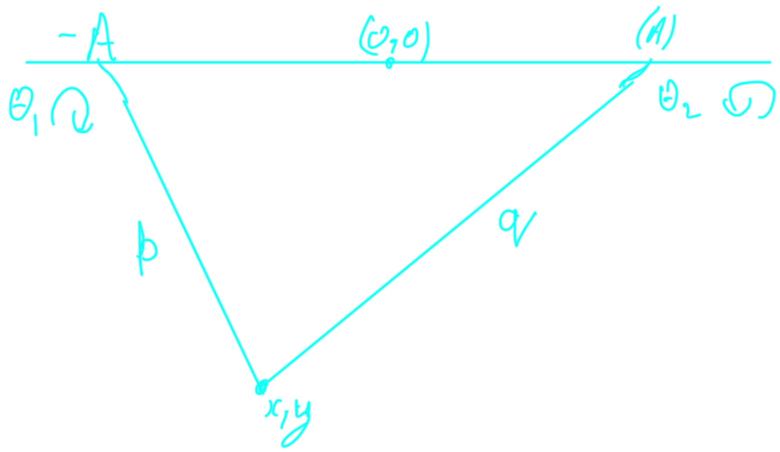
Reverse Kinematics (coordinates to angle)



p: $$(x-(-A))^2 + (y-0)^2 = p^2$$ (Distance formula)

$$p^2 = x^2 + y^2 + A^2 + 2Ax$$

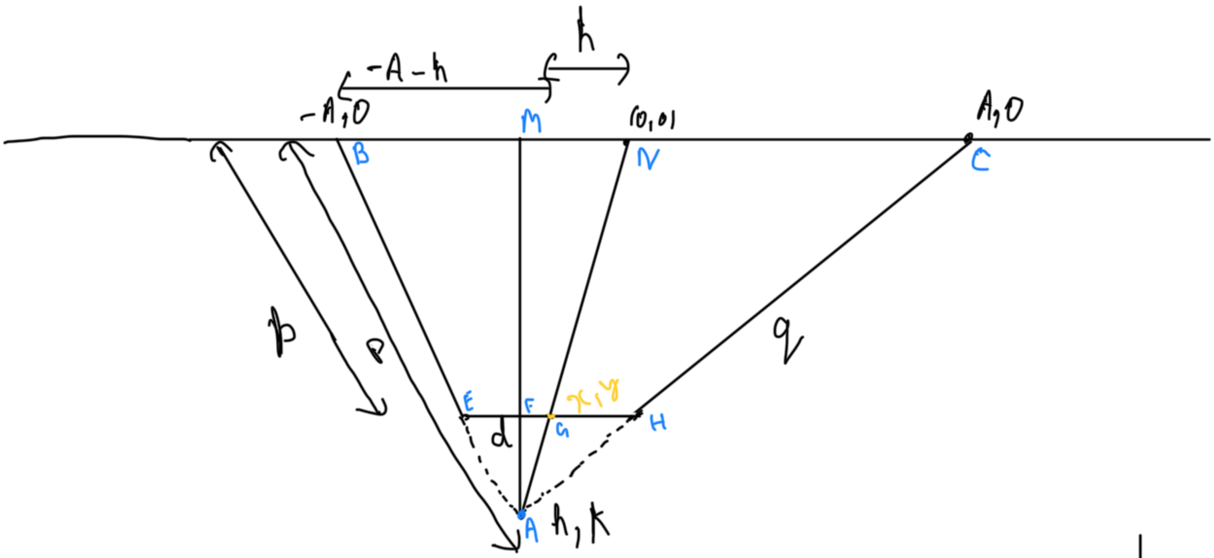
$$p^2 = (x+A)^2 + y^2$$

q: $$(x-A)^2 + (y-0)^2 = q^2$$

$$x^2 + y^2 + A^2 - q^2 - 2Ax = 0$$

$$q^2 = (x-A)^2 + y^2$$

$$p^2 = (x+A)^2 + y^2 \qquad q^2 = (x-A)^2 + y^2$$



ΔABC & ΔAEH

$$\Rightarrow \frac{d}{2A} = \frac{P-p}{P} = \frac{Q-q}{Q}$$

$$= 1 - \frac{p}{P} = 1 - \frac{q}{Q} \qquad \left(\frac{p}{P} = \frac{q}{Q}\right)$$

$$P = \frac{p}{1-\frac{d}{2A}} \; ; \; Q = \frac{q}{1-\frac{d}{2A}}$$

$$p = P\left(1-\frac{d}{2A}\right) \; ; \; q = Q\left(1-\frac{d}{2A}\right)$$

ΔAEF & ΔABM

$$\frac{P-p}{P} = \frac{K-y}{K}$$

$$\cancel{1} - \frac{p}{P} = \cancel{1} - \frac{y}{K}$$

$$K = \frac{yP}{p}$$

$$= \frac{y \cancel{P}}{\cancel{P}\left(1-\frac{d}{2A}\right)}$$

ΔAFG & ΔAMN

$$\frac{K-y}{K} = \frac{h-x}{h}$$

$$\frac{y}{K} = \frac{x}{h}$$

$$h = x\frac{K}{y}$$

$$= \frac{xP}{p}$$

$$\frac{x}{h} = \frac{y}{K} = \frac{p}{P} = \frac{q}{Q} = 1-\frac{d}{2A}$$

$$h = \frac{x}{1-\frac{d}{2A}} \; ; \; K = \frac{y}{1-\frac{d}{2A}}$$

$$P = \sqrt{(h+A)^2+K^2} \; ; \; Q = \sqrt{(h-A)^2+K^2}$$

$$p = P\left(1-\frac{d}{2A}\right) \; ; \; q = Q\left(1-\frac{d}{2A}\right)$$

$$\underline{p - R_p} = \theta_1 \; ; \; \underline{q - R_q} = \theta_2$$

$\because$ x/y : Coordinates of pen.

h/K : Coordinates of projected triangle vertice

P/Q : Length of projected arms

p/q : Actual length of gear belt

θ_1/θ_2 : Angle required to transform

γ γ

intial K_p to ϕ

K_p / K_q : initial length of gear belt or arms.

d : distance between two ties of gear belt on gondolla.

$2A$: Distance between two steppers.