Sequences and Series

Your soon-to-be new best friend ♡

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What is Sigma notation?

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Fun fact: Sigma is the 18th letter of the Greek alphabet, and is transliterated as “s”.

$$\sum_{n=1}^{4} n$$

The Sigma notation consists of several components. In the example above, we have:

n under Sigma Index of summation. Some people use i, k, or x

1 First value of n (can be anything)

4 Term we end on

n after Sigma Formula for each turn

Using Sigma notation

Summations

$$\sum_{n=1}^{4} n$$

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$$\sum_{n=1}^{4} n = 1 + 2 + 3 + 4 = 10$$

Let's try some quick maths!
We'll do this one together.
There's more on your worksheet.

Let's talk about why these work:

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Multiplying by a constant:

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Adding/subtracting:

$$\sum_{k=m}^{n} (a_k + b_k) = \sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k$$

Summation shortcuts ;)

Summing 1 equals n

$$\sum_{k=1}^{n} 1 = n$$

Summing the constant c equals $c \times n$

$$\sum_{k=1}^{n} c = nc$$

A shortcut when summing k (we will develop this later)

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

These ones are pretty cool and useful:

A shortcut when summing k^2

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

A shortcut when summing k^3

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Try the super cool practice word problem on your sheet!

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What is an arithmetic sequence?

- A sequence of numbers where there is a constant difference between successive terms
- Example: $3, 5, 7, 9, 11$

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We can define the n^{th} term with either of the following formulas:

$$a_n = a_1 + (n-1)d \qquad a_n = a_m + (n-m)d$$

Where we have:

a_n	the n^{th} term
n	the term number
d	the constant difference
m	the m^{th} term.
a_1	the first term of the series if we start counting from 1

How can we easily sum a finite arithmetic series?

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- Pair up your values and divide by 2!

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$$S_n = \frac{n}{2}(a_1 + a_n)$$

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- Follows a pattern where each term in found by multiplying the previous term by a constant called the common ratio
- Examples: $3, 6, 12, 24, 48$ or $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

What is a geometric sequence?

Geometric Sequences

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- Examples: $3, 6, 12, 24, 48$ or $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

We can define the n^{th} term with any of the following formulas:

$$a_n = a_{n-1} \times r \qquad a_n = a_1 \times r^{n-1}$$

Where we have:

a_n	the n^{th} term
a_{n-1}	the $(n-1)^{\text{th}}$ (previous) term
a_1	the first term of the series
n	the term number
r	the common ratio

Sum of finite geometric series

Geometric Sequences

$$
\begin{array}{lcccccccccccccc}
S_n & = & a & + & ar & + & ar^2 & + & ar^3 & + \dots & + & ar^{n-1} & & \\
rS_n & = & & + & ar & + & ar^2 & + & ar^3 & + \dots & + & ar^{n-1} & + & ar^n
\end{array}
$$

$$
\begin{aligned}
S_n - rS_n &= a - ar^n \\
S_n(1-r) &= a(1-r^n) \\
\therefore S_n &= a\left(\frac{1-r^n}{1-r}\right)
\end{aligned}
$$

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What if I told you that the sum of some **infinite** series wasn't infinite?

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What if I told you that we can solve this? Can you sense the excitement?

$$\sum_{k=1}^{\infty} 3\left(\frac{1}{2}\right)^{k-1}$$