Lambda Calculus

Now you can bring a computer to your tests!

Vincent Macri

William Lyon Mackenzie C.I. Math Club

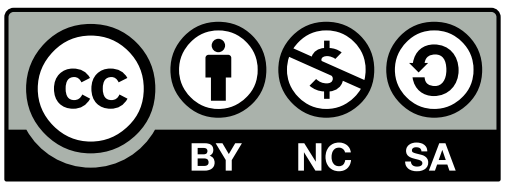

Table of Contents

What is lambda calculus?
Introduction

- Created by Alonzo Church

What is lambda calculus?
Introduction

- Created by Alonzo Church
- A way of representing pure mathematical functions

What is lambda calculus?
Introduction

- Created by Alonzo Church
- A way of representing pure mathematical functions
- Can represent any computer program

What is lambda calculus?
Introduction

- Created by Alonzo Church
- A way of representing pure mathematical functions
- Can represent any computer program
- Equivalent to Turing machines

In math class, we would define a function that accepts an argument x and outputs $x + 1$ as so:

$$f(x)$$

In math class, we would define a function that accepts an argument x and outputs $x + 1$ as so:

$$f(x) = x + 1$$

In math class, we would define a function that accepts an argument x and outputs $x + 1$ as so:

$$f(x) = x + 1$$

In lambda calculus, we do it like this:

$$\lambda$$

In math class, we would define a function that accepts an argument x and outputs $x + 1$ as so:

$$f(x) = x + 1$$

In lambda calculus, we do it like this:

$$\lambda x$$

In math class, we would define a function that accepts an argument x and outputs $x + 1$ as so:

$$f(x) = x + 1$$

In lambda calculus, we do it like this:

$$\lambda x.$$

In math class, we would define a function that accepts an argument x and outputs $x + 1$ as so:

$$f(x) = x + 1$$

In lambda calculus, we do it like this:

$$\lambda x.x + 1$$

In math class, we would define a function that accepts an argument x and outputs $x + 1$ as so:

$$f(x) = x + 1$$

In lambda calculus, we do it like this:

$$\lambda x.x + 1$$

If we wanted to find $4 + 1$, we could do this:

$$f(4) = 4 + 1 = 5$$

In math class, we would define a function that accepts an argument x and outputs $x + 1$ as so:

$$f(x) = x + 1$$

In lambda calculus, we do it like this:

$$\lambda x.x + 1$$

If we wanted to find $4 + 1$, we could do this:

$$f(4) = 4 + 1 = 5$$

In lambda calculus, we apply a value to a function like this:

$$(\lambda x.x + 1)4 = 4 + 1 = 5$$

In math class, we would define a function that accepts an argument x and outputs $x + 1$ as so:

$$f(x) = x + 1$$

In lambda calculus, we do it like this:

$$\lambda x.x + 1$$

If we wanted to find $4 + 1$, we could do this:

$$f(4) = 4 + 1 = 5$$

In lambda calculus, we apply a value to a function like this:

$$(\lambda x.x + 1)4 = 4 + 1 = 5$$

You can think of λ as f, and . as $=$.

Table of Contents

Currying
One Parameter

How would we define a function that outputs $x + y$ in math class?

Currying
One Parameter

How would we define a function that outputs $x + y$ in math class?

$$f(x, y) = x + y$$

Currying
One Parameter

How would we define a function that outputs $x + y$ in math class?

$$f(x, y) = x + y$$

In lambda calculus, functions are only allowed to have one parameter.

Currying
One Parameter

How would we define a function that outputs $x + y$ in math class?

$$f(x, y) = x + y$$

In lambda calculus, functions are only allowed to have one parameter.

So, to add two numbers, we have a function output another function, like this:

$$\lambda x.\lambda y.x + y$$

And we use it like this:

$$(\lambda x.\lambda y.x + y)(2, 3)$$

Currying
One Parameter

How would we define a function that outputs $x + y$ in math class?

$$f(x, y) = x + y$$

In lambda calculus, functions are only allowed to have one parameter.

So, to add two numbers, we have a function output another function, like this:

$$\lambda x.\lambda y.x + y$$

And we use it like this:

$$(\lambda x.\lambda y.x + y)(2, 3) = (\lambda y.2 + y)3$$

Currying
One Parameter

How would we define a function that outputs $x + y$ in math class?

$$f(x, y) = x + y$$

In lambda calculus, functions are only allowed to have one parameter.

So, to add two numbers, we have a function output another function, like this:

$$\lambda x.\lambda y.x + y$$

And we use it like this:

$$(\lambda x.\lambda y.x + y)(2, 3) = (\lambda y.2 + y)3 = 2 + 3 = 5$$

Why one argument?
One Parameter

- Very simple

Why one argument?
One Parameter

- Very simple
- Very powerful

Why one argument?
One Parameter

- Very simple
- Very powerful
- Functions can only have one variable

But I'm lazy

One Parameter

Me too!

But I'm lazy

One Parameter

Me too!

We have some shortcuts to help us write down lambda calculus expressions, but it's important to remember what they represent, without the shortcuts.

But I'm lazy

One Parameter

Me too!

We have some shortcuts to help us write down lambda calculus expressions, but it's important to remember what they represent, without the shortcuts.

$$\lambda x.\lambda y.\lambda z.A$$

Can be abbreviated as:

$$\lambda xyz.A$$

But I'm lazy

One Parameter

Me too!

We have some shortcuts to help us write down lambda calculus expressions, but it's important to remember what they represent, without the shortcuts.

$$\lambda x.\lambda y.\lambda z.A$$

Can be abbreviated as:

$$\lambda xyz.A$$

Also, we assume that we evaluate a function with "multiple" arguments starting with the leftmost parameter.

This is stupid. It just makes everything harder.

One Parameter

you're right!

This is stupid. It just makes everything harder.

One Parameter

For those examples, you're right!

This is stupid. It just makes everything harder.

One Parameter

For those examples, you're right!
Let's get to the fun stuff now!