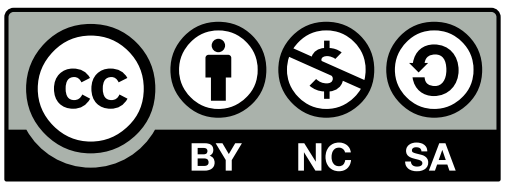
Euclid Preparation 3

Circle Geometry

Vincent Macri

William Lyon Mackenzie C.I. Math Club



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The central angle subtended by any arc is twice any of the inscribed angles on that arc.
This means that in the diagram, $\angle AOB = 2\angle ACB$.

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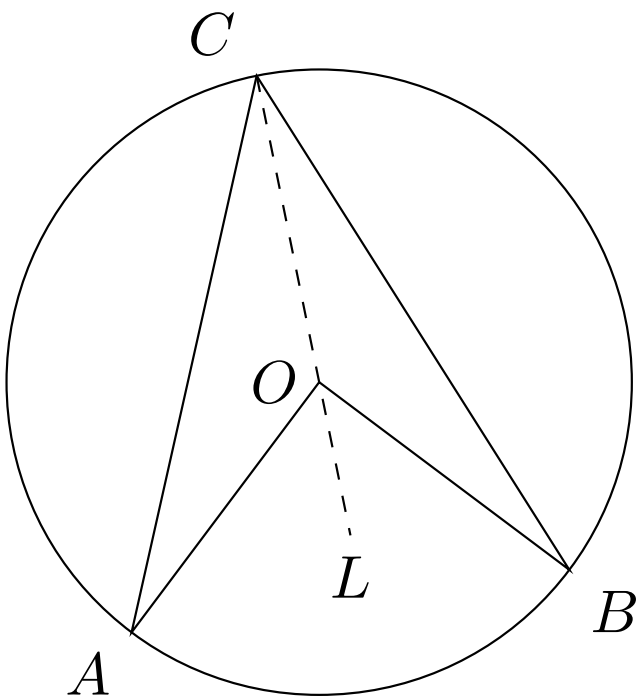

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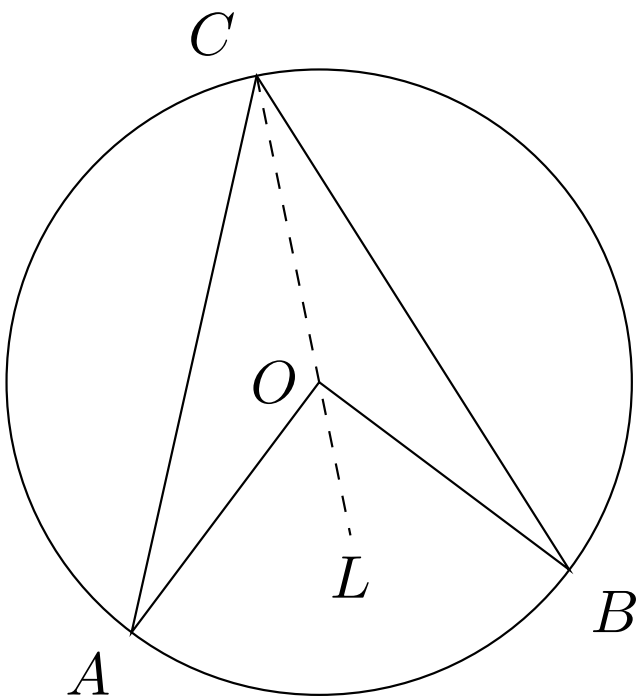

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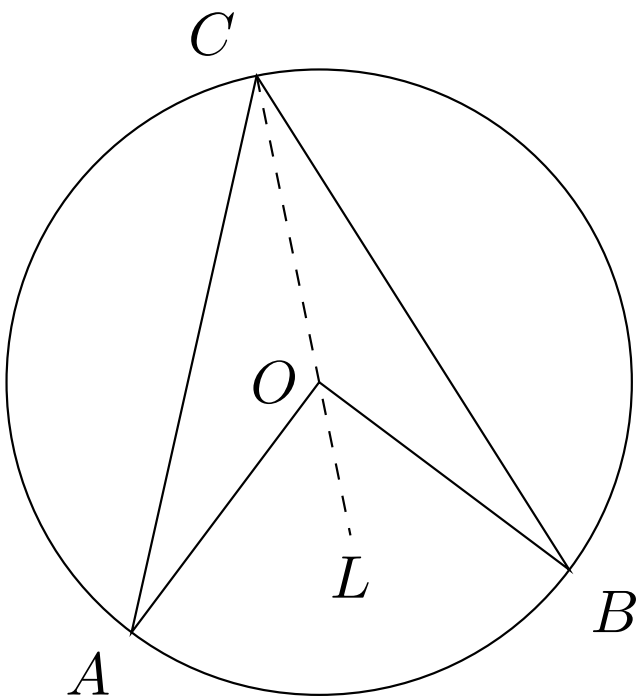

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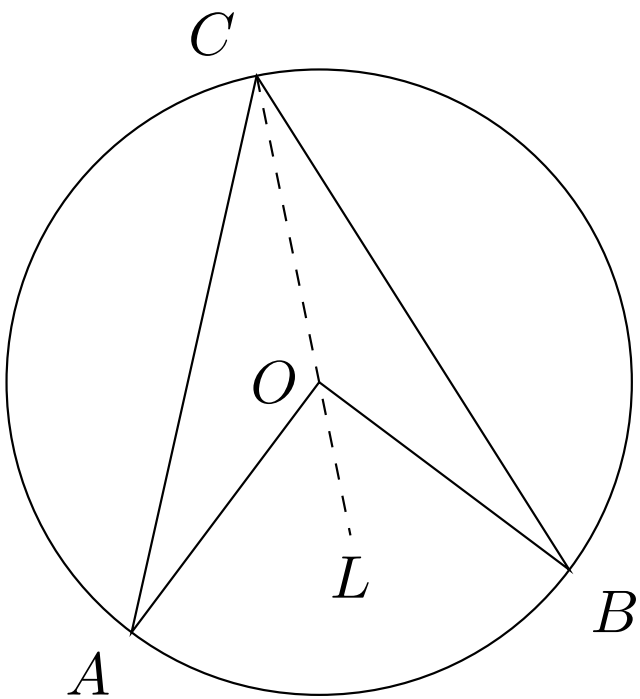

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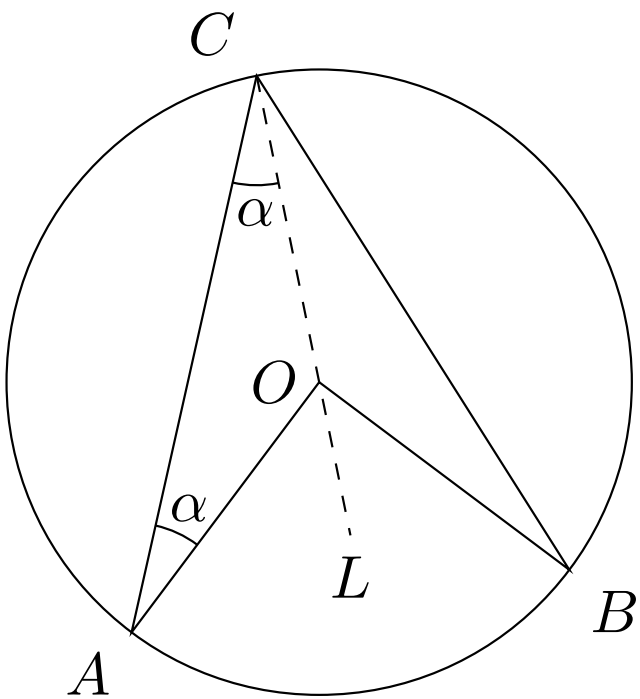

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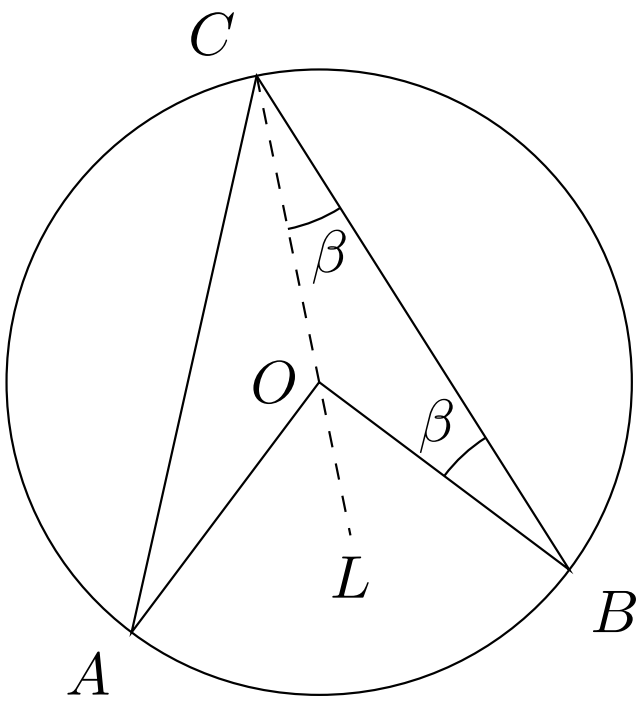

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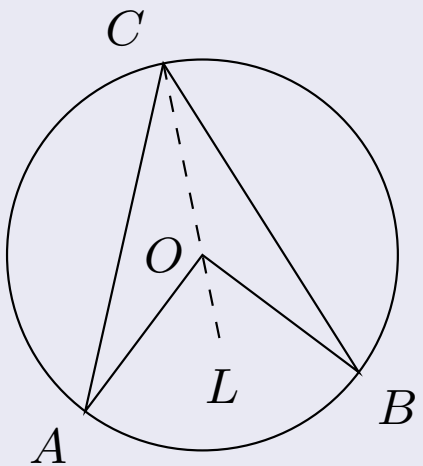
Also, note that $\triangle OAC$ and $\triangle OBC$ are isosceles. This is because OA, OB, and OC are all radii. So, $\angle OAC = \angle OCA$ and $\angle OCB = \angle OBC$.

Proof of the Star Trek Theorem

Star Trek Theorem

Prove that $\angle AOB = 2\angle ACB$.

We know that $\angle OAC = \angle OCA$.



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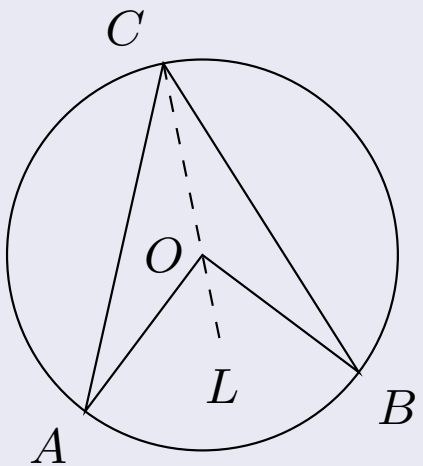
Prove that $\angle AOB = 2\angle ACB$.

We know that $\angle OAC = \angle OCA$. So: $2\angle OCA + \angle AOC = 180°$.



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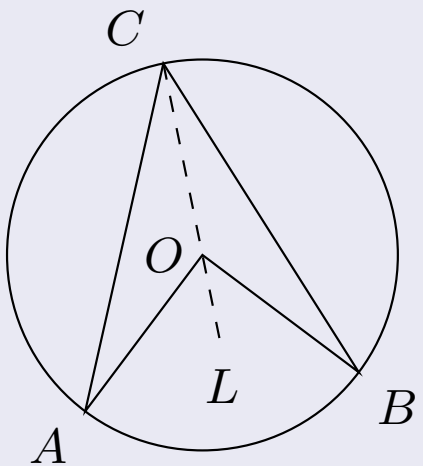
And we know that $\angle AOC + \angle AOL = 180°$.



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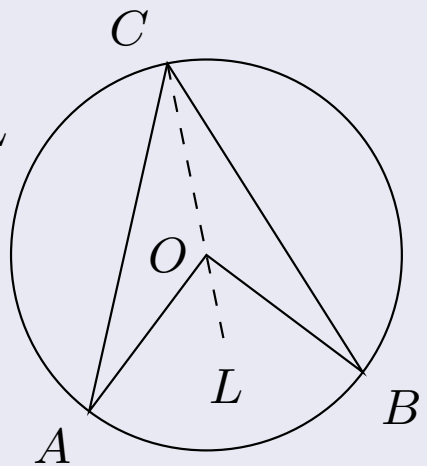
$$2\angle OCA + \angle AOC = \angle AOC + \angle AOL$$

$$\angle OCA = \frac{1}{2}\angle AOL$$



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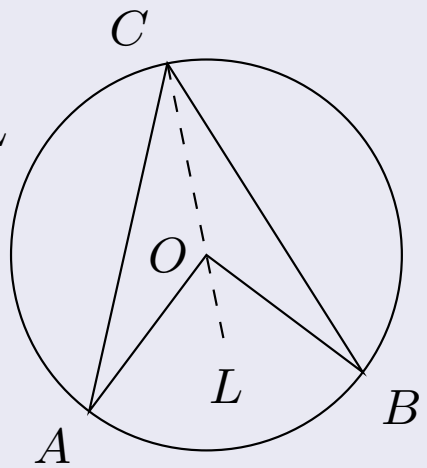
And similarly for $\triangle OBC$: $\angle OCB = \frac{1}{2}\angle BOL$.



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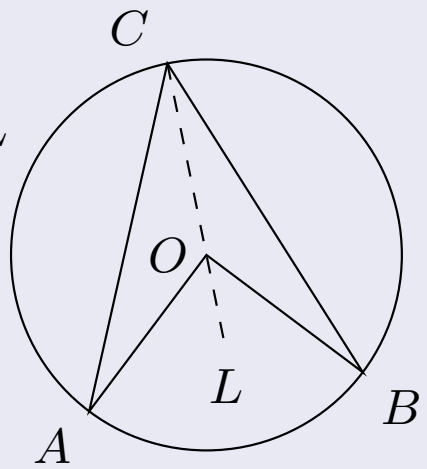

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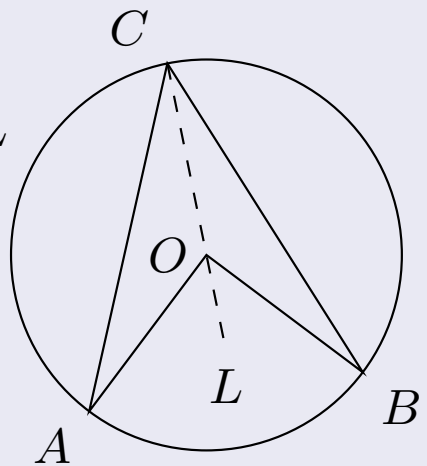

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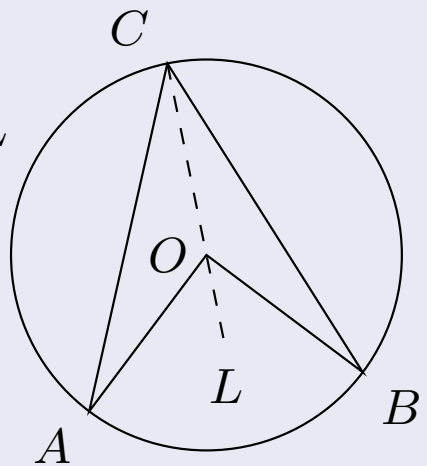

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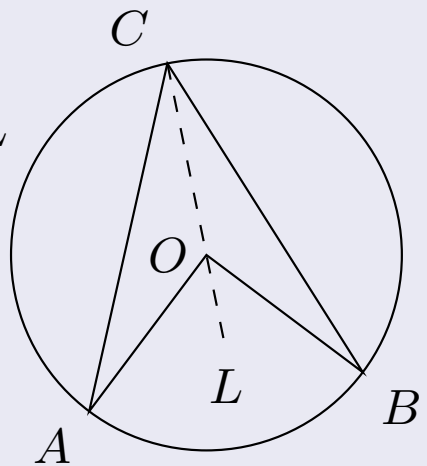

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$$2\angle ABC = \angle AOB$$

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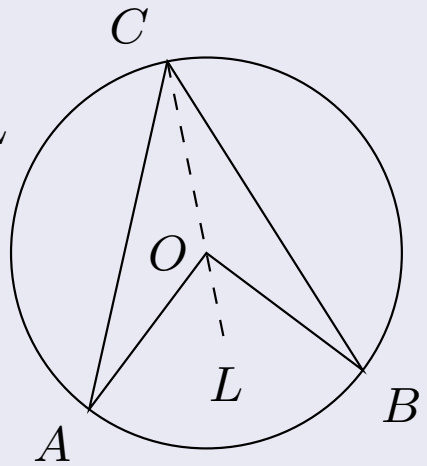

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□