The Riemann Hypothesis

Complex Maths

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What is an Imaginary Number?

Usually, we work with *real* numbers.
Real include all integers, rational numbers, and irrational numbers.
e.g. $1, \ 6.9, \ \pi, \ e, \ \sqrt{123}$

Definition of i

i is defined as $\sqrt{-1}$.
$\therefore i^2 = -1; (-i)^2 = -1$

What is a Complex Number?

Definition of a complex number

A complex number is the sum of a real and imaginary number.

$$\text{e.g. } 1+i,\ \pi+ei,\ 0+i,\ 1+0i,\ \text{etc}$$

Complex numbers have interesting properties, including how they and, multiply, and exponentiate.

Adding and Subtracting Complex Numbers

You can add and subtract complex numbers like you would add polynomials (combine like terms).

Example 1

$$(5 + 3i) + (6 + 4i) = 11 + 7i$$

Example 2

$$(3 + 6i) - (6 - 4i) = -3 + 2i$$

Congugates and Magnitudes

For some complex number $a + bi$, its **congugate** is $a - bi$.
The congugate of a complex number $z = a + bi$ is denoted with $\overline{z}$.

Example

What is the congugate of $3 + 5i$?

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What is the congugate of $3 + 5i$?

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For some complex number $a + bi$, its **magnitude** is $a^2 + b^2$.
The magnitude of a complex number $z = a + bi$ is denoted with $|z|$.

Example

What is the magnitude of $3 + 5i$?

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Example

What is the magnitude of $3 + 5i$?

$3^2 + 5^2$
$= 34$

Multiplying Complex Numbers

You can multiply complex numbers like you would multiply binomials (using FOIL).

Example

If $a = 5 + 3i$ and $b = 6 + 4i$, then

$$\begin{aligned} & (5+3i) \times (6+4i) \\ = & (5 \times 6) + (5 \times 4i) + (3i \times 6) + (3i \times 4i) \\ = & 30 + 20i + 18i + 12i^2 \\ = & 30 + 38i - 12 \\ = & 18 + 38i \end{aligned}$$

Multiplying Complex Numbers

A complex number multiplied by its congugate always gives its magnitude.

Example

$$
\begin{aligned}
& (5+3i) \times (5-3i) \\
= {} & 5^2 - (3i)^2 \\
= {} & 34
\end{aligned}
$$

Dividing Complex Numbers

To divide complex numbers, make the denominator into a real number by multiplying top and bottom by its congugate.

Example

$$\frac{1+2i}{2-3i}$$
$$=\frac{(1+2i)(2+3i)}{(2+3i)(2-3i)}$$
$$=\frac{2+3i+4i+6i^2}{2^2-(3i)^2}$$
$$=\frac{-4+7i}{13}$$
$$=\frac{-4}{13}+\frac{7}{13}i$$

Dividing Complex Numbers

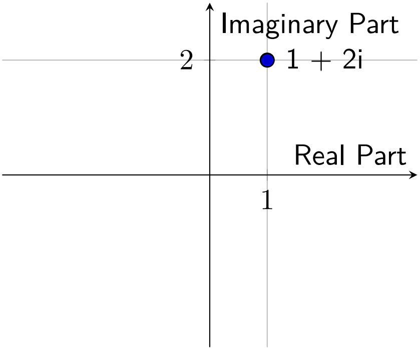
The general formula for dividing complex numbers a by b is:

Formula

$$\frac{a \times \overline{b}}{|b|}$$

Complex Plane

Complex points can be visualized on the complex plane.



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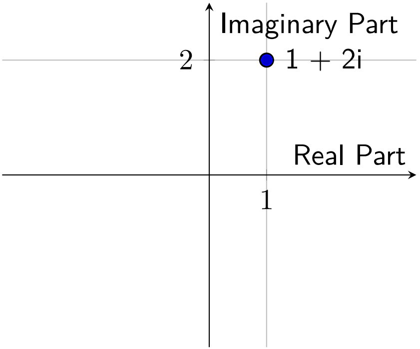

The **magnitude** of the number is the distance of the point from the origin.
The **argument** is the polar angle (angle counter-clockwise from the x-axis in the positive direction) of the point.