Euclid Preparation 4

Direct Proofs These proofs rely on only axioms, definitions, and other proven theorems.

1. Prove that the sum of two even integers is even. Prove that the sum of two odd integers is odd.

2. Prove that $a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$.

Proof by Contradiction These proofs rely on making a wrong assumption, and then showing that the assumption must be false.

1. Prove that $\sqrt{3}$ is irrational. Prove that $\sqrt{4}$ is irrational.

2. Prove that there is no smallest rational number greater than 0.

3. Prove that there is no greatest prime.

Proof by Exhaustion These proofs rely on verifying that the statement is true for all cases.

1. Prove that $(n+1)^3 \geq 3^n$ for $n \in \mathbb{N}, n \leq 4$.

2. Prove that no perfect square ends with an 8.

Nonconstructive Proof A proof which shows that something is true without providing a method for finding a specific example.

1. Prove that for $f(x) = x^3 - 3x^2 + 2x - 4$ there exists a real $r, 2 < r < 3$ such that $f(r) = 0$.

Proof by Induction A proof that shows that if the statement is true for some case, then it must be true for all other cases related to that case.

1. Prove that $x(x+1)(x+2)$ is always divisible by 3.

2. Prove that $a^n - 1$ is always divisible by $a - 1$ for $a, n \in \mathbb{N}$.

3. Prove that $n! > 2^n$ for $n \in \mathbb{N}, n \geq 4$.

