Basic Applied Maths ACSEE

Past Paper Questions and Answers by Topic

Contents

Form V
1.0 Calculating Devices

2021 PAST PAPERS

1. Use a non-programmable scientific calculator to:

(a) compute $\dfrac{\sqrt{\sqrt{19e^2}\ \ln 3}}{\sqrt{2}}$ correct to 5 significant figures.

(b) evaluate $\int_0^1 \sqrt{1-x^2}\,dx$ correct to 5 significant figures.

(c) find the mean and standard deviation of the following data correct to 4 decimal places.

Values	110	130	150	170	190
Frequency	10	31	24	2	2

(c) Mean = 136.9565

Standard deviation = 17.3041

Extract 1.1: A sample of correct response to part (c) of question 1

2020 PAST PAPERS

1. Use a non-programmable scientific calculator to compute:

5.5987
13.9

(a) the value of $\dfrac{3+3(\sqrt[3]{0.65})}{3-3(\sqrt[3]{0.65})}$ correct to 4 significant figures.

(b) the mean and standard deviation of 33, 28, 26, 35 and 38 correct to 2 decimal places.

(c) the value of $\dfrac{{}^5C_2+{}^9P_6}{11!}$ correct to 4 decimal places. — 1.51544×10^{-3}

$= 0.00151$

There are no answers for this question

2019 PAST PAPERS

1. Use a non-programmable calculator to:

(a) Compute the value of $\dfrac{\sin^{-1}(2/3)}{7.4(\ln\sqrt[3]{87})\div 2817\log 6289}$ correct to 4 decimal places.

(b) Evaluate $\int_0^1 (3x-2)^5 dx$.

(c) Solve the equation $x^2+6x-8=0$ correct to 3 decimal places.

(d) Find $2h(4)+t(4.5)$ correct to 4 decimal places, given that $h(x)=\dfrac{\sqrt{x+4}+(3+e^x)}{x+\sqrt[3]{x}}$

and $t(x)=\sqrt{\dfrac{(x-3)^{1/3}+(x+1)^6}{1+x}}$.

1.	(a)
	$40613.6009°$
	(b) $\int_0^1 (3x-\cancel{2})^5\,dx = -3.5$
	(c) $x = 1.123$ or -7.123.
	(d) $2h(4) + t(4.5) = 92.5736$.

Extract 1.1: *A sample of the candidate's correct responses in question 1*

2018 PAST PAPERS

1. Use a non-programmable calculator to:

(a) Compute the value of $\sqrt[3]{\dfrac{\log 122\times\ln 315}{e^{0.9}+\cos^{-1}0.5487}}$ to 6 significant figures.

(b) Find mean and standard deviation of the following data correct to 4 decimal places:

Length (cm)	110	130	150	170	190
Frequency	12	35	24	5	3

(c) Find the determinant of the matrix A, where $A=\begin{pmatrix}-1 & 3 & 1\\ 2 & 4 & 0\\ 0 & 5 & -3\end{pmatrix}$.

(d) Solve the quadratic equation $t^2-5t+3.31414=0$ giving the answer into 3 decimal places.

1	a) 1.51529
	b) mean is 137.8481
	standard deviation is 18.9383
	c) The determinant of matrix A, $\lvert A \rvert = 40$
	d) The values of t are:
	$t = 4.213$ and $t = 0.787$

Extract 1.1 shows a solution of a candidate who was able to use a non-programmable calculator correctly in performing the computations.

2017 PAST PAPERS

1. (a) Use a scientific calculator to find the values of each of the following expressions,

(i) $\dfrac{458.4^3 \times 0.00274 - 7560 \div 3567^3}{458.4^3 \times 0.00274 + 9681 \div 1516^2}$,

(ii) $\dfrac{547}{250}\left[\sum_{i=1}^{3} i(i+3)(i+4)\right]^{\frac{1}{2}}$.

(b) (i) Find $\log y$, if $y = \dfrac{-\sqrt[3]{3.14}}{\sin 45^\circ - \log 7}$ correct to six decimal places.

(ii) Determine the value of q if $2.37q^3 + 0.625e^{\pi} = 314$.

1a) i) $\underline{0.999999984}$

ii) $\underline{31.40370781}$

b i) $\Rightarrow y = \dfrac{-\sqrt[3]{3.14}}{\sin 45^\circ - \log 7}$

$\Rightarrow \log(10.61186312)$

$\Rightarrow \therefore \log y = \underline{1.025792}$

ii) $2.37q^3 + 0.625e^{\pi} = 314$

$0.625e^{\pi} - 314 = -2.37q^3$

$2.37q^3 = 314 - 0.625e^{\pi}$

$q = \sqrt[3]{\dfrac{314 - 0.625e^{\pi}}{2.37}}$

$\therefore q = \underline{5.018425}$

In Extract 1.1, the candidate was able to use a scientific calculator correctly in performing the computations.

2016 PAST PAPERS

1. (a) By using a scientific calculator evaluate,

(i) $\log_{0.75} 7.5 - \ln(5\sqrt{3})$ correct to five significant figures.

(ii) $\left(\begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & -1 \\ 1 & 3 & -2 \end{vmatrix}\right)^{2.356}$ correct to four decimal places.

1st find the determinant of matrix put into the power

(b) The following data are the weight of 37 members in a National Boxing Club.

62	78	40	70	58	65	54	69	71	67	74	64
65	59	68	70	66	80	54	62	83	77	51	72
79	66	83	63	67	61	71	64	59	76	67	58
64											

With the aid of a scientific calculator,

(i) Compute the mean weight

(ii) Find the variance

(iii) Calculate the standard deviation of the data.

1. (a) (i) $\log_{0.75} 7.5 - \ln(5\sqrt{3}) =$

$\frac{\log 7.5}{\log 0.75} - \ln 5\sqrt{3} = -9.1627.$

(ii) $\left(\begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & -1 \\ 1 & 3 & -2 \end{vmatrix}\right)^{2.356} = 906.6132.$

By using a calculator.

(b) (i) Mean weight = 66.4054 units

(ii) Variance = 80.9438 units

(iii) Standard deviation (σ) = 8.9969. units.

The responses in Extract 1.1 justify that the candidate used the correct functional keys of a calculator to compute the answers as required.

2015 PAST PAPERS

1. Evaluate the following expressions with the help of a calculator (write your answers correct to 2 decimal places).

(a) $\cos^{-1}\left(\frac{2}{3}\right)+\sin^{-1}\left(\frac{3}{4}\right)$.

(b) $\sqrt[3]{8\sin 25^\circ \cos 55^\circ}$.

(c) $\log_8 17-\ln\left(\frac{5}{12}\right)$.

(d) $T(t)=280+920e^{-0.9108t}$ at $t=10$ given that $e\cong 2.72$.

(e) The number of ways for 20 people to be seated on a bench if only 5 seats are available.

(f) The value of the function $f(x)=\left(1+\frac{1}{x}\right)^x$ when $x=10, 100, 1000, 10{,}000$ and hence comment on the value of $f(x)$ when x gets very large.

1. a) $\cos^{-1}\left(\frac{2}{3}\right)+\sin^{-1}\left(\frac{3}{4}\right) = 96.78^\circ$

b) $\sqrt[3]{8\sin 25 \cos 55} = 1.247 \simeq 1.25$

c) $\log_8 17 - \ln\left(\frac{5}{12}\right)$

$= \frac{\log 17}{\log 8} - \ln 5 + \ln 12$

$= 2.238$

$\log_8 17 - \ln\left(\frac{5}{12}\right) = 2.24$

1d. $T(t) = 280 + 920e^{-0.9108t}$ $\quad t = 10$, $e \cong 2.72$

$T(10) = 280.01$

e) $^{20}P_5 = \frac{20!}{(20-5)!} = \frac{20\times19\times18\times17\times16\times15!}{15!}$

$= 1860480$ ways

f) $f(x) = \left(1+\frac{1}{x}\right)^x$

$f(10) = 2.59$

$f(100) = 2.70$

$f(1000) = 2.72$

$f(10000) = 2.72$.

when x gets very large. $f(x)$ become $\simeq 2.72$

Extract 1.1 is a sample answer from one of the candidates who was able to use a scientific calculator correctly in working out the required answers in all the items of this question.

2.0 Functions

2021 PAST PAPERS

2. (a) Given that $g(x) = -3x+5$; Find the range of $g(x)$ for the domain $\{x : -2 \leq x \leq 3\}$.

(b) Find the turning point of $h(x) = \frac{1}{2}x^2 - x$.

(c) Find the inverse of $f(x) = 2x^2 - 5$.

(b) $h(x) = \frac{1}{2}x^2 - x$

Value of x:

from $x = -b/2a$

$x = \frac{-(-1)}{2(\frac{1}{2})}$

Qn 2 (b) $x = \frac{1}{1}$

$x = 1$.

Value of y:

from

$y = \frac{4ac - b^2}{4a}$

$y = \frac{(4 \times \frac{1}{2} \times 0) - (-1)^2}{4(\frac{1}{2})}$

$y = \frac{0 - 1}{2}$

$y = -\frac{1}{2}$

Hence turning point $(x, y) = (1, -\frac{1}{2})$

Extract 2.1: A sample of correct response to part (b) of question 2

2020 PAST PAPERS

2. (a) The function f is defined as $f(x)=\begin{cases}2x-1 & \text{if } -2<x\le 1\\ x^2 & \text{if } 1<x\le 2\\ 10-3x & \text{if } 2<x<3\end{cases}$

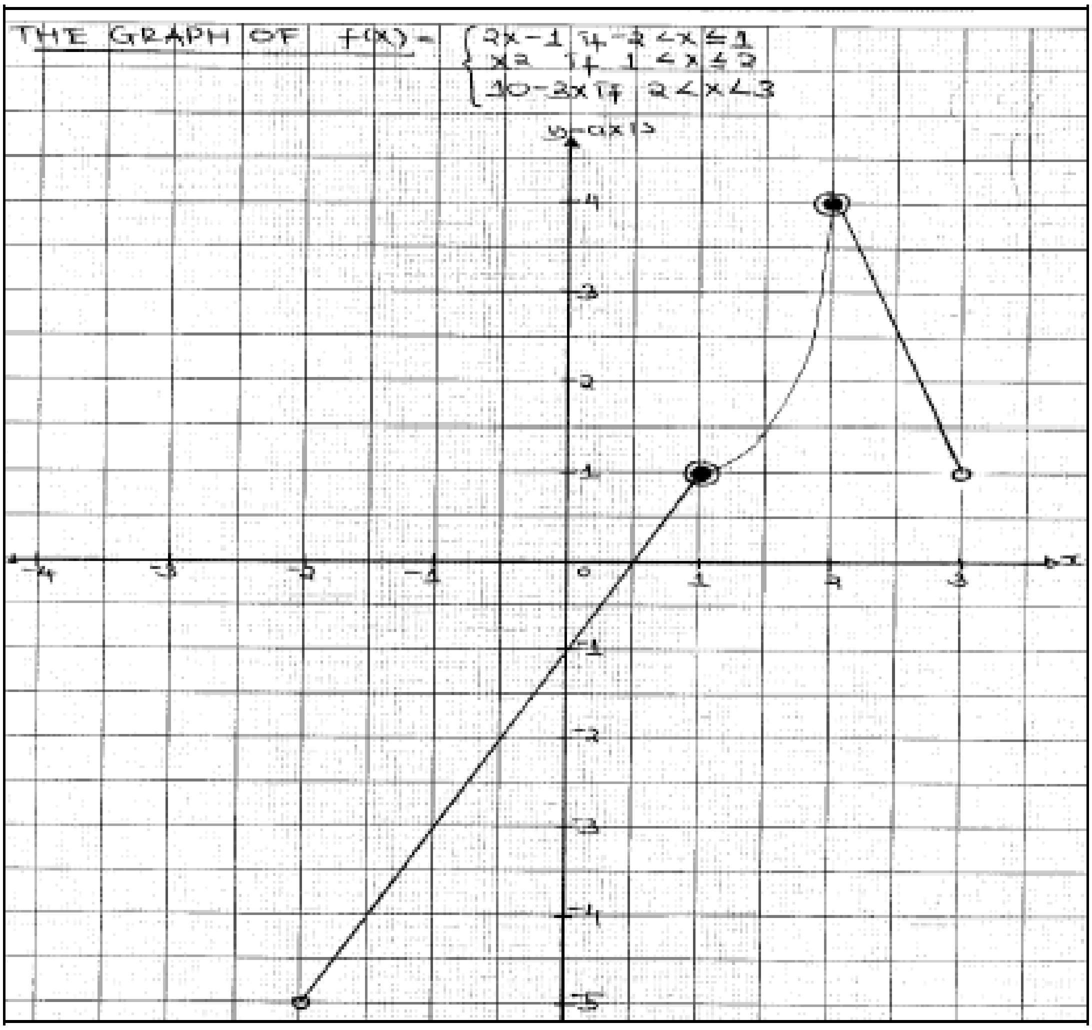
(i) Sketch the graph of $f(x)$.

(ii) State the domain and range of $f(x)$.

(b) Given that $f(x)=3x+3$ and $g(x)=x+3$, find:

(i) $(f\circ g)(x)$.

(ii) $(f\circ g)^{-1}(x)$.



Extract 2.1: *A sample of correct solution for part (a) of question 2.*

2019 PAST PAPERS

2. (a) The function is defined by $f(x) = \begin{cases} x^2+1 & \text{for } x>1 \\ |x| & \text{for } -2<x\le 1 \\ x+2 & \text{for } x\le -2 \end{cases}$

(i) Sketch the graph of $f(x)$.

(ii) Determine the domain and range $f(x)$.

(iii) Find the value of $f(-3)$, $f(0.5)$ and $f(2)$.

(b) If $f(x) = x + \frac{1}{x}$, show that $[f(x)]^3 = f(x^3) + 3f(x)$.

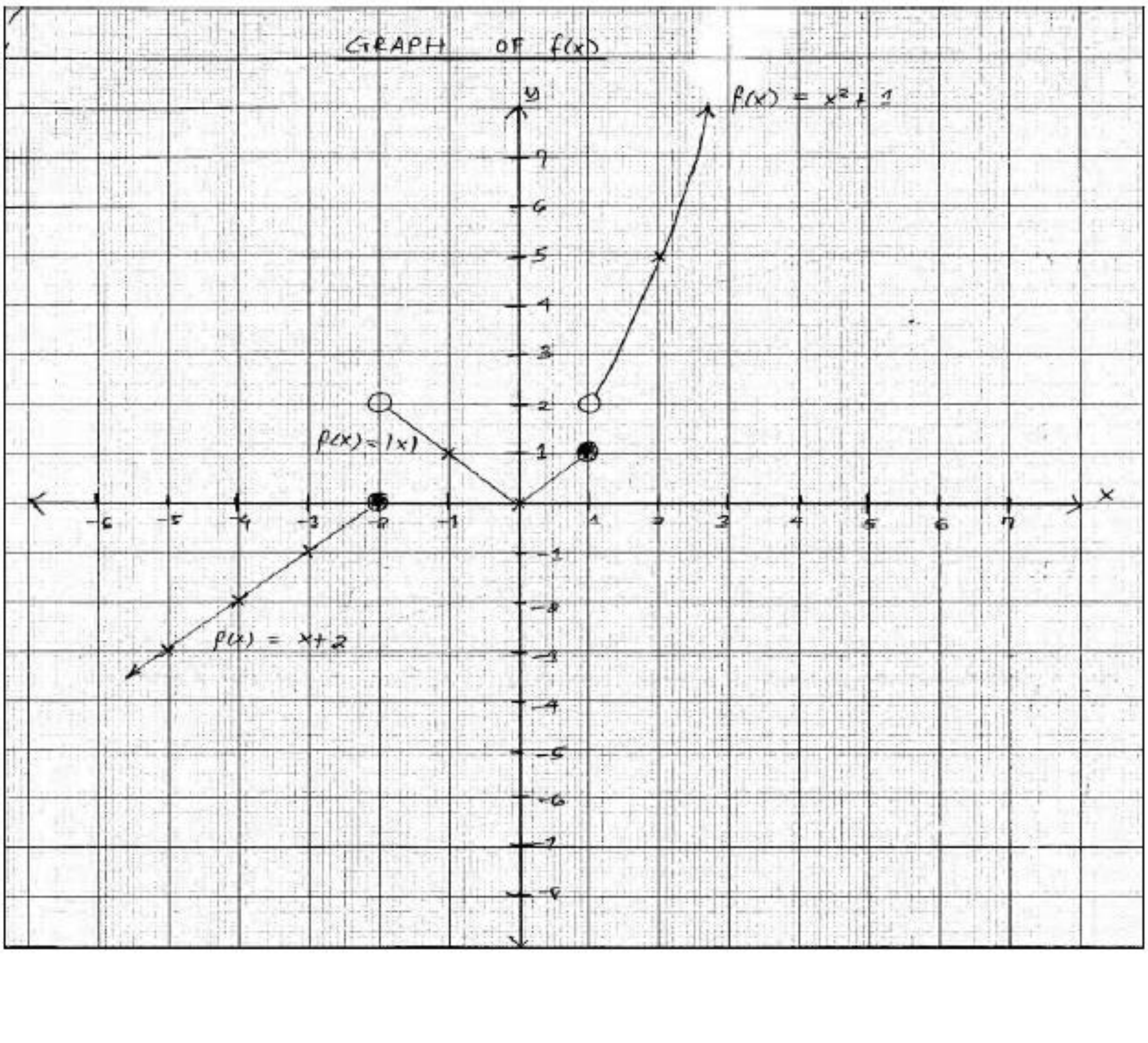
2. a/ Table of values

i/ For $f(x) = x^2 + 1$

x	1	2	3	4
x^2+1	2	5	10	17

ii/ Domain $= \{x : x \in R\}$

Range $= \{y : y \in R,\ y \neq 2\}$



2. a) iii) $f(-3)$ is obtained in third function,

$f(x) = x+2$

$f(-3) = -3+2$

$\therefore f(-3) = -1$

$f(0.5)$ is obtained in second function.

$f(x) = |x|$

$f(0.5) = |0.5|$

$\therefore f(0.5) = 0.5$

$f(2)$ is obtained in first function.

$f(x) = x^2+1$

$f(2) = 2^2+1$

$\therefore f(2) = 5$

b) Given, $f(x) = x + \frac{1}{x}$

Then, cubing both sides,

$[f(x)]^3 = [x + \frac{1}{x}]^3$

$= x^3 + 3x^2(\frac{1}{x}) + 3x(\frac{1}{x})^2 + \frac{1}{x^3}$

$[f(x)]^3 = x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$ --- (i)

But,

$f(x) = x + \frac{1}{x}$

$f(x^3) = x^3 + \frac{1}{x^3}$ ---- (ii)

substituting (ii) into (i),

$[f(x)]^3 = (x^3 + \frac{1}{x^3}) + 3[x + \frac{1}{x}]$

$= f(x^3) + 3[x + \frac{1}{x}]$

but $x + \frac{1}{x} = f(x)$,

$\therefore [f(x)]^3 = f(x^3) + 3f(x)$

shown!

Extract 2.1: *A sample of the candidate's correct responses in question 2*

2018 PAST PAPERS

2. (a) A step function f is defined on the set of real numbers such that $f(x)=\begin{cases}12x+5 \text{ if } x>1\\ x-4 \text{ if } x\le 1\end{cases}$. Find; $f(-\frac{1}{8})$, $f(2)$ and $f(-3)$.

(b) Sketch the graph of $f(x)=\dfrac{1}{2-x}$ and hence state its domain and range.

(c) The line that passes through point $A(-4,6)$ has a slope of -1. Draw the graph of this line in the interval $-4 \le x \le 4$.

2(a)	$f(x) = \begin{cases} 12x+5 & \text{if } x > 1 \\ x-4 & \text{if } x \leq 1. \end{cases}$
	$f(-1/8) = x-4$
	$f(-1/8) = -1/8 - 4$
	$\underline{f(-1/8) = -33/8}$
	$f(2) = 12x+5$
	$f(2) = 12 \times 2 + 5$
	$f(2) = 24+5$
	$\underline{f(2) = 29}$
	$f(-3) = x-4.$
	$f(-3) = -3-4$
	$\underline{f(-3) = -7}$
	$\therefore \quad f(-1/8) = -33/8, \; f(2) = 29, \; f(-3) = -7$

2(b)	Let $f(x) = y$
	$y = \dfrac{1}{2-x}.$
	For x-intercept, $y = 0.$
	$0 = \dfrac{1}{2-x}.$
	$(2-x)0 = 1.$
	No x-intercept
	For y-intercept, $x = 0$
	$y = \dfrac{1}{2-0}$
	$y = 1/2.$
	For V.A, $2-x=0$
	VA, $2 = x.$
	V.A, $x = 2.$

2(b) Domain = $\{x : x \neq 2\}$

Range = $\{y : y \neq 0\}$

2(c) $A(-4, 6)$.

$m = -1$

Take (x, y)

slope, $m = \frac{\Delta y}{\Delta x}$

$m = \frac{y - 6}{x - -4}$

$\frac{y - 6}{x + 4} = -1$

$y - 6 = -1(x + 4)$

$y - 6 = -x - 4$.

$y = -x - 4 + 6$

$y = -x + 2$

Table of values

$y = -x + 2$

x	-4	-3	-2	-1	0	1	2	3	4
y	6	5	4	3	2	1	0	-1	-2

For H.A, make x, the subject

$y(2 - x) = 1$

$2y - xy = 1$

$2y - 1 = xy$

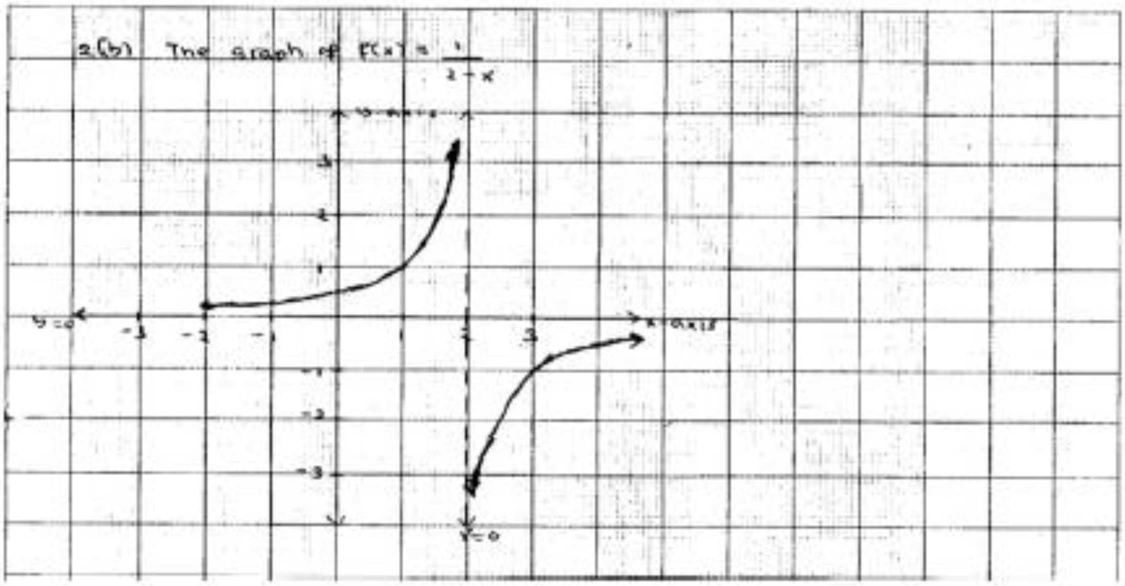
$xy = 2y - 1$

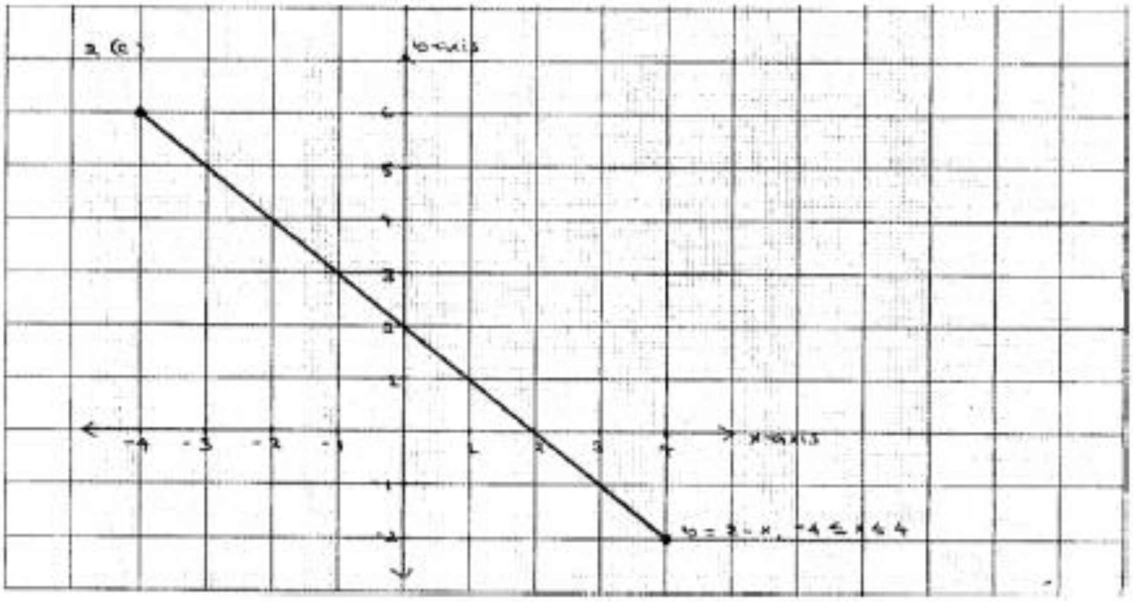
$x = \frac{2y - 1}{y}$

Interchanging variables

$x = \frac{2y - 1}{y}$

H.A, $y = 0$





Extract 2.1 shows a solution of a candidate who had an adequate knowledge on the tested concepts of functions and was able to apply it correctly.

2017 PAST PAPERS

2. (a) Given that $f(x) = 3x - 1$ and $g(x) = \sqrt{2x-1}$. Find,

(i) $f \circ g(25)$.

(ii) $g \circ f(14)$.

(b) (i) Verify that $x + 4$ is not a factor of the polynomial function $f(x) = x^3 - 9x^2 + 10x - 24$.

(ii) Describe the nature of the stationary points of the function $f(x) = 2x^3 - 15x^2 + 24x$, hence show them on the graph.

2(a) $fog(x) = f(g(x)) = 3(\sqrt{2x-1}) - 1$

Thus

$fog(25) = 3(\sqrt{50-1}) - 1$

$= 3\sqrt{49} - 1$

$= 20$

$\therefore fog(25) = 20$

(ii) $gof(14)$

From

$gof(x) = g(f(x))$

$gof(x) = g(f(x)) = \sqrt{2(3x-1) - 1}$

Then

$g \circ f(14) = \sqrt{2(3 \times 14 - 1) - 1}$

$= \sqrt{81}$

$= 9$

$\therefore$ The value of $g \circ f(14) = 9$

2(b) (i) Given

$f(x) = x^3 - 9x^2 + 10x - 24$

Required: To verify that $x+4$ is not a factor of $f(x)$

Then

for $x+4$ to be a factor of $f(x)$, the reminder

2b) should be equal to zero

let $x + 4 = 0$

$x = -4$

Then the reminder, $R(x)$ is calculated by substituting the value of $x = -4$ in $f(x)$

$R(x)$ = Reminder of $f(x)$

$\Rightarrow$ Reminder of $f(x) = (-4)^3 - 9(-4)^2 + 10(-4) - 24$

Reminder $= -64 - 144 - 40 - 24$

Reminder $= -272$

Since the reminder is not equal to zero

$\therefore$ $x + 4$ is not a factor of $f(x)$

hence verified

2b) (ii) Given;

$f(x) = 2x^3 - 15x^2 + 24x$.

For stationary point

Firstly, find $f'(x)$

$f'(x) = 6x^2 - 30x + 24$

let $f'(x) = 0$ -- for stationary points

$0 = 6x^2 - 30x + 24$

on simplifying the equation above.

$x^2 - 5x + 4 = 0$

2b(i) $x^2 - x - 4x + 4 = 0$

$x(x-1) - 4(x-1) = 0$

$(x-4)(x-1) = 0$

$x - 4 = 0$ or $x - 1 = 0$

$x = 4$ or $x = 1$

when $x = 4$, $y = ?$.

$y = f(4) = 2(4)^2 - 15(4)^2 + 24(4)$

$y = -16$

when $x = 1$

$y = f(1) = 2(1)^3 - 15(1)^2 + 24(1)$

$= 11$

$y = 11$

$\therefore (x, y) = (4, -16)$ or $(1, 11)$

For maximum and minimum points

$f''(x) = 12x - 30$

For maximum point

$f''(x) < 0$

$f''(1) = 12(1) - 30 = -18$

$f''(1) < 0$

$\therefore$ The ~~point~~ maximum point is $(1, 11)$

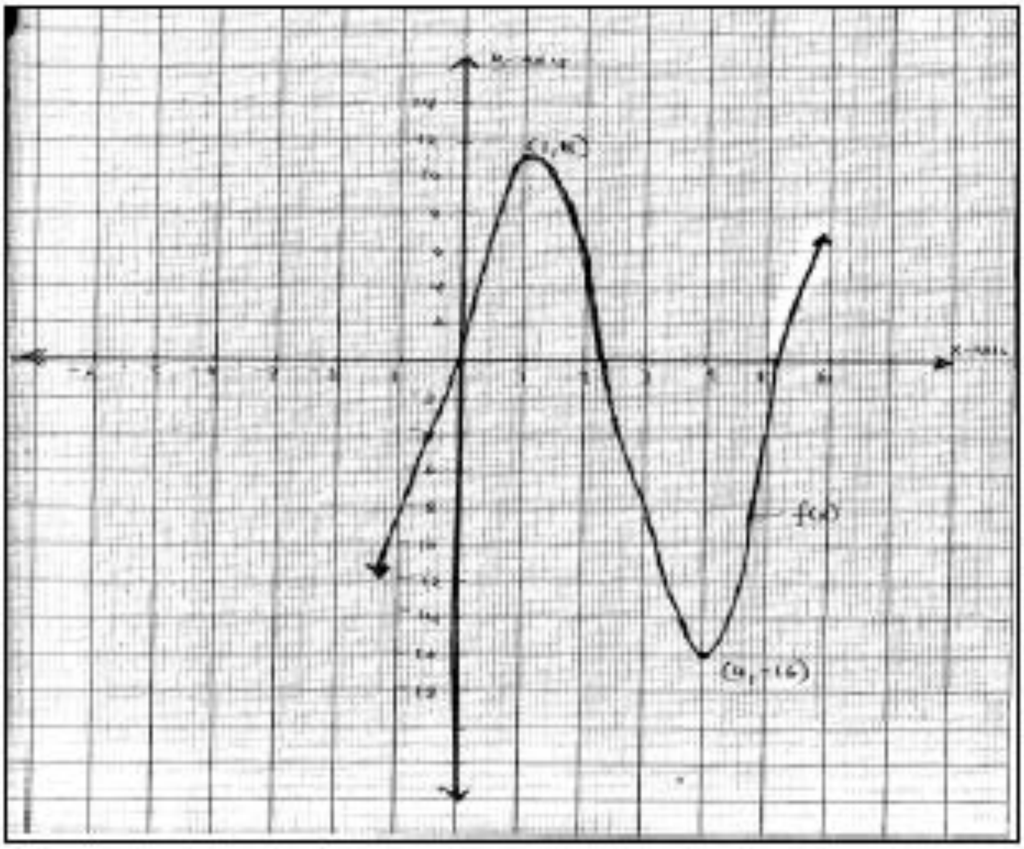
also for minimum point

$f''(x) > 0$

for $f''(4) = 12(4) - 30 = 18$

2b)(ii)

$\therefore$ The minimum stationary point is $(4, -16)$



Extract 2.2 shows that, the candidate answered the question correctly.

2016 PAST PAPERS

2. (a) A function is defined by the equation $f(x) = mx^2 + nx + k$. If $f(2) = 7$, $f(0) = -3$ and $f(-1) = 2$,

(i) Determine the values of m, n and k.

(ii) Find the domain and range of $f(x)$.

(b) (i) Sketch the graph of the rational function $g(x) = \frac{1}{4x-8}$.

(ii) What are the values of x and y for which $g(x)$ is defined?

Qn 2.

(a).i Solution.

from.

$f(x) = mx^2 + nx + K$.

given:

$f(2) = 7$.

thus:

$f(2) = m(2)^2 + n(2) + K = 7$.

$4m + 2n + k = 7$ --- --- eqn ①.

Also.

$f(0) = -3$.

Hence.

$f(0) = m(0)^2 + n(0) + K = -3$.

$K = -3$ - - -- eqn (2)

2 $f(-1) = m(-1)^2 + n(-1) + K = 2$

$m + -n + K = 2$ --- eqn (3).

from eqn 2 $K = -3$.

Hence.

$m - n + -3 = 2$.

$m - n = 5$ --- ④.

Also.

$4m + 2n - 3 = 7$.

$4m + 2n = 10$ --- eqn 5

Solve eqn 4 and 5 simultaneously

from $m = 5 + n$.

$4(5+n) + 2n = 10$.

$20 + 4n + 2n = 10$.

$20 + 6n = 10$.

$6n = 10 - 20$.

$6n = -10$

$n = -10/6$.

$n = -5/3$

Substitute the value of n on eqn 4

$m - \left(-5/3\right) = 5$.

$m = 5 - 5/3$

$m = 10/3$.

The values of $m = \underline{10/3}$.

$n = \underline{-5/3}$

$k = -3$

(ii) Solution.
from the equation.

$f(x) = \frac{10}{3}x^2 - \frac{5}{3}x - 3$.

Domain = $\{x : x \in R\}$.

let $f(x) = y$

$y = \frac{10}{3}x^2 - \frac{5}{3}x - 3$.

$\frac{dy}{dx} = \frac{20}{3}x - \frac{5}{3}$

at minimum point $\frac{dy}{dx} = 0$.

$\frac{20x}{3} - \frac{5}{3} = 0$.

$\frac{20}{3}x = \frac{5}{3}$.

$x = \frac{5}{20}$. $x = \frac{1}{4}$.

Thus $f(\frac{1}{4}) = \frac{10}{3}(\frac{1}{4})^2 - \frac{5}{3}(\frac{1}{4}) - 3$

$y = -\frac{77}{24}$.

Thus since point $y = -\frac{77}{24}$ is the minimum value.
Hence.

Range = $\{y : y \geq -\frac{77}{24}\}$.

Domain = $\{x : x \in R\}$.

In Extract 2.1(a), the candidate was able to formulate three equations from the given information and then solved them by substitution method to get the required values of m, n and k. The candidate was also able to state the domain and apply the concepts of differentiation to find the range of the function as required.

2(b)(i). Solution.

$$f(x) = \frac{1}{4x-8}$$

For vertical asymptote.

Equate the denominator with zero.

$$4x - 8 = 0.$$
$$4x = 8.$$
$$x = \frac{8}{4}$$
$$x = 2.$$

Vertical asymptote $x = 2$.

For Horizontal asymptote.

$$y = \frac{1}{4x-8}.$$

$$y = \frac{\frac{1}{x}}{\frac{4x}{x} - \frac{8}{x}}$$

$$y = \frac{\frac{1}{x}}{4 - \frac{8}{x}}$$

As $\frac{1}{x} \to 0.$

$$y = \frac{0}{4-0}$$

$$y = 0.$$

The Horizontal asymptote $y = 0$.

Y-intercept when $x = 0$:

$$y = \frac{1}{4(0) - 8}$$

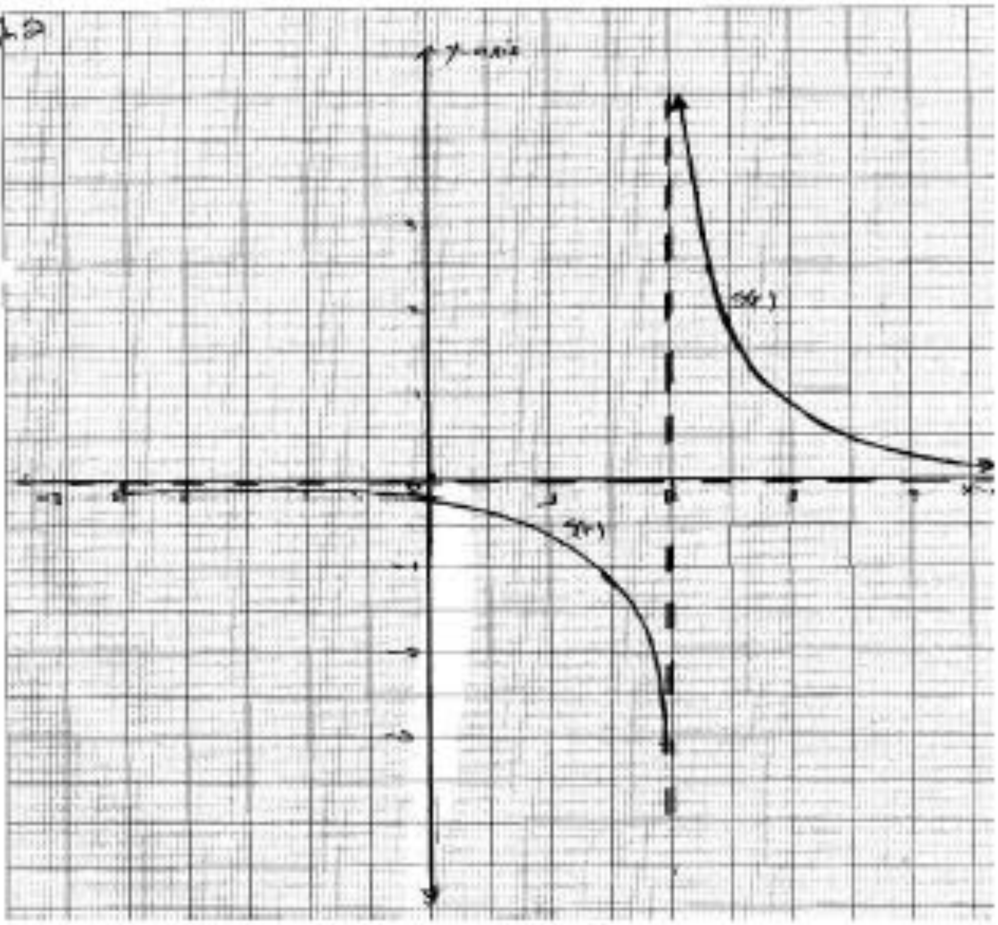
$$y = \frac{-1}{8}$$

x-intercept when $y = 0$:

no x-intercept.

The sketching is on the graph paper.

Q2



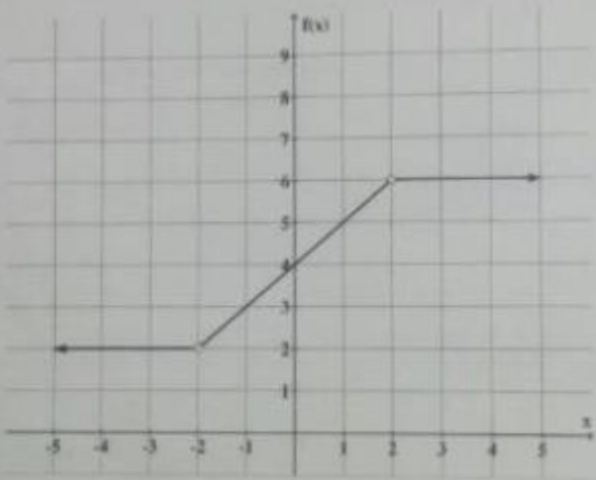
2(b)
(ii) Values of x and y when g(x) is define.
for values of (x) [Domain].
$= \{x : x \neq 2\}$
All values of x except 2.
for values of y [Range].
$= \{y : y \neq 0\}$
All values of y except $y = 0$.

In Extract 2.1(b), the candidate was able to sketch the graph of the rational function and state the values of x and y for which the function is defined.

2015 PAST PAPERS

2. (a) Find the coordinates of the points where the line $y - 2x + 5 = 0$ meets the curve $3x^2 - 4y^2 = 10 + xy$.

(b) The graph of a function $f(x)$ is given below.



Use the graph to determine:
(i) The function $f(x)$.
(ii) The domain and range of $f(x)$.

(c) Find the asymptotes and the intercepts of the function $f(x) = \dfrac{3x-7}{x+2}$ and then sketch its graph.

Qn 2 (a) $\begin{cases} 3x^2 - 4y^2 = 10 + xy & \text{— (i)} \\ y - 2x + 5 = 0. & \text{— (ii)} \end{cases}$

from equation (ii)

$y = 2x - 5$ — (iii)

Subsituting (iii) into (i).

$3x^2 - 4(2x-5)^2 = 10 + x(2x-5)$.

$3x^2 - 4(4x^2 - 20x + 25) = 10 + 2x^2 - 5x$.

$3x^2 - 16x^2 + 80x - 100 = 10 + 2x^2 - 5x$.

$3x^2 - 16x^2 - 2x^2 + 80x + 5x - 100 - 10 = 0$.

$-15x^2 + 85x - 110 = 0$.

$x = 2$ or $x = 11/3$.

∴ But for $y = 2x - 5$

$y = 2(2) - 5$ or $y = 2(11/3) - 5$

$y = -1$. $y = 7/3$

$(x, y) = (2, -1)$ or $(11/3, 7/3)$.

∴ Coordinates of point where lines meet are at $(2, -1)$ and $(11/3, 7/3)$.

In Extract 2.1 (a), the candidate had good algebraic skills that enabled him/her to obtain the correct points of intersection of the line and the curve.

2 (b) (i) $f(x) = \begin{cases} 2 & \text{if } x < -2. \\ x + 4 & \text{if } -2 < x < 2 \\ 6 & \text{if } x > 2 \end{cases}$

(ii) Domain is a set of all real values except -2 and 2.

Range is a set of all real values provided $2 \leq y \leq 6$

(c) Vertical asymptote.

$$x+2 = 0.$$
$$x = -2$$

$\therefore$ Vertical asymptote $= -2$

Horizontal asymptote

$$y = \frac{\frac{3x}{x} - 7/x}{\frac{x}{x} + 2/x.} \quad \lim x \to 0.$$

2c. $$y = \frac{3 - 7/x}{1 + 2/x} \quad \lim x \to 0.$$

Substituting $x = 0$.

$$y = \frac{3-0}{1+0}$$

$$y = 3$$

$\therefore$ Horizontal asymptote is 3.

; y-intercept ; $x = 0$.

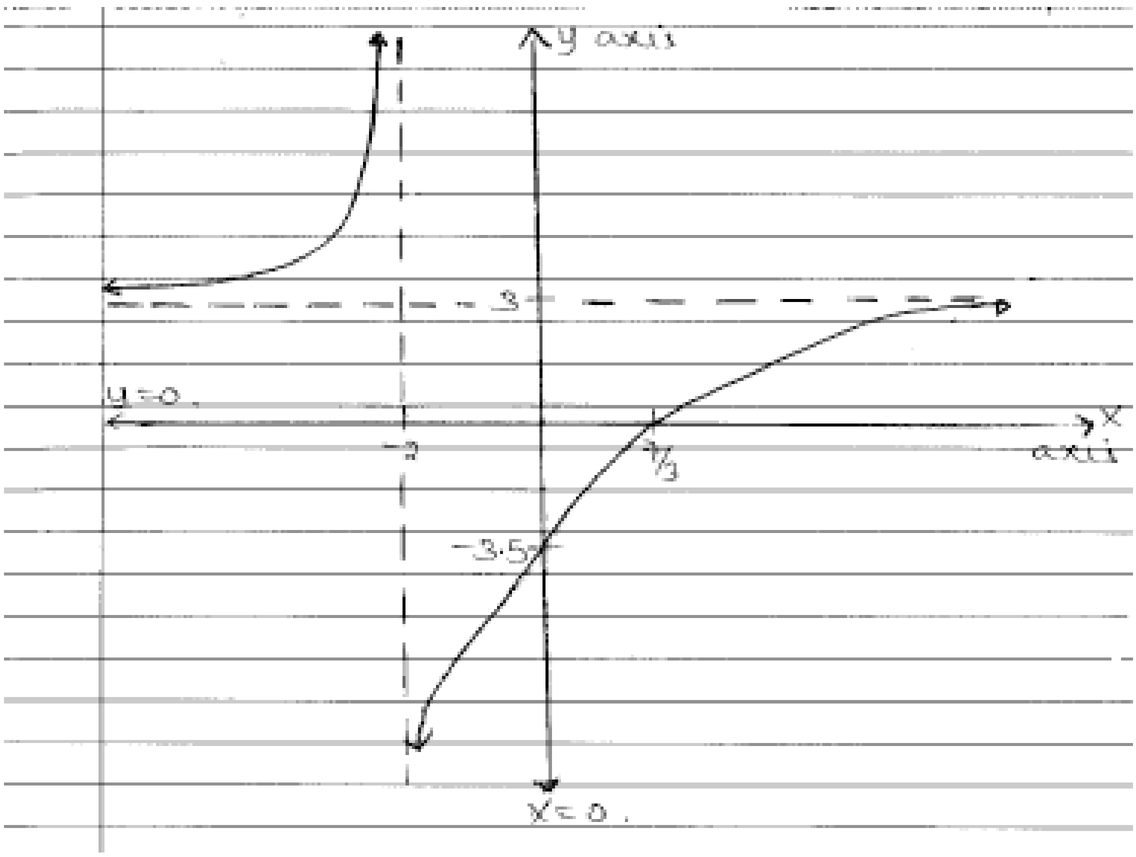
$$y = \frac{3(0) - 7}{0 + 2}$$
$$= \frac{0-7}{2.}$$
$$y = -7/2.$$

x-intercept ; $y = 0$.

$$y = \frac{3x - 7}{x+2}$$

$$0 = \frac{3x-7}{x+2}$$

$$0 = 3x - 7$$
$$7 = 3x.$$
$$x = 7/3.$$



Extract 2.2(a) shows that the candidate had sufficient knowledge that enabled him/her to identify the function and to state the domain and the range of the function correctly. The candidate also applied knowledge of intercepts and asymptotes correctly in sketching the graph of the rational function.

3.0 Algebra

2021 PAST PAPERS

3\. (a) In a sequence, the sum of the third and fourth terms is -12, the sum of the third and fifth terms is 60 and the sum of the fourth and fifth terms is 24.

(i) Show that the terms form geometrical sequence.

(ii) Find the sum of the first ten terms of the sequence.

(b) If the equation $(2k+3)x^2+2(k+3)x+(k+5)=0$ has equal roots, find the numerical value(s) of x.

3\. (b) given:

$$(2k+3)x^2+(2k+6)x+(k+5)=0 \quad \text{—(i)}$$

$a = 2k+3$, $b = 2k+6$, $c = k+5$

For equal roots:

$$b^2 = 4ac$$

$$(2k+6)(2k+6) = 4(2k+3)(k+5)$$

$$4k^2+24k+36 = 4(2k^2+13k+15).$$

$$4k^2+24k+36 = 8k^2+52k+60$$

$$k^2+7k+6=0$$

$$k^2+1k+6k+6=0$$

$$k(k+1)+6(k+1)=0$$

$$(k+1)(k+6)=0$$

Either:

$k+1=0$ or $k+6=0$

$k=-1$ $\qquad k=-6.$

Returning to equation (i)

EITHER: $k=-1$

$$x^2+4x+4=0$$

$$x^2+2x+2x+4=0$$

$$x(x+2)+2(x+2)=0$$

$$(x+2)(x+2)=0$$

Either: $x+2=0$ or $x+2=0$

$x=-2$ $\qquad x=-2.$

$\Rightarrow x_1=-2.$

3. (b) OR: $k = -6$

$$\left(-9x^2 - 6x - 1 = 0\right) \times -1$$

$$9x^2 + 6x + 1 = 0$$

$a = 9$, $b = 6$ and $c = 1$

From:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{(6)^2 - (4 \times 9 \times 1)}}{2(9)}$$

$$= \frac{-6 \pm \sqrt{0}}{18}$$

$$= -6/18$$

$$\Rightarrow x_2 = -1/3$$

$\therefore$ values of x are -2 and $-1/3$.

Extract 3.2: A sample of correct response to part (b) of question 3

2020 PAST PAPERS

3. (a) Use the substitution method to solve the following system of equations:

$$\begin{cases} 3x - y = 9 \\ x^2 + xy + 2 = 0 \end{cases}.$$

(b) Find the value(s) of x satisfying the equation $4^x - 6(2^x) - 16 = 0$.

(c) Find the sum of the first n terms of the series $1+3+5+\ldots$

3c.	Soln:
	$1+3+5+\ldots$
	$d = 3-1 = 5-3$
	$= 2.$
	$A_1 = 1.$
	$S_n = [2A_1 + d(n-1)]\frac{n}{2}.$
	$= [(2\times1) + 2(n-1)]\frac{n}{2}.$
	$= \frac{(2+2n-2)n}{2}$
	$= n^2$

Extract 3.1: *A sample of correct solution for part (c) of question 3.*

2019 PAST PAPERS

3. (a) The difference of two numbers is 1 and the difference of their squares is 7. Find the two numbers.

(b) If the first term of a GP exceeds the second term by 4 and the sum of the second and third terms is $2\frac{2}{3}$, find the first three terms of the GP.

3.	a) Let the numbers be x and y
	Then,
	The difference is 1, $x - y = 1$ ---- (i)
	Also,
	The difference of their squares is 7
	$x^2 - y^2 = 7$ --- (ii)
	solving (i) and (ii)
	(i) $x - y = 1$
	(ii) $x^2 - y^2 = 7$, by difference of two squares,
	From (ii) $(x+y)(x-y) = 7$
	$(x+y)\ 1 = 7$
	$x + y = 7$ -- (iii)
	solving (i) and (iii)
	(i) $x - y = 1$
	(iii) $x + y = 7$
	$2y = 6$
	$y = 3$
	substitute into (i), $x = y + 1$
	$x = 4$
	∴ The two numbers are 4 and 3
	b) Given; Let first term $= G_1$
	second term $= G_2$
	Then, third term $= G_3$
	$G_1 - G_2 = 4$ --- (i)
	$G_2 + G_3 = 2\frac{2}{3}$ --- (ii)

3\. b) (i) $G_1 - G_1 r = 4$

(ii) $G_1 r + G_1 r^2 = 2\frac{2}{3}$

Dividing (ii) by (i)

$$\frac{(ii)}{(i)} = \frac{G_1(r + r^2)}{G_1(1 - r)} = \frac{2\frac{2}{3}}{4}$$

$$4r + 4r^2 = 2\frac{2}{3} - 2\frac{2}{3}r$$

$$4r^2 + 6\frac{2}{3}r - 2\frac{2}{3} = 0$$

solving quadratically,

$r = \frac{1}{3}$ or -2

Then,

First term,

From (i) $G_1(1 - r) = 4$

$$G_1 = \frac{4}{1 - r}$$

when $r = \frac{1}{3}$, $G_1 = 6$

$r = -2$, $G_1 = \frac{4}{3}$

Second term

$G_2 = G_1 r$

when $r = \frac{1}{3}$, $G_2 = 2$

$r = -2$, $G_2 = -\frac{8}{3}$

Third term,

$G_3 = G_2 r$

when $r = \frac{1}{3}$, $G_3 = \frac{2}{3}$

$r = -2$, $G_3 = \frac{16}{3}$

∴ First term, ~~st~~ second term and third terms are $(6, 2$ and $\frac{2}{3})$ or $(\frac{4}{3}, -\frac{8}{3}$ and $\frac{16}{3})$

Extract 3.1: *A sample of the candidate's correct responses in question 3*

2018 PAST PAPERS

3. (a) Solve the simultaneous equations $\begin{cases} x^2 - 2y = 7 \\ x + y = 4 \end{cases}$ by substitution method.

(b) Calculate the sum of the series $\sum_{r=3}^{5} (-1)^{r+1} r^{-1}$.

(c) The second and fifth terms of an A.P are $x - y$ and $x + y$ respectively, find the third term.

3. a) Given the equations $x^2 - 2y = 7$ — (i)

$x + y = 4$ — (ii)

from eqn (ii) $x + y = 4$

$y = 4 - x$ — (iii)

Substitute equation (iii) into eqn (i)

$x^2 - 2y = 7$

$x^2 - 2(4 - x) = 7$

$x^2 - 8 + 2x = 7$

$x^2 + 2x - 8 - 7 = 0$

$x^2 + 2x - 15 = 0$

$x^2 - 3x + 5x - 15 = 0$

$x(x - 3) + 5(x - 3) = 0$

$(x + 5)(x - 3) = 0$

$x = -5$ or 3

3 a) To eqn (iii)

$y = 4 - x.$

When $x = -5$, $y = 4 - -5 = 9$

When $x = 3$, $y = 4 - 3 = 1.$

$\therefore$ The values of x and y are as follows, $(x, y) = (-5, 9)$ or $(x, y) = (3, 1)$

b) Given that $\sum_{r=3}^{5} (-1)^{r+1} r^{-1}.$

The sum, for, $r = 3$,

Let the sum to be S_n, by substitution of r as 3, 4 to 5

$$S_n = \left[(-1)^{3+1} 3^{-1}\right] + \left[(-1)^{4+1} 4^{-1}\right] + \left[(-1)^{5+1} 5^{-1}\right]$$

$$S_n = \left(\frac{1}{3} + -\frac{1}{4} + \frac{1}{5}\right) = \frac{17}{60}$$

$\therefore$ The sum of the series is $\frac{17}{60}.$

c) Given that the second term of an A.P, $A_2 = x - y$ and the fifth term $A_5 = x + y$.

Required to find the third term, A_3.

From $A_2 = (x - y)$, where $A_2 = A_1 + d$,

where A_1 = first term and d = common difference

Here $A_5 = A_1 + (5-1)d = A_1 + 4d = x + y$

and $A_3 = A_1 + 2d.$

from $A_2 = A_1 + d = x - y$

$A_1 + d = x - y$ —— (i)

$A_1 + 4d = x + y$ —— (ii)

3 (i)

$$A_1 + d = x - y$$
$$A_1 + 4d = x + y$$

$$(A_1 + d) - (A_1 + 4d) = x - y - x - y$$
$$-3d = -2y$$
$$d = \frac{2}{3}y$$

from eqn (i) $A_1 + d = x - y$

$$A_1 = x - y - d = x - y - \frac{2}{3}y = x - \frac{5}{3}y$$

$$A_1 = x - \frac{5}{3}y$$

But the third term, $A_3 = A_1 + 2d$.

So, $A_3 = A_1 + 2d = \left(x - \frac{5}{3}y\right) + 2\left(\frac{2}{3}y\right)$

$$= x - \frac{5}{3}y + \frac{4}{3}y$$
$$= x - \frac{y}{3}$$

$\therefore$ The third term is $x - \frac{y}{3}$

Extract 3.1 shows a solution from a candidate with competence in applying the necessary skills in algebra.

2017 PAST PAPERS

3. (a) A series is given by $S_n = \sum_{r=1}^{n}(2r-3)$,

(i) Determine the value of S_{50} in the series.

(ii) Find the value of n such that $S_n = 624$.

(b) Determine the values of x and y in the following simultaneous equations,

$$\begin{cases} \log(x+y)=1 \\ \log_2 x + 2\log_4 y = 4. \end{cases}$$

3 (a) (i) Given $S_n = \sum_{r=1}^{n} (2r-3)$

$S_{50} = \sum_{r=1}^{50} (2r-3)$

finding the formula for members.

for $r=1$ and $r=2$ and $r=3$

$S_n = 2(1)-3 + 2(2)-3 + 2(3)-3 + \cdots$

$= (2-3) + (4-3) + (6-3) + \cdots$

$S_n = -1 + 1 + 3 + \cdots$

This is arithmetic series with common difference $d = 1-(-1) = 3-1 = 2$

then from

$$S_n = \frac{n}{2}\left(2A_1 + (n-1)d\right) \text{ for}$$

arithmetic series where $n = 50$ $A_1 = -1$

$$S_{50} = \frac{50}{2}\left(2(-1) + (50-1)2\right)$$

$$= 25\left(-2 + 49(2)\right)$$

$$= 25\left(-2 + 98\right) = 25\left(96\right)$$

$$= 2400$$

$$\therefore S_{50} = 2400$$

2. (a) (ii) for $S_n = 624$

$$624 = \frac{n}{2}\left(2(-1) + (n-1)2\right)$$

$$624 = \frac{n}{2}\left(-2 + 2n - 2\right)$$

$$624 = \frac{n}{2}\left(2n - 4\right)$$

$$624 = n^2 - 2n$$

$$n^2 - 2n - 624 = 0$$

On solving for n, $n = 26$

$$\therefore n = 26.$$

(b) $\begin{cases} \log(x+y) = 1 & \text{— (i)} \\ \log_2 x + 2\log_4 y = 4 & \text{— (ii)} \end{cases}$

for (ii)

$$\frac{\log x}{\log 2} + \frac{2\log y}{\log 4} = 4.$$

$$= \frac{\log x}{\log 2} + \frac{2\log y}{\log 2^2} = 4$$

$$= \frac{\log x}{\log 2} + \frac{2\log y}{2\log 2} = 4$$

$$= \frac{\log x}{\log 2} + \frac{\log y}{\log 2} = 4$$

$$= \log_2 x + \log_2 y = 4.$$

$$\therefore \log_2(xy) = 4$$

and $\log(x+y) = 1$

writing in exponential form

$$2^4 = xy \quad \text{—(i)}$$

$$10^1 = x+y \quad \text{—(ii)}$$

making y the subject of the formula.

$$10 - x = y \quad \text{—— (iii)}$$

putting (iii) into (i)

3. (b) $2^4 = x(10-x) = 10x - x^2$

$16 = 10x - x^2 = 0 \quad x^2 - 10x + 16 = 0$

on solving for x.

$x^2 - 8x - 2x + 16 = 0.$

$(x^2 - 8x) - (2x - 16) = 0.$

$x(x-8) - 2(x-8) = 0.$

$(x-2)(x-8) = 0$

either $x - 2 = 0$ or $x - 8 = 0.$

$x = 2$ or $x = 8$

Then $y = 10 - x$

$= 10 - 2$ or $10 - 8$

$= 8$ or 2

$\therefore x = 2$ and $y = 8$

or $x = 8$ and $y = 2$

Extract 3.2 shows that, the candidate was able to answer the question correctly.

2016 PAST PAPERS

3. (a) The roots of the polynomial equation $P(x)=x^3-7x^2+Ax-8$ form a geometric progression. Find,

(i) The roots of the polynomial equation

(ii) The value of A

(iii) The abscissa at the turning points on the curve.

(b) Solve the following simultaneous equations by substitution method,

$$\begin{cases} xy=16 \\ x^2+y^2=32. \end{cases}$$

3. (a) $P(x) = x^3 - 7x^2 + Ax - 8$

let a, b, and c be the roots of the equation.

3. (a) $x^3 - 7x^2 + Ax - 8 = (x-a)(x-b)(x-c)$

~~x−a~~ But $b/a = c/b = r$ for geometric progression

By expanding the root equation

$(x-a)(x-b)(x-c)$

$= (x^2 - bx - ax + ab)(x-c)$

$= x^3 - x^2c - bx^2 - ax^2 + acx + abx + bcx - abc$

from the equation given.

$-7x^2 = -x^2c - bx^2 - ax^2$

$7 = c + b + a$ — (i)

$AX = acx + abx + bcx$

$A = ac + ab + bc$ — (ii)

also
$-8 = -abc$
$abc = 8$ — (iii)
But $ac = b^2$
$\therefore abc = 8$
$(ac)b = 8$

$\therefore (b^2)b = 8$
$b^3 = 8$
$b = 2$ — (i)
also $7 = c + 2 + a$
$5 = c + a$ — (iv)
But $ac = b^2$
$ac = (2)^2 = 4$
$a = 4/c$ — (v)
Substitute in (iv)
$5 = (4/c) + c$
$5 = \frac{4 + c^2}{c}$
$5c = c^2 + 4$
$c^2 - 5c + 4 = 0$
Using calculator, $c = 4$ or 1
$a = \frac{4}{c} = \frac{4}{4} = 1$
$a = \frac{4}{1} = 4$
$\therefore a = 1, b = 2, c = 4$ ans (A)
or $a = 4, b = 2, c = 1$ ans (B)

3. (a) $A = ac + ab + bc$

$A = (4\times1) + 4\times2 + 1\times2$

$= 4 + 8 + 2$

$A = 14$

(iii) The equation is

$y = x^3 - 7x^2 + 14x - 8$

Extract 3.2 (b)

3. (b) $xy = 16$

$x^2 + y^2 = 32$

$\left(\frac{16}{y}\right)^2 + y^2 = 32$

$\frac{16^2}{y^2} + y^2 = 32$

$16^2 + y^4 = 32y^2$

Let:

$u = y^2$

$16^2 + u^2 = 32u$

$u = 16$

Then

from:

$u = y^2$

$y^2 = 16$

$y = \pm\sqrt{16}$

$y = \pm 4$

$xy = 16$

$x = \pm 4$

$\therefore x = \pm 4$ and $y = \pm 4$

In Extracts 3.2 (a) and (b), the candidates demonstrated a good understanding on finding the roots of the polynomial equation and solving the simultaneous equations.

2015 PAST PAPERS

3. (a) Given the series $-1+1+3,...$

(i) Express it in the form $S_n = \sum_{r=1}^{n} f(r)$.

(ii) Give one reason as to whether the series is an arithmetic or a geometric progression.

(iii) Determine the value of n for which $S_n = 575$.

(b) If in a geometric progression, the second term exceeds the first term by 20 and the fourth term exceeds the second term by 15, find the possible values of the first term.

3(a) Given $-1+1+3$

(i) $S_n = \sum_{r=1}^{n} f(r)$

the series is Arithmetic progression since the common difference $= d = 2$.

from $A_n = A_1 + (n-1)d$

$= -1 + (n-1)2$

$= -1 + (n-1)2$

$= -1 + 2n - 2$

$= -1 - 2 + 2n$

$A_n = 2n - 3$

$A_n = f(r) = 2n - 3 = 2r - 3$

$\therefore S_n = \sum_{r=1}^{n} 2r - 3$

2(a) ii) The series is arithmetic since there is the common difference (d)

$$d = A_2 - A_1 = A_3 - A_2$$

$$= 1-(-1) = 3-1$$

$$d = 2 = 2$$

iii) $S_n = 575 = \frac{n}{2}(2A_1 + (n-1)d)$

$$575 = \frac{n}{2}\left(2(-1) + (n-1)2\right)$$

$$575 \times 2 = n\left(-2 + 2n - 2\right)$$

$$= n(-2 + 2n - 2)$$

$$575 \times 2 = n(2n - 4)$$

$$575 \times 2 = 2n^2 - 4n$$

$$0 = 2n^2 - 4n - 1150$$

on solving $n = 25$ or -23

Since number (n) can't be negative hence $n = 25$.

3(b) $G_2 - G_1 = 20.$

$G_4 - G_2 = 15$

for Geometric progression

$G_n = G_1 r^{n-1}$

$G_2 = G_1 r^{2-1} = G_1 r$

$G_4 = G_1 r^{4-1} = G_1 r^3$

$G_1 r - G_1 = 20 \quad - - \quad (i)$

$G_1 (r-1) = 20 \quad - - \quad (i)$

from $G_4 - G_2 = 15$

$G_1 r^3 - G_1 r = 15$

$G_1 (r^3 - r) = 15 \quad - \quad (ii)$

take $(i) \div (ii)$

$$\frac{\cancel{G_1}(r-1)}{\cancel{G_1}(r^3-r)} = \frac{20}{15}$$

$$\frac{r-1}{r(r^2-1)} = \frac{20}{15}$$

$$\frac{\cancel{r-1}}{r(\cancel{r-1})(r+1)} = \frac{20}{15}$$

3(b) $\frac{1}{r(r+1)} = \frac{20}{15}$

$15 = 20r(r+1)$

$15 = 20r^2 + 20r$

$0 = 20r^2 + 20r - 15$

On solving $r = 0.5$ or -1.5

from $a_2 = \cancel{a_1 r} = a_1 r^{0.5}$ or $\cancel{a_1 r^{-1.5}}$

$a_2 = 0.5a_1$ or $-1.5a_1 = a_2$

from $a_2 - a_1 = 20$

$0.5a_1 - a_1 = 20$

$-0.5a_1 = 20$

$a_1 = -40$

or

when $a_2 = -1.5a_1$

from $a_2 - a_1 = 20$

$-1.5a_1 - a_1 = 20$

$-2.5a_1 = 20$

$a_1 = -8$

$\therefore$ Possible values of a_1 are -8 or -40

In Extract 3.1, the candidate was able to recall and apply correctly the formula for the n^{th} term of arithmetic and geometrical progressions and the formula for the sum of the first n terms of arithmetic progression in answering question 3.

4.0 Differentiation

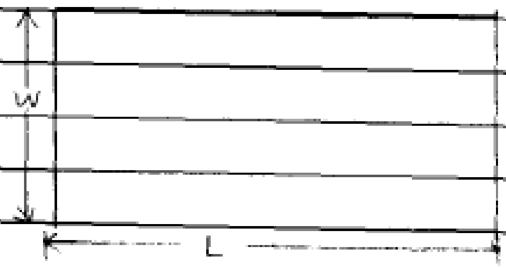
2021 PAST PAPERS

4. (a) (i) Find $\frac{dy}{dx}$ of $y = \sqrt{x^2 + 1}$.

(ii) Given the equations $x = (t^2 - 1)^2$, $y = t^3$. Find $\frac{dy}{dx}$ in terms of t.

(b) A metal wire whose length is 600 m is bent to make a rectangular fence. Calculate the dimensions of the fence that could give the maximum area.

(b)



Area $= WL$ — (i)

Total circumference $= (W + L)2$

$2W + 2L = 600m$

$W + L = 300m$ — (ii) $\Rightarrow W = 300 - L$

from (i)

$A = WL$

$A = (300 - L)L$

$A = 300L - L^2$

Differentiate dA/dL

$dA/dL = 300 - 2L$ — (iii)

for maximum area

$dA/dL = 0$

$0 = 300 - 2L$

4(b) $2L = 300m$

$L = 150m$

$W = 300 - L$

$W = (300 - 150)m$

$W = 150m$

∴ The Less Dimension of the fence are

length, $L = 150m$

Width $= 150m$

Extract 4.2: A sample of correct response to part (b) of question 4

2020 PAST PAPERS

4. (a) Find the first derivative of $f(x) = x^2$ from first principles.

(b) Find the slope of the tangent to the curve $8x^3 + xy^3 - 5y^2 = 0$ at $(1, -1)$.

(c) Use second derivative test to classify the stationary point(s) of the curve $f(x) = 2x^3 + 3x^2 - 12x - 5$.

4. (a) $f(x) = x^2$

$f(x+h) = (x+h)^2$

from the first principle;

$$\frac{dy}{dx} = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$\frac{dy}{dx} = \lim_{h \to 0} \left(\frac{(x+h)^2 - x^2}{h} \right)$$

$$\frac{dy}{dx} = \lim_{h \to 0} \left(\frac{x^2 + 2hx + h^2 - x^2}{h} \right)$$

$$\frac{dy}{dx} = \lim_{h \to 0} \left(\frac{2hx + h^2}{h} \right)$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} 2x + h$$

As $h \to 0$, $\frac{dy}{dx} = 2x$

Extract 4.1: *A sample of correct solution for part (a) of question 4.*

2019 PAST PAPERS

4. (a) Differentiate with respect to x the function $f(x) = e^{x^2+3x+2}$.

(b) Use implicit differentiation to find the derivative of $x^2 + y^2 - 6xy + 3x - 2y + 5 = 0$

(c) Find the stationary points of the function $f(x) = 2x^3 - 3x^2 - 36x + 14$ and determine the nature of each point.

4(a). $f(x) = e^{x^2+3x+2}$

$y = e^{x^2+3x+2}$

let $x^2+3x+2 = u$

$\frac{du}{dx} = 2x+3$

$y = e^u$

$\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

(a) $\frac{dy}{dx} = e^u \cdot (2x+3)$

But $u = x^2+3x+2$

$\frac{dy}{dx} = (2x+3)\, e^{x^2+3x+2}$

(b). $x^2+y^2-6xy+3x-2y+5=0$

On differentiating the above eqn with respect to x,

$2x + 2y\frac{dy}{dx} - \left(6y + 6x\frac{dy}{dx}\right) + 3 - 2\frac{dy}{dx} + 0 = 0$

$2x + 2y\frac{dy}{dx} - 6y + 6x\frac{dy}{dx} + 3 - 2\frac{dy}{dx} = 0$

$2x+3-6y + 2y\frac{dy}{dx} - 6x\frac{dy}{dx} - 2\frac{dy}{dx} = 0$

$2x+3-6y - (-2y+6x+2)\frac{dy}{dx} = 0$

$2x+3-6y - (6x+2-2y)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{2x+3-6y}{6x+2-2y}$

(c). $f(x) = 2x^3 - 3x^2 - 36x + 14$

$y = 2x^3 - 3x^2 - 36x + 14$

$\frac{dy}{dx} = 6x^2 - 6x - 36$

But at stationary points $\frac{dy}{dx} = 0$

$0 = 6x^2 - 6x - 36$

$x = 3$ and -2.

Thus the stationary points are obtained from:

$y = 2x^3 - 3x^2 - 36x + 14$

when $x = 3$.

$y = 2(3)^3 - 3(3)^2 - 36(3) + 14$

$y = -67$.

When $x = -2$

$y = 2(-2)^3 - 3(-2)^2 - 36(-2) + 14$

$y = 58$.

The stationary points are $(3, -67)$ and $(-2, 58)$

Nature of stationary points

from:

$\frac{dy}{dx} = 6x^2 - 6x - 36$

$\frac{d^2y}{dx^2} = 12x - 6$. But $x = 3$.

$\frac{d^2y}{dx^2} = 12(3) - 6 = 30$.

When $x = -2$.

$\frac{d^2y}{dx^2} = 12(-2) - 6 = -30$.

Thus the stationary points $(3, -67)$ are minimum

and $(-2, 58)$ are maximum

Extract 4.1: *A sample of the candidate's correct responses in question 4*

2018 PAST PAPERS

4. (a) If $f(x) = x$, find $\frac{dy}{dx}$ from first principles.

(b) Given the curve $f(x) = (x+1)(x-1)(2-x)$,

(i) Find x and y intercepts.

(ii) Determine the maximum and minimum points of $f(x)$.

(iii) Sketch the graph of $f(x)$.

4(a) From $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

$\frac{dy}{dx} = \lim_{h \to 0} \frac{x+h - x}{h}$

$\frac{dy}{dx} = \lim_{h \to 0} \frac{x+h-x}{h}$.

$\frac{dy}{dx} = \lim_{h \to 0} h/h$

$\frac{dy}{dx} = 1$.

$\therefore \frac{dy}{dx} = 1$.

4 (b) (ii) Maximum and Minimum points

$y = (x+1)(x-1)(2-x)$

$y = [(x+1)(x-1)](2-x)$

$y = [x^2 - x + x - 1](2-x)$

$y = (x^2 - 1)(2-x)$

$y = 2x^2 - x^3 - 2 + x$.

$y = -x^3 + 2x^2 + x - 2$.

For critical points, $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -3x^2 + 4x + 1$.

$-3x^2 + 4x + 1 = 0$.

on solving

$x = -0.215$ or $x = 1.549$ to 3 decimal places

but $y = (x+1)(x-1)(2-x)$

For $x = -0.215$

$y = (-0.215 + 1)(-0.215 - 1)(2 - -0.215)$

$y = -2.113$ to 3 decimal places

For $x = 1.549$

$y = (1.549 + 1)(1.549 - 1)(2 - 1.549)$

$y = 0.631$ to 3 decimal places

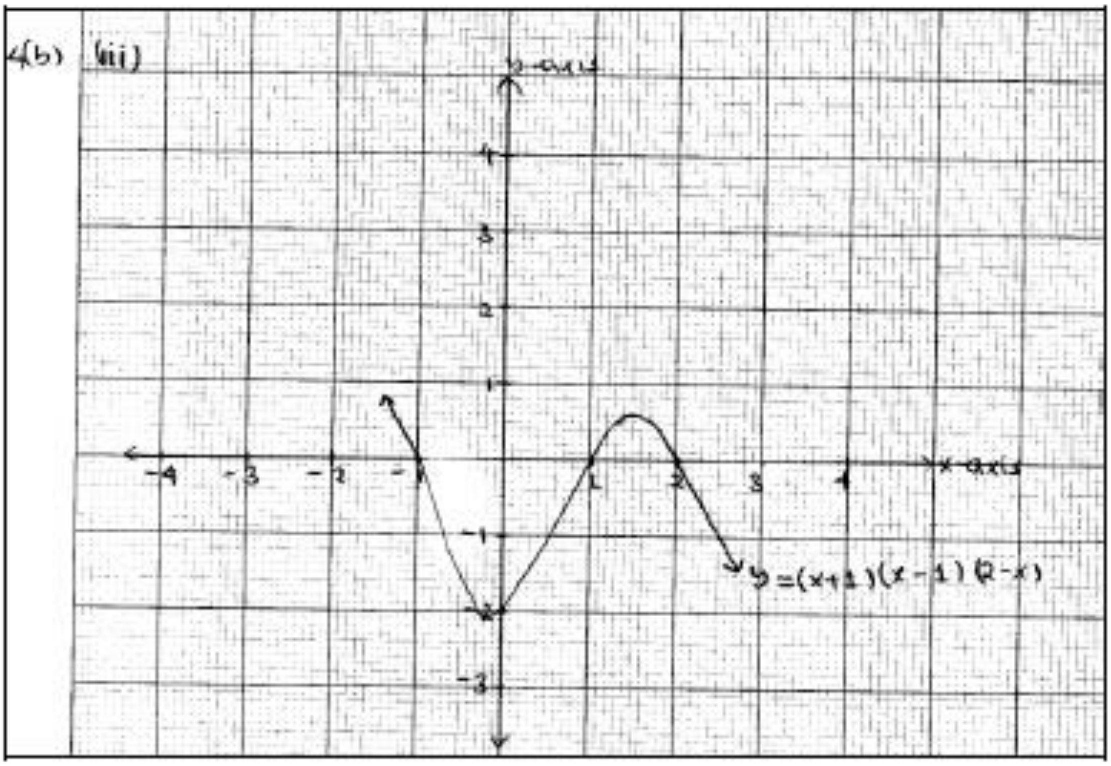
Therefore the critical points are

$(-0.215, -2.113)$ and $(1.549, 0.631)$

	$\frac{d^2y}{dx^2} = -6x + 4$
	For $x = -0.215$
	$\frac{d^2y}{dx^2} = -6(-0.215) + 4$
	$\frac{d^2y}{dx^2} = 1.29 + 4$
	$\frac{d^2y}{dx^2} = 5.29$

4(b)	(ii) Since $\frac{d^2y}{dx^2}$ is greater than 0, then
	The point $(-0.215, -2.113)$ is a Minimum point
	For $x = 1.549$
	$\frac{d^2y}{dx^2} = -6(1.549) + 4$
	$\frac{d^2y}{dx^2} = -9.294 + 4$
	$\frac{d^2y}{dx^2} = -5.294$
	Since $\frac{d^2y}{dx^2}$ is less than 0, then
	the point $(1.549, 0.631)$ is a Maximum point
	$\therefore$ The Minimum point is $(-0.215, -2.113)$
	The Maximum point is $(1.549, 0.631)$

4(b)	(i) $f(x) = (x+1)(x-1)(2-x)$, let $f(x) = y$
	For x-intercept, $y = 0$
	$0 = (x+1)(x-1)(2-x)$
	$x+1 = 0$, $x - 1 = 0$ or $2 - x = 0$
	$x = 0 - 1$, $x = 0 + 1$ or $2 = x$
	$x = -1$, $x = 1$ or $x = 2$.
	$\therefore$ x-intercepts $= -1$, 1 and 2.
	For y-intercept, $x = 0$
	$y = (0+1)(0-1)(2-0)$
	$y = (1)(-1)(2)$
	$y = (-1)(2)$
	$y = -2$
	y-intercept $= -2$.



Extract 4.1 shows how a candidate correctly applied knowledge of differentiation in answering question 4.

2017 PAST PAPERS

4. (a) Find $\frac{dy}{dx}$ in the following equations:

(i) $y = \frac{e^x\sqrt{\cos x}}{(2x+3)^2}$, when $x = 2\pi$.

(ii) $yx^2 - y^2x + 5y - 20x = 14$.

(b) Differentiate the function $f(x) = 4x^3 + 3x - 4$ from first principles.

(c) A 13 m long ladder leans against a wall. The bottom of the ladder is pulled away from the wall at the rate of 6m/s. How fast does the height on the wall decrease when the foot of the ladder is 5 m away from the base of the wall?

4. (a) (i) $y = \frac{e^x\sqrt{\cos x}}{(2x+3)^2}$

$y = \frac{e^x\sqrt{\cos x}}{(2x+3)(2x+3)}$

$y = \frac{e^x\sqrt{\cos x}}{(4x^2+12x+9)}$

Let

$u = e^x\sqrt{\cos x}$

$\frac{du}{dx} = e^x\sqrt{\cos x} + \frac{1}{2}(-\sin x)(\cos x)^{-1/2} e^x$

$\frac{du}{dx} = e^x\left((\cos x)^{1/2} - \frac{\sin x}{2\sqrt{\cos x}}\right)$

$\frac{du}{dx} = e^x\left(\sqrt{\cos x} - \frac{\sin x}{2\sqrt{\cos x}}\right)$

$v = (4x^2+12x+9)$

$\frac{dv}{dx} = (8x+12)$

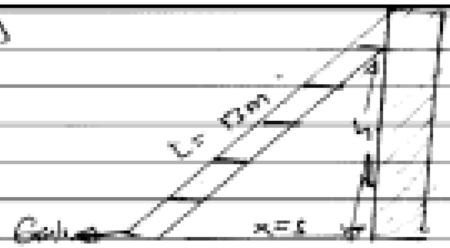
By quotient rule

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(2x+3)^2 e^x\left(\sqrt{\cos x} - \frac{\sin x}{2\sqrt{\cos x}}\right) - e^x\sqrt{\cos x}(8x+12)}{((2x+3)^2)^2}$

4(a)(ii)	Soln
	Given $yx^2 - y^2x + 5y - 20x = 14.$
	Differentiating each term with respect to x.
	$2xy + x^2\frac{dy}{dx} - \left(y^2 + 2xy\frac{dy}{dx}\right) + 5\frac{dy}{dx} - 20 = 0$
	$x^2\frac{dy}{dx} - 2xy\frac{dy}{dx} + 5\frac{dy}{dx} = y^2 - 2xy + 20$
	$\frac{dy}{dx}\left(x^2 - 2xy + 5\right) = y^2 - 2xy + 20$
	$\therefore \frac{dy}{dx} = \frac{y^2 - 2xy + 20}{x^2 - 2xy + 5}.$
4. (b)	Soln
	Given $f(x) = 4x^3 + 3x - 4.$
	If h is small increment of x.
	$f(x+h) = 4(x+h)^3 + 3(x+h) - 4.$
	By first principle
	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
	$= \lim_{h \to 0} \frac{4(x+h)^3 + 3(x+h) - 4 - (4x^3 + 3x - 4)}{h}$
	$= \lim_{h \to 0} \frac{4(x^3 + 3x^2h + 3xh^2 + h^3) + 3x + 3h - 4 - 4x^3 - 3x + 4}{h}$
	$= \lim_{h \to 0} \frac{4x^3 + 12x^2h + 12xh^2 + 4h^3 + 3x + 3h - 4 - 4x^3 - 3x + 4}{h}$
	$= \lim_{h \to 0} \frac{12x^2h + 12xh^2 + 4h^3 + 3h}{h}$
	$= \lim_{h \to 0} 12x^2 + 12xh + 4h^2 + 3.$
	As $h \to 0$
	$12xh = 12 \times x \times 0 = 0.$
	$4h^2 = 4(0)^2 = 0.$
	$\therefore 12x^2 + 12xh + 4h^2 + 3 = 12x^2 + 3.$
	$\therefore f'(x) = 12x^2 + 3$

4 (c)



$h^2 = L^2 - x^2$

$h^2 = 13^2 - x^2$

$h = \sqrt{13^2 - x^2}$

$2h\frac{dh}{dt} = -2x\frac{dx}{dt}$

$2h\frac{dh}{dt} = -2x\frac{dx}{dt}$

but $\frac{dh}{dt} = \frac{-x}{h}\frac{dx}{dt}$

but $dx/dt = 6\,m/s$

$x = 5$

$h = \sqrt{13^2 - x^2}$

$= \sqrt{13^2 - 5^2}$

$= 12$

$\frac{dh}{dt} = -\frac{5}{12}(6)$

$\frac{dh}{dt} = -2.5\,m/s$

∴ the height of wall decreases by 2.5 m/s.

In Extract 4.2, the candidate was able to apply correctly the concepts of differentiation in parts (a), (b) and (c).

2016 PAST PAPERS

4. (a) Show that $\frac{d}{dx}\left(\sin^{-1}(x-1)\right)=\frac{1}{\sqrt{2x-x^2}}$.

(b) A relation is defined by the equation $y^2-4x^3-4=0$. Find

(i) The slope of the curve at a point where $x=2$

(ii) The equations of the tangent to the curve at a point where $x=2$.

(c) Find $\frac{dy}{dx}$ if $y=x^2\left(1-\frac{1}{\sqrt{x}}\right)e^{\tan x}$.

4 (a) Soln.

$$\frac{d}{dx}\left(\sin^{-1}(x-1)\right)=\frac{1}{\sqrt{2x-x^2}}$$

Let

$$y=\sin^{-1}(x-1)$$

$$x-1=\sin y$$

To find $\frac{dy}{dx}$

$$\frac{d}{dx}(x-1)=\frac{d}{dx}(\sin y)$$

$$1=\cos y\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{1}{\cos y}\quad \text{But } \cos y=\sqrt{1-\sin^2 y}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-\sin^2 y}}$$

$$=\frac{1}{\sqrt{1-(x-1)^2}}$$

$$=\frac{1}{\sqrt{1-(x^2-2x+1)}}$$

4 (a) $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2+2x-1}}$

$\therefore \frac{d}{dx}\left(\sin^{-1}(x-1)\right) = \frac{1}{\sqrt{2x-x^2}}$ hence shown.

4 (b) soln.

(i) Given, $y^2 - 4x^3 - 4 = 0$

To find slope, $\frac{dy}{dx}$

$$\frac{d}{dx}(y^2 - 4x^3 - 4) = \frac{d}{dx}(0)$$

$$2y\frac{dy}{dx} - 4(3x^2) = 0$$

$$2y\frac{dy}{dx} - 12x^2 = 0.$$

$$\frac{dy}{dx} = \frac{12x^2}{2y}$$

when $x = 2$.

$$y^2 = 4x^3 + 4$$

$$= 4(2)^3 + 4$$

$$y = \pm 6$$

$$\therefore \frac{dy}{dx} = \frac{12(2)^2}{2 \times 6} = \pm 4$$

(i) $\therefore \frac{dy}{dx} = \text{slope} = \pm 4$

4 (b) (ii) equation of tangent

from $\frac{dy}{dx} = \frac{12x^2}{2y}$

$\frac{dy}{dx} = \frac{12 \times (2)^2}{12} = \pm 4$

slope $= \pm 4$

at $(x, y) = (2, \pm 6)$

$\frac{y-6}{x-2} = 4$

$y - 6 = 4(x-2)$

$y - 6 = 4x - 8$

$y - 4x = -8 + 6$

$y - 4x = -2$

$\underline{y = 4x - 2}$

Also when slope $= -4$

$\frac{y-(-6)}{x-2} = -4$

$\frac{y+6}{x-2} = -4$

$y + 6 = -4(x-2)$

$y + 6 = -4x + 8$

$y = -4x + 8 - 6$

$y = 2 - 4x$

$\therefore$ equations of tangent are

$\underline{\underline{y = 4x - 2}}$ and $\underline{\underline{y = 2 - 4x}}$

4 (c) Soln.

$y = x^2\left(1 - \frac{1}{\sqrt{x}}\right)e^{\tan x}$

$y = \left(x^2 - \frac{x^2}{x^{1/2}}\right)e^{\tan x}$

$y = \left[x^2 - \left(x^{2-1/2}\right)\right]e^{\tan x}$

$y = \left(x^2 - x^{3/2}\right)e^{\tan x}$

Let $a = \tan x$

$\frac{da}{dx} = \sec^2 x$

$b = e^{\tan x} = e^a$

$\frac{db}{da} = e^a$

$\frac{db}{dx} = \frac{db}{da} \cdot \frac{da}{dx}$

$\frac{db}{dx} = \sec^2 x \, e^{\tan x}$

Also

Let $c = x^2 - x^{3/2}$

$\frac{dc}{dx} = 2x - \frac{3}{2}x^{1/2}$

From $y = (x^2 - x^{3/2})e^{\tan x}$

$y = c \cdot b$

$\frac{d(y)}{dx} = c\frac{db}{dx} + b\frac{dc}{dx}$

4 (c) $\therefore \frac{d(y)}{dx} = \left(x^2 - x^{3/2}\right)\sec^2 x \, e^{\tan x} + e^{\tan x} \cdot \left(2x - \frac{3}{2}x^{1/2}\right)$

In Extract 4.2, the candidate managed to apply correctly the concepts of differentiation in answering the question.

2015 PAST PAPERS

4. (a) Find $\frac{dy}{dx}$ from first principle given $y = 2x^2$.

(b) If $x = 2t + 9$ and $y = (t+1)^4$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x.

(c) Given $f(x) = x^3 - 2x^2 + x - 7$:

(i) Find the stationary values of the function.

(ii) Find the equation of the tangent line to the curve at the point (0, -7).

(iii) Draw the graph of this function for $-2 \le x \le 3$ and indicate on the graph the stationary points and the equation of the tangent line obtained in part (c) (ii).

4. (a) Soln

$y = 2x^2$.

$f(x) = 2x^2$.

$f(x+\Delta x) = 2(x+\Delta x)^2$.

from first principle.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

$$= \lim_{\Delta x \to 0} \left(\frac{2(x+\Delta x)^2 - 2x^2}{\Delta x} \right)$$

$$= \lim_{\Delta x \to 0} \left(\frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 - 2x^2}{\Delta x} \right)$$

$$= \lim_{\Delta x \to 0} \left(\frac{4x\Delta x}{\Delta x} + \frac{2(\Delta x)^2}{\Delta x} \right)$$

$$= \lim_{\Delta x \to 0} \left(4x + 2\Delta x \right)$$

$\Delta x \to 0 \quad 2\Delta x \to 0$

$= 4x$.

$\therefore \frac{dy}{dx} = 4x$.

4. (b)

Soln

$x = 2t + 9, \quad y = (t+1)^4.$

from $x = 2t + 9.$

$t = \frac{x}{2} - \frac{9}{2}.$

$dt/dx = 1/2.$

$y = (t+1)^4$

let

$u = t + 1$

$du/dt = 1.$

$y = u^4$

$dy/du = 4u^3.$

$dy/dt = dy/du \cdot du/dt$

$= 4(t+1)^3.$

$dy/dx = dy/dt \cdot dt/dx.$

$= 4(t+1)^3 \cdot 1/2.$

$= 2\left(\frac{x-9}{2} + 1\right)^3$

$= 2\left(\frac{x-7}{2}\right)^3.$

$\therefore \; dy/dx = 2\left(\frac{x-7}{2}\right)^3$

4. (a) $\frac{dy}{dx} = 2\left(\frac{x-7}{2}\right)^3$.

$\frac{d^2y}{dx^2} = ?$

$f(x)' = 2\left(\frac{x-7}{2}\right)^3$.

Let $u = \frac{x-7}{2}$.

$\frac{du}{dx} = \frac{1}{2}$.

$f(x) = 2u^3$.

$\frac{d f(x)}{du} = 6u^2$.

$\frac{d^2y}{dx^2} = \frac{d f'(x)}{du} \cdot \frac{du}{dx}$.

$= 6\left(\frac{x-7}{2}\right)^2 \cdot \frac{1}{2}$.

$= 3\left(\frac{x-7}{2}\right)^2$

$\therefore \frac{d^2y}{dx^2} = 3\left(\frac{x-7}{2}\right)^2$

4. (b) $f(x) = x^3 - 2x^2 + x + 7$.

To the stationary point values.

$\frac{dy}{dx} = 3x^2 - 4x + 1$.

At stationary value $\frac{dy}{dx} = 0$.

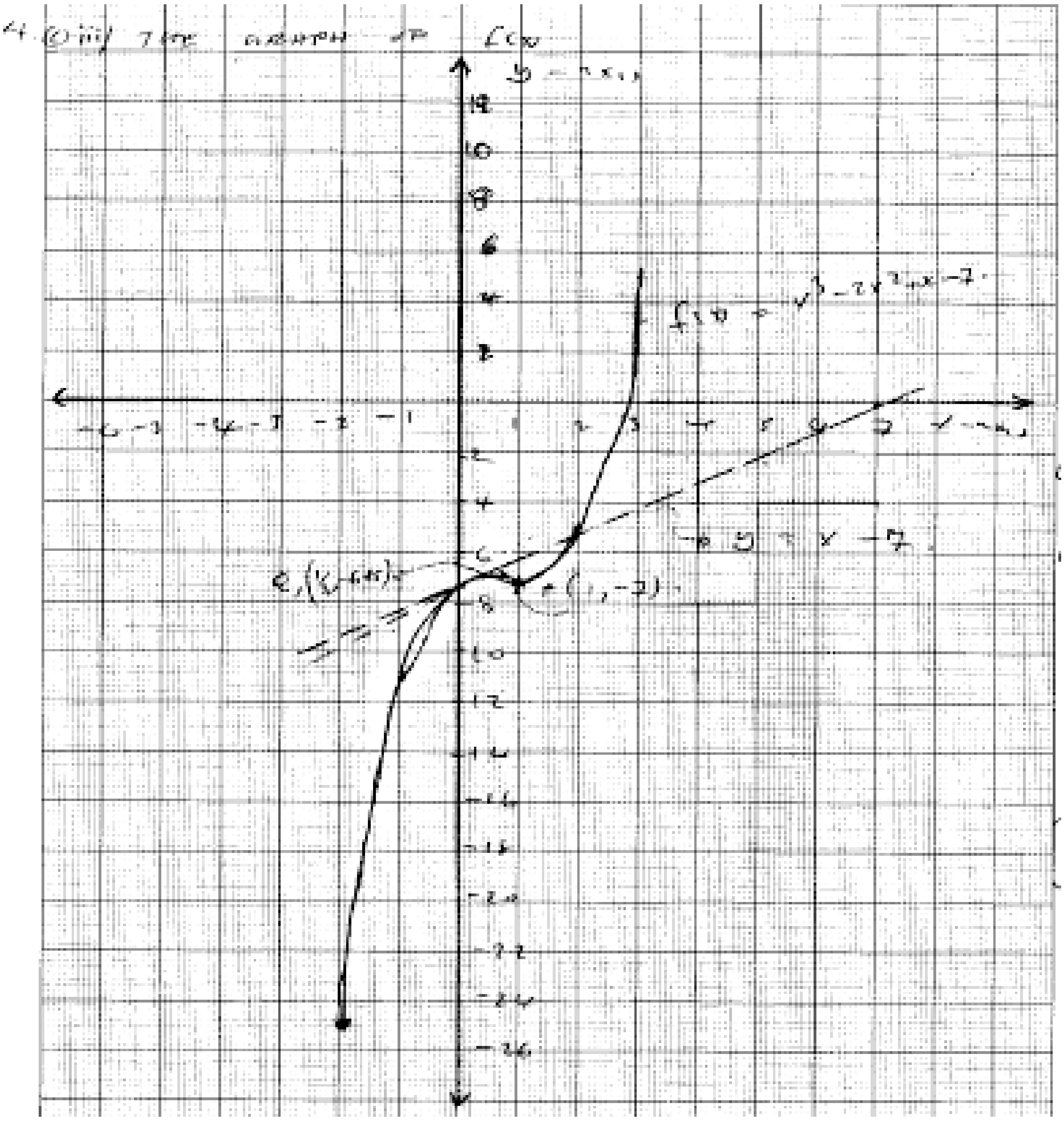
$3x^2 - 4x + 1 = 0$.

$3x^2 - 3x - x + 1 = 0$

$3x(x-1) - (x-1) = 0$

$(3x-1)(x-1) = 0$

$x = \frac{1}{3}$ or $x = 1$.



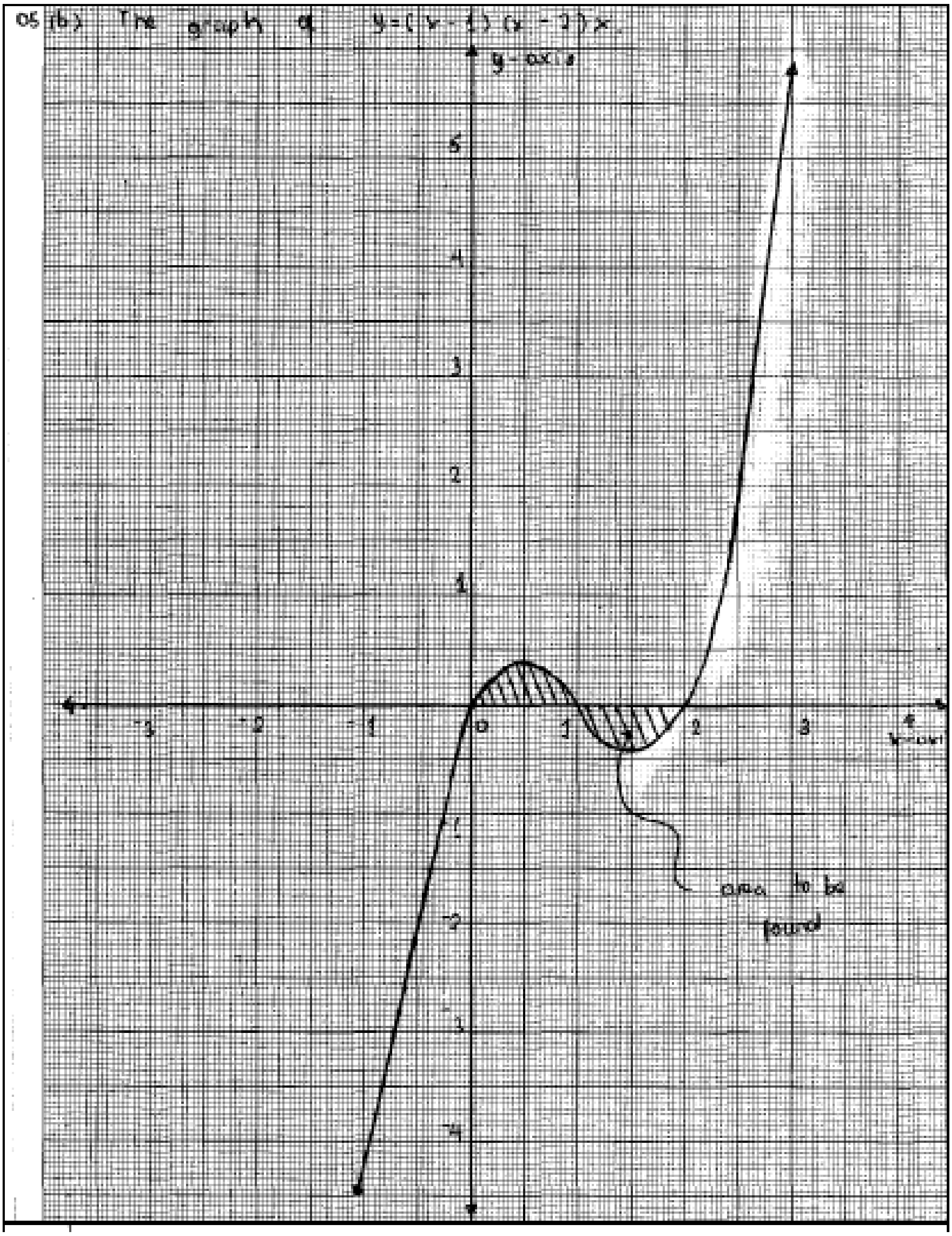
In Extract 4.1, the candidate demonstrated good understanding on the examined concepts of differentiation and also had good drawing skills.

5.0 Integration

2021 PAST PAPERS

5. (a) The first derivative of $f(t)$ is $f'(t) = 6t + 1$. Find $f(t)$ and the numerical value of $f(10)$ given that $f(0) = 2$.

   (b) Find the area enclosed between the curve $y = x(x-1)(x-2)$ and the x-axis.



05\. (b) from

$$\text{Area} = \int_a^b f(x)\,dx.$$

$$A = \int_0^1 x(x-1)(x-2)\,dx + \int_1^2 x(x-1)(x-2)\,dx$$

$$= \int_0^1 (x^3 - 3x^2 + 2x)\,dx + \int_1^2 (x^3 - 3x^2 + 2x)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2}\right]_0^1 + \left[\frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2}\right]_1^2$$

$$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 + \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2$$

$$0.25 + |-0.25|.$$

$$= 0.5 \text{ square units.}$$

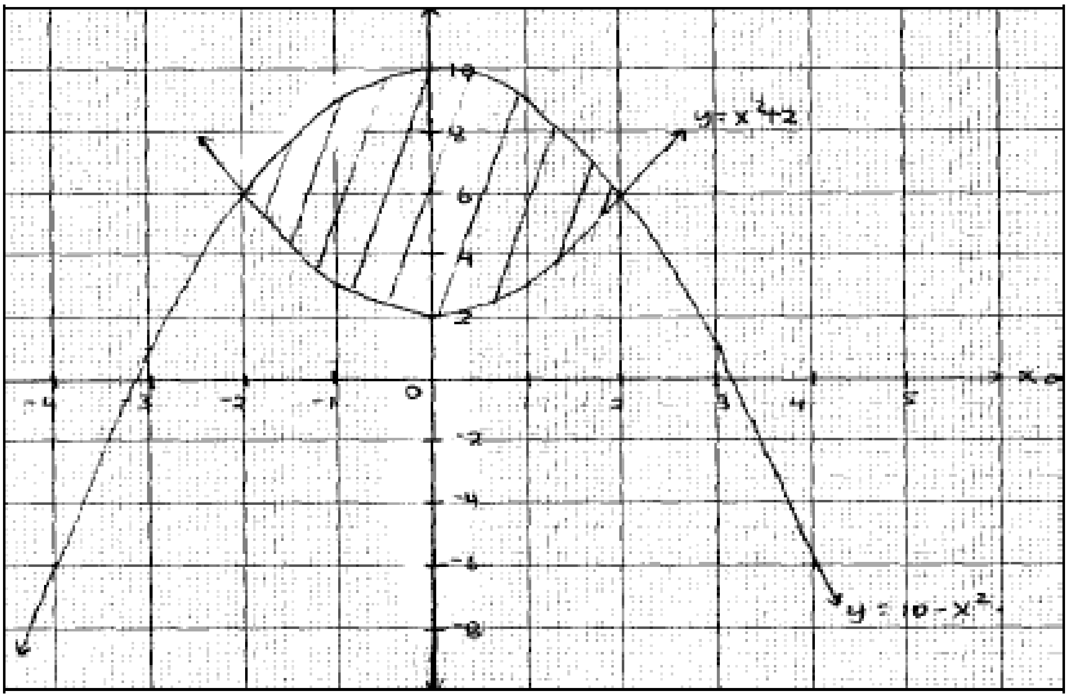
Extract 5.2: A sample of correct response to part (c) of question 5

$2 =$

5. (a) Integrate $\sin^2 2x \cos 2x$ with respect to x.

(b) Evaluate $\int_0^{\sqrt{a}} \frac{x}{x^2+a}\,dx$ (express your answer in the form $m\sqrt{n}$).

(c) Find the area enclosed between the curves $y = x^2 + 2$ and $y = 10 - x^2$.



Extract 5.2: *A sample of correct solution for part (c) of question 5.*

2019 PAST PAPERS

5. (a) Use substitution method to find the integral of each of the following functions:

(i) $\int x\sqrt{x^2+1}\,dx$

(ii) $\int \tan x\,dx$

(b) Find the area bounded by the curve $y = x^2 - 4x + 3$ and x- axis.

5. a) i) $\int x\sqrt{x^2+1}\quad dx$

let $u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$= \int\left(x\left(u^{1/2}\right)\right)\frac{du}{2\cancel{x}}$

$= \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\,\frac{u^{3/2}}{3/2} + c$

$= \frac{u^{3/2}}{3} + c$

But $u = x^2+1$

$\therefore \int x\sqrt{x^2+1}\,dx = \frac{\left(\sqrt{x^2+1}\right)^3}{3} + c$

5. a) ii) $\int \tan x \, dx$

$= \int \frac{\sin x}{\cos x} dx$

let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$= -\int \frac{\sin x}{u} \frac{du}{\sin x} = -\int \frac{du}{u} = -\ln u + c$

but $u = \cos x$

$= -\ln(\cos x) + c$

$\therefore \int \tan x \, dx = -\ln(\cos x) + c$

b) Area $= \left| \int_a^b f(x) dx \right|$

Given; $y = x^2 - 4x + 3$ and $y = 0$

$x^2 - 4x + 3 = 0$

$x^2 - x - 3x + 3 = 0$

$(x^2 - x) \mp (3x - 3) = 0$

$x(x-1) + 3(x-1) = 0$

$x - 1 = 0$ or $x + 3 = 0$

$x = +1$ or $x = +3$

5. b) $A = \left| \int_1^3 x^2 - 4x + 3 \, dx \right|$

$A = \frac{x^3}{3} - \frac{4x^2}{2} + 3x \Big]_1^3$

$A = \left| \left(\frac{3^3}{3} - (2 \times 3^2) + (3 \times 3) \right) - \left(\frac{1}{3} - \frac{4 \times 1^2}{2} + 3 \right) \right|$

$A = 4/3$ square units

$\therefore$ Area $= 1\frac{1}{3}$ square units

Extract 5.1: *A sample of the candidate's correct responses in question 5*

2018 PAST PAPERS

5. (a) Integrate $\int 2x\sqrt{x^2+3}\,dx$.

(b) Find the area of the region enclosed by the curve $y = x^2$ and the line $y = x$.

(c) Find the volume of revolution which is obtained when the area bounded by the line $y=2x$, $x-$axis, $x=1$ and $x=h$ is rotated about the x-axis.

05(a)

$\int 2x\sqrt{x^2+3}\ dx.$

$\int 2x\,(x^2+3)^{1/2}\ dx$

let $u = x^2+3.$

$du/dx = 2x.$

$dx = du/2x$

then,

$\int 2x\, u^{1/2}\,\frac{du}{2x} = \int u^{1/2}\,du.$

$= \frac{u^{1/2+1}}{1/2+1} + C$

$= \frac{u^{3/2}}{3/2} + C = 2/3\, u^{3/2} + C$

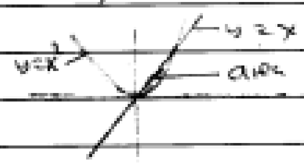
but $u = x^2+3$

then

$\int 2x\sqrt{x^2+3}\,dx = 2/3\,(x^2+3)^{3/2} + C$

05(b) Given $y = x^2$, and $y = x$

Then,



thus

$\text{Area} = \int_{x_1}^{x_2} y\,dx$

but

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, \quad x = 1$

05(b) Then,

Give $y = x^2$, $y_2 = x$

$$\text{Area} = \int_0^1 (x - x^2)\,dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3} + c \Big|_0^1$$

$$= \frac{1^2}{2} - \frac{1^3}{3} + c - \frac{0^2}{2} + \frac{0^3}{3} - c.$$

$$= \frac{1}{2} - \frac{1}{3}$$

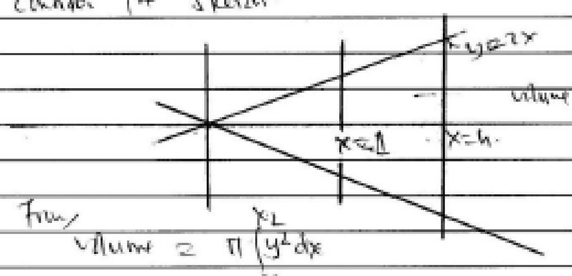
$$= \frac{1}{6} \text{ units square,}$$

Area will be $1/6$ unit square

05(c) Given

$y = 2x$, x - axis, $x = 1$, $x = h$,

Consider the sketch.



From,

$$\text{Volume} = \pi \int^{x_2} y^2\,dx$$

$$= \pi \int_1^{h} (2x)^2\,dx$$

$$= \pi \int_1^{h} 4x^2\,dx$$

05(c) $\text{Volume} = \pi \int_1^h 4x^2\,dx$

$$= \pi \left(\frac{4x^3}{3} + c\right)_1^h$$

$$= \pi \left[\frac{4h^3}{3} - \frac{4(1)^3}{3}\right]$$

$$= \pi \left[\frac{4h^3}{3} - \frac{4}{3}\right].$$

$$= \pi \left(\frac{4}{3}\right)(h^3 - 1)$$

$$= \frac{4\pi}{3}(h^3 - 1) \text{ cubic units}$$

∴ The volume will be $\frac{4\pi}{3}(h^3 - 1)$ cubic units when h is greater than 1.

or

Volume $= \frac{4\pi}{3}(1 - h^3)$ cubic units when h is less 1

Extract 5.1 indicates a sample work of a candidate who applied correctly the tested skills of integration in question 5.

2017 PAST PAPERS

5. (a) Evaluate the following integrals:

$$\int_0^{0.5\pi} \cos^3 x\,dx,$$

(b) The slope of a curve at any point is defined by the equation $\frac{dy}{dx} = 3x - \frac{1}{x^2}$, where $x \neq 0$. Find the equation of the curve.

(c) The area bounded by the lines $y = mx$, $y = h$, $y = 0$ and $x = 0$ is rotated about y-axis. If $x = r$ when $y = h$. Find the volume of the figure generated in terms of h and r.

5 a) $\int_0^{0.5\pi} \cos^3 x\,dx$

$= \int_0^{0.5\pi} \cos^2 x \cdot \cos x\,dx$ From $\sin^2 x + \cos^2 x = 1$.
$\cos^2 x = 1 - \sin^2 x$.

$= \int_0^{0.5\pi} (1 - \sin^2 x)\cos x\,dx$ let $\sin x = u$
$\frac{du}{dx} = \cos x$.
$\frac{du}{\cos x} = dx$.

$= \int_0^{0.5\pi} (1 - \sin^2 x)\frac{du}{\cancel{\cos x}} \cdot \cancel{\cos x}$

but $\sin x = u$

$= \int (1 - u^2)\,du = u - \frac{u^3}{3} + c.$

$= \sin x - \frac{\sin^3 x}{3}\Big|_0^{0.5\pi}$ $0.5\pi = 90°$.

$= \left[\sin 90° - \frac{\sin^3 90}{3}\right] - \left[\sin 0 - \frac{\sin^3 0}{3}\right]$

$= \left(1 - \frac{1}{3}\right] - 0 = \frac{2}{3}.$

9) $\frac{dy}{dx} = 3x - \frac{1}{x^2}$ when $x \neq 0$

$y = \int \left(3x - \frac{1}{x^2}\right) dx$.

$= \int \left(3x\,dx - \frac{dx}{x^2}\right)$

$= \left[\frac{3x^2}{2} + \frac{1}{x}\right] + c$.

$\therefore y = \frac{3x^2}{2} + \frac{1}{x} + c$.

$\therefore$ equation of the curve is $y = \frac{3x^2}{2} + \frac{1}{x} + c$.

10e) $V = \pi \int_a^b x^2 dy$

where $a = 0$, $b = h$.

$x^2 = \frac{y^2}{m^2}$.

$\therefore$

$V = \pi \int_0^h \frac{y^2}{m^2} dy$.

$= \frac{\pi}{m^2}\left[\frac{y^3}{3}\right]_0^h$

$= \frac{\pi}{3m^2}\left(h^3 - 0^3\right)$

$= \frac{\pi h^3}{3m^2}$.

but $h = mx$, $x = r$

$\Rightarrow m = \frac{h}{x} = \frac{h}{r}$.

$V = \frac{\pi h^3}{3\left(\frac{h^2}{r^2}\right)} = \frac{\pi h^3 r^2}{3h^2} = \frac{\pi h r^2}{3}$

$\therefore$ The volume generated is

$V = \frac{\pi h r^2}{3}$.

Extract 5.2 shows a solution from a candidate who was able to apply the concept of integration correctly.

2016 PAST PAPERS

5. (a) If $f'(z) = ze^{z^2}$ and $f(0) = \frac{9}{2}$, find $f(z)$.

(b) (i) Calculate the area of the region bounded by the curve $y = x^2 + 3x - 18$ and the line $y = 0$.

(ii) The marginal cost of producing x units of a product is given by the equation $c'(x) = 0.6x^2 + 4x$. If the fixed cost is 30000/-, find the cost function.

5 (a) Soln.

Given, $f'(z) = ze^{z^2}$ and $f(0) = 9/2$

$$\int f'(z) = \int ze^{z^2} dz$$

let $a = z^2$

$$\frac{da}{dz} = 2z$$

$$\frac{da}{2} = z\,dz$$

from

$$\int f'(z) = \int e^{z^2} \cdot z\,dz$$

$$= \int e^{a} \cdot \frac{da}{2}$$

$$= \frac{1}{2}\int e^{a} da$$

$$f(z) = \frac{1}{2}e^{a} + c \quad \text{But } a = z^2$$

$$f(z) = \frac{1}{2}e^{z^2} + c$$

$$f(0) = \frac{1}{2}e^{0^2} + c = 9/2$$

$$\frac{1}{2}e^{0} + c = 9/2$$

$$\frac{1}{2} + c = 9/2$$

5 (a) $\frac{1}{2} + c = 9/2$

$c = 9/2 - 1/2$

$c = \frac{9-1}{2} = 8/2$

$c = 4$

$\therefore f(x) = \frac{1}{2}e^{x^2} + 4$

5 (b). The area of the region bounded by the curve.

$y = x^2 + 3x - 18$.

$y = 0$ (x-axis).

To draw find the intercepts.

$y = x^2 + 3x - 18$

$x^2 + 3x - 18 = 0$.

Solve by using calculator.

$x_1 = 3$ $x_2 = -6$.

5(b)(i) To sketch the graph.

The y-intercept.

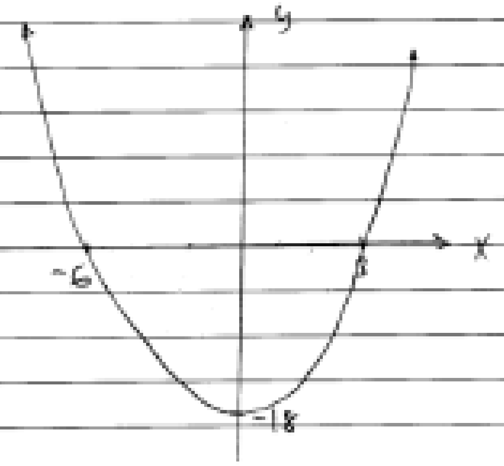
$x = 0$.

$y = x^2 + 3x - 18$.

$x = 0$.

$y = 0^2 + 3(0) - 18$

$y = -18$.



Then.

$$A = \int_a^b f(x)\,dx$$

$$A = \int_{-6}^{3} (x^2 + 3x - 18)\,dx.$$

$$A = \left[\frac{x^3}{3} + \frac{3x^2}{2} - 18x\right]_{-6}^{3}.$$

$$A = \left(\frac{3^3}{3} + \frac{3(3)^2}{2} - 18(3)\right) - \left(\frac{(-6)^3}{3} + \frac{3(-6)^2}{2} - 18(-6)\right)$$

5(b)(i) $A = (9 + 13.5 - 54) - \left(\frac{-216}{3} + 54 + 108\right)$.

$A = (-31.5) - (-72 + 54 + 108)$.

$A = (-31.5) - (90)$

$A = |-121.5|$

$A = 121.5$ square units.

5 (b) (ii) Firm

$c'(x) = 0.6x^2 + 4x$

$\int c'(x) = \int (0.6x^2 + 4x)\,dx$

$C(x) = \frac{0.6x^3}{3} + \frac{4x^2}{2} + C$

$C(x) = \frac{0.6x^3}{3} + 2x^2 + C$

But $c = 30{,}000$

$\therefore$ $C(x) = 0.2x^3 + 2x^2 + 30{,}000$

$\therefore$ Cost function $= \underline{0.2x^3 + 2x^2 + 30000}$

Extract 5.2 shows that, the candidate had an adequate knowledge of derivatives and anti-derivatives and was able to apply it appropriately.

2015 PAST PAPERS

5. (a) Evaluate the following integrals:

(i) $\int x(x+9)^{1/2}dx$,

(ii) $\int x\cos(5x+9)dx$.

(b) Given that $\int_1^4\left(3x^2 - ax - \frac{16}{x^2}\right)dx = 40$, find the value of the constant a.

(c) Sketch the graph of the curve $y = x^3 - 3x^2 + 2x$ and hence find the area bounded by the curve and the x-axis.

5.

(a)

(i) $\int x\,(x+9)^{1/2}\,dx$.

let

$(x+9)^{1/2} = u$

$x+9 = u^2$.

$x = u^2 - 9$

$dx = 2u\,du$

$\int (u^2-9)\,u \cdot 2u\,du$

$2\int (u^2-9)\,u^2\,du$

$2\int u^4 - 9u^2\,du$

$2\int u^4\,du - 18\int u^2\,du$

$\frac{2u^5}{5} - \frac{18u^3}{3} + C$. but $u = (x+9)^{1/2}$.

$\therefore \int x\,(x+9)^{1/2}\,dx = \frac{2}{5}(x+9)^{5/2} - 6(x+9)^{3/2} + C$

5(b)

$$\int_2^4 \left(3x^2 - ax - \frac{16}{x^2}\right) dx = 40$$

$$\int_2^4 3x^2\,dx - \int_2^4 ax\,dx - \int_2^4 \frac{16}{x^2}\,dx = 40$$

$$\left[x^3\right]_2^4 - a\left[\frac{x^2}{2}\right]_2^4 + 16\left[\frac{1}{x}\right]_2^4 = 40$$

$$(4^3 - 2^3) - \frac{a}{2}(4^2 - 2^2) + 16\left(\frac{1}{4} - \frac{1}{2}\right) = 40$$

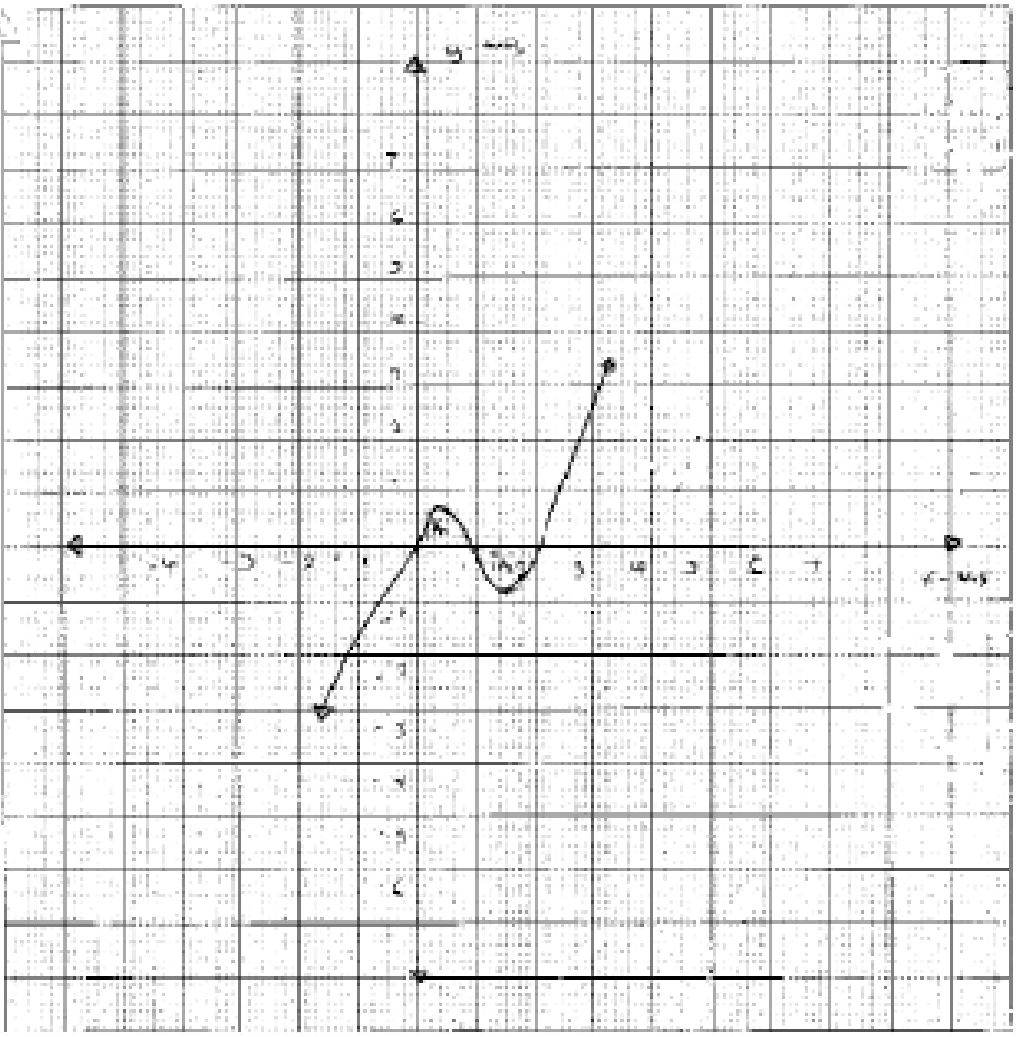
$$56 - 6a - 4 = 40$$

$$52 - 6a = 40$$

$$6a = 12$$

$$\therefore a = 2.$$

5(c)



5.(b)

Area bounded by the Curve

$A = \int_a^b y \, dx$

$A = \int_0^1 x^3 - 3x^2 + 2x\,dx + - \int_1^2 x^3 - 3x^2 + 2x\,dx.$

$A = \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 + - \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2$

$A = 0.25 + 0.25$ unit square

$\therefore A = 0.5$ unit square.

In Extract 5.2, the candidate demonstrated good understanding on how to evaluate definite and indefinite integrals and how to find area under a curve.

6.0 Statistics

2021 PAST PAPERS

6. A biology teacher asked each of her 20 students to bring a grasshopper as a specimen for practical and the length of each grasshopper was recorded in centimeters as follows:

1	3	5	4	5	2	4	2	4	2
4	2	5	3	2	3	2	3	1	3

(a) Prepare frequency distribution table (do not group the data).

(b) Calculate median of the data.

(c) Use assumed mean $A = 3$ and coding method to calculate mean and standard deviation correct to 2 decimal places.

b/ median of the data.

By using the ungrouped data formula

arranged order

1, 1, 2, 2, 2, 2, 2, 2, 3 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5

$$\text{median} = \frac{\left(\frac{1}{2}N\right)^{th} + \left(\frac{1}{2}N + 1\right)^{th}}{2}$$

$$= \frac{10^{th} + 11^{th}}{2} = \frac{3+3}{2} = 3$$

Hence the median is 3

Extract 6.1: A sample of correct response to part (b) of question 6

06. (c). Asumed mean $\bar{x} = 3.$

x	f_i	$d_i\,(x-\bar{x})$	$u = d_i/c$	$f_i u^2$	$f_i u_i$	$f_i u_i^2$
1	2	$\cancel{2}$ -2	-2	4	-4	8
2	6	$\cancel{6}$ -1	-1	1	-6	6
3	5	$\cancel{5}$ 0	0	0	0	0
4	4	$\cancel{4}$ 1	1	1	4	4
5	3	$\cancel{3}$ 2	2.	4.	6	12
summation	20				0	30

$$\text{mean} = A + c\,\frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f}$$

$$= 3 + \frac{1\,(0)}{20} = 3.$$

Hence mean $= 3.00$

$$\operatorname{Var}(x) = c^2\left[\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2\right]$$

$$= 1^2\left(\frac{1}{20} \times 30. - \frac{1}{20}\,(0)\right)$$

$$= 1.5$$

standard deviation $= \sqrt{\operatorname{Var}(x)}$

$$= \sqrt{1.5}$$

$$\sigma(x) = 1.22.$$

Extract 6.2: A sample of correct response to part (c) of question 6

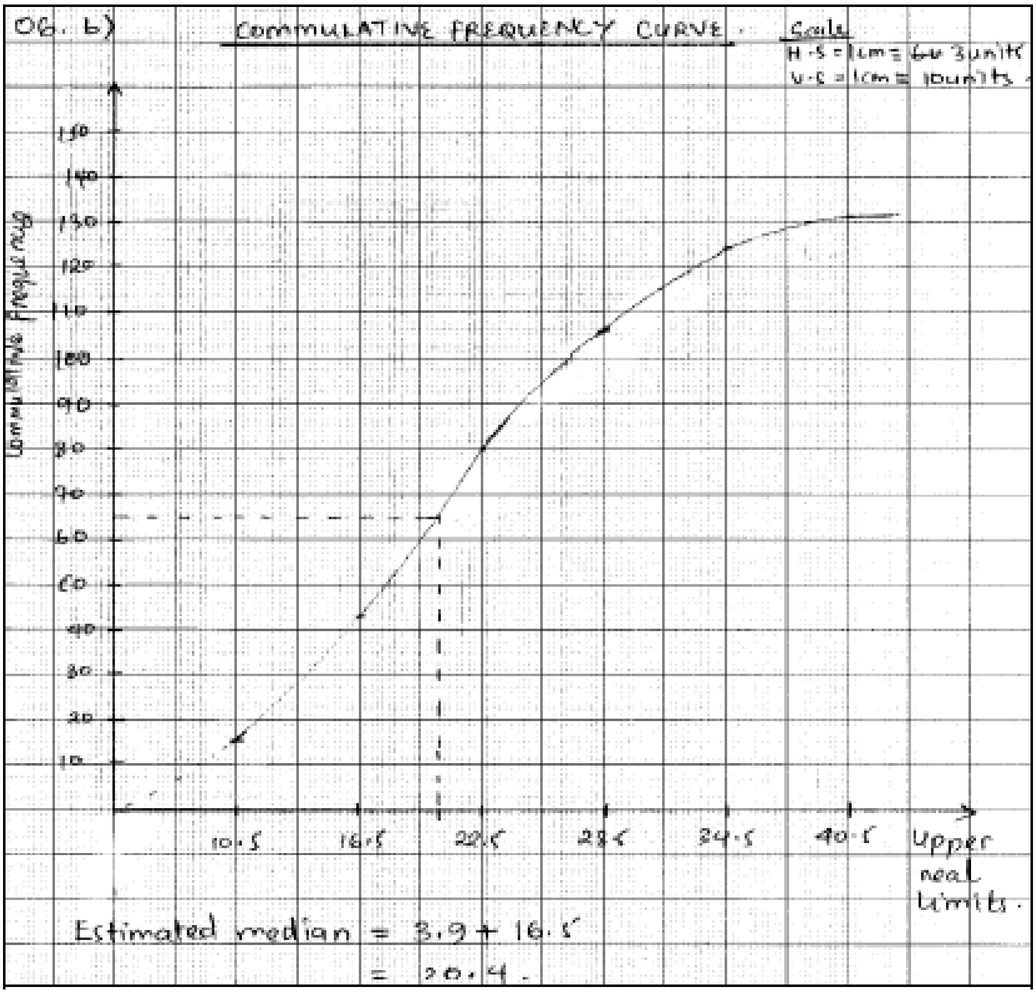
2020 PAST PAPERS

6. The following table shows litres of milk produced by 131 cows each day.

Litres of milk	5-10	11-16	17-22	23-28	29-34	35-40
Number of cows	15	28	37	26	18	7

(a) Estimate the mode.

(b) Draw the cumulative frequency curve and use it to estimate the median.



Extract 6.1: *A sample of correct solution for part (b) of question 6.*

2019 PAST PAPERS

6. The scores of 22 students in one of the Basic Applied Mathematics test are 49, 64, 38, 46, 60, 68, 46, 42, 62, 38, 68, 57, 63, 76, 51, 55, 66, 63, 58, 47, 59 and 54.

(a) Summarize the above data in a frequency distribution table using class interval of 5 and lowest limit of 35.

(b) By using assumed mean A = 57, find the mean score (give your answers to five significant figures).

(c) Find the interquartile range in one decimal place.

6. a) Frequency distribution table

class interval	f	x	d	fd	c.f
35 - 39	2	37	-20	-40	2
40 - 44	1	42	-15	-15	3
45 - 49	4	47	-10	-40	7
50 - 54	2	52	-5	-10	9
55 - 59	4	57	0	0	13
60 - 64	5	62	5	25	18
65 - 69	3	67	10	30	21
70 - 74	0	72	15	0	21
75 - 79	1	77	20	20	22
	$\Sigma f = 22$			$\Sigma fd = -30$	

6. b) Mean $(\bar{x}) = A + \frac{\Sigma fd}{\Sigma f}$

$\bar{x} = 57 + \frac{-30}{22}$

$\bar{x} = 55.636$

$\therefore$ Mean = 55.636

c) Interquartile Range = $Q_3 - Q_1$

$Q_3 = L + \left(\frac{\frac{3N}{4} - N_b}{N_w}\right)i$

$Q_3 = 59.5 + \left(\frac{16.5 - 13}{5}\right)5$

$Q_3 = 63.$

$Q_1 = L + \left(\frac{\frac{N}{4} - N_b}{N_w}\right)i$

$Q_1 = 44.5 + \left(\frac{5.5 - 3}{4}\right)5$

$Q_1 = 47.625$

$I.Q.R = 63 - 47.625 = 15.4$

$\therefore$ Interquartile Range = 15.4

Extract 6.1: *A sample of the candidate's correct responses in question 6*

2018 PAST PAPERS

6. The masses of 50 apples measured to the nearest grams are as follows:

86	101	114	118	87	92	93	116	105	102	97	93	101
111	96	117	100	106	118	101	107	96	101	102	104	92
99	107	109	105	113	100	103	108	92	98	95	100	103
110	113	99	106	116	101	105	86	88	108	92		

From these data;

(a) Construct a frequency distribution table using equal class intervals of width 5 grams taking the lower class boundary of the first interval as 84.5.

(b) Draw the histogram to illustrate the data.

(c) Calculate the mode by using the appropriate formula.

6 a) FREQUENCY DISTRIBUTION TABLE

Class Interval	Class mark (x)	Frequency (f)
85 - 89	87	4
90 - 94	92	6
95 - 99	97	7
100 - 104	102	13
105 - 109	107	10
110 - 114	112	5
115 - 119	117	5

b) The Histogram is on the Graph Papers

c) From $\text{mode} = L + \left(\frac{t_1}{t_1+t_2}\right)i$

where modal class → (100 - 104)

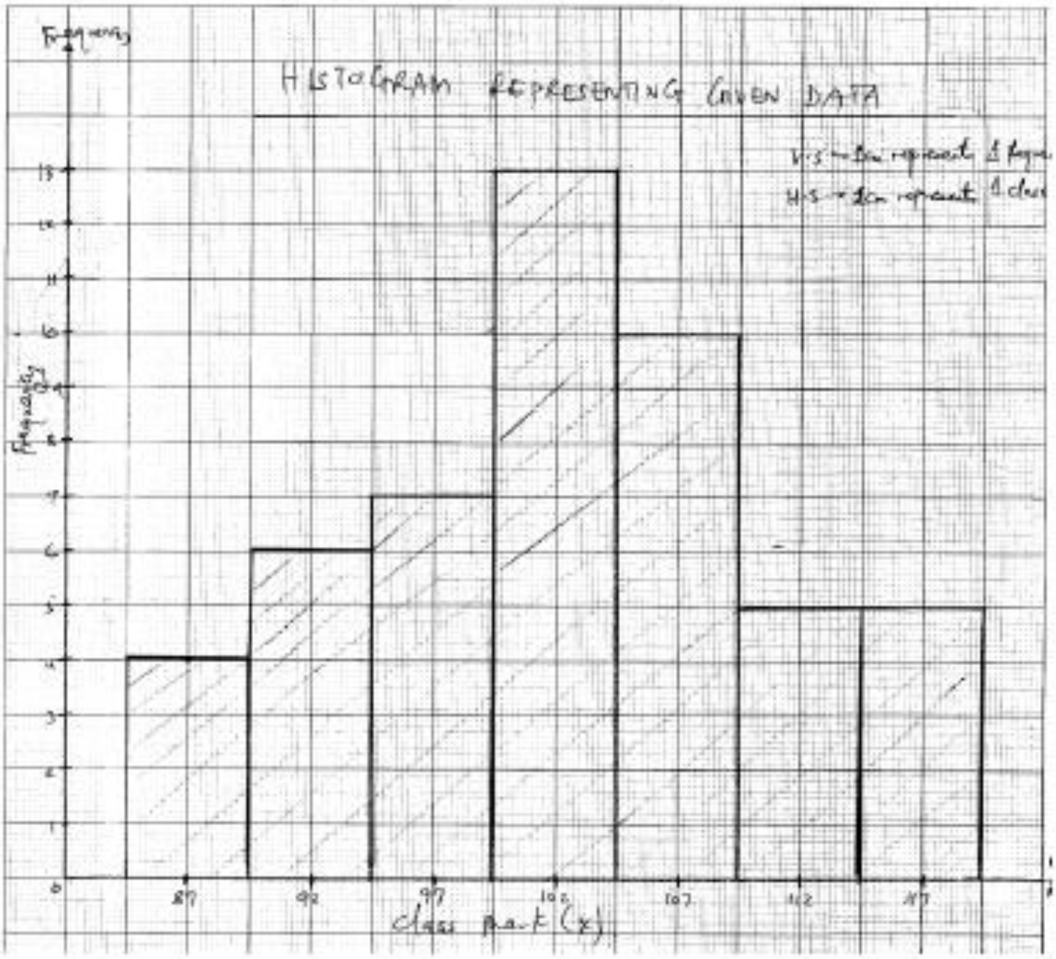
and L → lower class boundary = 99.5

$t_1 = 13-7 = 6 \quad i = 5$

$t_2 = 13-10 = 3$

The mode $= 99.5 + \left(\frac{6}{6+3}\right)5$

∴ The mode is 102.833



Extract 6.1 shows a sample solution of a candidate who correctly prepared the frequency distribution table; drew the histogram and applied the formula to find the mode.

2017 PAST PAPERS

6. (a) Define the following terms as they are used in statistics:
   (i) Range,
   (ii) Class size.

   (b) The manager of Gold Mining Company recorded the number of absent workers in 52 working days as shown in the table below;

Number of absent workers	5-9	10-14	15-19	20-24	25-29
Frequency	6	9	18	16	3

   Use these data to construct the cumulative frequency curve.

   (c) The following data shows time in seconds which was recorded by a teacher in a swimming competition of students from Precious Beach High School.

32	31	27	30	29	27	25	29	26	26	32
32	25	31	31	27	24	26	26	32	33	28
26	33	24	28	32	29	32	24	31	27	30
31	25	29	25	27	30	26				

   (i) Prepare the frequency distribution using the class intervals of 0-4, 5-9 etc.
   (ii) Determine the standard deviation.

6 (a) i/ Range

This is the difference between the highest value and the lowest value of given data.

Range = H − L

where H − Highest value

L − Lowest value.

ii/ Class size.

This is the difference between the upper boundary and the lower boundary of a given class interval.

It is the difference of upper real limit and the lower real limit.

(b) Table of workers.

Number of Absent workers	f	x	C.F	Upper Boundary
5 − 9	6	7	6	9.5
10 − 14	9	12	15	14.5
15 − 19	18	17	33	19.5
20 − 24	16	22	49	24.5
25 − 29	3	27	52	29.5

6 (c) Table of Values (Distribution Table).

C Interval	f	x	fx	$x-\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
20–24	3	22	66	−6.625	43.891	131.673
25–29	21	27	567	−1.625	2.641	55.461
30–34	16	32	512	3.375	11.391	182.256
	$\Sigma f = 40$		$\Sigma fx = 1145$			$\Sigma f(x-\bar{x})^2 = 369.39$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1145}{40} = 28.625.$$

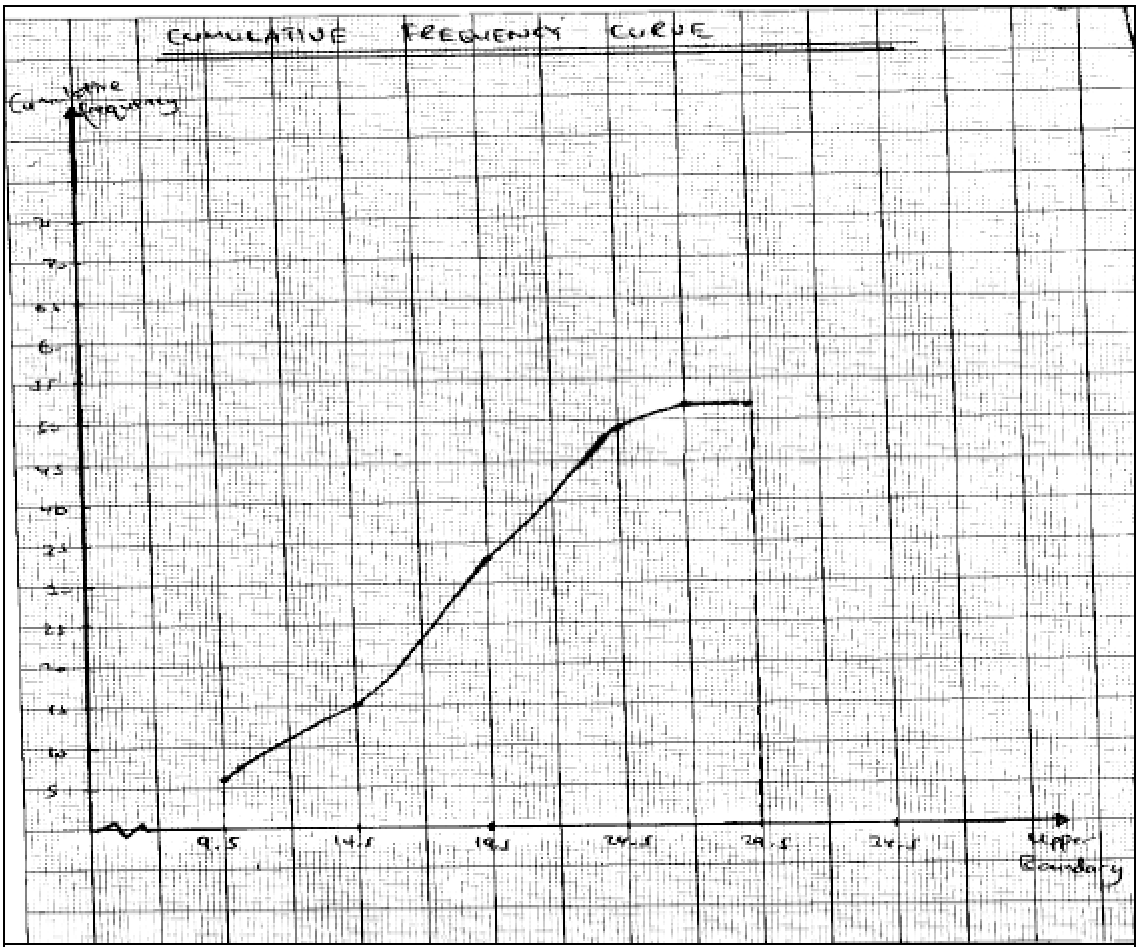
$$\text{S.Deviation} = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$$

$$= \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$$

$$= \sqrt{\frac{369.39}{40}}$$

$$= 3.0389$$

$\therefore$ Standard Deviation $= 3.0389.$



Extract 6.1 is a solution from a candidate who demonstrated good understanding on the topic of Statistics.

2016 PAST PAPERS

6. (a) The number of motorcycle accidents which were recorded in one region in Tanzania for seven weeks during November and December 2013 were 14, 2, 12, 4, 10, 6 and 8. Find,

(i) The mean number of accidents

(ii) The variance of the accidents.

(b) The table below shows the height of avocado trees in an Orchard,

Height $(\times 10^{-1} m)$	2-6	7-11	12-16	17-21	22-26	27-31
Frequency	12	14	18	15	4	8

(i) Use the data to draw the histogram

(ii) Estimate the mode from the histogram in b (i) above.

6 a (i) To find mean.

Given data,

14, 2, 12, 4, 10, 6, 8

From

$$\text{Mean}(\bar{x}) = \frac{\Sigma x}{N}$$

$$\bar{x} = \frac{14+2+4+12+10+6+8}{7}$$

$$\bar{x} = 8$$

$\therefore$ Mean $= 8$

(ii) Variance

From

$$\text{Variance} = \frac{\Sigma (x-\bar{x})^2}{N}$$

To prepare the table of values of $x, \bar{x}, x-\bar{x}$ and N

x	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
14	8	6	36
2	8	-6	36
12	8	4	16
4	8	-4	16
10	8	2	4
6	8	-2	4
8	8	0	0
Σ			112

$$\therefore \text{Variance} = \frac{\Sigma (x-\bar{x})^2}{N}$$

$$= \frac{112}{7}$$

$$\therefore \text{Variance} = 16$$

6 b (i) To draw the histogram

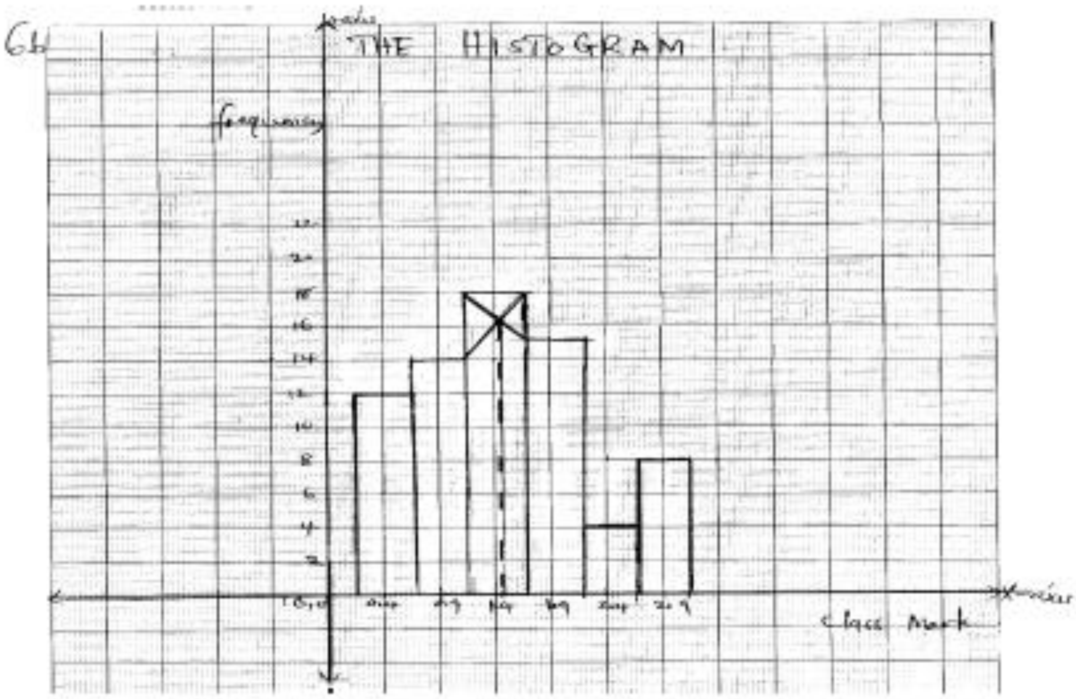
Histogram - is the graph drawn between frequency against class Mark

. Consider the table below

Height	class Mark (x)	frequency
0.2 – 0.6	0.4	12
0.7 – 1.1	0.9	14
1.2 – 1.6	1.4	18
1.7 – 2.1	1.9	15
2.2 – 2.6	2.4	4
2.7 – 3.1	2.9	8

(ii) From the histogram

Mode = 1.43



In Extract 6.1, the candidate answered all parts of the question correctly showing all important steps.

2015 PAST PAPERS

6. The following were the scores obtained by 22 students from Sarawak Secondary School in a mathematics classroom test:
49, 64, 38, 60, 46, 64, 68, 42, 38, 68, 57, 63, 76, 51, 54, 66, 62, 63, 58, 59, 47, 55.

(a) Summarize the scores in a frequency table with equal class intervals of size 5. Take the lowest limit to be 35.

(b) Find the mean score by using the data in part (a).

(c) Find the interquartile range.

(d) How many students scored above the mean score?

6 (a) i) frequency table.

Class Interval	frequency	Cum freq.
35 – 39	2	2
40 – 44	1	3
45 – 49	3	6
50 – 54	2	8
55 – 59	4	12
60 – 64	6	18
65 – 69	3	21
70 – 74	0	21
75 – 79	1	22
Total	22	

6. b)

class interval	frequ.	Cum freq	class mark	fx
35 – 39	2	2	37	74
40 – 44	1	3	42	42
45 – 49	3	6	47	141
50 – 54	2	8	52	104
55 – 59	4	12	57	228
60 – 64	6	18	62	372
65 – 69	3	21	67	201
70 – 74	0	21	72	0
75 – 79	1	22	77	77
Total	22			$\Sigma fx = 1239$

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{1239}{22}$$

$$\text{Mean}(\bar{x}) = 56.32$$

6c) Interquartile range (IQR).

IQR = Upper quartile − lower quartile.

Upper quartile = $L + \left(\frac{\frac{3N}{4} - n_a}{n_w}\right) i$

Upper quartile class $\frac{3}{4} \times 22 = 16.5$

is 60–64

$L = 59.5$, $n_a = 12$

$i = 5$, $n_w = 6$

Upper quartile $= 59.5 + \left(\frac{16.5 - 12}{6}\right) \times 5$

6c. Upper quartile $= 63.25$

Lower quartile $= L + \left(\frac{\frac{N}{4} - n_a}{n_w}\right) i$

$i = 5$

lower quartile class $\left(\frac{22}{4} = 5.5\right)$

is 45–49

$L = 44.5$

$n_a = 3$

$n_w = 3$

Lower quartile $= 44.5 + \left(\frac{\frac{22}{4} - 3}{3}\right) \times 5$

Lower quartile $= 48.67$

Interquartile range (IQR) = Upper quartile − lower quartile

$IQR = 63.25 - 48.67$

$IQR = 14.58$

Interquartile range is 14.58

6d. Students who scored above the mean score 56.32 are 13 students

In Extract 6.1, the candidate was able to prepare the frequency distribution table correctly and used appropriately in calculating the mean, interquartile range and the number of students who scored above the mean score.

Form VI

7.0 Probability

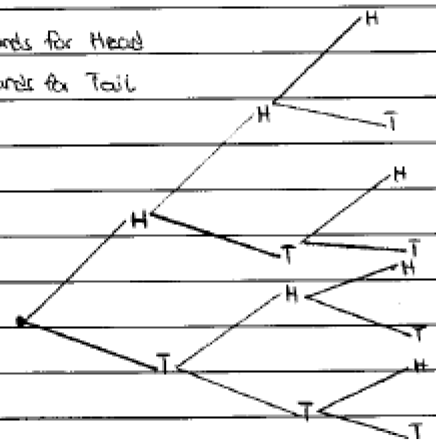
2021 PAST PAPERS

7. (a) A six sided die is thrown. Find the probability that an odd number will show up.
   (b) Three coins are tossed at once.
      (i) Use tree diagram to illustrate all possible outcomes.
      (ii) Find the probability of getting at least two heads.

b) Note

H stands for Head

T stands for Tail



i) $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$

ii) E = (atleast two heads) = $\{HHH, HHT, HTH, THH\}$

$n(E) = 4$

$P(E) = \frac{n(E)}{n(S)}$

$= 4/8 = 1/2$ ∴ Probability for at least two heads is $1/2$

Extract 7.1: A sample of correct response to part (a) of question 7

2020 PAST PAPERS

7. (a) A family has three children. Assuming equal chances for boys and g...

(i) use tree diagram to show all possible outcomes.

(ii) find the probability that two are girls and one is a boy.

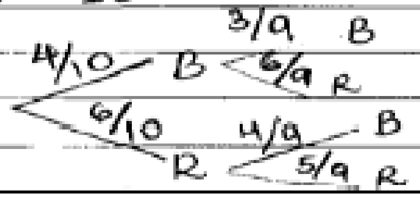
(b) A bag contains 6 red and 4 blue beads. A bead is picked out and retained and then a second bead is picked out. Find the probability that:

(i) both beads are red.

(ii) the beads are of different colours.

(c) How many odd numbers greater than 2000 and less than 3000 can be formed using the digits 1, 2, 3, 4, 5 and 6 if each digit is not repeated?

b) $n(S) = 10$



7.b)i) $P(R \cap R) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90}$

$\therefore$ Probability that both are red $= \frac{1}{3}$

ii) $P(R \cap B) = \left(\frac{4}{10} \times \frac{6}{9}\right) + \left(\frac{6}{10} \times \frac{4}{9}\right)$

$= \frac{8}{15}$

$\therefore$ Probability of different colours $= \frac{8}{15}$

Extract 7.1: *A sample of correct solution for part (a) of question 7.*

2019 PAST PAPERS

7. (a) Show that ${}^nC_r = {}^nC_{n-r}$.

(b) The events A and B are such that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cap B) = 0.1$. Use the appropriate formula to find $P(A' \cap B')$.

(c) A bag contains 8 marbles of which 3 are red and 5 are blue. One marble is drawn at random; its colour is noted and replaced in the bag. A marble is again drawn from the bag and its colour noted. Find the probability that the marble drawn will be red and blue in any order.

7. (a) Required to show

$${}^nC_r = {}^nC_{n-r}$$

from L.H.S of equation:

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

from R.H.S of equation

$${}^nC_{n-r} = \frac{n!}{(n-n+r)!\,(n-r)!}$$

$${}^nC_{n-r} = \frac{n!}{r!\,(n-r)!}$$

$${}^nC_{n-r} = \frac{n!}{(n-r)!\,r!}$$

$\therefore$ Thus L.H.S = R.H.S

$${}^nC_r = {}^nC_{n-r}$$

b) $P(A) = 0.3$ $P(B) = 0.4$ $P(A \cap B) = 0.1$

Required

$P(A' \cap B') = P(A \cup B)'$.

but

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.4 - 0.1$

$= 0.7 - 0.1$

$P(A \cup B) = 0.6$

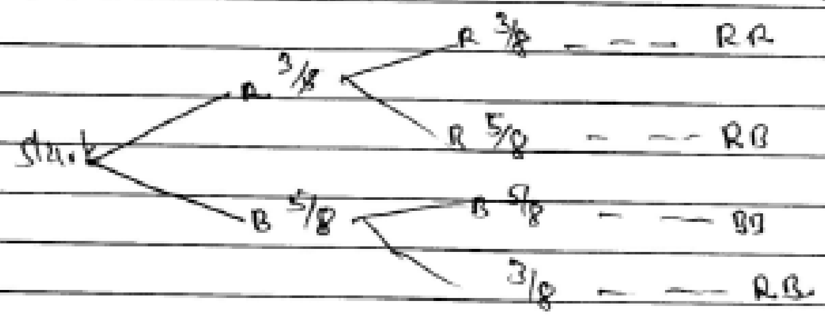
Then

$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$P(A' \cap B') = P(A \cup B)' = 1 - 0.6 = 0.4$

$\therefore P(A' \cap B') = 0.4$

7 c) by tree diagram outcome,



probability that marble drawn will be red and blue in any order is

$P(R \text{ and } B) = \left(\frac{3}{8} \times \frac{5}{8}\right) + \left(\frac{3}{8} \times \frac{5}{8}\right)$

$= \frac{15}{64} + \frac{15}{64}$

$= \frac{30}{64} = \frac{15}{32}$

$\therefore$ The probability $P(R \text{ and } B) = \frac{15}{32}$

Extract 7.2: *A sample of the candidate's correct responses in question 7*

2018 PAST PAPERS

7. (a) Verify that ${}^{8}C_3 + {}^{8}C_2 = {}^{9}C_3$.

(b) Two events A and B are such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{2}{7}$.

(i) Find $P(A \cup B)$ when A and B are mutually exclusive events.

(ii) $P(A \cap B)$ when A and B are independent events.

(c) Two students are chosen at random from a class containing 20 girls and 15 boys to form a student welfare committee. If replacement is allowed, find the probability that:

(i) both are girls.

(ii) one is a girl and the other is a boy.

7(a) Required to verify:

$${}^{8}C_3 + {}^{8}C_2 = {}^{9}C_3$$

From; L.H.S;

$${}^{n}C_r = \frac{n!}{(n-r)!\,r!}$$

$${}^{8}C_3 + {}^{8}C_2 = \frac{8!}{(8-3)!\,3!} + \frac{8!}{(8-2)!\,2!}$$

$$= \frac{8!}{5!\,3!} + \frac{8!}{6! \times 2!}$$

$${}^{8}C_3 + {}^{8}C_2 = 84 \text{ —— (i)}$$

From, R.H.S;

$${}^{9}C_3 = \frac{9!}{(9-3)!\,3!}$$

$${}^{9}C_3 = \frac{9!}{6! \times 3!}$$

$${}^{9}C_3 = 84 \text{ —— (ii)}$$

Since, L.H.S = R.H.S; Hence it verified.

7(b) Given; $P(A) = \frac{1}{3}$, $P(B) = 2/7$

(i) For Mutually Exclusive Events;

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = \frac{1}{3} + \frac{2}{7}$$

$$\therefore P(A \cup B) = 13/21$$

7 (b) (ii) For independent Events;

$P(A \cap B) = P(A) \times P(B)$

$P(A \cap B) = \frac{1}{3} \times \frac{2}{7}$

$\therefore P(A \cap B) = \frac{2}{21}$

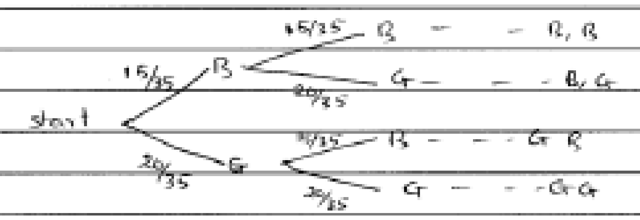
7 (c) Given;

→ 20 girls, 15 boys; let B = Boys, G = Girls;

then; $n(G) = 20$, $n(B) = 15$, $n(S) = 35$

Hence; $P(G) = \frac{20}{35}$, $P(B) = \frac{15}{35}$

Consider tree diagram below



(i) Both are Girls; $P(G, G)$

$P(G, G) = \frac{20}{35} \times \frac{20}{35} = \frac{16}{49}$

∴ The probability that Both are Girls is $\frac{16}{49}$.

(ii) One Girl and the other is Boy;

$P(B, G)$ or $P(G, B)$

$= \left(\frac{15}{35} \times \frac{20}{35}\right) + \left(\frac{20}{35} \times \frac{15}{35}\right)$

$P(BG \text{ or } GB) = \frac{24}{49}$

7(c) ii/ The probability that one girl and the other is a boy is $\frac{24}{49}$

Extract 7.1 illustrates a sample solution of a candidate who performed well in this question.

2017 PAST PAPERS

7. (a) If $P(n,4) = 42P(n,2)$

(i) Find n,

(ii) Evaluate $P(n,2)$ and $P(n,4)$.

(b) Events A, B and C are such that A and B are independent, while B and C are mutually exclusive. If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{3}$, find;

(i) $P(A \cap B)$,

(ii) $P(A \cup B)$,

(iii) $P(A \cup C)$.

7(a)(i)

$P(n,4) = 42P(n,2).$

${}^nP_4 = 42({}^nP_2)$

$\frac{n!}{(n-4)!} = 42 \times \frac{n!}{(n-2)!}$

$\frac{n!}{(n-4)!} = 42 \times \frac{n!}{(n-2)(n-3)(n-4)!}$

$\frac{1}{1} = \frac{42}{(n-2)(n-3)}$

$(n-2)(n-3) = 42.$

$n^2 - 3n - 2n + 6 = 42.$

$n^2 - 5n + 6 - 42 = 0.$

$n^2 - 5n - 36 = 0.$

$n^2 - 9n + 4n - 36 = 0.$

$n(n-9) + 4(n-9) = 0.$

$(n-9)(n+4) = 0.$

$n - 9 = 0.$ or $n + 4 = 0$

$n = 9.$ $\quad n = -4$

but n can't be negative

$\therefore n = 9$

7(a)(ii)

$= \frac{9!}{(9-2)!} = \frac{9!}{7!}$

$= \frac{9 \times 8 \times 7!}{7!}$

$= 9 \times 8 = 72$

$\therefore P(n,2) = 72.$

$P(n,4)$

$= {}^nP_4$

$= \frac{n!}{(n-4)!}$

$= \frac{9!}{(9-4)!}$

$= \frac{9!}{5!}$

$= \frac{9 \times 8 \times 7 \times 6 \times 5!}{5!}$

$= 3024.$

$\therefore P(n,4) = 3024$

7(b) $P(A) = 1/2$ $\quad P(B) = 1/4$ $\quad P(C) = 1/3$.

(i) $P(A \cap B) = P(A) \times P(B)$

$= \frac{1}{2} \times \frac{1}{4}. = \frac{1}{8}.$

$P(A \cap B) = 1/8$

7(b)

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{4} - \frac{1}{8}$

$P(A \cup B) = \frac{5}{8}.$

(iii) $P(A \cup C) = P(A) + P(C)$

$= \frac{1}{2} + \frac{1}{3}.$

$P(A \cup C) = \frac{5}{6}.$

In Extract 7.2, the candidate was able to correctly apply the permutation formula and probability concepts in answering this question.

2016 PAST PAPERS

7. (a) A fair coin is tossed once and the results are recorded, then a fair die is tossed.

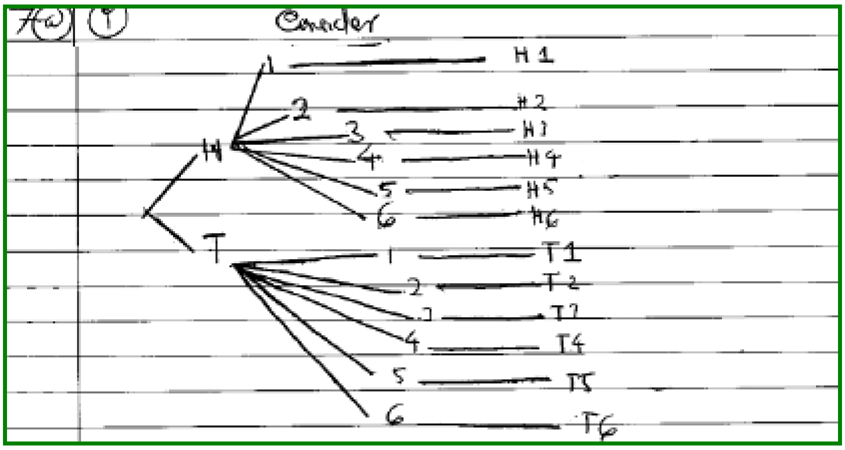
(i) Draw a tree diagram to show the possible outcomes.

(ii) Find the probability that, the outcome contains a head and an even number.

(b) Events X and Y are independent such that $P(X)=\frac{2}{3}$ and $P(X\cap Y)'=\frac{3}{4}$. Find,

(i) $P(X/Y)$

(ii) $P(X\cup Y)$.



6 —— T6

(ii) $P(\text{H and even number}) = \frac{n(\text{H and even number})}{n(S)}$

But,

$n(\text{H and even number}) = \{2H, 4H, 6H\}$

$n(S) \Rightarrow 12.$

$P(\text{H and even number}) = 3/12 = 1/4$

Extract 7.1 (a) shows that, the candidate was able to determine the sample space and used it to find the required probability.

7(b) Solution.

For independent event

$$P(X/Y) = \frac{P(X \cap Y)}{P(Y)}$$

But $P(X \cap Y) = 1 - P(X \cap Y)'$

$$= 1 - 3/4 = \underline{1/4}.$$

$$P(X/Y) = \frac{1/4}{\underline{P(Y)}}.$$

$$P(X \cap Y) = P(X) \cdot P(Y)$$

$$P(Y) = \frac{P(X \cap Y)}{P(X)} \Rightarrow \frac{1/4}{(2/3)} \Rightarrow \underline{3/8}.$$

$$P(X/Y) = \frac{1/4}{(3/8)} \Rightarrow \underline{2/3}$$

(ii) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$$= 2/3 + 3/8 - 1/4$$

$$\Rightarrow 19/24$$

In Extract 7.1 (b), the candidate correctly applied the formulae for conditional probability and independent events to answer part (b).

2015 PAST PAPERS

7. (a) If A and B are two events such that $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{8}$, find $P(A\cup B)$ and $P(A'\cap B')$.

(b) A fair die was rolled and the events A and B were recorded as follows: $A=\{1, 3, 5\}$ and $B=\{2, 3, 4, 5\}$. Find $P(A/B)$.

(c) In Section B of CSEE Basic Mathematics Examination each candidate has to choose and answer four out of six questions. How many choices are there for each candidate?

(d) A box contains 4 ripe mangoes and 9 none ripen mangoes. If two mangoes are randomly chosen from the box, find the probability that both will be ripe mangoes.

7. (a) Solution

Given; $P(A)=\frac{1}{4}$

$P(B)=\frac{1}{2}$.

$P(A\cap B)=\frac{1}{8}$.

(i) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$.

$P(A\cup B)=\frac{1}{4}+\frac{1}{2}-\frac{1}{8}$.

$=\frac{5}{8}$

$\therefore\ P(A\cup B)=\frac{5}{8}$

(ii) $P(A'\cap B')$

From De Morgan's rule

$P(A'\cap B')=P(A\cup B)'$.

From; $P(E)+P(E')=1$.

$P(A\cup B)+P(A\cup B)'=1$.

$\frac{5}{8}+P(A\cup B)'=1$.

$P(A\cup B)'=\frac{3}{8}$.

$\therefore\ P(A'\cap B')=\frac{3}{8}$.

(b) Solution

Given; A fair die with events A and B.

$A = \{1, 3, 5\}$.

$S = \{1, 2, 3, 4, 5, 6\}$

$B = \{2, 3, 4, 5\}$

$S = \{1, 2, 3, 4, 5, 6\}$.

7 (b)

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{6} = \frac{1}{2}.$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$P(B) = \frac{4}{6} = \frac{2}{3}.$$

$$P(A \cap B) = P(A) \times P(B)$$

$$= \frac{1}{2} \times \frac{2}{3}$$

$$= \frac{1}{3}.$$

$$P(A/B) = \frac{\frac{1}{3}}{\frac{2}{3}}$$

$$P(A/B) = \frac{1}{3} \times \frac{3}{2}$$

$$= \frac{1}{2}.$$

$$\therefore P(A/B) = \frac{1}{2}.$$

7 (c) Solution:

Given; Total number of questions = 6.

Questions required = 4

Number of choices are found by combination

$$^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

Where n - total number of items

r - taken items at a time.

$$^{6}C_{4} = \frac{6!}{(6-4)!\,4!}$$

$$= \frac{6!}{2! \times 4!}$$

$$= \frac{6 \times 5 \times 4!}{2 \times 1 \times 4!}$$

$$= \frac{6 \times 5}{2}$$

$= 15$ choices

$\therefore$ Number of choices for each candidate = 15.

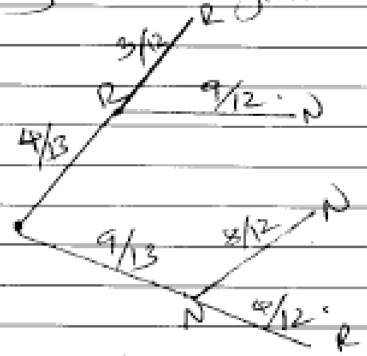
(d) Solution:

Given; Number of ripe mangoes = 4.

Number of non-ripe mangoes = 9.

Total number = 13.

7 (d) Using tree diagram.



Probability of both being ripe $= P(R) \times P(R)$

$$= \frac{4}{13} \times \frac{3}{12}$$

$$\frac{4}{13} \times \frac{1}{4} = \frac{1}{13}$$

$\therefore$ Probability $= \frac{1}{13}$

Extract 7.1 shows that the candidate understood the question and applied correctly the acquired knowledge and skills on the topic of probability to obtain the required solution.

8.0 Trigonometry

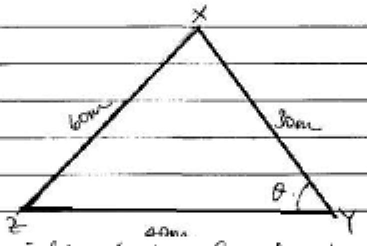
2021 PAST PAPERS

8. (a) If $\cos(x+\beta) = 2\sin(x-\beta)$, show that $\tan x = \dfrac{1+2\tan\beta}{2+\tan\beta}$.

(b) Solve the equation $2\sin\theta + \cos 2\theta = 1$ for $0^\circ \le \theta \le 180^\circ$.

(c) In a triangle XYZ, $\overline{XY} = 30$ m, $\overline{YZ} = 40$ m and $\overline{ZX} = 60$ m. Calculate the angle formed by the sides $\overline{XY}$ and $\overline{YZ}$.

8c)



Required: Angle formed by the sides $\overline{XY}$ and $\overline{YZ}$

from Cosine Rule

$(\overline{XZ})^2 = (\overline{YZ})^2 + (\overline{XY})^2 - 2(\overline{XY}\cdot\overline{YZ})\cos\theta$

$(60m)^2 = (40m)^2 + (30m)^2 - 2(40m \times 30m)\cos\theta$

$3600m^2 = 1600m^2 + 900m^2 - 2400\cos\theta$

$1100m^2 = -2400\cos\theta$

$\cos\theta = -0.458333333$

$\theta = 117.2796127^\circ \approx 117.28^\circ$

$\therefore$ The angle made by sides $\overline{XY}$ and $\overline{YZ}$ is 117.28°

Extract 8.3: A sample of correct response to part (b) of question 8

2020 PAST PAPERS

8. (a) (i) Evaluate the value of $\tan 15^0$ from sine and cosine of 45^0 and 30^0.

(ii) Given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{15}{17}$ where A and B are angles in the first and second quadrants respectively, find the exact value of $\sin(A+B)$.

(b) Prove that $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$.

8(a) $\cos A = 4/5$

and also

$\cos^2 B + \sin^2 B = 1$

$\sin^2 B = 1 - \cos^2 B$

$\sin B = \sqrt{1-\cos^2 B}$

$\sin B = \sqrt{1 - \left(\frac{15}{17}\right)^2}$

$\sin B = 8/17$

In the 2nd quandrants the sine of angle have positive values

Hence: cosine of B will be negative

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A+B) = \left(\frac{3}{5} \times \frac{-15}{17}\right) + \left(\frac{4}{5} \times \frac{8}{17}\right)$

$\sin(A+B) = \left(\frac{-9}{17}\right) + \frac{32}{85}$

$\sin(A+B) = \frac{-13}{85}$

Extract 8.2: *A sample of correct solution for part (a)(ii) of question 8.*

2019 PAST PAPERS

8. (a) Given a triangle XYZ, $\overline{XY} = 3.5$, $\overline{YZ} = 4.5$ and $\overline{ZX} = 6.5$. Calculate the size of angle Y.

(b) Solve the equation $1 + \cos\theta = 2\sin^2\theta$ for values of θ in the range 0 to 2π.

8. a/ From cosine rule,

$a^2 = b^2 + c^2 - 2bc.\cos\hat{A}$.

From the triangle,

$y^2 = x^2 + z^2 - 2xz.\cos\hat{Y}$

$\overline{XZ}^2 = \overline{YZ}^2 + \overline{XY}^2 - 2(\overline{YZ})(\overline{XY})\cos y^\circ$

$6.5^2 = 4.5^2 + 3.5^2 - 2(4.5)(3.5)\cos y^\circ$

$2(4.5)(3.5)\cos y^\circ = 4.5^2 + 3.5^2 - 6.5^2$

$y = \cos^{-1}\left[\dfrac{4.5^2 + 3.5^2 - 6.5^2}{2(4.5)(3.5)}\right]$

$\therefore$ Angle $y = 108.03^\circ$

b/ Given $1 + \cos\theta = 2\sin^2\theta$ $\qquad \sin^2\theta + \cos^2\theta = 1$

$1 + \cos\theta = 2[1 - \cos^2\theta]$

$1 + \cos\theta = 2 - 2\cos^2\theta$

$2\cos^2\theta + \cos\theta - 1 = 0$, solving,

$\cos\theta = 0.5$ or -1

$\Rightarrow$ when $\cos\theta = 0.5$

$\theta = \cos^{-1} 0.5$

$\theta_1 = 60^\circ$ ($= \pi/3$ rad)

Since cosine is positive in fourth quadrant,

$\theta_2 = 360^\circ - 60^\circ$

$\theta_2 = 300^\circ$ ($= 15/9\,\pi$ rad)

$\Rightarrow$ when $\cos\theta = -1$

$\theta = \cos^{-1} -1$

$\theta_3 = 180^\circ$ ($= \pi$ rad)

$\therefore$ Values of θ are π, $15/9\,\pi$ and $\pi/3$ rad.

Extract 8.2: *A sample of the candidate's correct responses in question 8*

2018 PAST PAPERS

8. (a) (i) Without using a calculator, find the value of $\cos 15^0$.

(ii) Prove that $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$.

(iii) Sketch the graph of $f(x)=\sin x$, where $-2\pi \le x \le 2\pi$.

(b) Solve the equation $\cos 2x+\sin^2 x=0$, where $0^0 \le x \le 360^0$.

8(a) (i) Required;

$\cos 15^\circ$

From; $\cos 15^\circ = \cos(45^\circ - 30^\circ)$

But; as per compound Angle formula;

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

let; $A = 45^\circ$, $B = 30^\circ$

$\cos(45^\circ - 30^\circ) = \cos 45 \cos 30 + \sin 45 \sin 30$

but from special angles;

$\cos 45^\circ = \frac{\sqrt{2}}{2}$, $\cos 30 = \frac{\sqrt{3}}{2}$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$, $\sin 30 = \frac{1}{2}$

Hence; $\cos 15 = \left(\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2} \times \frac{1}{2}\right)$

$\cos 15^\circ = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\therefore \cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$

8(a) i/ Required to prove;

$$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$$

From L.H.S;

$$\sin(A+B)\sin(A-B) = (\sin A\cos B + \sin B\cos A)(\sin A\cos B - \sin B\cos A)$$

$$\sin(A+B)\sin(A-B) = (\sin A\cos B)^2 - (\sin B\cos A)^2$$

$$\sin(A+B)\sin(A-B) = \sin^2 A\cos^2 B - \sin^2 B\cos^2 A$$

$$\sin(A+B)\sin(A-B) = \sin^2 A(1-\sin^2 B) - \sin^2 B(1-\sin^2 A)$$

$$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B$$

$$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = R.H.S$$

Hence proved!.

8(a) ii/ Required to sketch;

$f(x) = \sin x$ for $-2\pi \leq x \leq 2\pi$

Table of values;

x	-2π	$-\frac{3}{2}\pi$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
$f(x)=\sin x$	0	1	0	-1	0	1	0	-1	0

Refer to Graph paper;

8(b) Given; $\cos 2x + \sin^2 x = 0$ $x = ?$ at $0° \leq x \leq 360°$

From; $\cos 2x = 1 - 2\sin^2 x$

$\Rightarrow$ $1 - 2\sin^2 x + \sin^2 x = 0$

$1 - \sin^2 x = 0$

$\sin^2 x = 1$

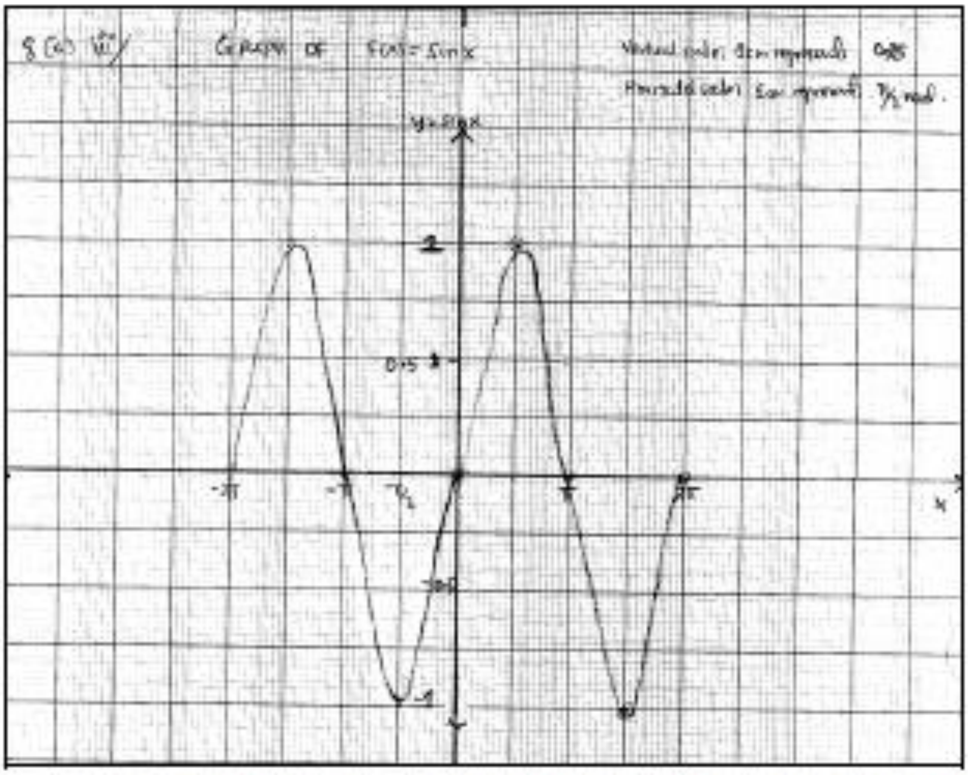
$\sin x = \pm\sqrt{1}$

$\sin x = +1$ or $\sin x = -1$

$x = \sin^{-1}(1)$; or $x = \sin^{-1}(-1)$

$x = 90°$ or $x = 270°$

Generally; $x = 90°, 270°$



Extract 8.1 shows a sample work of a candidate who answered all parts of this question correctly.

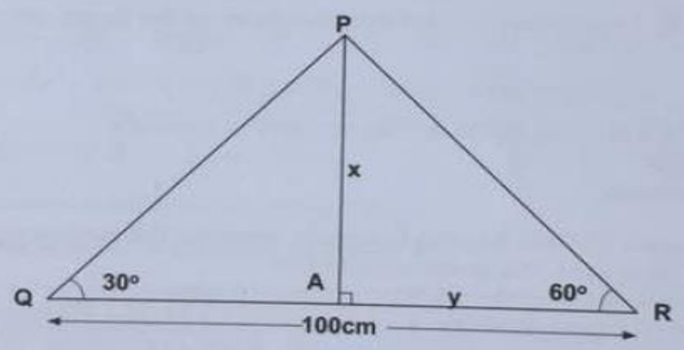
2017 PAST PAPERS

8. (a) (i) Express $\sin 3\theta$ in terms of $\sin\theta$.

(ii) Show that

$$\sqrt{\frac{(1-\cos\phi)}{1+\cos\phi}} = \operatorname{cosec}\phi - \cot\phi.$$

(b) Given the figure below,



(i) Determine the values of x and y,

(ii) Find $\sin\left(Q\hat{P}A\right)$.

8. a) (i) $\sin 3\theta$ in terms of $\sin\theta$

$\sin 3\theta = \sin(2\theta + \theta)$

$\sin(2\theta + \theta) = \sin 2\theta \cos\theta + \sin\theta\cos 2\theta$

from relation

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\therefore \sin(2\theta + \theta) = \sin 2\theta\cos\theta + \sin\theta\cos 2\theta$

But

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin(2\theta + \theta) = 2\sin\theta\cos\theta \times \cos\theta + \sin\theta(\cos^2\theta - \sin^2\theta)$

$\sin(2\theta + \theta) = 2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$

But $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin(2\theta + \theta) = 2\sin\theta(1 - \sin^2\theta) + \sin\theta(1 - \sin^2\theta) - \sin^3\theta$

$\sin(2\theta + \theta) = 2\sin\theta - 2\sin^3\theta + \sin\theta - \sin^3\theta - \sin^3\theta$

$\sin(2\theta + \theta) = 2\sin\theta + \sin\theta - 2\sin^3\theta - 2\sin^3\theta$

$\sin(2\theta + \theta) = 3\sin\theta - 4\sin^3\theta$

$\sin(2\theta + \theta) = \sin(3\theta)$

$\therefore \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

ii)

$$\sqrt{\frac{(1-\cos\phi)}{1+\cos\phi}} = \operatorname{cosec}\phi - \cot\phi$$

where $\operatorname{cosec}\phi = \frac{1}{\sin\phi}$

$\cot\phi = \frac{1}{\tan\phi}$

8. a) (ii) $\operatorname{cosec}\phi - \cot\phi = \frac{1}{\sin\phi} - \frac{1}{\tan\phi}$

But $\tan\phi = \frac{\sin\phi}{\cos\phi}$

$\frac{1}{\tan\phi} = \frac{\cos\phi}{\sin\phi}$

$\operatorname{cosec}\phi - \cot\phi = \frac{1}{\sin\phi} - \frac{\cos\phi}{\sin\phi}$

$= \frac{1-\cos\phi}{\sin\phi}$

$\sin^2\phi + \cos^2\phi = 1$

$\sin^2\phi = 1 - \cos^2\phi$

$\sin\phi = \sqrt{1-\cos^2\phi}$

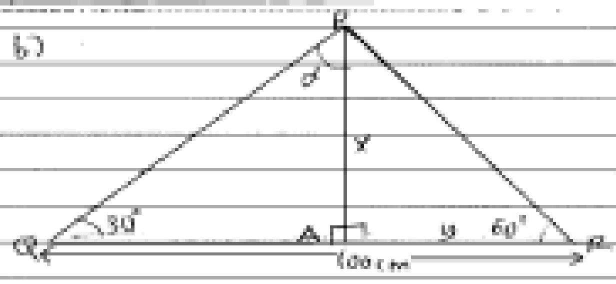
$\operatorname{cosec}\phi - \cot\phi = \frac{1-\cos\phi}{\sqrt{1-\cos^2\phi}}$

$\operatorname{cosec}\phi - \cot\phi = \sqrt{\frac{(1-\cos\phi)^2}{((1-\cos\phi)(1+\cos\phi))}}$

$= \sqrt{\frac{1-\cos\phi}{1+\cos\phi}}$

Hence shown

8. b)



$\sin = \frac{\text{Opposite}}{\text{Hypotenuse}}$, $\tan = \frac{\text{Opposite}}{\text{Adjacent}}$

$\tan 30° = \frac{x}{(100-y)}$

$x = \tan 30 (100-y)$

$x = 0.577(100-y)$

$x = 57.74 - 0.5774y$ ——— (i)

$\tan 60 = \frac{x}{y}$

8. b) $\tan 60 = \frac{x}{y}$

$x = y \tan 60°$ —— (ii)

$x = x$

$y \tan 60° = \tan 30 (100 - y)$

$y \tan 60 = 100 \tan 30 - y \tan 30$

$y \tan 60 + y \tan 30 = 100 \tan 30$

$\frac{y(\tan 60 + \tan 30)}{\tan 60 + \tan 30} = \frac{100 \tan 30}{\tan 60 + \tan 30}$

$y = \frac{100 \tan 30}{\tan 60 + \tan 30}$

$y = \frac{57.74}{1.732 + 0.5774}$

$y = 25$

$x = y \tan 60$

$x = 25 \times \tan 60$

$x = 43.30127019 \approx 43.3$

$\therefore$ $x = 43.3$ cm

$y = 25$ cm

ii) $\sin (Q\hat{P}A)$

let $Q\hat{P}A = \alpha$

$\sin \alpha = \frac{(100 - y)}{\text{hypotenuse}}$

8. b) ii) $100 - y = 100 - 25$

$= 75$

$\sin \alpha = \dfrac{75}{\text{Hypotenuse } (QP)}$

$QP = \sqrt{(r^2) + (QA)^2}$

$QP = \sqrt{(43.3)^2 + (75)^2}$

$QP = \sqrt{1875 + 5625}$

$QP = \sqrt{7500}$

$QP = 86.60254038 \approx 86.6 \text{ cm}$

$\sin \alpha = \dfrac{75}{86.6}$

$\sin \alpha = \sin(Q\hat{P}A)$

$\sin(Q\hat{P}A) = 0.866025403 \approx 0.866$

$\therefore \sin(Q\hat{P}A) = 0.866$

Extract 8.2 shows a solution from a candidate who demonstrated good understanding of the topic of Trigonometry.

2016 PAST PAPERS

8. (a) Define the following terms:

(i) Sine

(ii) Tangent.

(b) Evaluate $\tan 15^\circ + \cot 75^\circ$. Give the answer in simplest surd form.

(c) Prove that $(1 - \cos A)(1 + \sec A) = \sin A \tan A$.

Solution.

8. (a)(i). Sine – Is the trigonometric ratio which is given as the ratio of opposite side to hypotenuse side for a given angle in the right angled triangle

(a)(ii) Tangent – Is the trigonometric ratio which is given as the ratio opposite side to adjacent side for a given angle in a right angled triangle.

$$\tan = \frac{\text{opposite}}{\text{Adjacent}}$$

In Extract 8.2 (a), the candidate wrote correct definitions of sine and tangent as it was required.

8 (b) Solution

$$\tan 15^\circ + \cot 75 = \tan(45-30) + \frac{1}{\tan(45+30)}$$

$$\tan 15 + \cot 75 = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} + \frac{1 - \tan 45 \tan 30}{\tan 45 + \tan 30}$$

$$\tan 15 + \cot 75 = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} + \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$\tan 15 + \cot 75 = \frac{2 - \frac{2}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$\tan 15 + \cot 75 = \frac{2\sqrt{3} - 2}{\sqrt{3} + 1}$$

Rationalizing the denominator

$$\tan 15 + \cot 75 = \frac{2\sqrt{3} - 2}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$\tan 15 + \cot 75 = \frac{2 \times 3 - 2\sqrt{3} - 2\sqrt{3} + 2}{3 - \sqrt{3} + \sqrt{3} - 1}$$

$$\tan 15 + \cot 75 = \frac{6 - 4\sqrt{3} + 2}{2}$$

$$\tan 15 + \cot 75 = 4 - 2\sqrt{3}$$

In Extract 8.2 (b), the candidate correctly applied the compound angle formulae and substituted the correct values of special angles to find the required answer.

8 (c) $(1 - \cos A)(1 + \sec A) = (1 - \cos A)\left(1 + \frac{1}{\cos A}\right)$

$$(1 - \cos A)(1 + \sec A) = \frac{\cancel{\cos A} + 1 - \cos^2 A - \cancel{\cos A}}{\cos A}$$

$$(1 - \cos A)(1 + \sec A) = \frac{1 - \cos^2 A}{\cos A}$$

$$(1 - \cos A)(1 + \sec A) = \frac{\sin^2 A}{\cos A} \quad (\because \sin^2 A + \cos^2 A = 1)$$

$$(1 - \cos A)(1 + \sec A) = \frac{\sin A \cdot \sin A}{\cos A}$$

$$(1 - \cos A)(1 + \sec A) = \sin A \cdot \tan A$$

In Extract 8.2 (c), the candidate used correct trigonometric identities to prove that $(1-\cos A)(1+\sec A) = \sin A \tan A$.

2015 PAST PAPERS

8. (a) Without using a mathematical table or a calculator, evaluate:

(i) $\cos(165^\circ)$,

(ii) $\tan(A+B)$ given that A and B are acute angles having $\sin(A) = \frac{7}{25}$ and $\cos(B) = \frac{5}{13}$.

(b) (i) Find the values of x that satisfy the equation $\sin 2x + \cos x = 0$ for $0^\circ \leq x \leq 360^\circ$.

(ii) Verify that the solution of the equation in part (b) (i) can be obtained graphically by plotting the graph of $y = \sin 2x + \cos x$ for $0^\circ \leq x \leq 360^\circ$.

8 (a) soln

(i) $\cos 165^\circ$

$\cos 165^\circ = \cos(45 + 120)$

from

$\cos(A+B) = \cos A \cos B - \sin A \sin B$.

8 (a)

(i) $\cos 165 = \cos(45 + 120)$

$= \cos 45 \cos 120 - \sin 45 \sin 120$

but

$\cos 120 = -\cos 60^\circ$

$\sin 120 = \sin 60^\circ$

$\cos 165^\circ = -\cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$

$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$= -\frac{\sqrt{2}}{2} \times \frac{1}{2} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}$

$\cos 165^\circ = -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$\cos 165^\circ = \frac{-\sqrt{2} - \sqrt{6}}{4}$

or

$\cos 165 = \frac{-\sqrt{2}}{4}(1 + \sqrt{3})$

Therefore the value of

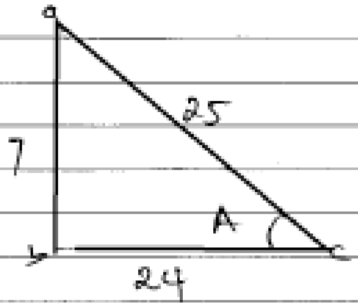
$\cos 165^\circ = \frac{-\sqrt{2} - \sqrt{6}}{4}$ or $\frac{-\sqrt{2}}{4}(1+\sqrt{3})$

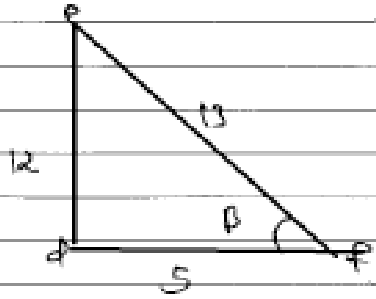
8 (a) (ii)

Data.

$\sin A = 7/25$

$\cos B = 5/13$





$\cos A = 24/25$ $\qquad$ $\sin B = 12/13$

$\sin A = 7/25$ $\qquad$ $\cos B = 5/13$

$\tan A = \frac{\sin A}{\cos A}$ $\qquad$ $\tan B = \frac{\sin B}{\cos B}$

$= 7/25 \div 24/25$ $\qquad$ $= \frac{12}{13} \div 5/13$

$\tan A = 7/24$ $\qquad$ $\tan B = 12/5$

from

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{7/24 + 12/5}{1 - 7/24 \times 12/5}$$

$$\tan(A+B) = 323/36$$

$$\therefore \tan(A+B) = 323/36$$

8 (b) (i) soln

from

$\sin 2x + \cos x = 0 \quad 0^\circ \le x \le 360^\circ$

from

$\sin 2x = 2\sin x \cos x$

$2\sin x \cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$

$2\sin x + 1 = 0$

for $\cos x = 0$

$x = \cos^{-1}(0)$

$x = 90^\circ, 270$

For

$2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \sin^{-1}(-\frac{1}{2})$

$x = 210^\circ, 330^\circ$

Thus, the value of x that satisfy the equation are

$x = 90^\circ, 210^\circ, 270^\circ$ and 330°.

8 (b) (ii) soln

Equation $y = \sin 2x + \cos x$

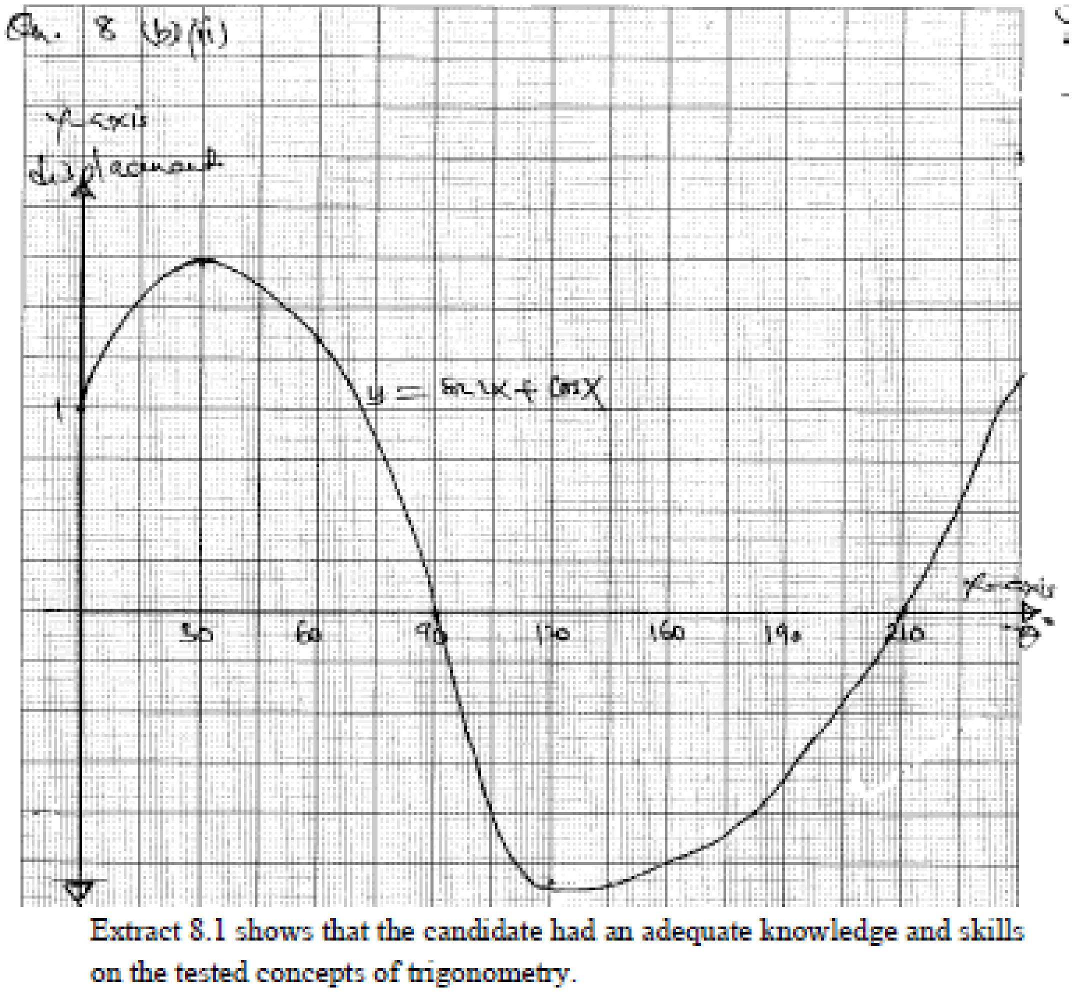
Table of values

X	90	210	270	330
$\sin 2x$	0	0.866	0	0.866
$\cos x$	0	-0.866	0	-0.866
y	0	0	0	0

For other angles

X	0°	30	45	60	120	180	270
$\sin 2x$	0	0.866	1	0.866	-0.866	0	0
$\cos x$	1	0.866	0.707	0.5	-0.5	-1	0
y	1	1.732	1.707	1.366	-1.366	-1	0

The graph is shown on the graph paper



Extract 8.1 shows that the candidate had an adequate knowledge and skills on the tested concepts of trigonometry.

9.0 Exponential and Logarithmic Functions

2021 PAST PAPERS

9. (a) If $f(x)=a^x$ where $x \in \mathbb{R}$;

(i) write down the condition(s) on a such that $f(x)$ is an exponential function.

(ii) show that $\dfrac{f(x+1)}{f(x)}=a$.

(b) Evaluate $\int_3^5 \frac{5}{(3x-8)}dx$ correct to 2 decimal places.

(c) Suppose Tsh 2000 is invested in an account which offers 7.125% compounded monthly.

(i) Express the amount A in the account as a function of the term of the investment t years.

(ii) How long will it take for the initial investment to double?

ii/ Solution

For double of investment

$2P = 2000(1.0059375)^{12t}$

$2 \times 2000 = 2000(1.0059375)^{12t}$

$2 = (1.0059375)^{12t}$

Applying log both sides.

9c

$\log 2 = 12t \log 1.0059375$

$\dfrac{\log 2}{\log 1.0059375} = 12t$

$\dfrac{117.086}{12} = \dfrac{12t}{12}$

$t = 9.76$ years.

$\therefore$ Time taken will be 9 years and 9 month

Extract 9.3: A sample of correct response to part (c) of question 9

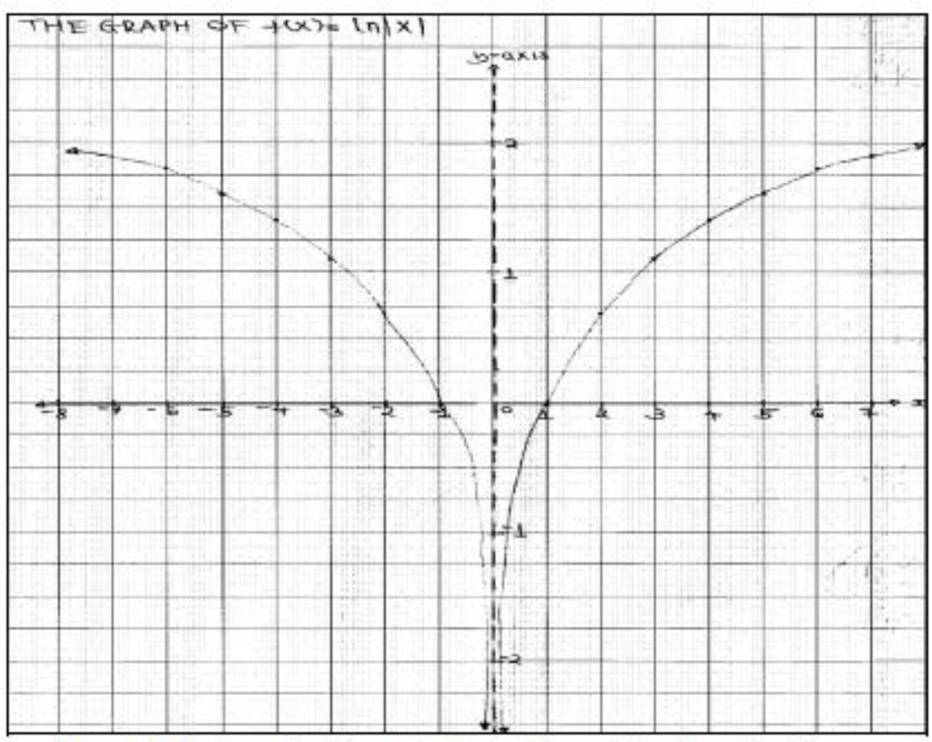
2020 PAST PAPERS

9. (a) Sketch the graphs of $f(x)=\ln|x|$ for $x \in \mathbb{R}$ and hence state its domain and range.

(b) The amount (A) of the radioactive isotope Carbon-14 at any time t is given by the formula $A(t)=A_0e^{kt}$ where A_0 is the initial amount of the element. If the half-life of the radioactive isotope Carbon-14 is about 5730 years;

(i) express the amount of Carbon-14 left from an initial N milligrams as a function of time t in years.

(ii) what percentage of the original amount of Carbon-14 left after 20,000 years?



Extract 9.3: *A sample of correct solution for part (a) of question 9.*

10.0 Matrices

2019 PAST PAPERS

9. (a) Given that $A=\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 2 & 1 & -1\end{pmatrix}$. Find:

(i) $|A|$.

(ii) A^{-1}.

(b) Using the result obtained in (a) (ii) solve $\begin{cases}x+y+z=7\\ x-y+2z=9\\ 2x+y-z=1\end{cases}$ simultaneously.

9a(i)) Solution

Given $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}$

$|A| = 1 \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} + 1 \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$

$|A| = (1-2) - (-1-4) + (1-(-2))$

$|A| = -1 + 5 + 3$

$|A| = 7$

9a(ii)) Minors of $A = \begin{pmatrix} \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \end{pmatrix}$

Minor of $A = \begin{pmatrix} -1 & -5 & 3 \\ -2 & -3 & -1 \\ 3 & 1 & -2 \end{pmatrix}$

Cofactor of $A = \begin{pmatrix} -1 & -5 & 3 \\ -2 & -3 & -1 \\ 3 & 1 & -2 \end{pmatrix} \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

Cofactor of $A = \begin{pmatrix} -1 & 5 & 3 \\ 2 & -3 & 1 \\ 3 & -1 & -2 \end{pmatrix}$

9a ii) Adjoint of $A = \begin{pmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} \times$ Adjoint matrix

$A^{-1} = \frac{1}{7} \begin{pmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix}$

9b) solution

Using:

$A^{-1} = \begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix}$

$\begin{cases} x + y + z = 7 \\ x - y + 2z = 9 \\ 2x + y - z = 1 \end{cases}$

In matrix form: $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix}$

Premultiplication:

$\begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix} \begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix}$

9b) But: $\begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/7 & 2/7 & 3/7 \\ 5/7 & -3/7 & -1/7 \\ 3/7 & 1/7 & -2/7 \end{pmatrix} \begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/7 \times 7 + 2/7 \times 9 + 3/7 \times 1 \\ 5/7 \times 7 + -3/7 \times 9 + -1/7 \times 1 \\ 3/7 \times 7 + 1/7 \times 9 + -2/7 \times 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 + 18/7 + 3/7 \\ 5 + -27/7 + -1/7 \\ 3 + 9/7 + -2/7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$

$\therefore x = 2,\ y = 1$ and $z = 4$

Extract 9.1: *A sample of the candidates' correct responses in question 9*

2018 PAST PAPERS

9. (a) Three entrepreneurs R_1, R_2 and R_3 sell seedlings of two species A and B. If the sales in one month and prices paid (in Tsh) for each type are $S = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 12 & 13 \\ 8 & 5 \\ 16 & 9 \end{bmatrix} \end{array}$ and $P = \begin{array}{c} A \\ B \end{array}\begin{bmatrix} 2500 \\ 3500 \end{bmatrix}$ respectively, find the total sales for each of the three entrepreneurs.

(b) Given matrix $A = \begin{bmatrix} 3 & -5 \\ 7 & -11 \end{bmatrix}$. Verify that $A^{-1}A = I$, where I is an identity matrix.

(c) Use Cramer's rule to solve $\begin{cases} x+y+z=6 \\ 2x+y-z=1 \\ x-y+z=2 \end{cases}$.

Qn 9. (a) Soln:

Given

Sales of Entrepreneurs

$$S = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 12 & 13 \\ 8 & 5 \\ 16 & 9 \end{bmatrix} \end{array}$$ and

Price paid (in Tsh)

$$P = \begin{array}{c} A \\ B \end{array}\begin{bmatrix} 2500 \\ 3500 \end{bmatrix}$$

Required total sales for each R_1, R_2 and R_3 which would be given as

$$SP = \begin{bmatrix} 12 & 13 \\ 8 & 5 \\ 16 & 9 \end{bmatrix}\begin{bmatrix} 2500 \\ 3500 \end{bmatrix}$$

$$= \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 30000 + 45500 \\ 20000 + 17500 \\ 40000 + 31500 \end{bmatrix}$$

$$= \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 75500 \\ 37500 \\ 71500 \end{bmatrix}$$

Hence

Total sales for R_1, R_2 and R_3 are 75500, 37500 and 71500 respectively.

9(b) Soln

Given

Matrix

$$A = \begin{bmatrix} 3 & -5 \\ 7 & -11 \end{bmatrix}$$

Required to verify that $A^{-1}A = I$

where I is Identity Matrix

9(b) Soln

Given

Matrix $A = \begin{bmatrix} -3 & -5 \\ 7 & -11 \end{bmatrix}$

Required to verify that $A^{-1}A = I$

where I is identity matrix

9(b)

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

lets find A^{-1}

Start with $|A|$

$\begin{vmatrix} 3 & -5 \\ 7 & -11 \end{vmatrix} = (-33) - (-35) = 2$

$A^{-1} = \frac{1}{|A|}\begin{bmatrix} -11 & 5 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} -11/2 & 5/2 \\ -7/2 & 3/2 \end{bmatrix}$

$A^{-1} \cdot A = \begin{bmatrix} -11/2 & 5/2 \\ -7/2 & 3/2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 7 & -11 \end{bmatrix}$

$= \begin{bmatrix} -33/2 + 35/2 & 55/2 - 55/2 \\ -21/2 + 21/2 & 35/2 - 33/2 \end{bmatrix}$

$= \begin{bmatrix} 2/2 & 0/2 \\ 0/2 & 2/2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Hence $A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ and

Verified

9(c) To get y Interchange $\begin{bmatrix}1\\1\\-1\end{bmatrix}$ with $\begin{bmatrix}6\\1\\2\end{bmatrix}$

find it's determinant and then divide by $|G|$.

$$y = \begin{vmatrix}1 & 6 & 1\\2 & 1 & -1\\1 & 2 & 1\end{vmatrix} \div (-6)$$

$$y = 1\begin{vmatrix}1 & -1\\2 & 1\end{vmatrix} - 6\begin{vmatrix}2 & -1\\1 & 1\end{vmatrix} + 1\begin{vmatrix}2 & 1\\1 & 2\end{vmatrix} \div (-6)$$

$$y = 3 - 18 + 3 \div (-6)$$

$$= (-12) \div (-6)$$

$$y = 2$$

To get z Interchange $\begin{bmatrix}1\\-1\\1\end{bmatrix}$ with $\begin{bmatrix}6\\1\\2\end{bmatrix}$

find it's determinant and then divide by $|G|$

$$z = \begin{vmatrix}1 & 1 & 6\\2 & 1 & 1\\1 & -1 & 2\end{vmatrix} \div (-6)$$

$$z = 1\begin{vmatrix}1 & 1\\-1 & 2\end{vmatrix} - 1\begin{vmatrix}2 & 1\\1 & 2\end{vmatrix} + 6\begin{vmatrix}2 & 1\\1 & -1\end{vmatrix} \div (-6)$$

$$z = 3 - 3 - 18 \div (-6)$$

$$z = (-18) \div (-6)$$

$$z = 3$$

Hence x, y and z values are 1, 2 and 3 respectively.

Extract 9.1 shows the responses from one of the candidates who did well in question 9.

2017 PAST PAPERS

9. (a) (i) Find a if $2^{2a+8}-32(2^a)+1=0$.

(ii) If $2\log_8 N = p$, $\log_2 2N = q$, and $q-p=4$, Find N.

(b) Given the system of linear equations below,

$x+y+z=7$

$x-y+2z=9$

$2x+y-z=1$

(i) Write the system of equations in matrix form.

(ii) Find the determinant and the inverse of the matrix.

(iii) Determine the values of x, y and z.

9. a)

i) Required to find a.

soln

$2^{2a+8} - 32(2^a) + 1 = 0$

$2^{2a+8} - (2^5)(2^a) + 2^0 = 0$

$(2^{2a}) \times 2^8 - (2^5)(2^a) + 1 = 0$

$(2^a)^2 \times 2^8 - (2^5)(2^a) + 1 = 0$

let $2^a = y$

then

$y^2 \times 2^8 - 2^5 y + 1 = 0$

$256y^2 - 32y + 1 = 0$

$y = 0.0625$

but

$2^a = y$

$2^a = 0.0625$

applying logarithm both side

$\log 2^a = \log 0.0625$

$a \log 2 = \log 0.0625$

$a = \frac{\log 0.0625}{\log 2}$

$a = -4$

9 a) Given

(ii) 11 $2\log_8 N = P$, $\log_2 2N = q$, $q - P = 4$

Required to find N.

soln

for

$$2\log_8 N = P$$

$$= \log_8 N^2 = P$$

$$N^2 = 8^P$$

from

$$q - P = 4$$

$$\log_2 2N - 2\log_8 N = 4$$

$$\log_2 2N - \log_8 N^2 = 4$$

from $\log_2 2 + \log_2 N - \log_8 N^2 = 4$

$$\log_2 2 + \log_2 N - 2\log_8 N = 4.$$

$$1 + \log_2 N - 2\left[\frac{1}{\log_N 8}\right] = 4.$$

$$1 + \log_2 N - 2\left[\frac{1}{\log_N 2^3}\right] = 4.$$

$$1 + \log_2 N - 2\left[\frac{1}{3\log_N 2}\right] = 4$$

$$1 + \log_2 N - 2/3\left[\frac{1}{\log_N 2}\right] = 4$$

9.

$$1 + \log_2 N - 2/3 \log_2 N = 4.$$

let $\log_2 N = X$

$$1 + x - 2/3\, x = 4$$

$$1 + 1/3\, x = 4$$

$$1/3\, x = 4 - 1$$

$$1/3\, x = 3$$

$$x = 9$$

$\log_2 N = x$

$\log_2 N = 9$

$N = 2^9$

$N = 512$

b) Given

$x + y + z = 7$

$x - y + 2z = 9$

$2x + y - z = 1$

i) In matrix form

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 1 \end{bmatrix}$$

9a) determinant of matrix

$$\text{determinant} = 1\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= 1((-1 \times -1) - 2) - 1((1 \times -1) - 4) + 1(1 - -2)$$

determinant $= 7$

Inverse of matrix

let it be matrix A

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Minor of

$$A = \begin{bmatrix} \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \end{bmatrix}$$

$$\text{Minor of } A = \begin{bmatrix} -1 & -5 & 3 \\ -2 & -3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$$

Cofactor of A $= \begin{bmatrix} -1 & 5 & 3 \\ 2 & -3 & 1 \\ 3 & -1 & -2 \end{bmatrix}$

Adjoint of A $= \begin{bmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$

Inverse $A^{-1} = \frac{1}{\det(A)} \times \mathrm{Adj}\, A$

$= \frac{1}{7}\begin{bmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$

(ii) Values of x, y, z

By multiplying A^{-1} both sides of the equation

$$\frac{1}{7}\begin{bmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -1 & 2 & 3 \\ 5 & -3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 9 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -1\times7+2\times9+3\times1 \\ 5\times7+-3\times9+-1\times1 \\ 3\times7+1\times9+-2\times1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ 7 \\ 28 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14/7 \\ 7/7 \\ 28/7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

Thus $x = 2$

$y = 1$

$z = 4$.

In Extract 9.1, the candidate correctly applied the laws of exponents and logarithms in answering part (a). In part (b) this candidate was able to solve the system of linear equations by using the inverse matrix method.

2016 PAST PAPERS

9. (a) If $f(m) = m^2 - 4m - k$, find $f(N)$ when $k = \begin{pmatrix} 11 & -5 \\ -4 & 12 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$.

(b) Use Cramer's rule to solve the following system of linear equations,

$5x - 7y + z = 11$

$6x - 8y - z = 15$

$3x + 2y - 6z = 7.$

9. (a) Soln.

$f(m) = m^2 - 4m - K \qquad K = \begin{pmatrix} 11 & -5 \\ -4 & 12 \end{pmatrix}$

$N = \begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$

$f(N) = \begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}^2 - 4\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} - \begin{pmatrix} 11 & -5 \\ -4 & 12 \end{pmatrix}$

$f(N) = \begin{pmatrix} 19 & 15 \\ 9 & 16 \end{pmatrix} - \begin{pmatrix} 8 & 20 \\ 12 & 4 \end{pmatrix} - \begin{pmatrix} 11 & -5 \\ -4 & 12 \end{pmatrix}$

$= \begin{pmatrix} 11 & -5 \\ -3 & 12 \end{pmatrix} - \begin{pmatrix} 11 & -5 \\ -4 & 12 \end{pmatrix}$

9(a). $\therefore f(N) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

In Extract 9.1 (a), the candidate made correct substitution of the values of k and N in the given equation and managed to obtain the required matrix for $f(N)$.

9 (b) Soln.

$$5x - 7y + z = 11$$

$$6x - 8y - z = 15$$

$$3x + 2y - 6z = 7$$

$$\begin{pmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 15 \\ 7 \end{pmatrix}$$

let $A = \begin{pmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{pmatrix}$

$$|A| = 5\begin{vmatrix} -8 & -1 \\ 2 & -6 \end{vmatrix} + 7\begin{vmatrix} 6 & -1 \\ 3 & -6 \end{vmatrix} + 1\begin{vmatrix} 6 & -8 \\ 3 & 2 \end{vmatrix}$$

$$|A| = 55$$

9 (b) $\underline{\text{Soln.}}$

$$5x - 7y + z = 11$$
$$6x - 8y - z = 15$$
$$3x + 2y - 6z = 7$$

$$\begin{pmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 15 \\ 7 \end{pmatrix}$$

$\underline{\text{let}}$ $A = \begin{pmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{pmatrix}$

$$|A| = 5\begin{vmatrix} -8 & -1 \\ 2 & -6 \end{vmatrix} + 7\begin{vmatrix} 6 & -1 \\ 3 & -6 \end{vmatrix} + 1\begin{vmatrix} 6 & -8 \\ 3 & 2 \end{vmatrix}$$

$$|A| = 55$$

$$\Delta x = \begin{pmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{pmatrix}$$

$$|\Delta x| = 11\begin{vmatrix} -8 & -1 \\ 2 & -6 \end{vmatrix} + 7\begin{vmatrix} 15 & -1 \\ 7 & -6 \end{vmatrix} + 1\begin{vmatrix} 15 & -8 \\ 7 & 2 \end{vmatrix}$$

$$|\Delta x| = 55$$

9 (b)

$$\Delta_y = \begin{pmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{pmatrix}$$

$$|\Delta_y| = 5\begin{vmatrix} 15 & -1 \\ 7 & -6 \end{vmatrix} - 11\begin{vmatrix} 6 & -1 \\ 3 & -6 \end{vmatrix} + 1\begin{vmatrix} 6 & 15 \\ 3 & 7 \end{vmatrix}$$

$$|\Delta_y| = -55$$

$$\Delta_z = \begin{pmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{pmatrix}$$

$$|\Delta_z| = 5\begin{vmatrix} -8 & 15 \\ 2 & 7 \end{vmatrix} + 7\begin{vmatrix} 6 & 15 \\ 3 & 7 \end{vmatrix} + 11\begin{vmatrix} 6 & -8 \\ 3 & 2 \end{vmatrix}$$

$$\Delta_z = -55$$

$$x = \frac{|\Delta_x|}{|A|} = \frac{55}{55} = 1$$

$$y = \frac{|\Delta_y|}{|A|} = \frac{-55}{55} = -1$$

$$z = \frac{|\Delta_z|}{|A|} = \frac{-55}{55} = -1,$$

$\therefore$ $x = 1$, $y = -1$ and $z = -1$.

In Extract 9.1 (b), the candidate correctly showed all the important steps of solving simultaneous equations by Cramer's rule.

2015 PAST PAPERS

9. (a) Given:

$$A=\begin{pmatrix}1 & 2\\ 3 & 4\\ -1 & 5\end{pmatrix},\quad B=\begin{pmatrix}-2 & 3 & 4\\ 3 & 2 & 1\end{pmatrix}\text{ and } C=\begin{pmatrix}3 & 5\\ 1 & 2\end{pmatrix}$$

(i) State with one reason as to whether the matrix operations AB, BA and BC are defined or not.

(ii) Find $2A+3B^T$.

(b) Verify that $\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix}=(a-b)(b-c)(c-a)$.

(c) If $D=\begin{pmatrix}a & -4 & -6\\ -8 & 5 & 7\\ -5 & 3 & 4\end{pmatrix}$ is the inverse of matrix $E=\begin{pmatrix}1 & 2 & -2\\ 3 & b & 1\\ -1 & 1 & -3\end{pmatrix}$, find the values of a and b.

9 (a) (i) - AB it is defined since number of columns of A equals to the number of rows of B

- BA it is defined since number of columns of B equals to the number of rows of A.

- BC it is not defined since number of columns of B is not equal to the number of rows of C

(ii) $2A + 3B^T$

$$2A = 2\begin{bmatrix}1 & 2\\ 3 & 4\\ -1 & 5\end{bmatrix}$$

$$2A = \begin{bmatrix}2 & 4\\ 6 & 12\\ -2 & 10\end{bmatrix}$$

9 (ii) $B^T =$

$$B = \begin{bmatrix}-2 & 3 & 4\\ 3 & 2 & 1\end{bmatrix}$$

$$B^T = \begin{bmatrix}-2 & 3\\ 3 & 2\\ 4 & 1\end{bmatrix}$$

$$3B^T = 3\begin{bmatrix}-2 & 3\\ 3 & 2\\ 4 & 1\end{bmatrix}$$

$$3B^T = \begin{bmatrix}-6 & 9\\ 9 & 6\\ 12 & 3\end{bmatrix}$$

$$2A + 3B^T = \begin{bmatrix}2 & 4\\ 6 & 12\\ -2 & 10\end{bmatrix} + \begin{bmatrix}-6 & 9\\ 9 & 6\\ 12 & 3\end{bmatrix}$$

$$= \begin{bmatrix}(2-6) & (4+9)\\ (6+9) & (12+6)\\ (-2+12) & (10+3)\end{bmatrix}$$

$$2A + 3B^T = \begin{bmatrix}-4 & 13\\ 15 & 18\\ 10 & 13\end{bmatrix}$$

$$(b)\quad (a-b)(b-c)(c-a) = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

taking the first row

$$1\begin{vmatrix} b & c \\ b^2 & c^2 \end{vmatrix} - 1\begin{vmatrix} a & c \\ a^2 & c^2 \end{vmatrix} + 1\begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix}$$

$$9\ (b) = (bc^2 - b^2c) + -(ac^2 - a^2c) + (ab^2 - a^2b)$$

$$= bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b$$

$$= a^2c - a^2b + ab^2 - b^2c + bc^2 - ac^2$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a)$$

$$c\quad \begin{bmatrix} 9 & -4 & -6 \\ -8 & 5 & 7 \\ -5 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ 3 & b & 1 \\ -1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 9-12+6 & 2a-4b-6 & -18-4+18 \\ -8+15-7 & 9a+5b+7 & 16-5-21 \\ -5+9-4 & -10+3b+4 & 10+3+12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2a-4b-6=0$$

$$9a+5b+7=0$$

$$a-12+6=1$$

$$a-6=1$$

$$\underline{a=7}$$

$$2a-4b-6=0$$

$$14-4b-6=0$$

$$\underline{b=2}$$

Extract 9.2 shows that the candidate had sufficient knowledge to carry out matrix operations. The only difficult the candidate faced was to factorize the expression for the determinant in part (b) and hence lost some few marks in this part.

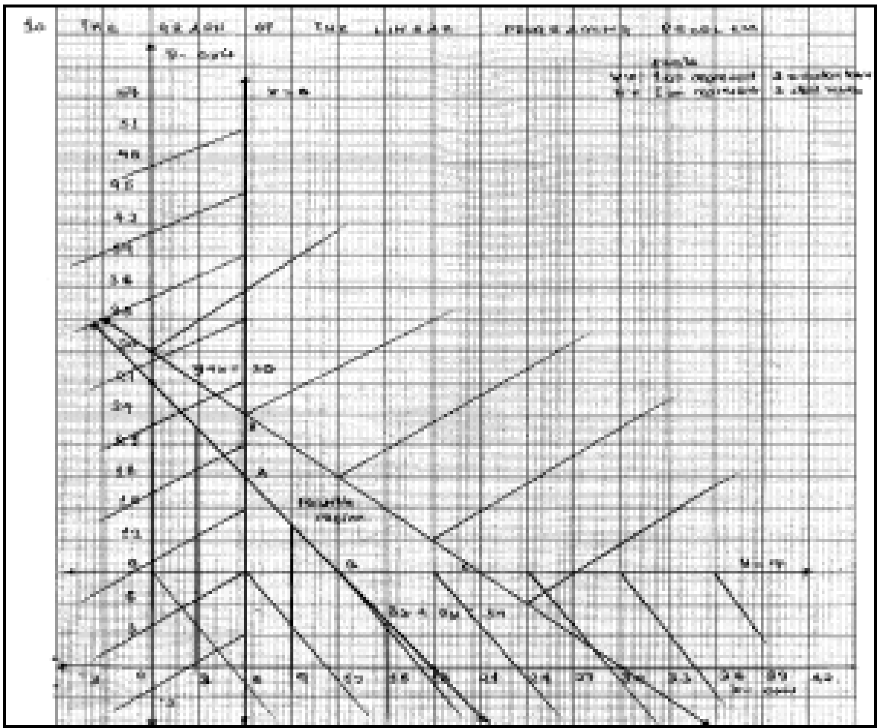
11.0 Linear Programming

2021 PAST PAPERS

10. An engineer wants to make at least 6 steel tables and 9 wooden tables every day. He does not want to make more than 30 tables per day. A steel table requires 3 units and wooden table 2 units of workshop space and there are at least 54 units of workshop space available. The profit of making a steel table is Tsh 5800 and a wooden table is Tsh 3600.

(a) How many steel and wooden tables should be manufactured per day to realize maximum profit?

(b) What is the maximum profit?



10. From the graph

Corner points	Objective function (5800x + 3600y)	net profit
A (6, 18)	6 × 5800 + 3600 × 18.	99 600
B (6, 24).	6 × 5800 + 3600 × 24.	121 200
C (21, 9).	21 × 5800 + 3600 × 9.	154, 200
D (12, 9)	12 × 5800 + 3600 × 9.	102, 000

Hence the optimum point is C (21, 9).

(i). Therefore the engineer should make 21 steel tables and 9 wooden table per day to realize maximum profit

(ii). The maximum profit is 154, 200/=

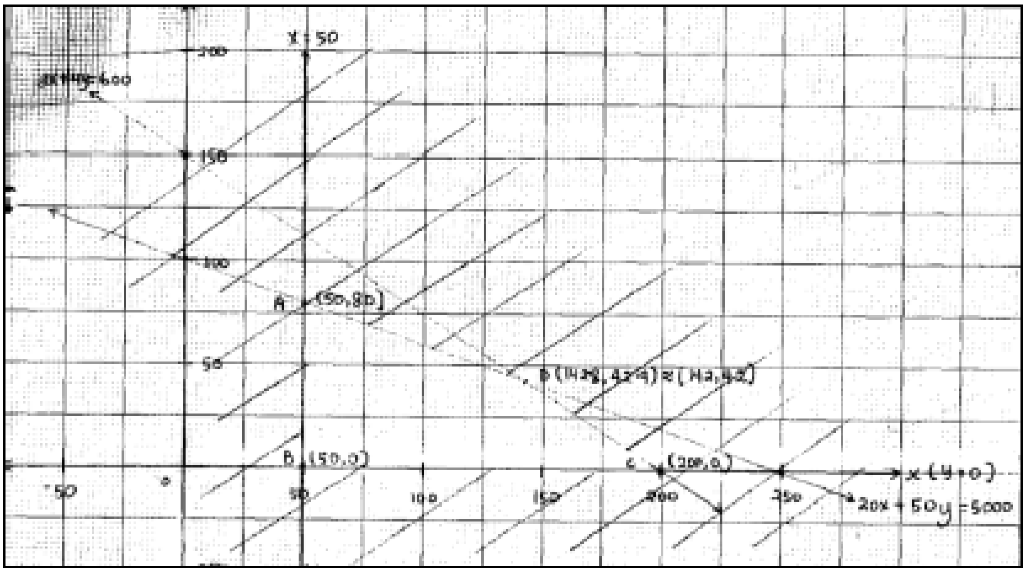
Extract 10.2: A sample of correct response for question 10

2020 PAST PAPERS

10. An aircraft has 600 m^2 of cabin space and can carry 5,000 kg of luggage. An economy class passenger gets 3 m^2 of space and is allowed to travel with 20 kg of luggage. The first class passenger gets 4 m^2 of space and is allowed to have 50 kg of luggage in the aircraft. In the aircraft, there is space for at least 50 economy class passengers. The profit per flight for the economy and first class passengers are 40,000/- and 100,000/- respectively.

(a) Write down all the constraints.

(b) Use graphical method to find the number of economy passengers and first class passengers which will give the maximum profit per flight.



Since number of people can never be in decimal D can be assumed as (142,42)

So:

RESULT TABLE

CORNER POINTS	f(x) = 40,000x + 100,000y	Values
A (50, 80)	40,000 x 50 + 100,000 x 80	10,000,000
B (50, 0)	40,000 x 50 + 100,000 x 0	2,000,000
C (200, 0)	40,000 x 200 + 100,000 x 0	8,000,000
D (142, 42)	40,000 x 142 + 100,000 x 42	9,880,000

∴ For maximum profit, 50 economy passengers and 80 first class passengers should board the flight.

Extract 10.1: *A sample of correct solution for question 10.*

2019 PAST PAPERS

10. In a workshop, each carpenter makes chairs and tables. Carpenter I is limited to 10 days a month, whereas carpenter II is limited to 15 days a month. The following table shows the duration of time it takes to manufacture a chair and a table and the profit on each item.

	Chair	Table
Carpenter I	2	1
Carpenter II	1	3
Profit(USD)	30	45

(a) Taking x and y to be the number of chairs and tables each should make respectively, write down the four inequalities involving x and y which satisfy the given problem.

(b) Find the number of chairs and tables each carpenter should make in a month so as to maximize the income.

10. Soln:

From the table,

	Chair (days)	Table (days)	Maximum no of days
Carpenter I	2	1	10
Carpenter II	1	3	15
Profit (USD)	30	45	

a/ Constraints (inequalities)

a/ $2x + y \leq 10$

b/ $x + 3y \leq 15$

c/ $x \geq 0$

d/ $y \geq 0$

where x is number of chairs

y is number of tables

10\. b/ From the data,

Objective function, $f(x,y) = 30x + 45y$

Required :

In order to maximize the income.

$\Rightarrow$ Maximize the value of $f(x,y) = 30x + 45y$

Tables of values

i/ $2x + y = 10$

x	0	5
y	10	0

ii/ $x + 3y = 15$

x	0	15
y	5	0

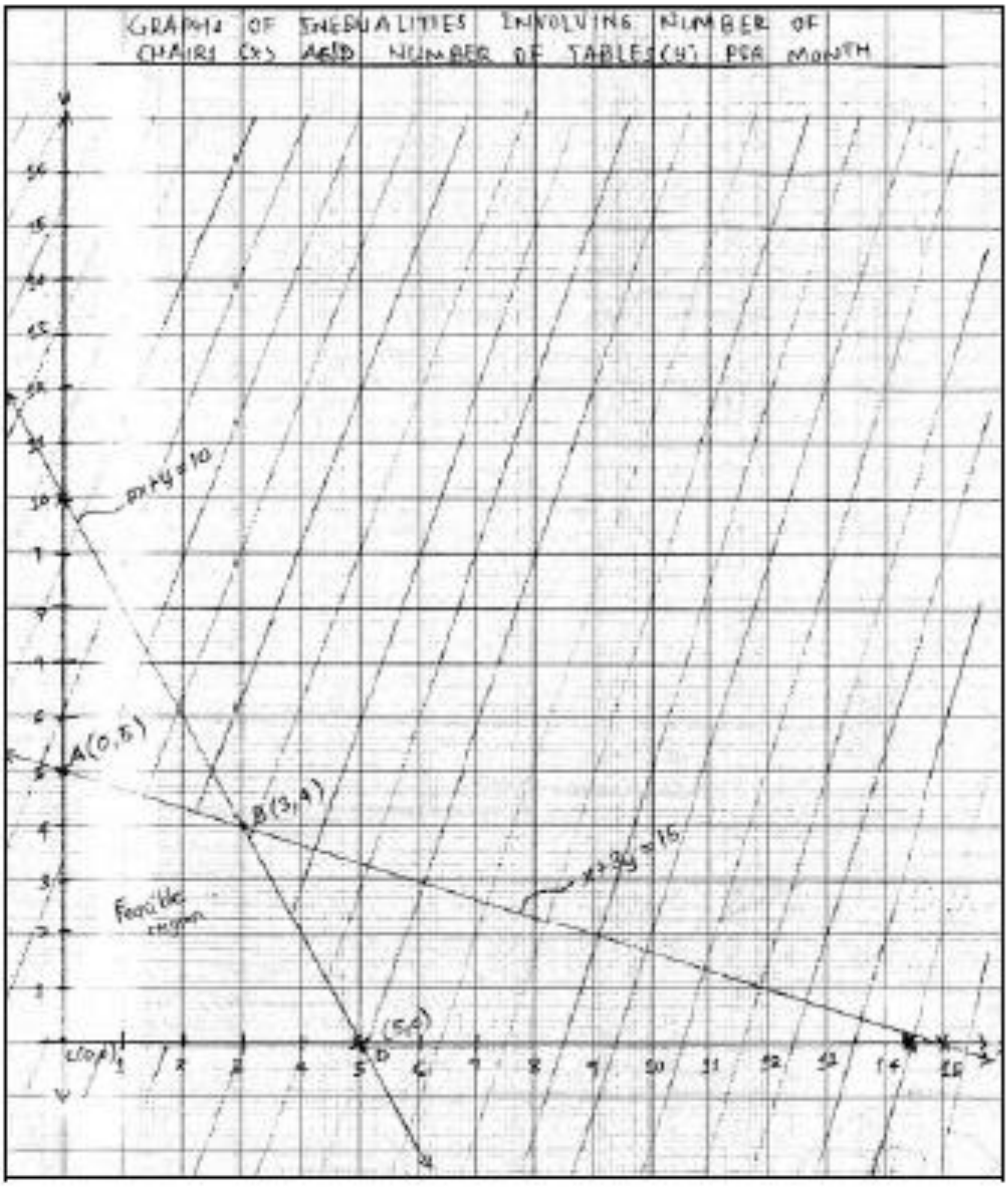
Table of results

Objective function, $f(x,y) = 30x + 45y$

Point	Objective function	Value (USD)
A (0, 5)	$f(0,5) = 30(0) + 45(5)$	225
B (3,4)	$f(3,4) = 30(3) + 45(4)$	270
C (0, 0)	$f(0,0) = 30(0) + 45(0)$	0
D(5,0)	$f(5,0) = 30(5) + 45(0)$	150

In order to maximize income, maximum of income obtained should be 270 USD.

$\therefore$ 3 chairs and 4 tables should be made in a month so as to maximize income.



Extract 10.1: *A sample of the candidate's correct responses in question 10*

2018 PAST PAPERS

10. (a) Mention any four applications of linear programming.

(b) Define the following terms in linear programming:

(i) Objective function

(ii) Constraints

(iii) Feasible region

(c) A special take away fast lunch of food and drinks contains 2 units of vitamin B and 5 units of iron. In each glass of drinks there are 4 units of vitamin B and 2 units of iron. A minimum of 8 units of vitamin B and 60 units of iron are served each day. If each serving of food cost 2000 Tshs and that of drinks cost 1600 Tshs; How much of the food and drinks are needed to be consumed in order to meet daily needs at a minimum cost?

10 (a) The four applications include,

(i) In hospitals;

- During the prescription of Medicine, nurses may give minimum amount of dosage to be given to patients; In such cases linear programming is used

(ii) In schools;

- When the school budget is limited, and the school wants to make new furnitures and tables at minimal cost, we utilize linear programming

10 (iii) At home;

- The organization of family budget may utilize the concept of linear programming so that needs of family may be met.

(iv) In industries;

- During manufacture of products, linear programming is used to identify which kind of products to be produced will yield maximum amount of profit.

10 (b) i/ Objective function;

- Is that linear function that gives the desired goal to be achieved either maximization/minimization

i.e. $f(x, y) = ax + by$

ii/ Constraints;

- These are linear inequalities which describe the condition of the problem.

iii/ Feasible Region;

- A region bounded by inequalities and all possible points that will lead to solution pertaining the problem.

10 (c) Let: Number of foods be x

Number of drinks be y

SUMMARY.

	FOOD	DRINKS	total
VITAMIN B	2 units	4 units	8 units
IRON	5 units	2 units	60 units
Cost	2000 Tsh	1600 Tsh	

10.10 Objective function;

Minimize; $2000x + 1600y = f(x,y)$

Subject to Constraint;

$2x + 4y \geq 8$ — (i)

$5x + 2y \geq 60$ — (ii)

Also; $x \geq 0$ — (iii)

$y \geq 0$ — (iv)

The equations that are;

$$\begin{cases} x + 2y = 4 \\ 5x + 2y = 60 \\ x = 0 \\ y = 0 \end{cases}$$

Table of values for; $x + 2y = 4$

x	0	4
y	2	0

Table of values for; $5x + 2y = 60$

x	0	12
y	30	0

⇒ Then Repo to Graph paper;

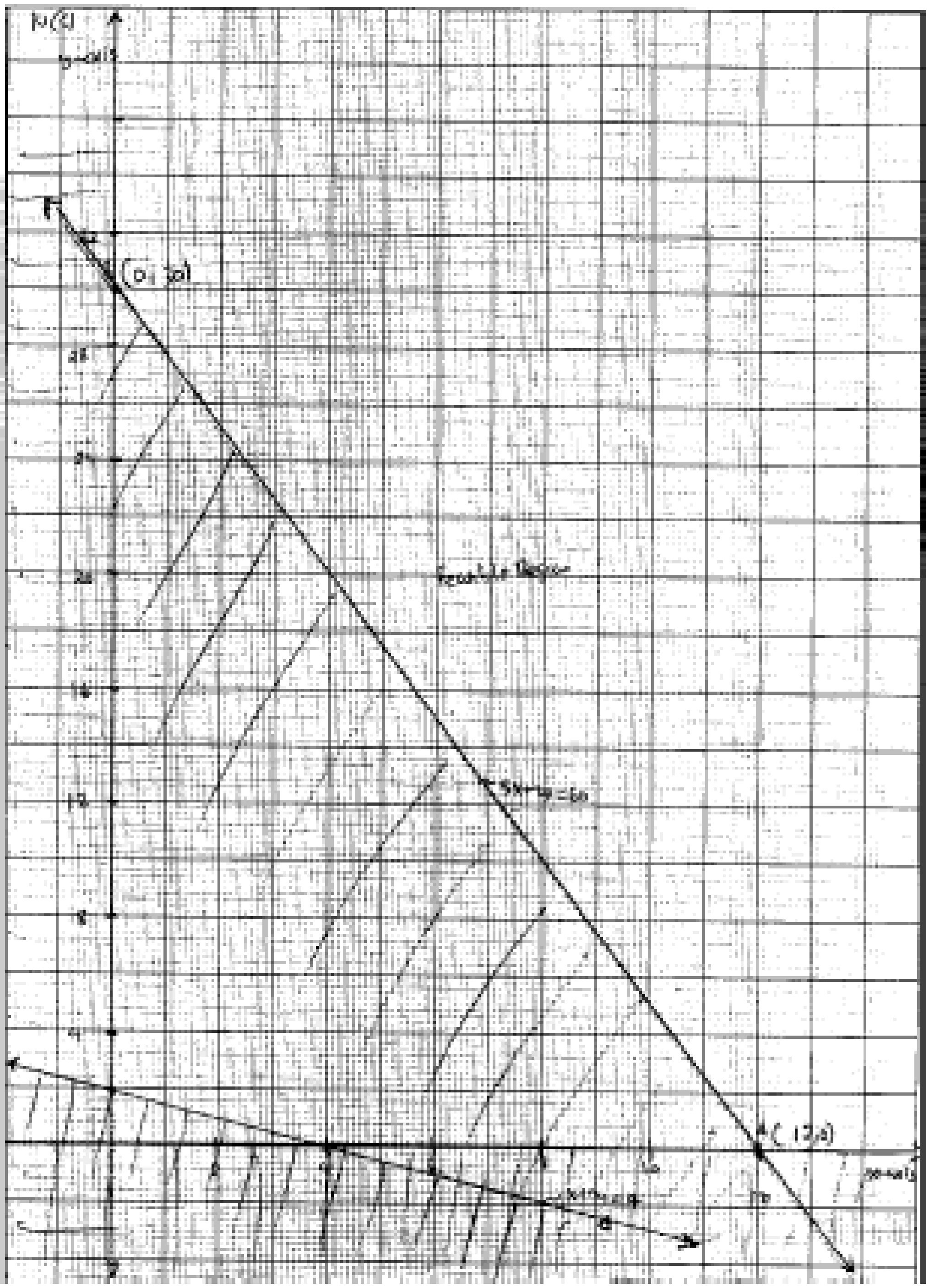
points are; (12,0) and (0,30)

	(x,y)	2000x + 1600y	values
A	(12,0)	2000(12) + 0(1600)	24,000
B	0,30	0(2000) + 30(1600)	48,000

optimality; Since minimum values are (12,0) which give 24000₦

Hence; It is advised that the person should take 12 packages of food with no drinks so as to meet daily requirement and minimise the costs.

→ The optimal point = (12,0)



Extract 10.1 shows a sample solution of a candidate who answered this question as required.

2017 PAST PAPERS

10. (a) Define the following terms:

(i) Linear programming

(ii) Constraints.

(b) A trader has 15000, 9000 and 1920 units of ingredients X, Y and Z for production of cakes and loaves. The requirements of units of a loaf of bread and a cake are indicated in the table below.

Foodstuffs	Units		
	X	Y	Z
Bread	25	10	30
Cake	15	18	30

A loaf of bread is sold at 4200/- shillings and a cake is sold at 2000/- shillings.

(i) Sketch the graph to illustrate this information.

(ii) What is the maximum amount of money obtained if both cakes and loaves of bread must be prepared?

(iii) How should the trader do to obtain that maximum profit?

10 (a)(i) linear programming is the branch of mathematics which deals with linear inequalities so as to maximize profits and minimize costs.

10(a) (ii) Constraints are linear inequalities used in solving the linear programming problems.

10 (b) Soln

Let x represents quantities of loaf bread

y represents quantities of cake

Constraints

$25x + 15y \leq 15000$

$10x + 18y \leq 9000$

$30x + 30y \leq 1920$

$x \geq 0$

$y \geq 0$.

Objective function.

$f(x, y) = 4200x + 2000y$.

Thus solving $f(x, y) = 4200x + 2000y$

under constraints

$25x + 15y \leq 15000$

$10x + 18y \leq 9000$

$30x + 30y \leq 1920$.

$x \geq 0$

$y \geq 0$

10 (b) Let $25x + 15y \leq 15000$ be $25x + 15y = 15000$

x	0	600
y	1000	0

Let $10x + 18y \leq 9000$ be $10x + 18y = 9000$

x	0	900
y	600	0

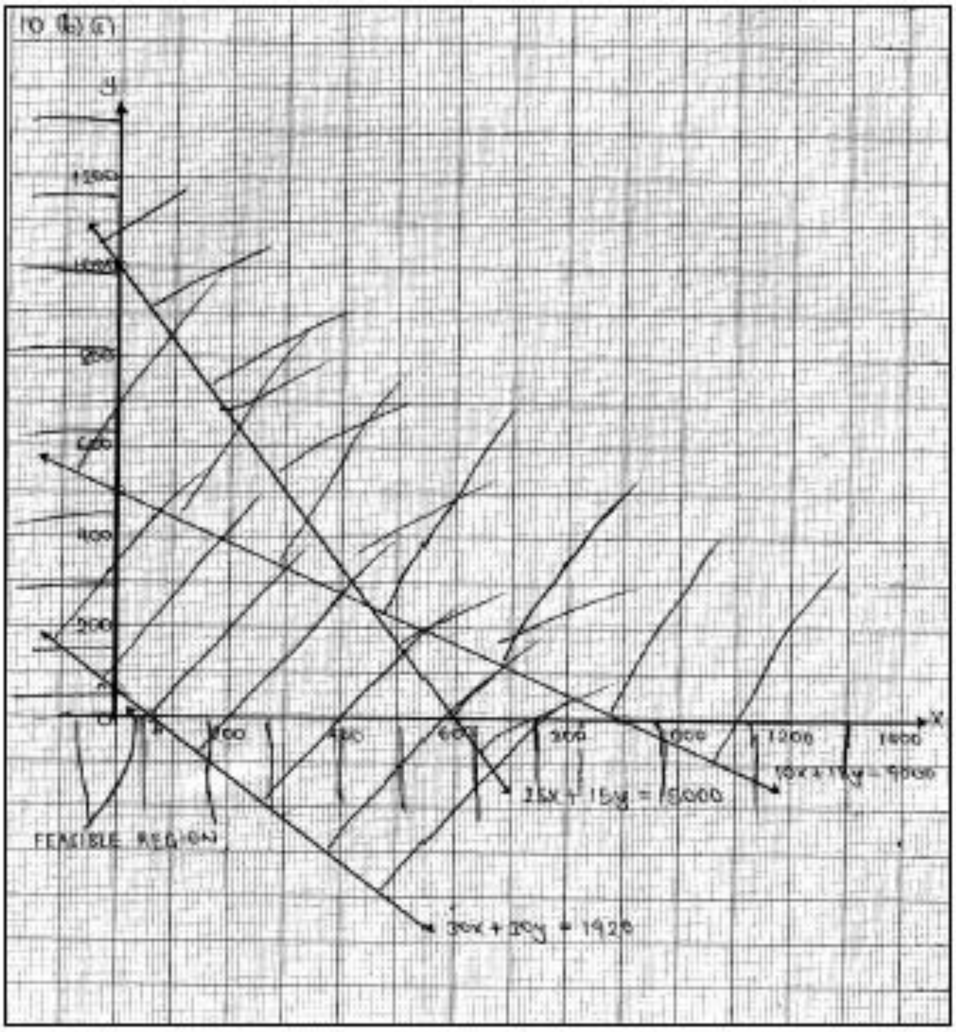
Let $30x + 30y \leq 1920$ be $30x + 30y = 1920$

x	0	64
y	64	0

10 (b) (ii) From the graph.

Corner points of the feasible region	Objective function $f(x, y) = 4200x + 2000y$	Value
O (0, 0)	4200(0) + 2000(0)	0
A (0, 64)	4200(0) + 2000(64)	128,000
B (64, 0)	4200(64) + 2000(0)	268,800

10:	(b) (ii) The trader obtains only maximum amount of money when only loaves of bread are prepared
	= 268,800
	(iii) The trader should prepare only 64 loaves of bread to maximize the profit.



Extract 10.1 shows that, the candidate was able to solve the given linear programming problem correctly.

2016 PAST PAPERS

10. (a) Given the linear inequalities: $2y \leq 4x$, $x \leq 6$, $y \geq 2$ and $2x+3y \leq 30$

(i) Draw the corresponding graph.

(ii) List the corner points of the feasible region.

(b) The daily profit obtained by Fruits Beverages Company in its business is given by the objective function $f(x,y)=250x+350y-2200$ and the constraints;

$x+y \geq 5.5$

$4x+2y \geq 16$

$x+2.5y \geq 9$

(i) Represent the linear programming problem graphically.

(ii) Determine the minimum and maximum profit of the company.

10. a/. i/ Given.

$2y \leq 4x$, $x \leq 6$, $y \geq 2$, $2x+3y \leq 30$

Let $2y = 4x$.

$y = 2x$.

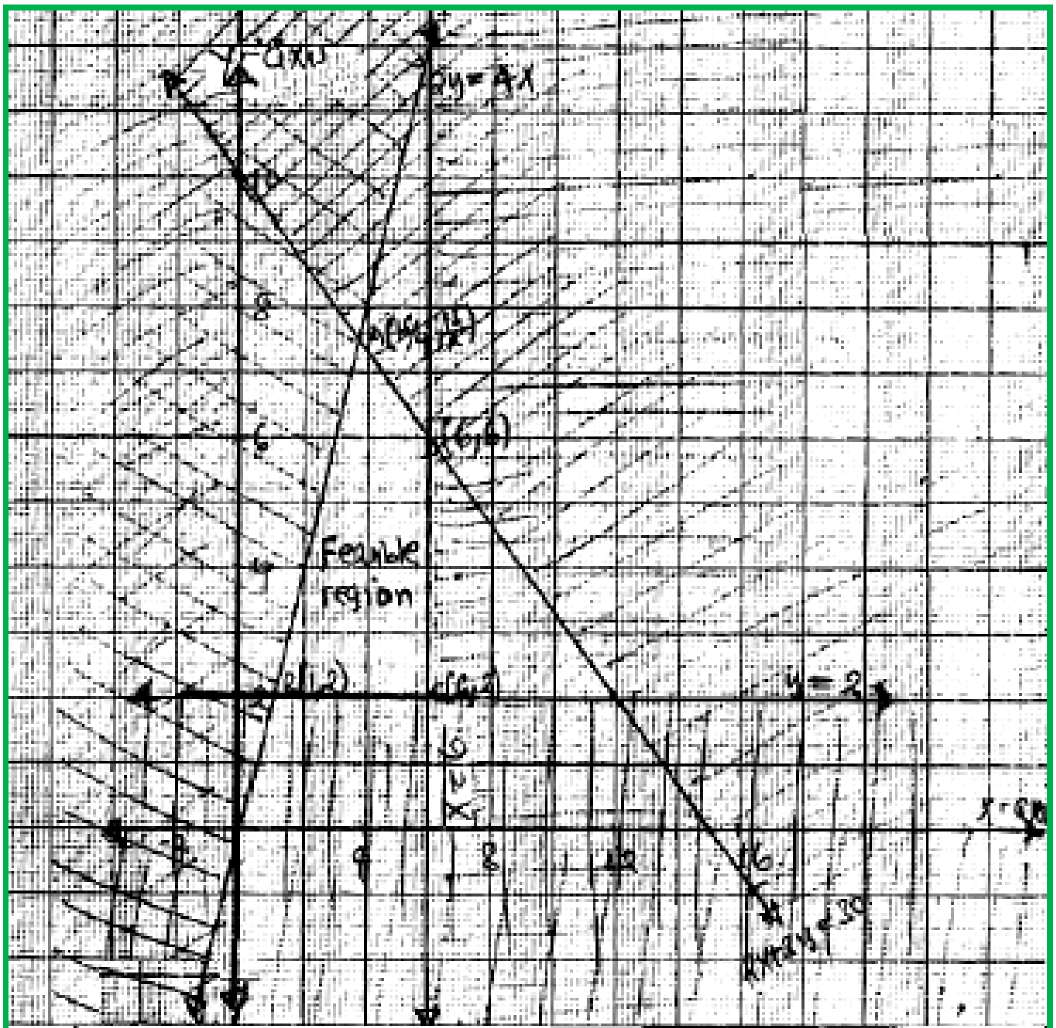
x- and y-intercept for $y=2x$.

x	0	0
y	0	0

Let $x=6$, $y=2$. and $2x+3y=30$.

x-and y-intercepts of $2x+3y=30$.

x	0	15
y	10	0



ii/ From the graph drawn.
Corner points A, B, C and D are
A = (15/24, 15/2).
B = (1, 2)
C = (6, 2)
D = (6, 6).

Extract 10.1 (a) shows that, the candidate was able to represent the linear inequalities graphically, identify the feasible region and indicate the corner points as it was required.

10. b/. Given.

$x+y \geq 5.5$

$4x+2y \geq 16$

$x+2.5y \geq 9$.

objective function: $f(x,y) = 250x + 350y = 2200$

Required inequalities

$x+y \geq 5.5$

$4x+2y \geq 16$.

$x+2.5y \geq 9$.

$x \geq 0$

$y \geq 0$.

Let $x+y = 5.5$

x- and y-intercept for $x+y=5.5$

x	0	5.5
y	5.5	0

Let $4x+2y = 16$.

x - y - intercept. for $4x+2y=16$.

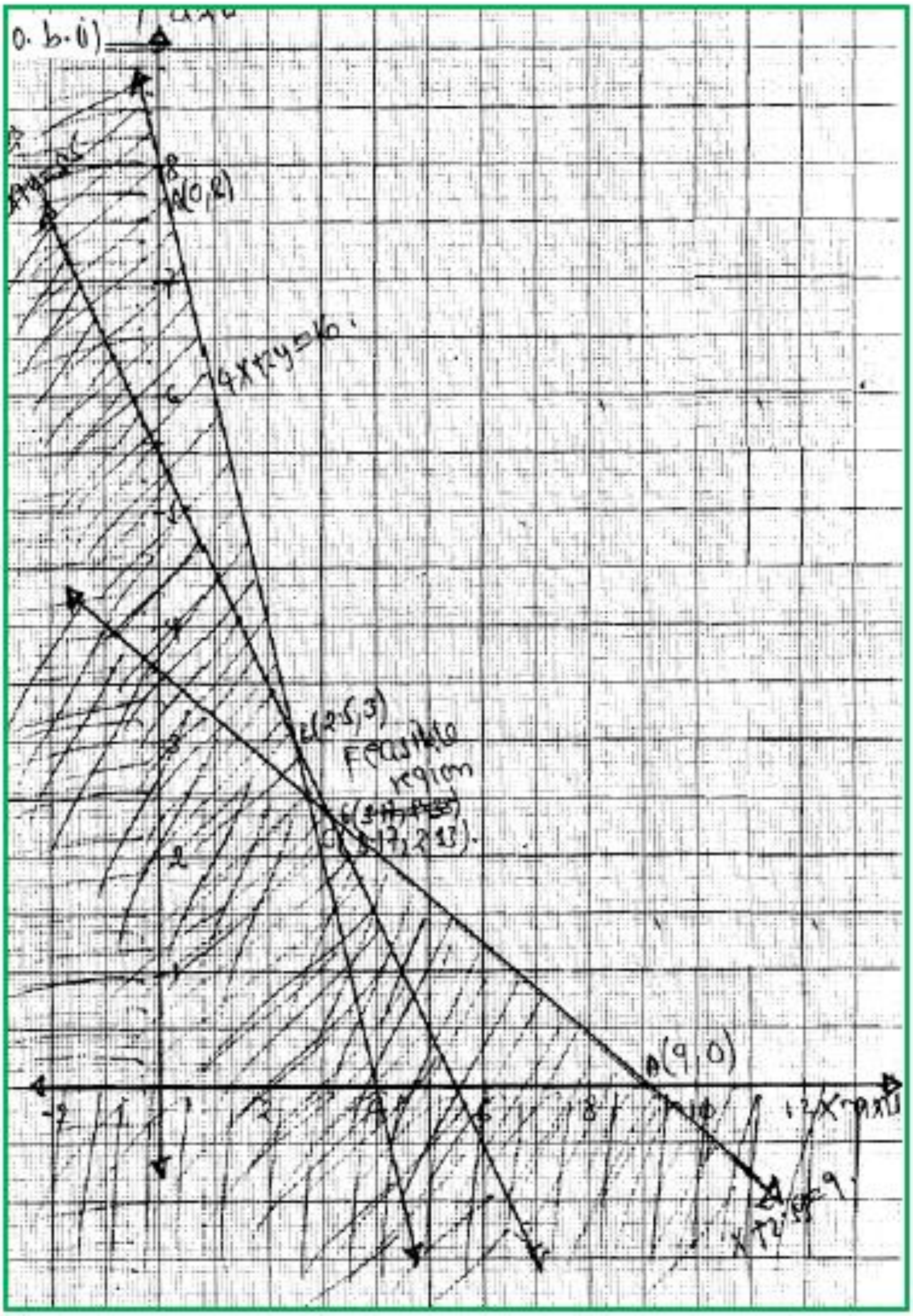
x	0	4
y	8	0

Let $x+2.5y = 9$.

x and y-intercept for $x+2.5y=9$.

x	0	9.
y	3.6	0

0. b. (i)



ii/. From the graph plotted, corner points of the feasible region are:

A(0,8), B(2.5,3), C(3.17, 2.33), D(9,0)

consider table

Corner points	$f(x,y) = 250x + 350y - 2200$	Total.
A(0,8)	250(0) + 350(8) − 2200	600
B(2.5,3)	250(2.5) + 350(3) − 2200	−525
C(3.17, 2.33)	250(3.17) + 350(2.33) − 2200	−592
D(9,0)	250(9) + 350(0) − 2200	50

(ii) Minimum profit is 50, and maximum profit is 600

Extract 10.1 (b) shows that, the candidate was able to represent the linear inequalities graphically and determine the maximum and minimum profit of the company as it was asked.

2015 PAST PAPERS

10. Mr. Taramise owns 480 acres of land on which he grows either maize or beans during the farming period. He normally expects a profit of Tshs 40,000/= per acre on maize and Tshs 30,000/= per acre on beans and he has 800 hours of labour available. If maize requires 2 hours per acre to raise and beans require 1 hour per acre to raise, find how many acres of maize and beans he should plant to get maximum profit.

10 Let x be acres of maize

Let y be acres of beans

Objective function $f(x,y) = 40,000x + 30,000y$

$x + y \leq 480$

$2x + y \leq 800$

$x \geq 0 \quad y \geq 0$

For function $x + y \leq 480$

$x + y = 480$

$y = 480 - x$

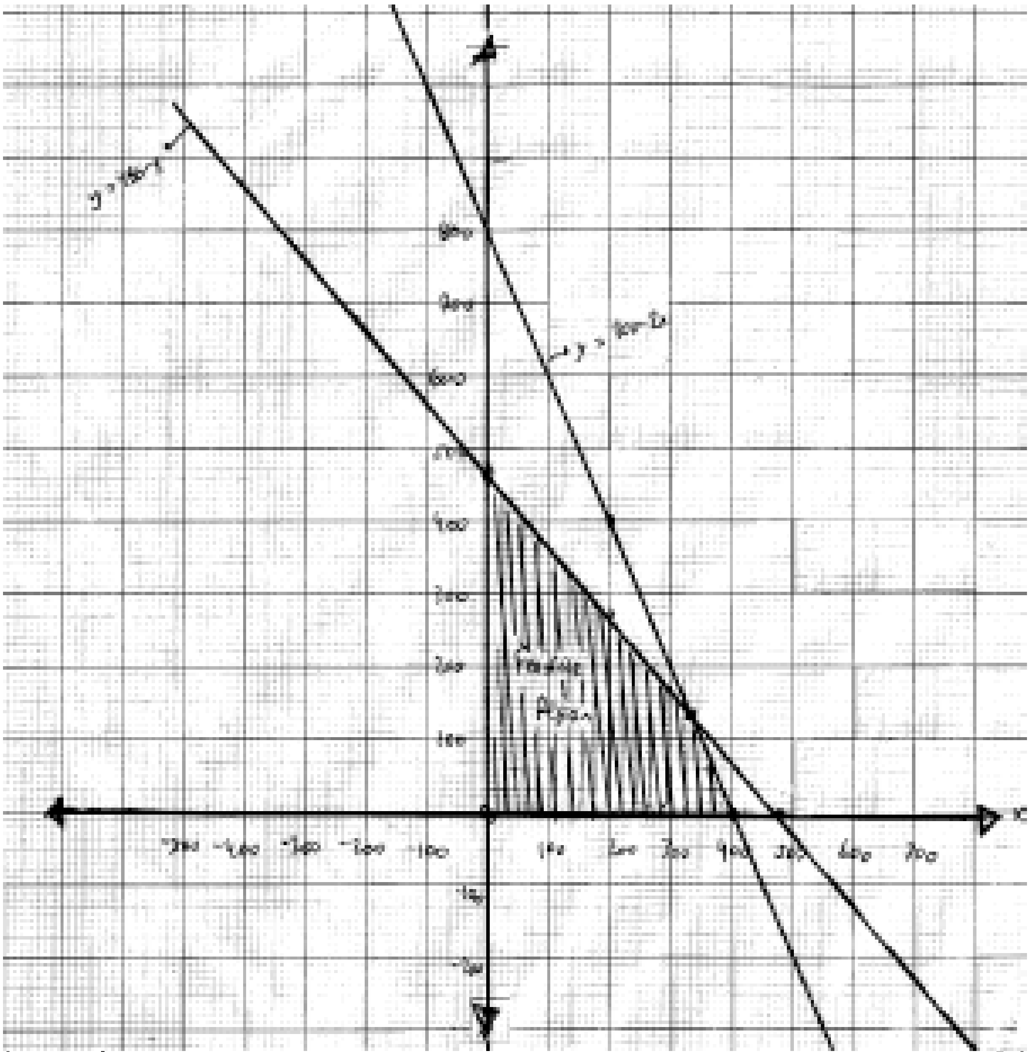
x	200	480	0
y	280	0	480

For function $2x + y \leq 800$

$2x + y = 800$

$y = 800 - 2x$

x	400	200	0
y	0	400	800



From graph (In previous answer sheet)

Corner points are

(0, 480) (0, 0) (400, 0) (320, 160)

Intersection point of the two graphs

Corner Points	Objective function	Profit
0, 480	40000(0) + 480(30,000)	14,400,000
(0, 0)	40,000(0) + 30,000(0)	0
(400, 0)	40,000(400) + 30,000(0)	16000000
(320, 160)	40,000(320) + 30,000(160)	17,600,000

∴ He should plant 320 acres of maize and 160 acres ~~of~~ beans

In Extract 10.1, the candidate formulated the constraints and the objective function correctly and drew correctly the graph which was used to obtain number of acres of maize and beans that was required for maximum profit.