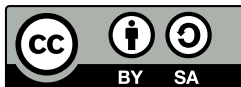


Boolean Algebra

Vincent Macri



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What is boolean algebra

Basics

- A branch of mathematics dealing only with true and false values (usually called 1 and 0, respectively)



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Basics

- A branch of mathematics dealing only with true and false values (usually called 1 and 0, respectively)
- Useful while considering logic
- Useful in computer science



Main operations

Basics

Boolean algebra has four important¹ operations.

¹There are other operations, but they are not as commonly used.



Main operations

Basics

Boolean algebra has four important¹ operations.

$\neg A$ NOT (negation). Also written as \overline{A} .

$A \wedge B$ AND (conjunction). Also written as $A \cdot B$ or AB .

$A \oplus B$ XOR (exclusive or).

$A \vee B$ OR (disjunction). Also written as $A + B$.

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Order of operations is **BNAO**: brackets, NOT, AND, then OR.

Note on exclusive or's placement

There is no generally agreement on where to put XOR in the order of operations. It is commonly put between AND and OR (BNAXO), but you should always use brackets to avoid ambiguity.

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What is a truth table

Truth Tables

A truth table is table of all possible input and output values of a boolean algebra statement.



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A truth table is table of all possible input and output values of a boolean algebra statement.

They are similar to the multiplication tables you used in elementary school, but are much more powerful.



Truth tables of main operations

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Truth Tables

Table: AND

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1



Truth tables of main operations

Truth Tables

Table: AND

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0	1	0
1	0	0
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Table: OR

A	B	$A \vee B$
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Truth tables of main operations

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Table: NOT

A	\overline{A}
1	1
1	0



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Associativity, commutativity, and distributivity

Laws and Identities

Boolean algebra has many similar laws as regular algebra. For example, both \wedge and \vee follow the associative law and commutative laws, just like \times and $+$. They also follow the distributive law.



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$$A + (B + C) = (A + B) + C$$



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Distributive law

$$A \cdot (B + C) = AB + AC$$



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- $A \cdot 0 = 0$

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- $A + A = A$

- $A \cdot A = A$



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- $A \cdot 1 = A$

- $A \cdot 0 = 0$

However, some of the identities which are true in boolean algebra do not work in regular algebra.

- $A + 1 = 1$
- $A \cdot (A + B) = A$

- $A + A = A$

- $A \cdot A = A$



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Some of the identities in boolean algebra are the same as in regular algebra.

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- $A \cdot 1 = A$

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- $A + A = A$
- $A + AB = A$

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Identities

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Some of the identities in boolean algebra are the same as in regular algebra.

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- $A \cdot 1 = A$

- $A \cdot 0 = 0$

However, some of the identities which are true in boolean algebra do not work in regular algebra.

- $A + 1 = 1$
- $A \cdot (A + B) = A$

- $A + A = A$
- $A + AB = A$

- $A \cdot A = A$
- $A + BC = (A + B) \cdot (A + C)$



Identities with NOT

Laws and Identities

These are some identities involving NOT:



Identities with NOT

Laws and Identities

These are some identities involving NOT:

$$\overline{\overline{A}} = A$$



Identities with NOT

Laws and Identities

These are some identities involving NOT:

$$\overline{\overline{A}} = A$$

$$\overline{A} + A = 1$$



Identities with NOT

Laws and Identities

These are some identities involving NOT:

$$\overline{\overline{A}} = A$$

$$\overline{A} + A = 1$$

$$\overline{A} \cdot A = 0$$



De Morgan's laws

Laws and Identities

Another set of identities useful in boolean algebra are De Morgan's laws.



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Another set of identities useful in boolean algebra are De Morgan's laws.

NOT OR

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

A	B	$\overline{A + B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0



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Laws and Identities

Another set of identities useful in boolean algebra are De Morgan's laws.

NOT OR

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

A	B	$\overline{A + B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

NOT AND

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

A	B	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



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$$A + AB \cdot B + C \cdot \overline{C}$$



$$A + AB \cdot B + C \cdot \overline{C}$$
$$A + (AB \cdot B) + (C \cdot \overline{C})$$



$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned} & A + (AB \cdot B) + (C \cdot \overline{C}) \\ &= A + (AB) + (0) \end{aligned}$$



$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + (AB \cdot B) + (C \cdot \overline{C})$$

$$= A + (AB) + (0)$$

$$= A + AB$$



$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + (AB \cdot B) + (C \cdot \overline{C})$$

$$= A + (AB) + (0)$$

$$= A + AB$$

$$= A$$



$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + (AB \cdot B) + (C \cdot \overline{C})$$

$$= A + (AB) + (0)$$

$$= A + AB$$

$$= A$$

$$\therefore A + AB \cdot B + C \cdot \overline{C} = A$$

