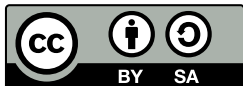


# Boolean Algebra

Vincent Macri



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# Basics

## What is boolean algebra

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- Useful while considering logic
- Useful in computer science

# Basics

## Main operations

Boolean algebra has four important<sup>1</sup> operations.

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# Basics

## Main operations

Boolean algebra has four important<sup>1</sup> operations.

$\neg A$  NOT (negation). Also written as  $\overline{A}$ .

$A \wedge B$  AND (conjunction). Also written as  $A \cdot B$  or  $AB$ .

$A \oplus B$  XOR (exclusive or).

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### Note on exclusive or's placement

There is no generally agreement on where to put XOR in the order of operations. It is commonly put between AND and OR (BNAXO), but you should always use brackets to avoid ambiguity.

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# Truth Tables

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A truth table is table of all possible input and output values of a boolean algebra statement.

They are similar to the multiplication tables you used in elementary school, but are much more powerful.

# Truth Tables

Truth tables of main operations

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Table: AND

$A$	$B$	$A \wedge B$
0	0	0
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## Truth tables of main operations

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$A$	$B$	$A \vee B$
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0	1	1
1	0	1
1	1	1

Table: NOT

$A$	$\overline{A}$
1	1
1	0

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# Laws and Identities

Associativity, commutativity, and distributivity

Boolean algebra has many similar laws as regular algebra.

For example, both  $\wedge$  and  $\vee$  follow the associative law and commutative laws, just like  $\times$  and  $+$ .

They also follow the distributive law.

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$$A + (B + C) = (A + B) + C$$

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## Distributive law

$$A \cdot (B + C) = AB + AC$$

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- $A \cdot 0 = 0$

However, some of the identities which are true in boolean algebra do not work in regular algebra.

- $A + 1 = A$
- $A \cdot (A + B) = A$

- $A + A = A$

- $A \cdot A = A$

# Laws and Identities

## Identities

Some of the identities in boolean algebra are the same as in regular algebra.

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- $A \cdot 1 = A$

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However, some of the identities which are true in boolean algebra do not work in regular algebra.

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## Identities

Some of the identities in boolean algebra are the same as in regular algebra.

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- $A \cdot 1 = A$

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However, some of the identities which are true in boolean algebra do not work in regular algebra.

- $A + 1 = A$
- $A \cdot (A + B) = A$

- $A + A = A$
- $A + AB = A$

- $A \cdot A = A$
- $A + BC = (A + B) \cdot (A + C)$

# Laws and Identities

## Identities with NOT

These are some identities involving NOT:

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$$\overline{\overline{A}} = A$$

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$$\overline{\overline{A}} = A$$

$$\overline{A} + A = 1$$

# Laws and Identities

## Identities with NOT

These are some identities involving NOT:

$$\overline{\overline{A}} = A$$

$$\overline{A} + A = 1$$

$$\overline{A} \cdot A = 0$$

# Laws and Identities

## De Morgan's laws

Another set of identities useful in boolean algebra are De Morgan's laws.

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## De Morgan's laws

Another set of identities useful in boolean algebra are De Morgan's laws.

NOT OR

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

$A$	$B$	$\overline{A + B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

# Laws and Identities

## De Morgan's laws

Another set of identities useful in boolean algebra are De Morgan's laws.

### NOT OR

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

A	B	$\overline{A + B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

### NOT AND

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

A	B	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



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# Practice

Simplify

$$A + AB \cdot B + C \cdot \overline{C}$$

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$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + (AB \cdot B) + (C \cdot \overline{C})$$

# Practice

Simplify

$$A + AB \cdot B + C \cdot \overline{C}$$

$$\begin{aligned} & A + (AB \cdot B) + (C \cdot \overline{C}) \\ &= A + (AB) + (0) \end{aligned}$$

# Practice

Simplify

$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + (AB \cdot B) + (C \cdot \overline{C})$$

$$= A + (AB) + (0)$$

$$= A + AB$$

# Practice

Simplify

$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + (AB \cdot B) + (C \cdot \overline{C})$$

$$= A + (AB) + (0)$$

$$= A + AB$$

$$= A$$

# Practice

Simplify

$$A + AB \cdot B + C \cdot \overline{C}$$

$$A + (AB \cdot B) + (C \cdot \overline{C})$$

$$= A + (AB) + (0)$$

$$= A + AB$$

$$= A$$

$$\therefore A + AB \cdot B + C \cdot \overline{C} = A$$