Problem Chosen	2022	Team Control Number
C	MCM/ICM Summary Sheet	2213860

Don't be blind: The more scientific strategy, the more rational investment

The prices of gold and bitcoin fluctuate greatly. If we can grasp their change trend and make reasonable decisions, we may make a substantial profit from the original funds. To make use of fixed assets to obtain the maximum return in a limited time, we use the daily price fluctuation of bitcoin and the fluctuation data of gold price on the market opening day. Firstly, we use the second-order moving average method, second exponential sliding prediction method and gray prediction model to predict the price, and then use the deep learning model to improve the model. Finally, the multivariable optimization model is used to make the investment decision-making scheme.

For problem 1, the prediction model is established first, and then the decision-making investment model is established. In the first part, to get more realistic prediction results, we first integrate the second-order moving average method, second exponential sliding prediction method and gray prediction model. Then, we use deep learning to continuously modify the characteristics of the data for prediction, to improve the model. In the second part, we use a multivariable optimization model to make investment decisions through programming. The comparison shows that under the optimal investment mode, the initial $1000 is worth $4723.0234 on September 10, 2021.

For problem 2, we made a specific analysis of the investment method and made a reasonable decision-making scheme by comparing the transaction cost with the additional income (loss). The existing assets under the two prediction models are calculated respectively, and the scheme with larger amount is selected as the best strategy.

For problem 3, by adjusting the Commission α the original 10 times and 0.1 times are substituted into the two prediction models and investment decisions to obtain different income time graphs. After comparison, we conclude that there is a lag in the model's perception of data changes under the first prediction model. Therefore, the increase of transaction cost avoids the blind transaction caused by small price fluctuation, and the income is higher. In the second prediction model, the increase of transaction cost increases the cost input and reduces the final income. In short, with the rise of transaction costs, the probability of not investing according to the strategy increases, but the specific amount of income is related to the characteristics of the prediction model.

For problem 4, we briefly express the trading strategy obtained from the above model, and then summarize and summarize it in the form of a memorandum.

In conclusion, our model can better predict the changing trend of gold and bitcoin and make the existing assets close to 5 times of the original assets through reasonable investment strategies. Therefore, our model has certain reference significance for the choice of investment mode in real life.

Keywords: Second-order moving average method, the Second exponential sliding prediction method, Gray prediction model, Neural networks, Multivariable optimization model

Memo

From:Team # 2213860
To:Market trader
Date:February 20, 2022
Subject:What trading strategy can make the most profit

Dear market trader, we are honored to inform you of our achievement after data analysis and modelling.

First, we use the second-order moving average method, the second exponential sliding prediction method and the gray prediction model, and predict the first price of the first T-day Gold (Bitcoin) price. Since these models are linear, we can integrate the average method.

Secondly, we attach a nonlinear factor, using the neural network, from historical gold prices and Bitcoin prices to predict prices. Our model makes more actual price by repeated correction and low noise The transaction method is determined by comparing the size relationship between the completion fee and the loss.

For example, the market trading day is in the gold trading strategy:

- When we predict that gold(bitcoin) prices rose, additional income will be obtained. If the transaction cost is larger, it should remain in the gold and do not continue to buy. If you purchase additional income, you should continue to buy gold.
- When we predict that gold(bitcoin) prices are reduced, comparative transaction costs will result in loss . If the transaction cost is larger, it should be saved in gold, do not choose to sell. If you don't sell gold, it will lead to greater losses, you should sell it.
- When we predict the price of gold(bitcoin) that usually holds gold, it should hold gold. Because although it does not bring benefits or losses due to transactions, you need to pay additional commissions for purchases or sales.
- In addition, we should also consider the effect of both price fluctuations on the overall optimal strategy. For example, when the price of gold rises, we should consider not only the size of the surplus and transaction costs that gold brings, but also the impact if the price of Bitcoin rises more. Therefore, at this time, the multivariate optimization model is used to determine the optimal strategy.

By comparing the results of the decision-making method under the neural network prediction model, the total assets obtained after 5 years were the US $ 4723.0234.

According to our fund data observation and thinking, we analyze the Neural networks model is better than The first one, but there is a lag in the reaction of the price fluctuations.

The above models and results demonstrate our investment strategy. I hope our research can help your investment.

Thank you.

Contents

1 Introduction

1.1 Problem Background

Everyone yearns for wealth. For idle assets, we are often not satisfied with just relying on deposits in the bank to obtain interest, but hope to invest reasonably and obtain the maximum benefits. Gold and bitcoin have become fashionable investment products with price fluctuations. It is difficult for us to make a decision based on the expected price fluctuation of bitcoin for some time. The market trader is a profession that gains profit from price differences through buying and selling. By buying and selling unstable gold and bitcoin assets, they hope to maximize the total income of the client. In days, we know the bitcoin price data from November 9, 2016, to October 21, 2021 and the gold price data on the market trading day. Now we are required to establish a mathematical model and use the past daily price flow to determine whether we should buy, hold or sell assets in cash, gold and bitcoin portfolios every day.

1.2 Restatement of the Problem

First, our investment will start with $1000 in cash on November 9, 2016. On each trading day during the five-year trading period, traders will have a portfolio of cash [C, G, B], which are US dollars, troy ounces and bitcoin. The initial state is [1000, 0, 0], which means that our initial principal is $1000. The Commission for each transaction (purchase or sale) is% of the transaction amount α%, No commission is required for holding assets.

Considering that bitcoin can be traded every day, but gold is only traded on the day when the market is open. We use the document of pricing data to establish the investment decision-making model, and give the best daily trading strategy only based on the price data as of the day.

(1) Using your model and strategy and substituting the known data, what is the value of the initial $1000 investment on September 10, 2021?

(2) Provide evidence that your model provides the best strategy to illustrate the rationality of such modelling.

(3) Test the sensitivity of the model to determine how sensitive the strategy is to transaction costs. Explain why transaction costs affect strategy and results.

(4) Introduce your strategy, model and results to traders in a maximum two-page memo.

1.3 Our Work

In order to solve the optimal investment strategy and maximize the benefits of limited assets, we need to further carry out specific analysis on the basis of predicting the price trend. In fact, due to the large fluctuation of gold price and bitcoin price. We first consider predicting their future price trend through historical prices and then determine how to invest. In the first part, we first use the second-order moving average method, second exponential sliding prediction method and gray prediction model[1] to predict the price of gold (bitcoin). Calculate their forecast curve and compare it with the actual price curve.

In the second part, we use the neural network to modify the error repeatedly to improve the model, get the prediction curve consistent with the actual price, and improve the model.

Next, we rely on the predicted price to establish a daily decision-making model through planning. Hold, buy or sell gold and bitcoin according to the classified discussion of changes in indicators.

Then analyze the special conditions of each question. Specifically, substitute relevant data into the first question to calculate the final result of the decision. In the third question, analyze the Commission in the change of α in detail and the sensitivity of the model.

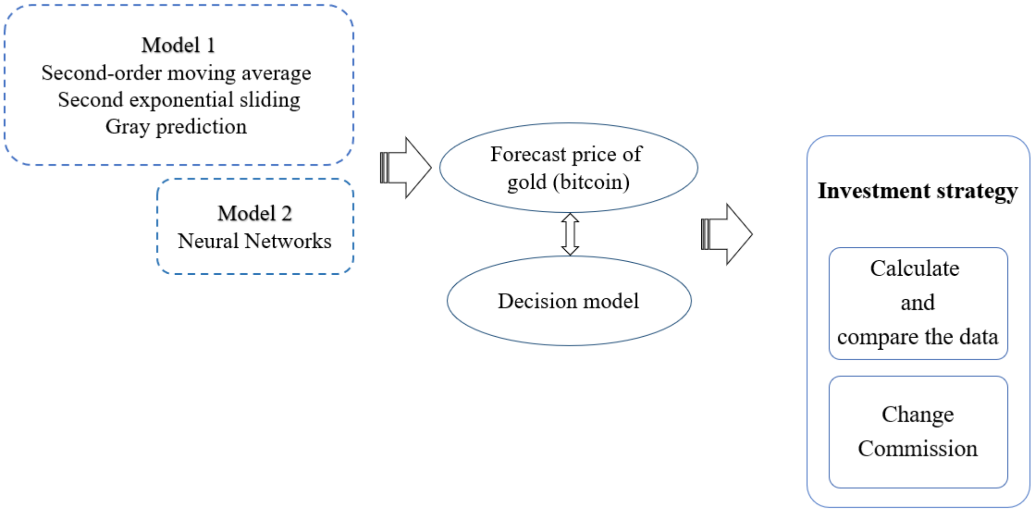

Figure 1: Steps for solving specific sub-questions

- Problem 1 requires us to use the established decision-making model of daily capital flow, substitute the given price data, and cumulatively calculate the corresponding value of the final portfolio.

Solution: In order to decide the investment method, we first need to predict the price trend of gold and bitcoin. Firstly, the second-order moving average method, second exponential sliding prediction method and gray prediction model are used. In addition, considering the characteristics of neural networks repeatedly correcting errors, it is convenient to be used for fitting. In the selection of network architecture, by comparing the applicability, the single-layer BP neural network is finally selected as the training model. Then through error correction, training model and noise reduction, the predicted prices of gold and bitcoin are obtained respectively. Finally, the predicted data are used to plan the decision-making scheme and substitute the actual price to calculate the asset value after 5 years.

- Problem 2 requires us to prove the rationality of the established decision-making scheme.

Solution: In question 1, we get the predicted prices of gold and bitcoin calculated by different models. Through comparison, we find that the prediction results obtained by the neural network are closer to the actual value. Therefore, the investment decision made on the basis of this prediction method is more reasonable. In addition, our decision model adopts a programming model and a multivariable optimization model. When predicting that the profit brought by buying is greater than the Commission, consider buying. Under such a decision-making method, our decision-making scheme is conducive to asset maximization.

- Problem 3 requires us to analyze the sensitivity of the model.

Solution: Considering the cost of changing every day, it is difficult to change the commission due to a large amount of data and long experience time. After changing α to the original 10 times and 0.1 times, the different prediction models and the final multivariable optimization model are reused for calculation, and the conclusion is drawn by comparing the income time curve.

- Problem 4 requires us to introduce strategies, models and results to market traders.

Solution: We introduce our models and strategies to market traders in the form of a memorandum, compare the asset amount calculated by different prediction models and decision-making models, and determine a more reasonable investment scheme.

2 Assumptions and Justifications

1. Assume that the closing price of a troy ounce in US dollars is the gold trading price for that day.

The data given in the question is limited, and only 1 gold price is given for each day in the market trading day. Therefore, we assume that the price of gold is traded at the closing price given for each day.

2. On the same day, gold and bitcoin can be held, bought, or sold in only one case.

Since the price fluctuations of gold and bitcoin are measured in days, if we choose to sell A and buy B, it is equivalent to selling A-B. (when A>B)

3. We assume that the transaction is done instantaneously.

Because the price of gold and bitcoin fluctuates irregularly over a certain period, it is completely meaningless to predict price changes when the observation time is too short. So for a certain period, we do not invest and keep our cash holdings. Next, let's assume that we can get a more accurate prediction from day m onwards.

3 Notations

Table 2: Notations used in this paper

Symbol	Description	Unit
C(n)	The cash on day n	dollar
G(n)	Amount of gold on day n	troy ounce
B(n)	The number of bitcoins on day n	/
Vg(n)	The closing price of one troy ounce on day n	dollar
Vb(n)	The price of one bitcoin on day n	dollar
Vg′(n)	The closing price of one troy ounce predicted on day n	dollar
Vb′(n)	The closing price of one bitcoin predicted on day n	dollar
t	Current days	/
T	The number of days between the current day and the forecast day	/
x_t	The value of one troy ounce of gold or one bitcoin converted to US dollars on day t	dollar
$\widehat{x_t}$	The value of one troy ounce of gold (bitcoin) converted to U.S. dollars on day t as predicted by the forecasting model	dollar
e_{t+1}	The deviation from the original prediction on day t + 1 at day t of the ex post facto is found at day t + 1	dollar

Table 3: Parameters used in this paper

Symbol	Description	Unit
β	Smoothing parameters	/
$\hat{\beta}$	Prediction smoothing parameters	/
σ^2	Random disturbance variance	/
$\widehat{\sigma_\beta}$	Standard error of β	/
MSE	Mean Square Error	/
h	Developmental grayscale	/
u	Control gray scale	/

4 Price forecasting models

4.1 Second-order moving average method[2]

Let the current number of days from the start of the investment be the day t and $\{x_t\}$ be the time series of the value of one troy ounce of gold or one bitcoin. And the model is:

$$\hat{x}_{1,t+T} = a_t + Tb_t .$$

(T is the number of days between the day and the forecast day, a_t, b_t are the parameters.)
First moving average:

$$M_t' = \frac{x_t + x_{t-1} + \ldots + x_{t-n+1}}{n} .$$

Second moving average:

$$M_t'' = \frac{M_t' + M_{t-1}' + \ldots + M_{t-n+1}'}{n} .$$

In order to maintain the accuracy of prediction and keep the prediction starting point of the neural network model consistent, let n = 5. According to the above formula, $x_t = 2M_t' - M_t''$.

Let $a_t = x_t$, then $a_t = 2M_t' - M_t''$, $b_t = \frac{2}{n-1}(M_t' - M_t'')$, and let T = 1, then the value of a unit of gold (bitcoin) on day t + 1 can be predicted. Record the predicted price of gold (bitcoin) on day t as $\hat{x}_{1,t+1}$.

4.2 Second exponential sliding prediction method[3]

First, a quadratic exponential prediction model is developed.

$$\begin{cases} \hat{x}_{2,t+T} = a_t + b_t T \\ a_t = 2S_t^{(1)} - S_t^{(2)} \\ b_t = \dfrac{\beta}{1-\beta}(S_t^{(1)} - S_t^{(2)}) \\ S_t^{(1)} = \beta x_t + (1-\beta) S_{t-1}^{(1)} \\ S_t^{(2)} = \beta S_t^{(1)} + (1-\beta) S_{t-1}^{(2)} \\ S_1^{(1)} = S_1^{(2)} = x_1 \end{cases} , \quad t = 2, 3, \cdots \tag{1}$$

We determine the value of smoothing parameters β (0< β < 1).

The logical process of setting the optimal prediction parameters of the quadratic exponential smoothing model by regression method is divided into three steps:

(1) construct the quadratic exponential smoothing random process according to the quadratic exponential smoothing prediction model;

(2) The quadratic exponential smoothing regression model is established according to the quadratic exponential smoothing random process;

(3) The regression coefficient of the quadratic exponential smoothing regression model is estimated to obtain the optimal prediction parameters.

a. Quadratic exponential smoothing stochastic process

In order to make the predicted value $\hat{x}_{2,\ t+T}$ of the model in Formula 1) and the conditional expectation $E(x_{t+1}|x_t,...,x_1;\beta)$. Establish equivalence relation $E(x_{t+1}|x_t,...,x_1;\beta)$ = $\hat{x}_{2,t+1}$ .We establish a quadratic exponential random smoothing process as follows:

$$\begin{cases} x_{t+T} = (1, T) \begin{bmatrix} 2 & -1 \\ \dfrac{\beta}{1-\beta} & -\dfrac{\beta}{1-\beta} \end{bmatrix} \begin{bmatrix} S_t^{(1)} \\ S_t^{(2)} \end{bmatrix} + \epsilon_{t+T} \\ \begin{bmatrix} S_t^{(1)} \\ S_t^{(2)} \end{bmatrix} = \begin{bmatrix} \beta \\ \beta^2 \end{bmatrix} x_t + \begin{bmatrix} 1-\beta & 0 \\ \beta(1-\beta) & 1-\beta \end{bmatrix} \begin{bmatrix} S_{t_1}^{(1)} \\ S_{t-1}^{(2)} \end{bmatrix} \\ S_1^{(2)} = S_1^{(1)} = x_1 \end{cases} , \quad t = 2, 3, \cdots \tag{2}$$

b. Quadratic exponential smoothing regression model

For the time series $\{x_1, x_2, ..., x_n\}$ generated by the random process of formula (2), a regression model can be established to estimate the unknown parameters β:

$$\begin{cases} x_{t+1} = \dfrac{2-\beta}{1-\beta} S_t^{(1)} - \dfrac{1}{1-\beta} S_t^{(2)} + \epsilon_{t+1} \\ \begin{bmatrix} S_t^{(1)} \\ S_t^{(2)} \end{bmatrix} = \begin{bmatrix} \beta \\ \beta^2 \end{bmatrix} x_t + \begin{bmatrix} 1-\beta & 0 \\ \beta(1-\beta) & 1-\beta \end{bmatrix} \begin{bmatrix} S_{t-1}^{(1)} \\ S_{t-1}^{(2)} \end{bmatrix} \\ S_1^{(2)} = S_1^{(1)} = x_1, \epsilon_{t+1} \sim N(0, \sigma^2) \end{cases} , \quad t = 2, \cdots, n-1 \tag{3}$$

In the model, the basic trend smoothing factors $\{(S_1^{(1)}, S_1^{(2)}),\ldots,(S_n^{(1)}, S_n^{(2)})\}$ that directly determine the observable time series $\{x_1, x_2,\ldots, x_n\}$ is the unobservable time series.

c. Quadratic exponential smoothing parameter estimation

The time series $\{x_1, x_2,\ldots, x_n\}$ is estimated by the least square method, and the optimal prediction parameter $\hat{\beta}$ is:

$$\hat{\beta} = \arg\min_{\beta} \sum_{t=2}^{n-1} \left(x_{t+1} - E(x_{t+1}|x_t, \cdots, x_1; \beta)\right)^2 .$$

The mean squared error *MSE* for the previous n-2 days is an estimate of the random perturbation term $\epsilon_{t+1} \sim N(0, \sigma^2)$ variance σ^2.

That is

$$\hat{\sigma}_\epsilon^2 = MSE = \frac{1}{n-3} \sum_{t=2}^{n-1} e_{t+1}^2 .$$

Where the residual sum of squares $\sum_{t=2}^{n-1} e_{t+1}^2$ is the sum of squares of deviations from the previous n-2 days' forecasts. The residual $e_{t+1}(t = 2,\ldots,n-1)$ is the deviation of the previous posterior day t+1 from the original prediction on day t+1 made on day t beforehand.

The standard error $\hat{\sigma}_\beta$ of the parameter β is estimated as follows:

$$\hat{\sigma}_\beta = \frac{1}{J'J} MSE.$$

Where the component of n − 2-dimensional Jacobian vector $J = (J_3,\ldots, J_n)^T$ is:

$$J_{t+1} = \frac{d\hat{x}_{2,t+1}}{d\beta} = \frac{dE(x_{t+1}|x_t,\ldots,x_1;\beta)}{d\beta}, t = 2,\ldots n-1$$

The regression model (3) is estimated by the least square method. The recursive relationship between the components of the Jacobian vector in the iterative process of the algorithm is as follows:

$$\begin{cases} J_{t+1} = \dfrac{1}{(1-\beta)^2} S_t^{(1)} - \dfrac{1}{(1-\beta)^2} S_t^{(2)} + \dfrac{2-\beta}{1-\beta} \dfrac{\mathrm{d}S_t^{(1)}}{\mathrm{d}\beta} - \dfrac{1}{1-\beta} \dfrac{\mathrm{d}S_t^{(2)}}{\mathrm{d}\beta} \\ \begin{bmatrix} S_t^{(1)} \\ S_t^{(2)} \\ \dfrac{\mathrm{d}S_t^{(1)}}{\mathrm{d}\beta} \\ \dfrac{\mathrm{d}S_t^{(2)}}{\mathrm{d}\beta} \end{bmatrix} = \begin{bmatrix} \beta \\ \beta^2 \\ 1 \\ 2\beta \end{bmatrix} x_t + \begin{bmatrix} 1-\beta & 0 & 0 & 0 \\ \beta(1-\beta) & 1-\beta & 0 & 0 \\ -1 & 0 & 1-\beta & 0 \\ 1-2\beta & -1 & \beta(1-\beta) & 1-\beta \end{bmatrix} \begin{bmatrix} S_{t-1}^{(1)} \\ S_{t-1}^{(2)} \\ \dfrac{\mathrm{d}S_{t-1}^{(1)}}{\mathrm{d}\beta} \\ \dfrac{\mathrm{d}S_{t-1}^{(2)}}{\mathrm{d}\beta} \end{bmatrix} \end{cases} \quad (4)$$

Recall that by day t+ 1, the result is $\hat{x}_{2,t+1}$.

4.3 Gray prediction model[4]

Let the original data series $S_{(0)} = \{X_{(0)}(i): i = 1,2,...,t\}$, where $X_{(0)}(i) = x_i$. First, accumulate the processing. We get

$$X_{(1)}(k) = \sum_{i=1}^{k} X_{(0)}(i) = X_{(1)}(k-1) + X_{(0)}(k)$$

Establish the differential equation for the new series $X_{(1)}$:

$$\frac{\mathrm{d}X_{(1)}}{\mathrm{d}t} + hX_{(1)} = u.$$

(h is the development grayscale, and u is the control grayscale.)

When the initial condition t= 1, $X_{(1)} = X_{(1)}(1)$.

The solution of the differential equation is

$$X_{(1)}(k+1) = (X_{(0)}(1) - \frac{u}{h})e^{-hk} + \frac{u}{h}.$$

Afterwards, the model is fitted to the series of gold (bitcoin) values up to that date and the predicted value at day t+ 1 is denoted as $\widehat{x}_{3,t+1}$.

4.4 Integration

All the above 3 model methods are linear, so they can be integrated by averaging. After integrating the results obtained from the above three models, we obtain the prediction formula for the price of gold (bitcoin) as

$$\hat{x}_{t+1} = \frac{\hat{x}_{1,t+1} + \hat{x}_{2,t+1} + \hat{x}_{3,t+1}}{3}.$$

Substitute the known gold (bitcoin) price data to make the prediction curve and compare it with the actual curve as follows.

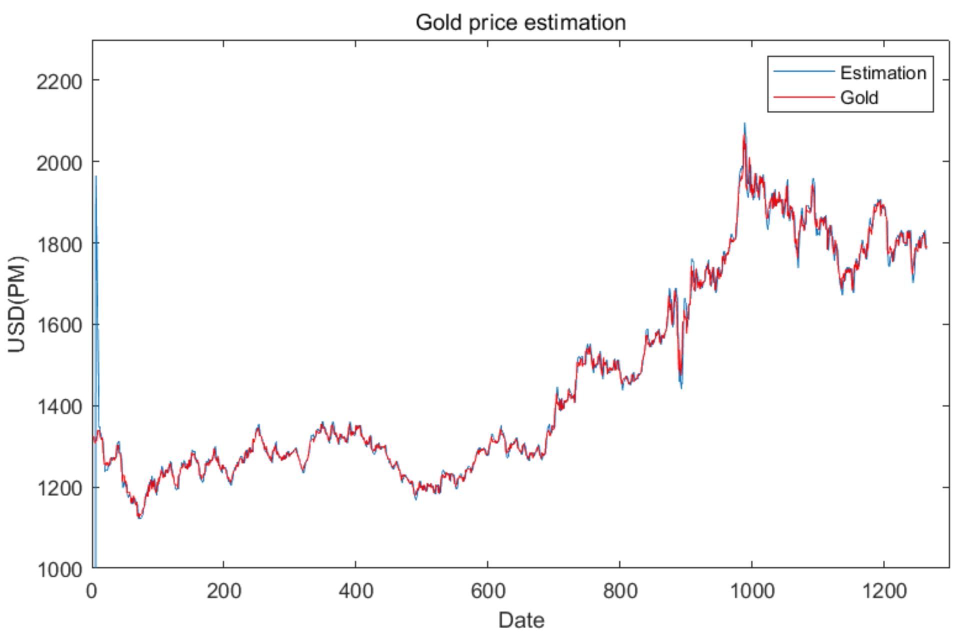

Figure 4: Comparison between predicted price and the actual price of gold

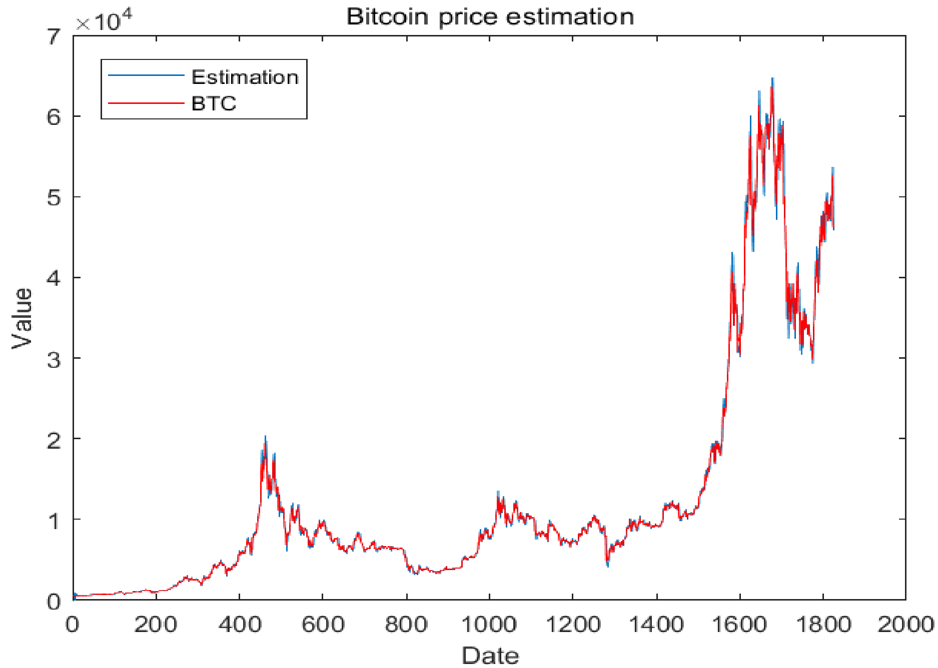

Figure 5: Comparison between predicted price and the actual price of bitcoin

5 Neural networks

In order to consider the possible influence of nonlinear factors, we establish a neural network model for improvement.

Due to the oscillatory nature of gold price fluctuations, it is difficult to fit with a specific function, while the future gold price is closely related to the historical gold price. Therefore, to find the connection between the current gold price and past gold price and to avoid the impact of short-term rapid and small oscillations on the fit, a neural network is trained to find the pattern of gold price changes.

Let the price of gold (bitcoin) on day n be. If the price on the day i is predicted, the sampling interval is set to inter and the number of samples is dim before the day i. We obtain the sampling vector.

$$x(i) = (price(i-1), price(i-1-inter), \dots, price(i-1-inter \times (dim-1)))$$

In the selection of network architecture, both the RBF neural network with local approximation and the BP neural network with global approximation are better choices. After practice, it is found that due to the fast small fluctuations of the gold price, the RBF neural network will have the problem that the singular matrix cannot be solved. And the multi-layer BP network will bring the problem of overfitting. So we finally choose a single-layer BP neural network as the most suitable training model.

5.1 Single layer neural network forward transfer

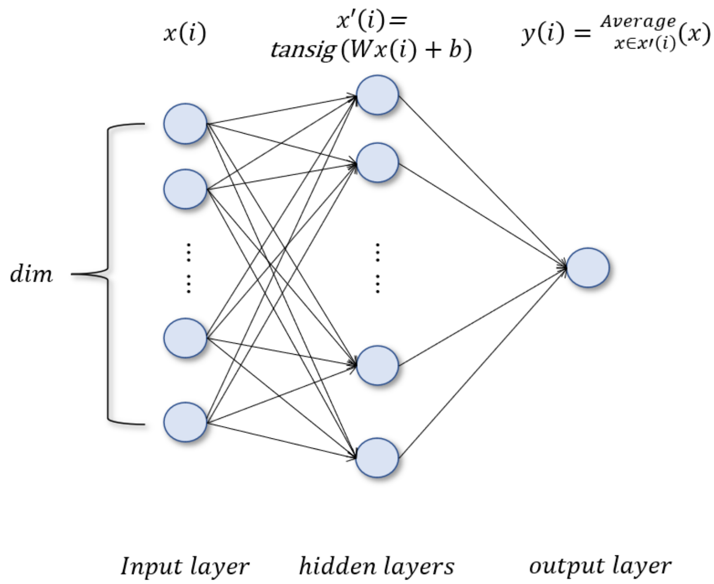

Figure 6: Schematic diagram of forward transmission of information between different layers

The input is a vector $x(i)$. The hidden layer $x'(i)$ is obtained by doing a linear transformation $Wx(i) + b$ followed by an activation function $tansig$. The final linear output is the predicted value $y(i)$. Where the activation function $tansig(x)$ is:

$$tansig(x) = \frac{2}{1 + e^{-2x}} - 1$$

(In the forward pass process, the trainable parameters are W, b)

5.2 Back propagation of single layer neural network

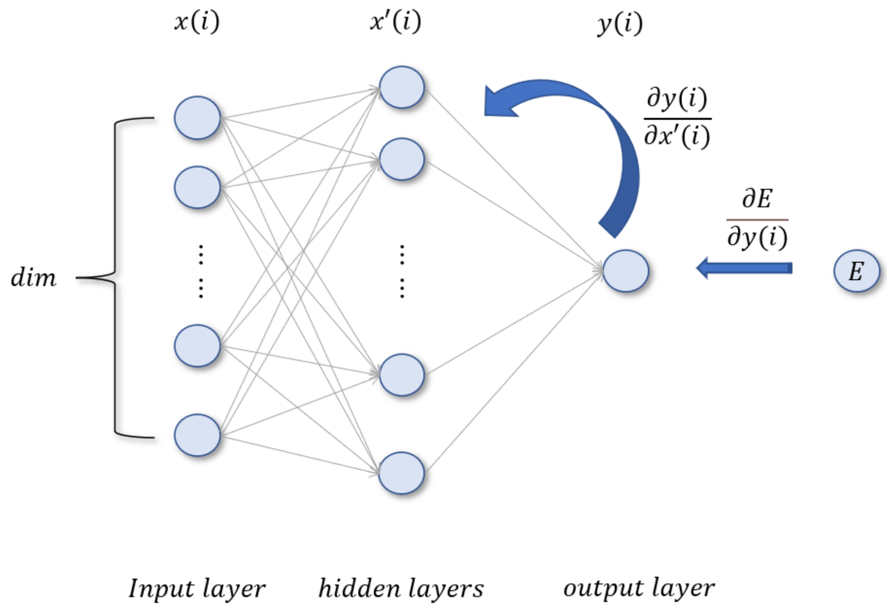

Figure 7: Schematic diagram of reverse transmission of information between different layers

We need to pass the error backwards from the output layer through the hidden layer to the input layer, and train the model by iteratively correcting the error at the end of each pass.

Errors： $E = \left(y(i) - price(i)\right)^2$

The effect of b on the overall error： $\frac{\partial E}{\partial W} = \frac{\partial E}{\partial y(i)} \cdot \frac{\partial y(i)}{\partial x\prime(i)} \cdot \frac{\partial x\prime(i)}{\partial W}$

Similarly, the effect of B on the overall error： $\frac{\partial E}{\partial b} = \frac{\partial E}{\partial y(i)} \cdot \frac{\partial y(i)}{\partial x\prime(i)} \cdot \frac{\partial x\prime(i)}{\partial b}$

is corrected by the error generation W, b pair at the end of a training session.

$$W' = W - \alpha \frac{\partial E}{\partial W}$$

$$b' = b - \alpha \frac{\partial E}{\partial b}$$

(where α is the learning rate of this neural network)

This training is repeated until the error reaches a satisfactory result.

5.2.1 Selection of parameters

The following table shows the results of hyperparameter selection for the single-layer neural network, using gold price fluctuations as training data.

Table 8: Selected hyperparameters

Hyperparameter	**Description**	**Value**
dim	Dimensionality of input data	5
Hidden dim	Dimensionality of hidden layers	1
inter	Sampling interval	1
Group	Number of groups of training samples	200
α	Learning Rate	0.05

5.3 Training results

5.3.1 Results of training regression analysis

The figure above shows that the trained model performs well on the test set and has a high fitting degree. However, the performance in the validation set is not satisfactory, which may be due to the interference of noise generated by training.

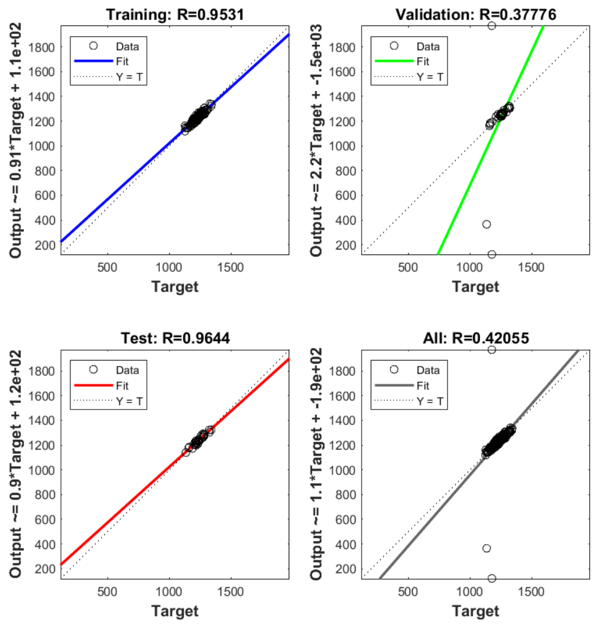

Figure 9: Model performance on the test set

The trained neural network model is used to predict the trend of gold, and the results are as follows:

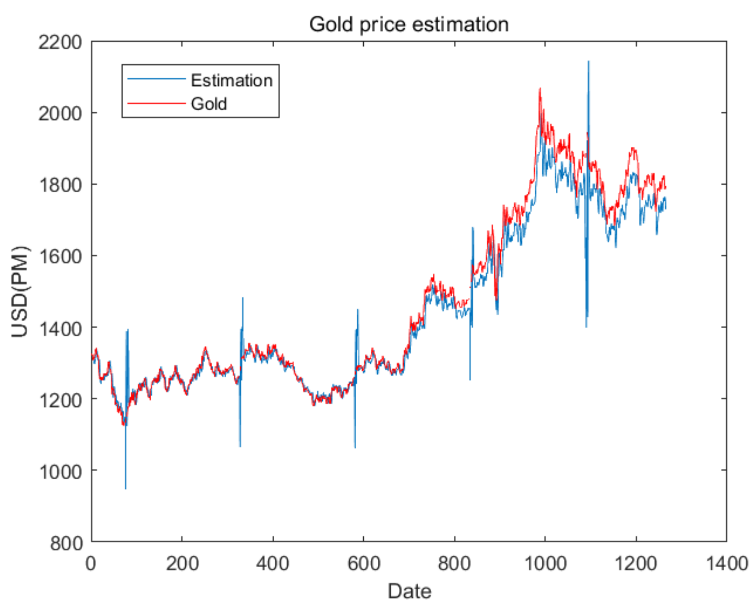

Figure 10: Preliminary gold price forecast results

5.3.2 Noise reduction treatment

It can be seen that although the gold price trend predicted by the neural network is basically consistent with the actual gold price trend, there is noise interference.

Therefore, the method of moving average filter (window size is 5) is used to smooth the prediction sequence. The specific principle is as follows.

Set the original sequence as $Y = (y_1, y_2, \cdots\cdots, y_{n-1}, y_n)$

Make the following transformation:

$$y'_1 = y_1$$

$$y'_2 = \frac{y_1 + y_2 + y_3}{3}$$

$$\vdots$$

$$y'_i = \frac{y_{i-2} + y_{i-1} + y_i + y_{i+1} + y_{i+2}}{5}$$

We have $Y' = (y'_1, y'_2, \cdots\cdots, y'_{n-1}, y'_n)$

The smoothed predicted gold price and daily actual gold price are used to draw the following results.

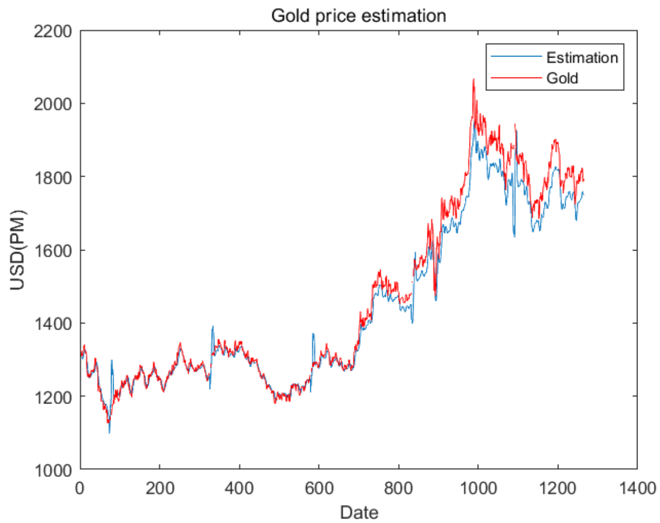

Figure 11: Prediction results of gold price after noise reduction

Through observation, we can find that the new prediction curve fits the actual gold price better, and it is smoother at the position of large fluctuation.

The neural network is trained in the same way to obtain the training regression analysis results of bitcoin price prediction and the predicted price are as follows.

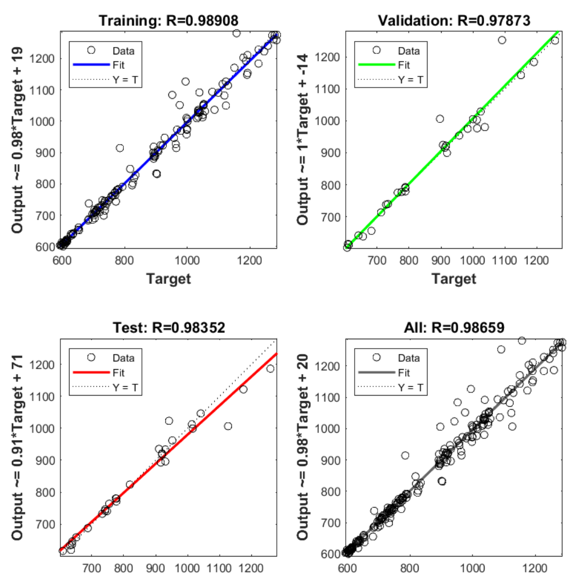

Figure 12: Model performance on the test set

The figure above shows that the trained model performs well on the test set and has a high fitting degree.

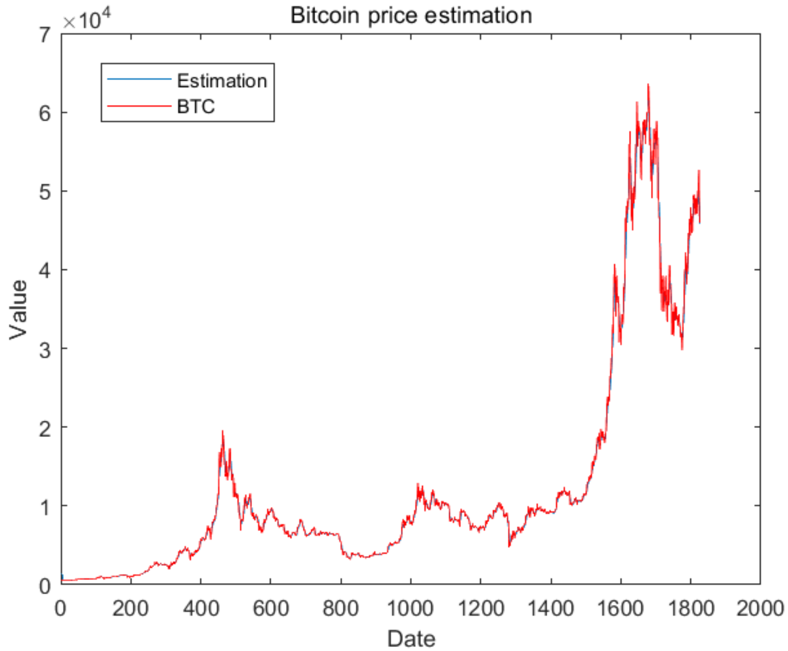

Figure 13: Prediction results of bitcoin price after noise reduction

It is observed that the trend of bitcoin price after noise reduction is good, which is consistent with the actual bitcoin price.

In conclusion, the use of a neural network can better predict the price of gold and bitcoin according to the historical price.

6 Decision model

First, we declare the meaning of some symbols.

Let $\Delta C = C(n + 1) - C(n)$.

$\Delta G = G(n + 1) - G(n)$.

$\Delta B = B(n + 1) - B(n)$.

ΔCg: denotes the fraction of ΔC associated with gold trading.

ΔCb: denotes the fraction of ΔC associated with bitcoin transactions.

$C(k) = 1000, G(k) = 0, B(k) = 0, k = 1, 2, \ldots, m$

Now consider the number of days m, when $Vg(m)$, $Vb(m)$ are known and $Vg(m + 1)'$, $Vb(m + 1)'$ can be derived from the prediction model.

In addition, since gold is not directly convertible with bitcoin, there are only two cases for each investment:

i. cash to gold

ii. cash and bitcoin transactions.

The next two scenarios are open market days and non-open market days.

1.On market open days, there are four scenarios where the forecast is compared to the data of the day.

ΔGi, ΔBj denote the ΔG, ΔB in different cases.

(First note that 'consider buying A' in the following indicates maintaining a hold or buy on A, but necessarily not selling A. 'consider selling' has the same meaning.)

(a) Buying gold.

$$\Delta G_1 = \frac{-\Delta C_g(1-\alpha_{gold})}{V_g(m)}$$

(b) Selling gold.

$$-\Delta G_2 V_g(m)(1-\alpha_{gold}) = \Delta C_g$$

(c) Buying bitcoin.

$$\Delta B_1 = \frac{-\Delta C_b(1-\alpha_{bitcoin})}{V_b(m)}$$

(d) Selling bitcion.

$$-\Delta B_2 V_b(m)(1-\alpha_{bitcoin}) = \Delta C_b$$

Let

$$\Delta G = \frac{-\Delta C_g(1-\alpha_{gold})}{V_g(m)} k_1 + \frac{-\Delta C_g}{V_g(m)(1-\alpha_{gold})} k_2$$

$$\Delta B = \frac{-\Delta C_b(1-\alpha_{bitcoin})}{V_b(m)} k_3 + \frac{-\Delta C_b}{V_b(m)(1-\alpha_{bitcoin})} k_4$$

$$\Delta C = \Delta C_g + \Delta C_b$$

$$C(m+1) = C(m) + \Delta C$$

$$G(m+1) = G(m) + \Delta G$$

$$B(m+1) = B(m) + \Delta B$$

At this point buying or selling gold or bitcoin can be divided into 4 scenarios. Considering all of them, we can get the programming (where $\Delta C_g, \Delta C_b, k_i$ is the independent variable):

$$\max S' = C(m+1) + V_g'(m+1)G(m+1) + V_b'(m+1)B(m+1)$$

$$\text{s.t.}\begin{cases} C(m+1) \geqslant 0 \\ G(m+1) \geqslant 0 \\ B(m+1) \geqslant 0 \\ k_i \in \{0,1\}, i=1,2,3 \\ k_1 k_2 = 0 \\ k_3 k_4 = 0 \\ k_1 \neq k_2\text{时}, (k_1-k_2)\Delta C_g \leqslant 0 \\ k_1 = k_2\text{时}, \Delta C_g = 0 \\ k_3 \neq k_4\text{时}, (k_3-k_4)\Delta C_b \leqslant 0 \\ k_3 = k_4\text{时}, \Delta C_b = 0 \end{cases}$$

Then substitute the solved $\Delta C_g, \Delta C_b, k_i$ into $S = C(m+1) + V_g(m+1)G(m+1) + V_b(m+1)B(m+1)$ to get the total asset value on the next day if the investment is made according to the strategy.

2.On days when the market is not open, gold cannot be traded, but bitcoin can be traded, and the situation is similar, except that k1 = k2 = 0. (where $\Delta C_g, \Delta C_b, k_i$ is the independent variable)

$$\max S = C(m+1) + V_g{}'(m+1)G(m+1) + V_b{}'(m+1)B(m+1)$$

$$\text{s.t.}\begin{cases} C(m+1) \geqslant 0 \\ G(m+1) \geqslant 0 \\ B(m+1) \geqslant 0 \\ k_i \in \{0,1\}, i = 1,2,3,4 \\ k_1 = k_2 = 0 \\ k_3 k_4 = 0 \\ k_1 \neq k_2\text{时},(k_1 - k_2)\Delta C_g \leqslant 0 \\ k_1 = k_2\text{时},\Delta C_g = 0 \\ k_3 \neq k_4\text{时},(k_3 - k_4)\Delta C_b \leqslant 0 \\ k_3 = k_4\text{时},\Delta C_b = 0 \end{cases}$$

Then substitute the solved $\Delta C_g, \Delta C_b, k_i$ into $S = C(m+1) + V_g(m+1)G(m+1) + V_b(m+1)B(m+1)$ to get the total asset value on the next day if the investment is made according to the strategy.

The analysis of the number of days m applies to the rest of the cases, just recursively.

7 Rationality analysis of investment strategy and model

Based on the first question model, we discuss the investment decision-making scheme in detail. Firstly, it is stated that the decision-making method generated by the multivariable optimization model we use is aimed at the change of one of gold and bitcoin. Since there is only one investor in the model of [C, G, B].

First of all, let's take gold as an example and start the discussion on the open market day (same for bitcoin)

(a) Gold is forecast to rising$V_g{}'(k+1) > V_g(k)$.

To get more money, one should consider buying gold at this time, assuming that buying gold costs ΔCg, and the transaction cost at this time is $\Delta C_g \alpha_{gold}$

The predicted value of additional gold is obtained as

$$\frac{\Delta C_g (1-\alpha_{gold})}{V_g(k)} V_g{}'(k+1)$$

Then, whether buying is profitable or not is related to the size of $\frac{1-\alpha_{\text{gold}}}{V_g(k)}V_g'(k+1)$ and α_{gold}.

i. When $\frac{1-\alpha_{\text{gold}}}{V_g(k)}V_g'(k+1)>\alpha_{\text{gold}}$, it means that the part of gold purchased is profitable and should be bought, unless there exist a sharp increase of bitcoin.

ii. When $\frac{1-\alpha_{\text{gold}}}{V_g(k)}V_g'(k+1)\leqslant\alpha_{\text{gold}}$, we can not make money, strictly less than the time will also lose money, so we should not buy, unless there exist a sharp decrease of bitcoin.

(b) Gold is forecast to fall $V_{\text{g}}'(k+1) < V_{\text{g}}(k)$.

At this point, to reduce losses, consider selling gold. Assume that selling ΔG troy ounces of gold will result in a loss due to transaction costs of $\Delta G V_{\text{g}}(k)\alpha_{gold}$.

At the same time, if this part is not sold, it will lose $\Delta G(V_g(k)-V_g(k+1))$.

Thus the ability to sell for more stop loss is related to the size of $\frac{V_{\text{g}}(k)-V_g'(k+1)}{V_g(k)}$ and α_{gold}.

i. When $\frac{V_{\text{g}}(k)-V_g'(k+1)}{V_g(k)}>\alpha_{\text{gold}}$, it will cost a lot if not to sell at this time. So we should sell.

ii. When $\frac{V_{\text{g}}(k)-V_g'(k+1)}{V_g(k)}\leqslant\alpha_{\text{gold}}$, the loss is the same when taking equal, strictly less than when not selling, the loss is smaller.

(c) The gold forecast is flat $V_{\text{g}}'(k+1) = V_{\text{g}}(k)$.

At this time, whether buying or selling, only the loss is caused by transaction costs. So we should keep hold.(The only difference is that gold cannot be traded on non-opening days, only bitcoin can be traded.)

Our strategy is generated under the reasonable prediction of gold (bitcoin) price, and measures factors such as commission, which is reasonable. The results of the model are shown below:

(1) Integrated model of quadratic moving average prediction, quadratic exponential smoothing prediction and grey prediction

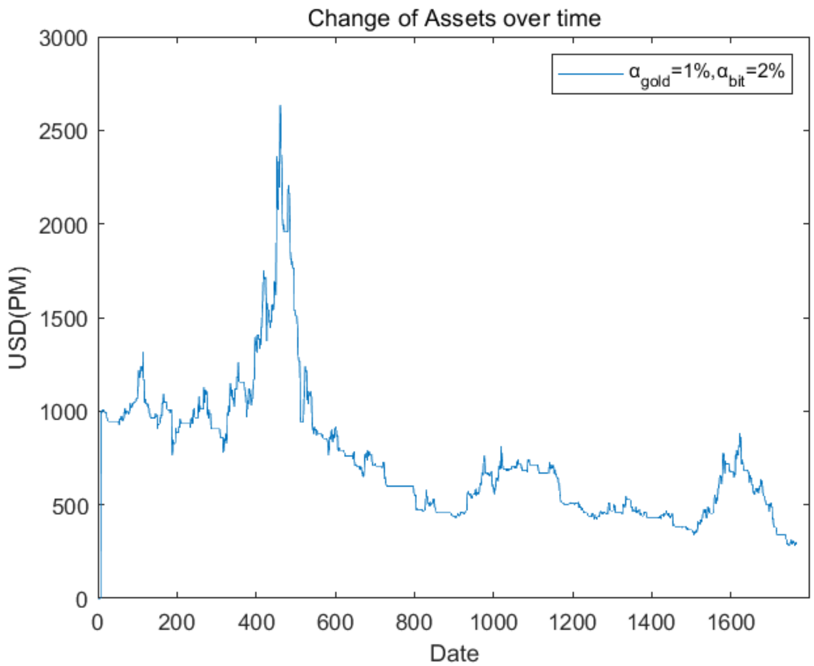

Figure 14: Return on investment - time

When $\alpha_{gold} = 1\%, \alpha_{bit} = 2\%$, the final asset value is 285.4871dollars.

(2) Neural network model

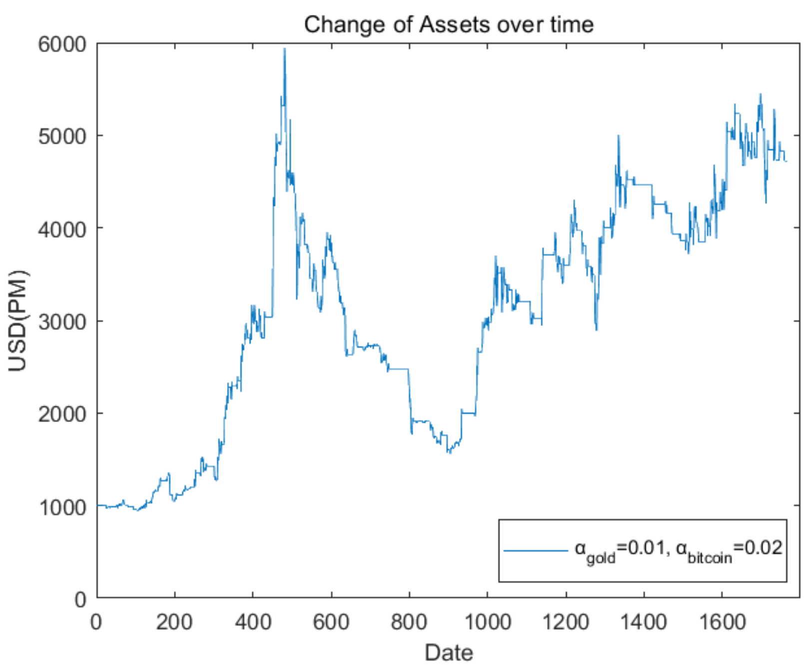

Figure 15: Return on investment - time

When $\alpha_{gold} = 1\%, \alpha_{bit} = 2\%$, the final asset value is 4723.0234dollars.

8 Sensitivity analysis of strategy

For the linear method integration model and neural network model, we substitute specific data for analysis.

Control Commission α Change to the original 0.1/10 times (i.e. order), and draw the income time chart as follows

(1) Integrated model of quadratic moving average prediction, quadratic exponential smoothing prediction and grey prediction.

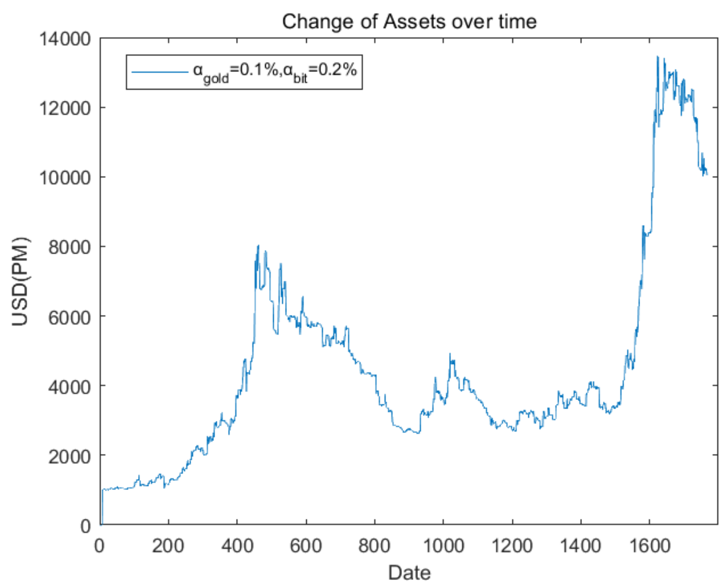

Figure 16: Investment income under 0.1x Commission

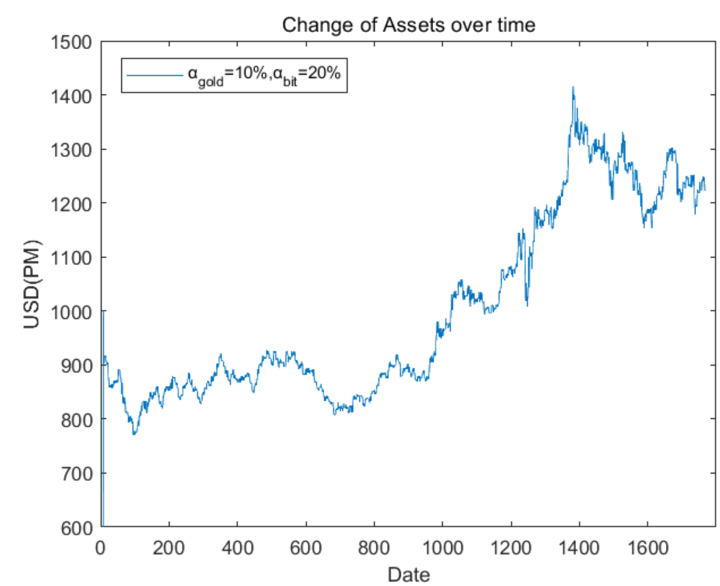

Figure 17: Investment income under 10X Commission

When $\alpha_{gold} = 10\%, \alpha_{bit} = 20\%$, the final asset value is 1224.0571 dollars.

When $\alpha_{gold} = 1\%, \alpha_{bit} = 2\%$, the final asset value is 285.4871 dollars.

When$\alpha_{gold} = 0.1\%, \alpha_{bit} = 0.2\%$, the final asset value is 10035.6275 dollars.

(2) Neural network model

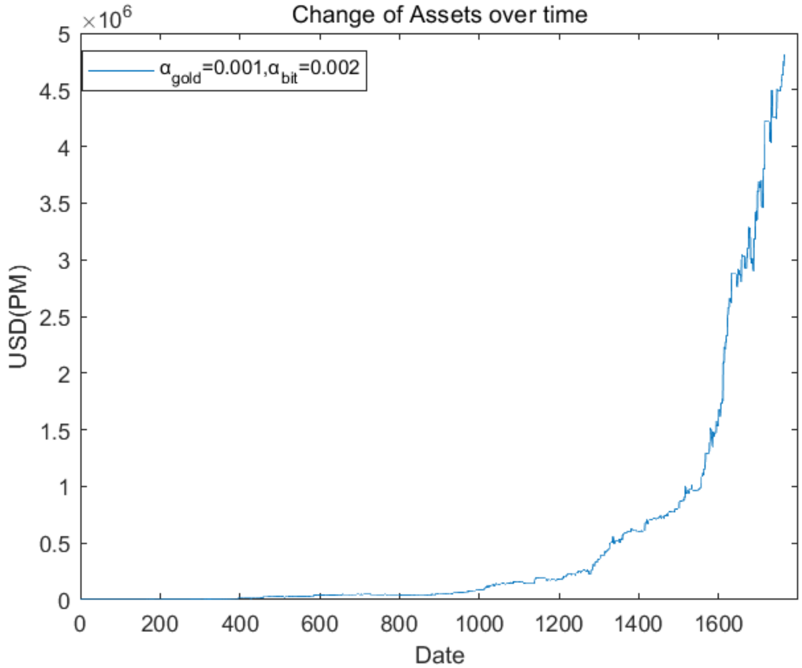

Figure 18: Investment income under 0.1X Commission

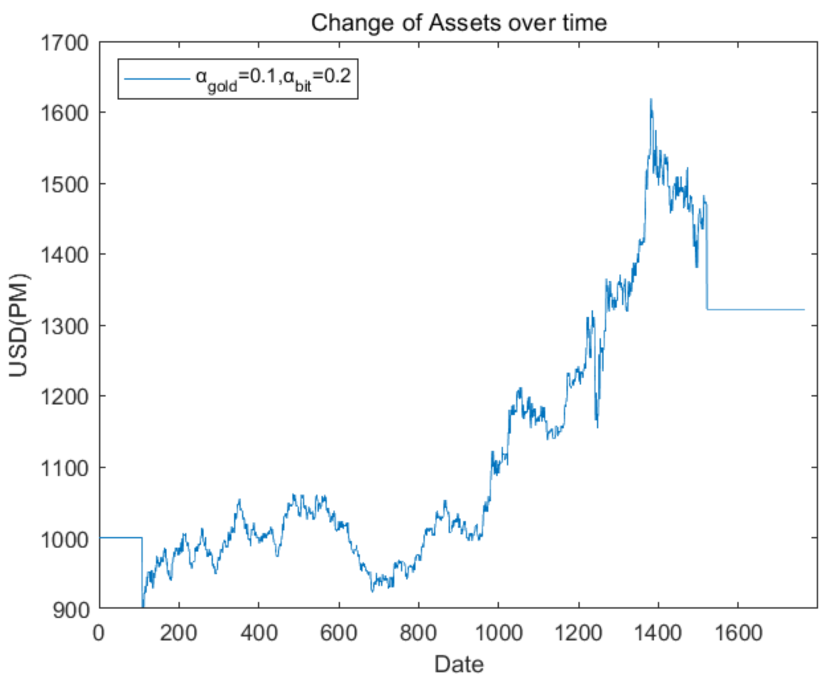

Figure 19: Investment income under 10X Commission

When $\alpha_{gold} = 10\%, \alpha_{bit} = 20\%$, the final asset value is 1322.2469 dollars.
When $\alpha_{gold} = 1\%, \alpha_{bit} = 2\%$, the final asset value is 4723.0234 dollars.
When$\alpha_{gold} = 0.1\%, \alpha_{bit} = 0.2\%$, the final asset value is 4797534.5209 dollars.

9 Model analysis

We find that the neural network model performs better than the first model. Under different commissions, the final asset value is greater. Explanation: Although the first model is closer to price changes, it may not be able to predict future changes well at the inflection point due to the limitation of linearity. When the price of gold rose for some time, the first model showed a rise. When the price of gold falls for a while, the first model can show a decline. This model shows an obvious lag in price prediction.

It is for this reason that the higher handling fee avoids the blind buying and selling caused by small price fluctuations, and weakens the negative impact caused by prediction delay to a certain extent. Therefore, in the first model, when the handling fee is 10 times the original, the final income is higher than that under the original handling fee.

Observing the details of the forecast price chart below shows the lag problem of model 1.

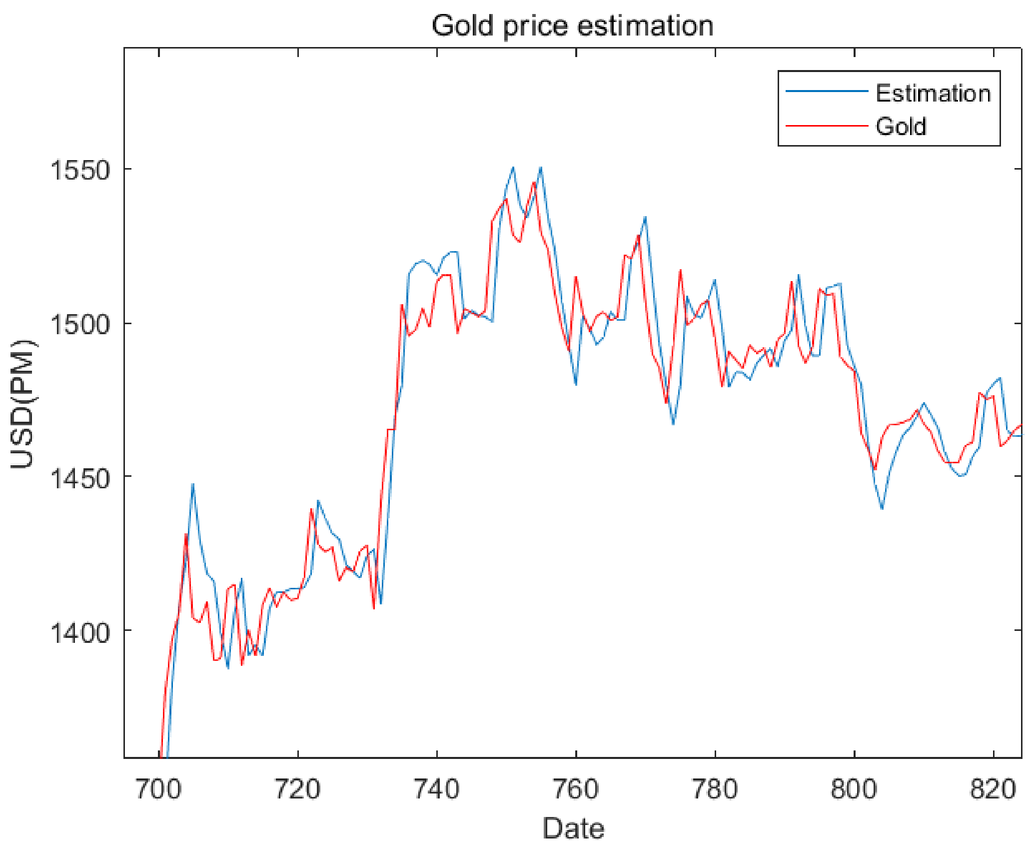

Figure 20: Local structure of linear prediction model

Compared with the price predicted by the neural network at the same time, we find that although there is a certain gap between the overall predicted price and the actual price, the delay problem at the inflection point is much better.

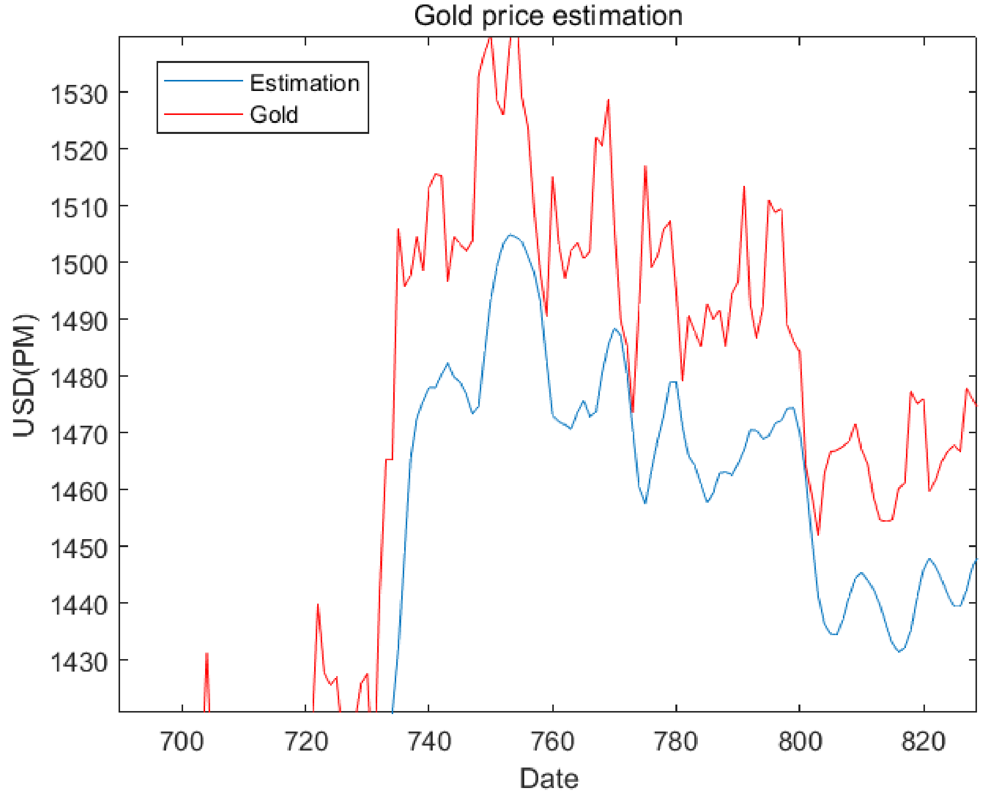

Figure 21: Local structure of nonlinear prediction model

We only analyze the change in commission for gold. (the same for bitcoin)

On the one hand, if the cost of trading goes up, when the predicted value of gold goes up, $\frac{1-\alpha_{\text{gold}}}{V_g(k)}V_g{}'(k+1)$ will go down and α_{gold} will go up. At this point, the trader is more inclined not to buy gold.

On the other hand, when the forecast price of gold falls, $\frac{1}{V_g(k)}(V_g(k)-V_g{}'(k+1))$ is flat. At this time we prefer not to sell gold.

In general, when transaction costs rise, the strategy is more likely not to invest. In a nutshell, our investment strategy becomes more conservative.

10 Model Evaluation

10.1 Strengths

1. We use mathematical tools related to optimization to make decisions based on accurately predicting the trend of gold and bitcoin, which can ensure the maximization of daily benefits.
2. Our decision model can accurately analyze the impact of single transaction cost change on transaction strategy.

10.2 Weaknesses

1. Take the daily closing price of gold as the trading price of gold in a whole day, but in fact, the trading price of gold is also changing in a day.
2. We assume that the transaction is completed in an instant, but in reality, the transaction cannot be completed in an instant. At this time, gold and bitcoin can be traded at the same time, but this is not considered in the strategy.

11 References

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