Game Theory

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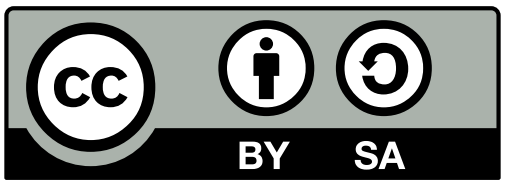

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What is game theory?
Introduction

"Game theory is about finding a way to rig games in your favour."

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— Vincent Macri

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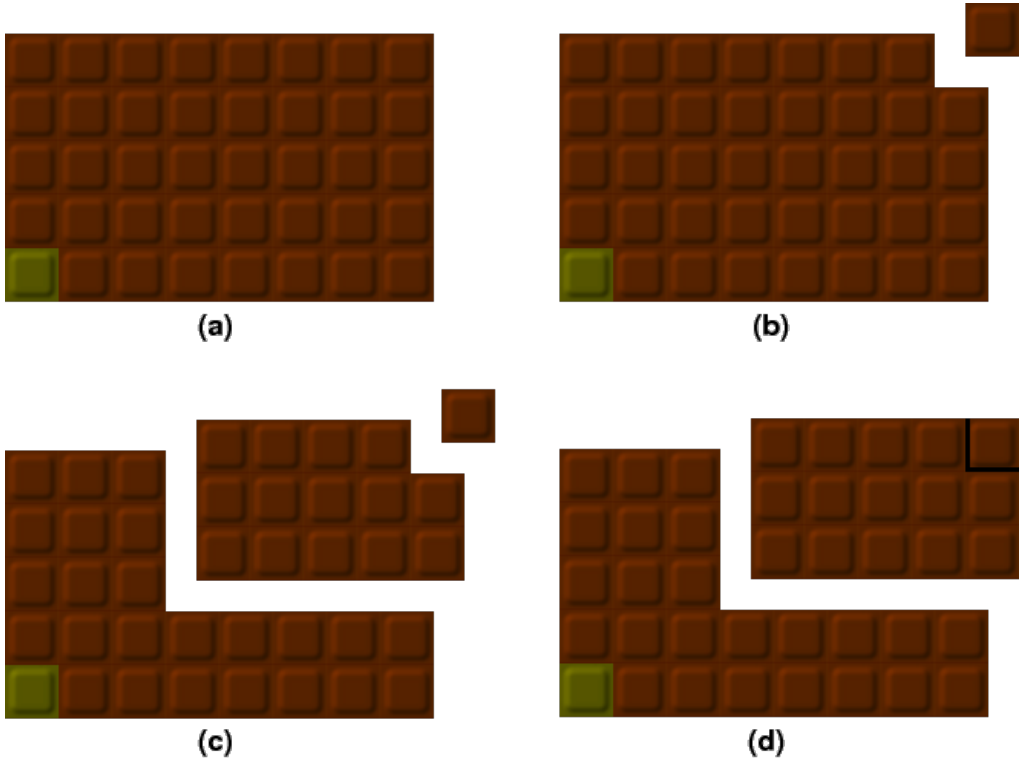
The second game

Chomp!

Math has never been this exciting (or delicious) 

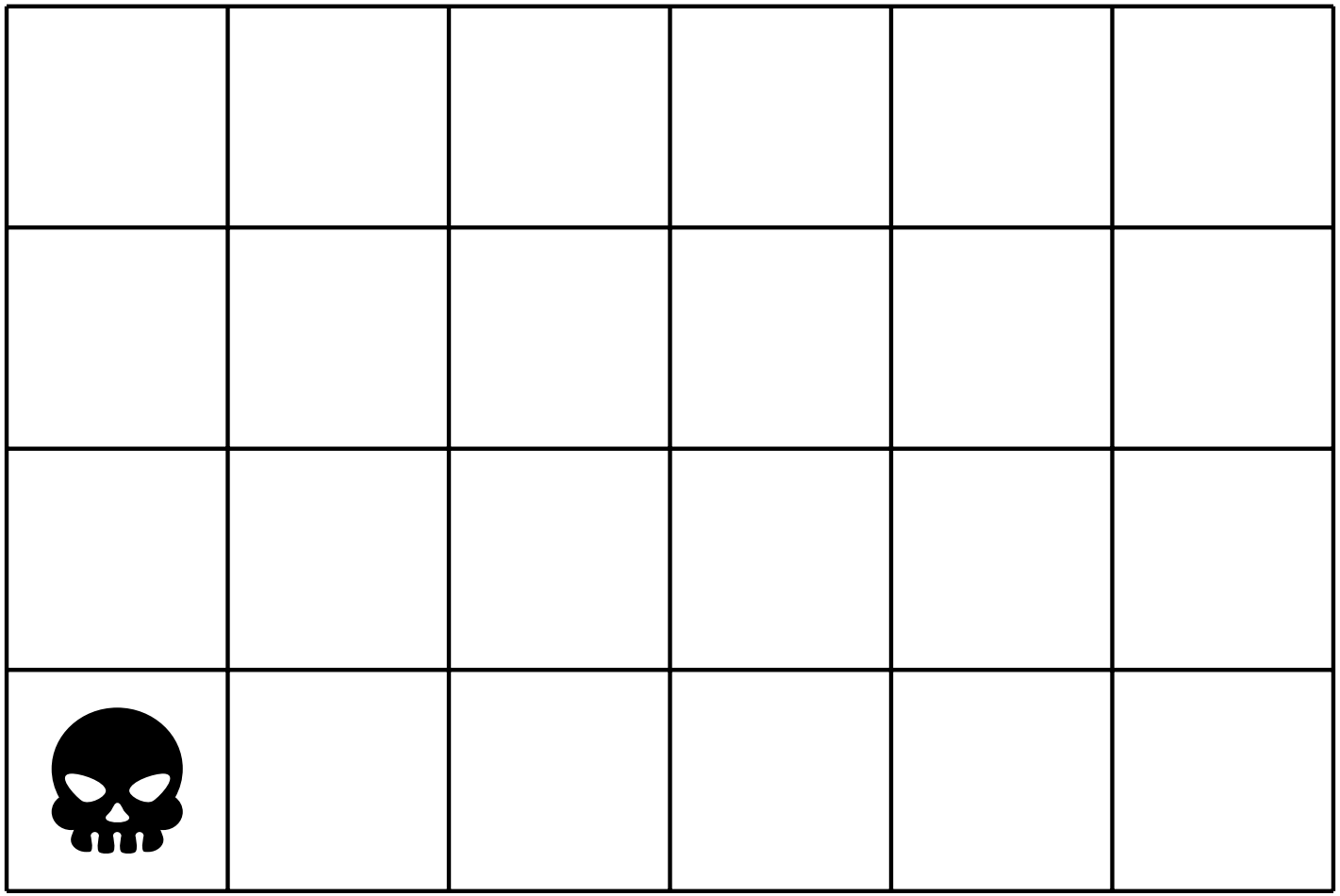
What's the game?
Chomp!

Chomp is played on a rectangular grid, such as squares of a candy bar. The lower left square is considered poison. Players take turns picking a square. With each choice, all squares above and to the right of the picked square are no longer available – they are eaten. The person forced to take the poison square loses.



Let's play! (4×6 board)

Chomp!



Is there a winning strategy? (Or, why this game so cool.)
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The real question is. . .

What is the winning strategy?
Chomp!

- Does anyone know one right away?

What is the winning strategy?
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- Does anyone know one right away?
- No!

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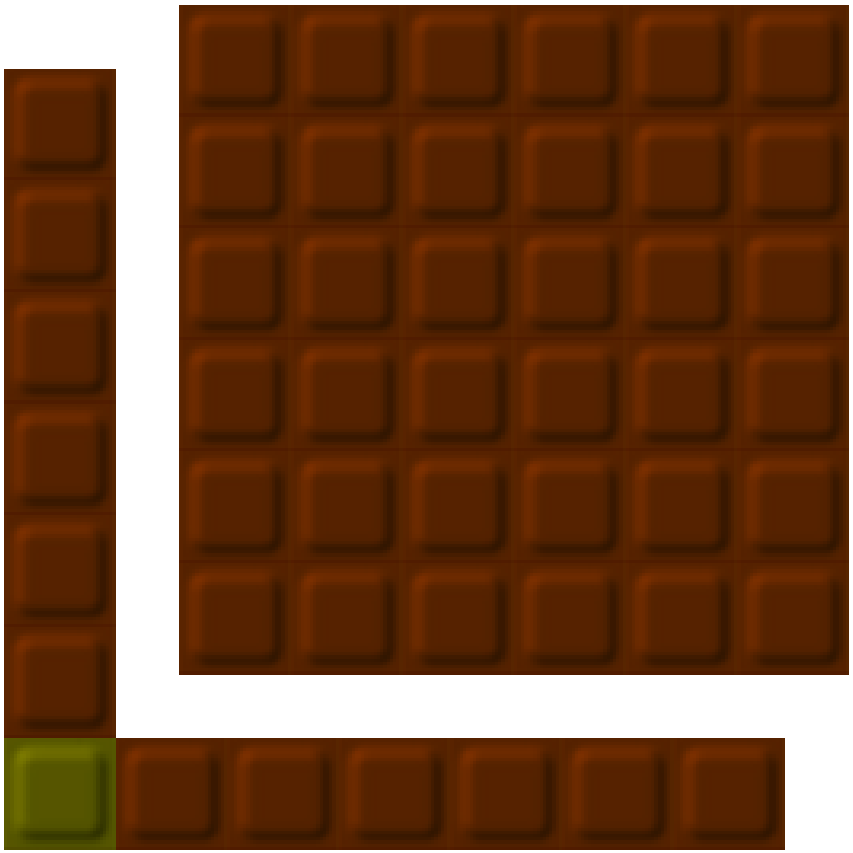
- Does anyone know one right away?
- No!
- Let's analyze some cases!

$n \times n$ grid
Chomp!

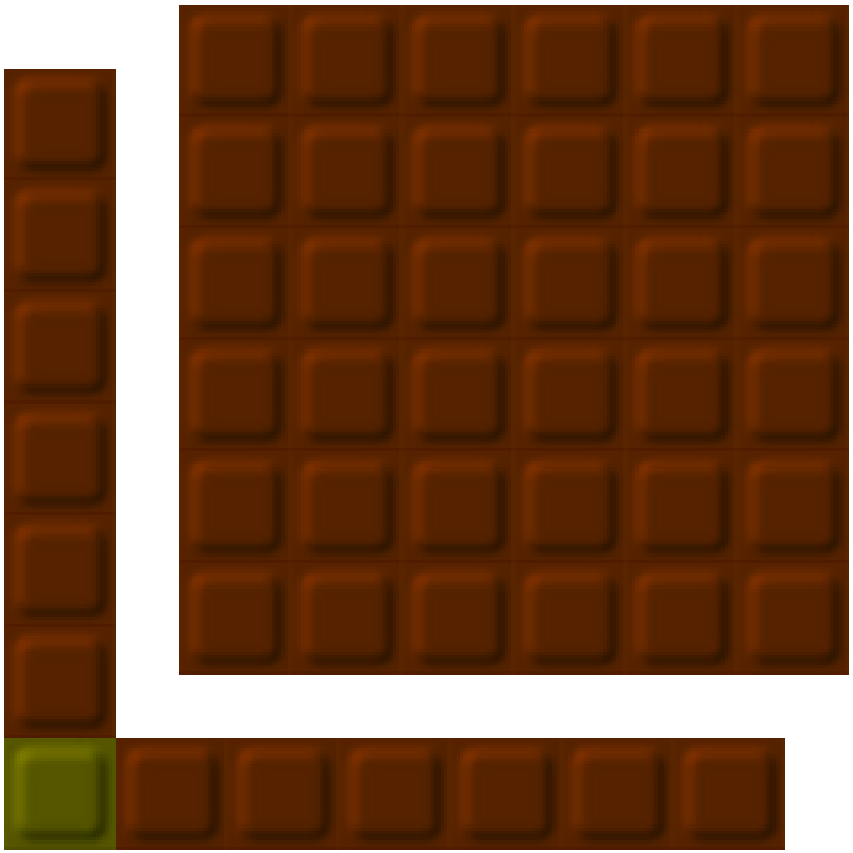
- What's the strategy here?



$n \times n$ grid

Chomp!

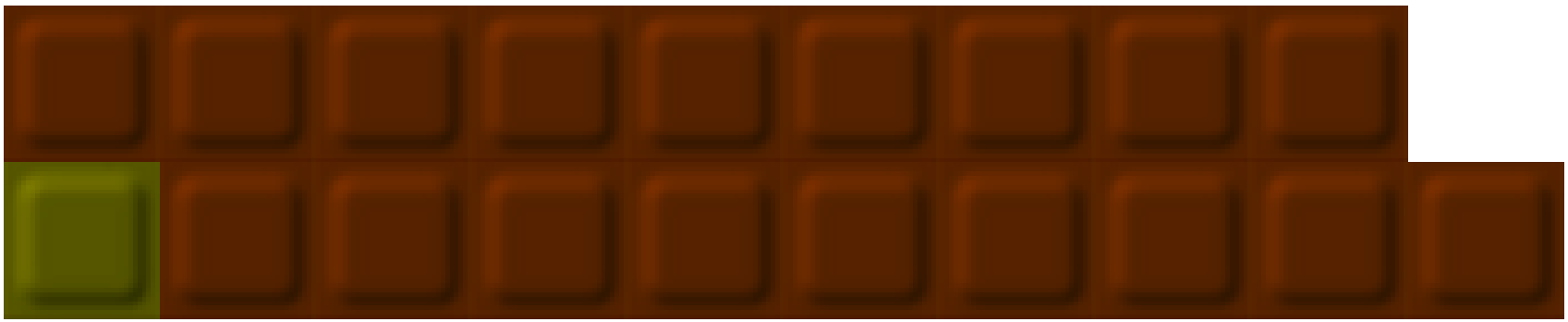
- What's the strategy here?
- Make an "L", and then take symmetrical moves!



$2 \times n$ grid

Chomp!

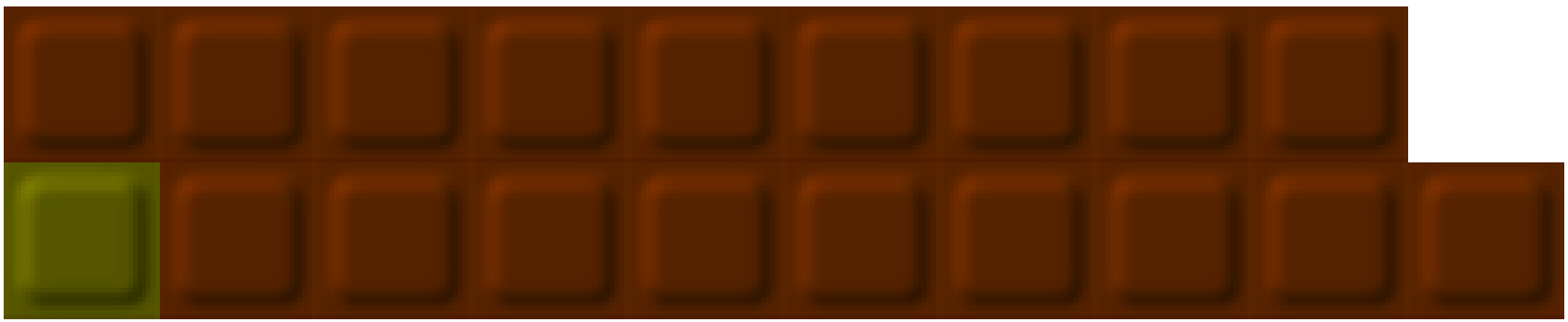
- What's the strategy here?



$2 \times n$ grid

Chomp!

- What's the strategy here?
- Make sure that player 2 encounters a rectangle... with a square missing!



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  - In 2002, Steven Byrnes (a high school senior!!) solved the $3 \times n$ case and won over $\$100\,000$
  - Computers can calculate winning moves for grids of reasonable size

Some cool extensions
Chomp!

- 3D or nD chomp
- Infinite/ordinal chomp:
  - Here is how player "Too" can win on a $2 \times \omega$ board