

# Lambda Calculus

Now you can bring a computer to your tests!

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# What is lambda calculus?

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- Created by Alonzo Church



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- A way of representing **pure** mathematical functions
- Can represent any computer program
- Equivalent to Turing machines



# $x + 1$

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In math class, we would define a function that accepts an argument  $x$  and outputs  $x + 1$  as so:

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You can think of  $\lambda$  as  $f$ , and  $.$  as  $=$ .



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# Currying

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So, to add two numbers, we have a function output another function, like this:

$$\lambda x. \lambda y. x + y$$

And we use it like this:

$$(\lambda x. \lambda y. x + y)(2, 3)$$



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# Why one argument?

One Parameter

- Very simple



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# Why one argument?

## One Parameter

- Very simple
- Very powerful
- Functions can only have one variable



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One Parameter

Me too!



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Can be abbreviated as:

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Also, we assume that we evaluate a function with “multiple” arguments starting with the leftmost parameter.



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One Parameter

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For those examples, you're right!



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For those examples, you're right!  
Let's get to the fun stuff now!

