Introduction to Sets

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Basic types
Types of Numbers

- $\mathbb{N}$ Natural numbers $(0, 1, 2, 3, \ldots)$
- $\mathbb{Z}$ Integers ($\mathbb{N}$ and $-1, -2, -3, \ldots$)
- $\mathbb{Q}$ Rational numbers ($\mathbb{Z}$ and $4.2, -\frac{2}{3}, \ldots$)
- $\mathbb{R}$ Real numbers ($\mathbb{Q}$ and $\pi, e, \sqrt{2}, \ldots$)
- $\mathbb{C}$ Complex numbers ($\mathbb{R}$ and $i, 2i+1, \ldots$)
- $\mathbb{P}$ Prime numbers $(2, 3, 5, 7, \ldots)$

Some mathematicians include 0 in $\mathbb{N}$, and some do not.
While it is generally accepted that $\mathbb{N}$ includes 0, we have notation to specify:

- $\mathbb{N}^0$ Natural numbers including 0
- $\mathbb{N}^*$ Natural numbers not including 0
- $\mathbb{N}^+$ Positive natural numbers (does not include 0)

We can also use this notation with other types of numbers:

- $\mathbb{Z}^-$ Negative integers (does not include 0)
- $\mathbb{R}^+$ Positive real numbers (does not include 0)

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Open and closed

Intervals

Open intervals

To denote numbers in an open (inclusive) range, we write: $[a, b]$
This means all the real numbers from a to b, including a and b.

Closed intervals

To denote numbers in a closed (exclusive) range, we write: (a, b)
This means all the real numbers from a to b, not including a and b.

$[0, 10)$ means all real numbers from 0 to 10, including 0, but not including 10.

$(-\infty, +\infty)$ means all real numbers.

$[0, \infty)$ means all real numbers that are not negative.

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A set is an unordered collection of distinct elements.
A set with a finite number of elements can be written in braces such as $\{a, b, c, \dots\}$
For example, we can define the set of math club co-presidents as:

$$M = \{\text{Vincent}, \text{Samantha}, \text{Caroline}\}$$

By convention, the names of sets are denoted in capital letters.
Since the elements of a set are distinct:

$$\{a, b, c\} \equiv \{a, a, b, c, b\}$$

($\equiv$ means equivalent)

The empty set is the set that contains no elements. It is denoted as $\emptyset$.

Examples

$\emptyset$ is the set of all 4-sided triangles.

$\emptyset$ is the set of all prime numbers divisible by 10.

The empty set can be thought of as an empty bag. It may be empty, but it still exists.

Definition

$$\emptyset = \{\}$$

A set A is a subset of a set B if all elements of A are in B.
A set B is a superset of a set A if all elements of A are in B.

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Quick trick

A set with k elements has 2^k different subsets.

Cardinality
Sets

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For example, the cardinality of the set $A = \{u, 1, c\}$ is 3.
We write this as $n(A) = 3$.

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Set membership
Notation

Recall our set of co-presidents:

$$M = \{\text{Vincent}, \text{Samantha}, \text{Caroline}\}$$

To say that an element is in a set, we use the symbol $\in$, meaning "is an element of", "belongs to", or (informally) "in".

$$\therefore \text{Vincent} \in M$$

To say that an element is not in a set, we use the symbol $\notin$, meaning "is not an element of", "does not belong to", or (informally) "not in".

$$\therefore \text{Euler} \notin M$$

For more complex sets, we can define them using set-builder notation.
For example, we can define the set of even numbers as so:

$$E = \{x \mid (\exists k \in \mathbb{Z})[x = 2k]\}$$

$\mid$ reads as "such that"
$\exists$ reads as "there exists"
This reads as: "E is the set of x values such that there exists an integer k such that $x = 2k$" (the second "such that" is implied).
Sometimes, set-builder notation can get complicated. Using words to define a set is also valid, but you must be careful that you are not ambiguous with your wording!

The universe of discourse
Notation

The universe of discourse (commonly shortened to universe) is the set of all values under consideration. It is similar to the domain of a function.
The universe set is commonly denoted as U.

Example (Defining $\mathbb{P}$)

$$U = \{x \mid x \in \mathbb{N}^*, x \neq 1\}$$

$$\mathbb{P} = \{x \mid x \in U \text{ and the only positive divisors of } x \text{ are } 1 \text{ and } x\}$$

If we had instead defined U as $U = \mathbb{Z}$, then our definition of primes would include negative numbers, which would be wrong. This is why it is important to consider the universe of discourse.

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Set union
Operations with Sets

The union of two sets is a set containing all of the elements of both sets.
The set union operator is: $\cup$
Formally:

$$A \cup B = \{x \mid x \in A \lor x \in B\} \qquad (\lor \text{ means "or"})$$

For example:

$$A = \{1, 2, 3\}$$
$$B = \{3, 4, 5\}$$
$$A \cup B = \{1, 2, 3, 4, 5\}$$

Set intersection
Operations with Sets

The intersection of two sets is a set containing only the elements that are in both sets.
The set intersection operator is: $\cap$
Formally:

$$A \cap B = \{x \mid x \in A \wedge x \in B\} \qquad (\wedge \text{ means "and"})$$

For example:

$$A = \{1, 2, 3\}$$

$$B = \{3, 4, 5\}$$

$$A \cap B = \{3\}$$

The complement of the set A is all elements not in A.
If the universe, U, is defined, the absolute complement of A is all elements in U that are not in A.
The relative complement of A with respect to B is written as $B \setminus A$. This is the set of elements in B, but not in A.

Definition

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Definition

$$B \setminus A = \{x \in B \mid x \notin A\}$$

Examples

$$\{1, 2, 3, 4, 5\} \setminus \{4, 5, 6, 7, 8\} = \{1, 2, 3\}$$
$$\mathbb{R} \setminus \mathbb{Q} = \{x \mid x \text{ is irrational}\}$$

Cartesian product
Operations with Sets

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The Cartesian product of a set A with a set B is defined as:

$$A \times B = \{(a, b) \mid a \in A \land b \in B\}$$

For example, if $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$, what is $A \times B$?

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c)\}$$