

Definitions

Booleans

 $\text{TRUE} = \lambda xy.x$ $\text{FALSE} = \lambda xy.y$ $\text{NOT} = \lambda b.b(\text{FALSE } \text{TRUE})$ $\text{AND} = (\lambda pq.p)(q \text{ } p)$ $\text{IFTHENELSE} = (\lambda btf.b)(t \text{ } f)$

Church Numerals

 $0 = \lambda fx.x$ $1 = \lambda fx.f(x)$ $2 = \lambda fx.f(f(x))$ $n = \lambda fx.f^n(x)$ $\text{SUCC} = \lambda nfx.f(nfx)$

1 One Argument

1.1 Write the function $f(a, b) = a^2 + b^2$ as a lambda calculus expression

2 Booleans

2.1 Write a lambda calculus expression for NAND

2.2 Write a lambda calculus expression for XOR

3 Church Numerals

3.1 What is the numerical value of the Church numeral whose lambda expression is $\lambda f x. f(f(f(f(f(x))))))$?

3.2 What is the lambda expression of the Church numeral whose numerical value is 7?

3.3 Compute $\text{SUCC } \lambda f x. f(f(f(x)))$ and write its numerical value.