Boolean Algebra

Vincent Macri

Table of Contents

1 Basics

2 Truth Tables

3 Laws and Identities

4 Practice

What is boolean algebra
Basics

- A branch of mathematics dealing only with true and false values (usually called 1 and 0, respectively)

- A branch of mathematics dealing only with true and false values (usually called 1 and 0, respectively)
- Useful while considering logic

What is boolean algebra

Basics

- A branch of mathematics dealing only with true and false values (usually called 1 and 0, respectively)
- Useful while considering logic
- Useful in computer science

Main operations
Basics

Boolean algebra has four important[1] operations.

[1]There are other operations, but they are not as commonly used.

Main operations
Basics

Boolean algebra has four important[1] operations.

- $\neg A$ NOT (negation). Also written as $\overline{A}$.
- $A \wedge B$ AND (conjunction). Also written as $A \cdot B$ or AB.
- $A \oplus B$ XOR (exclusive or).
- $A \vee B$ OR (disjunction). Also written as $A + B$.

[1] There are other operations, but they are not as commonly used.

Main operations
Basics

Boolean algebra has four important[1] operations.

- $\neg A$ NOT (negation). Also written as $\overline{A}$.
- $A \wedge B$ AND (conjunction). Also written as $A \cdot B$ or AB.
- $A \oplus B$ XOR (exclusive or).
- $A \vee B$ OR (disjunction). Also written as $A + B$.

Order of operations is BNAO: brackets, NOT, AND, then OR.

[1] There are other operations, but they are not as commonly used.

Boolean algebra has four important[1] operations.

- $\neg A$ NOT (negation). Also written as $\overline{A}$.
- $A \wedge B$ AND (conjunction). Also written as $A \cdot B$ or AB.
- $A \oplus B$ XOR (exclusive or).
- $A \vee B$ OR (disjunction). Also written as $A + B$.

Order of operations is BNAO: brackets, NOT, AND, then OR.

Note on exclusive or's placement

There is no generally agreement on where to put XOR in the order of operations. It is commonly put between AND and OR (BNAXO), but you should always use brackets to avoid ambiguity.

[1] There are other operations, but they are not as commonly used.

Table of Contents

1 Basics

2 Truth Tables

3 Laws and Identities

4 Practice

A truth table is table of all possible input and output values of a boolean algebra statement.

A truth table is table of all possible input and output values of a boolean algebra statement.
They are similar to the multiplication tables you used in elementary school, but are much more powerful.

Table: AND

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

Truth tables of main operations

Truth Tables

Table: AND

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

Table: OR

A	B	$A \vee B$
0	0	0
0	1	1
1	0	1
1	1	1

Truth tables of main operations

Truth Tables

Table: AND

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

Table: XOR

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Table: OR

A	B	$A \vee B$
0	0	0
0	1	1
1	0	1
1	1	1

Truth tables of main operations
Truth Tables

Table: AND

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

Table: XOR

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Table: OR

A	B	$A \vee B$
0	0	0
0	1	1
1	0	1
1	1	1

Table: NOT

A	$\neg A$
0	1
1	0

Table of Contents

1 Basics

2 Truth Tables

3 Laws and Identities

4 Practice

Boolean algebra has many similar laws as regular algebra.
For example, both $\wedge$ and $\vee$ follow the associative law and commutative laws, just like $\times$ and $+$.
They also follow the distributive law.

Associativity, commutativity, and distributivity

Laws and Identities

Boolean algebra has many similar laws as regular algebra.
For example, both $\wedge$ and $\vee$ follow the associative law and commutative laws, just like $\times$ and $+$.
They also follow the distributive law.

Associative law

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$
$$A + (B + C) = (A + B) + C$$

Boolean algebra has many similar laws as regular algebra.
For example, both $\wedge$ and $\vee$ follow the associative law and commutative laws, just like $\times$ and $+$.
They also follow the distributive law.

Associative law

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$
$$A + (B + C) = (A + B) + C$$

Commutative law

$$A \cdot B = B \cdot A$$
$$A + B = B + A$$

Boolean algebra has many similar laws as regular algebra.
For example, both $\wedge$ and $\vee$ follow the associative law and commutative laws, just like $\times$ and $+$.
They also follow the distributive law.

Associative law

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$
$$A + (B + C) = (A + B) + C$$

Commutative law

$$A \cdot B = B \cdot A$$
$$A + B = B + A$$

Distributive law

$$A \cdot (B + C) = AB + AC$$

Some of the identities in boolean algebra are the same as in regular algebra.

Some of the identities in boolean algebra are the same as in regular algebra.

- $A + 0 = A$

Some of the identities in boolean algebra are the same as in regular algebra.

- $A + 0 = A$
- $A \cdot 1 = A$

Identities

Some of the identities in boolean algebra are the same as in regular algebra.

- $A + 0 = A$
- $A \cdot 1 = A$
- $A \cdot 0 = 0$

Some of the identities in boolean algebra are the same as in regular algebra.

- $A + 0 = A$
- $A \cdot 1 = A$
- $A \cdot 0 = 0$

However, some of the identities which are true in boolean algebra do not work in regular algebra.

Some of the identities in boolean algebra are the same as in regular algebra.

- $A + 0 = A$
- $A \cdot 1 = A$
- $A \cdot 0 = 0$

However, some of the identities which are true in boolean algebra do not work in regular algebra.

- $A + 1 = 1$

Some of the identities in boolean algebra are the same as in regular algebra.

- $A + 0 = A$
- $A \cdot 1 = A$
- $A \cdot 0 = 0$

However, some of the identities which are true in boolean algebra do not work in regular algebra.

- $A + 1 = 1$
- $A + A = A$

Some of the identities in boolean algebra are the same as in regular algebra.

- $A + 0 = A$
- $A \cdot 1 = A$
- $A \cdot 0 = 0$

However, some of the identities which are true in boolean algebra do not work in regular algebra.

- $A + 1 = 1$
- $A + A = A$
- $A \cdot A = A$

Some of the identities in boolean algebra are the same as in regular algebra.

- $A + 0 = A$
- $A \cdot 1 = A$
- $A \cdot 0 = 0$

However, some of the identities which are true in boolean algebra do not work in regular algebra.

- $A + 1 = 1$
- $A + A = A$
- $A \cdot A = A$
- $A \cdot (A + B) = A$

Some of the identities in boolean algebra are the same as in regular algebra.

- $A + 0 = A$
- $A \cdot 1 = A$
- $A \cdot 0 = 0$

However, some of the identities which are true in boolean algebra do not work in regular algebra.

- $A + 1 = 1$
- $A + A = A$
- $A \cdot A = A$
- $A \cdot (A + B) = A$
- $A + AB = A$

Some of the identities in boolean algebra are the same as in regular algebra.

- $A + 0 = A$
- $A \cdot 1 = A$
- $A \cdot 0 = 0$

However, some of the identities which are true in boolean algebra do not work in regular algebra.

- $A + 1 = 1$
- $A + A = A$
- $A \cdot A = A$
- $A \cdot (A + B) = A$
- $A + AB = A$
- $A + BC = (A + B) \cdot (A + C)$

These are some identities involving NOT:

These are some identities involving NOT:

$$\overline{\overline{A}} = A$$

Identities with NOT

These are some identities involving NOT:

$$\overline{\overline{A}} = A$$

$$\overline{A} + A = 1$$

These are some identities involving NOT:

$$\overline{\overline{A}} = A$$

$$\overline{A} + A = 1$$

$$\overline{A} \cdot A = 0$$

Another set of identities useful in boolean algebra are De Morgan's laws.

De Morgan's laws

Laws and Identities

Another set of identities useful in boolean algebra are De Morgan's laws.

NOT OR

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

A	B	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

De Morgan's laws

Laws and Identities

Another set of identities useful in boolean algebra are De Morgan's laws.

NOT OR

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

A	B	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

NOT AND

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

A	B	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Table of Contents

1 Basics

2 Truth Tables

3 Laws and Identities

4 Practice

Simplify
Practice

$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + (AB \cdot B) + (C \cdot \overline{C})$$

$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned} & A + (AB \cdot B) + (C \cdot \overline{C}) \\ = \; & A + (AB) + (0) \end{aligned}$$

Simplify
Practice

$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned} & A + (AB \cdot B) + (C \cdot \overline{C}) \\ = {} & A + (AB) + (0) \\ = {} & A + AB \end{aligned}$$

$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned}
& A + (AB \cdot B) + (C \cdot \overline{C}) \\
= {} & A + (AB) + (0) \\
= {} & A + AB \\
= {} & A
\end{aligned}$$

Simplify

Practice

$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned} & A + (AB \cdot B) + (C \cdot \overline{C}) \\ = & A + (AB) + (0) \\ = & A + AB \\ = & A \end{aligned}$$

$\therefore A + AB \cdot B + C \cdot \overline{C} = A$