Boolean Algebra

Vincent Macri

Table of Contents

Basics
What is boolean algebra

- A branch of mathematics dealing only with true and false values (usually called 1 and 0, respectively)

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- Useful while considering logic
- Useful in computer science

Boolean algebra has four important[1] operations.

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Boolean algebra has four important[1] operations.

- $\neg A$ NOT (negation). Also written as $\overline{A}$.
- $A \wedge B$ AND (conjunction). Also written as $A \cdot B$ or AB.
- $A \oplus B$ XOR (exclusive or).
- $A \vee B$ OR (disjunction). Also written as $A + B$.

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Note on exclusive or's placement

There is no generally agreement on where to put XOR in the order of operations. It is commonly put between AND and OR (BNAXO), but you should always use brackets to avoid ambiguity.

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Table of Contents

A truth table is table of all possible input and output values of a boolean algebra statement.

Truth Tables

What is a truth table

A truth table is table of all possible input and output values of a boolean algebra statement.

They are similar to the multiplication tables you used in elementary school, but are much more powerful.

Truth Tables

Truth tables of main operations

Truth Tables
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Table: AND

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

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Truth tables of main operations

Table: AND

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Table: OR

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0	0	0
0	1	1
1	0	1
1	1	1

Table: NOT

A	$\overline{A}$
1	1
1	0

Table of Contents

Laws and Identities

Associativity, commutativity, and distributivity

Boolean algebra has many similar laws as regular algebra.

For example, both $\wedge$ and $\vee$ follow the associative law and commutative laws, just like $\times$ and $+$.

They also follow the distributive law.

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Associative law

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$
$$A + (B + C) = (A + B) + C$$

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Distributive law

$$A \cdot (B + C) = AB + AC$$

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- $A \cdot A = A$
- $A \cdot (A + B) = A$

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- $A + AB = A$

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- $A + A = A$
- $A \cdot A = A$
- $A \cdot (A + B) = A$
- $A + AB = A$
- $A + BC = (A + B) \cdot (A + C)$

These are some identities involving NOT:

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$$\overline{\overline{A}} = A$$

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$$\overline{\overline{A}} = A$$

$$\overline{A} + A = 1$$

These are some identities involving NOT:

$$\overline{\overline{A}} = A$$

$$\overline{A} + A = 1$$

$$\overline{A} \cdot A = 0$$

Another set of identities useful in boolean algebra are De Morgan's laws.

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NOT OR

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

A	B	$\overline{A + B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Laws and Identities
De Morgan's laws

Another set of identities useful in boolean algebra are De Morgan's laws.

NOT OR

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

A	B	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

NOT AND

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

A	B	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Table of Contents

Practice

Simplify

$$A + AB \cdot B + C \cdot \overline{C}$$

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$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + (AB \cdot B) + (C \cdot \overline{C})$$

Practice
Simplify

$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned} & A + (AB \cdot B) + (C \cdot \overline{C}) \\ = & A + (AB) + (0) \end{aligned}$$

Practice
Simplify

$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned}
& A + (AB \cdot B) + (C \cdot \overline{C}) \\
= {} & A + (AB) + (0) \\
= {} & A + AB
\end{aligned}$$

Practice
Simplify

$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned} & A + (AB \cdot B) + (C \cdot \overline{C}) \\ = \; & A + (AB) + (0) \\ = \; & A + AB \\ = \; & A \end{aligned}$$

Practice
Simplify

$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned} & A + (AB \cdot B) + (C \cdot \overline{C}) \\ = {} & A + (AB) + (0) \\ = {} & A + AB \\ = {} & A \end{aligned}$$

$\therefore A + AB \cdot B + C \cdot \overline{C} = A$