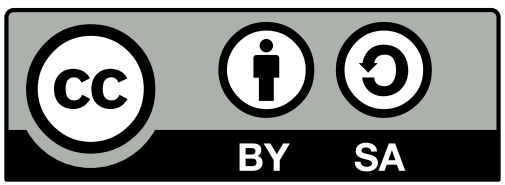
Geometry 1

Caroline Liu



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Polygons
Introduction

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Sides	Prefix	Name
3	Tri	Triangle
4	Quad	Quadrilateral
5	Penta	Pentagon

Angles
Introduction

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$$180(n-2)$$

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(It's always 360°.)

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(It's always 360°.)

These formulas are very useful for contests.

Shapes to look out for: without 3 sides

Introduction

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Trapezoids Can often be split into 2 triangles and a rectangle. Questions include determining a dimension given other info, or calculating area. Very common contest question.

Parallelograms, squares, and rectangles May be used in conjunction with circles, or you will be tasked with finding a dimension given some info. Also a common contest question.

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Facts, formulas, and things to look out for

Triangles

Triangles are one of the most common shapes found on contests. Whether its determining angles or sides, or deducing similar triangles, they are almost a guarantee. Basic concepts needed are the sum of the interior angles being 180°, and that the sum of 2 sides should never be larger than the 3rd side

Facts, formulas, and things to look out for

Triangles

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Next, we'll go over some important concepts.

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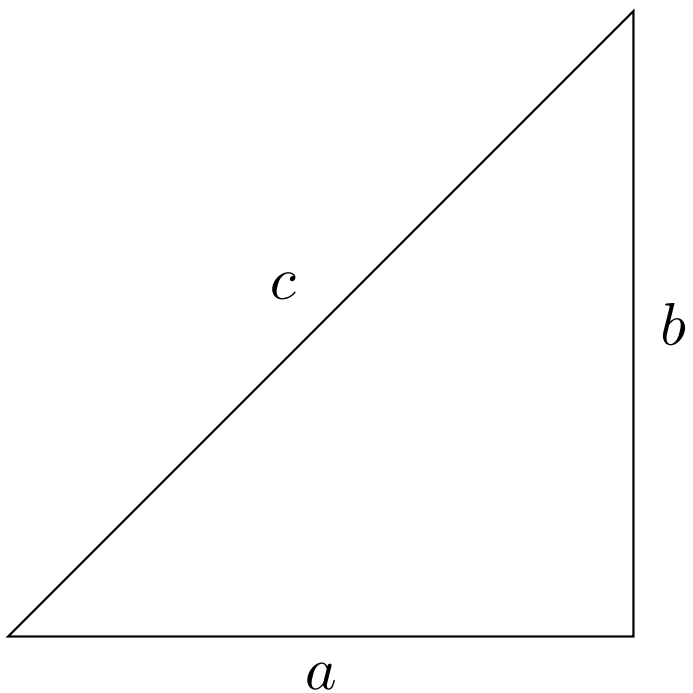
Your best friend

Pythagorean theorem

The Pythagorean theorem states that the relationship between the lengths of the sides of a right triangle is:

$$a^2 + b^2 = c^2$$

Where a and b are the legs of the triangle, and c is the hypotenuse.



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Pythagorean triplets

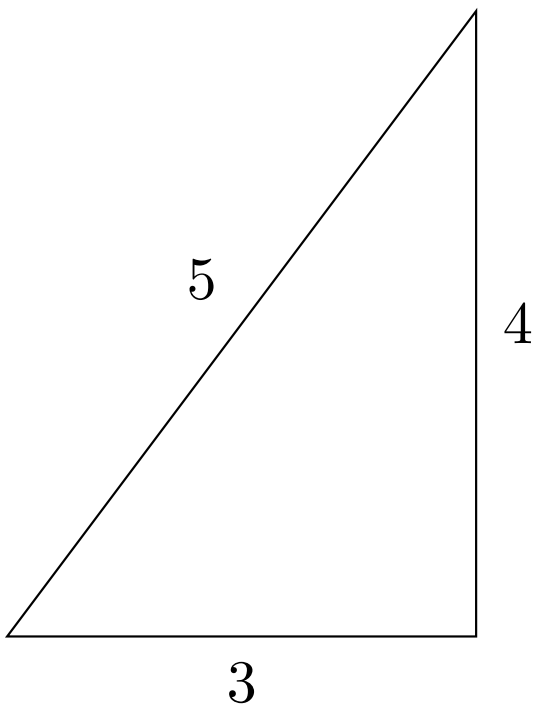
Important/special triangles

Pythagorean triplets are right triangles with three positive integer side lengths. A common example of this is a triangle whose side lengths have a ratio of $3 : 4 : 5$, or a multiple of that.

Pythagorean triplets are a quick way to determine if certain triangles are a right triangle, and also useful for finding the side lengths of triangles.

Pythagorean triplets example

Important/special triangles



Proof.

$$a^2 + b^2 = c^2$$

Let LS be the left side of the equation: $a^2 + b^2$
Let RS be the right side of the equation: c^2
Then $LS = RS$

$$LS = a^2 + b^2 \qquad RS = c^2$$
$$LS = 3^2 + 4^2 \qquad RS = 5^2$$
$$LS = 9 + 16 \qquad RS = 25$$
$$LS = 25$$

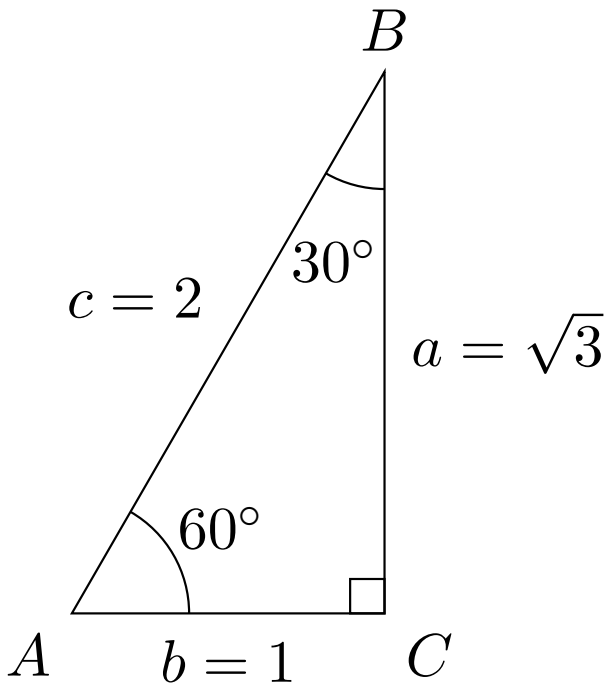
$$LS = RS$$

□

$30^\circ - 60^\circ - 90^\circ$ triangles

Important/special triangles

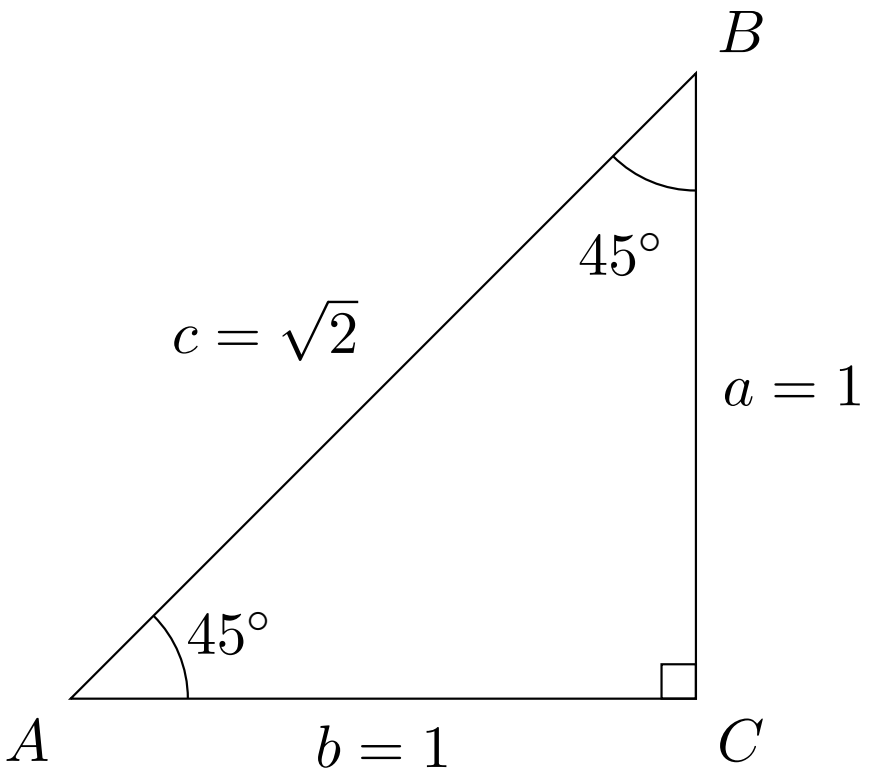
The ratio of the sides of a triangles with angles 30°, 60°, and 90° will always be $1 : \sqrt{3} : 2$.



$45^\circ - 45^\circ - 90^\circ$ triangles

Important/special triangles

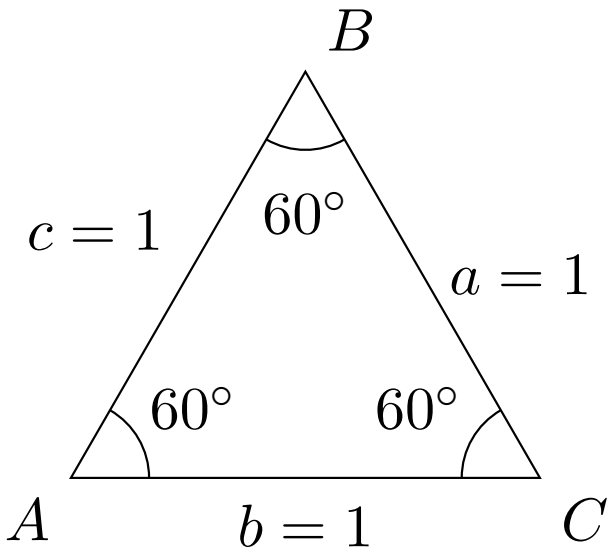
The ratio of the sides of a triangles with angles 45°, 45°, and 90° will always be $1 : 1 : \sqrt{2}$.



Equilateral triangles

Important/special triangles

The ratio of the sides of a triangles with angles 60°, 60°, and 60° will always be $1 : 1 : 1$.



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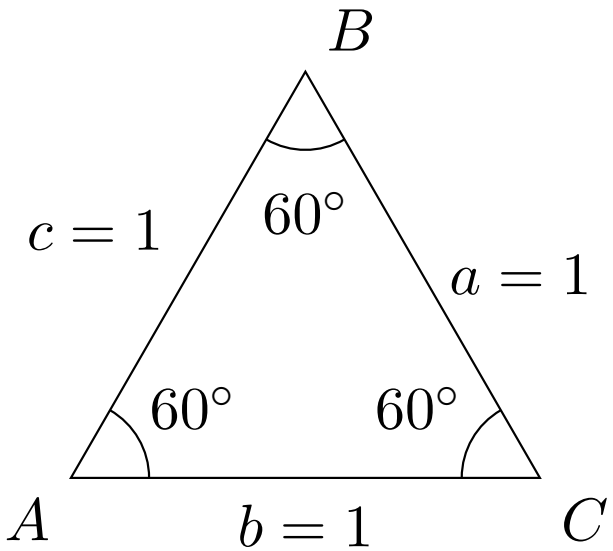

The area of an equilateral triangle with side length s is:

$$A = \frac{\sqrt{3}s^2}{4}$$

We can prove this.

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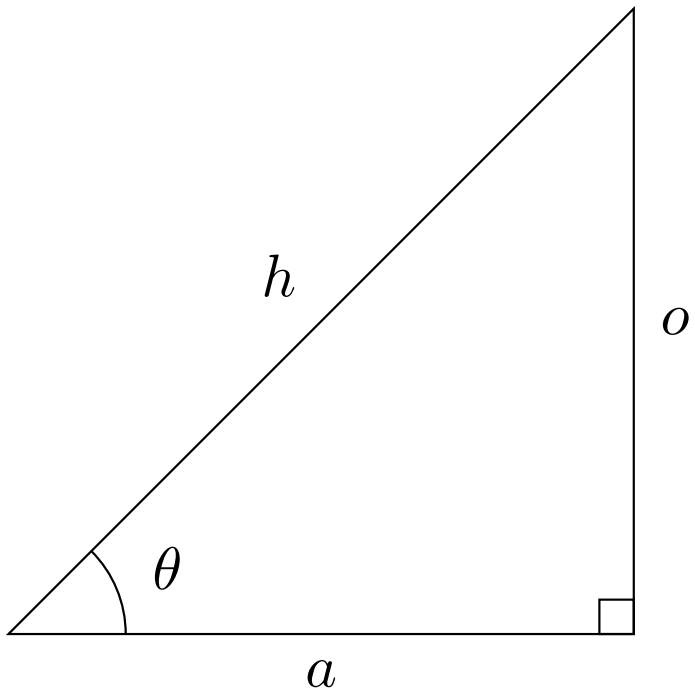
SOHCAHTOA

Trigonometric identities

$$\sin(\theta) = \frac{o}{h} \qquad \cos(\theta) = \frac{a}{h} \qquad \tan(\theta) = \frac{o}{a}$$



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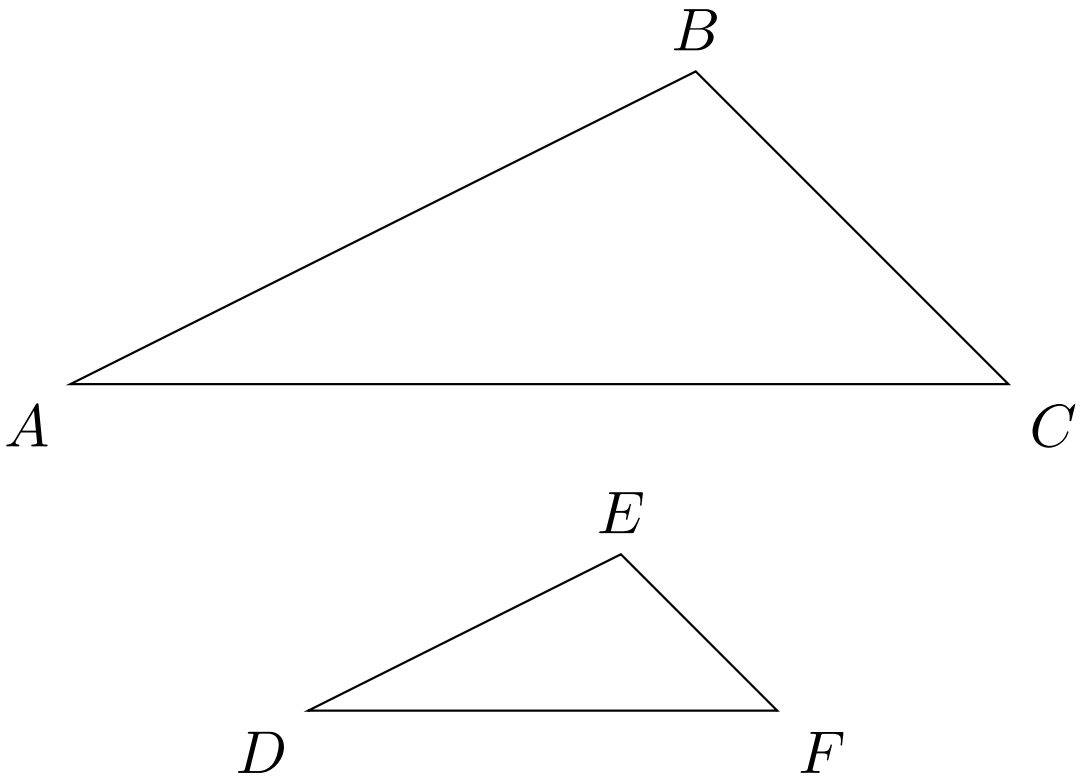
What similar triangles are

Similar triangles

Similar triangles have the same angles but different side lengths.



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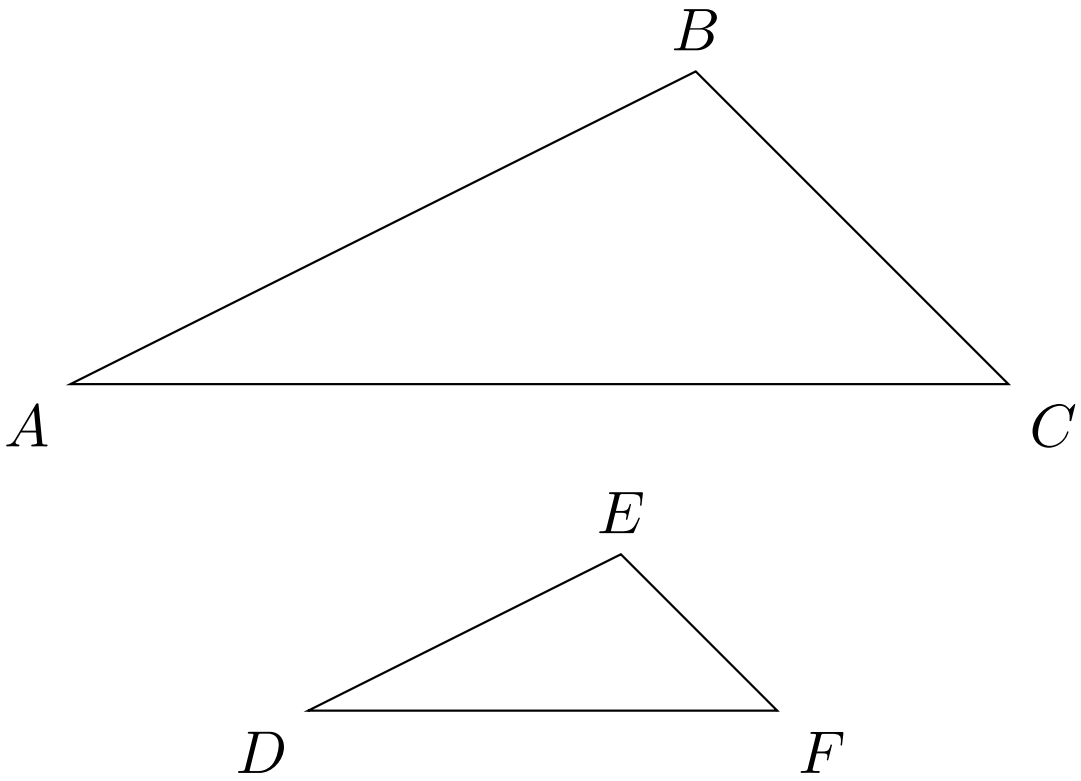


$$\triangle ABC \sim \triangle DEF$$

Conditions for similarity

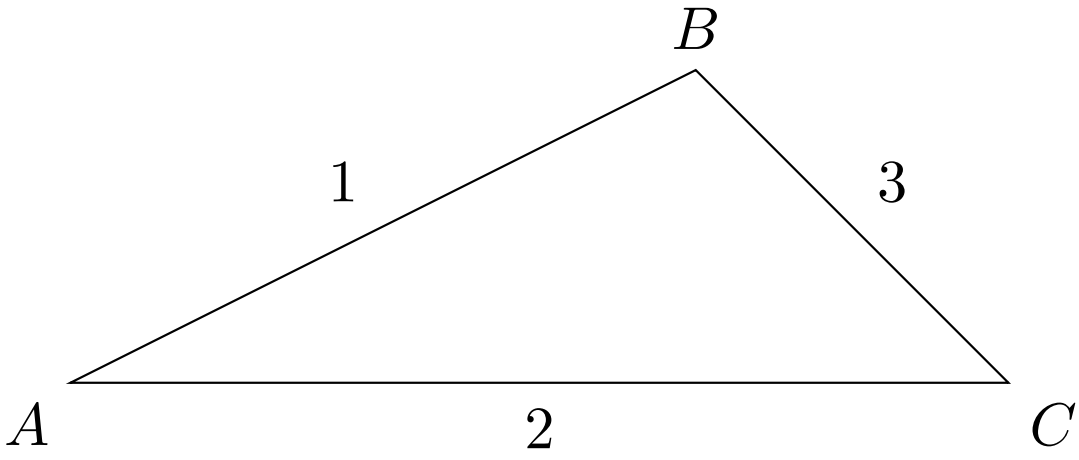
Similar triangles

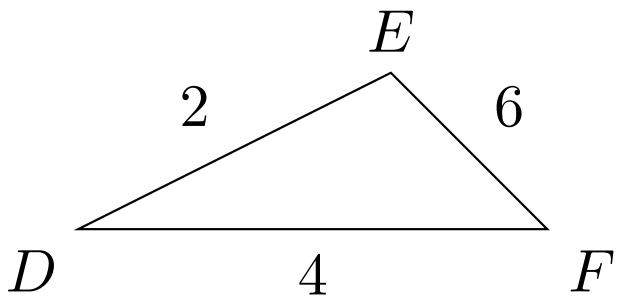
Side, side, side (SSS) If all 3 sides have the same ratio.

Angle, angle, angle (AAA) If all 3 angles are the same.

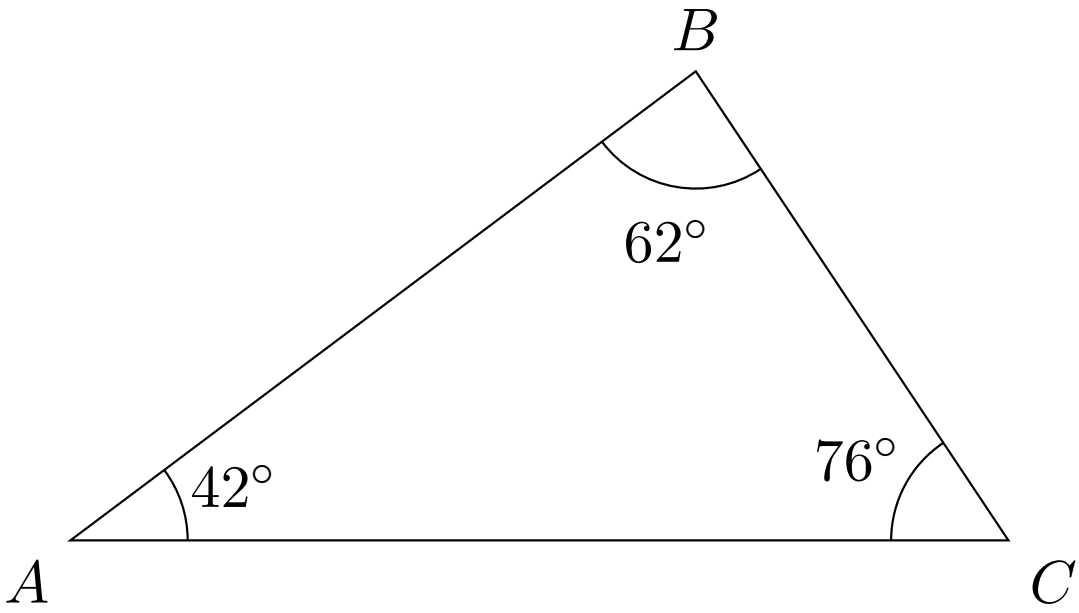
Side, angle, side (SAS) If 2 sides have the same ratio and 1 angle is the same.

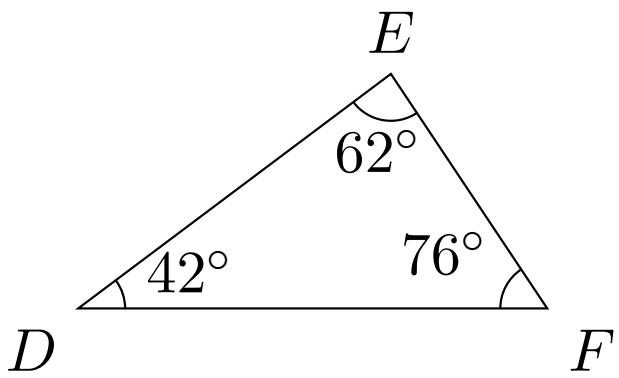
SSS
Similar triangles



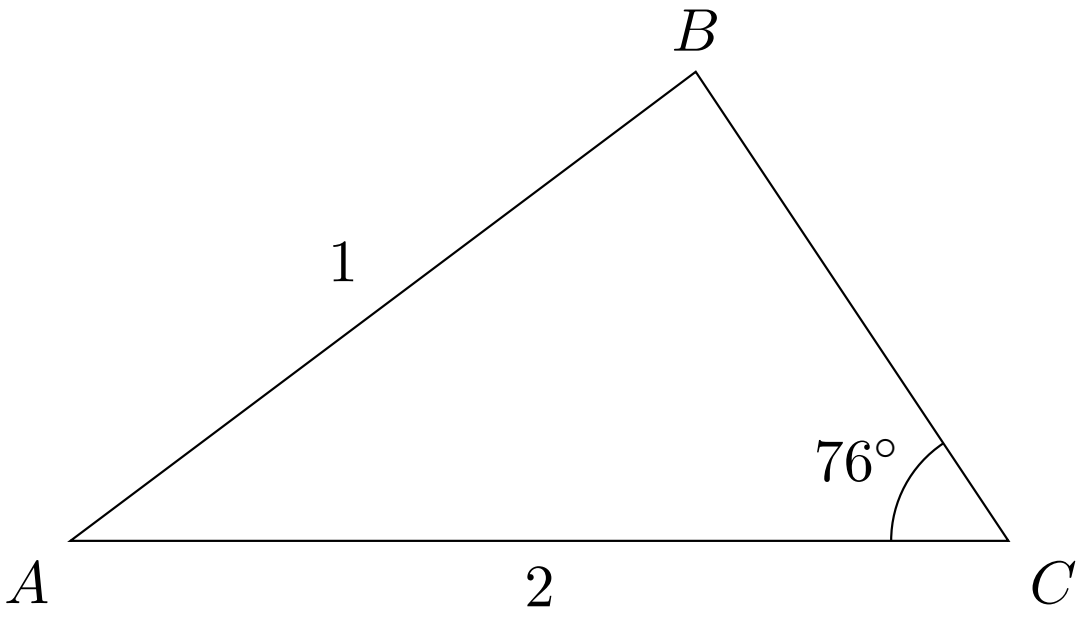


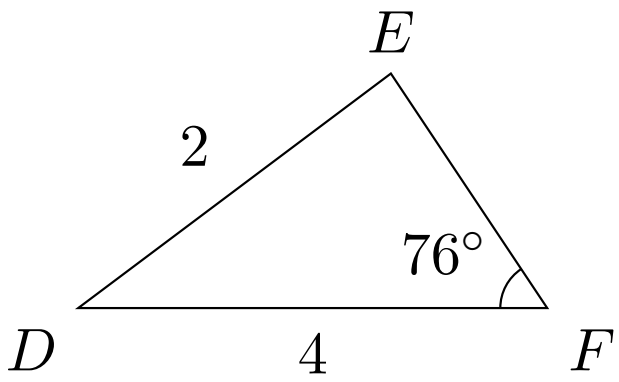
AAA
Similar triangles





SAS
Similar triangles





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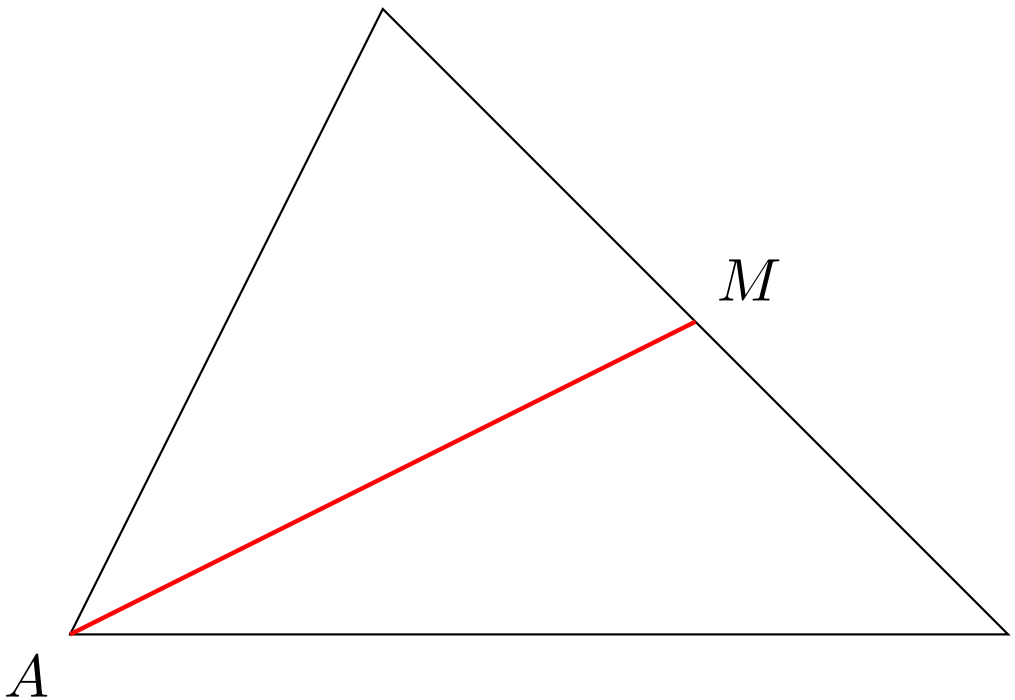
Bisectors/medians

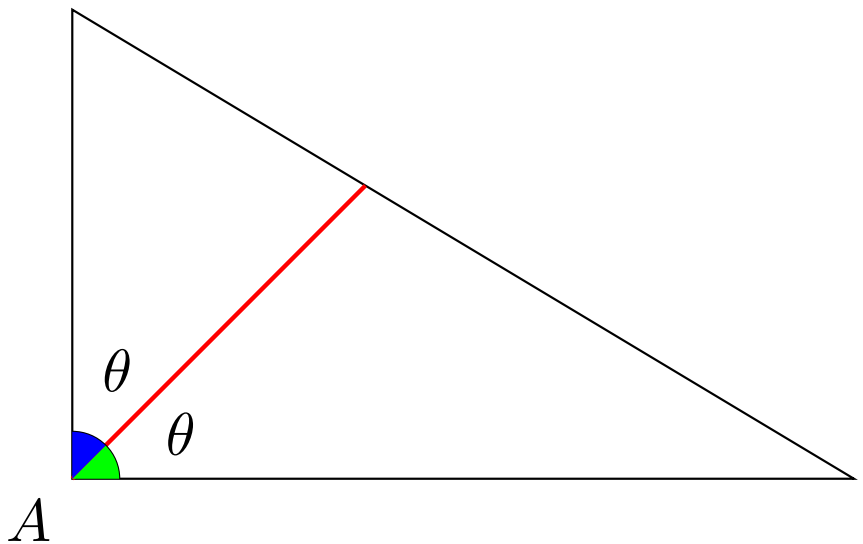
Special lines

Bisectors or medians are lines from a vertex to the middle of the opposite side.



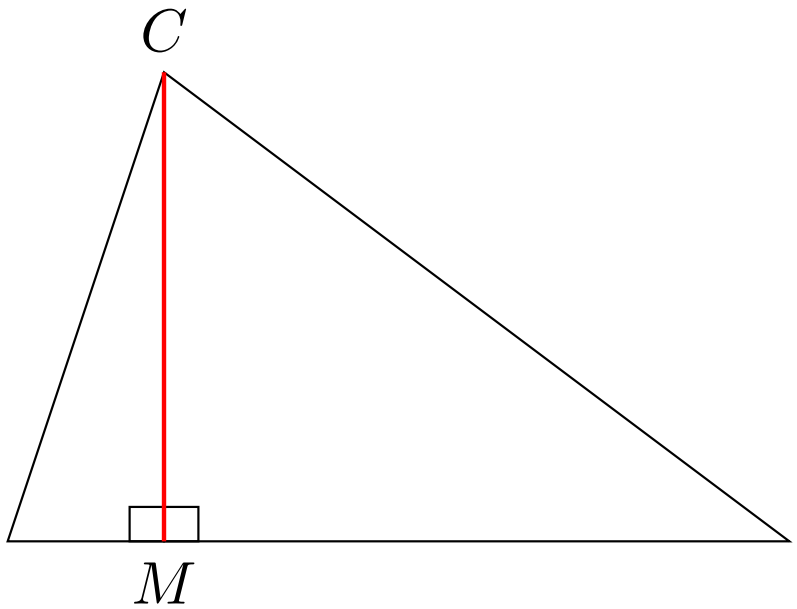
Angle bisectors
Special lines

Angle bisectors are lines bisecting an angle.



Heights/altitudes
Special lines

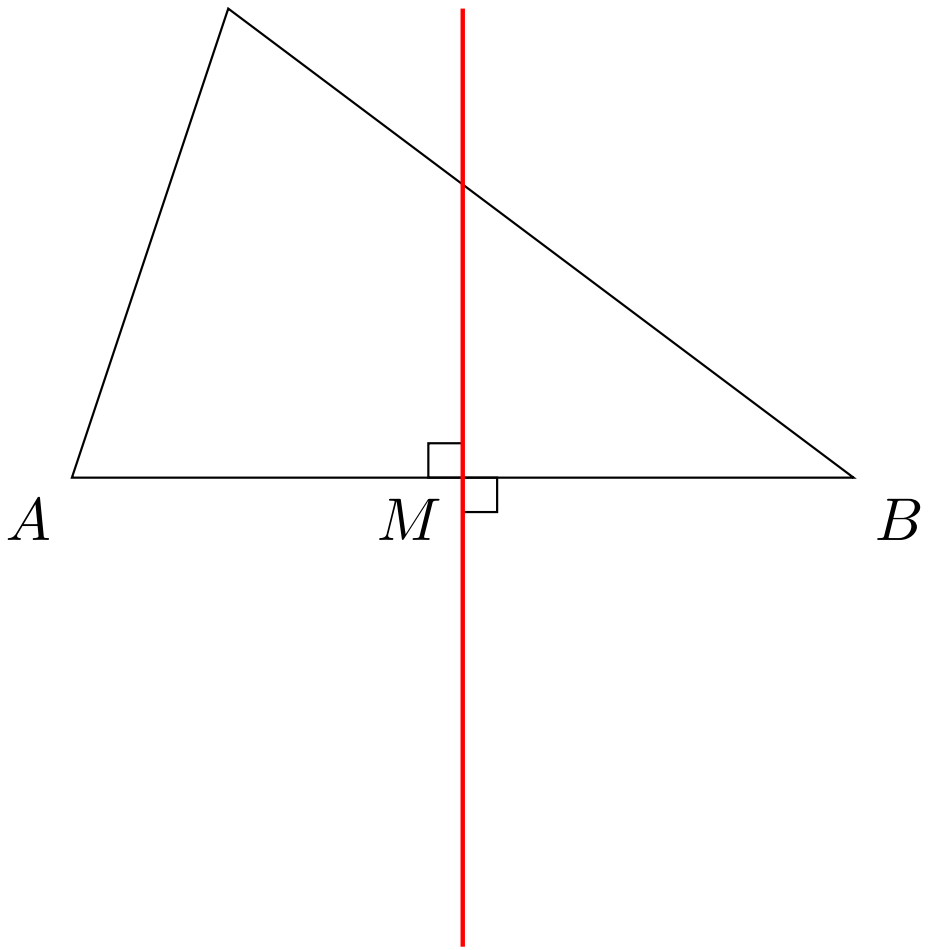
Heights or altitudes are lines perpendicular to a side that intersect the opposite vertex.



Perpendicular bisectors

Special lines

Perpendicular bisectors are lines perpendicular to a side that also bisect it.



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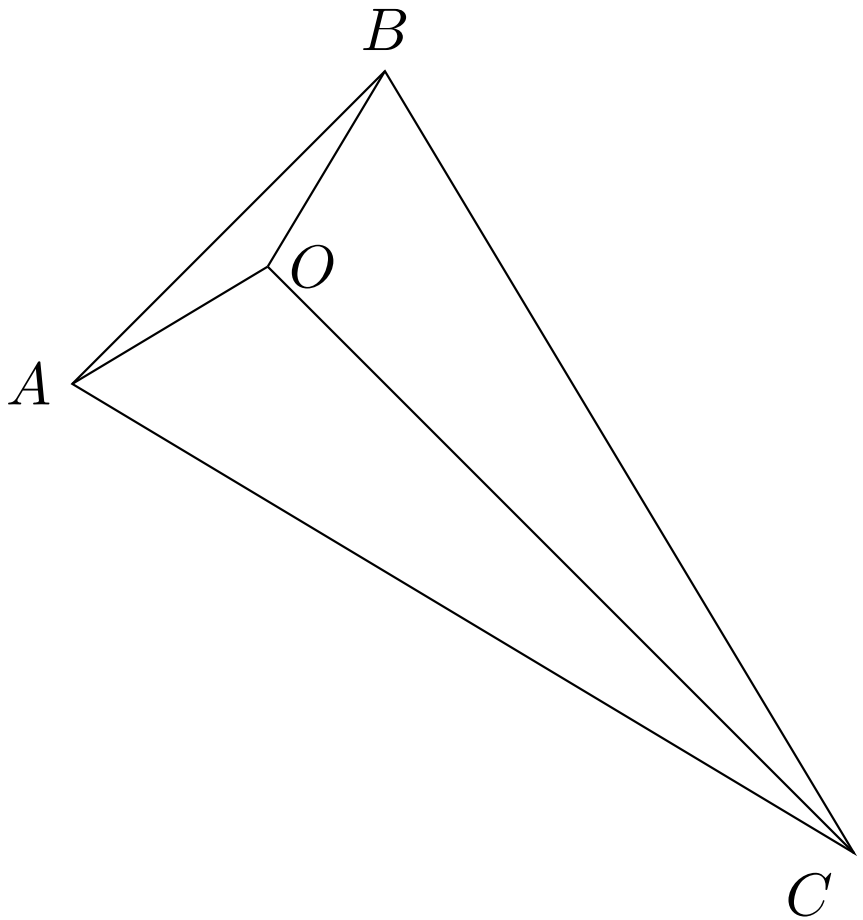
Orthocenter

Special points of intersection

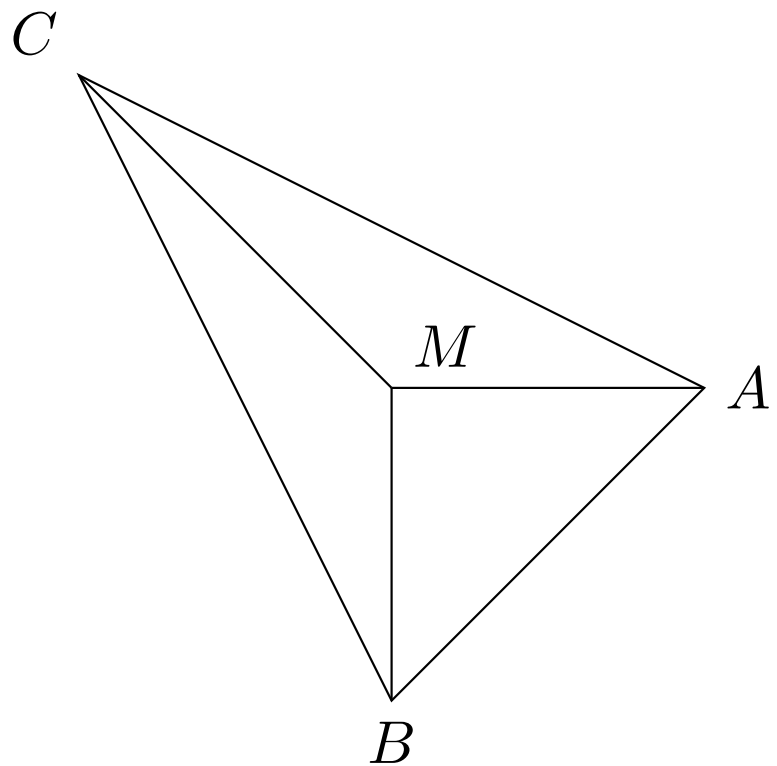
The orthocenter is the point of intersection of the 3 altitudes of a triangle.



Centroid

Special points of intersection

The centroid is the point of intersection of the 3 medians of a triangle.



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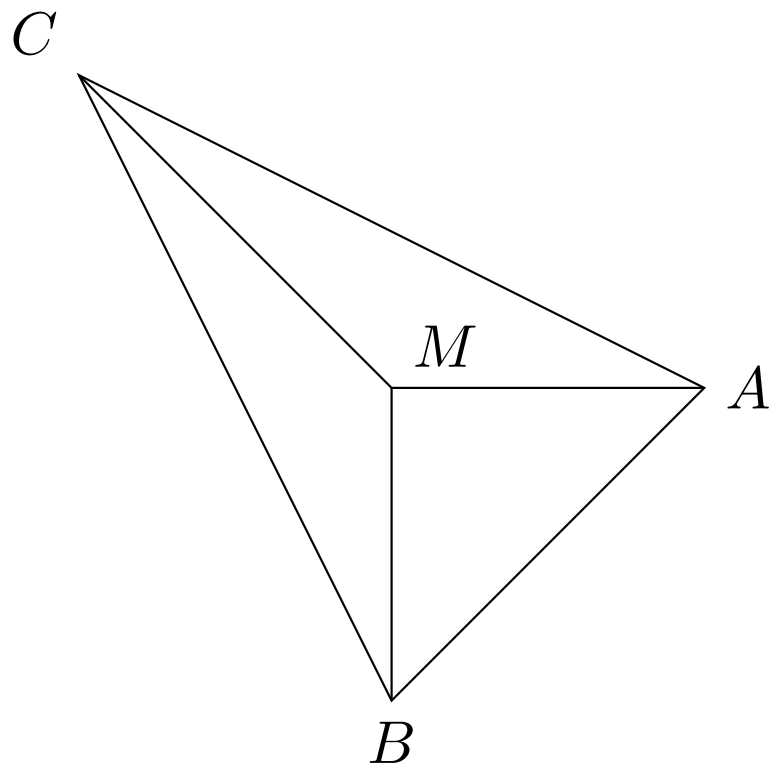
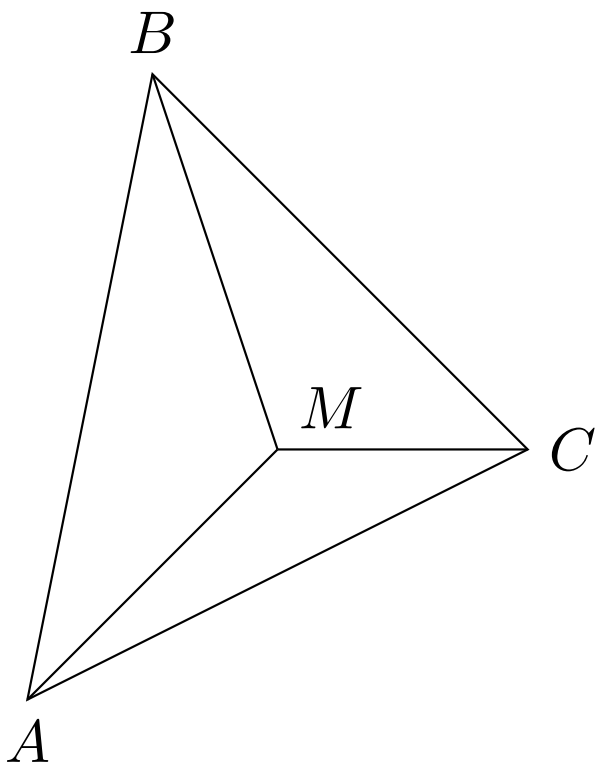


The centroid splits each median into the ratio $2:1$.

Incenter

Special points of intersection

The incenter is the point of intersection of the 3 angle bisectors of a triangle.



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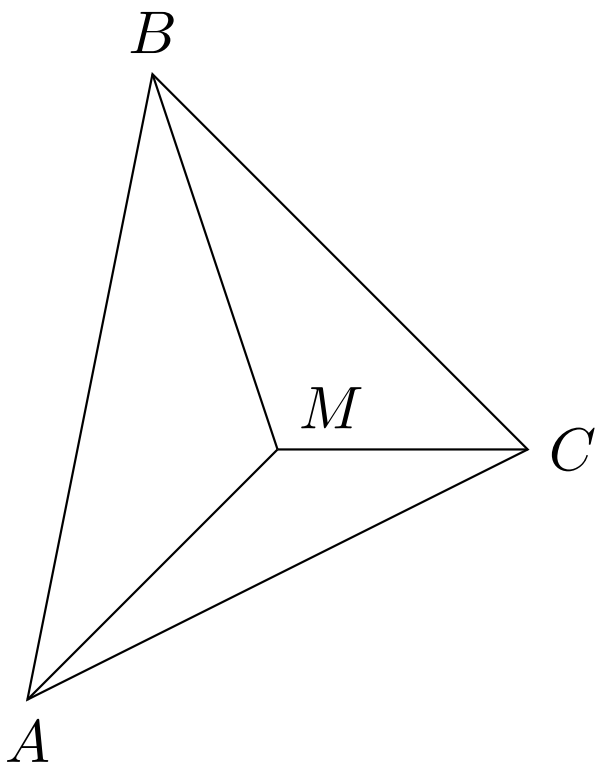
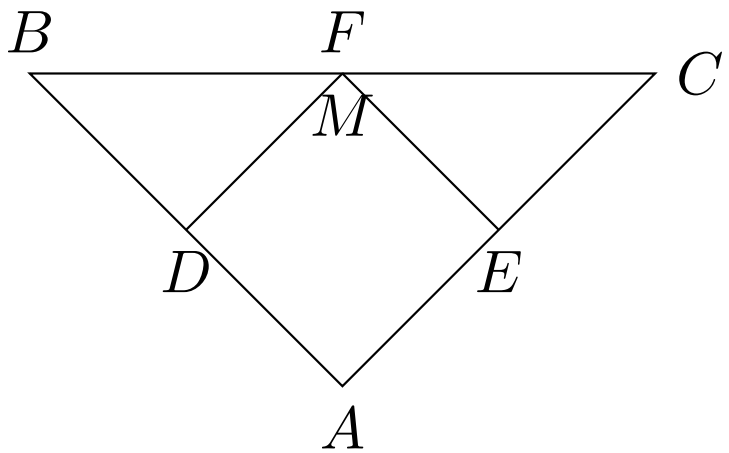


The incenter is the centre of a triangle's inscribed circle.

Circumcenter

Special points of intersection

The circumcenter is the point of intersection of the 3 perpendicular bisectors of a triangle.



Circumcenter
Special points of intersection

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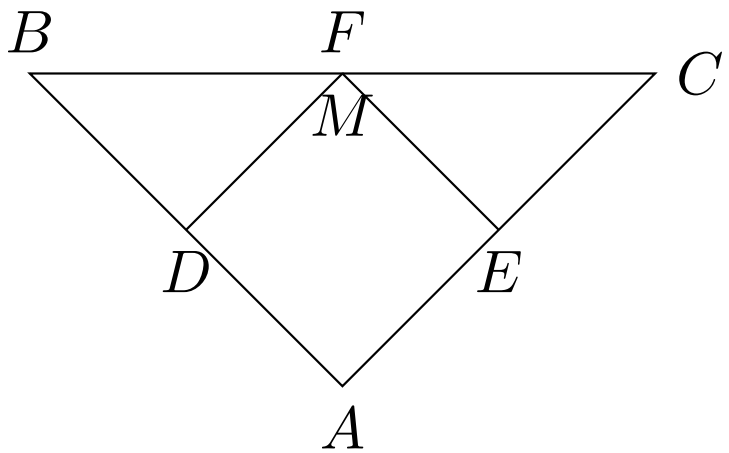

The circumcenter is the centre of triangle's incircle, the circle which is enclosed on all sides by the triangle. It is equidistant from all 3 sides of the triangle.