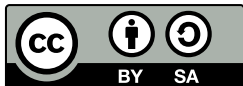


Introduction to Sets

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Types of Numbers

Basic types

\mathbb{N} Natural numbers $(0, 1, 2, 3, \dots)$

\mathbb{Z} Integers $(\mathbb{N}$ and $-1, -2, -3, \dots)$

\mathbb{Q} Rational numbers $(\mathbb{Z}$ and $4.2, -\frac{2}{3}, \dots)$

\mathbb{R} Real numbers $(\mathbb{Q}$ and $\pi, e, \sqrt{2}, \dots)$

\mathbb{C} Complex numbers $(\mathbb{R}$ and $i, 2i + 1, \dots)$

\mathbb{P} Prime numbers $(2, 3, 5, 7, \dots)$

Types of Numbers

Indexes

Some mathematicians include 0 in \mathbb{N} , and some do not.

While it is generally accepted that \mathbb{N} includes 0, we have notation to specify:

\mathbb{N}^0 Natural numbers including 0

\mathbb{N}^* Natural numbers not including 0

\mathbb{N}^+ Positive natural numbers (does not include 0)

We can also use this notation with other types of numbers:

\mathbb{Z}^- Negative integers (does not include 0)

\mathbb{R}^+ Positive real numbers (does not include 0)

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Intervals

Open and closed

Open intervals

To denote numbers in an open (inclusive) range, we write: $[a, b]$

This means all the real numbers from a to b , including a and b .

Closed intervals

To denote numbers in a closed (exclusive) range, we write: (a, b)

This means all the real numbers from a to b , **not** including a and b .

Intervals

Examples

$[0, 10)$ means all real numbers from 0 to 10, including 0, but not including 10.

$(-\infty, +\infty)$ means all real numbers.

$[0, \infty)$ means all real numbers that are not negative.

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A set is an unordered collection of distinct elements.

A set with a finite number of elements can be written in braces such as $\{a, b, c, \dots\}$

For example, we can define the set of math club co-presidents as:

$$M = \{\text{Vincent, Samantha, Caroline}\}$$

By convention, the names of sets are denoted in capital letters.

Since the elements of a set are distinct:

$$\{a, b, c\} \equiv \{a, a, b, c, b\}$$

(\equiv means equivalent)

Sets

The empty set

The empty set is the set that contains no elements. It is denoted as \emptyset .

Examples

\emptyset is the set of all 4-sided triangles.

\emptyset is the set of all prime numbers divisible by 10.

The empty set can be thought of as an empty bag. It may be empty, but it still exists.

Definition

$$\emptyset = \{\}$$

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Notation

Set membership

Recall our set of co-presidents:

$$M = \{\text{Vincent}, \text{Samantha}, \text{Caroline}\}$$

To say that an element is in a set, we use the symbol \in , meaning “is an element of”, “belongs to”, or (informally) “in”.

$$\therefore \text{Vincent} \in M$$

To say that an element is not in a set, we use the symbol \notin , meaning “is not an element of”, “does not belong to”, or (informally) “not in”.

$$\therefore \text{Euler} \notin M$$

Notation

Set-builder notation

For more complex sets, we can define them using set-builder notation.

For example, we can define the set of even numbers as so:

$$E = \{x \mid (\exists k \in \mathbb{Z})[x = 2k]\}$$

\mid reads as “such that”

\exists reads as “there exists”

This reads as: “ E is the set of x values such that there exists an integer k such that $x = 2k$ ” (the second “such that” is implied).

Sometimes, set-builder notation can get complicated. Using words to define a set is also valid, but you must be careful that you are **not** ambiguous with your wording!

Notation

The universe of discourse

The universe of discourse (commonly shortened to universe) is the set of all values under consideration. It is similar to the domain of a function.

The universe set is commonly denoted as U .

Example (Defining \mathbb{P})

$$U = \{x \mid x \in \mathbb{N}^*, x \neq 1\}$$

$$\mathbb{P} = \{x \mid x \in U \text{ and the only positive divisors of } x \text{ are } 1 \text{ and } x\}$$

If we had instead defined U as $U = \mathbb{Z}$, then our definition of primes would include negative numbers, which would be wrong. This is why it is important to consider the universe of discourse.

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Operations with Sets

Set union

The union of two sets is a set containing all of the elements of both sets.

The set union operator is: \cup

Formally:

$$A \cup B = \{x \mid x \in A \vee x \in B\} \quad (\vee \text{ means "or"})$$

For example:

$$A = \{1, 2, 3\}$$

$$B = \{3, 4, 5\}$$

$$A \cup B = \{1, 2, 3, 4, 5\}$$

Operations with Sets

Set intersection

The intersection of two sets is a set containing only the elements that are in both sets.

The set intersection operator is: \cap

Formally:

$$A \cap B = \{x \mid x \in A \wedge x \in B\} \quad (\wedge \text{ means "and"})$$

For example:

$$A = \{1, 2, 3\}$$

$$B = \{3, 4, 5\}$$

$$A \cap B = \{3\}$$

Operations with Sets

Set complement

The **complement** of the set A is all elements not in A .

If the universe, U , is defined, the **absolute complement** of A is all elements in U that are not in A .

The **relative complement** of A with respect to B is written as $B \setminus A$. This is the set of elements in B , but not in A .

Definition

$$B \setminus A = \{x \in B \mid x \notin A\}$$

Examples

$$\begin{aligned}\{1, 2, 3, 4, 5\} \setminus \{4, 5, 6, 7, 8\} &= \{1, 2, 3\} \\ \mathbb{R} \setminus \mathbb{Q} &= \{x \mid x \text{ is irrational}\}\end{aligned}$$