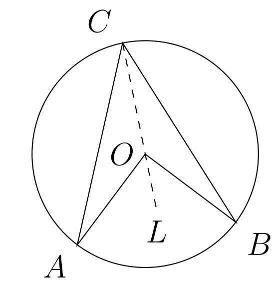
Mackenzie Math Club

Euclid Preparation 3

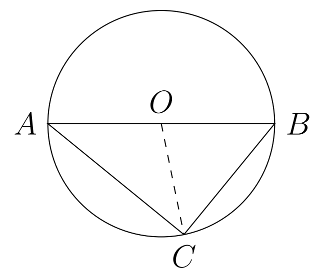
March 19, 2017

Theorem 1 (“Star Trek” theorem).
The central angle *subtended* by any arc is twice any of the inscribed angles on that arc.

$$\angle AOB = 2\angle ACB$$

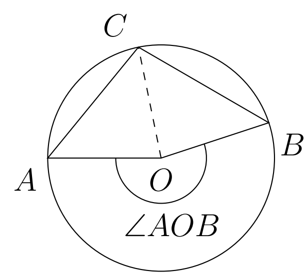


Extension 1.1 (Diameters and right angles).
If the chord AB is a diameter then $\angle ACB = 90°$.



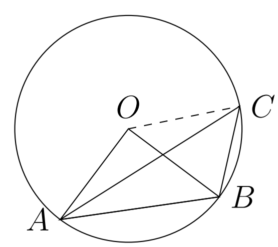
Extension 1.2 (On the major arc).
Theorem 1 is still true if $\angle AOB > 180°$.

$$\angle AOB = 2\angle ACB$$

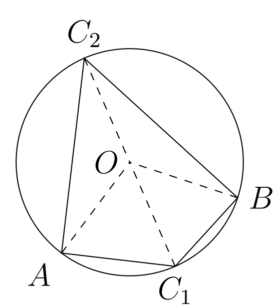


Extension 1.3 (Intersecting).
Theorem 1 is still true if the point C is chosen so that AB and OB intersect.

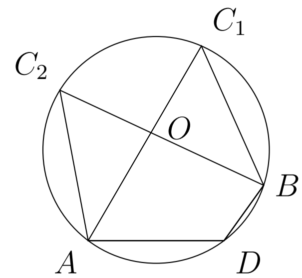
$$\angle AOC = 2\angle ACB$$



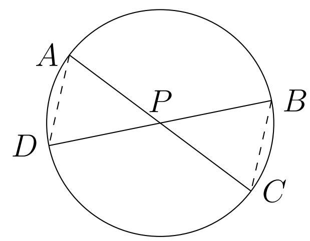
Extension 1.4 (Cyclic quadrilaterals).
If C_1 and C_2 are two points on the circle, one on the minor arc AB and the other on the major arc, then $\angle AC_1B + \angle AC_2B = 180°$. This is equivalent to proving that the opposite angles of a cyclic quadrilateral are supplementary.



Extension 1.5 (Angles subtended by the same arc).
If C_1 and C_2 are two different choices for the position of the point C along the same arc AB then $\angle AC_1B = \angle AC_2B$.
This is equivalent to saying that angles subtended by the same arc are equal.



Theorem 2 (Crossed chord theorem).
If two chords AB and CD of a circle intersect at point P, then $(PA)(PB) = (PC)(PD)$.

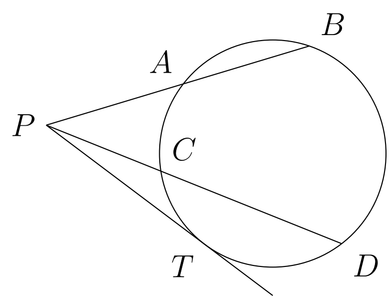


Extension 2.1 (Secant and tangents).
If PAB and PCD are two secants of the same circle and they intersect at a point P outside the circle then:

$$(PA)(PB) = (PC)(PD)$$

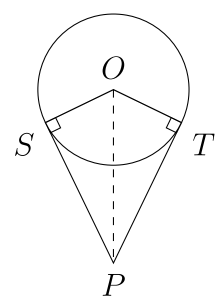
Additionally, if PT is a tangent to the circle, then:

$$(PA)(PB) = (PT)^2$$



Properties (Properties of two tangents).
If P is a point outside of a circle and PT and PS are two tangents to the circle, then the following are true:

1. A tangent at a point on a circle is perpendicular to the radius draw to the point.
2. $PS = PT$: tangents to a circle from an external point are equal.
3. OP bisects the angle between the tangents.



Theorem 3 (Tangent chord theorem).
Given that TA is any chord of a circle and PT is a tangent to the circle at T. If C is a point on the circle chosen to be on the side of the chord opposite to the tangent then $\angle TCA = \angle PTA$.