Euclid Preparation 3

Circle Geometry

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Theorem ("Star Trek" Theorem)

The central angle subtended by any arc is twice any of the inscribed angles on that arc.
This means that in the diagram, $\angle AOB = 2\angle ACB$.

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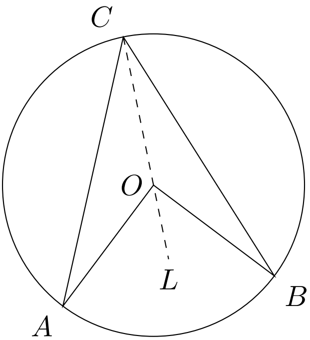

Here, $\angle AOB$ is subtended by the minor arc from A to B.

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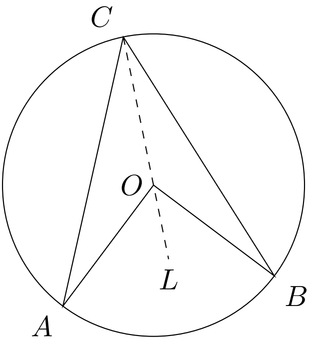

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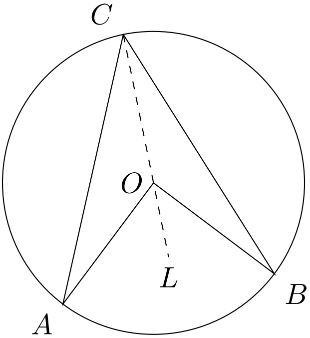

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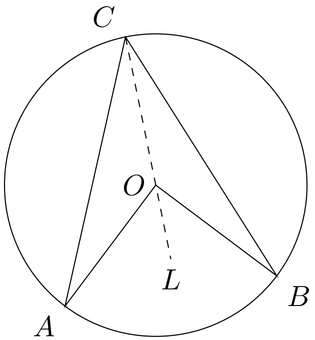

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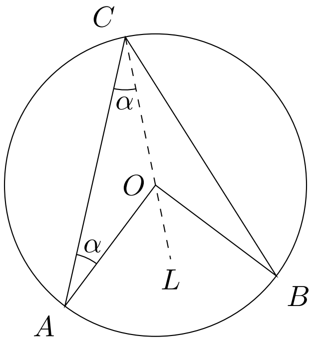

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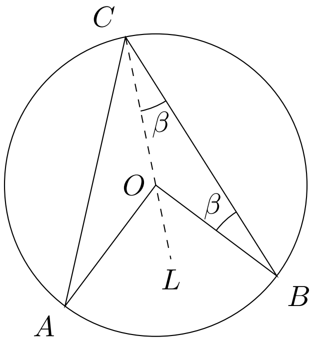

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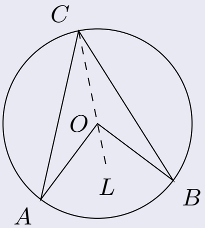
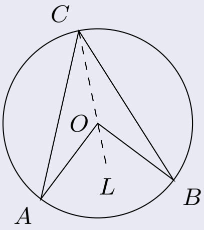
Proof that $\angle AOB = 2\angle ACB$.

We know that $\angle OAC = \angle OCA$.



Proof that $\angle AOB = 2\angle ACB$.

We know that $\angle OAC = \angle OCA$. So: $2\angle OCA + \angle AOC = 180°$.



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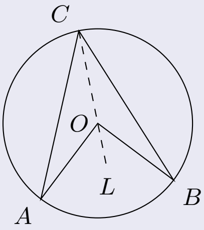
We know that $\angle OAC = \angle OCA$. So: $2\angle OCA + \angle AOC = 180°$.

And we know that $\angle AOC + \angle AOL = 180°$.



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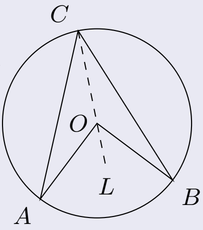

$$2\angle OCA + \angle AOC = \angle AOC + \angle AOL$$

$$\angle OCA = \frac{1}{2}\angle AOL$$

Proof of the Star Trek Theorem

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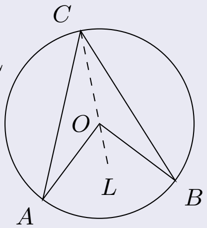

$$2\angle OCA + \angle AOC = \angle AOC + \angle AOL$$
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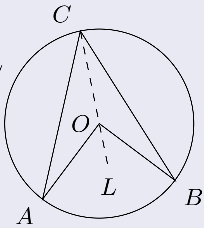

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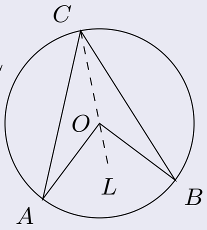

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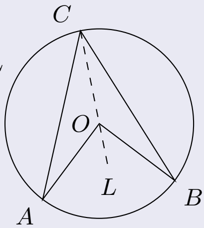

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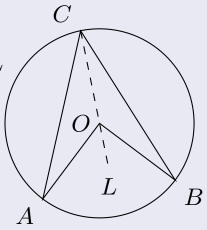

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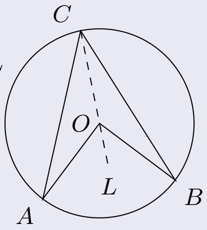

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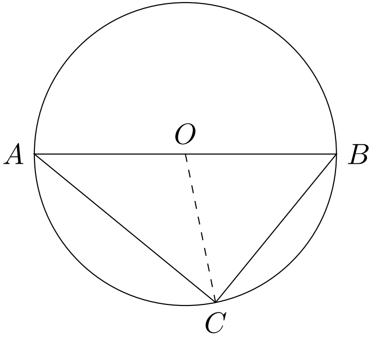
□

Extending

Example

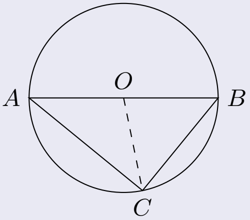
Show that if the chord AB is a diameter then $\angle ACB = 90^\circ$.
In other words, show that the angle subtended by a diameter is a right angle.



Proof that $\angle ACB = 90^\circ$.

We know that $\angle ACO = \angle CAO$. So:

$$2\angle ACO + \angle AOC = 180^\circ \qquad (1)$$



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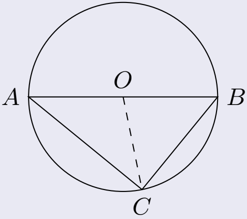
Similarly:

$$2\angle BCO + \angle BOC = 180^\circ \qquad (2)$$



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Similarly:

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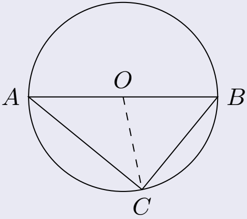
We also know that $\angle AOC = 180^\circ - \angle BOC$.



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Similarly:

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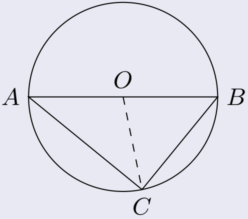
We also know that $\angle AOC = 180^\circ - \angle BOC$.
We substitute this into (1) to get $2\angle ACO = \angle BOC$.



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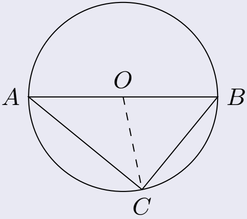

We also know that $\angle AOC = 180^\circ - \angle BOC$.
We substitute this into (1) to get $2\angle ACO = \angle BOC$.
We substitute this into (2) to get:

$$2\angle BCO + 2\angle ACO = 180^\circ$$
$$\angle BCO + \angle ACO = 90^\circ$$

Proof that $\angle ACB = 90^\circ$.

We know that $\angle ACO = \angle CAO$. So:

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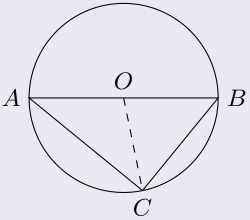

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Since $\angle BCO + \angle ACO = \angle ACB$, we arrive at:

Proof that $\angle ACB = 90^\circ$.

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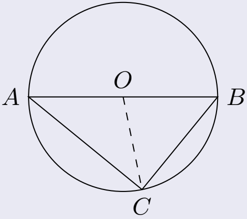

We also know that $\angle AOC = 180^\circ - \angle BOC$.
We substitute this into (1) to get $2\angle ACO = \angle BOC$.
We substitute this into (2) to get:

$$2\angle BCO + 2\angle ACO = 180^\circ$$
$$\angle BCO + \angle ACO = 90^\circ$$

Since $\angle BCO + \angle ACO = \angle ACB$, we arrive at:

$$\angle ACB = 90^\circ$$

□