Boolean Algebra

Vincent Macri

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What is boolean algebra

Basics

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- Useful while considering logic
- Useful in computer science

Main operations
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Boolean algebra has four important[1] operations.

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Main operations
Basics

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- $\neg A$ NOT (negation). Also written as $\overline{A}$.
- $A \wedge B$ AND (conjunction). Also written as $A \cdot B$ or AB.
- $A \oplus B$ XOR (exclusive or).
- $A \vee B$ OR (disjunction). Also written as $A + B$.

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Note on exclusive or's placement

There is no generally agreement on where to put XOR in the order of operations. It is commonly put between AND and OR (BNAXO), but you should always use brackets to avoid ambiguity.

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What is a truth table
Truth Tables

A truth table is table of all possible input and output values of a boolean algebra statement.

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A truth table is table of all possible input and output values of a boolean algebra statement.
They are similar to the multiplication tables you used in elementary school, but are much more powerful.

Truth tables of main operations

Truth Tables

Table: AND

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

Truth tables of main operations
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Table: OR

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0	1	1
1	0	1
1	1	1

Table: NOT

A	$\neg A$
0	1
1	0

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For example, both $\wedge$ and $\vee$ follow the associative law and commutative laws, just like $\times$ and $+$.
They also follow the distributive law.

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Associative law

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$
$$A + (B + C) = (A + B) + C$$

Associativity, commutativity, and distributivity

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Associativity, commutativity, and distributivity

Laws and Identities

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Distributive law

$$A \cdot (B + C) = AB + AC$$

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- $A \cdot A = A$
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- $A + AB = A$
- $A + BC = (A + B) \cdot (A + C)$

Identities with NOT

These are some identities involving NOT:

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$$\overline{\overline{A}} = A$$

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$$\overline{\overline{A}} = A$$

$$\overline{A} + A = 1$$

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$$\overline{\overline{A}} = A$$

$$\overline{A} + A = 1$$

$$\overline{A} \cdot A = 0$$

Another set of identities useful in boolean algebra are De Morgan's laws.

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NOT OR

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

A	B	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
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0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

NOT AND

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

A	B	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

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Simplify

Practice

$$A + AB \cdot B + C \cdot \overline{C}$$

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$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + (AB \cdot B) + (C \cdot \overline{C})$$

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$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned} & A + (AB \cdot B) + (C \cdot \overline{C}) \\ = {} & A + (AB) + (0) \end{aligned}$$

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$$A + AB \cdot B + C \cdot \overline{C}$$

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Simplify

Practice

$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned} & A + (AB \cdot B) + (C \cdot \overline{C}) \\ = {} & A + (AB) + (0) \\ = {} & A + AB \\ = {} & A \end{aligned}$$

$$\therefore A + AB \cdot B + C \cdot \overline{C} = A$$