Lambda Calculus

Now you can bring a computer to your tests!

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William Lyon Mackenzie C.I. Math Club

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What is lambda calculus?

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- Created by Alonzo Church

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- A way of representing pure mathematical functions
- Can represent any computer program
- Equivalent to Turing machines

$x+1$

Introduction

In math class, we would define a function that accepts an argument x and outputs $x+1$ as so:

$$f(x)$$

$x + 1$

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You can think of λ as f, and . as $=$.

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So, to add two numbers, we have a function output another function, like this:

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And we use it like this:

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Why one argument?

One Parameter

- Very simple

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- Very powerful

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- Very simple
- Very powerful
- Functions can only have one variable

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One Parameter

Me too!

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Also, we assume that we evaluate a function with "multiple" arguments starting with the leftmost parameter.

This is stupid. It just makes everything harder.

One Parameter

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For those examples, you're right!

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Let's get to the fun stuff now!