Summations

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Example

Add up all of the integers from 1 to 1000. Show your work.

This is a fairly easy question, but it takes a long time to do (assuming you aren't using any clever tricks).

Typing $1 + 2 + 3 + 4 + 5 + \cdots + 996 + 997 + 998 + 999 + 1000$ into your calculator will take a long time.

Surely there is a better way to do this.

Basic Sums

Summation notation

We can use the Greek letter sigma to show a summation:

$$\sum_{n=1}^{1000} n$$

This means add up all values of n, starting from $n = 1$ and going up to $n = 1000$.

Most newer scientific calculators can do summations. If we type the above into our calculator, we will find that:

$$\sum_{n=1}^{1000} n = 500500$$

$\therefore$ the answer to our question is 500500.

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Advanced Sums

Another example

Example

Add up the first 10 positive even numbers.

The nth positive even number can be written as $2 \times n$.

We can use this and our calculator to compute our answer:

$$\sum_{n=1}^{n=10} 2n = 110$$

Since we're only going up to 10, we can check our answer manually:

$$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 = 110$$

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Sum 1 to 100

The legend of Gauss

Example

Find the sum of the integers from 1 to 100.

Legend has it that the teacher of the famous mathematician Gauss told the class to add up all the numbers from 1 to 100, hoping it would distract them and the teacher would get some time to rest.

Unfortunately for the teacher, Gauss was very smart and did it in a few minutes.

Let's look at how.

Sum 1 to 100
Recognizing patterns

If write out the summation twice, but once in reverse, we see an interesting pattern.

$$\begin{array}{rrrrrrrrrrrrrr} & 1 & + & 2 & + & 3 & +\dots & + & 98 & + & 99 & + & 100 \\ + & 100 & + & 99 & + & 98 & +\dots & + & 3 & + & 2 & + & 1 \\ \hline & 101 & + & 101 & + & 101 & +\dots & + & 101 & + & 101 & + & 101 \end{array}$$

This is 101×100, which we know is 10100.

But we added up the numbers twice, so we need to divide by 2.

$$10100 \div 2 = 5050 \qquad \therefore \sum_{n=1}^{100} n = 5050$$

Sum 1 to 100

Making a formula

Example

Create a formula to find the sum of all the integers from 1 to n for any positive n, without using sigma notation.

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Sum 1 to 100

Making a formula

Example

Create a formula to find the sum of all the integers from 1 to n for any positive n, without using sigma notation.

We can do something similar as we did in the last example.

$$\begin{array}{crcrcrcrcr}
 & 1 & + & 2 & +\dots & + & n-1 & + & n \\
+ & n & + & n-1 & +\dots & + & 2 & + & 1 \\
\hline
 & n+1 & + & n+1 & +\dots & + & n+1 & + & n+1
\end{array}$$

Sum 1 to 100

Making a formula

Example

Create a formula to find the sum of all the integers from 1 to n for any positive n, without using sigma notation.

We can do something similar as we did in the last example.

$$\begin{array}{rrrrrrrrrr} & 1 & + & 2 & +\dots & + & n-1 & + & n \\ + & n & + & n-1 & +\dots & + & 2 & + & 1 \\ \hline & n+1 & + & n+1 & +\dots & + & n+1 & + & n+1 \end{array}$$

$$\sum_{x=1}^{n} x = \frac{n(n+1)}{2}$$