) of 20%. The sales manager offers a discount of 25% before adding the VAT. Calculate:
   (a) The list price including VAT.
   (b) The amount of discount before VAT is added.
   (c) The reduced final price of the computer.

7a List price = 2 500 000

VAT = 20% List price

$= \frac{20}{1\cancel{00}} \times 2\,500\,0\cancel{00}$

= 500 000

List price + vat = 2 500 000 + 500 000

= 3000000/=

∴ The price will be 3 000 000/=

b Discount = 25%

$= \frac{25}{1\cancel{00}} \times 2500\,0\cancel{00}$

= 625000

∴ The discount amount is 625000/=

e Final price

$= 2\,500\,000 - 625\,000 \cancel{+ 500\,000}$

$= 1\,875\,000$

VAT = 20%

$= \frac{20}{100} \times 1\,875\,000$

$= 375\,000$

Final price $= 1\,875\,000 + 375\,000$

$= 2\,250\,000/=$

$\therefore$ The final price is $2\,250\,000/=$

e Final price

$= 2\,500\,000 - 625\,000 \cancel{+ 500\,000}$

$= 1\,875\,000$

VAT = 20%

$= \frac{20}{100} \times 1\,875\,000$

$= 375\,000$

Final price $= 1\,875\,000 + 375\,000$

$= 2\,250\,000/=$

$\therefore$ The final price is $2\,250\,000/=$

2016

7. (a) Mariam, Selina and Moses contributed 800,000; 1,200,000 and 850,000 shillings respectively while starting their business.

   (i) Find the ratio of their contributions in its simplest form.

   (ii) If the business made a profit of 1,900,000 shillings; find how much each got if the profit was shared in the same ratio as their contributions.

   (b) A dealer bought 10 books for 200,000. He sold $\frac{2}{5}$ of them at 30,000 shillings each and the remaining at 25,000 shillings each. What was his percentage profit?

2015

7. (a) A shopkeeper makes a 20% profit by selling a radio for sh. 480,000.
   (i) Find the ratio of the buying price to the selling price.
   (ii) If the radio would be sold at 360,000, what would be the percentage loss?

   (b) A farmer sold a quarter of his maize harvest and gave one third of the remaining to his relatives. If the farmer remained with 25 bags of maize find how many bags of maize did the farmer harvest.

7 (a).

B·P ⟶ 100%

480,000 ⟶ 120%

$$B\cdot P = \frac{\cancel{480},\cancel{000} \times 100}{\cancel{120}}$$

Buying price = 400000

Ratio of buying price to selling price

$$= \frac{B\cdot P}{S\cdot P}$$

$$= \frac{\cancel{400,000}}{\cancel{480,000}}$$

$$= \frac{5}{6}$$

$$= 5/6$$

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(ii) 400,000 ⟶ 100.

360,000 ⟶ x.

$$x = \frac{\cancel{360,000} \times 100}{\cancel{400,000}}$$

Percentage loss = 100 − 90

= 10%

= 10%

b) Soln.

let his maize harvest be y.

$$\frac{1}{4}y + \cancel{\tfrac{1}{3}}\left(\tfrac{3}{4} \times \tfrac{1}{3}\right)y = y - 25$$

$$\frac{1}{4}y + \frac{1}{4}y = y - 25$$

$$\frac{1}{2}y = y - 25$$

$$25 = \frac{1}{2}y$$

$$y = 50$$

2014

7. (a) Kieku has to share 80 books with his younger sisters Upendo and Okuli. He decided that for every 2 books that Okuli gets, Upendo gets 3 and he gets 5 books. Find the number of books each gets.

   (b) Nyaumwa invested a certain amount of money in a bank which pays interest rate of 6 percent after every 6 months. After 5 years, his total savings were sh 9,600,000. Determine the amount of money Nyaumwa invested initially.

7a. total number of books = 80 books.
Ratio = Okuli : Upendo : Kieku = 2 : 3 : 5 — 01
$2+3+5 = 10$
Okuli = $\frac{2}{10} \times 80 = 16$ books
Upendo = $\frac{3}{10} \times 80 = 24$ books — 0½
Kieku = $\frac{5}{10} \times 80 = 40$ books. — 0½
∴ Okuli gets 16 books, Upendo get 24 books and Kieku gets 40 books. — 0½

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7b. Interest rate = 6% — 02½
time (T) = 2 times per year.
In five years = $2 \times 5 = 10$
$I = \frac{PRT}{100}$ where P = amount invested initially (principal)
R = Rate interest rate
T = time
I = Interest — 0½
but
Amount = Interest (I) + Principal (P)
$A = I + P$
$I = A - P$ — 01
$A - P = \frac{PRT}{100}$
$9{,}600{,}000 - P = \frac{P \times 6 \times 10}{100}$ — 01
$960000000 - 100P = 60P$
$\frac{960000000}{160} = \frac{160P}{160}$
$P = 6000000/=$ — 01
∴ Amount invested initially was 60,000,000 sh. — 06

2013

7. (a) A radio is bought for sh 400,000 and sold for sh 500,000. Find:
   (i) The profit made
   (ii) The percentage profit.

   (b) Find the time in which sh 300,000 will earn an interest of sh 60,000 if the interest rate is 10% per annum.

7. (a) Soln.

given that.

Buying price = 400,000sh.

Selling price = 500,000sh.

(i) Profit made.

From: Profit made = Selling price − buying price

= 500,000 − 400,000

= 100,000 sh.

∴ Profit made = Sh 100,000 Answer.

(ii) Percentage profit.

From: Percentage profit = $\frac{\text{profit made}}{\text{buying price}} \times 100\%$

= $\frac{100,000}{400,000} \times 100\%$

01 01 01½ 02½

QUESTION NUMBER: 7a. cont | SUBJECT NAME: BASIC MATH | INDEX NUMBER: S0860/0021

= $\frac{1}{4} \times 100\% = 25\%$

∴ Percentage profit is 25% Answer.

(b) Soln.

given.

Amount principle = 300,000 sh.

Interest = 60,000

Rate = 10%

Time = ?

From: $I = \frac{PRT}{100}$

where:

I = Interest, P = principle and T = time, R = Interest rate.

$I = \frac{PRT}{100}$

By cross multiplication.

$\frac{100I}{PR} = \frac{PRT}{PR}$

$T = \frac{100I}{PR} = \frac{100 \times 60,000}{300,000 \times 10}$

T = 2 years.

∴ The time to be earn is 2 years. Answer.

02½ 00½ 01 01 01

2012

7. (a) By selling an article at shs. 22,500/= a shopkeeper makes a loss of 10%. At what price must the shopkeeper sell the article in order to get a profit of 10% ?

(b) An alloy consists of three metals A, B and C in the proportion $A : B = 3 : 5$ and $B : C = 7 : 6$ Calculate the proportion $A : C$.

a. $100\% = ?$ (x)

$90\% = 22{,}500$

$\frac{90x}{90} = \frac{22500 \times 10\cancel{0}}{9\cancel{0}}$

$x = 25000/=$

$100\% = 25{,}000$

$110\% = ?$

$\frac{110 \times 25{,}000}{100} = \frac{100x}{100}$

$x = 27500/=$

$= 27{,}500/=$

7. b. $A/B = 3/5$ $B/C = 7/6$ $A/C = ?$

$\frac{5A}{5} = \frac{3B}{5}$ $\frac{6B}{7} = \frac{7C}{7}$

$A = \frac{3}{5}B$ $C = \frac{6}{7}B$

$\frac{A}{C} = \frac{\frac{3}{5}B}{\frac{6}{7}B}$ $\frac{3}{5} \times \frac{7}{6} = \frac{7}{10}$

$A : C = 7 : 10$

2011

7. The ratio of men : women : children living in Mkuza village is 6 : 7 : 3. If there are 42,000 women, find how many:
   (a) (i) children live in Mkuza village
   (ii) people altogether live in Mkuza village.

   (b) The 42,000 women is an increase of 20% on the number of women 10 years ago. How many women lived in the village?

2010

7. (a) An amount of Tshs. 12,000 is to be shared among Ali, Anna and Juma in the ratio 2:3:5 respectively. How much will each get?

   (b) A certain worker used his salary as follows: 20% on house rent, 45% on food, 10% on refreshment and 15% on school fees. If he/she was left with Tsh.22,000, determine:

   (i) The salary of this worker.
   (ii) The amount of money which he/she spent on food. **(6 marks)**

2009

7. (a) Express $2\frac{1}{2}$: 3 as integers in a simplified form.

   (b) The sides of a rectangle are in the ratio 3:5. If the perimeter of this rectangle is 800 cm; find the dimensions of the rectangle.

(6 marks)

2008

7. (a) Sixty people working 8 hours a day take 4 days to cultivate a village farm. How long will it take twenty people to cultivate the same farm if they work 15 hours a day?

   (b) Neema bought a tray of eggs (containing 30 eggs) for shs 2,000/=. She boiled the eggs using a litre of kerosene costing shs. 400/=. and sold each egg at a price of shs.100/= each. Find her percentage profit.

(6 marks)

8. Sequence and Series

2018

8. (a) If an arithmetic progression has A_1 as the first term and d as the common difference,
   (i) write the second, third, fourth and fifth terms.
   (ii) Establish the formula for the sum of the first five terms of the arithmetic progression by using the results in part (i).

   (b) The first and second terms of a geometric progression are 3 and 9 respectively.
   (i) Find the third, fourth and fifth terms.
   (ii) Verify that the sum of the first 5 terms is given by $S_n = G_1 \frac{r^n - 1}{r-1}$ by using the results in part (i).

8 (a) solution.

(i) Given: A_1 and d.

Then; from:

$$A_n = A_1 + (n-1)d$$
$$A_2 = A_1 + (2-1)d$$
$$\therefore A_2 = A_1 + d$$

Then:

$$A_n = A_1 + (n-1)d$$
$$A_3 = A_1 + (3-1)d$$
$$\therefore A_3 = A_1 + 2d$$

Then;

$$A_n = A_1 + (n-1)d$$
$$A_4 = A_1 + (4-1)d$$
$$\therefore A_4 = A_1 + 3d$$

Then;

$$A_n = A_1 + (n-1)d$$
$$A_5 = A_1 + (5-1)d$$
$$\therefore A_5 = A_1 + 4d$$

(ii) solution:

$$S_5 = A_1 + A_2 + A_3 + A_4 + A_5$$
$$S_5 = A_1 + A_1 + d + A_1 + 2d + A_1 + 3d + A_1 + 4d$$
$$S_5 = 5A_1 + 10d$$

$\therefore$ The formula for the sum of the first five terms of the arithmetic progression is given by; $S_5 = 5A_1 + 10d$

8 (b) solution:

Given: $G_1 = 3$, $G_2 = 9$.

from: $r = \frac{G_2}{G_1}$

$r = \frac{9}{3} = 3$.

(i) from:

$G_3 = G_1 r^2$

$G_3 = 3 \times 3^2$

$G_3 = 3 \times 9$

$\therefore G_3 = 27$.

Then:

$G_4 = G_1 r^3$

$G_4 = 3 \times 3^3$

$G_4 = 3 \times 27$

$\therefore G_4 = 81$.

Then;

$G_5 = G_1 r^4$

$G_5 = 3 \times 3^4$

$G_5 = 3 \times 81$

$\therefore G_5 = 243$

(ii) solution.

case I:

$S_5 = G_1 + G_2 + G_3 + G_4 + G_5$

$S_5 = 3 + 9 + 27 + 81 + 243$

$S_5 = 363$.

Case II:

from: $S_n = G_1 \frac{r^n - 1}{r - 1}$

$S_5 = 3 \frac{(r^5 - 1)}{5 - 1}$

(ii) Then: $S_5 = 3 \frac{(r^5 - 1)}{r - 1}$

$S_5 = 3 \frac{(3^5 - 1)}{3 - 1}$

$S_5 = 3 \frac{(243 - 1)}{2}$

$S_5 = 3 \times \frac{242}{2}$

$S_5 = 3 \times 121$

$S_5 = 363$.

Then; since the results obtained in case I and case II is equal $= 363$.

$S_n = G_1 \frac{r^n - 1}{r - 1}$ Hence Proved.

2017

8. (a) If the sum of n terms of a geometric progression with first term 1 and common ratio $\frac{1}{2}$ is $\frac{31}{16}$, find the number of terms.

(b) How many integers are there between 14 and 1,000 which are divisible by 17?

8. a) $S_n = 31/16$ $S_n = G_1 \cdot \frac{1-r^n}{1-r}$

$G_1 = 1$ $\frac{31}{16} = 1 \times \frac{1-(1/2)^n}{1-1/2} = 1 \times \frac{1-2^{-n}}{1/2}$

$r = 1/2$

$\frac{31}{16} = \frac{1-2^{-n}}{1/2}$

$\frac{31 \times 1/2}{16} = 1 - 2^{-n}$

$\frac{31}{32} = 1 - 2^{-n}$

$2^{-n} = 1 - \frac{31}{32} = \frac{32}{32} - \frac{31}{32} = 1/32$

$2^{-n} = 1/2^5 = 2^{-5}$

$2^{-n} = 2^{-5}$

$-n = -5$

$n = 5$

$\therefore$ number of terms is 5

b.) it will be an arthimetic progresion.

$A_1 = 17$ $d = 17$ $17\sqrt{1000} = 58$ rem 14

$A_2 = 34$

$58 \times 17 = 986$ So, $A_n = 986$.

$59 \times 17 = 1003$

$A_n = A_1 + (n-1)d$

$986 = 17 + 17(n-1) = 17 + 17n - 17$

$\frac{986}{17} = \frac{17n}{17}$

$58 = n$.

$\therefore$ There are 58 intergers.

2016

8. (a) The 8th term of an arithmetic progression is 9 greater than the 5th term and the 10th term is 10 times the 2nd term. Find the common difference and the first term of the arithmetic progression.

(b) The sum of the first two terms of a geometric progression is 18 whereas the sum of the second and the third term is 54, find the first term and the common ratio.

2015

8. (a) How many terms of the series $3+6+9+12+\dots$ are needed for the sum to be 630?

(b) Jennifer saved sh. 6 million in a Savings Bank whose interest rate was 10% compounded annually. Find the amount in Jennifer's savings account after 5 years.

8. a) $3+6+9+12+\dots$

3^{rd} term $-$ 2^{nd} term
$= 9-6$
$= 3$

2^{nd} term $-$ 1^{st} term
$= 6-3$
$= 3$

They have common difference

$\therefore$ The series is arithmetic.

$a_1 = 3$

$d = a_2 - a_1$
$= 6-3$
$= 3$

$$S_n = \frac{n}{2}\{2a_1 + (n-1)d\}$$

$$630 = \frac{n}{2}\{2\times 3 + (n-1)3\}$$

$$630 = \frac{n}{2}\{6 + 3n - 3\}$$

$$2\times 630 = \frac{n}{2}\{3n+3\}\times 2$$

$$1260 = n\{3n+3\}$$

$$1260 = 3n^2 + 3n$$

$$3n^2 + 3n - 1260 = 0$$

$$n^2 + n - 1260 = 0$$

$$n = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

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Question Number	SUBJECT NAME: MATHEMATICS INDEX NUMBER: S0298/0024	For Examiners' use only
8.	$= \frac{-1 \pm \sqrt{1+1680}}{2}$	
	$= \frac{-1 \pm 41}{2}$	
	$= \frac{-1-41}{2}$ or $\frac{-1+41}{2}$	
	$= -21$ or 20	
	But, the answer can't be negative	
	$\therefore n = 20$.	
	$\therefore$ 20 terms of the series are needed to make the sum 630.	
	b) $A_n = P\left[1 + \frac{RT}{100}\right]^n$	
	$A_n = 6\times10^6\left[1 + \frac{10\times1}{100}\right]^5$	
	$A_n = 6\times10^6(1.1)^5$	
	$\log A_n = \log(6\times10^6\times(1.1)^5)$	
	$\log A_n = \log6 + \log10^6 + 5\log1.1$	
	$\log A_n = 0.7782 + 6 + 5\times0.0414$	
	$\log A_n = 6.7782 + 0.2070$	
	$\log A_n = 6.9852$	
	$A_n = 9.665\times10^6$	
	$A_n = 9665000$ shs	

2014

8. (a) The 20^{th} term of an arithmetic progression is 60 and the 16^{th} term is 20. Find the sum of the first 40 terms.

(b) A shopkeeper invested sh 4,800,000 for 5 years. If the amount of money accumulated is sh 7,730,450, calculate the compound interest rate.

Question Number | SUBJECT NAME .... INDEX NUMBER [illegible] | For Examiners' use only

8. (a)

data

from

$A_{20} = 60$

$A_{16} = 20$

from

$A_n = A_1 + (n-1)d$

$A_{20} = A_1 + 19d$

$A_1 + 19d = 60$ —(i) 00½

also

$20 = A_{16} = A_1 + 15d$ —(ii)

solve them by elimination method 00½

$A_1 + 19d = 60$ (i)

$A_1 + 15d = 20$ (ii)

$4d = 40$

$d = 10$ 00½

also

$A_1 + 15d = 20$

$A_1 + 150 = 20$

$A_1 = -130$ 00½

from

$S_n = \frac{n}{2}[2A_1 + (n-1)d]$

$S_{40} = \frac{40}{2}[2\times -130 + 39\times 10]$ 00½

$= 20[-260 + 390]$

$= 20[130] = 2600$

The sum of the first 40 terms is $\underline{2600}$ 00½

03

8. (b) Given

$P = 4,800,000$
$A_n = 7,730,450$
$n = 5$

Given

$$A_n = P\left(1+\frac{r}{100}\right)^n$$

$$7,730,450 = 4,800,000\left(1+\frac{r}{100}\right)^5$$

$$\frac{7,730,450}{48,00000} = \left(1+\frac{r}{100}\right)^5$$

$$1.6105 = \left(1+\frac{r}{100}\right)^5$$

$$\left(1+\frac{r}{100}\right) = \sqrt[5]{1.6105}$$

$$1+\frac{r}{100} = 1.1$$

$$\frac{r}{100} = 0.1$$

$$r = 10\%$$

fulmit

Compound interest rate is 10%

2013

8. (a) The first term of an arithmetic progression is 12 and the common difference is 10. Find the n^{th} term.

(b) Find the amount of money accumulated at the end of 2 years after investing 500,000 shillings at a compound interest rate of 10% annually.

Question Number	SUBJECT NAME Mathematics INDEX NUMBER [illegible]	For Examiners use only
8 (a)	Given	
	$A_1 = 12$	
	$d = 10$	
	but	
	$A_n = A_1 + (n-1)d$	01
	$A_n = 12 + (n-1)10$	00½
	$= 12 + 10n - 10$	
	$= 12 - 10 + 10n$	
	$= 2 + 10n$	
	∴ The n^{th} term is $2 + 10n$.	01
(b)	Data given	
	$n = 2$ years	
	$P = 500,000$	
	$R = 10\%$	
	formula	
	$A_n = P(1 + R/100)^n$	01
	$A_2 = 500,000(1 + 10/100)^2$	00½
	$= 500,000(1 + 0.1)^2$	
	$= 500,000(1.1)^2$	00½
	$= 500,000(1.21)$	00½
	$= 605,000$ /=	
	∴ The amount of money accumulated is 605,000 /=	01

2012

8. (a) If the 5[th] term of an arithmetic progression is 23 and the 12[th] term is 37, find the first term and the common difference.

   (b) Find the sum of the first four terms of a geometric progression which has a first term of 1 and a common ratio of $\frac{1}{4}$.

8a. $A_5 = 23 = A_1 + 4d$

$A_{12} = 37 = A_1 + 11d$

soln.

$$-\begin{cases} A_1 + 11d = 37 \\ A_1 + 4d = 23 \end{cases}$$

$7d = 14$

$d = 2.$

$A_1 + 4d = 23$

$A_1 = 23 - 8$

$A_1 = 15.$

$\therefore$ The first term, $A_1 = 15$ and common difference, $d = 2$,

b. $r = \frac{1}{4}$, $G_1 = 1$.

$$S_n = \frac{G_1(r^n - 1)}{r-1} \quad \text{or} \quad \frac{G_1(1-r^n)}{1-r}$$

$$S_4 = \frac{1\left(1-(\frac{1}{4})^4\right)}{1-\frac{1}{4}}$$

$$= \frac{1\left(1-4^{-4}\right)}{1-\frac{1}{4}} = \frac{1\left(1-\frac{1}{256}\right)}{\frac{3}{4}}$$

$$= \frac{1 \times \left(\frac{256-1}{256}\right)}{\frac{3}{4}}$$

$$= \frac{\cancel{255}^{85}}{\cancel{256}_{64}} \times \frac{\cancel{4}^{1}}{\cancel{3}_{1}} = \cancel{255}$$

$$= \frac{85}{64}$$

$$\therefore S_4 = \frac{85}{64}$$

2011

8. (a) If the first term of an arithmetic progression is 3 and the third term is 13, find the second term, the fourth term and the sum of the first ten terms.

(b) A certain geometric progression has a common ratio of 2 and the sum of the first five terms is 155. Find the first term and give the formula for the n^{th} term.

2010

8. (a) Find the general term and hence the 30^{th} term of the sequence 1, -2, 4, -8, ……….

(b) Given the series 100 + 92 + 84 + …….
Find
(i) the 20^{th} term
(ii) the sum of the first 20 terms. **(6 marks)**

2009

8. (a) If the third term of a geometric progression is 100 and the sixth term is 800, find the fifth term and the sum of the first two terms.

(b) A small business sells products worth 1,000,000 (Tshs) during its first year. The owner of the business has set a goal of increasing annual sales by 750,000 (Tshs) each year. Assuming this goal is met; find the total sales during the first 10 years of the business in operation. **(6 marks)**

2008

8. (a) Write down the next two terms in the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \ldots$

(b) (i) The n^{th} term of an AP is $12 - 4n$. find the first term and the common difference.

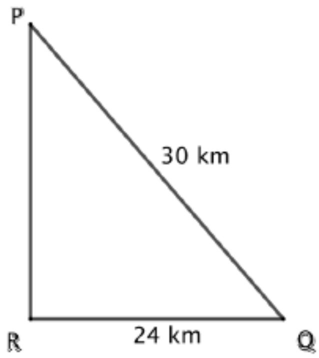
(ii) In an AP the 1^{st} term is -10, the 15^{th} term is 11 and the last term is 41. Find the sum of all the terms in the progression.

(c) The 5^{th} term of a GP is 8, the third term is 4 and the sum of the first ten terms is positive. Find the first term, the common ratio and the sum of the first ten terms. **(6 marks)**

9. Trigonometry and Pythagoras

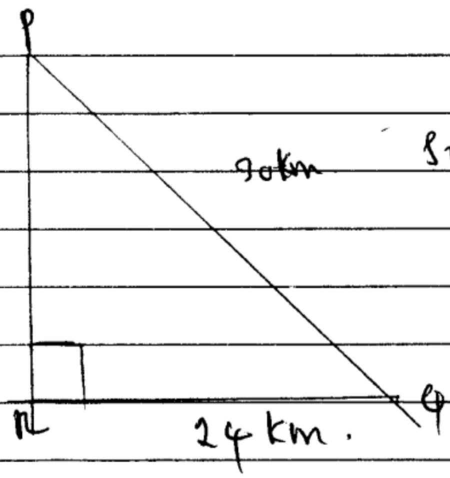
2018

9. (a) Find the distance PR in the following figure if the lines PR and RQ are perpendicular.



(b) A flagpole is 5 meters high. Find to the nearest cm, the length of its shadow when the elevation of the sun is 60°.

9 a)



Since PR ⊥ RQ.

From. $(PQ)^2 = (RQ)^2 + (PR)^2$ — pythagoras theorem.

$(PQ)^2 - (RQ)^2 = (PR)^2$

$(30)^2 - (24)^2 = (PR)^2$.

9a)
$900 - 576 = (\overline{PR})^2$

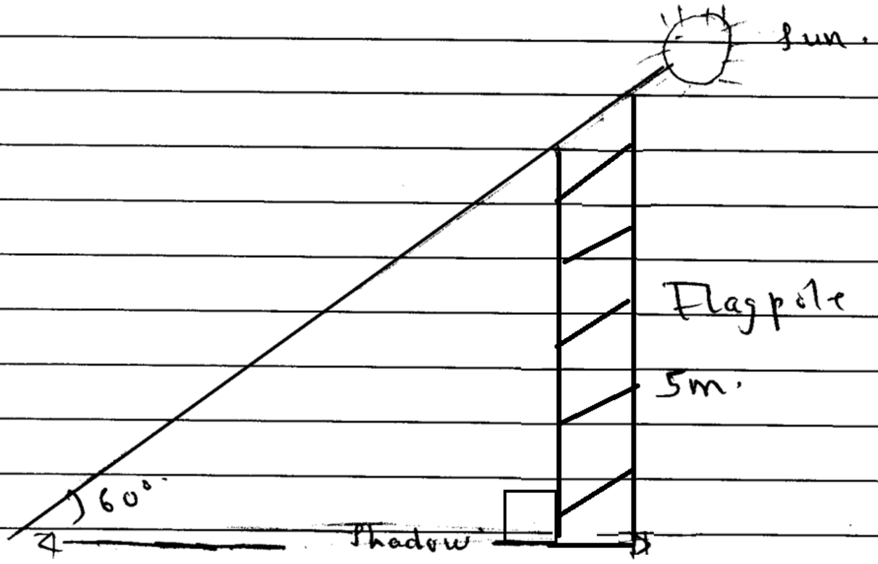
$(\overline{PR}) = 324$

$(\overline{PR}) = \sqrt{324^2}$

$(\overline{PR}) = 18km.$

$\overline{PR} = 18Km$

9b)



From $\tan \theta = \frac{\text{opposite}}{\text{Adjacent}}$

$\tan 60° = \frac{5m}{\text{Adjacent}}$

$\text{Adjacent} = \frac{5m}{\tan 60°}$

$\text{Adjacent} = \frac{5m}{1.732}$

$\text{Adjacent} = 2.79 \cancel{m}$

$\text{Adjacent} = 2.89m$ (2.d.p)

~~but~~

$1m = 100cm.$

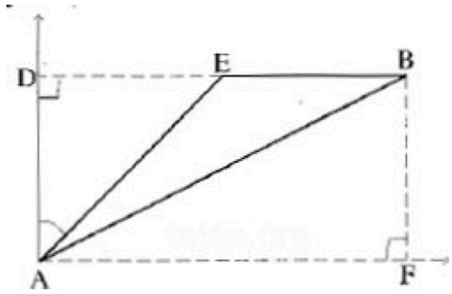
$2.89m = x$

$x = 289cm.$

The length of its shadow is 289cm.

2017

9. In the figure below, AE = 20 m, EB = $20\sqrt{2}$ m and $DAE = 45°$.



Find:

(a) The length: DE, AD and AB.

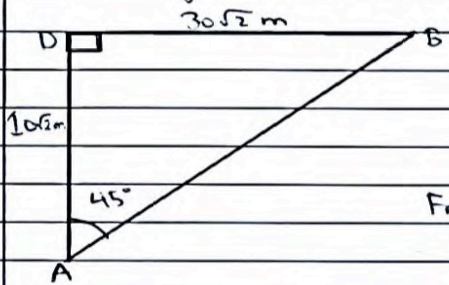
(b) The area of triangle ABE, leaving the answer in surd form.

$\cos 45° = \frac{AD}{20m}$

$AD = 20m \times 0.7071$ or $20m \times \sqrt{2}/2$

$AD = 14.142m$ or $10\sqrt{2}$ m

$\therefore$ The length AD is $10\sqrt{2}$ m or 14.142m.



$DB = DE + EB$

$DB = 10\sqrt{2}m + 20\sqrt{2}m$

$DB = 30\sqrt{2}$ m

From Pythagoras theorem;

$\overline{AB}^2 = \overline{AD}^2 + \overline{DB}^2$

$\overline{AB}^2 = (10\sqrt{2}\ m)^2 + (30\sqrt{2}\ m)^2$

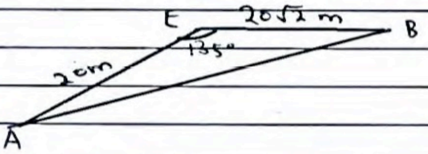
$\overline{AB}^2 = 200m^2 + 1800m^2$

$\overline{AB}^2 = 2000m^2$

$\overline{AB} = 20\sqrt{5}$ m or 44.72 m

$\therefore$ The length $\overline{AB}$ is 44.72 m or $20\sqrt{5}$ m;

(b)



From;

Area = $\frac{1}{2}ab\sin\hat{C}$

Area = $\frac{1}{2} \times 20m \times 20\sqrt{2}\,m \times \sin 135°$

Area = $200\sqrt{2}\,m^2 \times \sqrt{2}\ \sin 45°$

Area = $200\sqrt{2}\,m^2 \times \frac{\sqrt{2}}{2}$

Area = $(100 \times 2)\,m^2$

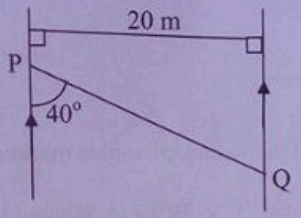
Area = $200m^2$ $= (10\sqrt{2} \times 10\sqrt{2})m^2$

$= (10\sqrt{2}m)^2$

$\therefore$ The area of triangle ABE is $(10\sqrt{2}\ m)^2$

2016

9. (a) A river with parallel banks is 20 m wide. If P and Q are two points on either side of the river, as shown in the figure below, find the distance PQ.



(b) In triangle LMN, LM = 5m, LN = 6m and angle MLN = 66°. Find MN.

2015

9. (a) Find the value of $\frac{\sin(150°)\cos(315°)}{\tan(300°)}$ without using mathematical tables.

(b) Calculate the angles of a triangle which has sides $4m$, $5m$ and $7m$.

9a. soln.

$\frac{\sin(150°)\cos(315°)}{\tan(300°)}$

Where

$\sin 150° = \sin 30°$

$\cos 315° = \cos 45°$

$\tan 300° = -\tan 60°$

Then

$\sin 30° = 1/2$

$\cos 45° = \sqrt{2}/2$.

$-\tan 60° = -\sqrt{3}$

Substituting $\frac{1/2 \times \sqrt{2}/2}{-\sqrt{3}}$.

$= \frac{\sqrt{2}/4}{-\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

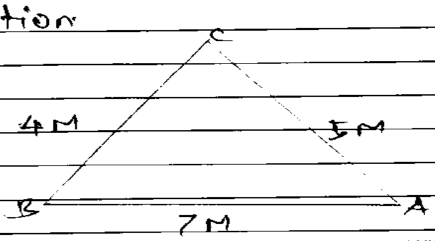
$= \frac{\sqrt{6}/4}{-3}$

$= \sqrt{6}/4 \times \frac{1}{-3}$.

$= -\sqrt{6}/12$.

$\therefore$ The value is $-\sqrt{6}/12$.

b. solution



9b. From.

$A^2 = B^2 + C^2 - 2CB\cos A$.

$4^2 = 5^2 + 7^2 - 2(5 \times 7 \times \cos A)$

$16 = 25 + 49 - 70\cos A$.

$16 = 74 - 70\cos A$.

$70\cos A = 74 - 16$.

$\frac{70\cos A}{70} = \frac{58}{70}$.

$\cos A = 0.8285$.

$A = \cos^{-1} 0.8285$

$A = 34^\circ 4'$

$B^2 = A^2 + C^2 - 2AC\cos B$.

$5^2 = 4^2 + 7^2 - 2(4 \times 7 \cos B)$

$25 = 16 + 49 - 56\cos B$.

$25 = 65 - 56\cos B$.

$56\cos B = 65 - 25$

$56\cos B = 40$.

$\cos B = 0.7143$.

$B = 44^\circ 24'$

$C = 180 - (A + B)$

$= 180 - (34^\circ.4' + 44^\circ 24')$

$= 180 - 78^\circ 28'$

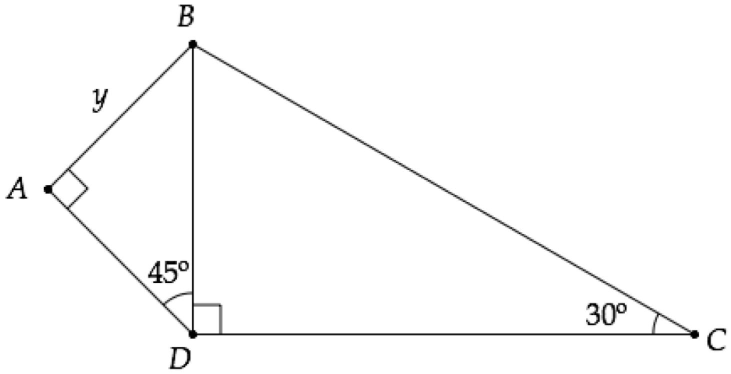
$= 101^\circ 32'$

$\therefore$ The angles are

$101^\circ 32'$, $44^\circ 24'$, $34^\circ\ 4'$

2014

9. (a) Find the length marked y in four significant figures.



(b) A $4m$ ladder rests against a vertical wall with its foot $2m$ from the wall. How far up the wall does the ladder reach? Give your answer in two decimal places.

9 (a) SOLUTION

To find length y

To get length y, find length $\overline{BD}$ first

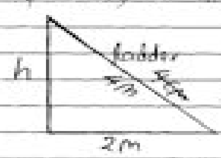
Where $\tan 30° = \frac{BD}{20\text{ cm}}$

$0.5774 \times 20\text{ cm} = BD$

$BD = 11548\text{cm}$

$BD = 11.55\text{ cm}$

Then $\sin 45° = \frac{y}{BD}$

Question Number	SUBJECT NAME: BASIC MATHS INDEX NUMBER: S0155/0030	For Examiners' use only
9	$\sin 45 = \frac{y}{11.55\text{ cm}}$	
	$0.7071 \times 11.55\text{cm} = y$	$01/2$
	$y = 8.167\text{ cm}$	
	Therefore, $y = 8.167\text{ cm}$	$01/2$
	(b) SOLUTION	
	By using a diagram	
		01
	From the Pythagoras' theorem	
	$h^2 + (2m)^2 = (4m)^2$	$01/2$
	$h^2 + 4m^2 = 16m^2$	
	$h^2 = 16m^2 - 4m^2$	$01/2$
	$\sqrt{h^2} = \sqrt{12m^2}$	
	$h = \sqrt{12}\text{ m}$	
	$h = 3.464\text{ m}$	01
	$h = 3.46\text{ m}$	
	Therefore the height of the wall reached by the ladder $= 3.46\text{ m}$	06

2013

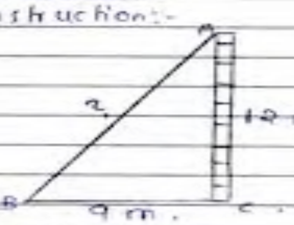
9. (a) A ladder leans against a wall. If the ladder reaches $12m$ up the wall and its foot is $9m$ from the base of the wall, find the length of the ladder.

(b) Given that A and B are complementary angles and $\sin A = \frac{3}{5}$, find $\tan B$ (Leave your answer as an improper fraction).

SUBJECT NAME B/ MATHEMATICS. INDEX NUMBER S0110/0097

9 (a) Solution:-

Construction:-



Let BA = Length of the ladder.

AC = height of the wall.

From pythagoras theorem,

$a^2 + b^2 = c^2$.

$(AC)^2 + (BC)^2 = (BA)^2$

$(12\,m)^2 + (9\,m)^2 = (BA)^2$

$144\,m^2 + 81\,m^2 = (BA)^2$

$\sqrt{225\,m^2} = \sqrt{(BA)^2}$

$15\,m = BA$

$BA = 15\,m$

$\therefore$ The length of the ladder $= 15\,m$.

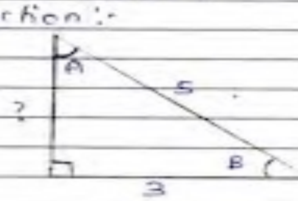
(b) Solution:-

Complementary angles $\rightarrow A + B = 90^\circ$.

$\sin A = \frac{3}{5}$ → opposite / → hypotenuse.

$\tan B = ?$

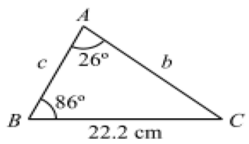
Construction:-



Question Number	SUBJECT NAME B/ MATHEMATICS	INDEX NUMBER S0110/0097	For Examiner's use
9	(b) $b = 3,\ c = 5,\ a = ?$		02
	By pythagorus theorem.		
	$a^2 + b^2 = c^2$.		
	$a^2 + 3^2 = 5^2$.		01/2
	$a^2 + 9 = 25$.		
	$a^2 = 25 - 9$.		
	$\sqrt{a^2} = \sqrt{16}$		
	$a = 4$.		01
	$\tan B = \frac{a}{b}$		
	$\therefore \tan B = \frac{4}{3}$.		01

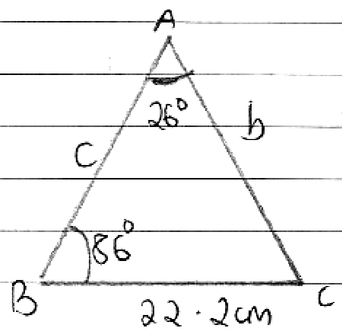
2012

9. (a) Find the length AC from the figure below:



(b) A ladder reaches the top of a wall $18m$ high when the other end on the ground is $8m$ from the wall. Find the length of the ladder.

9a



$= \frac{\sin B}{b} = \frac{\sin A}{a}$ (sine rule)

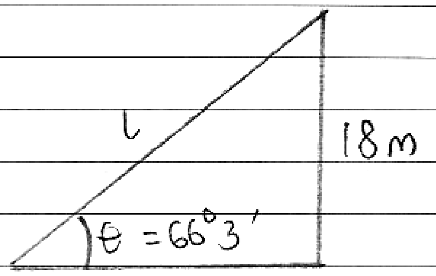
$= b = \frac{\sin B \cdot a}{\sin A}$

$= b = \frac{\sin 86^\circ \times 22{\cdot}2cm}{\sin 26^\circ} = \frac{0.9976 \times 22{\cdot}2cm}{0.4384}$

$= b = 50.53$ cm.

Therefore the length of $\overline{AC}$ is 50,53 cm

9 b.



$\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

$= \dfrac{18m}{8m} = 2.25$

$\sin\theta = \dfrac{\text{Opposite}}{\text{hypotenuse}}$

$\sin\theta = \dfrac{18m}{l}$

$l = \dfrac{18m}{\sin 66°3'}$

$l = \dfrac{18m}{0.914}$

$l = 19.70\,m$

Therefore the length of the ladder is 19.70m.

2011

9. Figure 2 represents plotting of two stations A and B which are 4,000m apart. T is a stationary target in the same vertical plane as A and B. When the distance from station A is 10,000m, the angle of elevation is 30°. Calculate
   - (a) The vertical height of the target, TX
   - (b) The distance AX, BX and TB
   - (c) The angle of elevation of the target, T, from B

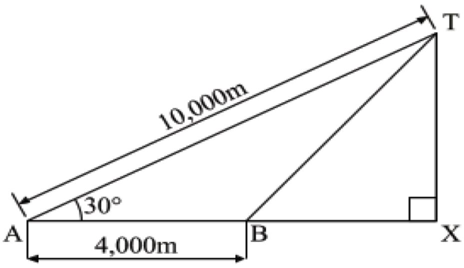
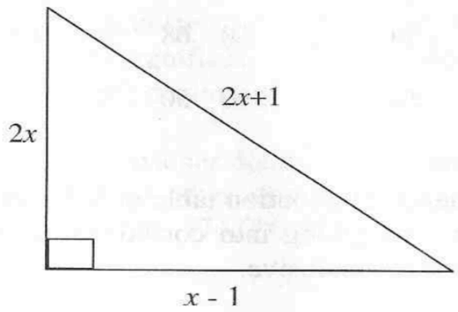


Figure 2

2010

9. (a) If $\tan A = \dfrac{3}{4}$ and A is acute, find $\cos A$, $\sin A$ and hence verify the identity $\cos^2 A + \sin^2 A = 1$

(b)



Given the right angled triangle above whose sides are measured in centimeter determine:

(i) the value of x

(ii) the area of the triangle **(6 marks)**

2009

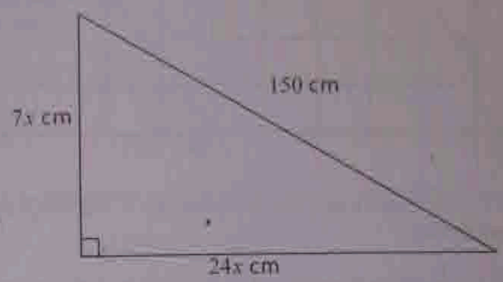
9. (a) Given that x is an acute angle and that $\sin x = \dfrac{p}{q}$, find the value of $\tan x$.

(b) An observer on the top of a cliff, 25 m above sea level, views a boat on the sea at an angle of depression of 60°. How far is the boat from the top of the cliff? **(6 marks)**

2008

9. (a) To find the height of a tower a surveyor sets up his theodolite 100 m from the base of the tower. He finds that the angle of elevation to the top of the tower is 30°. If the instrument is 1.5 m above the ground, what is the height of the tower?

(b) The right angled triangle in the diagram below has sides of length $7x$ cm, $24x$ cm and 150 cm.



(i) Find the value of x.

(ii) Calculate the area of the triangle. **(6 marks)**

10. Quadratic Equations

2018

10. (a) Use factorization method to solve the quadratic equation $x^2 - 9x + 14 = 0$.

(b) Find the values of x that satisfies the equation $\frac{1350}{x} - \frac{1350}{(x+3)} = 5$.

10	a). $x^2 - 9x + 14 = 0$
	$a = -2$
	$b = -7$
	$x^2 - 7x - 2x + 14 = 0$
	$x(x-7) - 2(x-7) = 0$
	$(x-2)(x-7) = 0$
	$x - 2 = 0$ or $x - 7 = 0$
	$x = 2$ or $x = 7$
	$\therefore$ The value of x is 2 or 7

10 b). $\frac{1350}{x} - \frac{1350}{(x+3)} = 5$

$\frac{1350x + 4050 - 1350x}{x^2 + 3x} = \frac{5}{1}$

$(1350x + 4050 - 1350x = 5x^2 + 15x) \times \frac{1}{5}$

$\cancel{270}x + 810 - \cancel{270}x = x^2 + 3x$

$810 = x^2 + 3x$

$x^2 + 3x - 810 = 0$

Recall general formular

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 1$

$b = 3$

$c = -810$

$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-810)}}{2}$

$x = \frac{-3 \pm \sqrt{9 + 3240}}{2}$

$x = \frac{-3 \pm \sqrt{3249}}{2} \Rightarrow x = \frac{-3 \pm 57}{2}$

$x = \frac{-3 + 57}{2}$ or $\frac{-3 - 57}{2}$

$x = 27$ or -30

$\therefore x = 27$ or -30

2017

10. (a) Solve the equation $4x^2 - 32x + 12 = 0$ by using the quadratic formula.

(b) Anna is 6 years younger than her brother Jerry. If the product of their ages is 135, find how old is Anna and Jerry.

10. Solution.

(a) Given; $4x^2 - 32x + 12 = 0$.

i.e $x^2 - 8x + 3 = 0$.

From the quadratic formula;

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{8 \pm \sqrt{64 - 12}}{2}$$

$$\Rightarrow x = \frac{8 \pm \sqrt{52}}{2}$$

$$\Rightarrow x = \frac{8 \pm 2\sqrt{13}}{2} = 4 \pm \sqrt{13}$$

$\therefore$ $x = 4 + \sqrt{13}$ or $4 - \sqrt{13}$.

(b) Let; x be Jerry's age.

$\Rightarrow x - 6 =$ Anna's age.

Then; $x(x-6) = 135$.

$\Rightarrow x^2 - 6x = 135$.

$\Rightarrow x^2 - 6x - 135 = 0$.

$\Rightarrow x^2 + 9x - 15x - 135 = 0$

$\Rightarrow x(x+9) - 15(x+9) = 0$.

$\Rightarrow (x-15)(x+9) = 0$.

$\Rightarrow x = 15$ or -9.

10.(b) But; Age is always positive.

$\Rightarrow x = 15$.

Then; Jerry's age $= 15$ years

and Anna's age $= 15 - 6$

$= 9$ years.

$\therefore$ Anna is 9 years old while Jerry is 15 years old.

2016

10. (a) If one of the roots of the quadratic equation $x^2 + bx + 24 = 0$ is $1\frac{1}{2}$, find the value of b.

(b) Two numbers differ by 3. If the sum of their reciprocals is $\frac{7}{10}$, find the numbers.

2015

10. (a) Factorize completely $2x^2 + x - 10$ by splitting the middle term.

(b) Solve the equation $\sqrt{x^2 - 7} = 7 + x$.

10 a) $2x^2 + x - 10$

$mn = 1 \quad | \; 5, -4$

$mn = -20$

$2x^2 + 5x - 4x - 10$ c1

$x(2x+5) - 2(2x+5)$

$(x-2)(2x+5)$ ANS c1

b) $\left(\sqrt{x^2 - 7}\right)^2 = (7+x)^2$

$x^2 - 7 = (7+x)(7+x)$ c1

$x^2 - 7 = 49 + 7x + 7x + x^2$ c1

Question Number	SUBJECT NAME MATHS INDEX NUMBER S0201/0432	For Examiners' use only
	b) $\cancel{x^2} - 7 = 49 + 14x + \cancel{x^2}$	
	$-7 - 49 = 14x$	c1
	$\frac{-56}{14} = \frac{14x}{14}$	
	$x = -4$ ANS	c1

2014

10. (a) Use the quadratic formula to solve $x^2 + 4x - 12 = 0$

(b) A garden measuring 12 by 16 metres is to have a pedestrian pathway of equal width constructed all around it, increasing the total area to 285 square metres. What will be the width of the pathway?

10. a) Soln.

To solve $x^2+4x-21=0$ by Quadratic formula.

From $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ where $a=1, b=4$ and $c=-21$ — $01/2$

$x = \frac{-4 \pm \sqrt{4^2-4(1)(-21)}}{2(1)}$ — $01/2$

(01)

Question Number	SUBJECT NAME S...B/MATHS INDEX NUMBER S0185/0019	For Examiners' use only
10.	$x = \frac{-4 \pm \sqrt{16+84}}{2}$	(a)
	$x = \frac{-4 \pm \sqrt{100}}{2}$	
	$x = \frac{-4 \pm 10}{2} = \frac{6}{2}, \frac{-14}{2}$	
	$x = 3$ or $x = -7$.	
	$\therefore x = 3$ or $x = -7$.	01

2013

10. (a) What must be added to $x^2 + 8x$ to make the expression a perfect square?

(b) Find two consecutive odd numbers whose product is 195.

Question Number	SUBJECT NAME: BASIC MATHEMATICS INDEX NUMBER: S0248/0062	For Examiners' use only
10.	a. For x^2+8x to be a perfect square:-	
	$\left(\frac{b}{2a}\right)^2$ must be added	01
	$b=8$, $a=1$, 2 is a constant	
	$\left(\frac{b}{2a}\right)^2 = \left(\frac{8}{2(1)}\right)^2$	
	$=(4)^2$	
	$=16$	01
	∴ For x^2+8x to be a perfect square 16 must be added.	01
	b. Let the two consecutive odd numbers be x and $x+2$	01
	$(x)(x+2)=195$	00½
	$x^2+2x=195$	
	$x^2+2x-195=0$	00½
	$x^2+15x-13x-195=0$	
	$x(x+15)-13(x+15)=0$	
	$(x+15)(x-13)=0$	
	$(x+15)=0$ or $(x-13)=0$	
	$x=-15$ or $x=13$	
	$x=13$	
	$x+2=13+2=15$	
	∴ The two consecutive odd numbers are 13 and 15.	01

2012

10. (a) Solve for x if $\frac{6}{x-4} = 1 + \frac{4}{x}$

(b) If the sum of two numbers is 3 and the sum of their squares is 29, find the numbers.

10a

$\frac{6}{x-4} = 1 + \frac{4}{x}$

$\frac{6}{x-4} = \frac{x+4}{x}$

$6x = (x+4)(x-4)$

$6x = x^2 - 4x + 4x - 16$

$6x = x^2 - 16$

$x^2 - 16 - 6x = 0$

$x^2 - 8x + 2x - 16 = 0$

$x(x-8) + 2(x-8) = 0$

$(x+2)(x-8) = 0$

$x = 8$ or -2

10b

let the first number be x

the second number be y

$x + y = 3$ ... (i)

$x^2 + y^2 = 29$ ... (ii)

$x = 3 - y$

$(3-y)(3-y) + y^2 = 29$

$9 - 3y - 3y + y^2 + y^2 = 29$

$2y^2 - 6y + 9 = 29$

$2y^2 - 6y + 9 - 29 = 0$

$2y^2 - 6y - 20 = 0$

$2y^2 + 4y - 10y - 20 = 0$

$2y(y+2) - 10(y+2) = 0$

$(2y - 10)(y+2) = 0$

$y = 5$ or -2

$x = 3 - y$

when $y = 5$	when $y = -2$
$x = 3 - 5 = -2$	$x = 3 - -2 = 5$

$\therefore$ The numbers are 5 and -2

2011

10. (a) Find the solution of the quadratic equation $8x^2 - 34x + 21 = 0$ by using the factorization method.

(b) Solve for x if $\frac{1}{x-2} - \frac{1}{x^2-4} = \frac{4}{5}$

2010

10. (a) Factorize each of the following expressions:

(i) $3a^2c - 5a^2d - 3b^2c + 5b^2d$

(ii) $3(2 - y^2) - 17y$

(b) Find the value of y which satisfies the equation $3(2 - y^2) - 17y = 0$

2009

10. (a) (i) By factorization, find the solution set for $x^2 - x - 6 = 0$

(ii) Solve the simultaneous equations given below by elimination method.

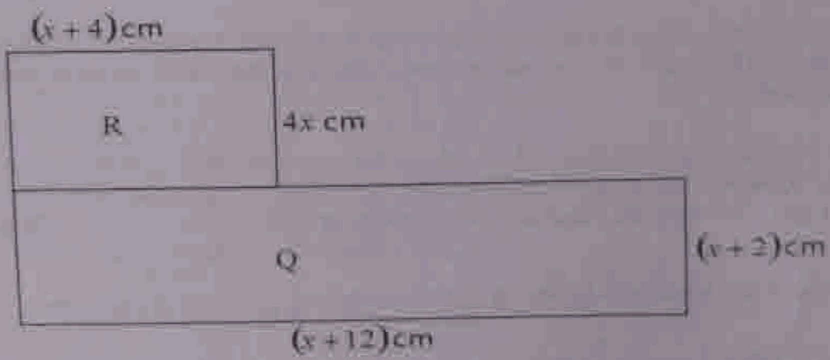
$$\begin{cases} 3x - y = 23 \\ 4x + 3y = 48 \end{cases}$$

(b) Solve for x if $5 - 2x \geq 7x - 4$

(6 marks)

2008

10. Study the following diagram carefully and answer the questions that follow.



(a) (i) Write down an expression for the area of rectangle R.

(ii) Show that the total area of rectangles R and Q is $(5x^2 + 30x + 24)$ cm^2.

(b) If the total area of R and Q is 64cm^2; calculate the value of x correct to 1 decimal place.

(6 marks)

2007

10. (a)

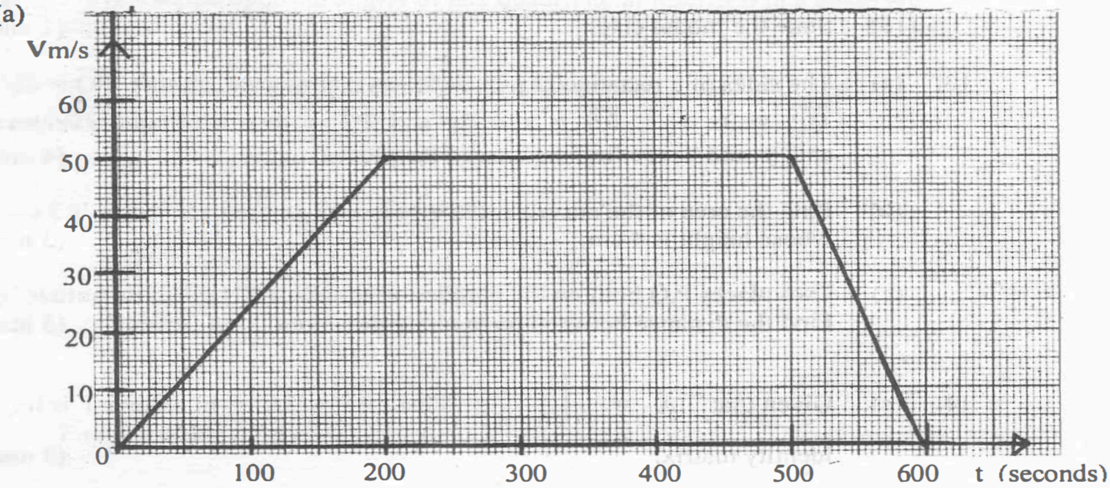


Fig. 4

The graph in figure 4 represents the journey made by a car between two sets of traffic lights. How far is it between the traffic lights? **(3 marks)**

(b) Solve the simultaneous equations

$$\begin{cases} x - y = 2 \\ 2x^2 - 3y^2 = 15 \end{cases}$$

(3 marks)

11. Linear Programming

2018

11. A farmer needs to buy up to 25 cows for a new herd. He can buy either brown cows at 50,000/= each or black cows at 80,000/= each and he can spend a total of not more than 1,580,000/=. He must have at least 9 cows of each type. On selling the cows he will make a profit of 5,000/= on each brown cow and 6,000/= on each black cow. How many of each type he should buy to maximize profit?

11\. Let x be number of brown cows

y be number of black cows

Constraints

$x + y \leqslant 25$

$50,000x + 80,000y \leqslant 1,580,000$

$\rightarrow 5x + 8y \leqslant 158$

$x \geqslant 9$

$y \geqslant 9$

objective function

maximize $(x, y) = 5000x + 6000y$

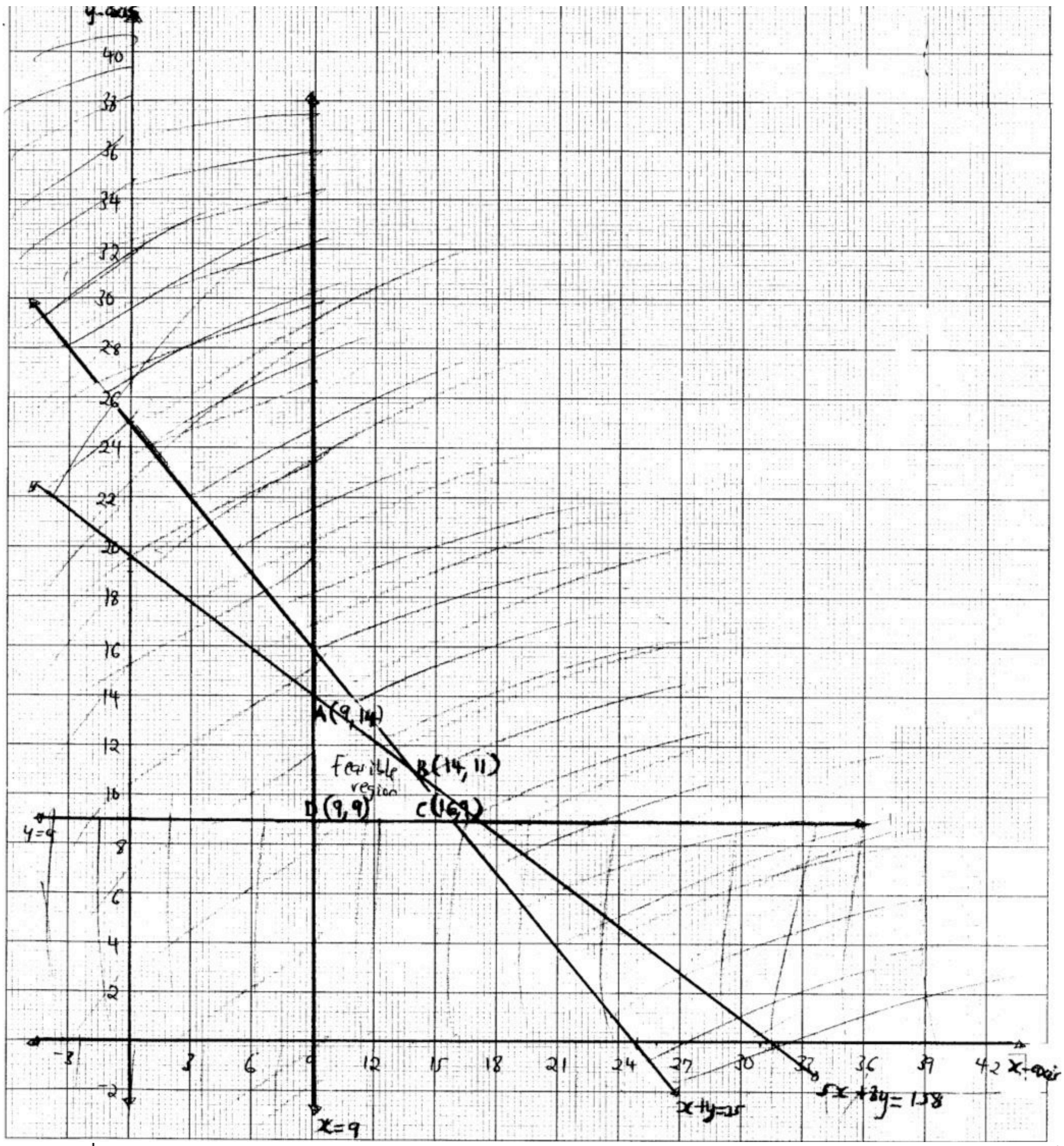
Tables of values

$x + y = 25$

x	0	25
y	25	0

$5x + 8y = 158$

x	0	31.6
y	19.75	0



corner points	objective function = 5000x + 6000y	value
A (9, 14)	5000 (9) + 6000 (14)	129,000
B (14, 11)	5000 (14) + 6000 (11)	136,000
C (16, 9)	5000 (16) + 6000 (9)	134,000
D (9, 9)	5000 (9) + 6000 (9)	99,000

∴ In order to maximize profit, he should buy 14 brown cows and 11 black cows.

2017

11. Zelda wants to buy oranges and mangoes for her children. The oranges are sold at sh. 150 each and mangoes at sh. 200 each. She must buy at least two of each kind of fruit but her shopping bag cannot hold more than 10 fruits. If the owner of the shop makes a profit of sh. 40 on each orange and sh. 60 on each mango, determine how many fruits of each kind Zelda must buy for the shop owner to realise maximum profit.

11. solution

Let the number of oranges be x

the number of mangoes be y

$x + y \leq 10$ - - (i)

$x \geq 2$ - - - (ii)

$y \geq 2$ - - - (iii)

Objective function:

$f(x,y) = 40x + 60y \rightarrow$ maximum

$x + y \leq 10$

$x = 10$

$y = 10$

$x \geq 2$

$y \geq 2$

From the graph; corner points are

$(2,8)$, $(8,2)$, $(2,2)$

$f(x,y) = 40x + 60y \rightarrow$ maximum

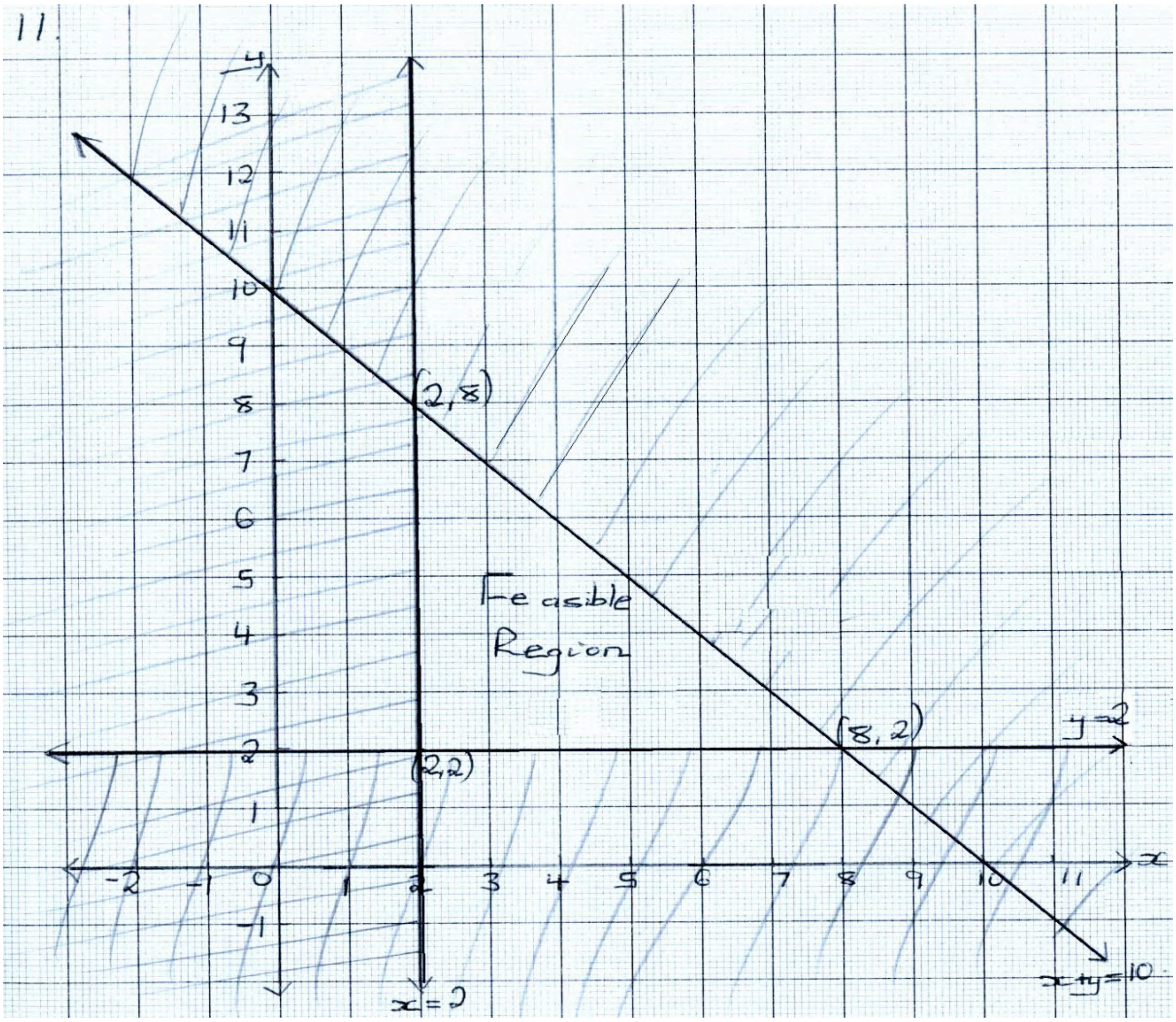
$f(2,8) = 40(2) + 60(8) = 560$

$f(8,2) = 40(8) + 60(2) = 440$

$f(2,2) = 40(2) + 60(2) = 200$

$(2,8)$ = maximum

$\therefore$ Zelda should buy 2 oranges and 8 mangoes for the shop owner to realise maximum profit.



2016

11. A shopkeeper sells refrigerators and washing machines. Each refrigerator takes up 1.8 m^2 of space and costs 300,000 shillings; whereas each washing machine takes up 1.5 m^2 of space and costs 500,000 shillings. The owner of the shop has 6,000,000 shillings to spend and has 27 m^2 of space.

(a) Write down all the inequalities which represent the given information.

(b) If he makes a profit of 30,000 shillings on each refrigerator and 40,000 shillings on each washing machine, find how many refrigerators and washing machine he should sell for maximum profit.

2015

11. A small industry makes two types of clothes namely type A and type B. Each type A takes 3 hours to produce and uses 6 metres of material and each type B takes 6 hours to produce and uses 7 metres of material. The workers can work for a total of 60 hours and there is 90 metres of material available. If the profit on a type A cloth is 4,000 shillings and on a type B is 6,000 shillings, find how many of each type should be made for maximum profit.

11. soln

Let; x – be number of clothes of type A made

y – be number of clothes of type B made

	Hours	Materials	Profit
Type A	3	6	4,000
Type B	6	7	6,000
Total	60	90	

Inequality	Equation
$3x + 6y \leq 60$	$3x + 6y = 60$ (0, 10) (20, 0)
$6x + 7y \leq 90$	$6x + 7y = 90$ (0, 12.9) (15, 0)
$x \geq 0$	$x = 0$
$y \geq 0$	$y = 0$

$f(x, y) = 4{,}000x + 6{,}000y$

Corner points (x, y)	$F(x, y) = 4{,}000x + 6{,}000y$	Results
A (0, 0)	$F(0, 0) = 4{,}000(0) + 6{,}000(0)$	0
B (15, 0)	$F(15, 0) = 4{,}000(15) + 6{,}000(0)$	60,000
C (8, 6)	$F(8, 6) = 4{,}000(8) + 6{,}000(6)$	68,000
D (0, 10)	$F(0, 10) = 4{,}000(0) + 6{,}000(10)$	60,000

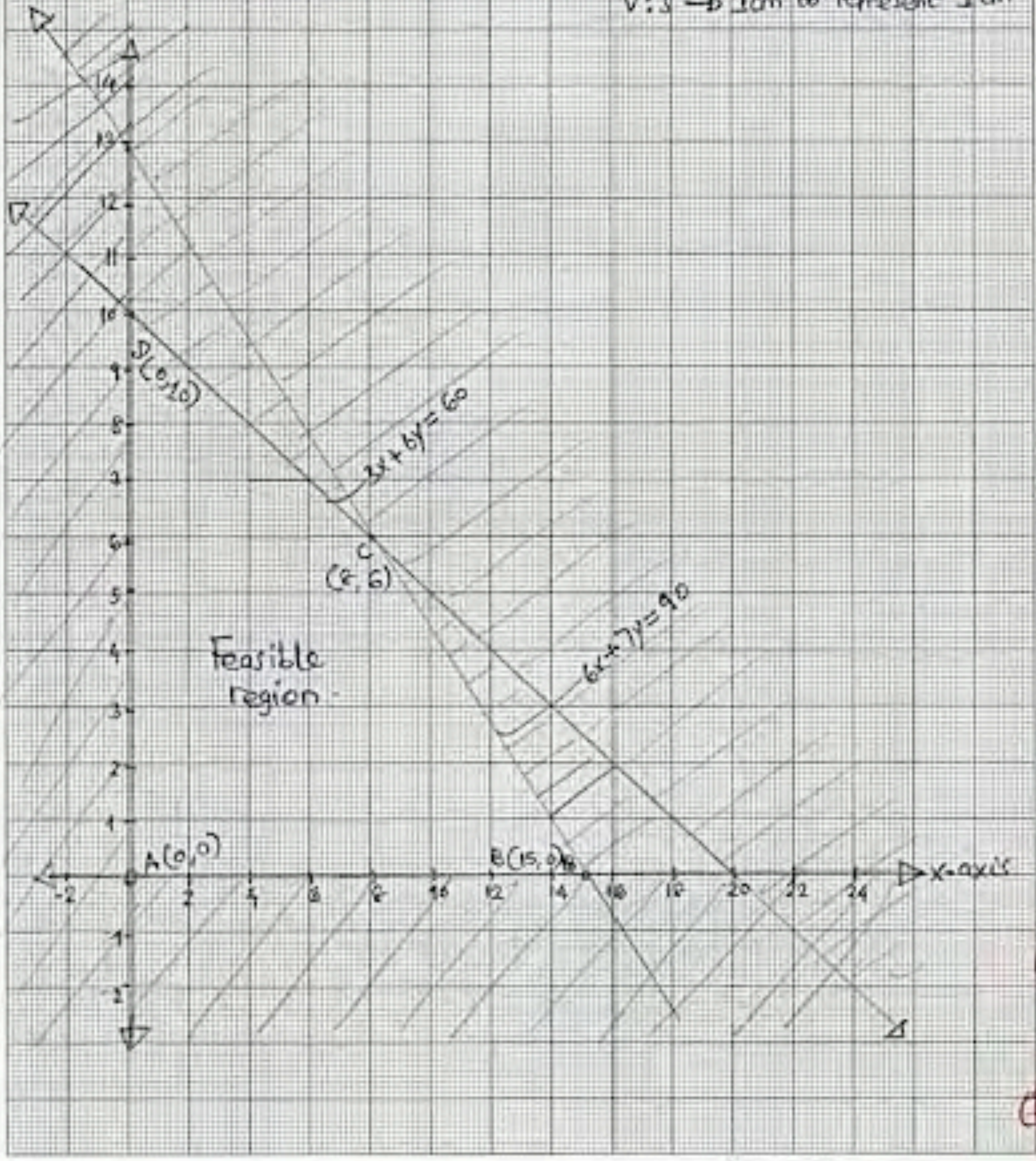
∴ For maximum profit; An Industry must make 8 clothes of type A and 6 clothes of type B.

SUBJECT NAME B/MATH INDEX NUMBER

THE GRAPH OF LINEAR PROGRAMMING

SCALE: H: S → 1cm to represent 2cm

V: S → 1cm to represent 1cm



07

01

01

D

010

2014

11. A farmer has 20 hectares for growing tomatoes and cabbages. The cost per hectare for tomatoes is sh 48,000 and for cabbages is sh 32,000. The farmer has budgeted sh 768,000. Tomatoes require one man-day per hectare and cabbages require two man-days per hectare. There are 36 man-days available. The profit on tomatoes is sh 160,000 per hectare and on cabbages is sh 192,000 per hectare. Find the number of hectares of each crop the farmer should plant to maximize the profit.

11\. Let x be the number of hectares for the tomatoes to be grown

y be the number of hectares for the cabbages to be grown

Inequalities

$48000x + 32000y \leq 768000$

$x + 2y \leq 36$

$x + y \leq 20$

$x \geq 0$

$y \geq 0$

Question Number | SUBJECT NAME .... | INDEX NUMBER S0121/0034 | For Examiners' use only

11 The inequalities can further be written as follows

$3x + 2y \leq 48$

$x + y \leq 20$

$x + 2y \leq 36$

$x \geq 0$

$y \geq 0$

Objective function, $f(x,y) = 160000x + 192000y$ — $a1/2$

x and y intercepts.

For $3x + 2y = 48$

x	0	16
y	24	0

For $x + y = 20$

x	0	20
y	20	0

For $x + 2y = 36$

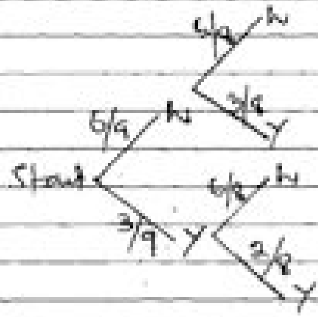
x	0	36
y	18	0

The GRAPH at the GRAPH papers

Corner points	Objective function: $f(x,y) = 160000x + 192000y$
A(0,0)	$f(0,0) = 160000(0) + 192000(0) = 0$
B(16,0)	$f(16,0) = 160000(16) + 192000(0) = 2560000$
C(8,12)	$f(8,12) = 160000(8) + 192000(12) = 3584000$
E ~~D~~(0,18)	$f(0,18) = 160000(0) + 192000(18) = 3456000$
D ~~E~~(4,16)	$f(4,16) = 160000(4) + 192000(16) = 3712000$

$a2\frac{1}{2}$

$05\frac{1}{2}$

Question Number	SUBJECT NAME .... INDEX NUMBER 50121/0030	For Examiners' use only
11	For maximizing profit	
	Optimal point is D(4,16)	
	Optimal solution is 3712000	
	∴ The farmer should plant 4 hectares of tomatoes and 16 hectares of cabbages in order to maximize profit	01
		06½
16 a.		
	Let W stand for the probability that the white balls will be selected	
	Y stand for the probability that the yellow balls will be selected.	
	Consider the tree diagram.	
		01 01

SUBJECT NAME: B. MATH

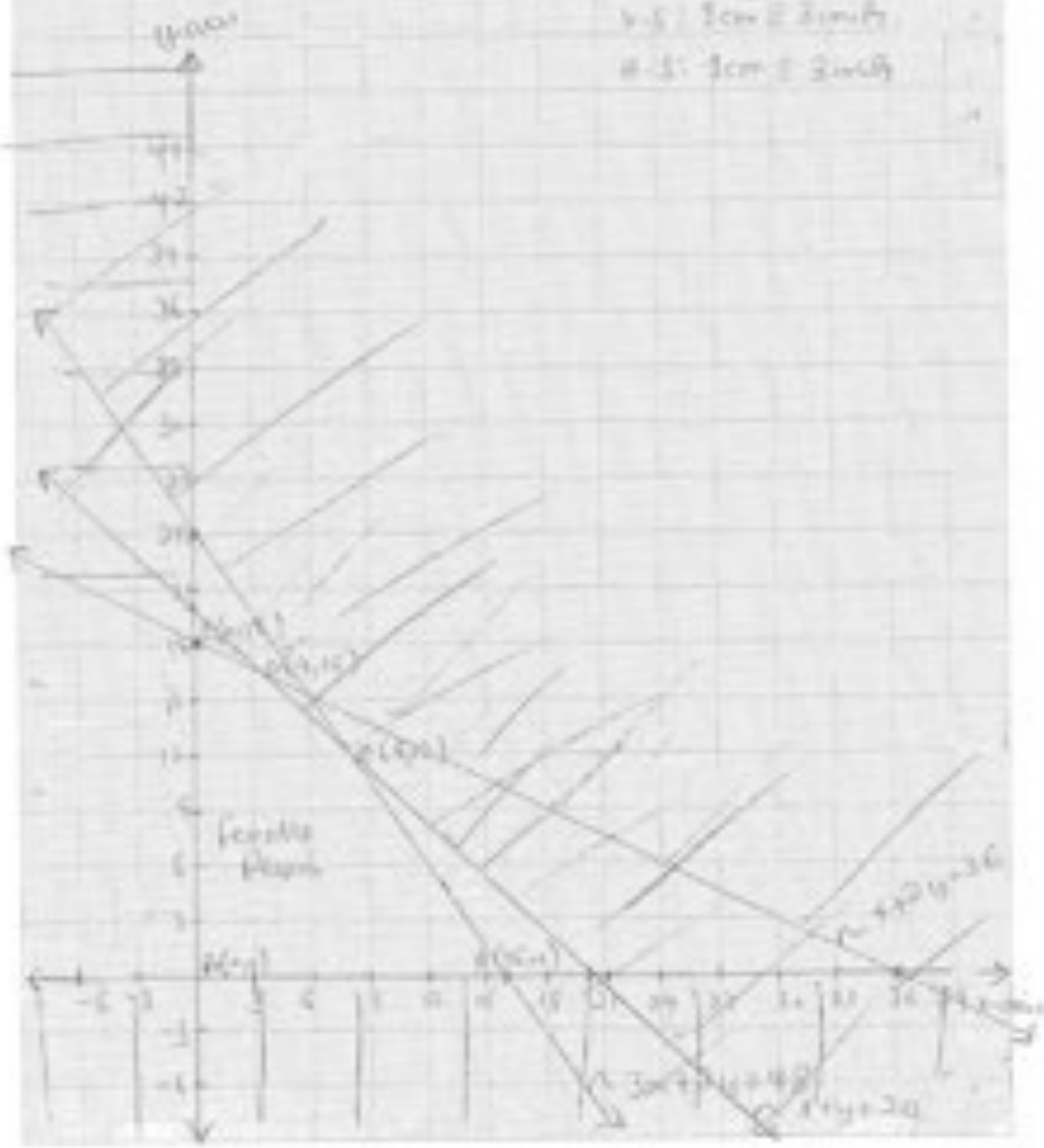
INDEX NUMBER: S0031/0030

11- THE GRAPH FOR LINEAR PROGRAMMING QUESTION

SCALE:

Y-S: 1cm = 3units

X-S: 1cm = 3units



LIF1

2013

11. (a) Solve by graphical method the following system of simultaneous equations:

$4x + y = 6$

$5x + 2y = 9$

(b) A farm is to be planted with sorghum and maize while observing the following constraints:

	Sorghum	Maize	Maximum total
Days labour per hectare	4	2	20
Labour cost per hectare	1400	1200	8400
Cost of fertilizer per hectare (shs)	600	800	4800

If sorghum yields a profit of 800,000 shillings per hectare while maize yields 600,000 shillings per hectare, how many hectares should be planted with each crop for maximum profit?

Question Number | SUBJECT NAME: BASIC MATH'S | INDEX NUMBER: S.0110/0033.

11. (a) By graphical method.

Soln.

$4x + y = 6.$

$5x + 2y = 9.$

Table of Value for $4x + y = 6$.

x	-3	-2	-1	0	1	2	3
y	18	14	10	6	+2	-2	-6

$0\frac{1}{2}$

Table of Value for $5x + 2y = 9$.

x	-3	-2	-1	0	1	2	3
y	24/2	19/2	14/2	9/2	4/2	-1/2	-6/2
y	12	9.5	7	4.5	2	-1/2	-3

$0\frac{1}{2}$

Note: Graph Drawn on graph paper.

Ans: From the Graph: Point of intersection $= (x, y) = (1, 2)$ $0\frac{1}{2}$

(b). Soln.

Constraints:

Let the number of hectares for Sorghum $= x$.

Let the number of hectares for Maize $= y$. $0\frac{1}{2}$

$4x + 2y \leq 20$ —— (i) $0\frac{1}{2}$

$1400x + 1200y \leq 8400$ —— (ii)

$600x + 800y \leq 4800$ —— (iii) $0\frac{1}{2}$

Also: $x \geq 0, \quad y \geq 0.$

11. (b) Drawing the Graph:

$X \geq 0, \Rightarrow X = 0.$ Also: $y \geq 0 \Rightarrow y = 0.$

$4x + 2y \leq 20, \Rightarrow \frac{4x}{2} + \frac{2y}{2} = \frac{20}{2}$

$2x + y = 10$ — (i)

Intercepts: $(0, 10)$ $(5, 0)$

$1400x + 1200y \leq 8400 \Rightarrow \frac{1400x}{100} + \frac{1200y}{100} = \frac{8400}{100}$

$14x + 12y = 84.$

$7x + 6y = 42$ — (ii)

Intercepts: $(0, 7)$ $(6, 0)$

$600x + 800y \leq 4800 \Rightarrow \frac{600x}{100} + \frac{800y}{100} = \frac{4800}{100}$

$\frac{6x}{2} + \frac{8y}{2} = \frac{48}{2}$

$3x + 4y = 24.$

Intercepts: $(0, 6)$ $(8, 0)$

Note: Graph on Graph paper.

Objective function:

$800,000x + 600,000y = f(x).$

Consider the table below:

Corner points	$f(x) = (800,000x + 600,000y)$	Max/Min
A(2.4, 4.2)	1920000 + 2520000	4440000
B(3.6, 2.8)	2880000 + 1680000	4560000
C(0, 6)	0 + 3600000	3600000
D(5, 0)	4000000 + 0	4000000
O(0, 0)	0 + 0	0.

SUBJECT NAME BASIC MATHS INDEX NUMBER S0[illegible]/0022.

11. (b) The Maximum Value is 4560000 Tsh.
∴ The hectares to be planted are: 3.6 hectares for Sorghum and 2.8 hectares for Maize for Maximum profit of 4560000 Tsh.

12. Frequency distribution table

C. Interval	C. Mark	f	$d = x - A$	fd
41 – 50	45.5	10	−30	−300
51 – 60	55.5	22	−20	−440
61 – 70	65.5	34	−10	−340
71 – 80	75.5	25	0	0
81 – 90	85.5	7	10	70
91 – 100	95.5	2	20	40
				−970

(a) Mean: from

$$\text{Mean} = A + \frac{\sum fd}{N}$$

$$\text{Mean} = 75.5 + \left(\frac{-970}{100}\right)$$

$$\text{Mean} = 75.5 + -9.7$$

$$\therefore \text{Mean} = 65.8$$

(b) Median = from:

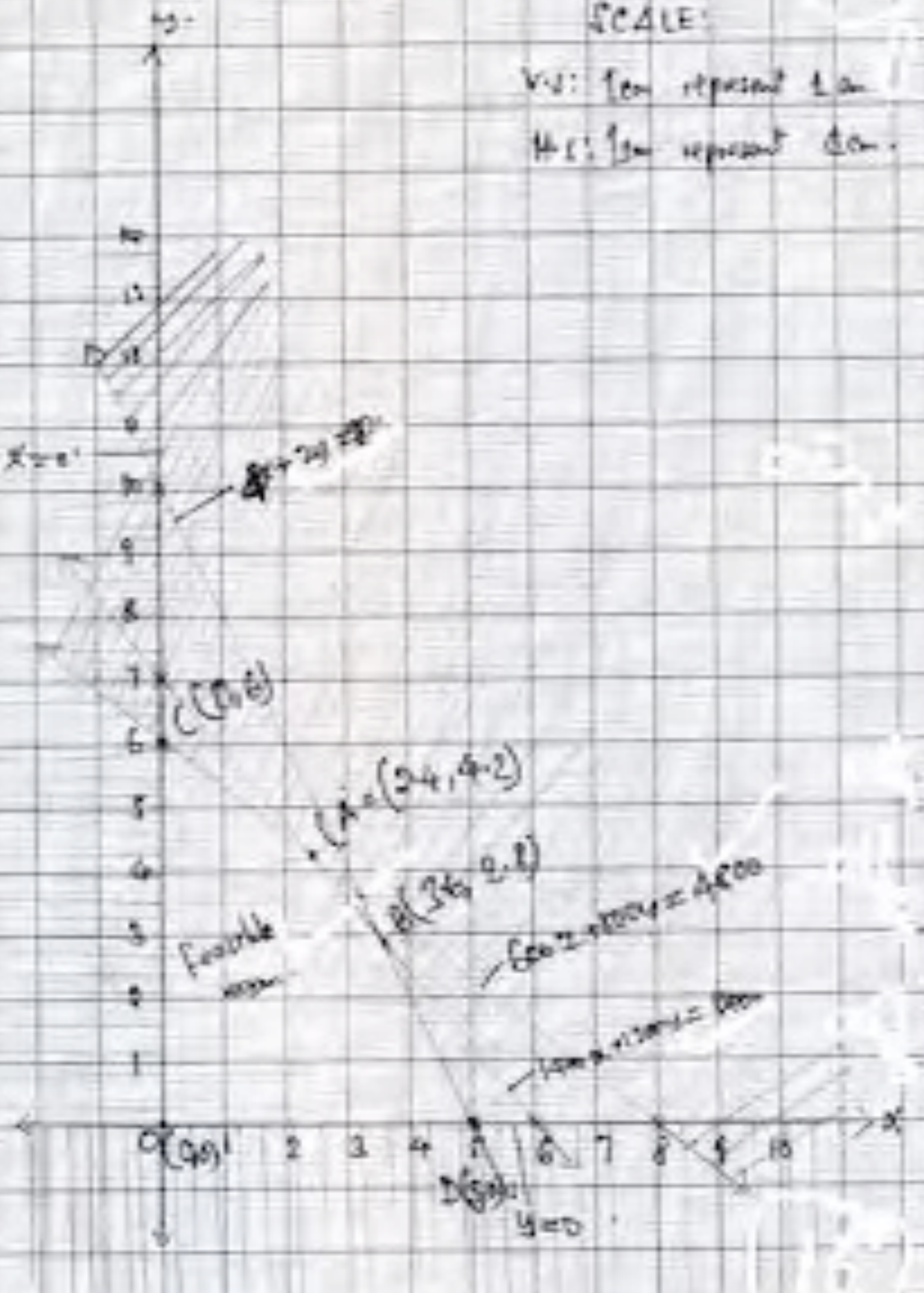
$$\text{Median} = L + \left(\frac{N/2 - Nb}{n_w}\right) i$$

$$\text{Median} = 60.5 + \left(\frac{100/2 - 32}{34}\right) 10$$

SUBJECT NAME BASIC MATH'S INDEX NUMBER S-0206/0023

1. (b).

A GRAPH.



2012

11. Anna and Mary are tailors. They make x blouses and y skirts each week. Anna does all the cutting and Mary does all the sewing. To make a blouse it takes 5 hours of cutting and 4 hours of sewing. To make a skirt it takes 6 hours of cutting and 10 hours of sewing. Neither tailor works for more than 60 hours a week.
    - (a) For sewing show that $2x+5y \le 30$
    - (b) Write down another inequality in x and y for the cutting.
    - (c) If they make at least 8 blouses each week, write down another inequality.
    - (d) Using $1cm$ to represent 1 unit on each axis, show the information in parts (a), (b) and (c) graphically. Shade only the required region.
    - (e) If the profit on a blouse is shs. 3,000/= and on a skirt is shs. 10,000/=, calculate the maximum profit that Anna and Mary can make in a week.

SECTION B

11 Let x - represent ~~cutting~~ blouses
y - represent ~~sewing~~ skirts

	Blouse	skirts	
cutting	$5x$	$6y$	60
Sewing	$4x$	$10y$	60

(a) for sewing

$$4x + 10y \le 60$$

$$\frac{4x}{2} + \frac{10y}{2} \le \frac{60}{2}$$

$$2x + 5y \le 30$$

(b) for cutting inequality

$$5x + 6y \le 60$$

(c) If they make atleast 8 blouses

$$x \geq 8$$

(d) (i) $2x + 5y \le 30$

(ii) $5x + 6y \le 60$

$x \geq 8$

By using intercepts

$2x + 5y = 30$

x	0	15
y	6	0

11. (d) (ii) $5x + 6y \leq 60$

x	0	12
y	10	0

(c) (iii) $x \geq 8$

$x = 8$

$x \geq 0$

$y \geq 0$

(e) To maximize the profit

$3000x + 10,000$

A	8, 0	24000 + 0 = 24000
B	8, 2	24000 + 20000 = 44000
C	9, 2	27000 + 20000 = 47,000
D	12, 0	36000 + 0 = 36,000

The maximum profit = 47000/=

at 27,000/ of blouses and 20,000 of skirts

2011

11. The number of units of proteins and starch contained in each of two types of food A and B are shown in the table below:

Type of Food	Units of Protein Per kg	Units of Starch Per kg	Cost per kg
A	8	10	400/=
B	12	6	500/=
Minimum Daily Requirement	32	22	

What is the cheapest way of satisfying the minimum daily requirement?

2010

11. (a) Maximize $f = 2y - x$ subject to the following constraints:

$x \geq 0$

$y \geq 0$

$2x + y \leq 6$

$x + 2y \leq 6$

(b) Sara had 300 shillings to buy erasers and pencils. An eraser cost 20 shillings while a pencil costs 30 shillings. If the number of erasers bought is at least twice the number of pencils, formulate the inequalities that represent this information. **(10 marks)**

2009

11. (a) Find the greatest value of the function $f(x, y) = 7x + 3y$ subject to the Constraints:

$2x + 3y \leq 12$

$x + 3y \geq 9$

$x \geq 0, \quad y \geq 0$

(b) The curve $y = ax^2 + bx + c$ passes through the points (1, 8), (0, 5) and (3, 20). Find the values of a, b and c and hence the equation of the curve. **(10 marks)**

2008

11. A shopkeeper buys two types of sugar: white sugar and brown sugar. The white sugar is sold at shs. 40,000/= per bag and the brown sugar is sold at shs. 60,000/= per bag. He has shs. 1,500,000/= available and decides to buy at least 30 bags altogether. He has also decided that at least one third of the bags should be brown sugar. He buys x bags of white sugar and y bags of brown sugar.

(a) Write down three (3) inequalities which will summarize the above information.

(b) Represent these inequalities graphically.

(c) The shopkeeper makes a profit of shs. 10,000/= from a bag of white sugar and shs. 20,000/= from a bag of brown sugar. Assuming he can sell his entire stock, how many bags of each type he should buy to maximize his profit? Find that profit. **(10 marks)**

2007

11. A person requires 15 and 14 units of chemical A and B respectively for his garden. A liquid product contains 5 and 2 units of A and B respectively, per jar; a dry product contains 1 and 4 units of A and B respectively, per carton. If the liquid product costs Tshs 3000/= per jar and the dry product costs Tshs. 2000/= per carton, how many of each should a person purchase to minimize the cost and meet the requirements? **(10 marks)**

Statistics

2018

12. The scores of 45 pupils in a Civics test were recorded as follows:

30	65	50	62	40	35	64	32	28	59	60	82	24	35	63
68	46	48	73	92	54	46	63	75	58	43	71	72	27	28
61	71	36	64	80	61	64	76	64	35	76	73	70	64	46

(a) Construct a frequency distribution table of the given data, taking equal class intervals 21 – 40, 41 – 60, …

(b) Calculate the mean score.

(c) Draw the cumulative frequency curve and use it to estimate the median.

12. a.) FREQUENCY DISTRIBUTION TABLE

Class Intervals	Tally marks	frequency (f)	Class mark (x)	fx	Cumulative frequency C.F
21 - 40	~~////~~ ~~////~~ /	11	30.5	335.5	11
41 - 60	~~////~~ ~~////~~	10	50.5	505.0	21
61 - 80	~~////~~ ~~////~~ ~~////~~ ~~////~~ //	22	70.5	1551.0	4~~2~~3
81 - 100	//	2	90.5	181.0	45
				2572.5	

b. Mean score $(\bar{x}) = \dfrac{\Sigma fx}{N}$

N = Total number of frequency

$N = 45$

$\Sigma fx = 2572.5$

$\bar{x} = \dfrac{2572.5}{45}$

$\bar{x} = 57.17$

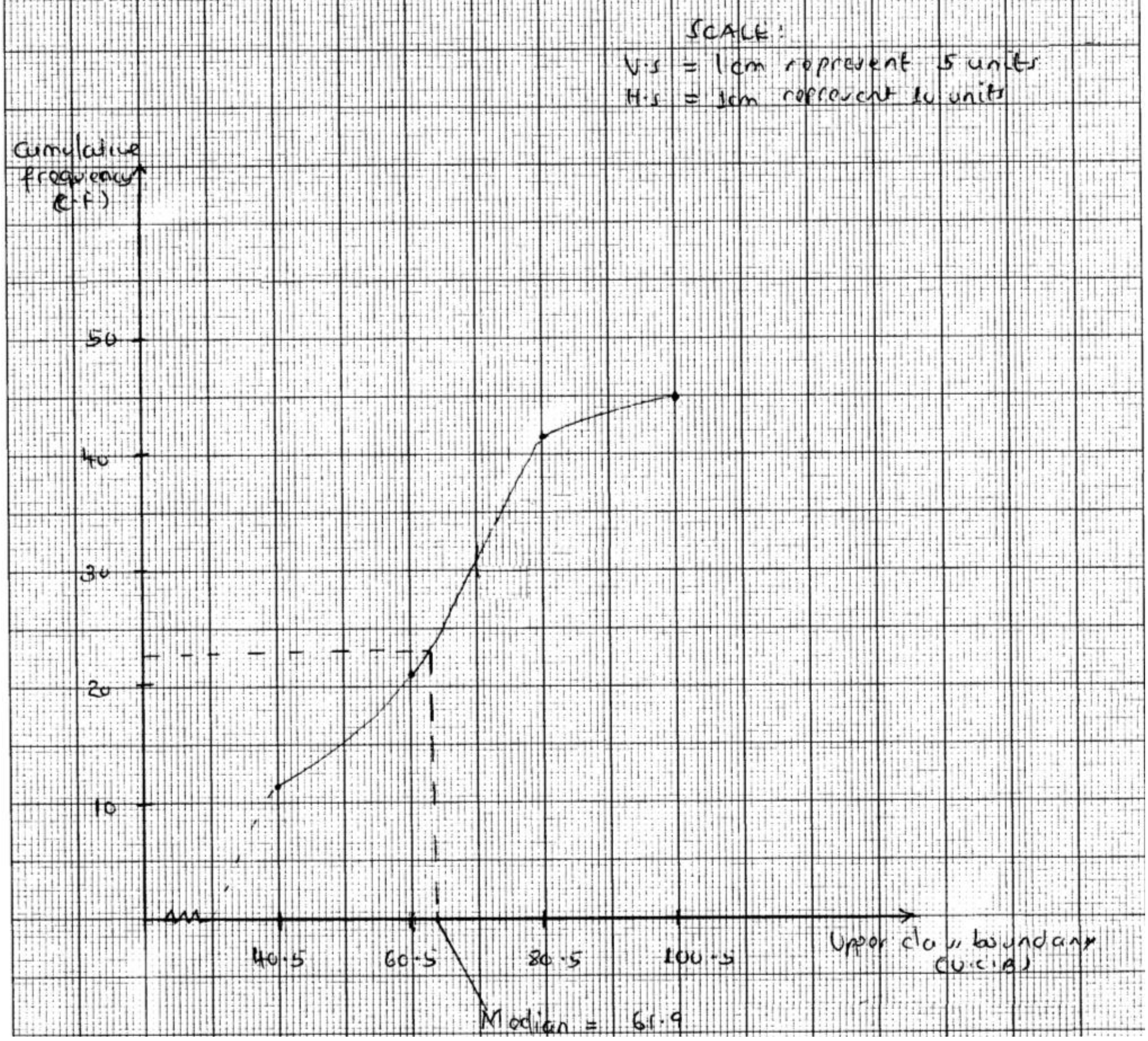
∴ Mean score is 57.17

c.

Cumulative frequency C.F	Upper class boundary (U.C.B)
11	40.5
21	60.5
43	80.5
45	100.5

12c.

THE CUMULATIVE FREQUENCY CURVE ESTIMATE THE MEDIAN



2017

12. The heights of 50 plants recorded by a certain researcher are given below:

56	82	70	69	72	37	28	96	52	88	41	42
50	40	51	56	48	79	29	30	66	90	99	49
77	66	61	64	97	84	72	43	73	76	76	22
46	49	48	53	98	45	87	88	27	48	80	73
54	79										

(a) Copy and complete this tally table for the data given above.

Height (cm)	Tally	Frequency
21-30		
31-40		
41-50		
51-60		
61-70		
71-80		
81-90		
91-100		

Use this table to:

(b) Draw a histogram for the height of the plants.

(c) Find the mean height of the plants (do not use the assumed mean method).

(d) Find the median of the heights of the plants.

12 a) FREQUENCY DISTRIBUTION TABLE

Height (cm)	Tally	Frequency
21 – 30	𝍸	5
31 – 40	II	2
41 – 50	𝍸 𝍸 I	11
51 – 60	𝍸 I	6
61 – 70	𝍸 I	6
71 – 80	𝍸 𝍸	10
81 – 90	𝍸 I	6
91 – 100	IIII	4

b).

Height (cm)	frequency	Classmark (x)	fx
21 - 30	5	25.5	127.5
31 - 40	2	35.5	71
41 - 50	11	45.5	500.5
51 - 60	6	55.5	333
61 - 70	6	65.5	393
71 - 80	10	75.5	755
81 - 90	6	85.5	513
91 - 100	4	95.5	382
	$\Sigma f = 50$		$\Sigma fx = 3075$

c). Mean $= \Sigma fx / \Sigma f$

$= 3075 / 50$

$= 61.5$

Mean is 61.5cm

12. d). Median $= Li + \left(\frac{N/2 - nb}{nw}\right) i$

Where Li = Real lower limit of median class = 60.5

N = Total frequency = 50 ∴ $50/2 = 25$

nb = frequency below median class = 24

nw = frequency within median class = 6

i = Class interval = 10

$= 60.5 + \left(\frac{25 - 24}{6}\right) 10$

$= 60.5 + \left(\frac{1}{6}\right) 10$

$= 60.5 + 1.67$

$= 62.17$

∴ Median is 62.17 cm

2016

12. The following were the scores of 35 students in a mathematics mock examination:
07, 19, 78, 53, 43, 67, 12, 54, 27, 22, 33, 80, 25, 58, 50, 36, 65, 33, 16, 19, 34, 20, 55, 27, 37, 41, 04, 32, 48, 28, 70, 31, 61, 08, 35.

(a) Prepare the frequency distribution table using the class intervals 0 – 9, 10 – 19, 20 – 29, etc.
(b) Which class interval has more students?
(c) Represent the information in a histogram and a frequency polygon and then find the mode.
(d) Calculate the median mark.

2015

12. The following marks were obtained by 32 students in a physics examination:
32, 35, 42, 50, 46, 29, 39, 38, 45, 37, 48, 52, 37, 58, 52, 48, 36, 54, 37, 42, 64, 37, 34, 28, 58, 64, 34, 57, 54, 62, 48, 67.

(a) Prepare a frequency distribution table using the class intervals: 24 - 29, 30 - 35 etc.
(b) Draw the histogram.
(c) Draw the cumulative frequency curve and use it to estimate the median.
(d) Find the mean mark.

Question Number | SUBJECT NAME B. MATHEMATICS | INDEX NUMBER S.1601/0014. | For Examiners' use only

12.(a)

class Interval	class mark (x)	f	f(x)	Tallies	upper class limit boundary	Cumulative frequency
24 – 29.	26.5	2	53.0	II	29.5	2
30 – 35	32.5	4	130.0	IIII	35.5	6
36 – 41.	38.5	7	269.5	~~IIII~~ II	41.5	13
42 – 47.	44.5	4	178.0	IIII	47.5	17
48 – 53.	50.5	6	303.0	~~IIII~~ I	53.5	23
54 – 59.	56.5	5	282.5	~~IIII~~	59.5	28
60 – 65.	62.5	3	187.5	III	65.5	31
66 – 71.	68.5		68.5	I	71.5	32.
		N=32.	Σfx=1472			

02

(c) From the graph the median is 46.

$00\frac{1}{2}$

(d) mean mark $= \frac{\Sigma fx.}{N.}$

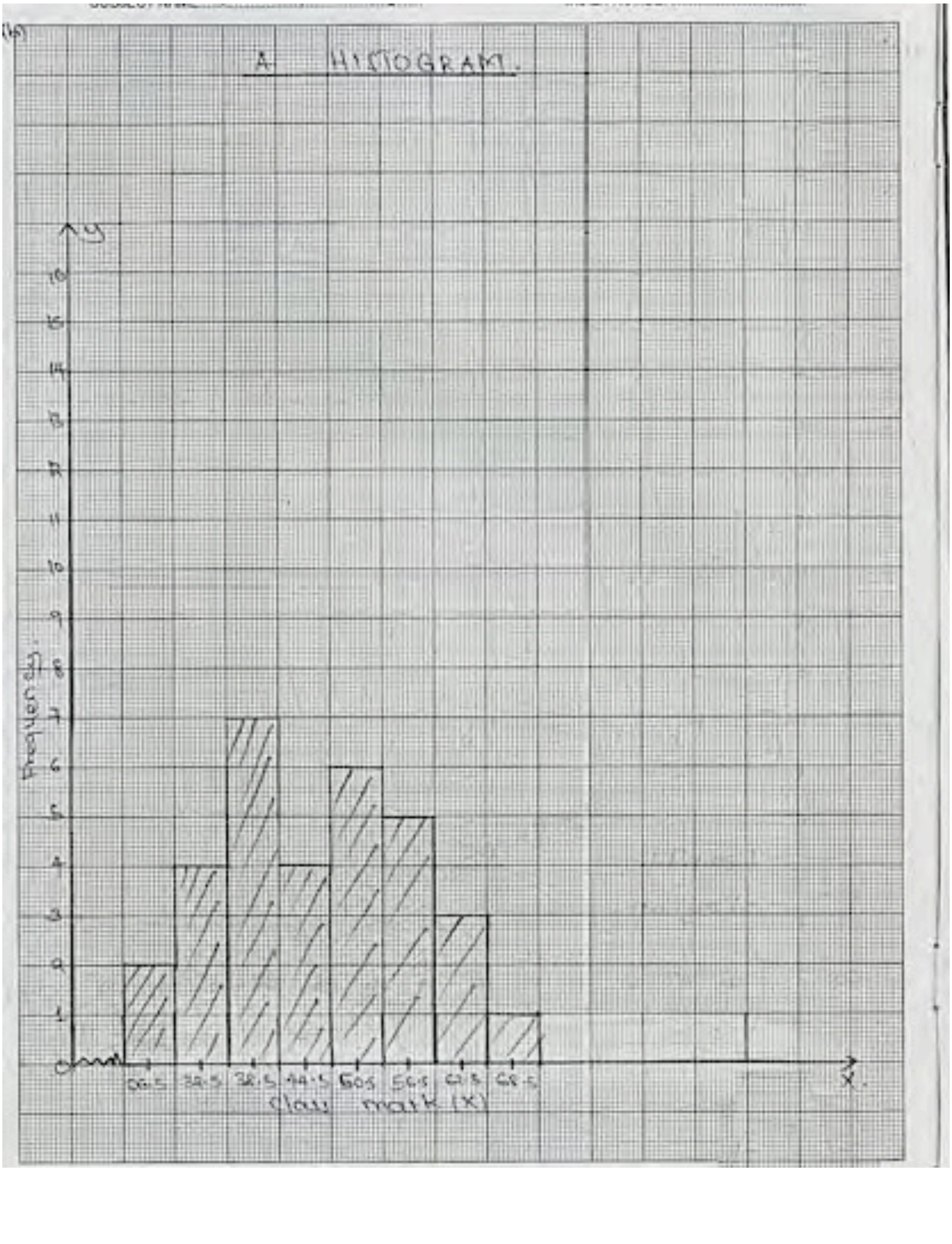
$0\frac{1}{2}$

$= \frac{1472.}{32.}$

01

mean mark = 46.

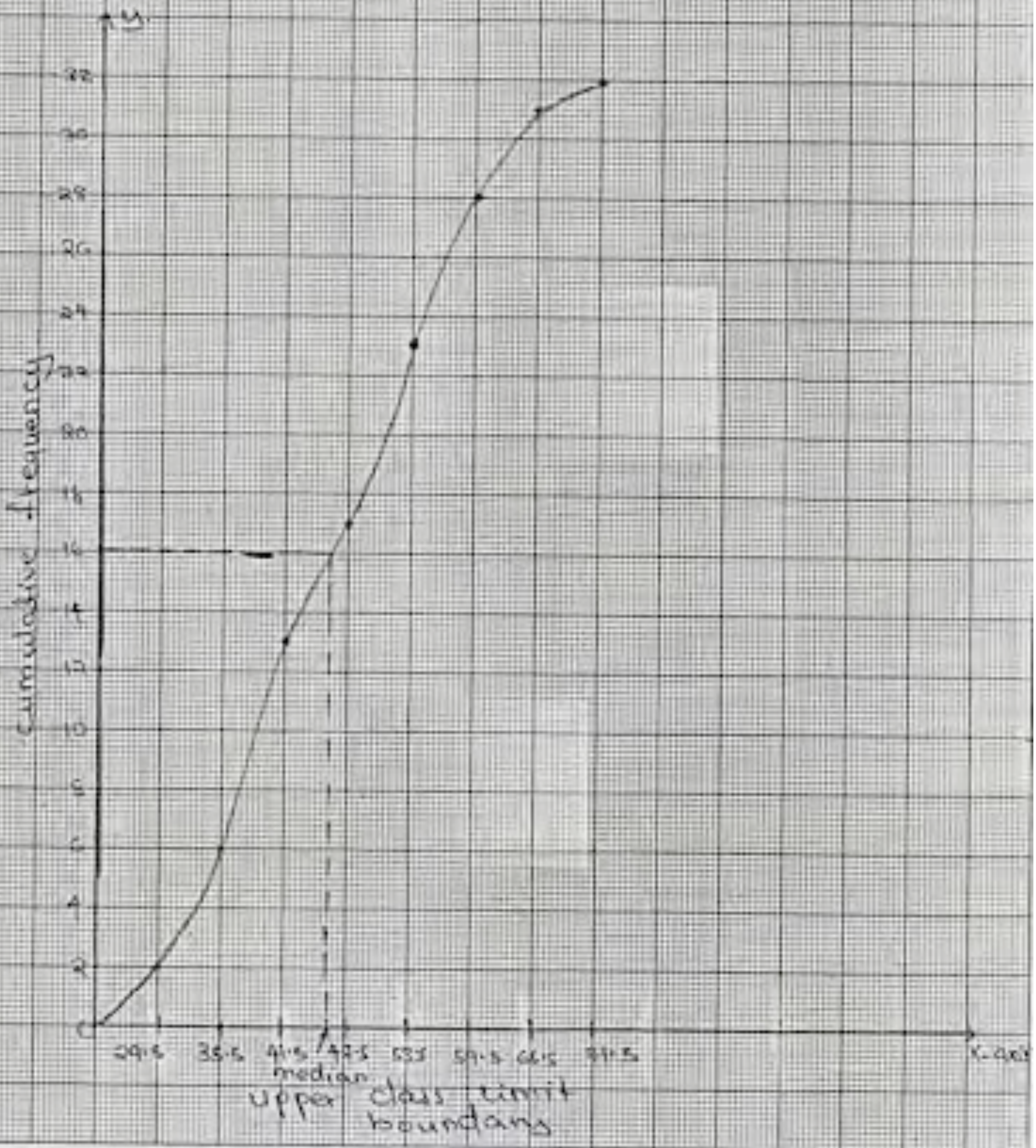
01


A HISTOGRAM.
Frequency
Class mark (X)

SUBJECT NAME B. MATHEMATICS. INDEX NUMBER S.1601/0014.

(c)

THE CUMULATIVE FREQUENCY CURVE GRAPH.



(10)

2014

12. The heights of some plants grown in a laboratory were recorded after 5 weeks. The results are shown in the following table:

Height (cm)	11 - 15	16 - 20	21 - 25	26 - 30	31 - 35	36 - 40
Frequency	4	8	20	21	12	3

(a) Calculate the mean and mode.

(b) Draw a cumulative frequency curve for the data.

(c) Estimate the median from the graph.

Question Number | SUBJECT NAME B) MATHEMATICS | INDEX NUMBER S2121/0031 | For Examiners' use only

15 b) ii) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\therefore$ The value of $x = 2$ and $y = 3$

12. Soln

Class Interval	f	x	fx
11 - 15	4	13	52
16 - 20	8	18	144
21 - 25	20	23	460
26 - 30	21	28	588
31 - 35	12	33	396
36 - 40	3	38	114
	N = 68		$\Sigma fx = 1754$

(a) from Mean $= \frac{\Sigma fx}{N}$ 01

$= \frac{1754}{68}$

$= 25.794$ 01

$= 25.79$

$\therefore$ Mean $= 25.79$

(ii) Mode

Modal class $= 26 - 30$

$L = 25.5$

$t_1 = 1$

$t_2 = 9$

$i = 5$ 01

from mode $= L + \left(\frac{t_1}{t_1 + t_2}\right) i$

$04\frac{1}{2}$

Question Number | SUBJECT NAME: B/MATHEMATICS | INDEX NUMBER: S0121/0031 | For Examiners' use only

12 (a) (ii) $Mode = 25.5 + \left(\frac{1}{1+9}\right)5$

$= 25.5 + \frac{1}{10} \times 5$

$= 25.5 + \frac{1}{2}$

$= 25.5 + 0.5$

$= 26$

$\therefore Mode = 26$

(b)

Class Interval	f	cf
11 - 15	4	4
16 - 20	8	12
21 - 25	20	32
26 - 30	21	53
31 - 35	12	65
36 - 40	3	68
	N=68	

Scale: Vertical scale 1cm ≡ 5 units

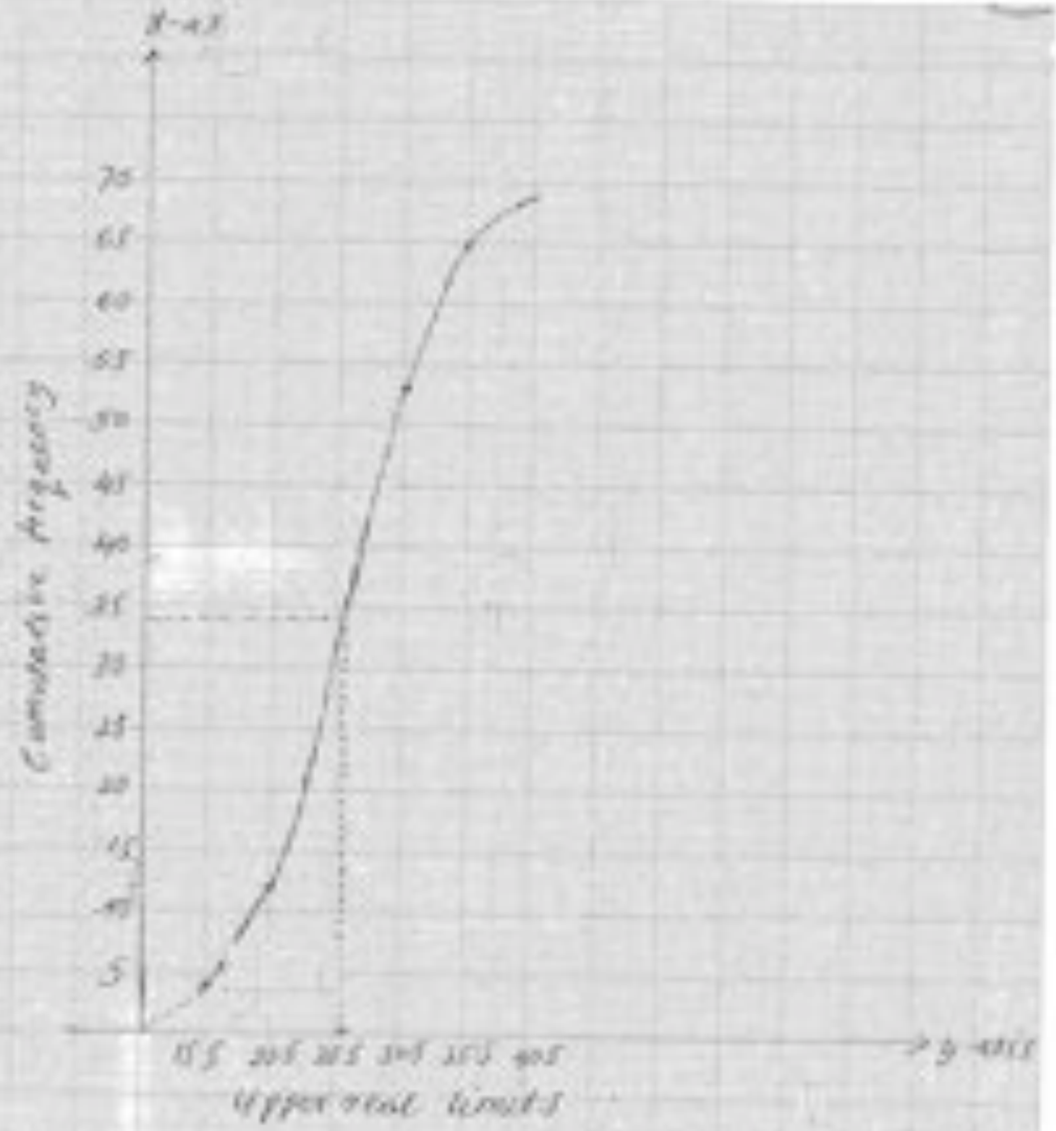
Horizontal scale 1cm ≡ 1 unit

(c) soln

$\therefore$ from the graph the estimated median is 26

SUBJECT NAME: B/MATHEMATICS INDEX NUMBER: 50121/0031 (07

12(b)



2013

12. Carefully study the frequency distribution table which shows the marks of 100 students in a Physics examination.

Marks	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90	91 - 100
Number of Students	10	22	34	25	7	2

Calculate

(a) the mean given the assumed mean is 75.5,

(b) the median in two decimal places,

(c) the mode in two decimal places.

12. Solution:-

FREQUENCY DISTRIBUTION TABLE FOR MARKS OF 100 STUDENTS.

Class interval	Class marks (x)	$d = x - A$	f	fd	Cumulative frequency	Class size
41 - 50	45.5	-30.0	10	-300	10	10
51 - 60	55.5	-20.0	22	-440	32	10
61 - 70	65.5	-10.0	34	-340	66	10
71 - 80	75.5	0.0	25	0	91	10
81 - 90	85.5	10.0	7	70	98	10
91 - 100	95.5	20.0	2	40	100	10
			$N = 100$	$\Sigma fd = -970$		

SUBJECT NAME B/ MATHEMATICS INDEX NUMBER S0110/0097

(a) Mean by Assumed mean.

$$\text{Mean} = A + \frac{\Sigma fd}{N}$$

$$\text{Mean} = 75.5 + \frac{-970}{100}$$

$$\text{Mean} = 75.5 + -9.7$$

$\therefore$ Mean $= 65.8$ marks.

(b) Median in two decimal places.

$\frac{1}{2} \times 100 = 50^{th}$

Median class $= (61 - 70)$.

$L = 60.5$

$N_b = 32$

$N_w = 34$

$i = 10$

$$\text{Median} = L + \frac{\left(\frac{N}{2} - N_b\right)i}{N_w}$$

$$= 60.5 + \frac{\left(\frac{100}{2} - 32\right)10}{34}$$

$$= 60.5 + \frac{(50 - 32)10}{34}$$

$$= 60.5 + \frac{18 \times 10}{34}$$

$$= 60.5 + \frac{90}{17}$$

$$= 60.5 + 5.294$$

$$= 65.794 \approx 65.79 \text{ (in 2 d.p)}$$

$\therefore$ The median $=$ ~~65.80 marks~~ 65.79 marks.

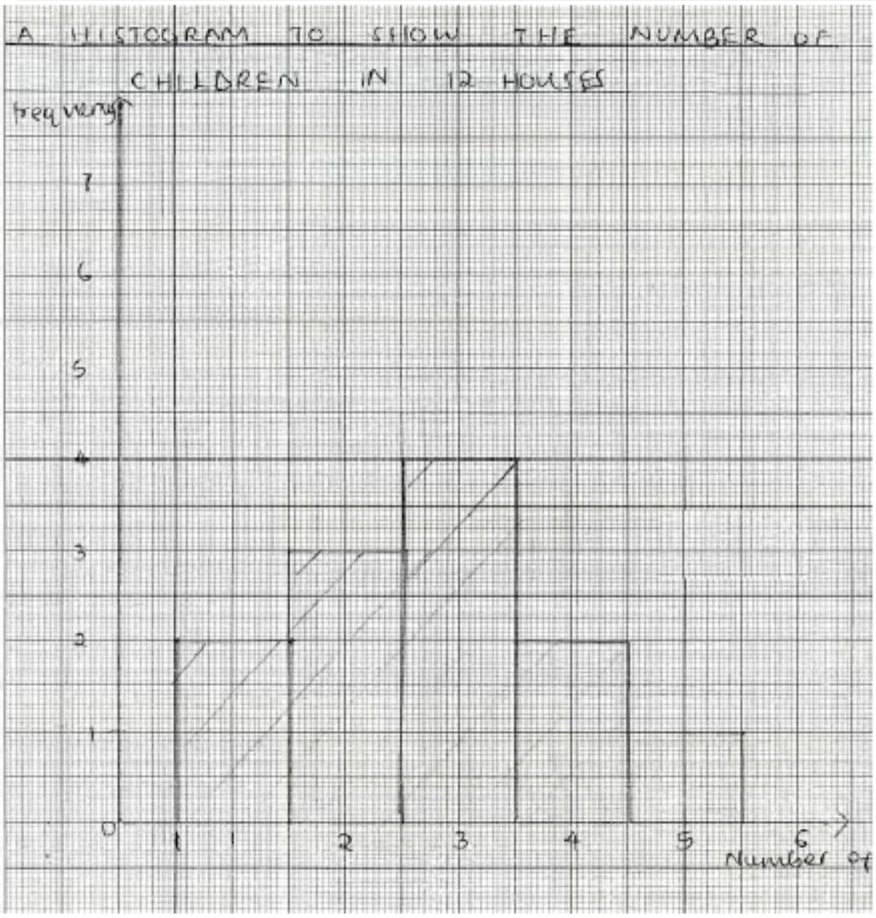
2012

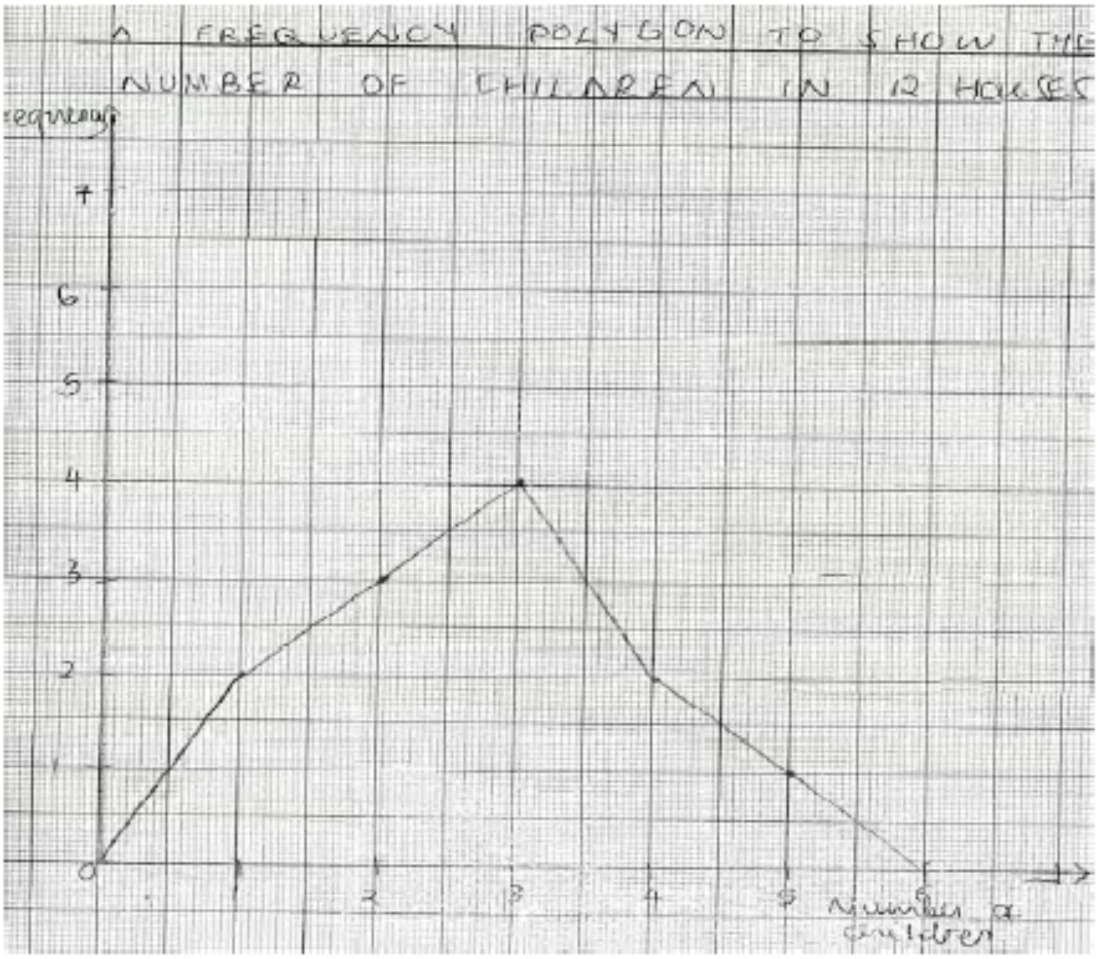
12. In a survey of the number of children in 12 houses, the following data resulted: 1, 2, 3, 4, 2, 2, 1, 3, 4, 3, 5, 3
   (a) Show this data in a frequency distribution table.
   (b) Draw a histogram and a frequency polygon to represent this data.
   (c) Calculate the mean and mode number of children per house.

FREQUENCY DISTRIBUTION TABLE.

Number of children (x)	1	2	3	4	5
Frequency (f)	2	3	4	2	1

Histogram and frequency polygon in graph





12c. Mean $= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{(2\times1)+(2\times3)+(3\times4)+(4\times2)+(5\times1)}{12}$

$= \dfrac{2+6+12+8+5}{12} = \dfrac{33}{12}$

$= 2.75$ children ≈ 3.

The mean is $2.75 \approx 3$ children

12c Mode = 3. Has the highest frequency.

2011

12. The following table gives the scores of sixty students in a Basic Mathematics test.

Scores	Frequency
0 – 10	5
10 – 20	7
20 – 30	15
30 – 40	25
40 – 50	8

Calculate:

(a) The mean score if the assumed mean is obtained from the mid mark of the modal class;

(b) The median;

(c) The range.

2010

12. The data below represent masses in kg of 36 men.

51	61	60	70	75	71	75	70	74	73	72	82
70	71	76	74	50	68	68	66	65	72	69	64
83	63	83	58	80	90	50	89	55	62	62	61

(i) Prepare a frequency distribution table of class interval of size 5 beginning with the number 50 taking into consideration that both lower limit and upper class limits are inclusive.

(ii) Calculate the mean and mode from the frequency distribution table prepared in (i) above by using assumed mean from the class mark of the modal class. **(10 marks)**

2009

12. Carefully study the frequency distribution table which shows marks for 40 students in History examination.

Marks	1 – 20	21 – 40	41 – 60	61 – 80	81 – 100
Number of students	3	11	12	8	6

Determine

(i) The mean, given the assumed mean is 50.5

(ii) The median

(iii) Modal class and its corresponding class mark. **(10 marks)**

2008

12. (a) The age at which a child first walked (to the nearest month) was recorded for eight (8) children. The results were 12,10,16,19,10,12,12 and 13. Calculate the Mean. Mode and Median of the data.

(b) A survey was made on the number of people attending conferences on one particular week. A random sample of 100 conference centres was taken and the results were as follows:

Number of people attending conference	150-154	155-159	160-164	165-169	170-174
Number of conference centres	8	16	43	29	4

(i) Draw a histogram and a cumulative frequency curve to represent these results.

(ii) Estimate the median of this data from the cumulative frequency curve in 12 (b)(i) above. **(10 marks)**

2007

12. The frequency distribution of the length of a sample of 100 nails, measured to the nearest mm is shown below.

Length	40 – 42	43 – 45	46 – 48	49 – 51	52 – 54	55 – 57	58 - 60
Frequency	4	9	13	20	34	18	2

(a) How many nails have length less than 51.5 mm? **(1 mark)**

(b) Calculate the mean length. **(4 marks)**

4.2
5 | 21
20
1

4.2
5
21.0

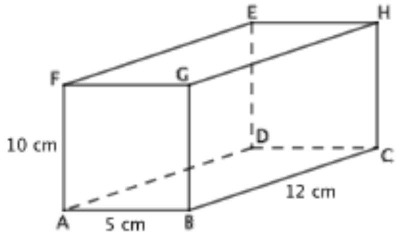
(c) Draw a histogram and use it to estimate the modal length. **(4 marks)**

(d) State the modal class. **(1 mark)**

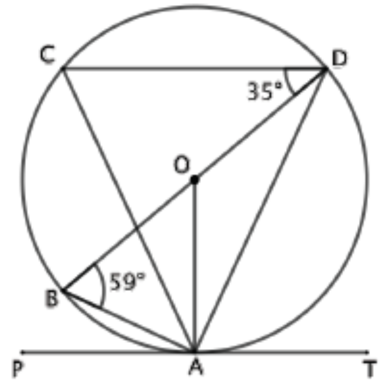
13. Circles, Three Dimensional Figures and the Earth as a Sphere

2018

13. (a) In the following cuboid, AB = 5 cm, BC = 12 cm and BG = 10 cm. Calculate:
(i) The length of AH (give your answer correct to one decimal place).
(ii) The angle CAH.

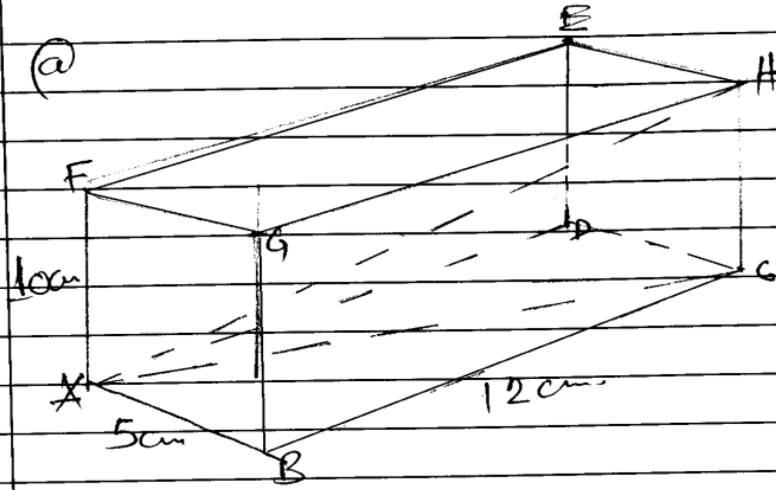


(b) In the following figure A, B, C and D lie on the circle; O is the centre of the circle, BD is its diameter and PAT is the tangent of the circle at A.



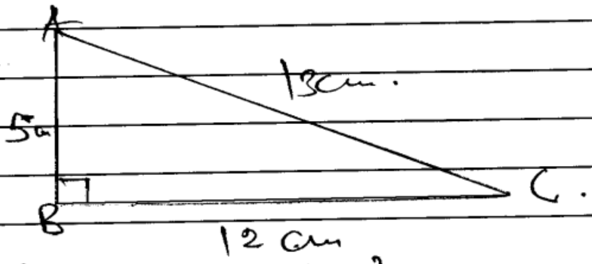
If angle $A\hat{B}D = 59°, C\hat{D}B = 35°$, find $A\hat{C}D, A\hat{D}B, D\hat{A}T$ and $C\hat{A}O$.

13 (a)



(i) Length AH.

Consider the triangle ABC



$(AC)^2 = (12)^2 + (5)^2.$

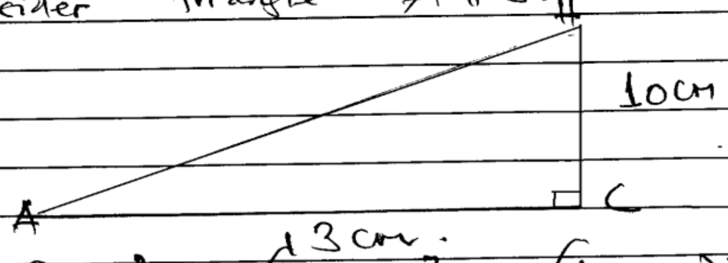
$(AC)^2 = 144 + 25$

$\sqrt{(AC)^2} = \sqrt{169}$

$AC = 13cm$

Solve for AH

Consider triangle AHC.



$(AH)^2 = (13cm)^2 + (10\,cm)^2$

13 (a) (i) $(AH)^2 = \sqrt{69cm^2 + 100cm^2}$

$\sqrt{(AH)^2} = \sqrt{269 cm^2}$

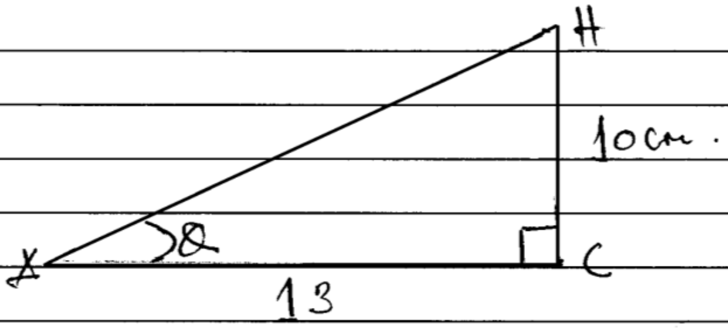
$AH = 16.4$ cm

$\therefore$ The length AH = 16.4 cm.

(ii) The angle CAH.

Soln

Consider Triangle CAH.



from

$\tan \theta = \frac{opp}{Adj}$

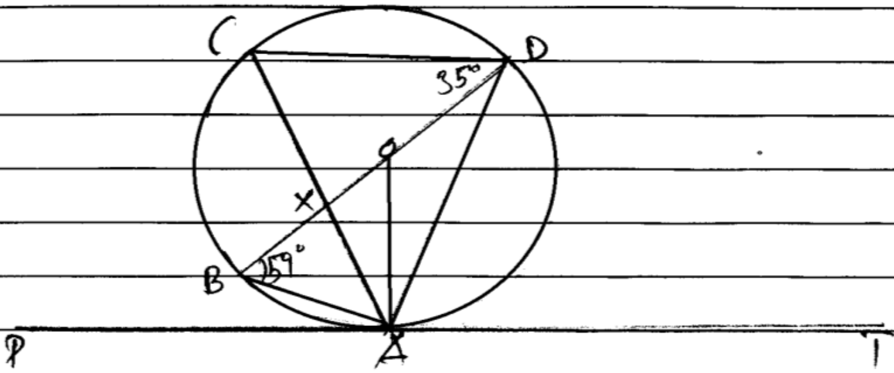
$\tan C\hat{A}H = \frac{10}{13} = 0.7692.$

$C\hat{A}H = \tan^{-1} 0.7692$

$C\hat{A}H = 37° 34'$

$\therefore$ The angle CAH = 37°34'.

(b)



18 (b) (i) $A\hat{C}D$

from $A\hat{O}D = 2(B\hat{A}O)$

$A\hat{O}D = 2(59°)$.

$A\hat{O}D = 118°$

Also

$A\hat{C}D = \frac{1}{2}(A\hat{O}D)$

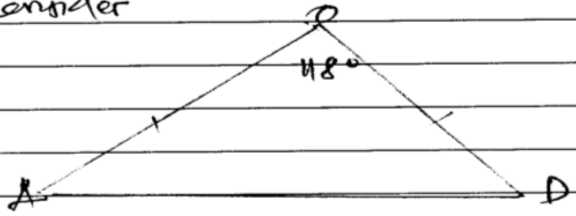
$A\hat{C}D = \frac{1}{2} \times 118°$

$A\hat{C}D = 59°$

$\therefore \quad A\hat{C}D = 59°$.

(ii) $A\hat{D}B$

Consider



$A\hat{D}O = \hat{A}DB = DAO$.

Thus $A\hat{D}B + D\hat{A}O + A\hat{O}D = 180$

$A\hat{D}B + A\hat{D}B + 118° = 180°$

$2A\hat{D}B = 180° - 118°$

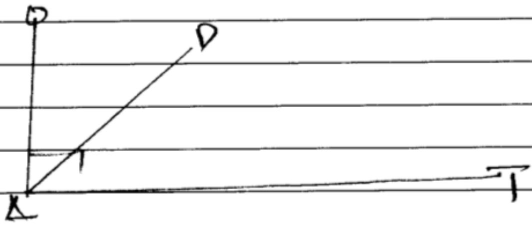
$\frac{2A\hat{D}B}{2} = \frac{62°}{2}$

$A\hat{D}B = 31°$

$\therefore \quad A\hat{D}B = 31°$

(iii) $D\hat{A}T$

Consider



13 (b) (iii) Since $D\hat{A}O = A\hat{D}B$

$D\hat{A}O = 31^\circ.$

But $T\hat{A}O = 90^\circ$

$D\hat{A}O + D\hat{A}T = 90^\circ$

$31^\circ + D\hat{A}T = 90^\circ$

$D\hat{A}T = 90^\circ - 31^\circ$

$D\hat{A}T = 59^\circ$

$\therefore\ D\hat{A}T = 59^\circ.$

(iv) $C\hat{A}O$

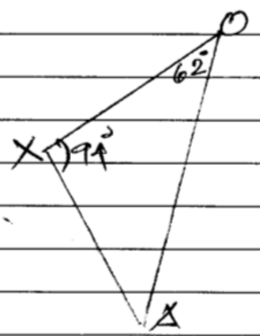
Soln

from $A\hat{C}D = 59^\circ$, $B\hat{D}C = 35^\circ$.

Thus $O\hat{X}A = 59^\circ + 35^\circ$

$O\hat{X}A = 94^\circ.$

Now Consider



$X\hat{A}O = C\hat{A}O.$

$C\hat{A}O + 94^\circ + 62^\circ = 180^\circ$

$C\hat{A}O + 156^\circ = 180^\circ$

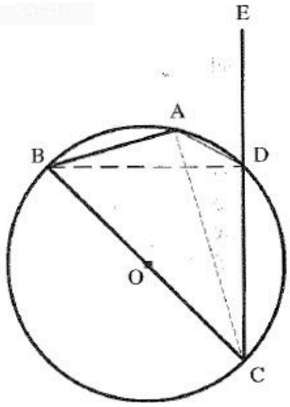
$C\hat{A}O = 180^\circ - 156$

$C\hat{A}O = 24^\circ$

$\therefore\ C\hat{A}O = 24^\circ.$

2017

13. In the figure below, BC is a diameter of the circle, O is the centre of the circle and side CD of the cyclic quadrilateral ABCD is produced to E.



(a) With reasons, name the right angles in this figure.

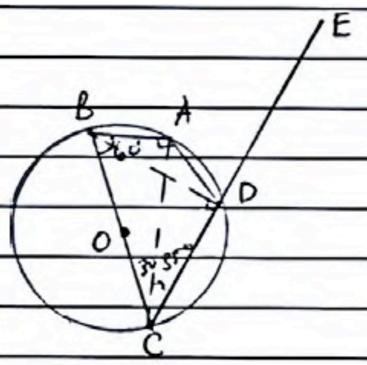
(b) Show that $A\hat{D}E = A\hat{B}C$.

(b) If $A\hat{D}E = 60$ and $C\hat{A}D = 25$, find:

(i) the value of $A\hat{B}D$,

(ii) the lengths AB and BD given that CB = 10cm.

13.



(a) $C\hat{A}B = 90^\circ$ (Is a right angle).

$C\hat{D}B = 90^\circ$ (Is a right angle)

This is because the angle subtended at the circumference by the diameter is 90°

Also. The angle subtended at the circumference is half that subtended at the centre. That is

$$A\hat{O}C = 2(C\hat{D}B)$$
$$180^\circ = 2C\hat{D}B$$
$$\underline{\underline{C\hat{D}B = 90^\circ}}$$

$$A\hat{O}C = 2(C\hat{A}B)$$
$$180^\circ = 2(C\hat{A}B)$$
$$\underline{\underline{C\hat{A}B = 90^\circ}}$$

Required to show $A\hat{D}E = A\hat{B}C$

(b). $C\hat{B}A + C\hat{D}A = 180^\circ$ -- sum of opposite angles in a cyclic

$$C\hat{D}A = 180^\circ - C\hat{B}A$$
$$C\hat{B}A = 180^\circ - C\hat{D}A \quad \ldots (i)$$
$$A\hat{B}C = 180^\circ - C\hat{D}A$$

$C\hat{D}A + A\hat{D}E = 180^\circ$ ... Sum of angles in a straight line

$$A\hat{D}E = 180^\circ - C\hat{D}A \quad \ldots (ii)$$

By comparing (i) and (ii)

$$180^\circ - C\hat{D}A = 180^\circ - C\hat{D}A$$

$\therefore A\hat{B}C = A\hat{D}E$, hence shown (c)(ii) $\cos 60^\circ = \frac{BA}{10}$

$\frac{1}{2} = \frac{BA}{10}$

$2BA = 10$

$\overline{BA} = 5$ cm

$\frac{\sin 65^\circ}{1} = \frac{BD}{10}$

$BD = 10 \sin 65^\circ$ cm

$BD = 9.063$ cm.

(i) $A\hat{B}D = A\hat{C}D = 35^\circ$

$\therefore A\hat{B}D = 35^\circ$.

Because

$\because A\hat{B}D = A\hat{C}D$

Then: $A\hat{B}D + 30^\circ + 90^\circ + 25^\circ = 180$ ... sum of opp of a circle

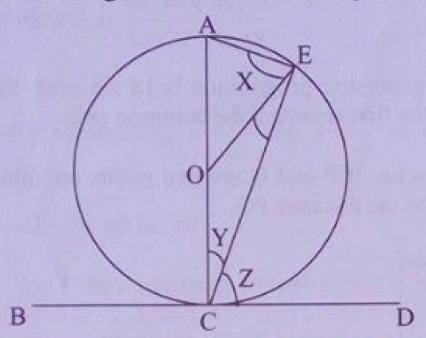
$A\hat{B}D = 180 - 90 - 55$

$A\hat{B}D = 90 - 55$

$A\hat{B}D = 35^\circ$

2016

13. (a) In the figure below, BD is a tangent to the circle having the centre O.



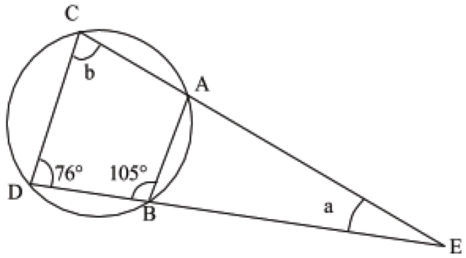
Given that angle OEC = 28°, find the values of angles marked X, Y and Z.

(b) Calculate the distance from Chagwe (5°S, 39°E) to Minga (12°S, 39°E) in kilometres. Use $\pi = 3.14$, the radius of the earth $R = 6370$ km and write the answer correct to 1 decimal place.

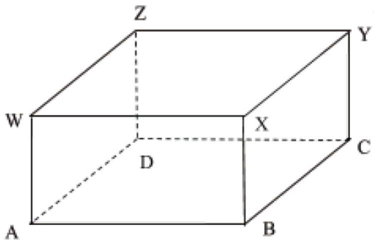
(c) If a bus leaves Chagwe at 8.00 am on Monday and travels at 40km/hour, at what time will it reach Minga?

2015

13. (a) Find the value of the angles a and b in the figure below.



(b) A rectangular box with top $WXYZ$ and base $ABCD$ has $AB = 9cm$, $BC = 12cm$ and $WA = 3cm$.



Calculate:

(i) The length of AC,

(ii) The angle between WC and AC.

(c) Two places P and Q both on the parallel of latitude $26°N$ differ in longitude by $40°$. Find the distance between them along their parallel of latitude.

13·a) $b + 105° = 180°$ ---- Sum of opposite angles in a cyclic quadrilateral 01

so:

$b = 180° - 105° = 75°$

$\therefore b = 75°$ 01

$a = \frac{1}{2}(\widehat{CD} - \widehat{AB})$

but:

$a + b + 76° = 180°$ ---- Sum of angles in ΔCED 01

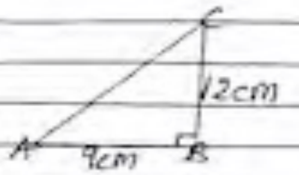
so:

$a = 180° - (75° + 76°) = 180° - 151° = 29°$

$\therefore a = 29°$ 01

10. b) i) consider $\triangle ABC$

13.



by pythagoras theorem:

$a^2 + b^2 = c^2$

$12^2 + 9^2 = \overline{AC}^2$

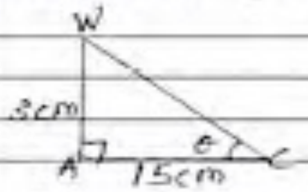
$144 + 81 = \overline{AC}^2$

$225 = \overline{AC}^2$

$\overline{AC} = \sqrt{225} = 15cm$

$\therefore \overline{AC} = 15cm$

ii) consider $\triangle AWC$



$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

$\tan \theta = \frac{3cm}{15cm} = 0 \cdot 2$

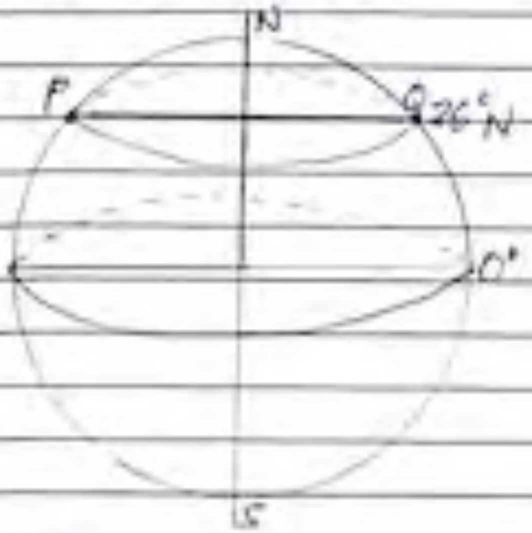
$\theta = \tan^{-1} 0 \cdot 2$

$\theta = 11°19'$

$\therefore$ The angle is $11°19'$

Question Number	SUBJECT NAME: MATHEMATICS	INDEX NUMBER: S0239/0051

13·c)



$\Delta\theta = 40°$

but:

$$d = \frac{\Delta\theta}{360°} 2\pi R \cos\theta$$

where: $\theta = 26°$

so:

$$d = \frac{40°}{360°} \times 2 \times 3.14 \times 6400\text{km} \times \cos 26°$$

$d = 40° \times 111.64\text{km} \times 0.8988$

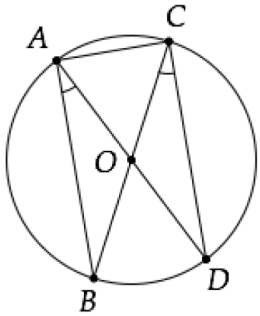
$d = 4013.68128\text{km} \approx 4013.68\text{km}$

$\therefore$ distance is 4013.68km

2014

13. (a) Prove that the sizes of the angles in the same segment of a circle are equal.

(b) In the figure below, O is the centre of the circle and $\overline{AD}$ bisects angle BAC. Find angle BCD.

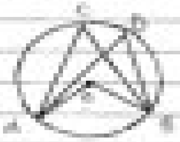


(c) Kicheko and Mtakuja are two villages on latitude $60°S$. The distance between Kicheko and Mtakuja measured along the parallel of latitude is $1111km$. Find the difference between their longitudes in two significant figures.

13(a) Solution:

Required to prove: Sizes of the angles in the same segment of a circle are equal.

Consider the circle below: ACB



with O the centre of a circle.

Required to prove $A\hat{C}B = A\hat{D}B$

From the figure:

$A\hat{O}B = 2A\hat{C}B$ {Central angle is twice angle at the point on the circumference} 01

Also $A\hat{O}B = 2A\hat{D}B$ {Central angle is twice angle at the point on the circumference}.

So

$2A\hat{C}B = 2A\hat{D}B = A\hat{O}B$ {Similar angles}

$\frac{2A\hat{C}B}{2} = \frac{2A\hat{D}B}{2}$ 01

$A\hat{C}B = A\hat{D}B$

But $A\hat{C}B$ and $A\hat{D}B$ are angles in the same segment. 01

Therefore sizes of the angles in the same segment of a circle are equal

Hence proved.

Question Number	SUBJECT NAME Q-MATHEMATICS	INDEX NUMBER C0140/0060	For Examiners' use only

13(b) Solution. (04)

Given: Circle ABDC.

O centre.

$\overline{AD}$ bisects $B\hat{A}C$.

Required to find: $B\hat{C}D$.

From given circle:

$B\hat{A}D = D\hat{A}C$ {Angle bisected by AD}.

But:

$B\hat{A}C = 90^\circ$ {Angle in a semi circle}. 01

also:

$B\hat{A}C = B\hat{A}D + D\hat{A}C$

$\frac{90^\circ}{2} = \frac{2B\hat{A}D}{2}$.

$B\hat{A}D = 45^\circ$ 01

But:

$B\hat{A}D = B\hat{C}D$ {Angles in the same segment of a circle}. $= 45^\circ$

Therefore angle BCD is 45°. 01

13(c) Solution. (07)

[illegible]

1.6[illegible]

32 | 77.2045

[illegible]

[illegible]

[illegible]

[illegible]

By using $R = 6400$ km and $\pi = 3.14$

$$\frac{1111 \text{ km}}{2(3.14)(6400)\cos 60^\circ} = \frac{\alpha}{360^\circ} \qquad 01$$

$$\frac{1111}{20096} = \frac{\alpha}{360}$$

$$\alpha = \frac{1111 \times 360^\circ}{20096}$$

$$= \frac{399960}{20096}$$

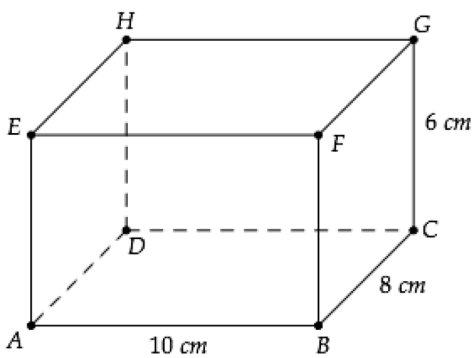
$$= 19.90^\circ \qquad 01$$

$$= 20^\circ \text{ correct to (2sf)}$$

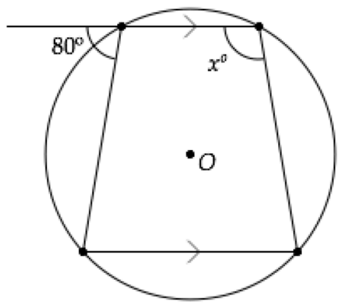
Therefore difference between their longitudes is 20° correct to two significant figures.

2013

13. (a) The figure below shows a rectangular prism in which $AB = 10cm$, $BC = 8cm$ and Calculate the length of DF (Leave your answer in surd form)



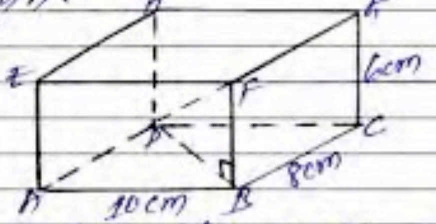
(b) If O is the centre of a circle given below, find the value of x.



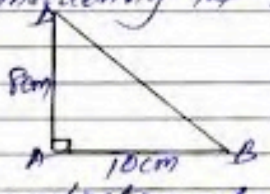
(c) Calculate the circumference of a small circle, in kilometres, along the parallel of latitude $30°N$ (Leave your answer in surd form).

Question Number | SUBJECT NAME BASIC MATHEMATICS | INDEX NUMBER S0461/0085 | For Examiners' use only

13. (a) Soln.
Given;



By considering the ΔABD so as to get $\overline{DB}$



By application of pythagoras' theorem.

$(DB)^2 = (AD)^2 + (AB)^2$

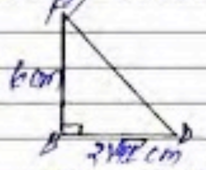
$(DB)^2 = (8^2 + 10^2)\,cm^2$ 01

$(DB)^2 = (64 + 100)\,cm^2$

$\sqrt{(DB)^2} = \sqrt{164}\,cm^2$

$DB = 2\sqrt{41}\,cm.$ 01

Also, by considering the ΔBDF so as to get $\overline{DF}$



Also, $(FD)^2 = (FB)^2 + (BD)^2$

$(DF)^2 = 6^2 + (2\sqrt{41})^2\,cm^2$ 01

$(DF)^2 = (36 + 164)\,cm^2$

$(DF)^2 = \sqrt{200}\,cm^2$

$DF = 10\sqrt{2}\,cm.$

$\therefore$ The length $\overline{DF}$ is $10\sqrt{2}\,cm$. 01

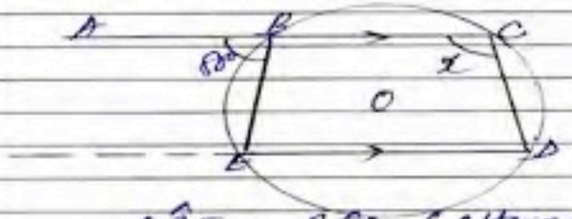
04

Question Number | SUBJECT NAME: BASIC MATHEMATICS | INDEX NUMBER: S0461/0085 | For Examiners' use only

(04)

13. (b) Soln.

Given that:



$A\hat{B}E = B\hat{E}D$ (Alternate interior angles).

$B\hat{E}D = 80°$

Also,

$D\hat{E}B + B\hat{C}D = 180°$ (Opposite angles in a cyclic quadrilateral polygon)

$80° + B\hat{C}D = 180°$

$B\hat{C}D = 180° - 80°$

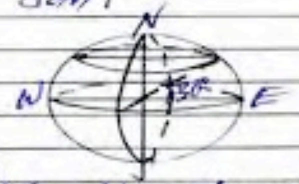
$B\hat{E}D = 100°$ — 01

But, $B\hat{C}D = x$

then, $x = 100°$.

$\therefore$ $x = 100°$. — 01

(c) Soln.



from, Circumference (C) $= 2\pi R \cos\theta$ but $R = 6370$ km and $\pi = \frac{22}{7}$ — 01

then, $C = 2 \times \frac{22}{7} \times 6370\text{km} \times \cos 30°$ — 01

$C = 20020\sqrt{3}$ km. — 01

$\therefore$ The circumference of a small circle is $20020\sqrt{3}$ km — 01

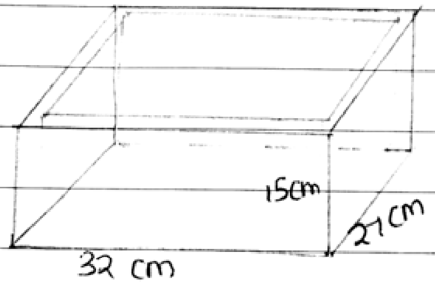
052013

2012

13. (a) An open rectangular box measures externally $32cm$ long, $27cm$ wide and $15cm$ deep. If the box is made of wood $1cm$ thick, find the volume of wood used.

(b) Find the distance (in km) between towns $P(12.4°S,\ 30.5°E)$ and $Q(12.4°S,\ 39.8°E)$ along a line of latitude, correctly to 4 decimal places.

13 a)



Total volume $= 32cm \times 27cm \times 15cm$

$= \cancel{4}12960 cm^3$

Internal volume $= (32cm - 2cm)(27cm - 2cm)(15cm - 1cm)$

$= 30 cm \times 25 cm \times 14 cm$

$= 10500 cm^3$

Volume of wood $= 12960 cm^3 - 10500 cm^3$

$= 2460 cm^3$

$\therefore$ The volume of wood is $\underline{2460 cm^3}$

b) Distance $= \frac{\Delta\alpha}{360^\circ} \times 2\pi R \cos\theta$

$= \frac{(39.8^\circ - 30.5^\circ)}{360^\circ} \times 2 \times \frac{22}{7} \times 6370 \times \cos 12.4$

$= \frac{9.3}{360} \times \frac{44}{7} \times 6370 \times \cos 12.4$

$= \frac{9.3}{360} \times 44 \times 910 \times 0.9767$

$= \frac{31}{120} \times 44 \times 91 \times 0.9767$

$= 1010.06$

$\therefore$ Distance $= \underline{1010.06 km}$

2011

13. In figure 3, $ABCD$ is a rectangle in which $AB = 3cm$ and $BC = 2cm$. V is a point such that $VA = VB = VC = VD = 6cm$ and $AO = OC$. Find:

(a) The angle VAD;

(b) The length of AC;

(c) The angle between VA and the plane $ABCD$.

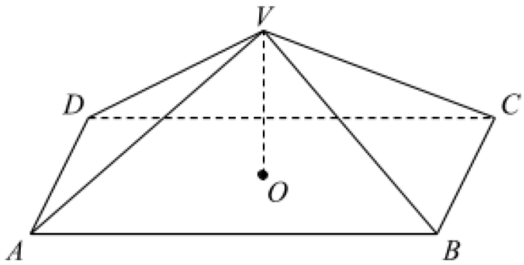


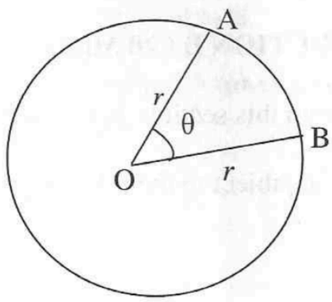
Figure 3

2010

13. (a) Below is a circle with centre O and radius r units. By considering the circumference of the circle, the area of the circle, the given angle θ and the degree measures of a circle (360°), develop the formula for finding:

(i) arc length AB

(ii) area of sector AOB.



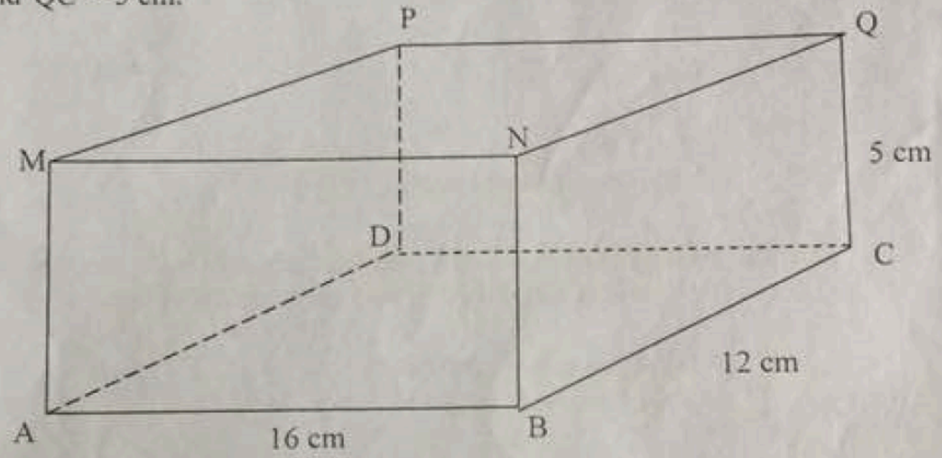
(b) Find (i) the length of arc AB

(ii) the area of the sector AOB

if θ is 57° and r is 5.4 cm $\left(\text{use } \pi = \frac{22}{7}\right)$ **(10 marks)**

2009

13. The figure below shows a rectangular prism in which $\overline{AB} = 16$ cm, $\overline{BC} = 12$cm and $\overline{QC} = 5$ cm.



Calculate

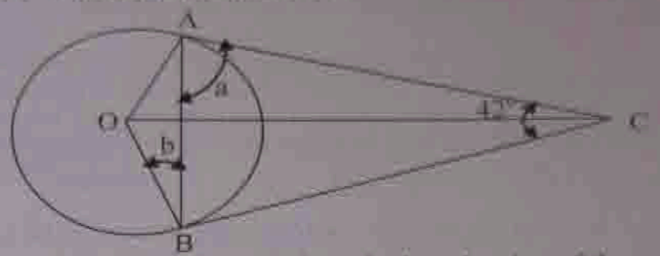
(a) its total surface area

(b) the angel between $\overline{PB}$ and the plane ABCD

(c) the volume in litres the prism can hold (1 litre = 1000 cm^3) **(10 marks)**

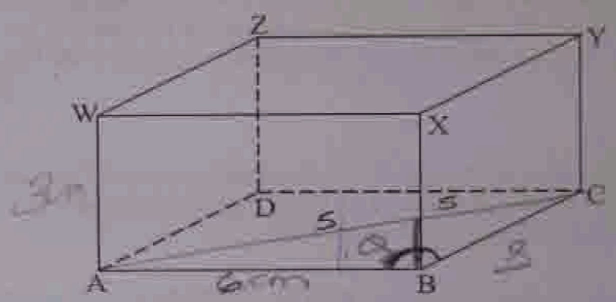
2008

13. (a) The two tangents AC and BC to the circle drawn below meet at C.



If O is the center of the circle, calculate the size of the angles marked a and b.

(b) A rectangular box with top WXYZ and base ABCD has AB = 6 cm, BC = 8 cm, and WA = 3 cm.



Calculate the

(i) length of AC

(ii) angle between WC and AC.

(c) A ship sails from port P to a distance 7 km on a bearing of 306°, and then a further 11 km on a bearing of 170° to arrive at X. Calculate the distance from P to X. **(10 marks)**

2007

13\. (a) The sides of a rectangular plot PQRS in metres are such that $\overline{PQ} = 4x + 3$, $\overline{QR} = 3x + 1$, $\overline{RS} = x + 6y$ and $\overline{PS} = 4x - y$. Find the values of x and y and hence find the area of the plot in metres. **(4 marks)**

(b) Find the area of the curved surface of a cone whose base radius is 3 cm and whose height is 4 cm. **(3 marks)**

(c) Two places P,Q both on the parallel of latitude 26°N differ in latitude by 40°. Find the distance between them along their parallel of latitude. **(3 marks)**

14. Accounts

2018

14. Mwanne commenced business on 1st April, 2015 with capital in cash 200,000/=

April 2 bought goods for cash 100,000/=
3 bought goods for cash 300,000/=
4 purchased shelves for cash 230,000/=
5 sold goods for cash 400,000/=
9 paid wages for cash 50,000/=
12 purchased goods for cash 70,000/=
13 sold goods for cash 600,000/=
16 paid rent for cash 100,000/=
20 bought goods for cash 60,000/=
25 sold goods for cash 300,000/=
27 paid salary for cash 70,000/=

Prepare the following:
(a) Cash account,
(b) Trial balance.

14. @

DR CASH ACCOUNT CR

Date	Particulars	Folio	Amount	Date	Particulars	Folio	Amount
2015				2015			
1.4	Capital		200,000	2.4	Purchases		100,000
5.4	Sales		400,000	3.4	Purchases		300,000
13 4	Sales		600,000	4.4	Shelves		230,000
25.4	Sales		300,000	9.4	Wages		50,000
				12.4	Purchases		70,000
				16.4	Rent		100,000
				20.4	Purchases		60,000
				27.4	Salaries		70,000
				30.4	Balance	c/d	520,000
			1,500,000				1,500,000
1.5	Balance	b/d	520,000				

MWANNE

(b) Trial balance as at 30th. April. 2015

S/NO	Particulars	Folio	Debit	Credit
1	Cash		520,000	
2	Capital			200,000
3	Sales			1,300,000
4	Purchases		530,000	
5	Shelves		230,000	
6	Wages		50,000	
7	Rent		100,000	
8	Salaries		70,000	
	TOTAL		1,500,000	1,500,000

2017

14. (a) What is a trial balance and what is its main purpose.

(b) On January 1st 2015 Semolina Women Group started a business with a capital in cash of 2,000,000/=

January	2	Purchased goods for cash 1,400,000/=
	3	Sold goods for cash 1,000,000/=
	6	Purchased goods for cash 600,000/=
	15	Paid rent for cash 220,000/=
	26	Paid wages for cash 220,000/=
	15	Sold goods for cash 620,000/=

Prepare:

(i) The cash account and balance it.

(ii) The Trial Balance.

14. (a) • Trial balance is a statement that shows a list of debit and credit balances of accounts extracted from various ledgers to check the arithmetical accuracy of double entry recording at any given date, usually at the end of the year

• The main purpose of Trial balance is to check the arithmetical accuracy of double entry recording at any given date also used to provide information to be used in creating final accounts and balance sheet.

b)

DR		CASH ACCOUNT L·1					CR
Date	Particulars	Folio	Amount	Date	Particulars	Folio	Amount
1·1·2015	Capital	2	2,000,000	2·1·2015	Purchases	3	1,400,000
3·1·2015	Sales	4	1,000,000	6·1·2015	Purchases	3	600,000
15·1·2015	Sales	4	620,000	15·1·2015	Rent	5	220,000
				26·1·2015	Wages	6	220,000
				30·1·2015	Balance	c/d	1,180,000
			3620,000				3,620,000
1·2·2015	Balance	b/d	1,180,000				

ii)

TRIAL BALANCE AS AT 30th Jan 2015

S/N	PARTICULARS	FOLIO	DR	CR
1.	Cash		1,180,000	
2.	Capital			2,000,000
3.	Purchases		2,000,000	
4.	Sales			1,620,000
5.	Rent		220,000	
6.	Wages		220,000	
	TOTAL		3,620,000	3,620,000

2016

14. (a) Given:

Opening stock 01 – 01 – 2012	34,430/=
Closing stock 31-12-2012	26,720/=
Net purchases during 2012	212,290/=
Expenses for the year	45,880/=

Gross Profit is 50% of cost of goods sold

Find: (i) Cost of goods sold (ii) The gross profit

(b) On 1st June , 2013 Mrs Lemisha started business with capital of 100,000/= and made the following transactions:

June	2 bought furniture	40,000/=
	7 bought goods	70,000/=
	11 sold goods	65,000/=
	16 paid Sundry expenses	30,000/=
	19 cash sales	80,000/=
	24 paid wages	50,000/=
	26 withdraw cash	30,000/=

(i) Prepare the cash account.

(ii) Prepare the balance sheet as at 30/06/2013.

(iii) Explain the importance of the balance sheet you have prepared in part (b)(ii) above.

2015

14. The following trial balance was extracted from the businessman books' of Chericho Ramaji, at 31st December 2006.

S/N	Details	Dr (Tshs.)	Cr (Tshs.)
1.	Capital		830,000
2.	Purchases	1,200,000	
3.	Sales		1,750,000
4.	Return inwards	55,000	
5.	Return outwards		64,000
6.	Plant and machine	240,000	
7.	Furniture and fittings	75,000	
8.	Sundry debtors	137,000	
9.	Sundry creditors		86,000
10.	Wages	228,000	
11.	Bad debts	36,000	
12.	Discount received		27,000
13.	Opening stock	500,000	
14.	Insurance	16,000	
15.	Commission receivable		43,000
16.	Trade expenses	22,000	
17.	Cash in hand	17,000	
18.	Cash at bank	274,000	
	Total	2,800,000	2,800,000

Prepare Trading, Profit and Loss account for the year ended 31st December 2006.

Question Number | SUBJECT NAME MATHEMATICS | INDEX NUMBER S0298/0024 | For Examiners' use only

14. DR TRADING, PROFIT AND LOSS ACCOUNT AS AT 31st DEC CR

Particulars	Amount	Particulars	Amount
Opening stock	500,000	Sales	1,750,000
Add: Purchases	1,200,000	Less: Return	
Net/total purchase	1,700,000	inwards	55,000
Less: Return		Net/total sales	1,695,000
Outwards	64,000		
Cost of goods available for sale	1,636,000		
Less: closing stock	–		
Cost of goods sold	1,636,000		
Gross profit c/d	59,000		
	1,695,000		1695,000
		Gross profit b/d	59,000
Wages	228,000	Discount received	27,000
Bad debts	36,000		
Insurance	16,000	Commission receivable	43,000
Trade expenses	22,000		
		Net loss c/d	173,000
	302,000		302,000
Net loss b/d	173,000		

2014

14. Mr. Kijembe started business on 16th March, 2011 with capital in cash 2,066,000/=

March 17 bought goods for Cash 1,000,000/=

19 bought shelves for Cash 1,100,000/=

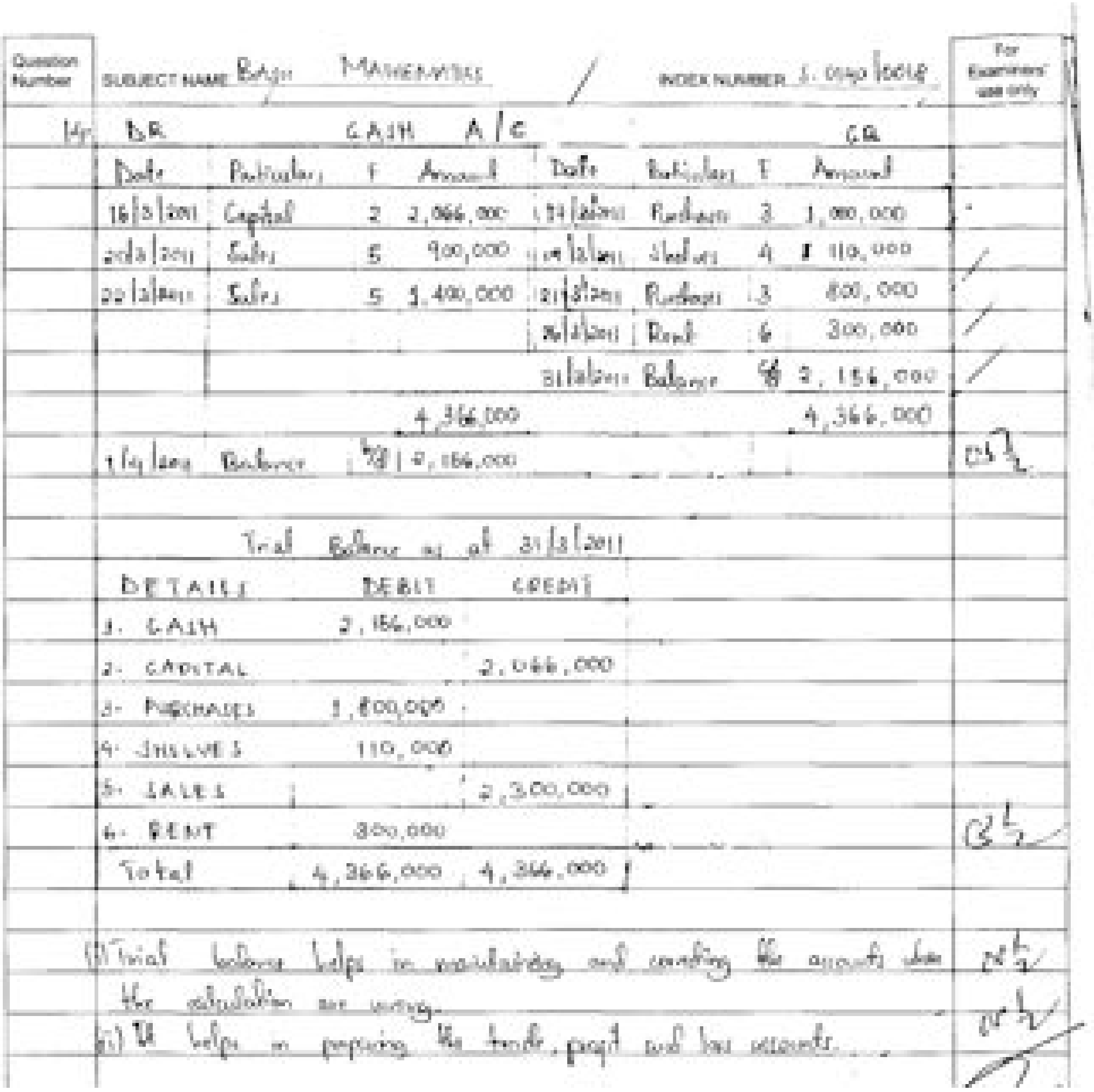
20 sold goods for Cash 900,000/=

21 purchases for Cash 800,000/=

22 sold for Cash 1,400,000/=

26 paid Rent 300,000/=

Record the above transactions in a cash account ledger and extract a trial balance. State two uses of the trial balance you have prepared.



Question Number | SUBJECT NAME Basic Mathematics | INDEX NUMBER | For Examiners' use only

14.

DR CASH A/C CR

Date	Particulars	F	Amount	Date	Particulars	F	Amount
16/3/2011	Capital	2	2,066,000	17/3/2011	Purchases	3	1,000,000
20/3/2011	Sales	5	900,000	19/3/2011	Shelves	4	110,000
22/3/2011	Sales	5	1,400,000	21/3/2011	Purchases	3	800,000
				26/3/2011	Rent	6	300,000
				31/3/2011	Balance	c/d	2,156,000
			4,366,000				4,366,000
1/4/2011	Balance	b/d	2,156,000				

Trial Balance as at 31/3/2011

DETAILS	DEBIT	CREDIT
1. CASH	2,156,000	
2. CAPITAL		2,066,000
3. PURCHASES	1,800,000	
4. SHELVES	110,000	
5. SALES		2,300,000
6. RENT	300,000	
Total	4,366,000	4,366,000

(i) Trial balance helps in maintaining and correcting the accounts when the calculation are wrong.

(ii) It helps in preparing the trade, profit and loss accounts.

2013

14. Mr. Chapakazi commenced business on 1st June, 2011 with capital in cash 5,500,000/=

June 2 Bought goods by Cash 2,000,000/=
3 Bought furniture for Cash 600,000/=
7 Sold goods for Cash 2,000,000/=
11 Cash purchases 700,000/=
13 Paid Rent for Cash 2,000,000/=
16 Cash Sales 950,000/=
20 Paid Transport for Cash 80,000/=
22 Sold goods for cash 250,000/=
26 Paid salaries for Cash 160,000=

Record the above transactions in the cash account, balance them and extract the Trial Balance.

Question Number | SUBJECT NAME MATHEMATICS | INDEX NUMBER S0110/0089 | For Examiners use only

14 DR MR. CHAPAKAZI CASH ACCOUNT AS AT 30th June 2011 CR

DATE	DETAILS	F	AMOUNT	DATE	DETAILS	F	AMOUNT
1·6	Capital	2	5,500,000	2·6	Purchases	3	2,000,000
7·6	Sales	5	2,000,000	3·6	Furniture	4	600,000
16·6	Sales	5	950,000	11·6	Purchases	3	700,000
22·6	Sales	5	250,000	13·6	Rent	6	200,000
				20·6	Transport	7	80,000
				26·6	Salaries	8	160,000
				30·6	Balance	c/d	4,960,000
			8,700,000				8,700,000
1·7	Balance	b/d	4960,000				

05½

TRIAL BALANCE AS AT 30·JUNE·2011

PARTICULARS	DEBIT	CREDIT
Cash	4,960,000	
Capital		5,500,000
Purchases	2,700,000	
Furniture	600,000	
Sales		3,200,000
Rent	200,000	
Transport	80,000	
Salaries	160,000	
	8,700,000	8,700,000

@ ½

04½

10

SA

2012

14. (a) The following balances were extracted from the ledgers of Mr. and Mrs. Mkomo business on 31st January. Prepare a trial balance.

Capital	30,000/=	Insurance	3,000/=
Furniture	25,000/=	Cash	18,000/=
Motor vehicle	45,000/=	Discount received	7,000/=
Sales	68,000/=	Discount allowed	4,000/=
Purchases	54,000/=	Drawing	12,000/=
Creditors	76,000/=	Electricity	5,000/=
Debtors	15,000/=		

(b) Determine the gross profit and the net profit from the information given below.

Sales	38,000/=
Opening stock	8,000/=
Purchases	25,000/=
Electricity	4,000/=
Discount allowed	2,000/=
Closing stock	5,000/=

14a)

MR AND MRS MKOMO

TRIAL BALANCE AS AT 31st JANUARY.

Date	Details	F	DR	CR
31, jan	capital			30,000
31, jan	Furniture		25000	
31, jan	Motor vehicle		45000	
31, jan	Sales			68000
31, jan	Purchases		54000	
31, jan	creditors			76000
31, jan	Debtors		15000	
31, jan	Insurance		3000	
31, jan	cash		18000	
31, jan	Discount received			7000
31, jan	Discount allowed		4000	
31, jan	Drawing		12000	
31 jan	Electricity		5000	
			181000	181,000

14(b) gross profit

= Sales - (opening stock + purchases - closing stock)

= 38000 - (8000 + 25000 - 5000)

= 38000 - 28000

= 10000

gross profit is 10,000/=

Net profit

= Gross profit - (electricity + discount allowed)

= 10,000 - (40,000 + 2,000)

= 10,000 - 6000

= 4,000

Net profit is 4000/=

2011

14. Study the given trial balance and answer questions that follow:

Trial Balance as of 31 December 2007

S/N	Details	Amount (Tshs)	Amount (Tshs)
1.	Cash	185,000.00	
2.	Capital		200,000.00
3.	Purchases	110,000.00	
4.	Sales		104,000.00
5.	Water bills	3,000.00	
6.	Advertising	2,000.00	
7.	Telephone bills	1,000.00	
8.	Salaries	3,000.00	
		304,000.00	304,000.00

Prepare the following for the year ending 31 December 2007:

(a) Trading account;

(b) Profit and loss account;

(c) Balance sheet.

2010

14. From 1st January to 29th January 2006 Mr. Bin decided to keep records of his business as follows:

Jan.	1	Mr. Bin started a business with capital in cash	500,000.00
	5	Purchased goods	254,000.00
	6	Sold goods	290,000.00
	9	Purchased goods	204,000.00
	10	Expenses	24,000.00
	29	Sold goods	320,000.00

You are required to:

(a) prepare the trial balance

(b) open capital and cash account.

N.B All payments and receipts were made in cash. **(10 marks)**

2009

14. The following information relates to Mr. Kazimoto, a trader, as at 30th July 2004:

Sales	shs. 340,000.00
Cost of sales	75% of sales
Opening stock	shs. 90,000.00
Net profit	20% of sales
Closing stock	20% of cost of goods sold

Calculate:

(a) Purchases

(b) Cost of sales

(c) Closing stock

(d) Net profit

(e) Expenses **(10 marks)**

2008

14. At the beginning of August 2008, Nguvumpya Secondary School started up a school project shop with a capital of Tshs. 1,800,000/=. The school project manager made the following transactions:

On August 6th she bought some stationeries for the shop worth Tshs. 180,000/=,
On August 9th she sold goods to the students worth Tshs. 270,000/=,
On August 11th she bought soft drinks for the shop from IPP Company worth Tshs. 630,000/=,
On August 13th she sold foodstuffs to teachers worth Tshs. 450,000/=,
On August 15th she sold foodstuffs to villagers worth Tshs. 360,000/=,
On August 17th she bought loaves of bread for the shop worth Tshs. 450,000/=,
On 19th paid transport charges Tshs. 50,000/= and the shop management paid wages to the shop manager Tshs. 90,000/= on August 28th.

(a) Enter these transactions in a cash book.

(b) Bring down the balance at the end of August 28th 2008.

(10 marks)

15. Matrices and Transformations

2018

15. (a) Find the point $P(x, y)$ if $\begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -23 \\ -11 \end{pmatrix}$.

(b) A translation T maps point $P(x, y)$ in part (a) into (3,2). Find where it takes the point (7,4).

(c) Find the image of the point obtained in part (b) under a rotation of 90° followed by another rotation of 180° anticlockwise.

15 a) $\begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -23 \\ -11 \end{pmatrix}$

soln.

let $\begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}$ be A. and find determinant of A.

$|A| = 2 \times -1 - 4 \times 3$

$|A| = -2 - 12$

$|A| = -14$

$A^{-1} = \dfrac{1}{|A|}\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

$A^{-1} = \dfrac{1}{-14}\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

$A^{-1} = \begin{pmatrix} -1/-14 & -3/-14 \\ -4/-14 & 2/-14 \end{pmatrix}$

$A^{-1} = \begin{bmatrix} 1/14 & 3/14 \\ 2/7 & -1/7 \end{bmatrix}$

Then premultiply by the inverse both sides of the matrix equation.

$\begin{pmatrix} 1/14 & 3/14 \\ 2/7 & -1/7 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/14 & 3/14 \\ 2/7 & -1/7 \end{pmatrix}\begin{pmatrix} -23 \\ -11 \end{pmatrix}$

but

$AA^{-1} = I$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -23/14 & -33/14 \\ -46/7 & 11/7 \end{pmatrix}$

$\begin{pmatrix} x + 0 \\ 0 + y \end{pmatrix} = \begin{pmatrix} -56/14 \\ -35/7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \end{pmatrix}$

$\therefore$ $x = -4$ and $y = -5$

$P(x, y) = P(-4, -5)$

b) $P(x, y) = P(-4, -5) \longrightarrow P'(3, 2)$

From

Translation $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$

$x_1, y_1 = 3, 2 \quad x, y = -4, -5 \quad a, b = ?$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} -4 \\ -5 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ -5 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 7 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$\therefore$ Translation vector is $(7, 7)$

Hence $x', y' = ?$ $\quad T(7, 7)$ and $(x, y) = (7, 4)$

Then

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \end{pmatrix} + \begin{pmatrix} 7 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 14 \\ 11 \end{pmatrix}$$

$\therefore$ It will take point $(7, 4)$ to $(14, 11)$

c) Point $(14, 11)$

R_{+90}

Matrix of rotation $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{pmatrix} \begin{pmatrix} 14 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 14 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -11 \\ 14 & 0 \end{pmatrix} = \begin{pmatrix} -11 \\ 14 \end{pmatrix}$$

$\therefore$ After rotation through $90°$ image is $(-11, 14)$

Then through $180°$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos 180 & -\sin 180 \\ \sin 180 & \cos 180 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -11 \\ 14 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 11 & 0 \\ 0 & -14 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 11 + 0 \\ 0 + -14 \end{pmatrix} = \begin{pmatrix} 11 \\ -14 \end{pmatrix}$$

$\therefore$ The image under R_{+90} and $R_{+180°}$ is $(11, -14)$

2017

15. (a) Find the inverse and identity matrix of $A = \begin{pmatrix} 6 & 4 \\ -2 & 5 \end{pmatrix}$.

(b) Triangle OAB has vertices at O(0,0), A(2,1) and B(-1,3). If the triangle is enlarged by $E = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and then translated by $T = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$, find the vertices of the triangle.

(c) Draw on the same x - y plane triangle OAB and the images after being:
(i) enlarged
(ii) translated

15 a, $A = \begin{bmatrix} 6 & 4 \\ -2 & 5 \end{bmatrix}$

$|A| = (6 \times 5) - (4 \times -2)$

$|A| = 30 - {}^{-}8$

$|A| = 30 + 8$

$|A| = 38$

$\frac{1}{38}\begin{bmatrix} 5 & -4 \\ 2 & 6 \end{bmatrix}$

$\therefore$ The inverse of A (A^{-1}) is $\frac{1}{38}\begin{bmatrix} 5 & -4 \\ 2 & 6 \end{bmatrix}$

Identity matrix $= A^{-1} \times A$

$\frac{1}{38}\begin{bmatrix} 5 & -4 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 6 & 4 \\ -2 & 5 \end{bmatrix}$

$\begin{bmatrix} 5/38 & -4/38 \\ 2/38 & 6/38 \end{bmatrix}\begin{bmatrix} 6 & 4 \\ -2 & 5 \end{bmatrix}$

$\begin{bmatrix} \frac{30}{38} + \frac{8}{38} & \frac{20}{38} + \frac{-20}{38} \\ 12/38 + {}^{-}12/38 & 8/38 + 30/38 \end{bmatrix}$

$\begin{bmatrix} \frac{38}{38} & 0/38 \\ 0/38 & 38/38 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore$ Identity matrix of $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

15. b) $O(0,0)$, $A(2,1)$, $B(-1,3)$

Enlargement $E = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Translation $= \begin{bmatrix} -3 \\ -5 \end{bmatrix}$

For, Enlarged $E = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$(0,0) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$(2,1) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4+0 \\ 0+2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

$(-1,3) \quad \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2+0 \\ 0+6 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$

Then translated. $(0,0)$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

For: $(4, 2)$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix} = \begin{pmatrix} 4-3 \\ 2-5 \end{pmatrix}$$

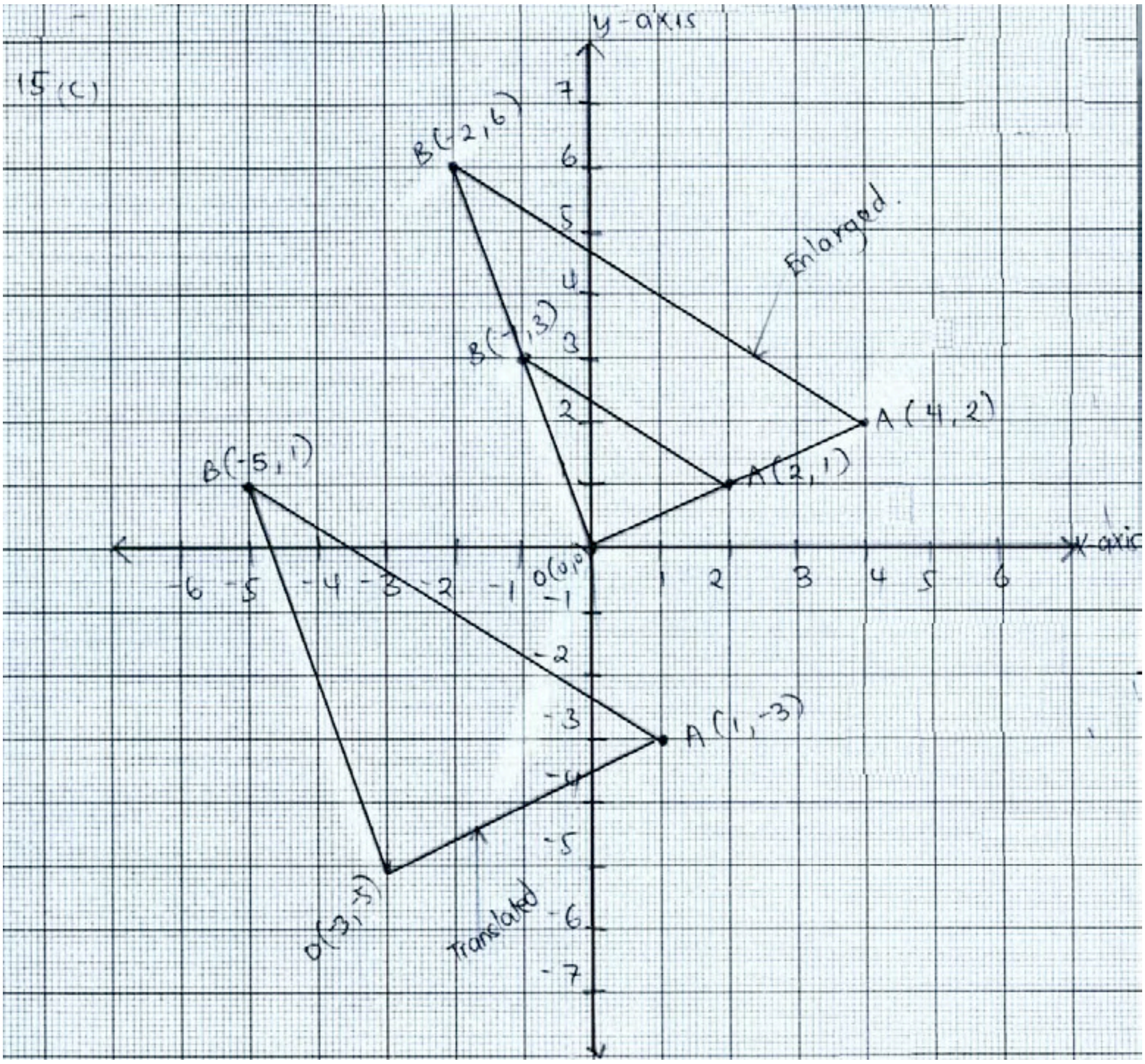
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

For; $(-2, 6)$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix} = \begin{pmatrix} -2-3 \\ 6-5 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$$

$\therefore$ The vertices of triangle will be at $O(-3,-5)$, $A(1,-3)$ and $B(-5,1)$



2016

15. (a) Given matrices $A=\begin{pmatrix}3 & 5\\ 4 & -2\end{pmatrix}$, $B=\begin{pmatrix}3 & -1\\ -3 & 2\end{pmatrix}$ and $C=\begin{pmatrix}k & -4\\ 3 & -2\end{pmatrix}$;

(i) Find A^2+2A,

(ii) Find t and y such that $B^2=tB+yI$ where I is an identity matrix.

(iii) Find the value of k if the determinant of C is 5.

(b) A linear translation Q carried point (x, y) into (x', y') such that $x'=5x-3y$ and $y'=-2x+4y$.

(i) Determine the transformation matrix Q.

(ii) Find Q(3, 3).

(iii) Find the image of the point obtained in part (b)(ii) under Q.

2015

15. (a) Given matrices $Q = \begin{pmatrix} -3 & 1 \\ 0 & 2 \end{pmatrix}$ and $P = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $QP = \begin{pmatrix} -2 & -2 \\ 2 & 2 \end{pmatrix}$, find the elements of matrix P.

(b) Determine the matrix A from the equation $\begin{pmatrix} 5 & 3 \\ 4 & 5 \end{pmatrix} - 2A = \begin{pmatrix} -2 & 1 \\ 3 & 5 \end{pmatrix}$

(c) Given a triangle with vertices $A(0,0)$, $B(3,0)$ and $C(3,1)$; find its image under:

(i) a translation by the vector $(2,3)$,

(ii) the enlargement matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

(d) Sketch the triangle and the images in parts (c)(i) and (ii) on the same pair of axes and comment on their sizes.

15. $Q = \begin{pmatrix} -3 & 1 \\ 0 & 2 \end{pmatrix}$ $P = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Given $QP = \begin{pmatrix} -2 & -2 \\ 2 & 2 \end{pmatrix}$

$QP = \begin{pmatrix} -3 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} -2 & -2 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} -3a+c & -3b+d \\ 2c & 2d \end{pmatrix}$

From $2c = 2$

$c = 1$

$2d = 2$

$d = 1$

$-3a + c = -2$

$-3a + 1 = -2$

$-3a = -2-1$

$-3a = -3$

$a = 1$

$-3b + d = -2$

$-3b + 1 = -2$

$-3b = -2-1$

$-3b = -3$

$b = 1$

Elements of matrix P; $a=1, b=1, c=1, d=1$

$P = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

15 (b) $\begin{pmatrix} 5 & 3 \\ 4 & 5 \end{pmatrix} - 2A = \begin{pmatrix} -2 & 1 \\ 3 & 5 \end{pmatrix}$

$\begin{pmatrix} 5 & 3 \\ 4 & 5 \end{pmatrix} - \begin{pmatrix} -2 & 1 \\ 3 & 5 \end{pmatrix} = 2A$

$\begin{pmatrix} 7 & 2 \\ 1 & 0 \end{pmatrix} = 2A$

$$\frac{1}{2}\begin{pmatrix} 7 & 2 \\ 1 & 0 \end{pmatrix} = \frac{2A}{2}$$

$$\begin{pmatrix} 7/2 & 1 \\ 1/2 & 0 \end{pmatrix} = A$$

$$\therefore A = \begin{pmatrix} 7/2 & 1 \\ 1/2 & 0 \end{pmatrix}$$

(c)(i) $(p', q') = (a, b) + (p, q)$

$(p', q') = (2, 3) + (0, 0)$

$p'q' = (2, 3)$

$A' = (2, 3)$

$(p', q') = (2, 3) + (3, 0)$

$B' = (5, 3)$

$(p', q') = (2, 3) + (3, 1)$

$C' = (5, 4)$

The Image is $A'(2,3)$, $B'(5,3)$, $C'(5,4)$

(ii) $\begin{pmatrix} p' \\ q' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$A' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

15 (c) (ii) $B' = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

$C' = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

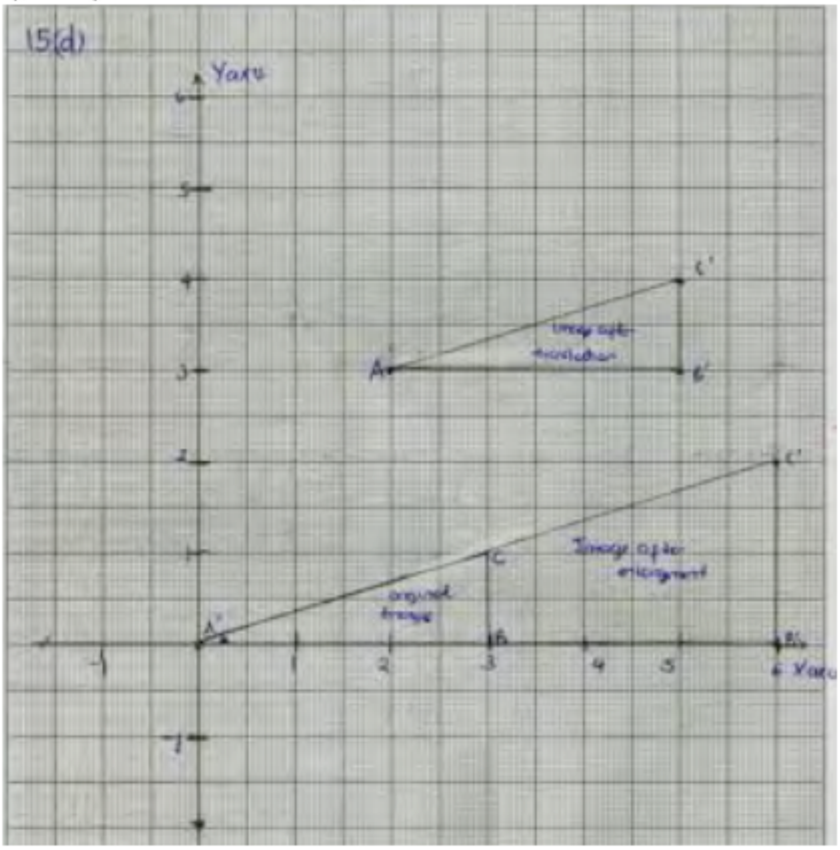
$= \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$\therefore$ The Image after enlargement $A'(0,0)$, $B'(6,0)$, $C'(6,2)$

(d) On the graph

The size of the Image after translation is the same as

	the size of original triangle
	The size of the Image after enlargement is large
	than the size of original triangle by a scale factor of 2



2014

15. (a) (i) Determine a matrix M which represents a reflection in the line $y-x=0$

(ii) Find the image of the line $x+2y-4=0$ after a reflection in the line $y-x=0$.

(b) (i) If $A=\begin{pmatrix}3 & 2\\ 4 & -1\end{pmatrix}$ find $|A|$ and A^{-1}.

(ii) Use the inverse matrix obtained in (b)(i) to solve $3x+2y=12$ and $4x-y=5$.

15 (i) Line $y-x=0$.

$y=x$.

$\theta=45$.

$$\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin\theta & -\cos 2\theta\end{pmatrix}$$

$$\begin{pmatrix}\cos 90 & \sin 90\\ \sin 90 & -\cos 90\end{pmatrix}$$

$\cos 90 = 0$

$\sin 90 = 1$.

$$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

∴ The Matrix M representing the reflection on line $y-x=0$

is $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

15 (II) line $x+2y-4=0$

$x+2y-4=0$

$2y=4-x$

15 $y=2-x/2$

x	y
0	2
4	0

$x/2 = 2 \times 2$

$x=4$

Hence we have the following points:

$(x \; y) = (0 \; 2)$

$(x \; y) = (4 \; 0)$

Then

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0+2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0+0 \\ 4+0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

Hence we have:

$(2, 0)$ and $(0, 4)$

the gradient:

$$\frac{\Delta y}{\Delta x}$$

$$= \frac{4-0}{0-2} = \frac{4}{-2} = -2$$

then:

$$\frac{4-y}{0-x} = -2$$

$$4-y = -2(-x)$$

$$4-y = 2x$$

$$4-2x = y$$

or

$$2x+y-4=0$$

∴ the image of the line is

$$\underline{2x+y-4=0}$$

(b) $A = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$

$$|A| = (3\times -1) - (4\times 2)$$

$$= -3 - 8$$

$$= -11$$

∴ $|A| = -11$

15

(b) $A^{-1} = \frac{1}{|A|}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$= -\frac{1}{11}\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}$

$= -\frac{1}{11}\begin{pmatrix} -1 & -2 \\ -4 & 3 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & \frac{-3}{11} \end{pmatrix}$

$\therefore$ the A^{-1} is $\begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & -\frac{3}{11} \end{pmatrix}$

(ii) $3x+2y=12$

$4x-y=5.$

$\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$

Multiply both side by the inverse of matrix matrix

$\begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & -\frac{3}{11} \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & -\frac{3}{11} \end{pmatrix}\begin{pmatrix} 12 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{12}{11} + \frac{10}{11} \\ \frac{48}{11} - \frac{15}{11} \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{22}{11} \\ \frac{33}{11} \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\therefore\ x = 2$ and $y = 3.$

2013

15. (a) Given $A = \begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}$ find A^{-1}.

(b) Use the results obtained in part (a) to find the point of intersection of the following system of simultaneous equations:
$2x + 6y = 22$
$4x + 8y = 32$

(c) Find the image of the point obtained in part (b) above under a rotation $90°$ anticlockwise.

15 (a) Soln.

Given $A = \begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}$

$|A| = (8 \times 2) - (6 \times 4)$.

$|A| = (16) - (24)$.

$|A| = -8$.

As from: $A^{-1} = \frac{1}{|A|}$ (Adj. Matrix).

$A^{-1} = \frac{1}{-8}\begin{pmatrix} 8 & -6 \\ -4 & 2 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} -1 & +6/8 \\ 1/2 & -1/4 \end{pmatrix}$

$\therefore \quad A^{-1} = \begin{pmatrix} -1 & 3/4 \\ 1/2 & -1/4 \end{pmatrix}$.

(b) Soln.

$2x + 6y = 22$.

$4x + 8y = 32$.

In Matrix form:

$\begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 22 \\ 32 \end{pmatrix}$

Let $A = \begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}$

As from part (a) above: $A^{-1} = \begin{pmatrix} -1 & 3/4 \\ +1/2 & -1/4 \end{pmatrix}$

15 (b). Multiply by A^{-1} throughout the eqn that is in matrix form.

$$\begin{pmatrix} -1 & 3/4 \\ 1/2 & -1/4 \end{pmatrix}\begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 3/4 \\ 1/2 & -1/4 \end{pmatrix}\begin{pmatrix} 22 \\ 32 \end{pmatrix}$$

$$\begin{pmatrix} -2+3 & -6+6 \\ 1-1 & 3-2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -22+24 \\ 11-8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$\therefore$ The point of intersection is: $(x, y) = (2, 3)$.

(c) Soln.

From: $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$.

Also: from part (b) $(x, y) = (2, 3)$.

$$R_{90^\circ} = \begin{pmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$R_{90^\circ} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$R_{90^\circ} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}.$$

$\therefore$ Image of $(2, 3)$ Under Rotation of 90° Anticlockwise $= (-3, 2)$.

2012

15. (a) Find the value of k such that the matrix $\begin{pmatrix} 2k+2 & k \\ 4k-3 & k+3 \end{pmatrix}$ is singular.

(b) The vertices of ABC are $A(1,2)$, $B(3,1)$ and $C(-2,1)$. If triangle ABC is reflected on the x-axis, find the coordinates of the vertices of its image.

(c) Solve the following simultaneous equations by matrix method.

$$\begin{cases} 2x + 3y - 2 = 0 \\ -9y + 8x - 1 = 0 \end{cases}$$

15. a) $\begin{vmatrix} 2K+2 & K \\ 4K-3 & K+3 \end{vmatrix}$

$(2K+2)(K+3) - ((4K-3)(K)) = 0$

$2K^2 + 6K + 2K + 6 - (4K^2 - 3K) = 0$

$2K^2 + 8K + 6 - 4K^2 + 3K = 0$

$2K^2 - 4K^2 + 8K + 6 + 3K = 0$

$-2K^2 + 8K + 3K + 6 = 0$

$-2K^2 + 11K + 6 = 0$

$\underline{-12K^2}$ <u>sum of factors</u>

$-K$ $12K$ $11K$

$-2K^2 - K + 12K + 6 = 0$

$(-2K^2 - K)(12K + 6) = 0$

$-K(2K+1)\ 6(2K+1) = 0$

$(6-K)(2K+1) = 0$

$6 - K = 0 \quad 2K + 1 = 0$

$K = 6 \quad 2K = -1$

$K = 6$ or $K = -\frac{1}{2}$

<u>$\therefore$ The value of $K = 6$ or $-\frac{1}{2}$.</u>

15. b) A $(1, 2)$

B $(3, 1)$

C $(-2, 1)$

$M_x\ (x, -y)$

Reflected on x-axis $(x, y) \longrightarrow (x, -y)$

15. b) $A(1,2)$
$A'(1,-2)$

$B(3,1)$
$B'(3,-1)$

$C(-2,1)$
$C'(-2,-1)$

$\therefore$ The coordinates of the vertices of its image are $A'(1,-2)$ $B'(3,-1)$ and $C'(-2,-1)$ when reflected in the x-axis.

15.(c.

$$2x + 3y = 2$$
$$-9y + 8x = 1$$

$$2x + 3y = 2$$
$$8x - 9y = 1$$

$$\begin{pmatrix} 2 & 3 \\ 8 & -9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

det.

$$\begin{vmatrix} 2 & 3 \\ 8 & -9 \end{vmatrix}$$

$$= 2 \times -9 - 8 \times 3$$
$$= -18 - 24$$
$$= -42$$

15. c)

$$\frac{1}{-42}\begin{pmatrix}-9 & -3\\ -8 & 2\end{pmatrix}$$

$$\begin{pmatrix}-9/42 & -3/-42\\ -8/42 & -2/42\end{pmatrix}$$

$$\begin{pmatrix}9/42 & 3/42\\ 8/42 & -2/42\end{pmatrix}$$

$$\begin{pmatrix}9/42 & 3/42\\ 8/42 & -2/42\end{pmatrix}\begin{pmatrix}2 & 3\\ 8 & -9\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}9/42 & 3/42\\ 8/42 & -2/42\end{pmatrix}\begin{pmatrix}2\\ 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}9/42 & 3/42\\ 8/42 & -2/42\end{pmatrix}\begin{pmatrix}2\\ 1\end{pmatrix}$$

$$\begin{matrix}x\\ y\end{matrix} = \begin{pmatrix}9/42 \times 2 + 3/42 \times 1\\ 8/42 \times 2 + -2/42 \times 1\end{pmatrix}$$

$$\begin{matrix}x\\ y\end{matrix} = \begin{pmatrix}18/42 + 3/42\\ 16/42 \quad -2/42\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}21/42\\ 14/42\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}1/2\\ 1/3\end{pmatrix}$$

$x = 1/2$ and $y = 1/3$ Ans

2011

15. (a) Reflect the point $(1,2)$ in the line $x+y=0$.

(b) Find the enlargement matrix which maps the point $(-3,4)$ into $(18,-24)$.

(c) It is given that $A=\begin{pmatrix}2 & 4\\ 3 & 1\end{pmatrix}$, $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ and k is a real number.

(i) Find the matrix $A-kI$

(ii) Show that the matrix in c(i) above has no inverse if $k^2-3k-10=0$

2010

15. (a) A transformation T has the matrix $T=\begin{bmatrix}1 & x\\ r & -2\end{bmatrix}$. Under the same transformation T, the point (-4, 1) is mapped onto the point (6, 3). Find x and r.

(b) For what values of n will the matrix $\begin{pmatrix}n-1 & n+3\\ 1 & 6n\end{pmatrix}$ be non-singular?

2009

15. (a) A translation takes the point (8, 5) to (12, -4). Find where it will take the point (5, 4).

(b) A linear transformation T maps (x, y) onto (x', y') such that

$$\begin{pmatrix}x'\\ y'\end{pmatrix}=\begin{pmatrix}2 & -4\\ -1 & 3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}+\begin{pmatrix}8\\ -4\end{pmatrix}$$

Find the image of (2, -3) under T.

(c) A point (x, y) is reflected on the line $y=x$ followed by a rotation through an angle of 180° clockwise about the origin. Find the image of (2, 3) under this double transformation.

2008

15. (a) R is the point (1, 2). It is translated onto the point S by the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

Write down

(i) the coordinates of S

(ii) the vectors which translates S onto R.

(b) The matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a single transformation.

(i) Describe fully this transformation.

(ii) Find the coordinates of the image of the point (5, 3) after this transformation.

(c) If M_2 denotes a reflection in the y-axis and R_{180} a rotation about the origin through an angle of $180°$ for any point (x, y).

(i) Find $R_{180}M_2(x, y)$ and $M_2R_{180}(x, y)$.

(ii) Is $R_{180}M_2$ commutative? Give a reason. **(10 marks)**

2007

15. (a) (i) Draw the graphs of the functions $f(x) = x^2 - 4$ and $g(x) = x + 2$ in the same coordinate system.

(ii) Shade the region enclosed by the graphs in (i) indicating the intercepts for both graphs. **(7 marks)**

(b) From the graphs in (a) write the coordinates of the points where $f(x) = g(x)$. **(2 marks)**

(c) State the domain and range of $f(x)$. **(1 mark)**

16. Functions and Probability

2018

16. (a) A bag contains 6 white shirts and 3 blue shirts. Three shirts are picked at random one after another with replacement. Determine the probability that:
 (i) All three shirts are blue in colour,
 (ii) Two shirts are white and one shirt is blue,
 (iii) One shirt is white and two shirts are blue.

 (b) The function f is defined by $f(x) = \begin{cases} -2 \text{ if } x < -1 \\ 0 \text{ if } x = -1 \\ x + 2 \text{ if } x \geq -1 \end{cases}$
 (i) Sketch the graph of f.
 (ii) Use the graph to determine the domain and range of f.

16(a) Let W – set of white shirts

B – set of blue shirts.

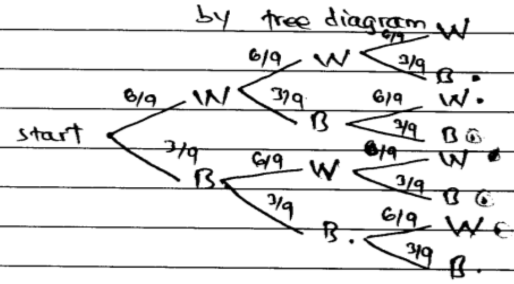
Given $n(W) = 6$

$n(B) = 3$

$n(U) = 9.$

(i) Probability that all are blue.

by tree diagram



Probability of all three having blue colour $= 3/9 \times 3/9 \times 3/9 = \frac{27}{729} = 1/27.$

$\therefore \cancel{= 27/729}$. $\therefore = 1/27.$

(ii) Two shirts are white and One blue = Event (E).

From tree diagram above.

$P(E) = (6/9 \times 6/9)(3/9) + (6/9 \times 3/9 \times 6/9) + (3/9 \times 6/9 \times 6/9)$.

$= \frac{108}{729} + \frac{108}{729} + \frac{108}{729} = \frac{108 + 108 + 108}{729}$

$= \frac{324}{729} = 4/9.$

$\therefore$ The probability is $4/9$.

(iii) Let E = One shirt is white two shirts are blue.

$P(E) = (6/9 \times 3/9 \times 3/9) + (6/9 \times 3/9 \times 3/9) + (3/9 \times 3/9 \times 6/9)$

$= \frac{54}{729} + 54/729 + 54/729 = \frac{162}{729} = 2/9$

16 (a)(ii) The probability for two blue shirts and one white is 2/9.

(b) Tables of values for.

-2 if $x < -1$.

x	-1	-2	-3	-4.
Y	-2	-2	-2	-2.

0 if $x = -1$

x	-1
y	0

$x+2$ if $x \geq -1$.

x	-1	0	1	2	3.
Y	1	2	3	4	5.

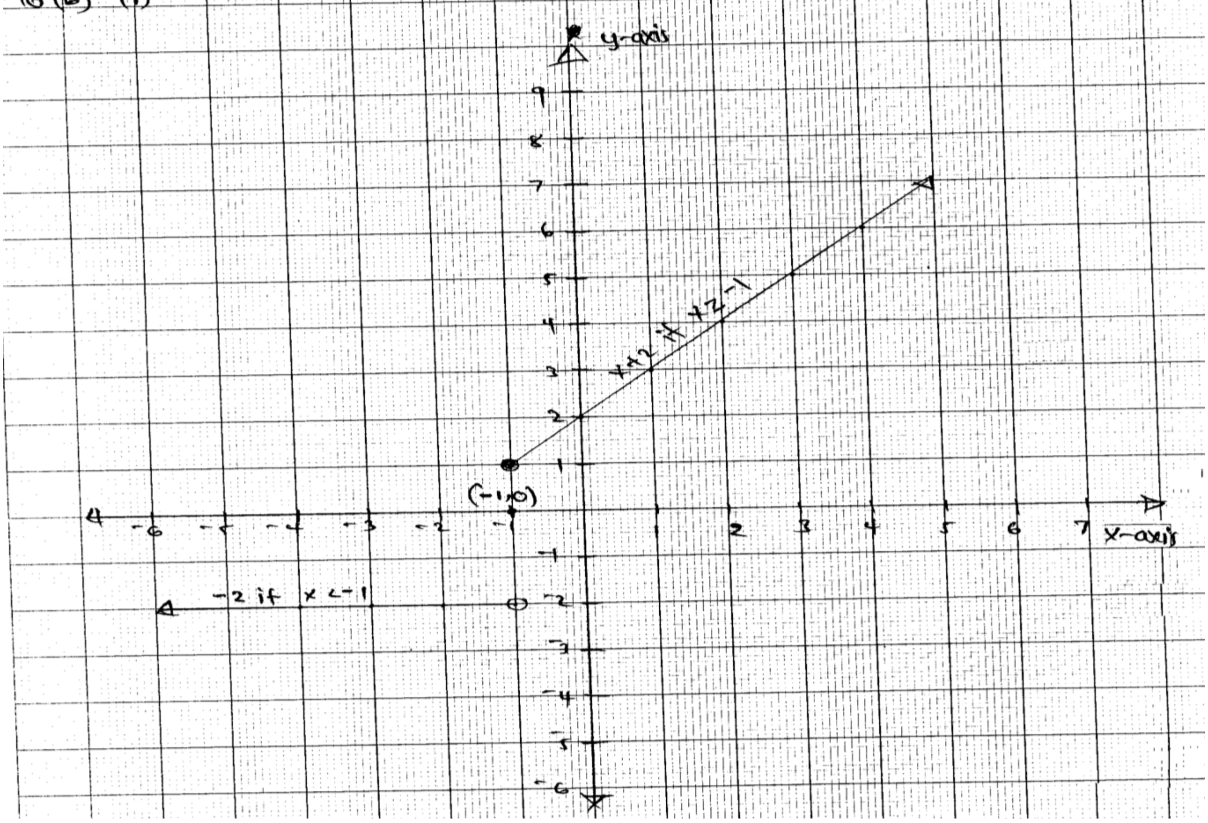
(i) Graph on graph paper.

(ii) From the graph:

Domain = $\{x : x = \mathbb{R}\}$

Range = $\{y : y = 0, y = -2\ y \geq 1\}$

16 (b) (i)



2017

16. (a) A function f is defined on the set of integers as follows:

$$f(x) = \begin{cases} 1 + x & 1 \leq x \leq 2 \\ 2x - 1 & 2 \leq x \leq 4 \\ 3x - 10 & 4 \leq x \leq 6 \end{cases}$$

(i) Draw a pictorial diagram for $f(x)$.

(ii) Find the domain and range of $f(x)$.

(b) Given that $f(x) = \frac{5x+7}{x+2}$, find $f^{-1}(4)$.

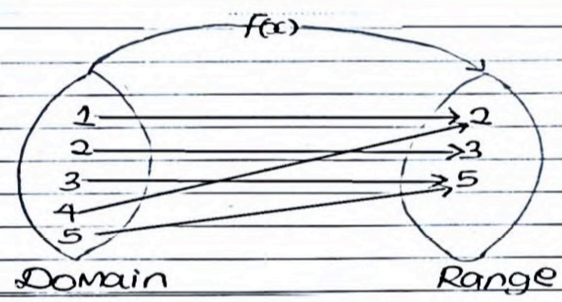
(c) In a yard there are 500 vehicles, of which 160 are cars, 130 are vans and the remaining are lorries. If every vehicle has an equal chance to leave, find the probability of:

(i) A van leaving first,

(ii) A lorry leaving first,

(iii) A car leaving second if either a lorry or van had left first.

16a)



Domain = {1, 2, 3, 4, 5}

Range = {2, 3, 5}.

b) $f(x) = \frac{5x+7}{x+2}$.

$y = \frac{5x+7}{x+2}$

$f^{-1} \rightarrow \quad x = \frac{5y+7}{y+2}$

$\frac{x}{1} \times \frac{5y+7}{y+2}$

$x(y+2) = 5y + 7$

$xy + 2x = 5y + 7$

$2x - 7 = (5y - xy)\, y$

$\frac{2x - 7}{5 - x} = \frac{(5 - x)y}{5 - x}$

$$y = \frac{2x-7}{5-x}.$$

$$f^{-1}(4) = \frac{2(4)-7}{5-4}$$

$$= \frac{8-7}{5-4}$$

$$= \frac{1}{1}.$$

$$\therefore \quad f^{-1}(4) = 1.$$

16 c) Solution:

Given:

Total number of vehicles (Sample space) = 500

Number of cars = 160

Number of vans = 130

Number of lorries = ?

$n(\psi) = 500$

$n(C) = 160$

$n(V) = 130$

$n(L) = ?$

Then; No of lorries $= 500 - (160 + 130)$

$= 500 - 290$

$= 210$.

i) Probability of a van to leave first.

$n(\psi) = 500$

$n(V) = 130$.

$P(E) = \frac{130}{500}$

$= 0.26$.

$\therefore$ The probability of van to leave first is 0.26.

ii) A lorry to leave first.

$n(\psi) = 500$

$n(L) = 210$

Probability $= \frac{210}{500} \simeq 0.42$

$\therefore$ Probability of lorry to leave first is 0.42.

16 c (iii) If the lorry or van will leave

Then $n(\psi) = 500 - 1$

$= 499$

Then; Probability of car to leave

$= \frac{160}{499} \simeq 0.32$.

$\therefore$ Probability that the car will leave after a van or lorry has left is 0.32.

2016

16. (a) The function f is defined as follows:

$$f(x) = \begin{cases} 1 & \text{if } x \leq 0 \\ x^2 + 1 & \text{if } 0 < x \leq 2 \\ 5 & \text{if } x \geq 2 \end{cases}$$

(i) Sketch the graph of $f(x)$,

(ii) Use the graph to determine the domain and range of $f(x)$.

(b) (i) Two numbers are chosen at random from 1, 2 and 3. What is the probability that their sum is an odd number if repetition is not allowed?

(ii) If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find $P(A \cup B)'$.

2015

16. (a) The function f is defined as follows:

$$f(x)=\begin{cases} x & \text{if} \quad x>2 \\ 2 & \text{if} \quad -2<x\le 2 \\ x+4 & \text{if} \quad x\le -2 \end{cases}$$

(i) Sketch the graph of $f(x)$,

(ii) Determine the domain and range of $f(x)$.

(b) Jeremia has two shirts, a white one and a blue one. He also has 3 trousers, a black, green and a yellow one. What is the probability of Jeremia putting on a white shirt and a black trouser?

(c) If a number is to be chosen at random from the integers $1,\ 2,\ 3,\ \ldots,\ 11,\ 12$; find the probability that:

(i) It is an even number,

(ii) It is divisible by 3.

(d) If in part 16(c) above, E_1 is the set of even numbers and E_2 the set of numbers that are divisible by 3, show whether E_1 and E_2 are mutually exclusive events.

16. a)

$$f(x)=\begin{cases} x & \text{if} \quad x>2 \\ 2 & \text{if} \quad -2<x\le 2 \\ x+4 & \text{if} \quad x\le -2 \end{cases}$$

$x+4=y$

x	-3	-2	-1	0
y	1	2	3	4

$x=y$

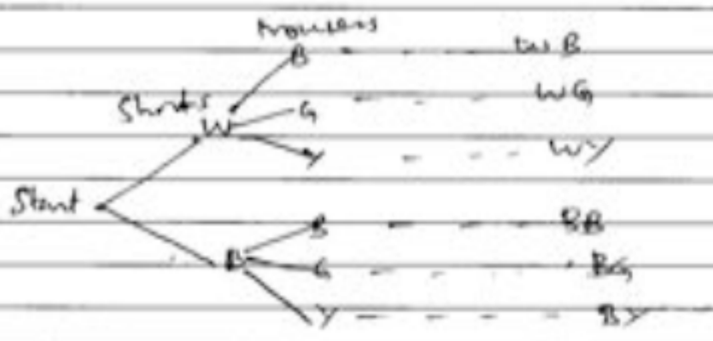
x	-2	-1	0	1	2
y	-2	-1	0	1	2

16 ii) ~~Domain = {x~~

Domain = $\{x : x \in \mathbb{R}\}$

Range = $\{y : y \in \mathbb{R}\}$

b)



$S = \{WB, WG, WY, BB, BG, BY\}$

$n(S) = 6$

$E = \{WB\}$

$n(E) = 1$

$P(E) = \frac{nE}{nS} = \frac{1}{6}$

$\therefore$ probability that Juma will put on a white shirt and a black trousers $= 1/6$.

c) $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$n(S) = 12$

i) Even number, $E = \{2, 4, 6, 8, 10, 12\}$
$n(E) = 6$

$P(E) = \frac{n(E)}{n(S)}$

$= \frac{6}{12} = \frac{1}{2}$

$\therefore$ the probability that it's an even number $= \frac{1}{2}$

16 (ii) $E = \{3, 6, 9, 12\}$
$n(E) = 4$
$P(E) = \frac{n(E)}{n(S)} = \frac{4}{12} = \frac{1}{3}$

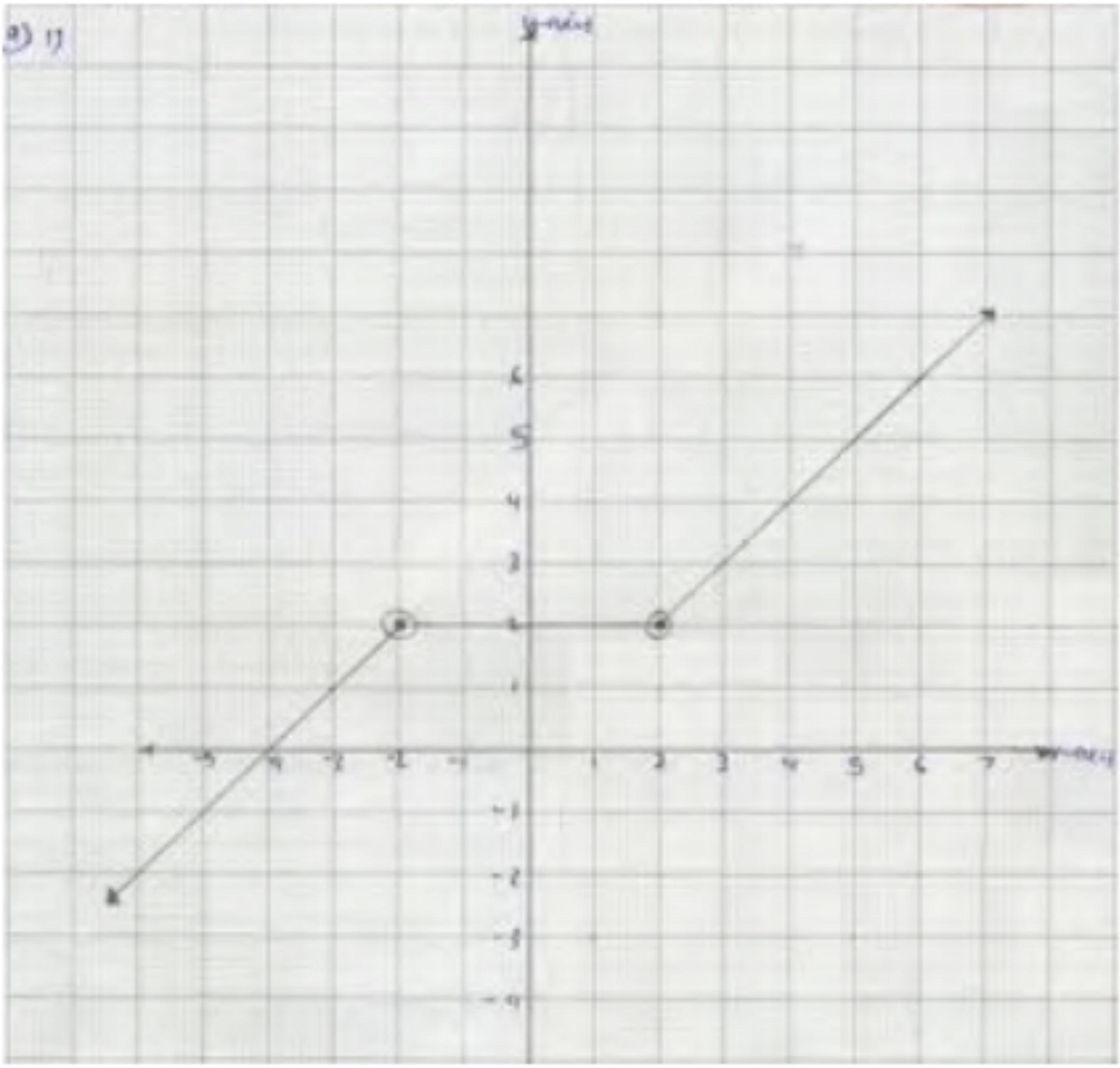
$\therefore$ the probability that the number is divisible by $3 = \frac{1}{3}$.

d) $E_1 = \{2, 4, 6, 8, 10, 12\}$
$E_2 = \{3, 6, 9, 12\}$
$n(E_1) = 6$
$n(E_2) = 4$
$n(E_1 \cap E_2) = 2$
$n(E_1 \cup E_2) = 8$

Mutually exclusive events. $n(E_1 \cap E_2) = 0$.
$n(E_1 \cup E_2) = n(E_1) + n(E_2) - n(E_1 \cap E_2)$
$8 = 6 + 4 - 2$
$8 = 10 - 2$
$8 = 8$

but mutually exclusive $n(E_1 \cap E_2) = 0$
$n(E_1 \cup E_2) = n(E_1) + n(E_2) - n(E_1 \cap E_2)$
$8 = 6 + 4 - 0$
$8 = 10$

$\therefore$ E_1 and E_2 are not mutually exclusive.



2014

16. (a) A bag contains 6 white balls and 3 yellow balls. A ball is selected at random and not replaced. Another ball is then selected. Find the probability of selecting one white ball and one yellow ball.

(b) Given $f(x) = \begin{cases} -4 & \text{when} \quad x < -1 \\ x^2 + 1 & \text{when} \quad -1 \le x \le 2 \\ 5 & \text{when} \quad x \ge 2 \end{cases}$

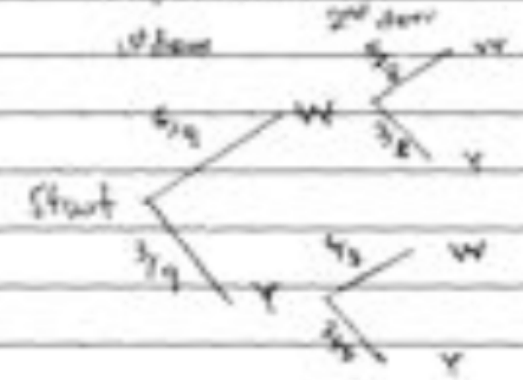
(i) Sketch the graph of $f(x)$.

(ii) State the domain and range of $f(x)$.

(iii) Is $f(x)$ a one-to-one function? Give reason(s).

16. a) 6 white balls

3 yellow balls



Probability of white and yellow is $P(W \cap Y)$.

$P(W \cap Y) + P(Y \cap W)$

$\left(\frac{2}{3} \times \frac{3}{8}\right) + \left(\frac{1}{3} \times \frac{3}{4}\right)$

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\therefore$ Probability of selecting one white ball and one yellow ball is $\frac{1}{2}$

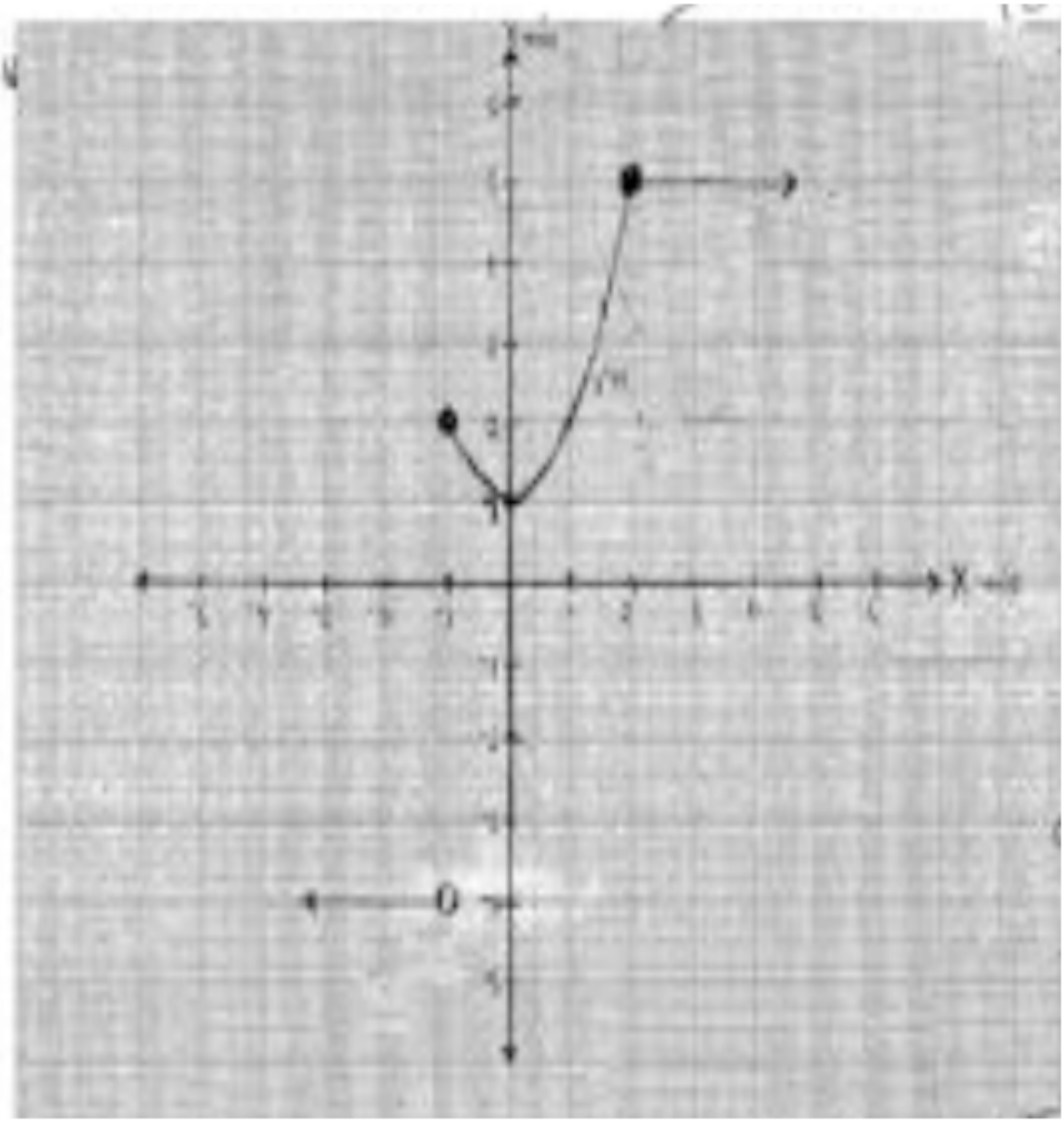
b) $f(x) = \begin{cases} -4 \text{ when } x \leq -1 \\ x^2+1 \text{ when } -1 \leq x \leq 2 \\ 5 \text{ when } x \geq 2 \end{cases}$

Table of values

x^2+1

x	-1	0	1	2
x^2+1	2	1	2	5

16 b)



2013

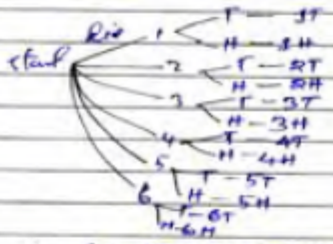
16. (a) A die and a coin are tossed together. What is the probability of getting a tail and an even number?

(b) Draw the graph of the inverse of $R = \{(x, y) : y \geq 0 \ and \ y \leq x\}$. Find its domain and range.

(c) Without using a table of values, draw the graph of $y = x^2 - 4x + 2$ and use it to solve the equation $x^2 - 4x = 5$.

16(a) Solution

By using a tree diagram



$\therefore$ S = 1T, 1H, 2T, 2H, 3T, 3H, 4T, 4H, 5T, 5H, 6T, 6H

Then $n(S) = 12$.

$n(E) = 3$

Then

$P(E) = \frac{n(E)}{n(S)} = \frac{3}{12} = \frac{1}{4}$.

$\therefore P(E) = \frac{1}{4}$.

(b) Solution

Given $R = [(x, y) : y \geq 0 \text{ and } y \leq x]$.

Hence

$R^{-1} = [(x, y) : x \geq 0 \text{ and } x \leq y]$.

Then

equations: $x = 0$, $x = y$

Table of values for $x = y$

x	-3	-2	-1	0	1	2
y	-3	-2	-1	0	1	2

Refer to the graph paper.

16(b) From the graph on the graph paper

Domain of R^{-1} = ~~{(x,y): x ≤ 0}~~ = $[(x:y): x \geq 0]$

Range of R^{-1} = $[(x,y): y \geq 0]$

(c) Solution

(i) To find the turning point of the equation

$y = x^2 - 4x + 2$

where

Turning point $(x,y) = \left[\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right]$

$= \left[\frac{-(-4)}{2\times1}, \frac{4\times1\times2-(-4)^2}{4(1)}\right]$

$= \left[\frac{4}{2}, \frac{8-16}{4}\right] = \left[2, -\frac{8}{4}\right] = (2, -2)$

∴ The turning point is $(2, -2)$

(ii) To find the y-intercept of the equation

Let, $x = 0$

$y = (0)^2 - 4(0) + 2$

$= 0 - 0 + 2 = 2$

y-intercept $= (0, 2)$.

Then

Subtract eqn (ii) from eqn (i)

$y = x^2 - 4x + 2 - (x^2 - 4x - 5)$

$= x^2 - 4x + 2 - x^2 + 4x + 5$

$\therefore y = 7$

Then, refer to the graph paper.

16(c) Therefore from the graph

the values of x are -1 and 5.

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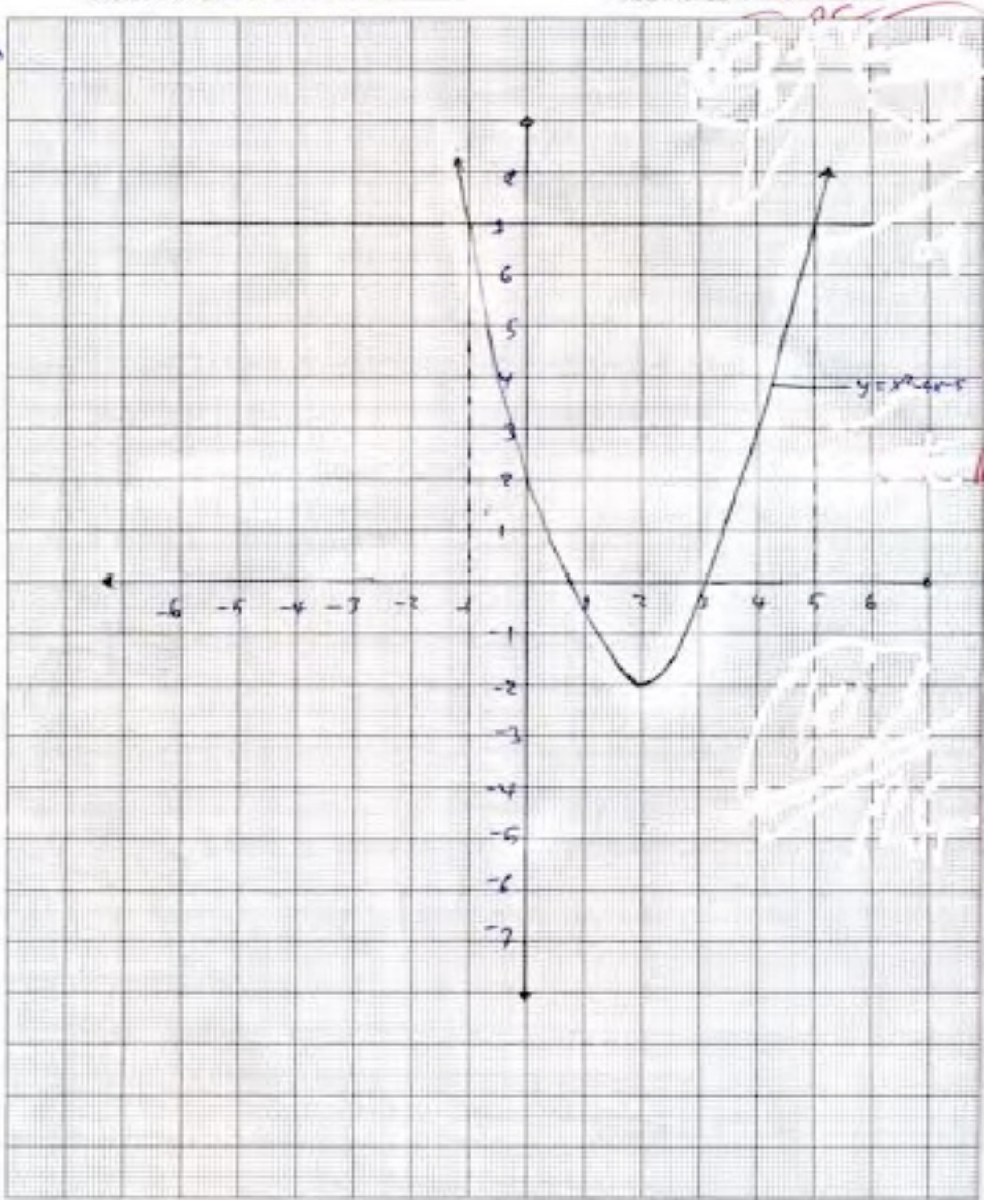
10(a)



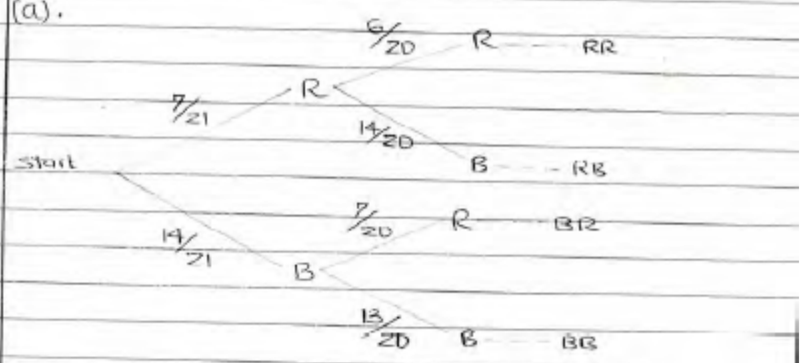
2012

16. A box contains 7 red balls and 14 black balls. Two balls are drawn at random without replacement.
   (a) Draw a tree diagram to show the results of the drawing.
   (b) Find the probability that both are black.
   (c) Find the probability that they are of the same colour.
   (d) Find the probability that the first is black and the second is red.

   (e) Verify the probability rule $P(A)+P(A')=1$ by using the results in part (b).

16. Given that $n(R) \to$ red balls $= 6+1 = 7$
$n(B) \to$ black balls $= 14$
2 Balls are drawn without replacements.

(a).



(b) Find probability that both are black
$P(B \cap B)$
$P(B) \times P(B) = \frac{14}{21} \times \frac{13}{20}$
$= \frac{2}{3} \times \frac{13}{20} = \frac{13}{30} = \frac{13}{30}$

Probability of both blacks $= \frac{13}{30}$

(c) Probability of same colour
$P(R \cap R) + P(B \cap B) = \left(\frac{7}{21} \times \frac{6}{20}\right) + \left(\frac{14}{21} \times \frac{13}{20}\right)$

$= \left(\frac{1}{3} \times \frac{3}{10}\right) + \left(\frac{2}{3} \times \frac{13}{20}\right)$

$= \frac{1}{10} + \frac{13}{30}$

$\frac{3+13}{30} = \frac{16}{30} = \frac{8}{15}$

$\therefore$ Probability of being of same colour $= \frac{8}{15}$

(d) Probability

of the first (B) and second (R)

$P(BR) = P(B) \times P(R) = \frac{14}{21} \times \frac{7}{20}$

$= \frac{2}{3} \times \frac{7}{20} = \frac{7}{30}$

Probability of black and red $= \frac{7}{30}$

(e) $P(A) + P(A') = 1$

Hence we have

$P(B \cap B) = \frac{13}{30}$

Probability $P(B \cap B)' = P(RR) + P(RB) + P(BR)$

$= \frac{1}{10} + \frac{7}{30} + \frac{7}{30}$

$= \frac{3+7+7}{30} = \frac{17}{30}$

$P(B \cap B)' = \frac{17}{30}$

let $B \cap B$ be A

and $(B \cap B)'$ be A'

Hence $P(B \cap B) = P(A) = \frac{13}{30}$

$P(B \cap B)' = P(A)' = \frac{17}{30}$

$P(A) + P(A)' = \frac{13}{30} + \frac{17}{30} = \frac{30}{30}$

$\therefore$ $\underline{P(A) + P(A)' = 1.}$

2011

16. Draw a graph of the function $y = x^2 - 3x + 2$ for the values of x from -2 to 5.
From your graph, find:
(a) The range of the function;
(b) The minimum value of y and the value of x at which this minimum value occurs;
(c) The solution of the equation $x^2 - 3x - 4 = 0$
(d) The solution of the inequality $x^2 - 3x + 2 > 0$

2010

16. (a) If $f(x) = -2x + 3$ find $f^{-1}(3)$

(b) Draw the graph of $f(x) = |x-1|$ for $-4 \le x \le 4$

(c) State the domain and range of $f(x) = |x-1|$

(d) The probability that Rose and Juma will be selected for A – level studies after completing their O – level studies are 0.4 and 0.7 respectively. Calculate the probability that:
(i) both of them will be selected.
(ii) either Rose or Juma will be selected.

2009

16. (a) If $f(x) = x^2 - 4x + 3$
Find (i) $f^{-1}(x)$
(ii) the domain and range of $f(x)$

(b) If the probability that Ali will pass Mathematics is 0.3 and the probability that he will pass Biology is 0.6, find the probability that:
(i) He will pass both subjects.
(ii) He will fail both subjects.

(c) If A is the event 'Ali will pass Mathematics' and B is the event 'Ali will pass Biology' show whether or not A and B are independent events . [use the information given in part (b) above]

2007

16\. (a) Box A has 10 light bulbs of which 4 are defective and box B has 6 light bulbs of which 1 is defective, if a box is selected at random and then a bulb is randomly drawn, calculate the probability that the bulb drawn is defective. **(3 marks)**

(b) A pair of fair dice is thrown. Find the probability that the sum is 10 or greater if a 5 appears on the first die. **(3 marks)**

(c) Given that $P(A') = \frac{1}{6}$, $P(B') = \frac{3}{5}$ and $P(A \cap B) = \frac{1}{4}$.

Find $P(A \cup B)$. **(4 marks)**