Restricted Perceptions and the Zero Lower Bound Episode *

(Preliminary and incomplete, not for distribution)

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Abstract

In recent years, two general classes of time-varying DSGE models that have gained attraction in the literature: The first one is the class of Markov-switching models that allow for time-variation in the structural system parameters, which offer a tractable way of studying regime shifts. The second one is adaptive expectation models that allow agents' beliefs to deviate from the underlying Rational Expectations Equilibrium (REE) and instead vary over time based on historical data. In this paper, we bring these two classes of models together and construct a framework that allows for the estimation of Markov-switching DSGE models under adaptive expectations. In particular, we consider scenarios where the agents' perceived law of motion (PLM) is based on small forecasting rules that do not take into account the regime shifts in the underlying system. We build on this idea to estimate DSGE models under adaptive expectations during the recent zero lower bound (ZLB) episode that started with the onset of the 2007-08 crisis.

We start by considering the E-stability principle in Markov-switching models: An Equilibrium is said to be E-stable if the agents can learn the underlying equilibrium by starting from an arbitrary point and updating their beliefs based on simple recursive algorithms, such as recursive least squares. If the PLM coincides with the closed-form MSV solution, then the system will converge to the underlying REE. However, the underlying equilibrium does not coincide with the REE whenever agents' PLM are misspecified. In such scenarios, the resulting misspecification equilibrium is called a Restricted Perceptions Equilibrium (RPE) in general. We show the E-stability of the underlying RPE in two cases: (i) The PLM coincides with the closed-form MSV solution, but the regime shifts are unknown to the agent. (ii) The PLM is based on a parsimonious backward-looking AR(1) rule.

Our results show that, in both cases, there is an E-stable RPE that deviates from the underlying REE. Furthermore, the E-stability of these equilibria holds even if one of the underlying Markov regimes is E-unstable, as long as the exit probability from the E-unstable regime is sufficiently large. In such cases, the system is characterized by erratic behaviour during the unstable regime, while reverting back to its stable behaviour during the stable regime. We denote this as the **long-run E-stability principle.**

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Next we turn to estimation of our class of DSGE models: A popular method in the state-space Markov-switching literature is the Kim & Nelson (1999) filter, which is based on a "collapsing" approximation to make the models tractable. We extend this filter to accommodate adaptive expectations, and consider the Bayesian estimation of two standard DSGE models: The first one is the 3-equation NKPC model along the lines of Woodford (2003), which is too simplistic for any actual policy analysis, but provides a good starting point for exposing our main results. The second one is the more complex Smets-Wouters (2007) model, which has become very popular among central banks and policy makers during the last decade. Based on our preliminary estimations, our results can be summarized as follows: The Markov-switching adaptive expectation models are able to outperform the REE benchmark in all cases, and the Regime-switching REE model in some cases. Furthermore, we observe important differences in the impulse response and shock propagation structure of the models under consideration. Particularly, a financial shock of the same size (which can be thought of as the financial crisis of 2007-08) typically has a longer lasting impact under adaptive learning, which suggests that Rational Expectations models may severely underestimate the crisis' impact and the system's vulnerability to another shock during the Zero lower bound episode.

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1 Introduction

With the onset of the Global Financial Crisis of 2007-08 and the subsequent drop of interest rates to near-zero levels among the leading central banks, there has been increased interest among policymakers and central bankers in the zero lower bound episode. There is still ongoing debate about the precise impact of the zero lower bound constraint on the economy as a whole, and in particular about its macroeconomic cost in terms of aggregate GDP levels. Furthermore, the implications on the best government spending strategies are also mixed: while some researchers recommend a fiscal austerity programme to exit the ZLB episode, others think it is best to adapt a fiscal consolidation strategy. A common approach in studies examining the ZLB episode is the assumption of Rational Expectations: Agents are assumed to perfectly know the underlying regime along with all other cross-correlations of the relevant macroeconomic variables and form their expectations accordingly. In standard DSGE models, this typically leads to very short periods of anticipated ZLB episodes, which deteriorates their empirical performance considerably.

In recent years, adaptive learning gained popularity in DSGE models as a plausible alternative to the standard Rational Expectations Hypothesis. Accordingly, the assumption that agents perfectly know the underlying economic relations and the corresponding cross-correlations is relaxed; instead they have their own sub-models that does not necessarily coincide with the REE-implied law of motion. They act as econometricians and update their models each period as new observations become available. Our key contribution in this paper is to estimate DSGE models during the ZLB episode under adaptive learning. We then examine the consequences of deviating from the REE during this period, particularly how it might contribute to a prolonging of the crisis and how it might change implications of standard DSGE models about the potential impact of a government spending shock during this episode.

There have been various different approaches to incorporate the ZLB constraint into benchmark DSGE models: while some researchers use a perfect foresight/ endogenous duration approach, others use a Rational Expectations/ Markov switching framework. (??) shows that, in general, the implications of using different approaches do not lead to substantial differences in results, as long as the ZLB constraint is accounted for. In this paper, we focus on a Markov-switching framework to account for the constraint: while Markov-switching and adaptive learning have been two very popular classes of time-variation in DSGE models, there is surprisingly little work on DSGE models that combine both approaches. Therefore, our paper is also the first one to explicitly unify these two approaches, where we examine Markov-switching DSGE models under adaptive learning.

Our key assumption in this paper is that, the underlying regimes are unobserved to economic agents: agents only indirectly become aware of regime changes to the extent that these shifts have an observable and strong enough impact on their information set. To illustrate this, consider the following example: A central bank follows a simple Taylor rule that reacts to inflation in setting interest rates. This will be known to economic agents to the extent that the central bank discloses its goal of inflation targeting, but the agents never know the exact reaction coefficient. Accordingly, the agents will not find out if the central bank suddenly and discreetly decides to change its reaction coefficient. Instead, the agents will slowly find out about this regime shift to the extent that it leads to observable consequences in the interest rate and inflation levels.

(??) explore the class of RE equilibria in Markov-switching models. Since we assume regimes are never observed, any equilibrium concept in our framework can never coincide with a Rational Expectations Equilibrium. Instead, in limited information environments, there are so-called Restricted Perceptions Equilibria (RPE), where the agents' misspecification of the economy becomes self-fulfilling and the system stabilizes on a non-rational equilibrium. We compute these equilbria for two classes of PLMs: In the first case we assume that agents' PLM has the MSV-form, except that the PLM does not take into account the possibility of regime-switches. In the second case, we take this idea further and consider VAR-type PLMs, where the information set of the agent can be even smaller due to unobserved shocks or ignored cross-correlations. We show that standard E-stability conditions apply to these equilibria, and therefore the systems will converge to the underlying equilibria under standard recursive algorithms such as least-squares. Furthermore, the E-stability and convergence results will continue to hold if one of the underlying regimes is unstable or even explosive, as long as the the remaining regimes are strongly stable enough. This is a simple extension of the long-run determinacy result of Davig and Leeper (2007), which they call the long-run Taylor principle. We therefore denote our result as the long-run E-stability.

We next build a variant of the Kim & Nelson (1999) filter to estimate our MS-DSGE models under adaptive learning, and we apply the filter to two standard workhorse models: The first one is the 3-equation NKPC model along the lines of Woodfort (2003), which provides a good starting point to expose our main results. The second one is the more complex Smets-Wouters (2007) model, which has become very popular among central bankers and policy makers as a benchmark during the last decade. Our estimation results can be summarized as follows: The MS-AL models outperform the standard REE benchmark in all cases, and the also the regime-switching REE models in a majority of cases. Furthermore, we observe important differences in the impulse response and shock propagation structure of the models under consideration. For instance, a financial shock of the same size typically has a longer-lasting impact under adaptive learning, while a government spending shock may have a larger impact. These results suggest that benchmark REE models may severely underestimate the crisis duration, as well as underestimate the impact of a fiscal stimulus package.

The paper is organized as follows: Section (i) illustrates the main concepts in a simple framework with one-forward looking variable. Section (ii) shows the computation and E-stability results of the two classes of Restricted Perceptions Equilibria in DSGE models. Section (iii) provides the filter used in our estimations, while sections (iv) and (v) discuss the estimations results in the 3-equation NKPC and SW models respectively. Finally Section (vi) concludes.

2 Preliminaries: Fisherian Model of Inflation Determination

2.1 Rational Expectations Equilibria and Determinacy

To set the ideas, following Davig & Leeper (??), we first consider the simple model of Fisherian inflation determination without regime switching:

$$\begin{cases} i_t = E_t\pi_{t+1} + r_t \\ r_t = \rho r_{t-1} + v_t \\ i_t = \alpha\pi_t \end{cases}$$

where r_t is the exogenous AR(1) ex-ante real interest rate, i_t is the nominal interest rate and π_t is inflation. We assume monetary policy follows a simple rule by adjusting nominal interest rate to inflation, denoted by α[1]. We can re-write the model in terms of inflation as follows:

$$\begin{cases} \pi_t = \frac{1}{\alpha}(E_t\pi_{t+1} + r_t) \\ r_t = \rho r_{t-1} + v_t \end{cases}$$

In this case the standard MSV-solution takes the form $\pi_{i,t} = \beta_i r_t$, where the solution is pinned down by iterating this forward to obtain the one-step ahead expectations, plugging this back into the ALM and computing the underlying fixed point, which yields $\beta = \frac{1}{\alpha-\rho}$. Accordingly, the solution is given by $\pi_t = \frac{1}{\alpha-\rho}r_t$. In this benchmark case, the equilibrium is determinate if $\alpha > 1$, i.e. monetary policy is sufficiently aggressive.
Davig & Leeper (2007) consider the scenarios where interest rate coefficient α is subject to regime switches. Accordingly, assume that α changes stochastically between two regimes, $s_t = \{1, 2\}$ and transition matrix:

$$Q = \begin{pmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{pmatrix}$$

In this case inflation dynamics are given as:

$$\begin{cases} \pi_t = \frac{1}{\alpha(s_t)}(E_t\pi_{t+1} + r_t) \\ r_t = \rho r_{t-1} + v_t \end{cases}$$

Denoting $\pi_{i,t} = \pi_t(s_t = i)$, we can re-cast the model into a multivariate form:

$$\begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \pi_{1,t} \\ \pi_{2,t} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} E_t\pi_{1,t+1} \\ E_t\pi_{2,t+1} \end{bmatrix} + \begin{bmatrix} r_t \\ r_t \end{bmatrix}$$

Since expectations are regime-dependent by assumption, the corresponding PLM is regime-dependent:

$$\pi_{i,t} = \beta_i r_t$$

And therefore the 1-step ahead expectations are also regime-dependent:

Regime-contingent expectations:

$$\begin{cases} E_t\left[\pi_{t+1} \middle| s_t = 1\right] = (p_{11}a_1 + p_{12}a_2)\rho r_t \\ E_t\left[\pi_{t+1} \middle| s_t = 2\right] = (p_{21}a_1 + p_{22}a_2)\rho r_t \end{cases}$$

Davig & Leeper (??) show that, in this case, the equilibrium is determinate as long as the long-run Taylor principle condition is satisfied:

$$(1-\alpha_2)p_{11} + (1-\alpha_1)p_{22} + \alpha_1\alpha_2 > 1$$

.

A key insight of this principle is that, the long-run dynamics of the model can be determinate even if one of the underlying regimes is indeterminate, provided there is at least one regime

[1] For the remainder, we assume $Var(r_t) = 1$ to simplify the exposure.

that is sufficiently determinate or the probability of entering into the indeterminate regime is sufficiently small. In the following, we extend this insight into the learnability, i.e. the E-stability of equilibria.

2.2 Restricted Perceptions, E-stability and Least-Squares Learning

In this paper, we deviate from previous literature by assuming that agents are not aware of regime shifts; therefore their PLMs and the implied 1-step ahead expectations are not regime contingent:

$$\pi_t = \beta r_t \Rightarrow E_t \pi_{t+1} = \beta E_t r_{t+1} = a\rho r_t$$

The implied ALM is then given by:

$$\begin{cases} \pi_t = \frac{1}{\alpha(s_t)}(\beta\rho + 1) r_t \\ r_t = \rho r_{t-1} + v_t \end{cases}$$

Since the assumed PLM does not nest the regime-dependent MSV solution, it can never coincide with the underlying Rational Expectations Equilibrium. However, there is a non-rational equilibrium associated with the above PLM: this is commonly referred to as a Restricted Perceptions Equilibrium (RPE) in the adaptive learning literature. Unlike a REE, one cannot use the method of undetermined coefficients to pin down the value of β associated with an RPE. Instead, following (??), we can use an unconditional moment restriction as follows: the coefficient β determines the perceived correlation between inflation and real rate of interest in the PLM, i.e. $\beta = \frac{E[\pi_t r_t]}{E[r_t r_t]}$. In an equilibrium, the unconditional moment $\frac{E[\pi_t r_t]}{E[r_t r_t]}$ implied by the ALM should be equal to β. In an equilibrium as such, the agents make systematic mistakes to the extent that they do not use the statistically optimal forecasting rule. However, the RPE pins down the best forecasting rule given their class of PLMs. Computing the associated moment in ALM, we have:

$$\frac{E[\pi_t r_t]}{E[r_t r_t]} = E[\frac{1}{\alpha(s_t)}\beta\rho + \frac{1}{\alpha(s_t)}]$$

The unconditional moment in the expression above requires the ergodic distribution of the Markov chain denoted by q, which solves $q'Q = q$.

We get $q = [\frac{1-p_{22}}{2-p_{11}-p_{22}}, \frac{1-p_{11}}{2-p_{11}-p_{22}}]$.

This yields $\beta = \frac{1}{\sum_i q_i \alpha_i - \rho}$, and the solution reduces to:

$$\pi_t = \frac{1}{\sum_i q_i \alpha_i - \rho} r_t$$

Instead of the standard determinacy of Rational Expectations models, our main concept of interest in this case is E-stability. This governs whether the agents can learn the above fixed-point above by starting from an arbitrary point β_0, and updating their beliefs about the coefficient each period as new observations become available. As shown in Evans & Honkapohja (??), E-stability is governed by the mapping from agents' PLM to the implied ALM, defined as the T-map. In our example, the T-map is given by:

$$T : \beta \rightarrow \frac{\beta\rho + 1}{\sum_i q_i \alpha_i}$$

The T-map is locally stable if its Jacobian matrix has eigenvalues with real parts less than one. When this local stability condition is satisfied, the equilibrium is said to be E-stable. In our example, the Jacobian is given by:

$$\frac{DT(\beta)}{D(\beta)} = \frac{\rho}{\sum_i q_i \alpha_i}$$

. Hence the equilibrium is E-stable if the largest eigenvalue has real part less than one, i.e. $\frac{\rho}{\sum_i q_i \alpha_i} < 1$.

Assuming there are only two regimes, In general, in order to satisfy E-stability, we need a more aggressive monetary policy rule α_1 whenever:

-The average time spent in regime 1 (q_1) decreases, the average time spent in regime 2 (q_2) increases or the monetary policy rule in regime 2 (α_2) becomes less aggressive. An interesting case to consider is two regimes with one pegged interest rate rule, e.g $\alpha_2 \to 0$, in which case the E-stability condition reduces to:

$$\alpha_1 > \frac{\rho}{q_1}$$

These results suggest that, it is possible to have E-stability despite having an E-unstable regime, if there is a sufficiently E-stable regime and the model does not spend too much time in the unstable regime on average. This is intuitively an extension of Davig & Leeper's insight on long-run determinacy to the learnability of equilibria, therefore we call this the principle of long-run E-stability.

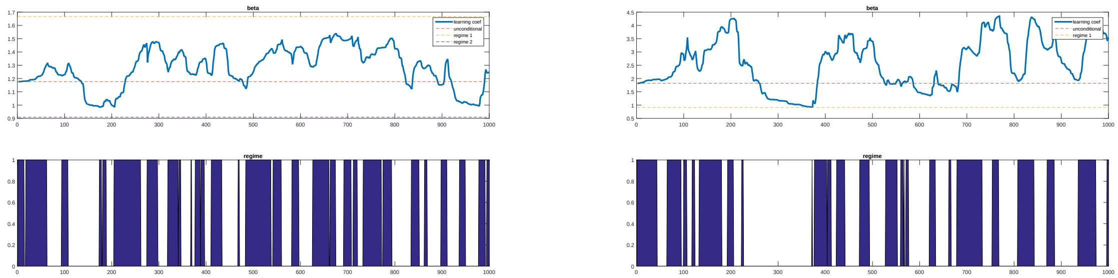
Agents' forecasts along the RPE imply systematic forecast errors: since they do not know the underlying regimes and only use a univariate rule, they essentially have a weighted average based on the ergodic distribution at the RPE, where they minimize the unconditional forecast errors. In general, we assume agents do not simply remain the RPE-consistent values but instead update their beliefs each period as new observations become available, using the baseline constant-gain least squares a la Evans & Honkapohja (??). Accordingly in our example, the coefficient β is updated as follows:

$$\begin{cases} R_t = R_{t-1} + \gamma(r_t^2 - R_{t-1}) \\ \beta_t = \beta_{t-1} + \gamma R_t^{-1} r_t(\pi_t - \beta_{t-1} r_t) \end{cases}$$

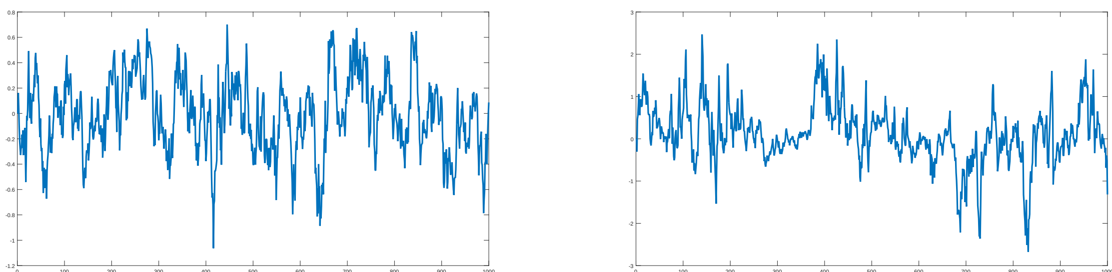
where γ denotes the constant gain value, i.e. the weight that agents put into the most recent observation. The learning scheme allows the agents to put more weight into recent observations, thereby giving them more flexibility. This means agents can update their beliefs along regime-specific directions as the unobserved regime keeps switching; but unconditionally the beliefs oscillate around the RPE-consistent value over time. Figure (??) shows two simulations of our model with the calibration $\rho = 0.9, p_{11} = 0.95, p_{22} = 0.95, \eta_\sigma = 0.1, \gamma = 0.05$. Under these parameter values, the ergodic distribution is given by $(q_1, q_2) = (0.5, 0.5)$, i.e the system spends an equal amount of time in each regime. We consider two scenarios in our simulations: In the first case, monetary policy switches between $\alpha_1 = 1.5$ and $\alpha_2 = 2$. In this case both underlying regimes are E-stable, and it can be readily seen that the coefficient moves towards the regime-specific value as it switches back and forth between the regimes; however in fluctuates around the RPE-consistent value unconditionally. In the second case we change the monetary policy coefficient of the first-regime to $\alpha_1 = 0.89$, effectively making the first regime E-unstable. In this case the learning coefficient moves in the regime-specific value while in the E-stable regime, while it can follow any erratic pattern while in the E-unstable regime. Unconditionally, however, in still fluctuates around the RPE-consistent value. Hence, the learning coefficient never settles on the RPE-consistent value as long as the gain parameter is sufficiently large, but the presence of the RPE guarantess that the time series does not become explosive once it enters into the E-unstable regime.

Figure 1: Left panel: Case (i) with two E-stable regimes. Right panel: Case (ii) with one E-stable and one E-unstable regime.

Learning coefficient and the underlying regime:



Inflation:



3 Restricted Perceptions Equilibria in MS-DSGE Models: General Case

Our simple example in the previous section illustrates well our idea of restricted perceptions in Markov-switching DSGE models, where we can analytically compute the Restricted Perceptions Equilibrium. In this section, we generalize our results to cases with multiple forward-looking variables, where the underlying equilibrium quickly becomes intractable. We consider two benchmark cases for the information set (i.e. the PLM) of the agent: the first one assumes that agents PLM coincides with the form of the underlying MSV-solution, except that the regime-shifts are not taken into account. In the second case, we consider the more general VAR(1)-type learning rules, which assumes observed shocks and the cross-correlations may be misspecified.

3.1 MSV-consistent RPE

In the above examples considered, lagged state variables do not enter into the model. In these cases, the underlying RPE takes a simple form that can be computed analytically. However, in empirically relevant models, lagged state variables typically enter into the model equation (via habit formation, price / wage indexation, interest rate smoothing, etc.). In this section we consider MSV-learning where the lagged state variables are included in the model. Consider the model of the form:

$$\begin{cases} X_t = A(s_t) + B(s_t)X_{t-1} + C(s_t)E_tX_{t+1} + D(s_t)\epsilon_t \\ \epsilon_t = \rho\epsilon_{t-1} + \eta_t \end{cases}$$

PLM takes the form of MSV solution but is regime-independent, which is given by (along with the one-step ahead expectations) :

$$\begin{cases} X_t = a + bX_{t-1} + d\epsilon_t \\ E_t X_{t+1} = (a + ba) + b^2 X_{t-1} + (bd + d\rho)\epsilon_t \end{cases}$$

which yields the implied ALM:

$$X_t = (A(s_t) + C(s_t)(a + ba)) + (C(s_t)b^2 + B(s_t))X_{t-1} + (C(s_t)(bd + d\rho) + D(s_t))\epsilon_t$$

For the remainder, we use the notation $\tilde{m} = \sum_i \pi_i m_i$ for the weighted average of any matrices m_i. Imposing the moment restrictions in this case yields:

$$\begin{cases} a = E[A(s_t) + C(s_t)(a + ba)] = \tilde{A} + \tilde{C}(a + ba) \\ b = E[C(s_t)b^2 + B(s_t)] = \tilde{C}b^2 + \tilde{B} \\ d = E[C(s_t)(bd + d\rho) + D(s_t)] = \tilde{C}(bd + d\rho) + \tilde{D} \end{cases}$$

In the above expression, a and d can be obtained for a given b. However, the second equation is quadratic in b. With n endogenous variables, this can in principle have up to $\binom{2n}{n}$ solutions in b (Check this!).

In this case, the T-map is given by:

$$\begin{pmatrix} a \\ b \\ d \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{A} + \tilde{C}(a + ba) \\ \tilde{C}b^2 + \tilde{B} \\ \tilde{C}(bd + d\rho) + \tilde{D} \end{pmatrix}$$

Denoting $\theta = (a, b, d)'$, the associated Jacobian is :

$$\frac{DT}{D\theta} = \begin{bmatrix} \tilde{C} + \tilde{C}b & vec^{-1}_{n,n}(a' \otimes \tilde{C}) & 0 \\ 0 & 2\tilde{C}b & 0 \\ 0 & vec^{-1}_{n,n}(d' \otimes \tilde{C}) & \tilde{C}b + vec^{-1}_{n,n}(\rho' \otimes \tilde{C}) \end{bmatrix}$$

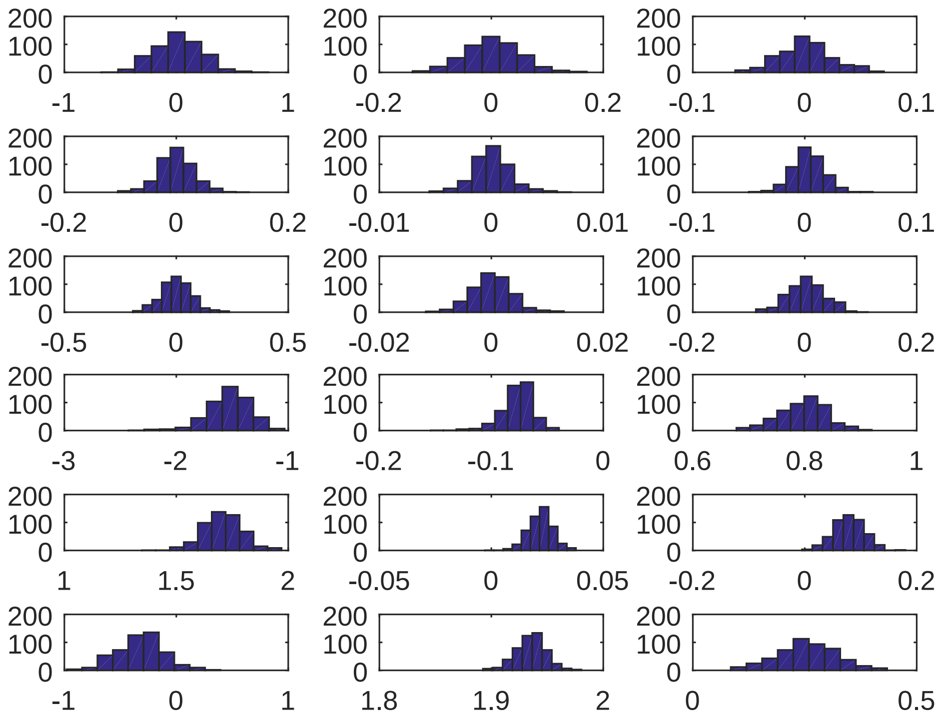
where $vec^{-1}_{n,n}$ denotes the matricization of a vector to an (n, n) matrix.

The eigenvalues of the Jacobian above are given by the terms on the diagonal. Hence the underlying RPE is E-stable if these eigenvalues have real parts less than one.

(these eigenvalues include the b term itself, i.e. b will converge to something if b satisfies a certain condition. Does this make sense? Cars' paper had something similar and the referees didn't like it, is it the same thing?)

Although we cannot obtain an explicit expression for the underlying equilibrium, we can still simulate the model and check whether the coefficients converge somewhere. The figure below provides the same exercise as in the previous section, where we add an interest rate smoothing coefficient of 0.9 to the interest rate rule in the first regime.

Figure 2: Distributions from 500 simulations of length 10000 for the two-regime NKPC. First row: intercept terms (should converge to zero). Second and third rows: lagged inflation and output gap coefficients (should converge to zero as they do not belong in the MSV-rule). Fourth row: Lagged interest rate: should converge to non-zero values given non-zero interest rate smoothing. Fifth and sixth rows: Coefficients on output gap and inflation shocks: should converge to non-zero values. Overall, we observe similar distributions compared to the switching (and also non-switching) case of previous section, indicating convergence towards an equilibrium.



3.2 VAR-consistent RPE

So far we considered MSV-type of learning, where the only source of misspecification in PLM comes from the fact that expectations are not regime-specific. In general, however, we can consider any type of misspecification in the PLM and there may exist an E-stable RPE associated with the given PLM. In this section, we extend our analysis to VAR(1)-type rules with unobserved shocks. Accordingly, consider again models of the form:

$$\begin{cases} X_t = A(s_t) + B(s_t)X_{t-1} + C(s_t)E_tX_{t+1} + D(s_t)\epsilon_t \\ \epsilon_t = \rho\epsilon_{t-1} + \eta_t \end{cases}$$

Since shocks are assumed to be unobserved, we can stack up the endogenous variables X_t and exogenous shocks ϵ_t into a vector $S_t = [X_t' \ \ \epsilon_t']'$ to obtain models of the form:

$$S_t = \gamma_0(s_t) + \gamma_1(s_t)S_{t-1} + \gamma_2(s_t)E_tS_{t+1} + \gamma_3(s_t)\eta_t$$

Assume the PLM, and the one-step ahead expectations take the form:

$$\begin{cases} S_t = a + bS_{t-1} + u_t \\ E_tS_{t+1} = (a + ba) + b^2S_{t-1} \end{cases}$$

Implied ALM is given by:

$$S_t = (\gamma_0(s_t) + \gamma_2(s_t)(a + ba)) + (\gamma_1(s_t) + \gamma_2(s_t)b^2)S_{t-1} + \gamma_3(s_t)\eta_t$$

In cases where the matrix b does not exactly coincide with the functional form of the MSV solution, the consistency requirements do not simplify.

$E[S_t] = (I - b)^{-1}a$ in PLM.

$E[S_t] = (I - \tilde{\gamma_1} - \tilde{\gamma_2}b^2)^{-1}(\tilde{\gamma_0} + \tilde{\gamma_2}(a + ba))$ in ALM.

The vector a in the expression above can be solved for a given b. Next we turn to the consistency requirement for the matrix b. Note that in PLM, we have :

$E[\tilde{S}_t\tilde{S}_t{}']^{-1}E[\tilde{S}_t\tilde{S}_{t-1}{}'] = b$, where $\tilde{S}_t = S_t - E[S_t]$. Hence we turn to computing these moments in ALM. Denoting by Γ_0 and Γ_1 the variance and autocovariance matrices respectively, the first two Yule-Walker equations of the ALM are given as :

$$\begin{cases} \Gamma_1 = \tilde{M}(b)\Gamma_0 \\ \Gamma_0 = \tilde{M}(b)\Gamma_0\tilde{M}(b)' + \tilde{\gamma_3}\Sigma_\eta\tilde{\gamma_3}' \end{cases}$$

where $\tilde{M}(b) = \tilde{\gamma_1} + \tilde{\gamma_2}b^2$. Solving the second expression above yields

$$vec(\Gamma_0) = (I - \tilde{M}(b) \otimes \tilde{M}(b))^{-1}(\tilde{\gamma_3} \otimes \tilde{\gamma_3})vec(\Sigma_\eta)$$

Hence for each term $b(i,j)$, we have $b(i,j) = \frac{vec(\Gamma_1)_{(j-1)N+j}}{vec(\Gamma_0)_{(j-1)N+j}} \Rightarrow b = \Gamma_1 \oslash \Gamma_0$.

In the special case where b takes the form of the corresponding MSV matrix on lagged endogenous terms, the solution for b reduces to the quadratic equation $b = \tilde{\gamma_1} + \tilde{\gamma_2}b^2$ as in the previous case.

The T-map in this case is given by the following:

$$T : \begin{pmatrix} a \\ b \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{\gamma_0} + \tilde{\gamma_2}(a + ba) \\ \tilde{\gamma_1} + \tilde{\gamma_2}b^2 \end{pmatrix}$$

Denoting $\theta = (a, b)$, the corresponding Jacobian is

$$\frac{DT}{D\theta} = \begin{bmatrix} \tilde{\gamma_2}(I + b) & vec^{-1}_{n,n}(a' \otimes \tilde{\gamma_2}) \\ 0 & 2b\tilde{\gamma_2} \end{bmatrix}$$

Hence the corresponding RPE will be E-stable if the eigenvalues $\tilde{\gamma_2}(I + b)$ and $2b\tilde{\gamma_2}$ have real parts less than one.

Bayesian Estimation of Markov-Switching DSGE Models under Adaptive Learning: Filtering Algorithm

We extend Kim & Nelson (1999) algorithm with least squares updating for expectations. In Markov-switching models with m regimes, there are m^t different timelines at each period t, which quickly make the standard Kalman filter algorithm intractable. The standard way of dealing with this issue is to "collapse" the state variables and covariance matrices at each iteration to reduce the number of timelines. In our approach, we carry only a single lag of the state variables. This means, if there are m regimes in the model, we carry m different timelines in each period. There are m^2 different sets of variables in the forecasting and updating steps of each iteration. These are collapse at the end of each iteration to reduce to m sets of variables. In order to introduce adaptive learning into this framework, we further collapse the m sets of variables into a single vector based on the filtered probabilities. This gives us the filtered states at each iteration. The expectations are then updated once, based on the single set of filtered states. These expectations are then used in each timeline in the following iterations. See Kim & Nelson (1999) for the details of Kalman and Hamilton Filter blocks.

State-space representation of the model, where S_t and y_t denote (unobserved) states and observable variables respectively:

$$\begin{cases} S_t = \gamma_2 + \gamma_1 S_{t-1} + \gamma_3 \epsilon_t, & , \epsilon_t \sim N(0,\sigma) \\ y_t = E + F S_t \end{cases}$$

0) Initial States :

$\tilde{S}^i_{0|0}, \tilde{P}^i_{0|0}, Pr[S_0 = i|\Phi_0], \Phi_0$ given.

1) Kalman Filter Block with the standard measurement and transition equations:

For $t = 1 : N$ For $\{S_{t-1} = i, S_t = j\}$

$$\begin{cases} S^{(i,j)}_{t|t-1} = \gamma^{(j)}_1 S^{(i)}_{t-1|t-1} + \gamma^{(j)}_2 \\ P^{(i,j)}_{t|t-1} = \gamma^{(j)}_1 P^{(i)}_{t-1|t-1} \gamma^{(j)}_1 + \gamma^{(j)}_3 \Sigma^{(j)} (\gamma^{(j)}_3)' \\ v^{(i,j)}_{t|t-1} = (y_t - F^{(j)} S^{(i,j)}_{t|t-1}) \\ Fe^{(i,j)} = F^{(j)} P^{(i,j)}_{t|t-1} F^{(j)} \\ S^{(i,j)}_{t|t} = S^{(i,j)}_{t|t-1} + P^{(i,j)}_{t|t-1} (F^{(j)})' (Fe^{(i,j)})^{-1} v^{(i,j)} \\ P^{(i,j)}_{t|t} = P^{(i,j)}_{t|t-1} (F^{(j)})' (Fe^{(i,j)})^{-1} F^{(j)} P^{(i,j)}_{t|t-1} \end{cases}$$

2) Hamilton Block for transition probabilities:

Denote:

$Pr[S_{t-1} = i, S_t = j|\Phi_{t-1}] = pp^{i,j}_{t|t-1}$,

$f(y_t|\Phi_{t-1})$,

$Pr[S_{t-1} = i, S_t = j|\Phi_t] = pp^{i,j}_{t|t}$,

$Pr[S_t = j|\Phi_t] = \tilde{pp}^j_{t|t}$

$$\begin{cases} pp^{(i,j)}_{t|t-1} = Q(i,j)pp^{(i)}_{t-1|t-1} \\ f(y_t|\Phi_{t-1}) = \sum_{j=1}^{M}\sum_{i=1}^{M} f(y_t|S_{t-1}=i, S_t=j, \Phi_{t-1})pp^{(i,j)}_{t|t-1} \\ pp^{(i,j)}_{t|t} = \frac{f(y_t|S_{t-1}=i,S_t=j,\Phi_{t-1})pp^{(i,j)}_{t|t-1}}{f(y_t|\Phi_{t-1})} \\ p^j_{t|t} = \sum_i^M pp^{(i,j)}_{t|t-1} \end{cases}$$

3) Collapsing to reduce the number of states from m^2 to m:

$$\begin{cases} S^{(i)}_{t|t} = \frac{\sum_{i=1}^{M} pp^{(i,j)}_{t|t} S^{(i,j)}_{t|t}}{p^{(j)}_{t|t}} \\ P^{(i)}_{t|t} = \frac{\sum_{i=1}^{M} pp^{(i,j)}_{tt}(P^{(i,j)}_{t|t} + (S^{(j)}_{t|t} - S^{(i,j)}_{t|t})(S^{(j)}_{t|t} - S^{(i,j)}_{t|t})')}{p^{(j)}_{t|t}} \end{cases}$$

4) Update expectations based on filtered states:

Filtered states:

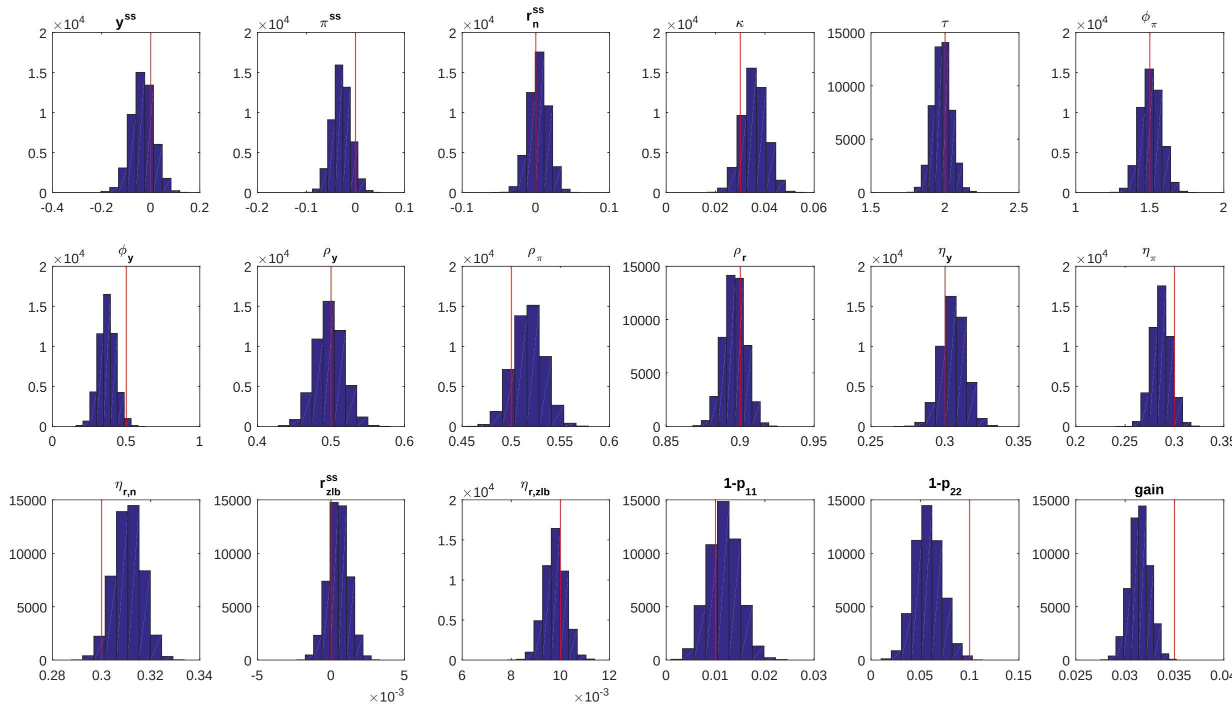
$$\tilde{S}_{t|t} = \sum_{j=1}^{M} p^{(j)}_{t|t} S^{(j)}_{t|t}$$

Expectations:

$$\begin{cases} \Phi_t = \Phi_{t-1} + \gamma R_t^{-1}\tilde{S}_{t-1|t-1}(\tilde{S}_{t|t} - \Phi^T_{t-1}\tilde{S}_{t-1|t-1})^T \\ R_t^{-1} = R_{t-1} + \gamma(\tilde{S}_{t-1|t-1}\tilde{S}^T_{t-1|t-1} - R_{t-1}) \end{cases}$$

NKPC Estimation-Monte Carlo Exercise

Before moving onto estimation with real historical data, we first check the performance of the filter on a short simulated dataset of 200 periods based on the 3-equation NKPC model. The length of the dataset is chosen to be close to our historical dataset of the U.S data over period 1966-I:2016:IV. The red lines denote the true parameter values of the simulation, accompanied by the resulting distributions of the Metropolis-Hastings MCMC with 100000 draws. We observe small biases with the exit probability of the ZLB regime, and the gain parameter of the expectations updating. However, the filter performs reasonably well overall, and all other parameters have the true parameter in their estimated 90 % HPD interval.



y^{ss}
π^{ss}
r_n^{ss}
κ
τ
ϕ_π
ϕ_y
ρ_y
ρ_π
ρ_r
η_y
η_π
$\eta_{r,n}$
r_{zlb}^{ss}
$\eta_{r,zlb}$
1-p$_{11}$
1-p$_{22}$
gain

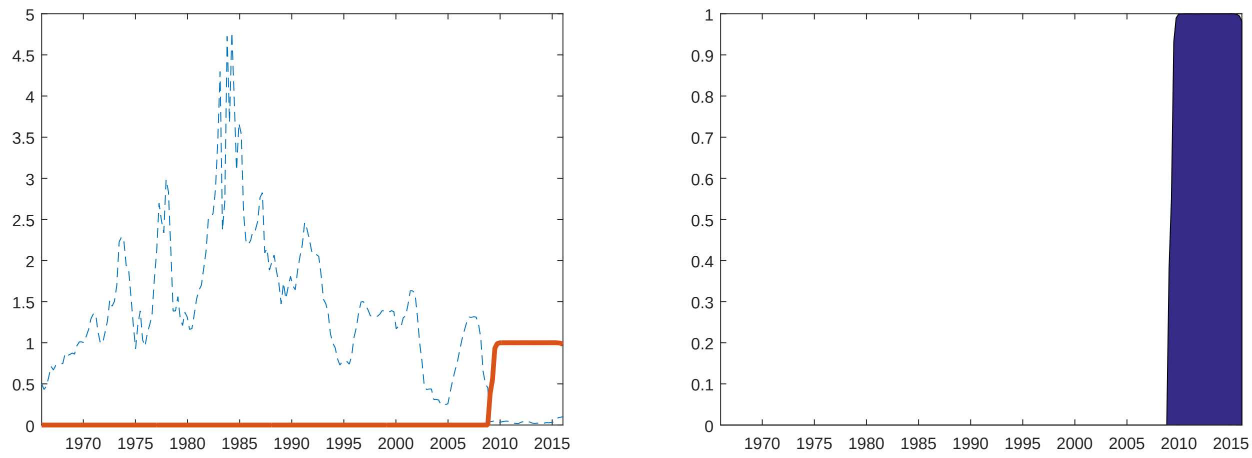
Estimating the Baseline NKPC: REE-Based initial beliefs

Table 1: Estimation Sample: 1966:I-2016:IV based on the U.S data, where the observables are the output gap (based on CBO's historical estimates), inflation and interest rate.The Markov-switching REE model is based on J. Maih's RISE toolbox, while the standard REE case is provided by Dynare. The AR(1), VAR(1) and MSV-learning cases are based on our algorithm above.

Parameter	Prior			Posterior AR(1)	VAR(1)	MSV	REE-MS	REE
	Dist	Mean	St. Dev	Mode	Mode	Mode	Mode	Mode
$\bar{y}$	Normal	0	0.25	-0.21	0.09	-0.32	-0.17	0.24
$\bar{\pi}$	Gamma	0.62	0.25	0.53	0.9	0.7	0.39	0.17
$\bar{r_1}$	Gamma	1	0.25	1.05	1.17	0.84	0.68	1.11
κ	Beta	0.3	0.15	0.033	0.018	0.005	0.004	0.006
τ	Gamma	2	0.5	2.54	3.01	3.02	2.75	4.57
ϕ_π	Gamma	1.5	0.25	1.25	1.3	1.56	1.56	1.42
ϕ_y	Gamma	0.5	0.25	0.59	0.51	0.45	0.27	0.27
ρ_y	Beta	0.5	0.2	0.33	0.48	0.89	0.92	0.93
ρ_π	Beta	0.5	0.2	0.04	0.07	0.85	0.92	0.89
ρ_r	Beta	0.5	0.2	0.96	0.96	0.9	0.8	0.8
η_y	Inv. Gamma	0.1	2	0.77	0.73	0.1	0.1	0.1
η_π	Inv. Gamma	0.1	2	0.26	0.28	0.03	0.03	0.04
η_{r_1}	Inv. Gamma	0.1	2	0.32	0.33	0.32	0.32	0.3
$\bar{r_2}$	Normal	0.1	0.25	0.04	0.0	0.03	0.03	-
η_{r_2}	Uniform	0.005	0.05	0.02	0.02	0.01	0.01	-
$1-p_{11}$	Beta	0.1	0.05	0.02	0.02	0.02	0.02	-
$1-p_{22}$	Beta	0.3	0.1	0.1	0.11	0.13	0.17	-
$gain$	Gamma	0.035	0.015	0.0392	0.009	0.0246	-	-
Laplace				-296.49	-305.37	-289.07	-317.02	-368.49

Figure 3: **AR(1)**

Interest rates and regime probability:



Expectation coefficients. Intercepts on the left, first-order autocorrelations on the right.

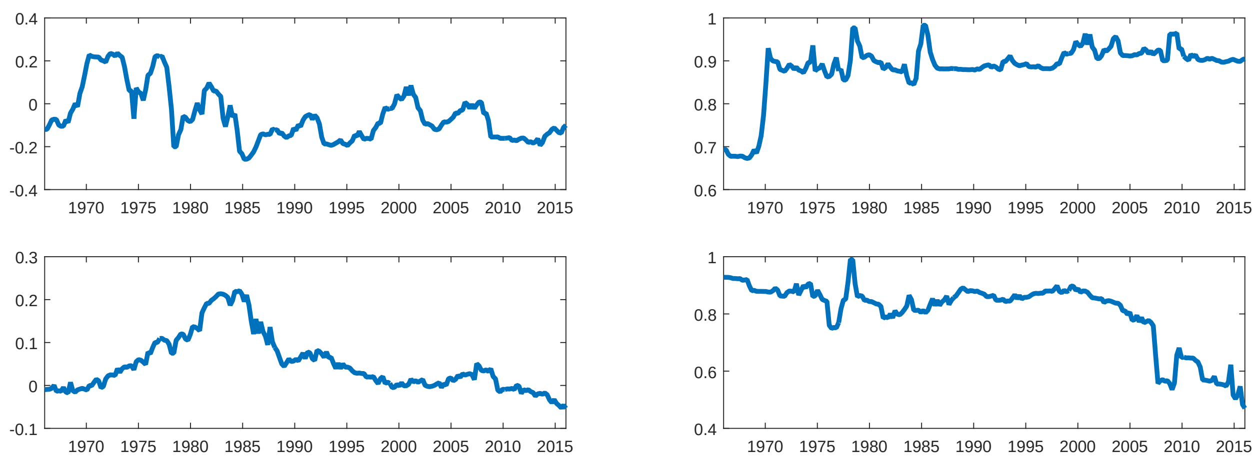
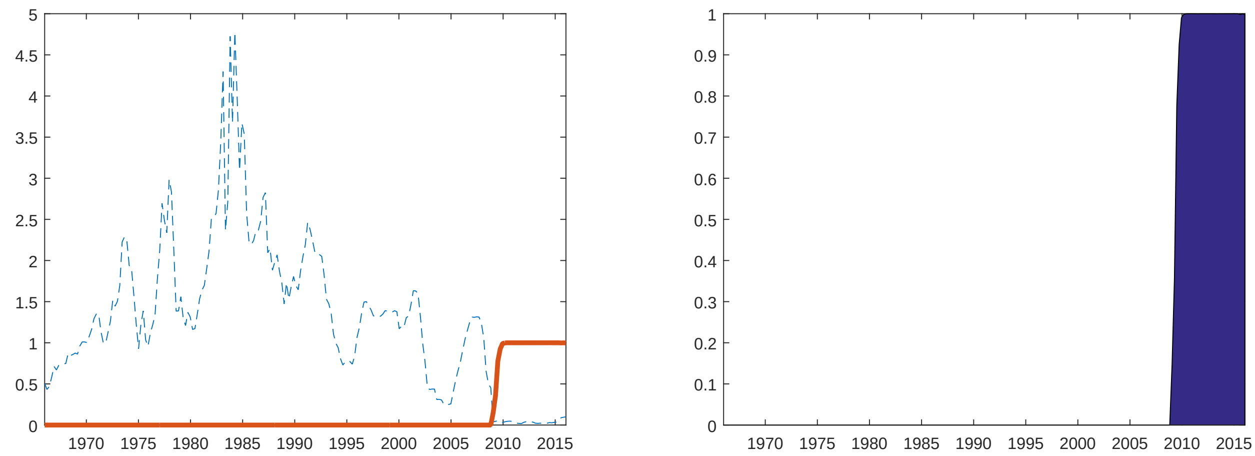


Figure 4: **VAR(1)**

Interest rates and regime probability:



Expectation coefficient. Intercepts on the left, first-order autocorrelations on the right.

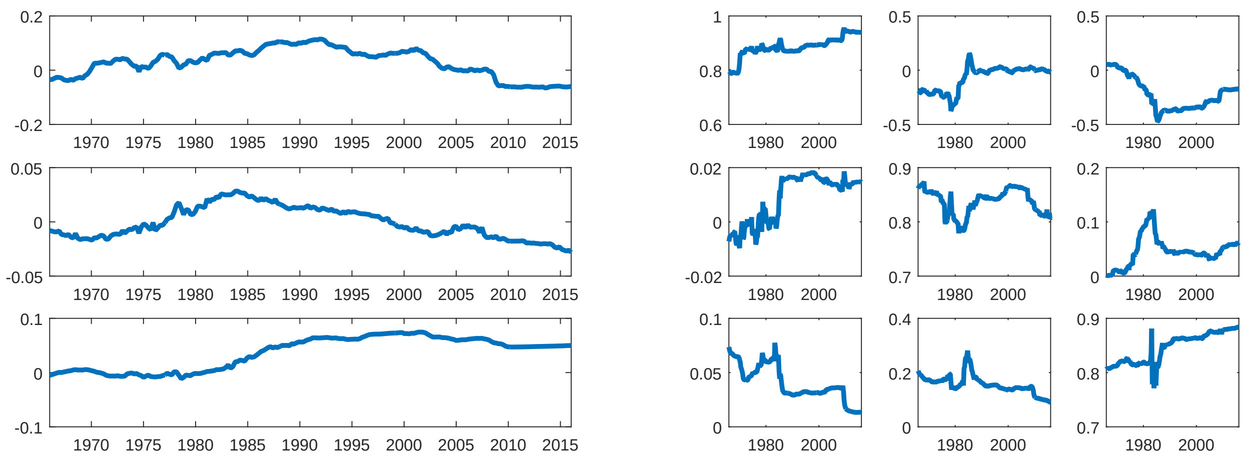
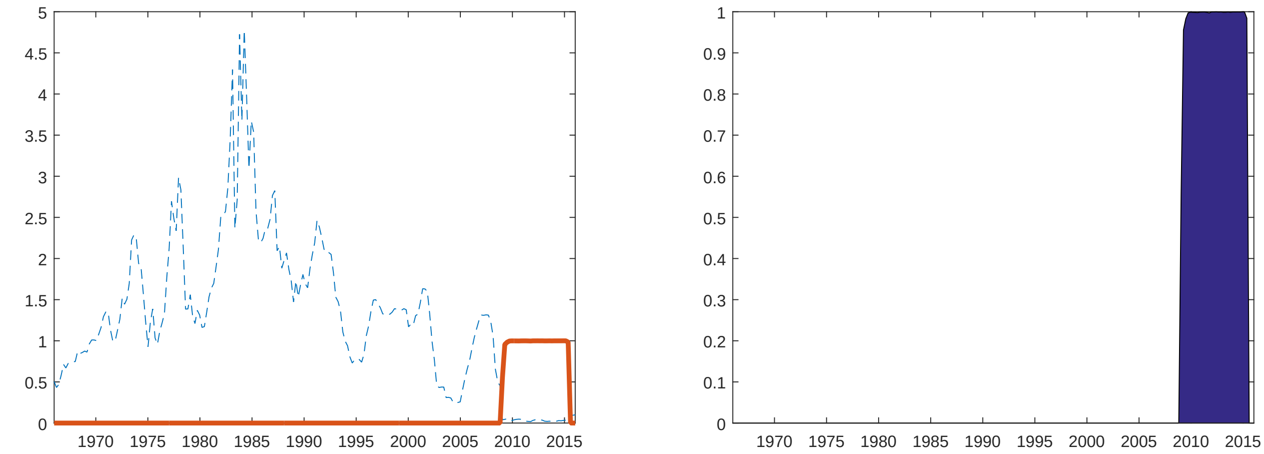
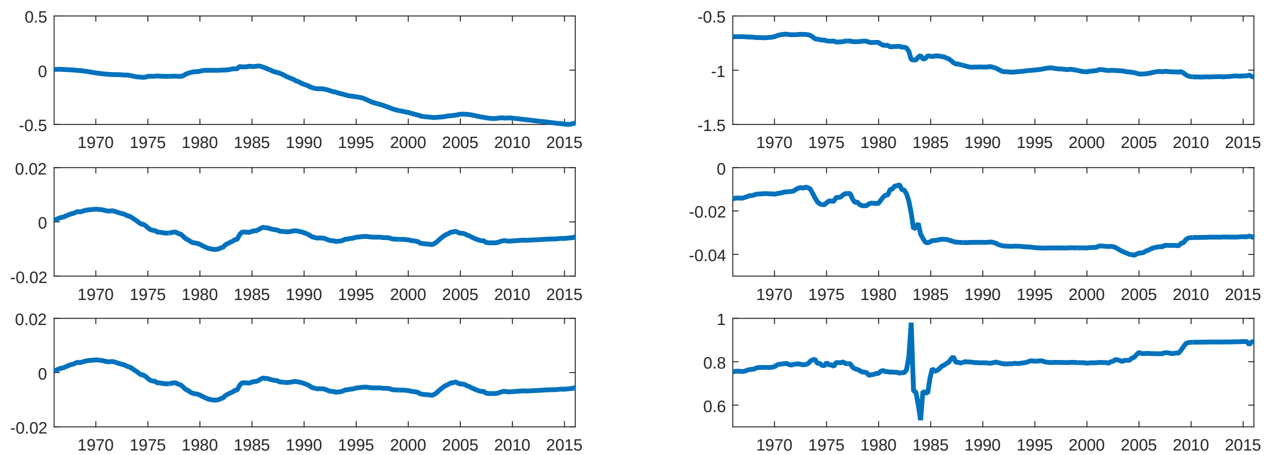


Figure 5: **MSV**

Interest rates and regime probability:



Expectation coefficients. Intercepts on the left, first-order autocorrelations on the right



Shock coefficients:

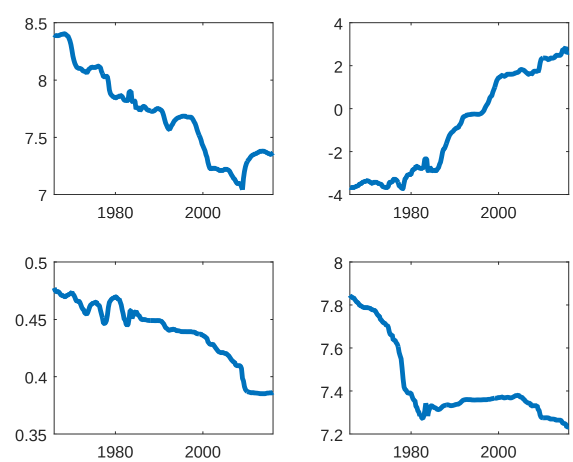
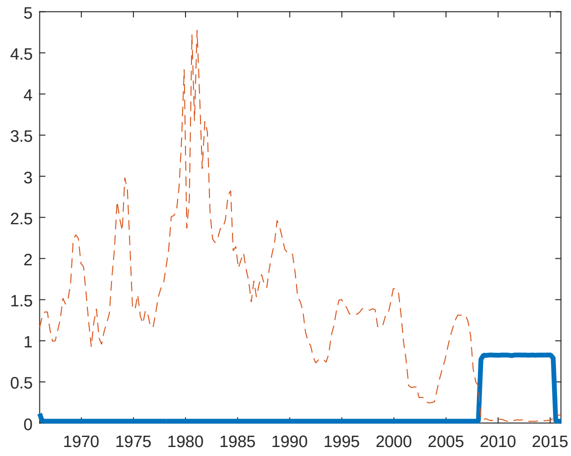
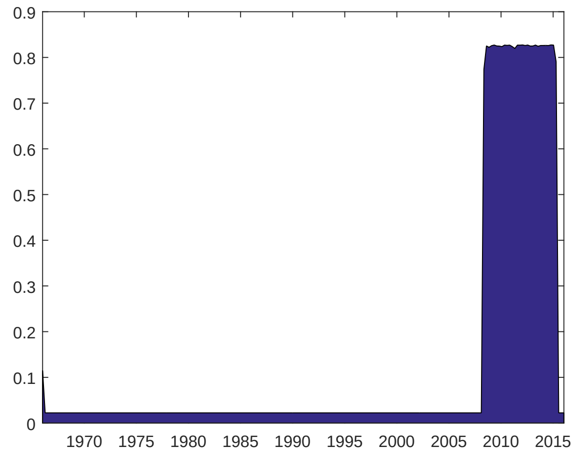


Figure 6: **MSV-REE**

Interest rates and regime probability:



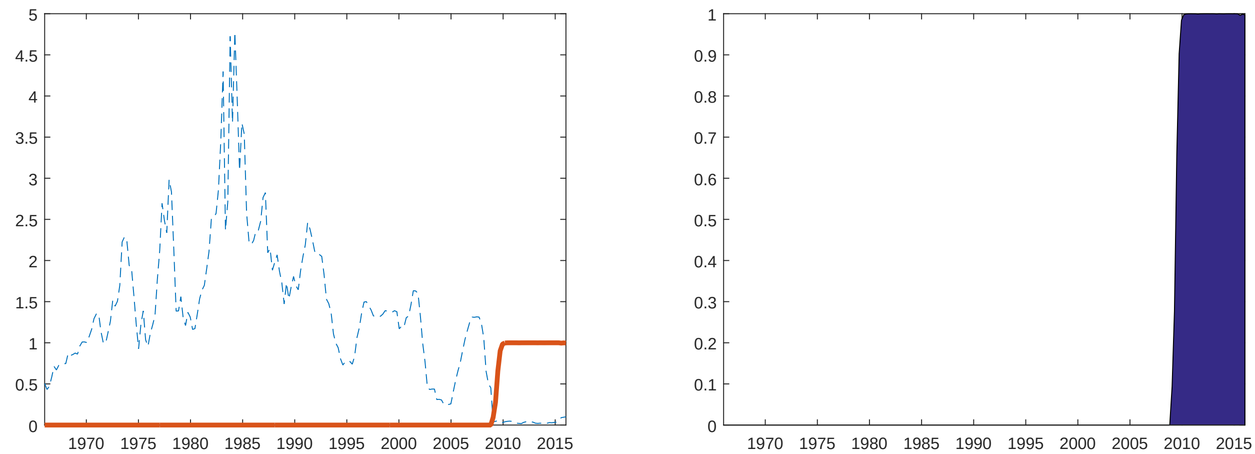


Estimating the Baseline NKPC: Optimized initial beliefs

Parameter	Prior			Posterior				
				AR(1)	VAR(1)	MSV	REE-MS	REE
	Dist	Mean	St. Dev	Mode	Mode	Mode	Mode	Mode
$\bar{y}$	Normal	0	0.25	-0.15	0.05	-0.35	-0.17	0.24
$\bar{\pi}$	Gamma	0.62	0.25	0.49	0.96	0.69	0.39	0.17
$\bar{r_1}$	Gamma	1	0.25	1.03	1.34	0.88	0.68	1.11
κ	Beta	0.3	0.15	0.03	0.03	0.038	0.004	0.006
τ	Gamma	2	0.5	2.52	3.11	2.6	4.73	4.57
ϕ_π	Gamma	1.5	0.25	1.25	1.32	1.55	1.56	1.42
ϕ_y	Gamma	0.5	0.25	0.59	0.36	0.37	0.27	0.27
ρ_y	Beta	0.5	0.2	0.3	0.39	0.91	0.92	0.93
ρ_π	Beta	0.5	0.2	0.05	0.05	0.81	0.92	0.89
ρ_r	Beta	0.5	0.2	0.95	0.97	0.89	0.8	0.8
η_y	Inv. Gamma	0.1	2	0.76	0.75	0.09	0.1	0.1
η_π	Inv. Gamma	0.1	2	0.26	0.26	0.04	0.03	0.04
η_{r_1}	Inv. Gamma	0.1	2	0.32	0.33	0.32	0.32	0.3
$\bar{r_2}$	Normal	0	0.25	0.04	0.04	0.03	0.03	-
η_{r_2}	Inv. Gamma	0.01	0.2	0.02	0.02	0.01	0.01	-
$1-p_{11}$	Beta	0.1	0.05	0.02	0.02	0.02	0.02	-
$1-p_{22}$	Beta	0.3	0.1	0.1	0.1	0.13	0.17	-
$gain$	Gamma	0.035	0.015	0.04	0.0382	0.0085	-	-
Laplace				-294.123	-308.92	-282.62	-317.02	-368.49

Figure 7: **AR(1)**

Interest rates and regime probability:



Expectation coefficients. Intercepts on the left, first-order autocorrelations on the right.

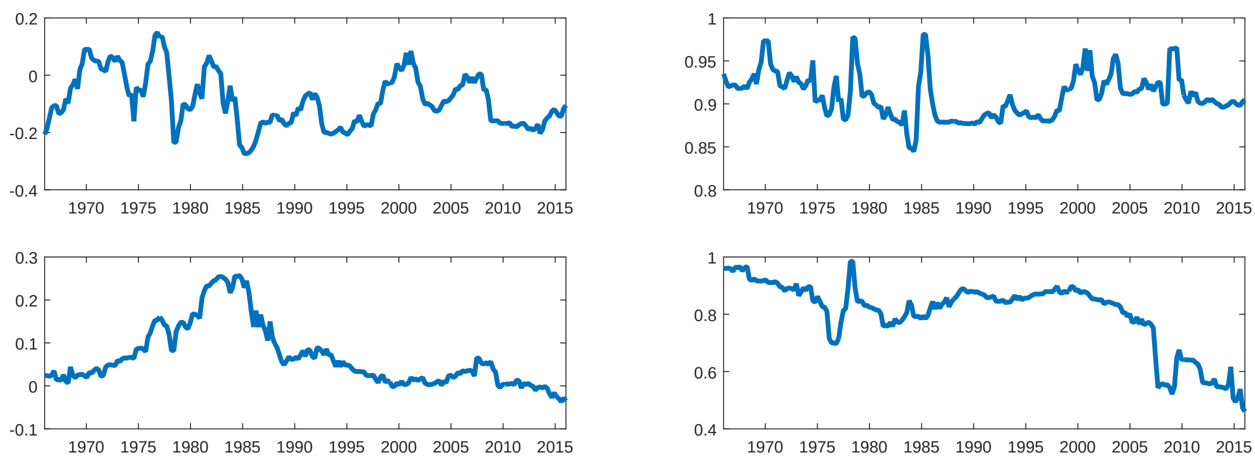
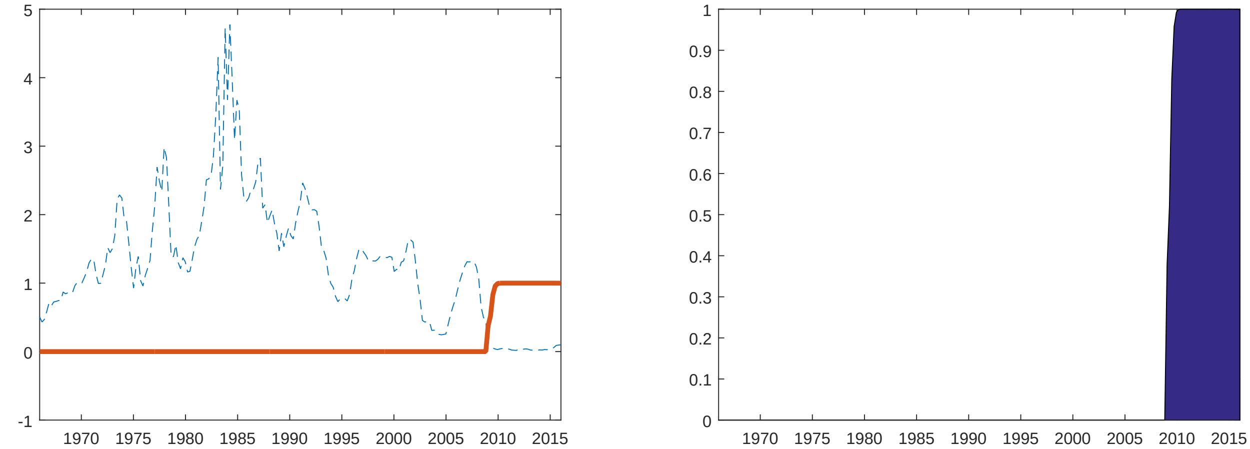


Figure 8: **VAR(1)**

Interest rates and regime probability:



Expectation coefficient. Intercepts on the left, first-order autocorrelations on the right.

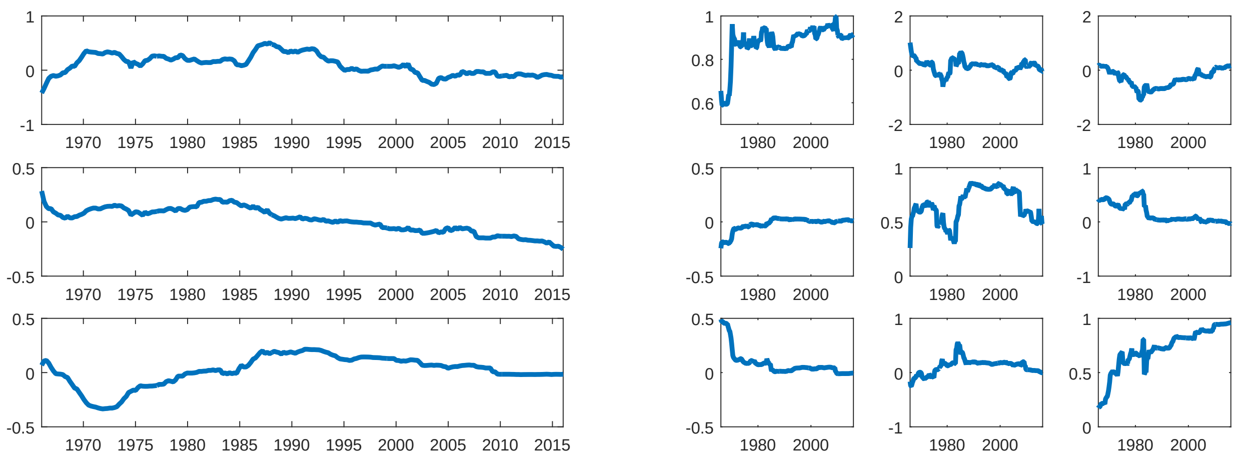
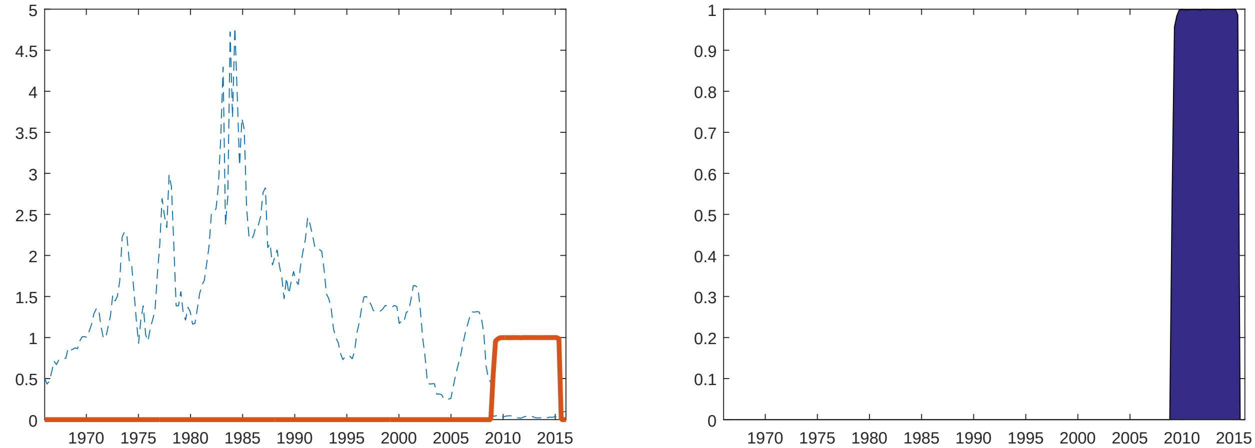
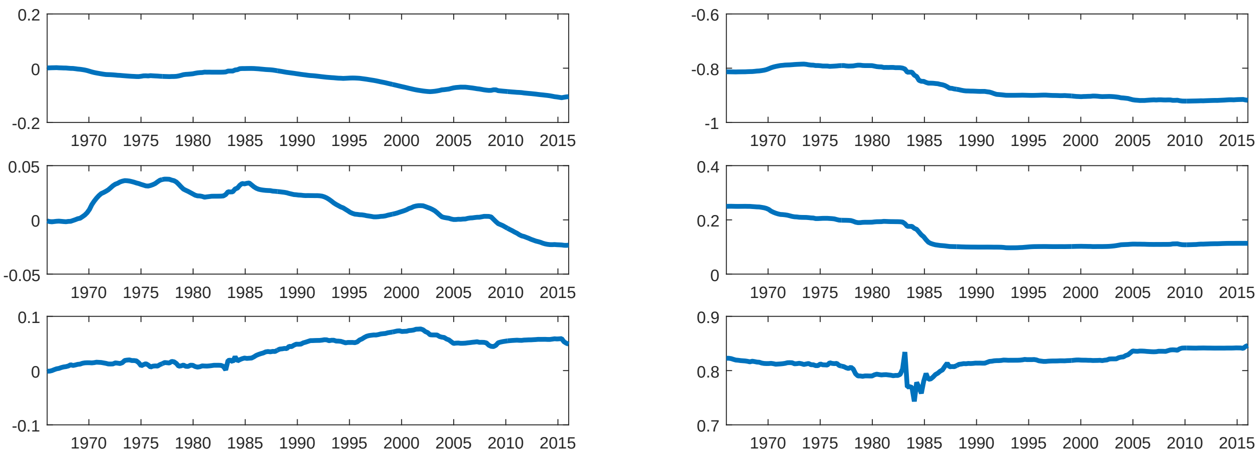


Figure 9: **MSV**

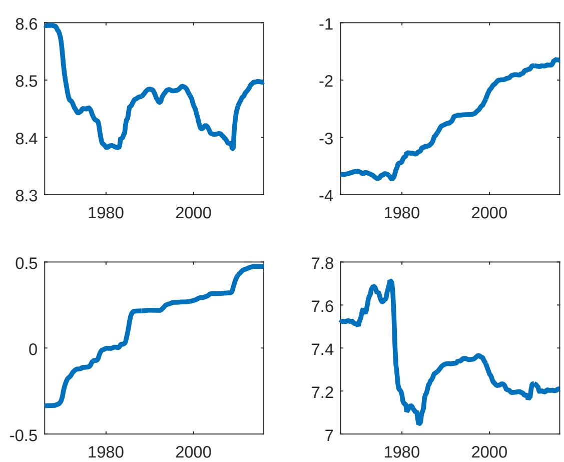
Interest rates and regime probability:



Expectation coefficients. Intercepts on the left, first-order autocorrelations on the right



Shock coefficients:

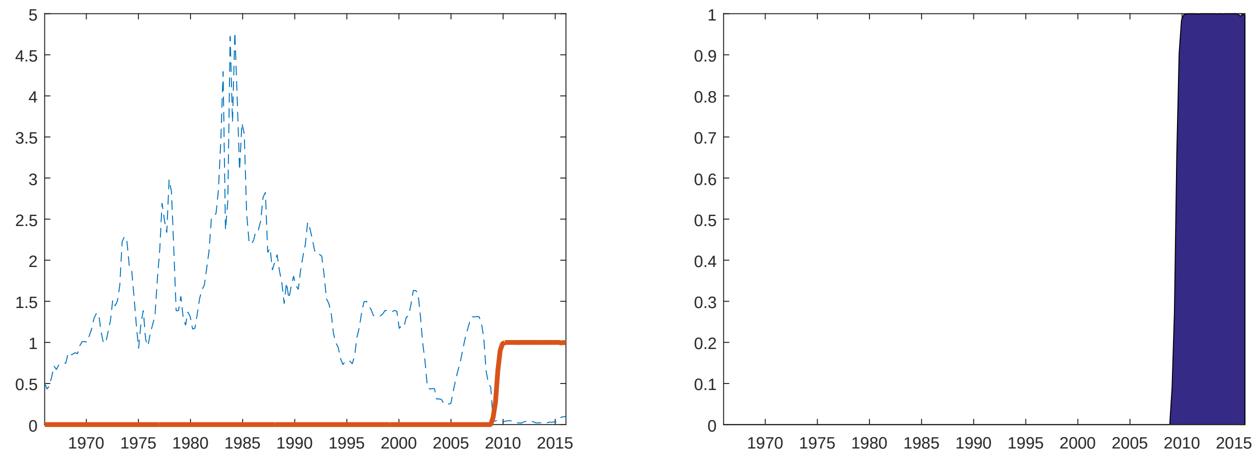


Estimating the Baseline NKPC: Filter-based initial beliefs

Parameter	Prior			Posterior				
				AR(1)	VAR(1)	MSV	REE-MS	REE
	Dist	Mean	St. Dev	Mode	Mode	Mode	Mode	
$\bar{y}$	Normal	0	0.25	-0.16	-0.01	-0.26	-0.17	0.24
$\bar{\pi}$	Gamma	0.62	0.25	0.47	0.46	0.54	0.39	0.17
$\bar{r_1}$	Gamma	1	0.25	0.99	1.03	0.86	0.68	1.11
κ	Beta	0.3	0.15	0.03	0.0081	0.006	0.004	0.006
τ	Gamma	2	0.5	2.51	2.92	2.79	4.73	4.57
ϕ_π	Gamma	1.5	0.25	1.25	1.26	1.52	1.56	1.42
ϕ_y	Gamma	0.5	0.25	0.59	0.56	0.33	0.27	0.27
ρ_y	Beta	0.5	0.2	0.3	0.52	0.92	0.92	0.93
ρ_π	Beta	0.5	0.2	0.05	0.09	0.9	0.92	0.89
ρ_r	Beta	0.5	0.2	0.95	0.96	0.87	0.8	0.8
η_y	Inv. Gamma	0.1	2	0.76	0.74	0.12	0.1	0.1
η_π	Inv. Gamma	0.1	2	0.26	0.27	0.04	0.03	0.04
η_{r_1}	Inv. Gamma	0.1	2	0.32	0.32	0.32	0.32	0.3
$\bar{r_2}$	Normal	0	0.25	0.04	0.04	0.03	0.03	-
η_{r_2}	Inv. Gamma	0.01	0.2	0.02	0.02	0.01	0.01	-
$1-p_{11}$	Beta	0.1	0.05	0.02	0.02	0.02	0.02	-
$1-p_{22}$	Beta	0.3	0.1	0.1	0.1	0.13	0.17	-
$gain$	Gamma	0.035	0.015	0.0421	0.0064	0.0075	-	-
Laplace				-294.24	-307.34	-290	-317.02	-368.49

Figure 10: **AR(1)**

Interest rates and regime probability:



Expectation coefficients. Intercepts on the left, first-order autocorrelations on the right.

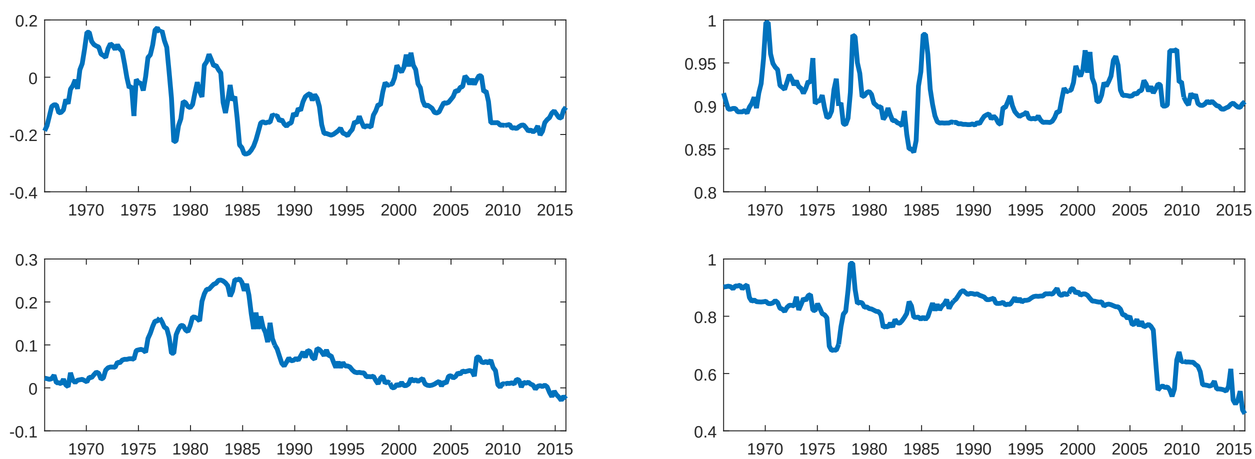
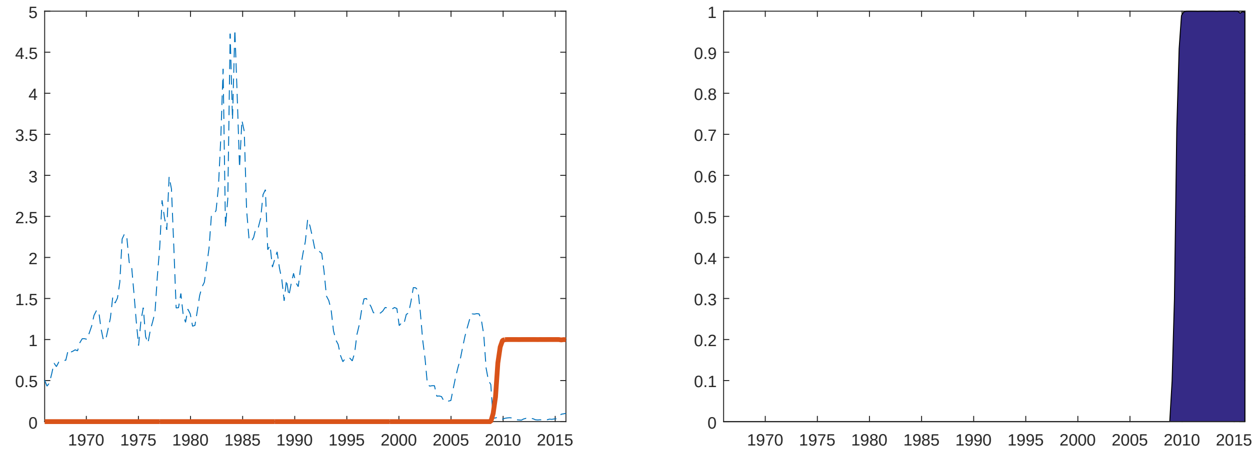


Figure 11: **VAR(1)**

Interest rates and regime probability:



Expectation coefficient. Intercepts on the left, first-order autocorrelations on the right.

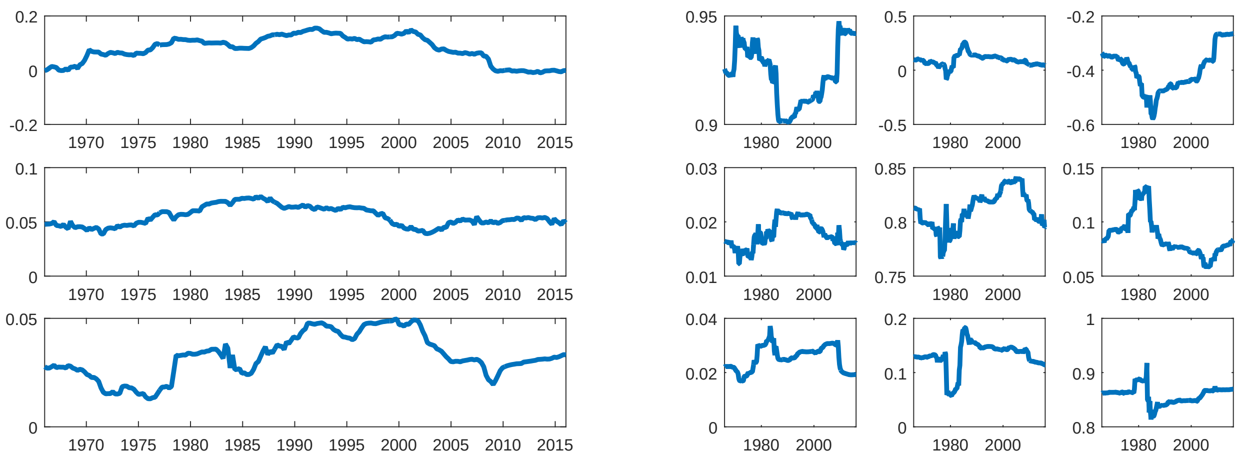
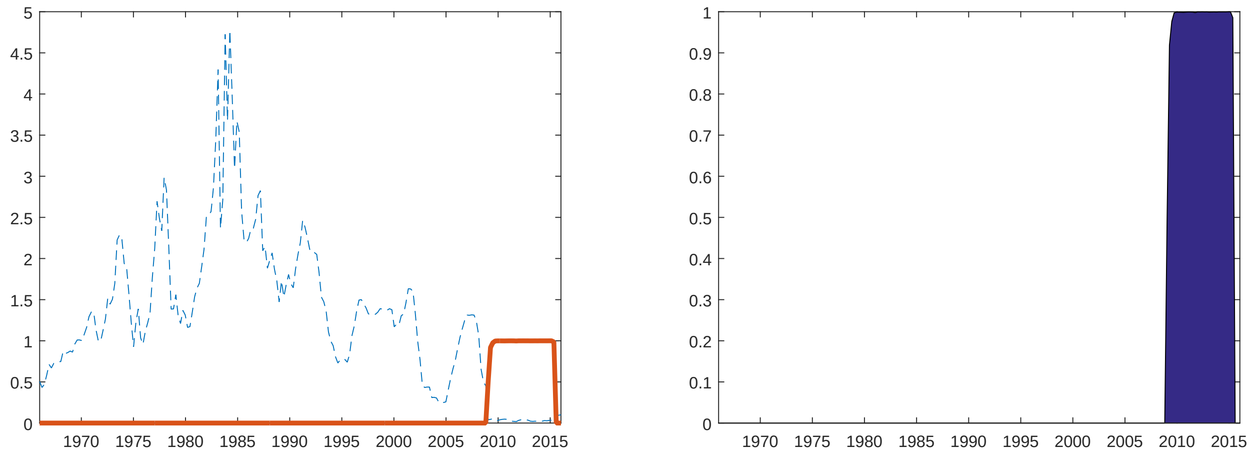
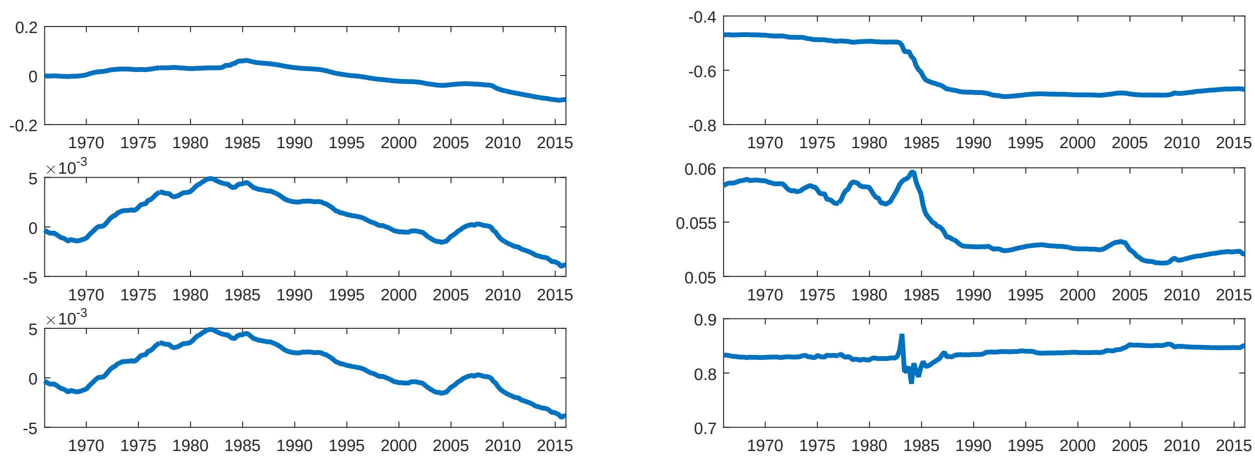


Figure 12: **MSV**

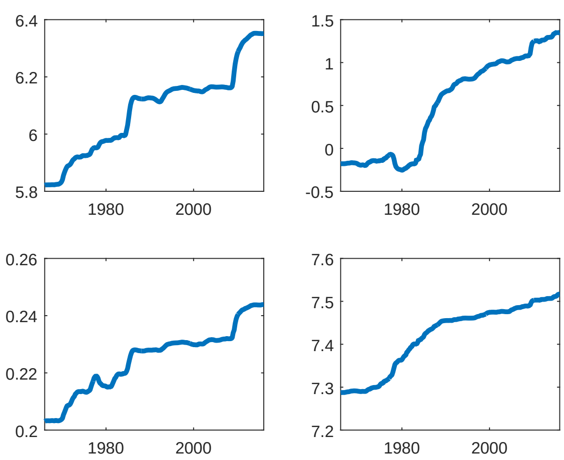
Interest rates and regime probability:



Expectation coefficients. Intercepts on the left, first-order autocorrelations on the right



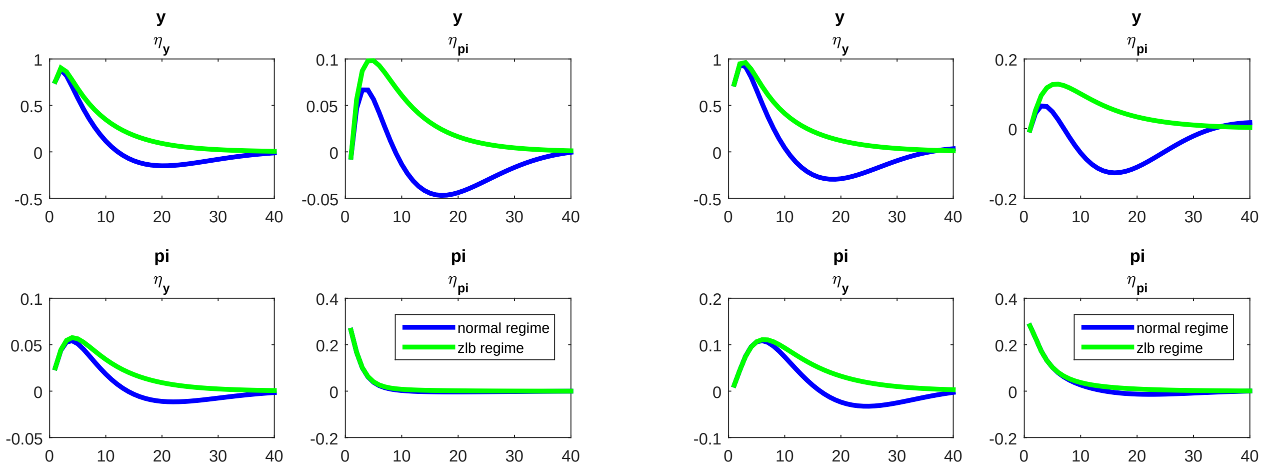
Shock coefficients:



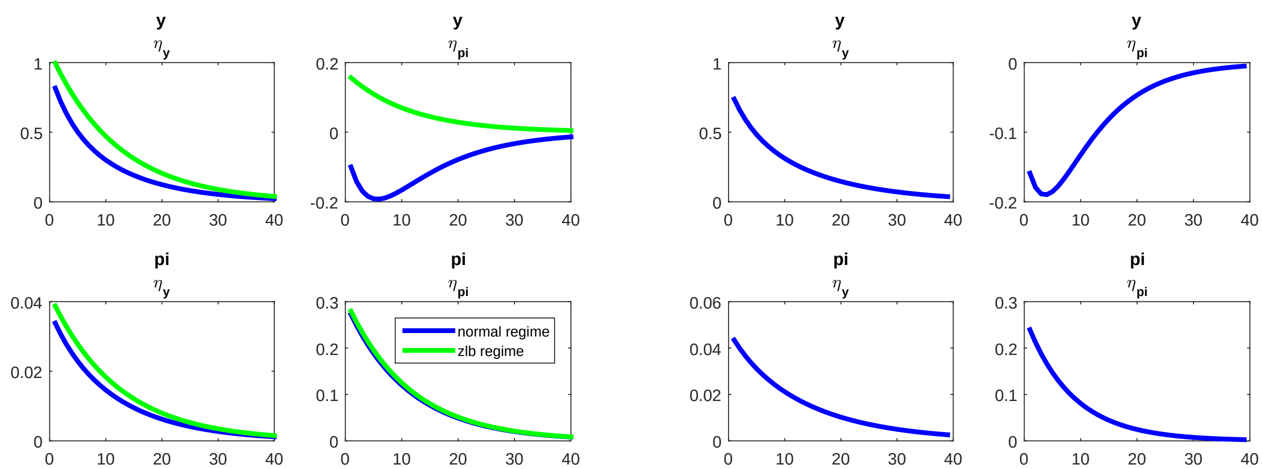
Impulse Responses (using the REE-based initial beliefs setup)

Figure 13: Impulse responses of time-varying PLMs are based on an arbitrary period.

AR(1) and VAR(1) beliefs:



MS and standard REE benchmarks:



MSV beliefs:

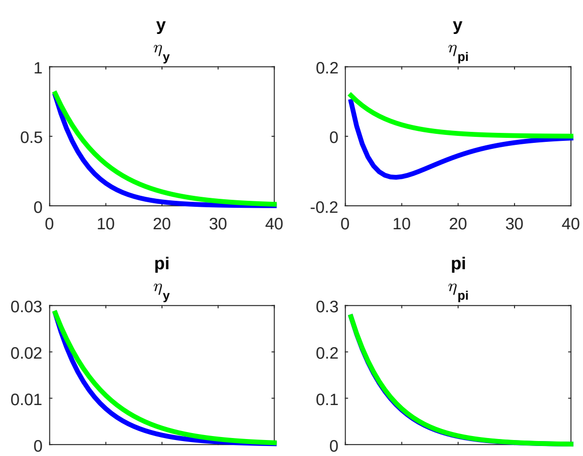


Figure 14: Time-varying impulse responses, final 50 periods of the estimation sample. **AR(1)**:

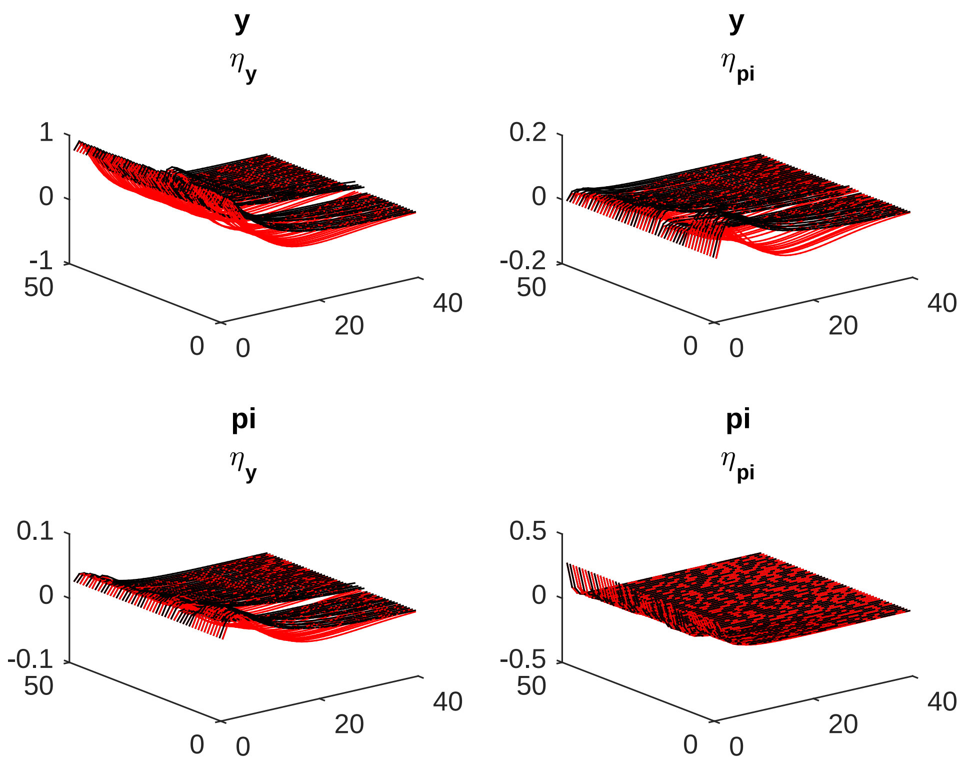


Figure 15: Time-varying impulse responses, final 50 periods of the estimation sample. **VAR(1)**:

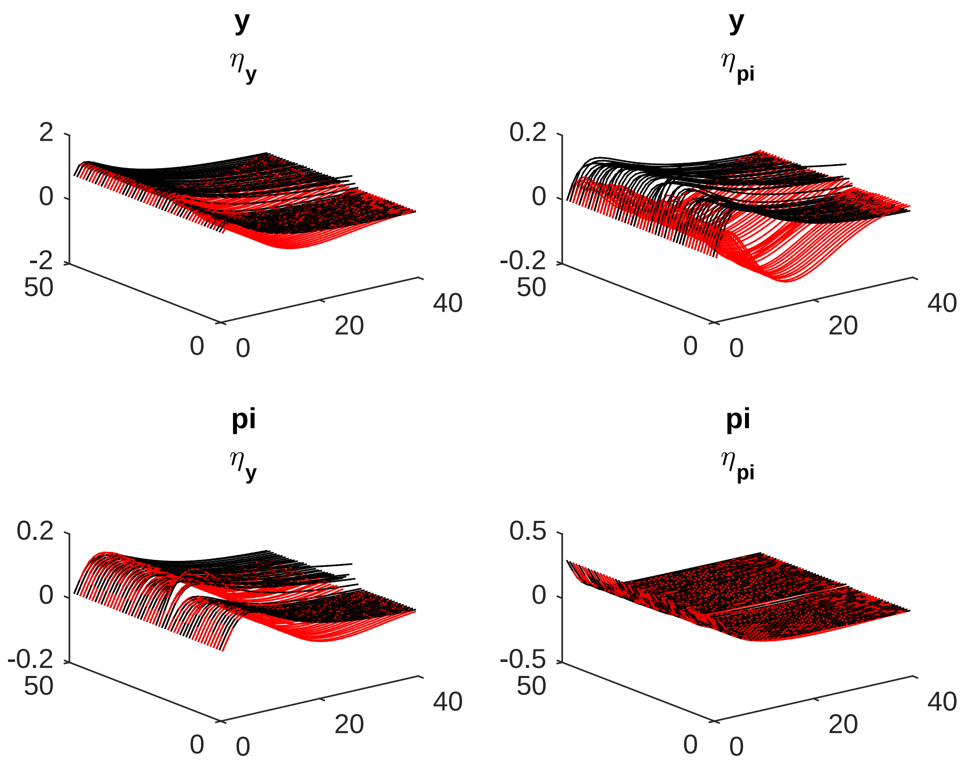
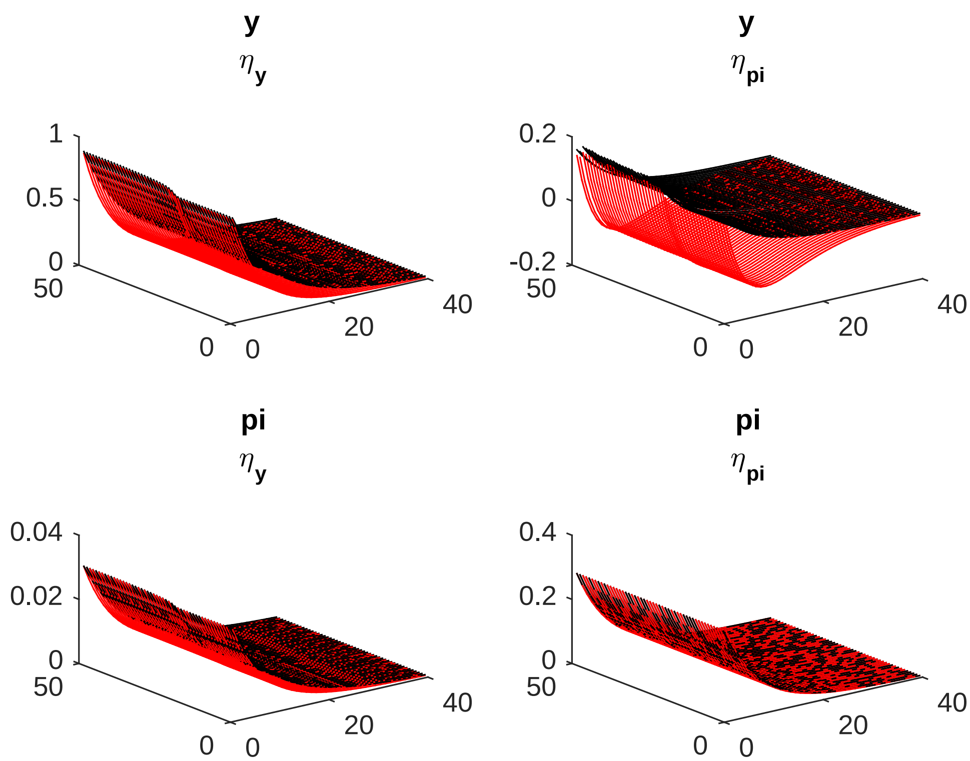


Figure 16: Time-varying impulse responses, final 50 periods of the estimation sample. **MSV**:



SW Estimation-I

Table 2: Estimation period: 1966:I-2016:IV

Prior			AR(1)	MSV	REE	RISE
Param	Dist	Mean	Mode	Mode	Mode	Mode
ϕ	Normal	4	1.02	3.75	4.87	6.4
σ_c	Normal	1.5	0.88	1.48	1.36	1.14
λ	Beta	0.7	0.55	0.73	0.75	0.83
ξ_w	Beta	0.5	0.7	0.75	0.93	0.95
σ_l	Normal	2	2.41	2.18	1.98	1.74
ξ_p	Beta	0.5	0.62	0.67	0.8	0.83
ι_w	Beta	0.5	0.43	0.63	0.84	0.81
ι_p	Beta	0.5	0.40	0.23	0.07	0.08
ψ	Beta	0.5	0.49	0.69	0.83	0.69
ϕ_p	Normal	1.25	1.49	1.57	1.59	1.56
r_π	Normal	1.25	1.64	1.62	1.5	1.35
ρ	Beta	0.75	0.88	0.88	0.85	0.86
r_y	Normal	0.125	0.13	0.12	0.05	0.06
r_{dy}	Normal	0.125	0.14	0.14	0.17	0.19
$\bar{\pi}$	Gamma	0.625	0.71	0.86	0.75	0.76
$\bar{\beta}$	Gamma	0.25	0.14	0.19	0.21	0.25
$\bar{l}$	Normal	0	0.73	1.59	-1.2	0.15
$\bar{\gamma}$	Normal	0.4	0.4	0.43	0.4	0.41
α	Normal	0.3	0.15	0.16	0.17	0.18
ρ_a	Beta	0.5	0.98	0.95	0.96	0.95
ρ_b	Beta	0.5	0.35	0.42	0.36	0.29
ρ_g	Beta	0.5	0.99	0.99	0.98	0.98
ρ_i	Beta	0.5	0.48	0.83	0.83	0.76
ρ_r	Beta	0.5	0.15	0.07	0.08	0.16
ρ_p	Beta	0.5	0.03	0.64	0.81	0.78
ρ_w	Beta	0.5	0.04	0.19	0.06	0.05
ρ_{ga}	Beta	0.5	0.52	0.5	0.53	0.51
η_a	Inv. Gamma	0.1	0.46	0.43	0.44	0.45
η_b	Inv. Gamma	0.1	0.68	0.21	0.21	0.23
η_g	Inv. Gamma	0.1	0.48	0.48	0.49	0.48
η_i	Inv. Gamma	0.1	1.36	0.36	0.36	0.34
$\eta_{r,N}$	Inv. Gamma	0.1	0.21	0.21	0.21	0.23
$\eta_{r,ZLB}$	Gamma	0.03	0.01	0.01	-	0.01
η_p	Inv. Gamma	0.1	0.27	0.07	0.05	0.06
η_w	Inv. Gamma	0.1	0.73	0.35	0.37	0.37
$gain$	Gamma	0.035	0.012	0.001	-	-
$1-p_{11}$	Beta	0.1	0.02	0.02	-	0.01
$1-p_{22}$	Beta	0.1	0.13	0.13	-	0.29
$r\bar{}_{zlb}$	Normal	0.05	0.03	0.03	-	0.03
Laplace			-1135	-1168	-1213	-1171

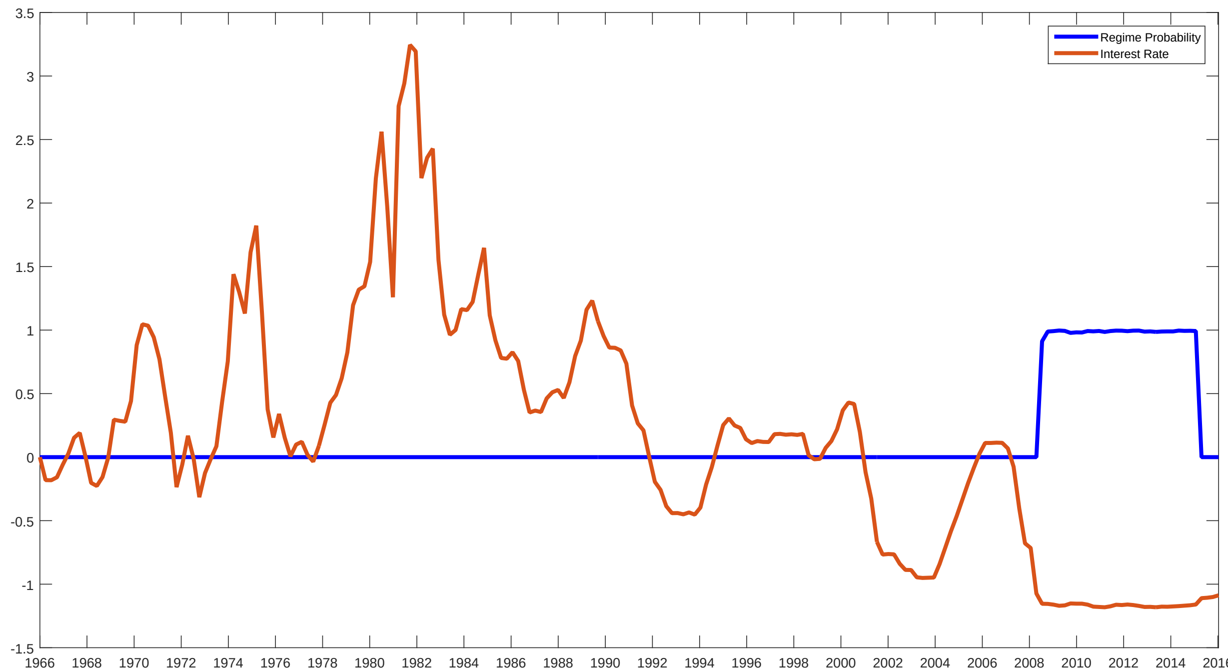
SW Estimations-II

Table 3: Estimation period: 1985:I-2016:IV

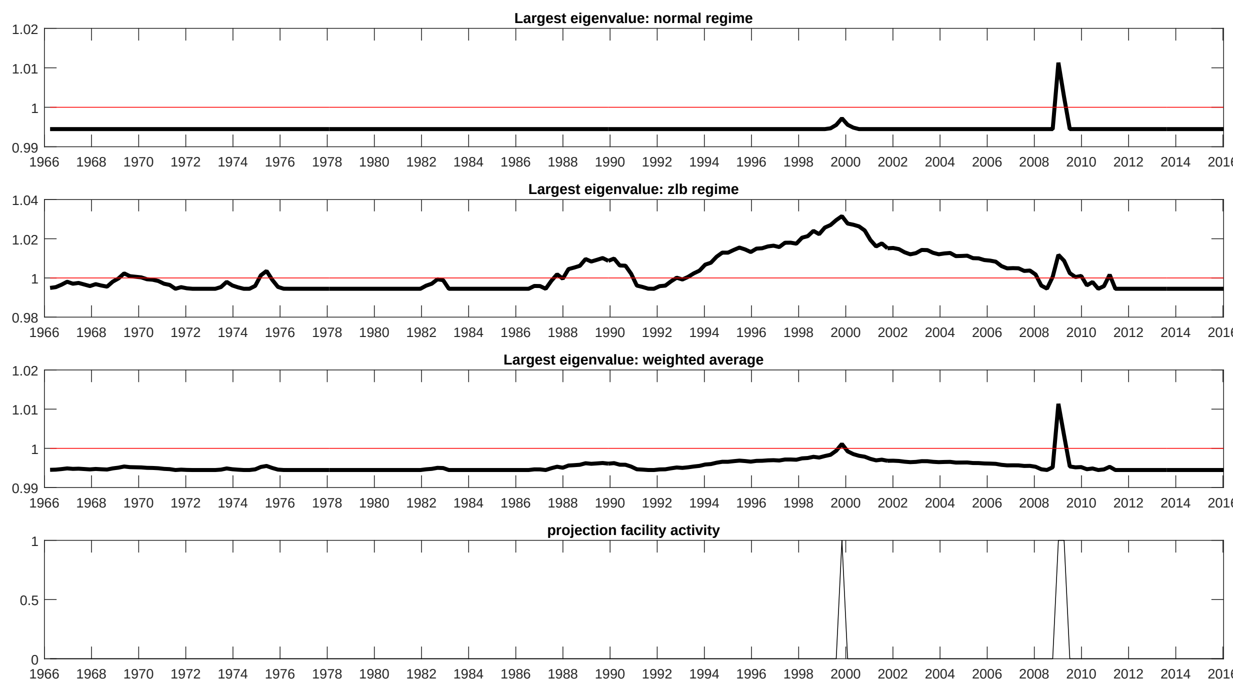
Prior			AR(1)	MSV	REE	RISE	
Param	Dist	Mean	Mode	Mode	Mode	Mode	
ϕ	Normal	4	0.86	5	6.72	4.91	
σ_c	Normal	1.5	0.35	0.82	1.93	1.12	
λ	Beta	0.7	0.6	0.82	0.7	0.55	
ξ_w	Beta	0.5	0.63	0.69	0.8	0.84	
σ_l	Normal	2	2.14	2.52	1.08	1.65	
ξ_p	Beta	0.5	0.77	0.76	0.83	0.92	
ι_w	Beta	0.5	0.48	0.59	0.5	0.46	
ι_p	Beta	0.5	0.27	0.42	0.13	0.19	
ψ	Beta	0.5	0.66	0.82	0.88	0.8	
ϕ_p	Normal	1.25	1.31	1.48	1.51	1.4	
r_π	Normal	1.25	1.42	1.49	1.63	1.48	
ρ	Beta	0.75	0.91	0.9	0.87	0.86	
r_y	Normal	0.125	0.17	0.07	0.07	0.24	
r_{dy}	Normal	0.125	0.11	0.07	0.07	0.09	
$\bar{\pi}$	Gamma	0.625	0.68	0.61	0.67	0.61	
$\bar{\beta}$	Gamma	0.25	0.2	0.27	0.21	0.25	
$\bar{l}$	Normal	0	3.03	5.69	1.24	3.82	
$\bar{\gamma}$	Normal	0.4	0.45	0.5	0.34	0.47	
α	Normal	0.3	0.11	0.15	0.17	0.15	
ρ_a	Beta	0.5	0.99	0.99	0.99	0.98	
ρ_b	Beta	0.5	0.60	0.58	0.34	0.96	
ρ_g	Beta	0.5	0.99	0.99	0.95	0.98	
ρ_i	Beta	0.5	0.40	0.79	0.85	0.73	
ρ_r	Beta	0.5	0.66	0.52	0.46	0.48	
ρ_p	Beta	0.5	0.03	0.61	0.55	0.34	
ρ_w	Beta	0.5	0.07	0.19	0.09	0.09	
ρ_{ga}	Beta	0.5	0.39	0.4	0.45	0.4	
η_a	Inv. Gamma	0.1	0.4	0.38	0.39	0.05	
η_b	Inv. Gamma	0.1	0.8	0.16	0.17	0.37	
η_g	Inv. Gamma	0.1	0.38	0.39	0.4	0.3	
η_i	Inv. Gamma	0.1	1.15	0.31	0.29	0.09	
$\eta_{r,N}$	Inv. Gamma	0.1	0.09	0.08	0.08	0.09	
$\eta_{r,ZLB}$	Gamma	0.03	0.01	0.01	-	-	
η_p	Inv. Gamma	0.1	0.18	0.07	0.08	0.11	
η_w	Inv. Gamma	0.1	0.86	0.4	0.43	0.42	
$gain$	Gamma	0.035	0.013	0.008	-	-	
$1-p_{11}$	Beta	0.1	0.03	0.03	-	0.01	
$1-p_{22}$	Beta	0.1	0.13	0.13	-	0.28	
$\bar{r_{zlb}}$	Normal	0.05	0.04	0.03	-	-	-
Laplace			-515	-566	-583	-552	

Figures-AR(1) learning (Based on Full Sample Results)

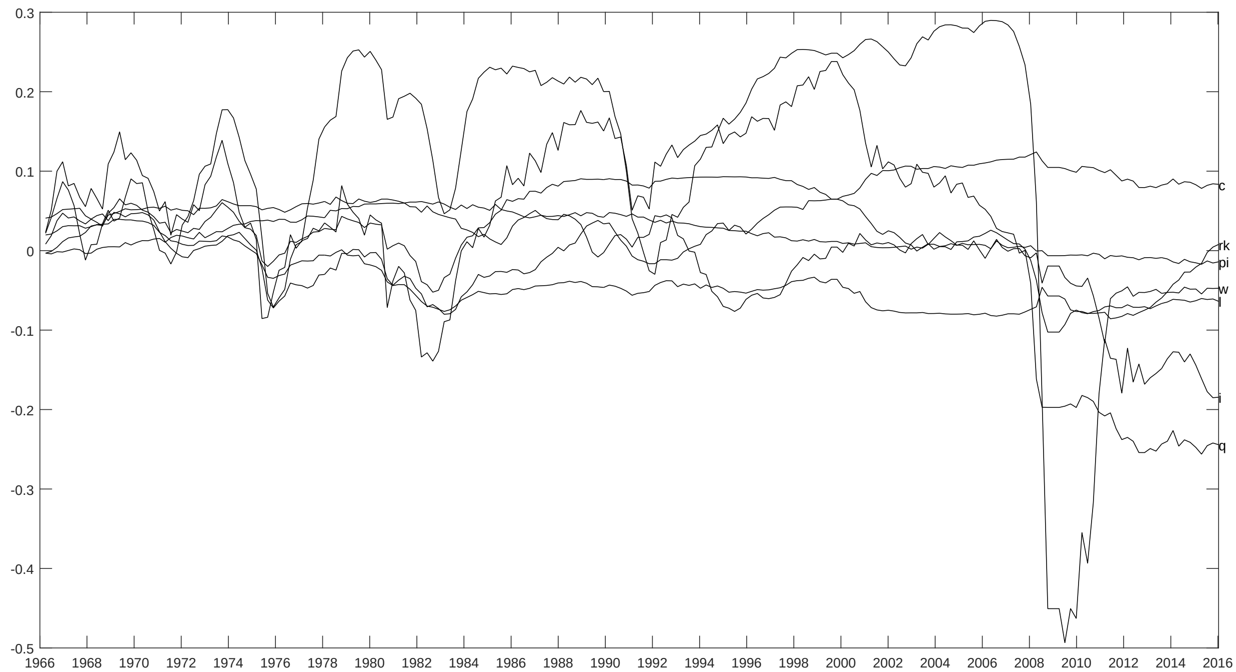
Filtered ZLB regime probability:



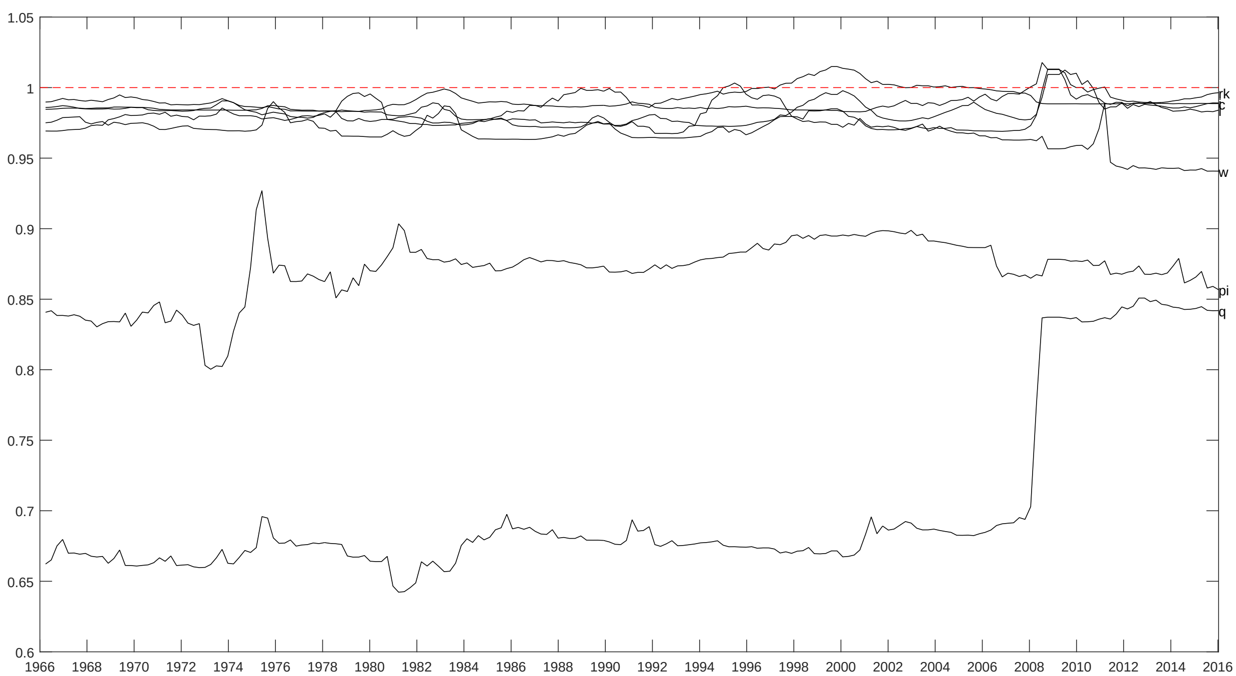
Eigenvalues and Projection Facility



Intercept coefficients

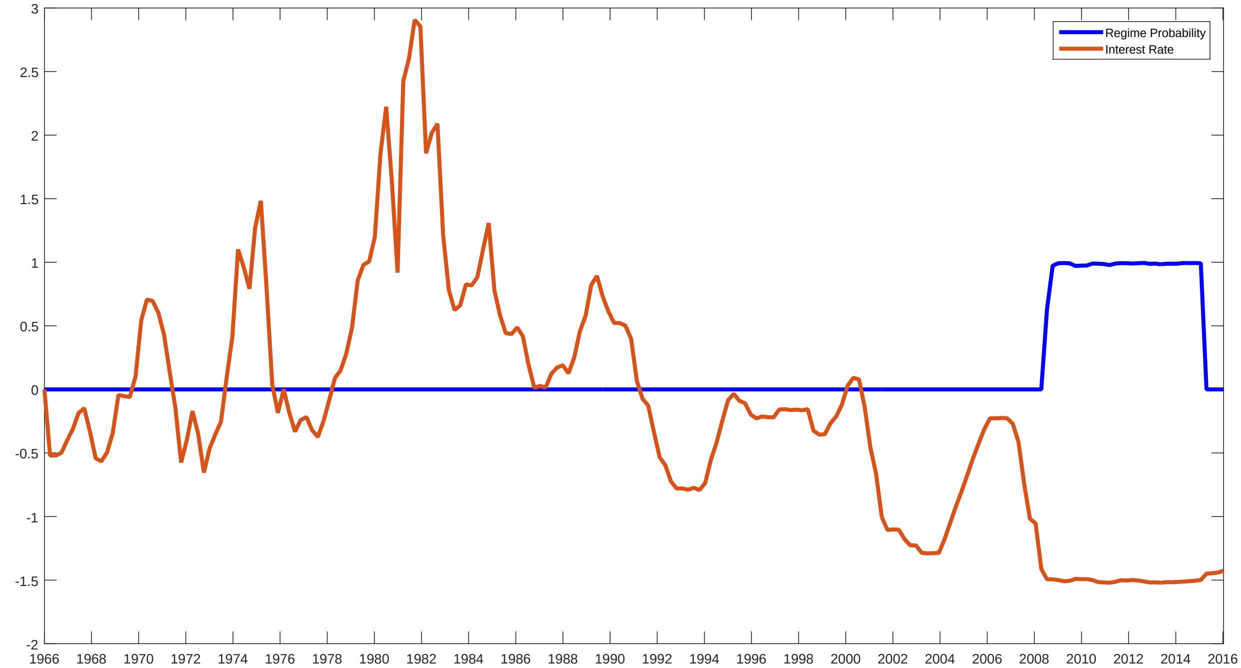


Persistence coefficients:

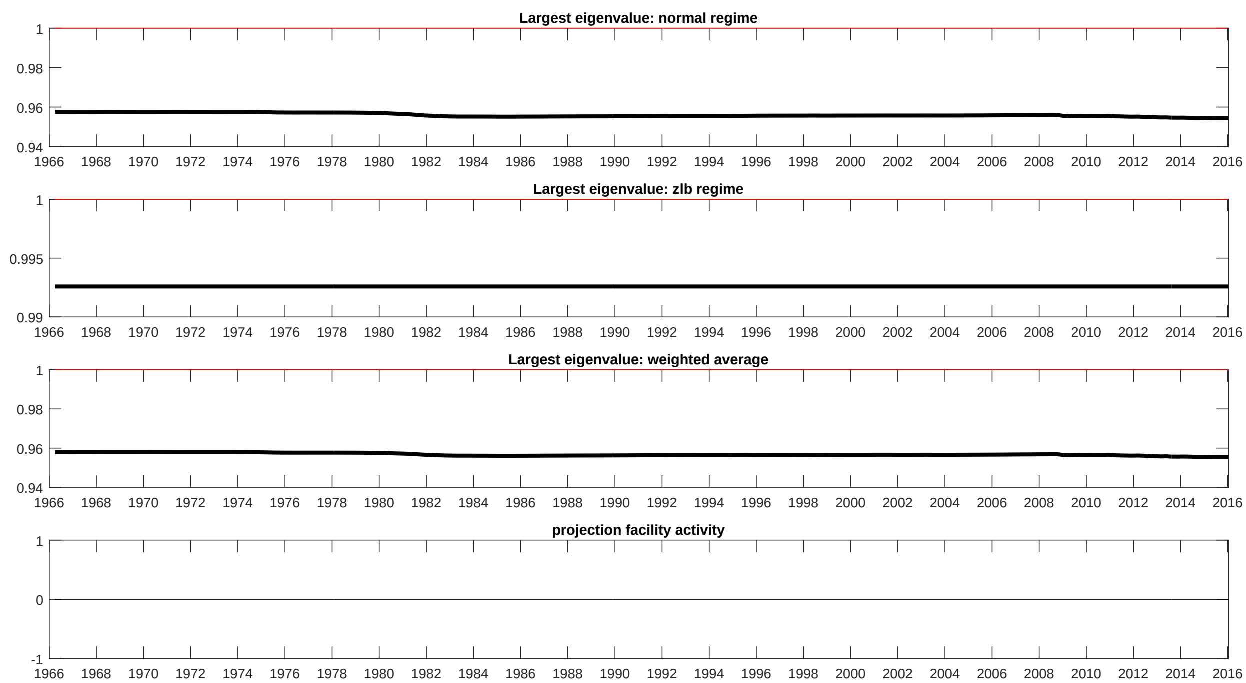


Figures-MSV learning (Based on Full Sample Results)

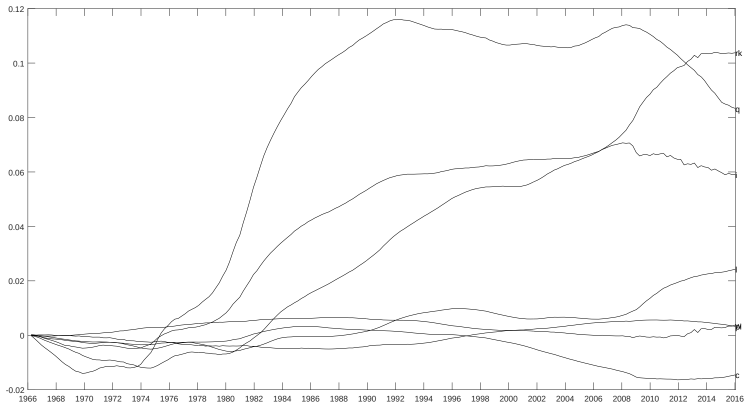
Filtered ZLB regime probability:



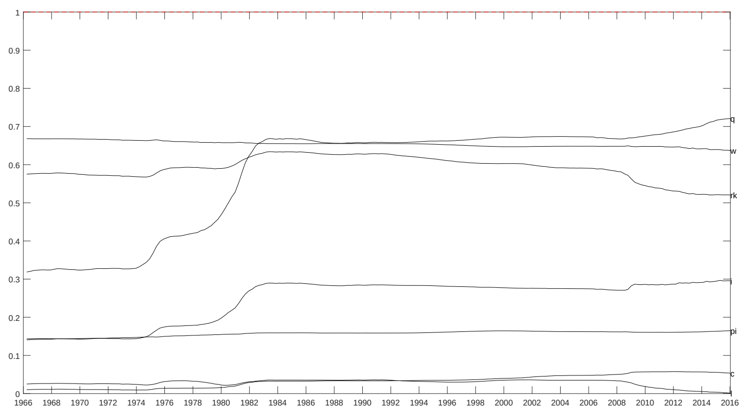
Projection Facility active or not:



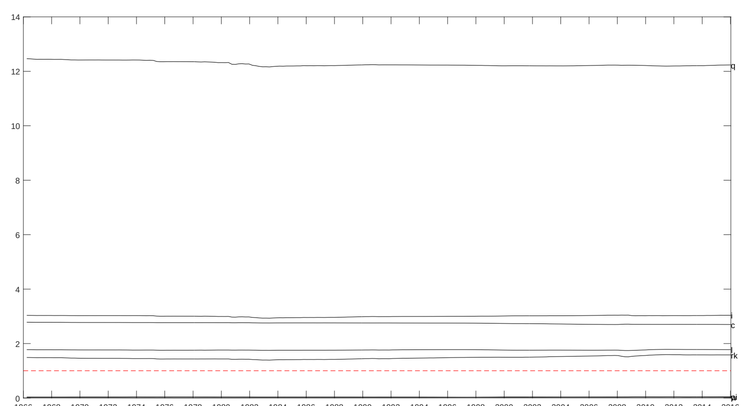
Intercept coefficients:



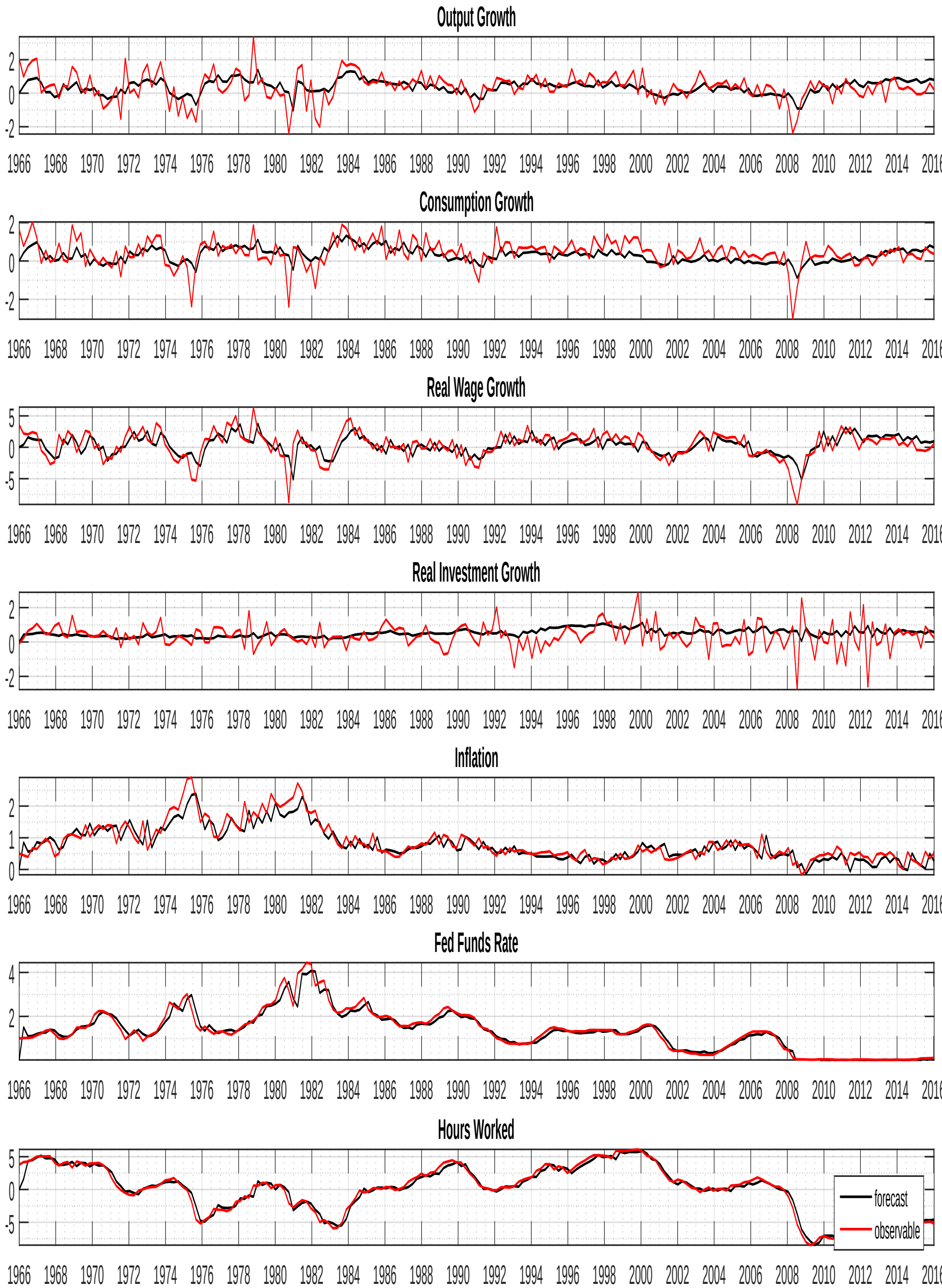
Learning coefficient on lagged inflation:



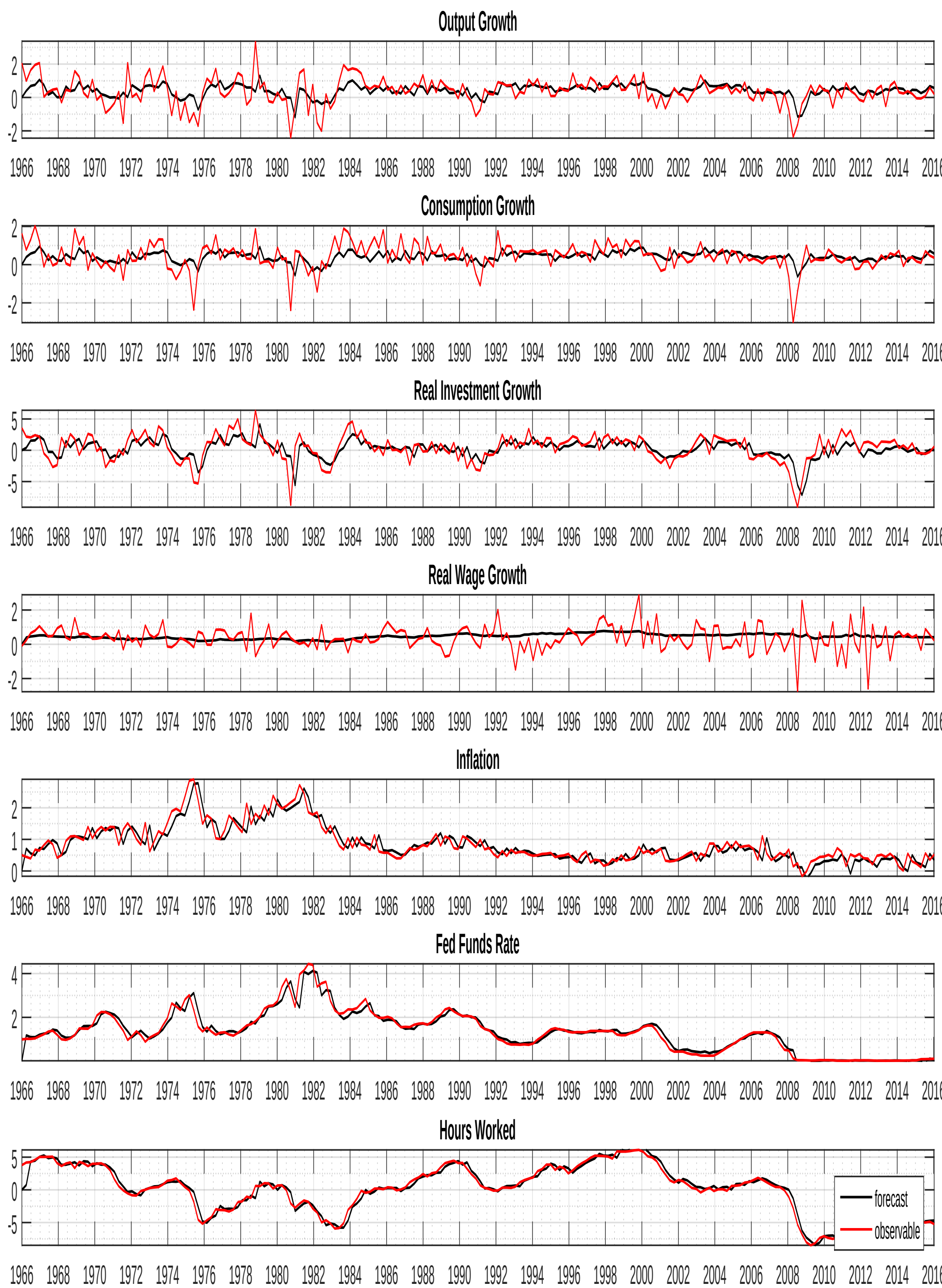
Learning coefficient on b-shock:



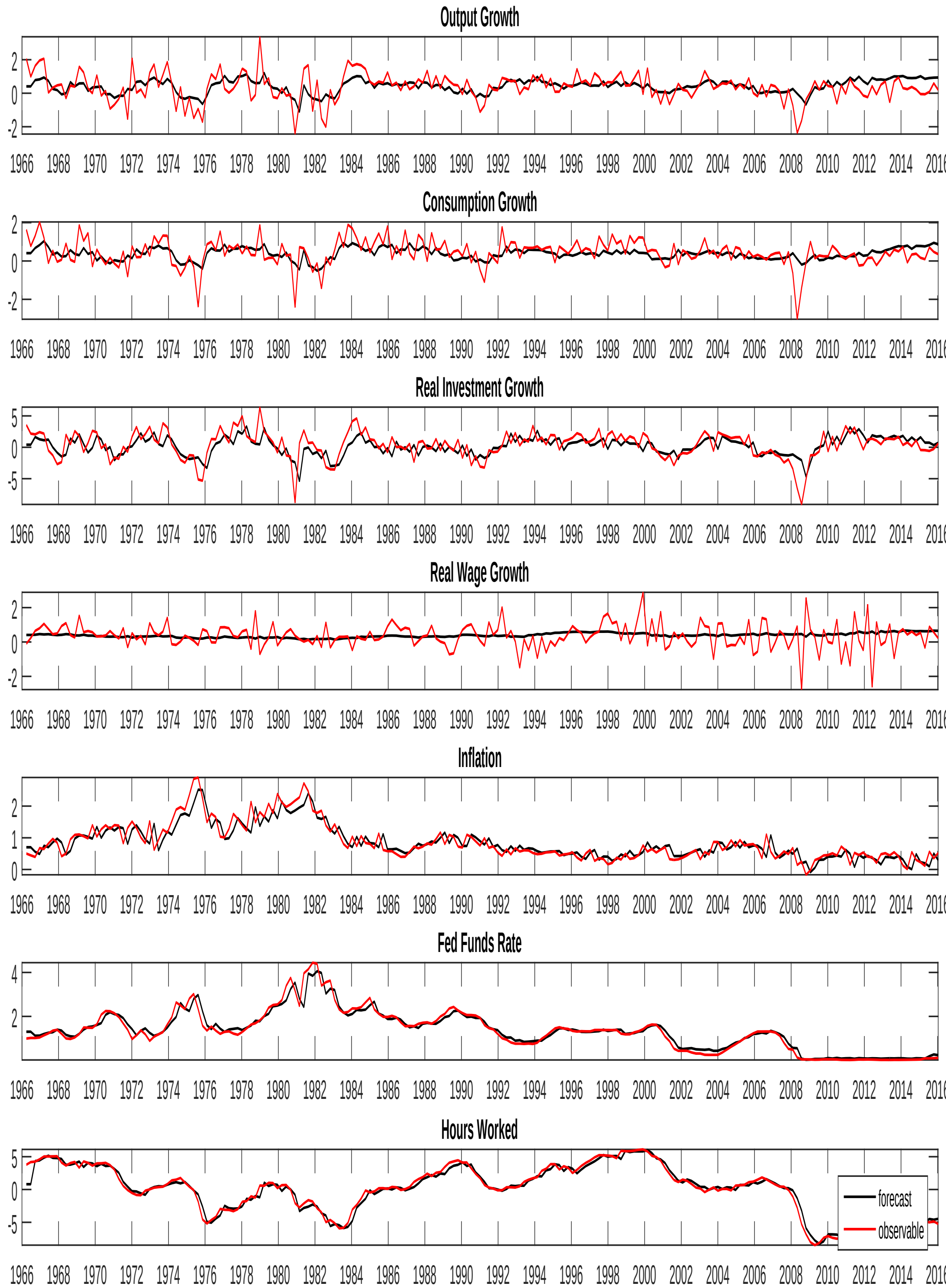
In-Sample forecasts (forecasting step of the filter): MSV learning



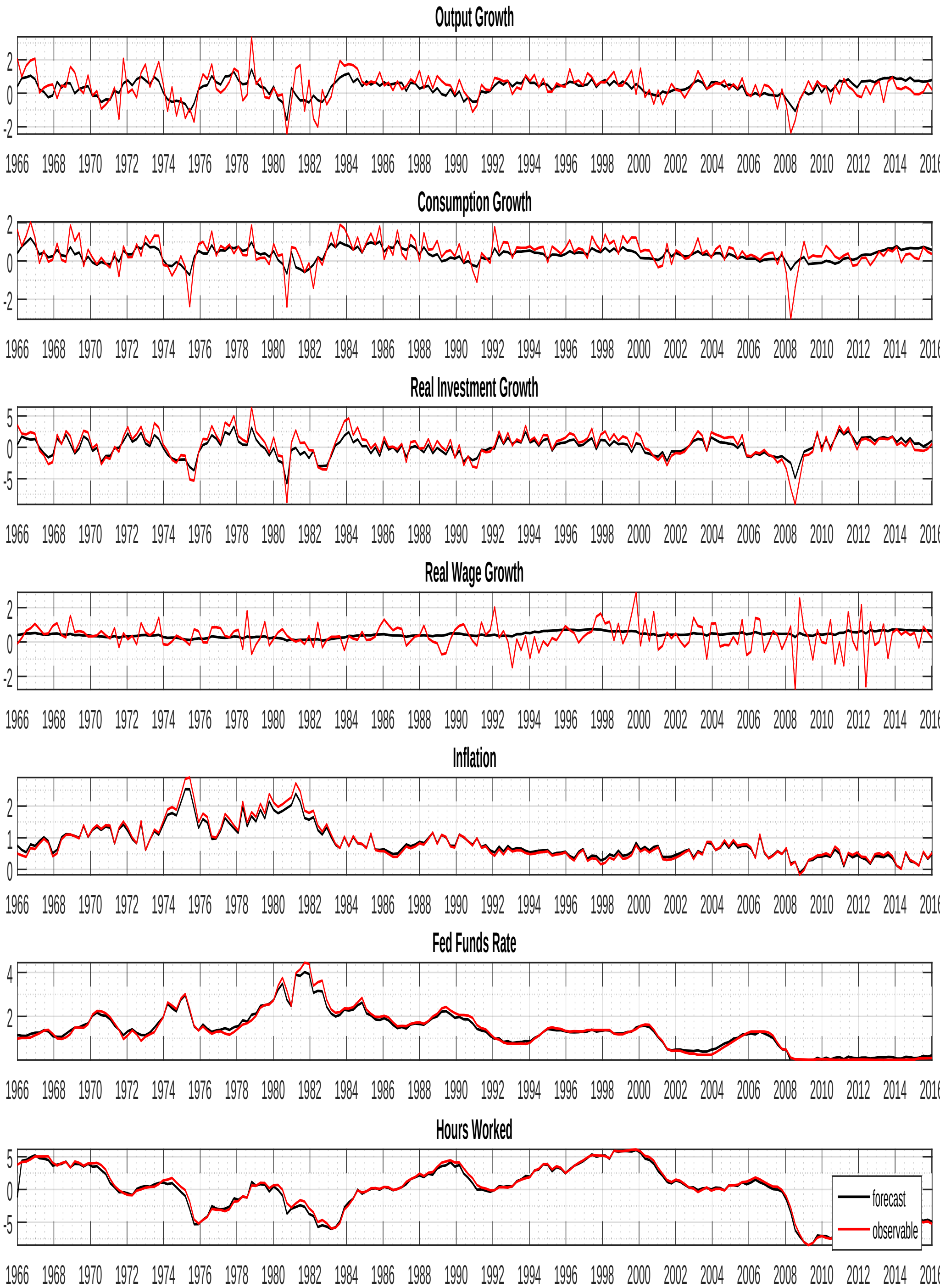
In-Sample forecasts (forecasting step of the filter): AR(1) learning



In-Sample forecasts (forecasting step of the filter): RISE



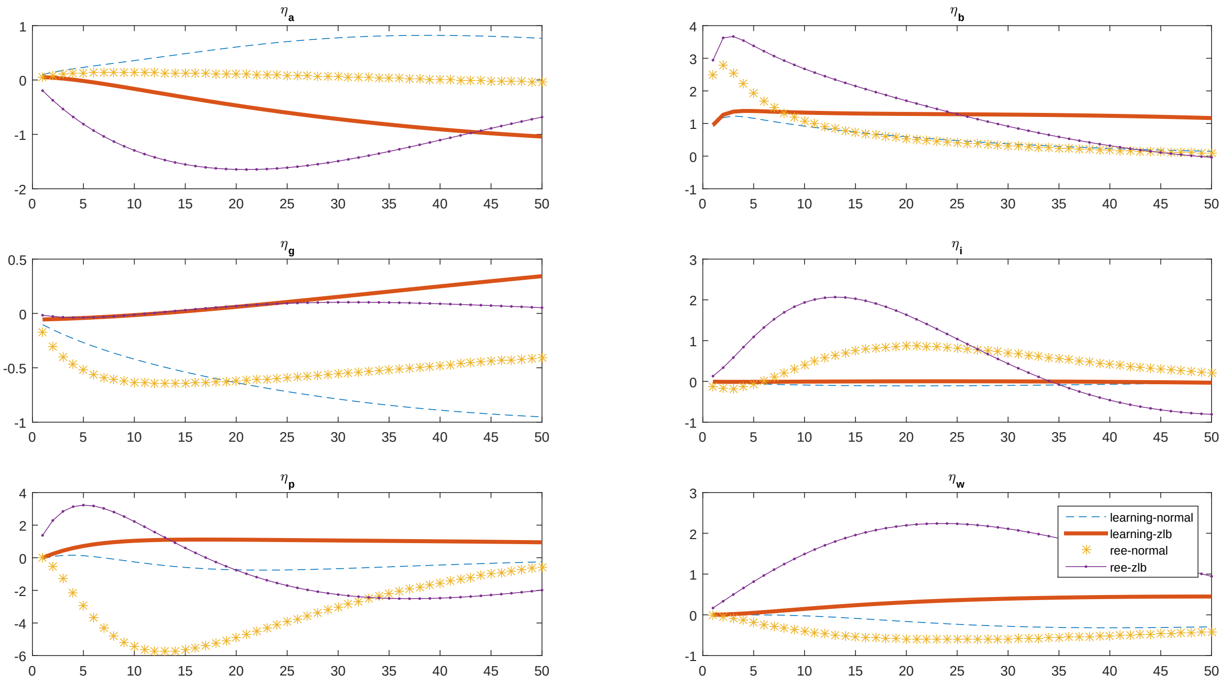
In-Sample forecasts (forecasting step of the filter): REE



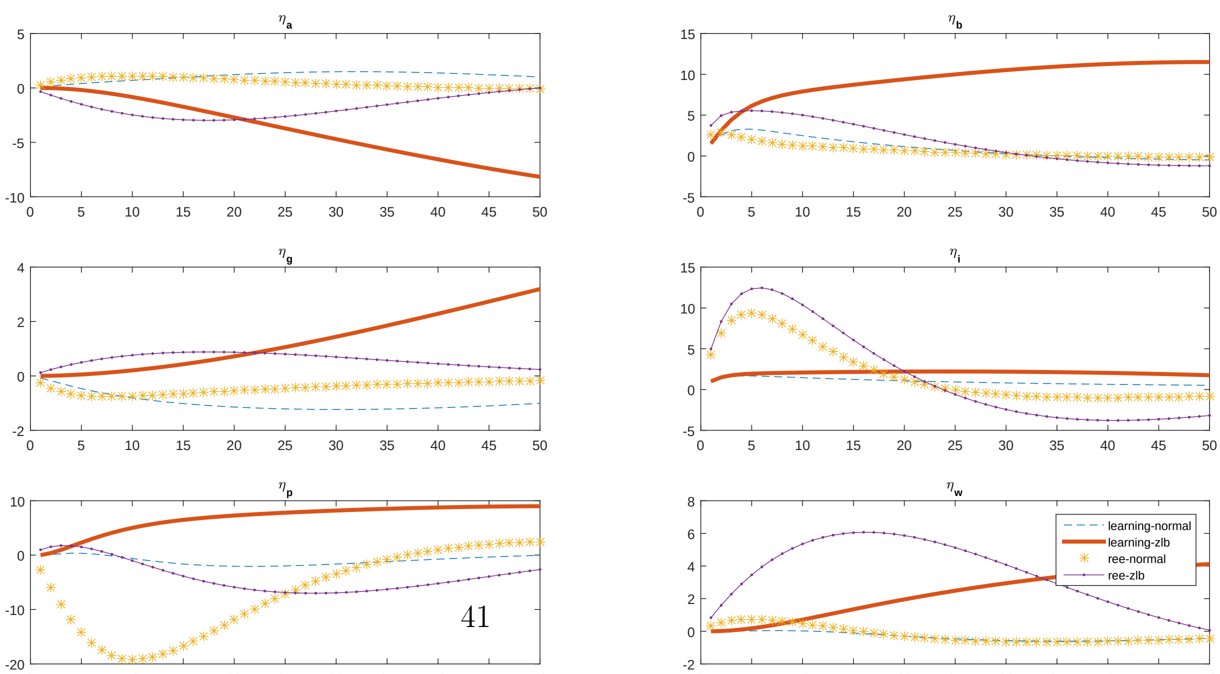
Impulse Responses- Comparison with the REE benchmark: Normal regime and zlb impulse responses are based on 2006Q1 and 2011Q1 respectively

Figure 17: Comparison of AR(1) learning IRFs with RISE IRFs. One unit shocks of η_a,η_b,η_g,η_i,η_p,η_w respectively.

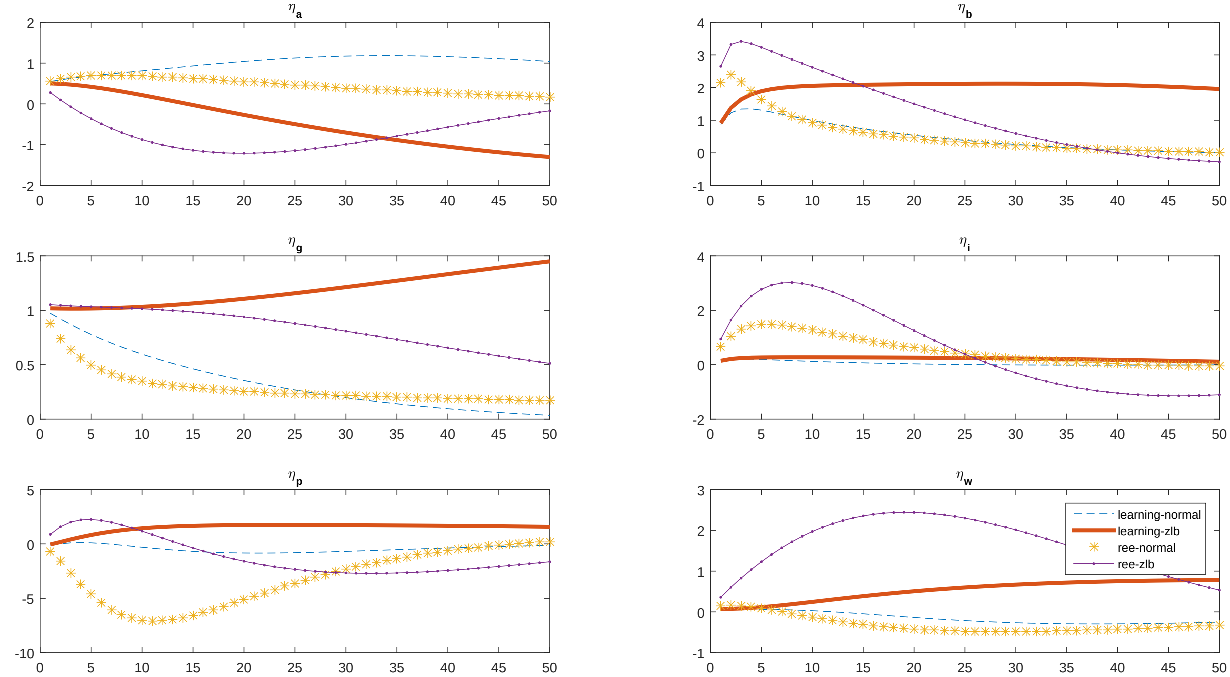
Consumption:



Investment:



Output:



Inflation:

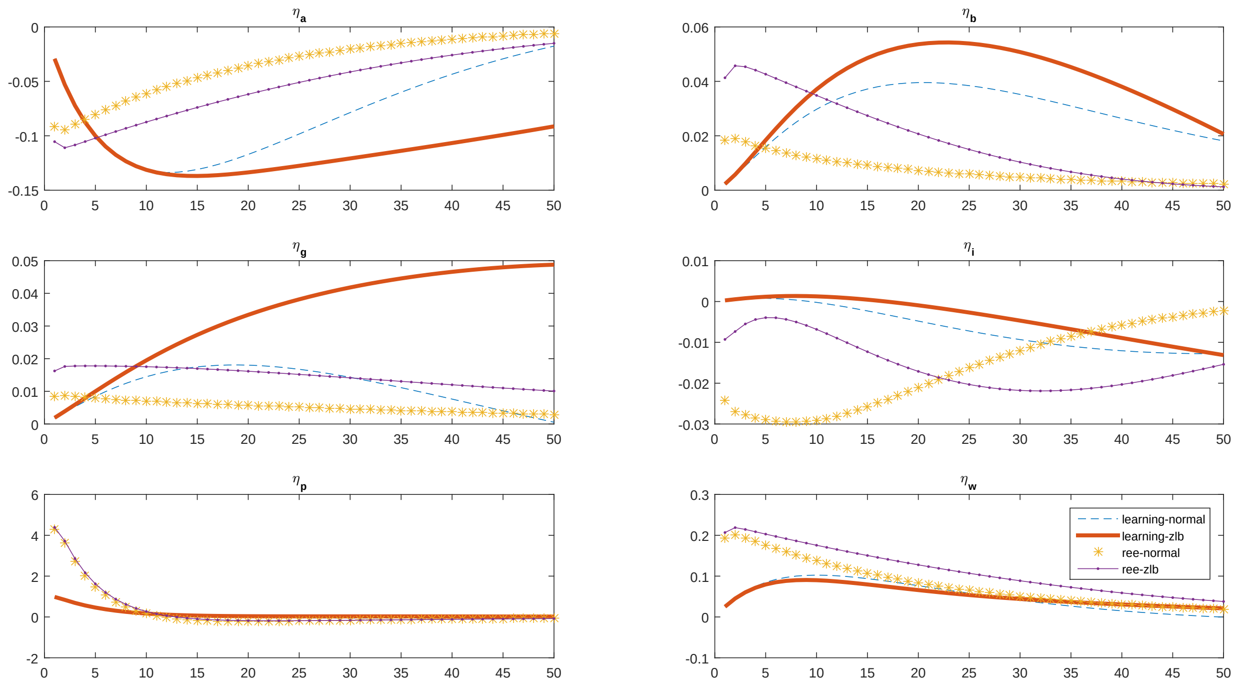
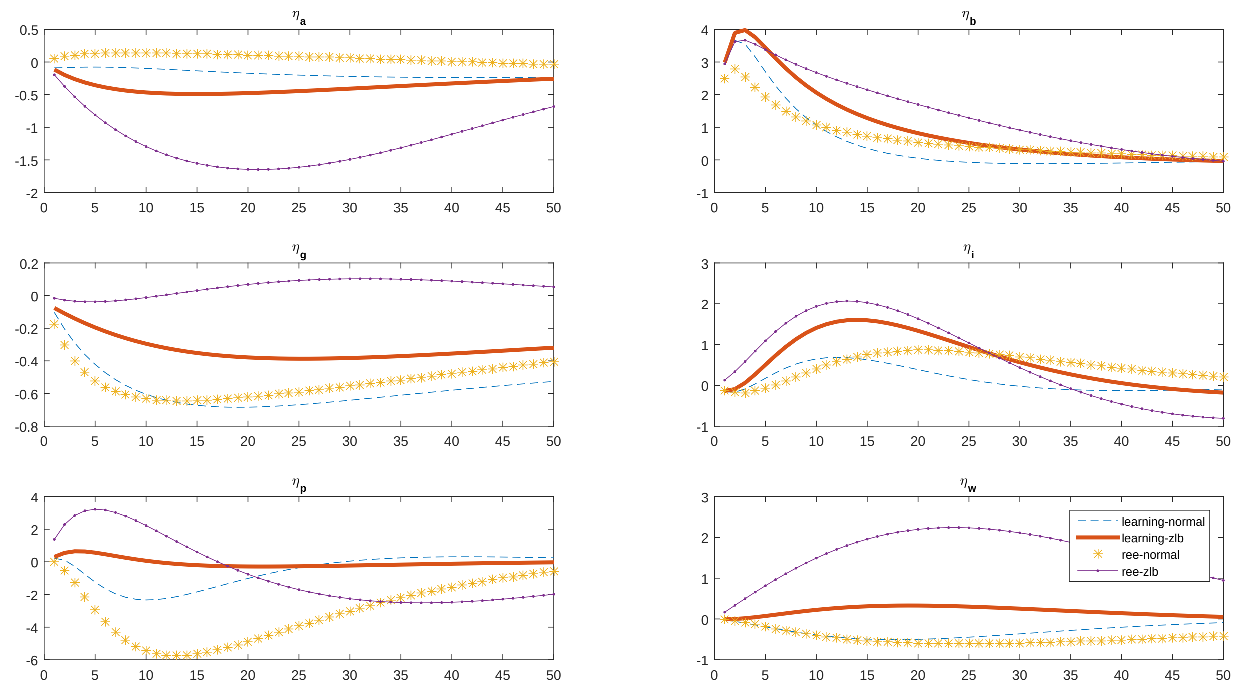
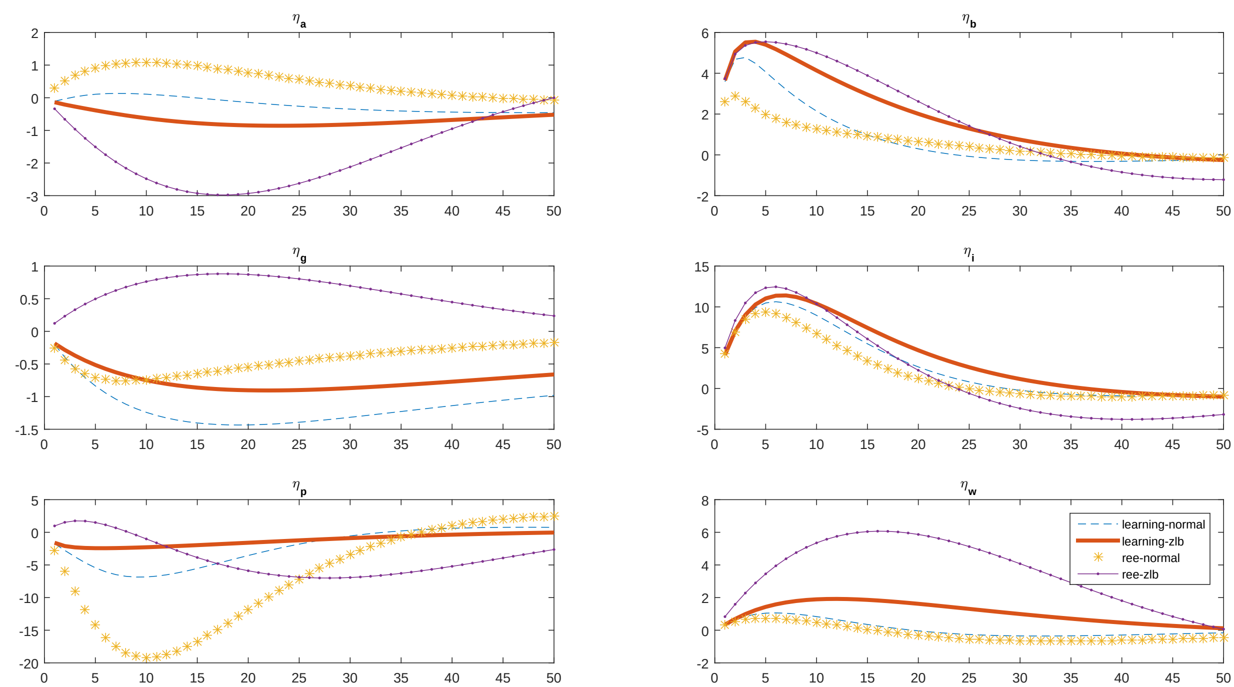


Figure 18: Comparison of MSV learning IRFs with RISE IRFs. One unit shocks of η_a,η_b,η_g,η_i,η_p,η_w respectively.

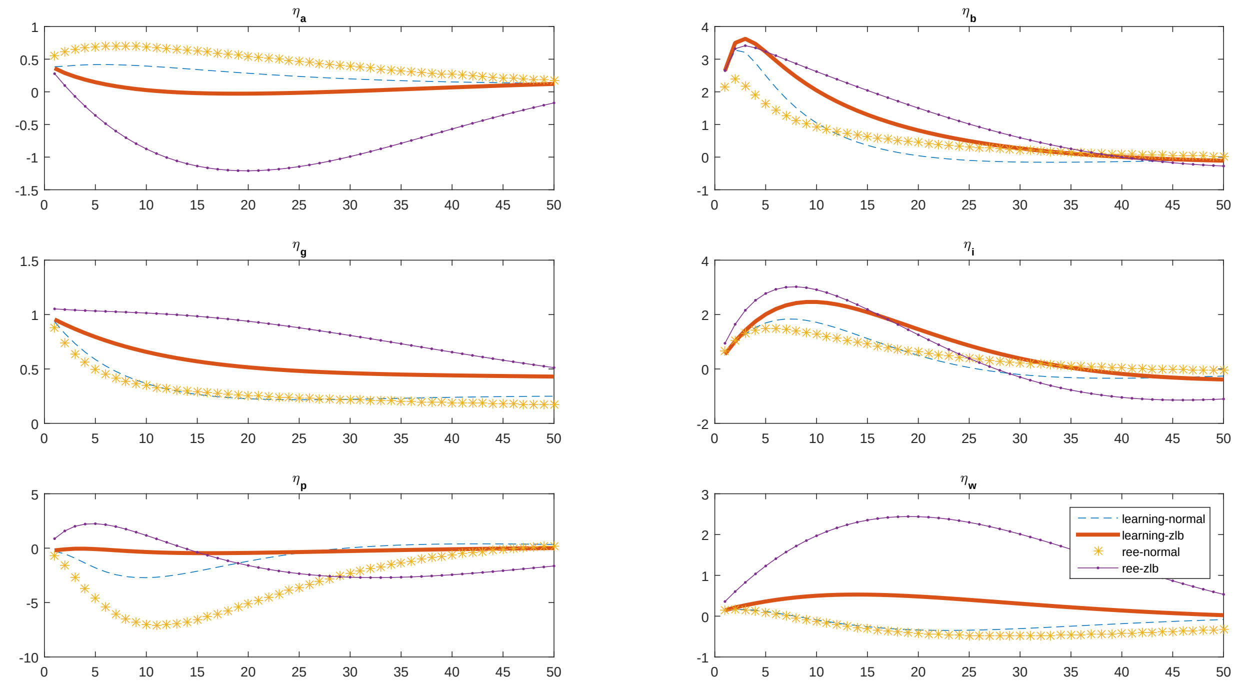
Consumption:



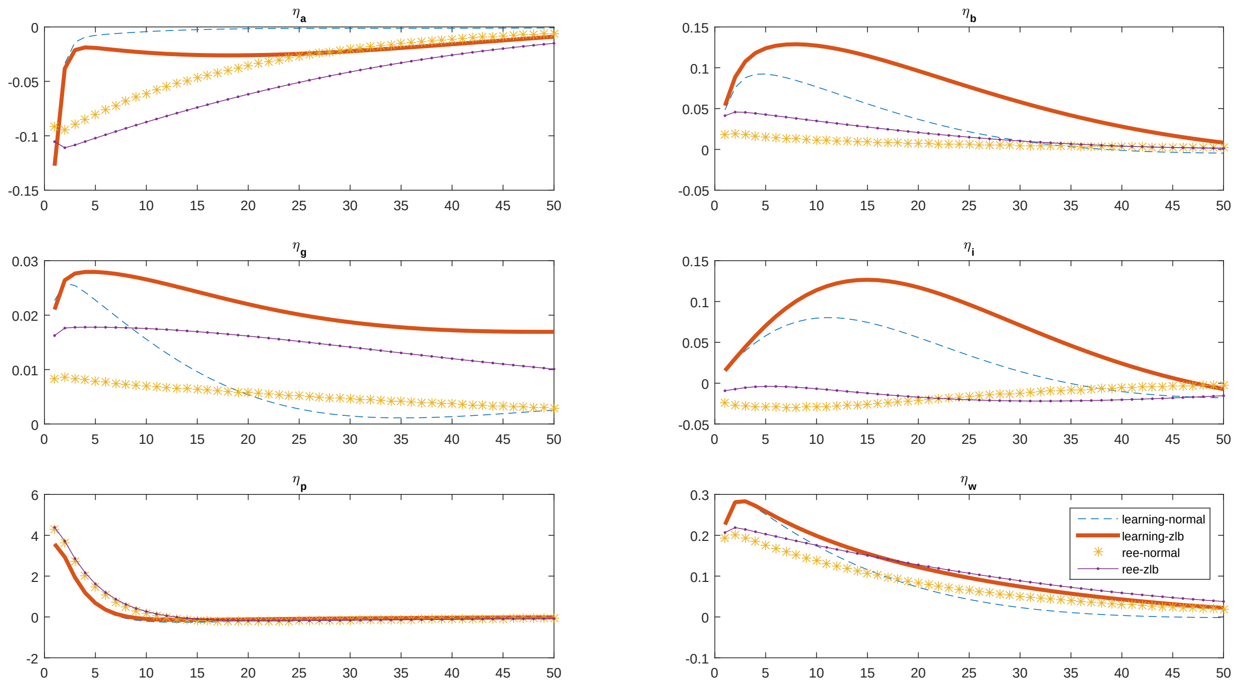
Investment:



Output:

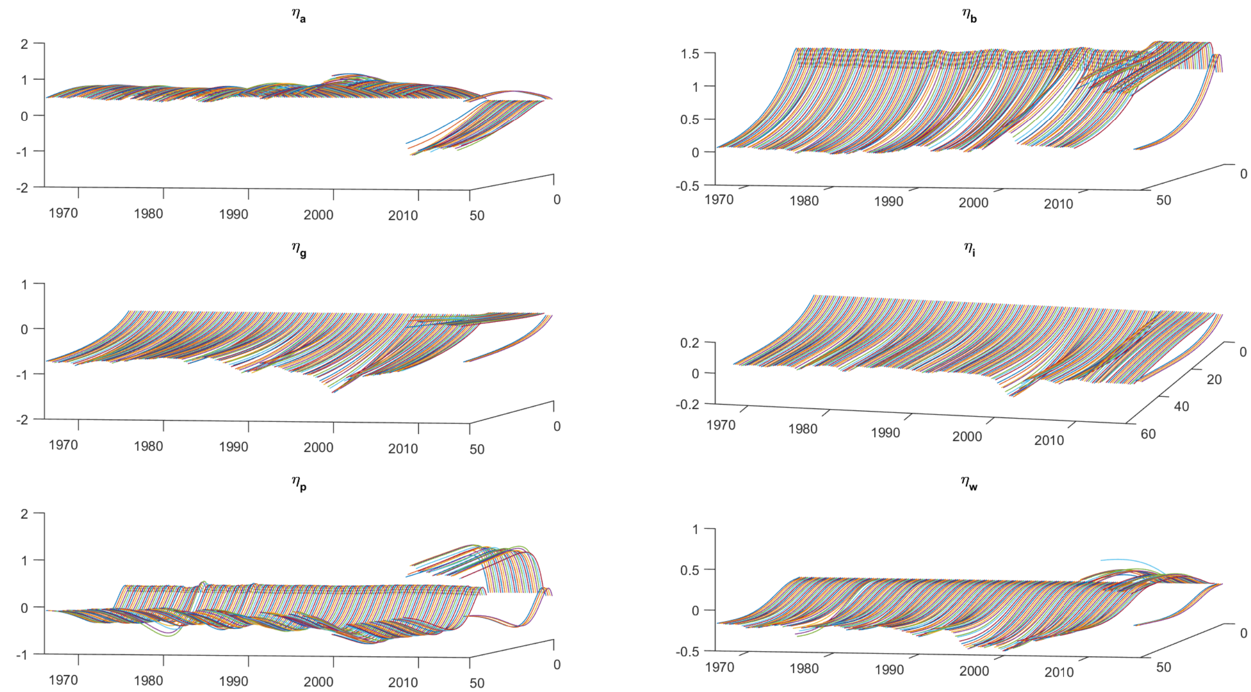


Inflation:

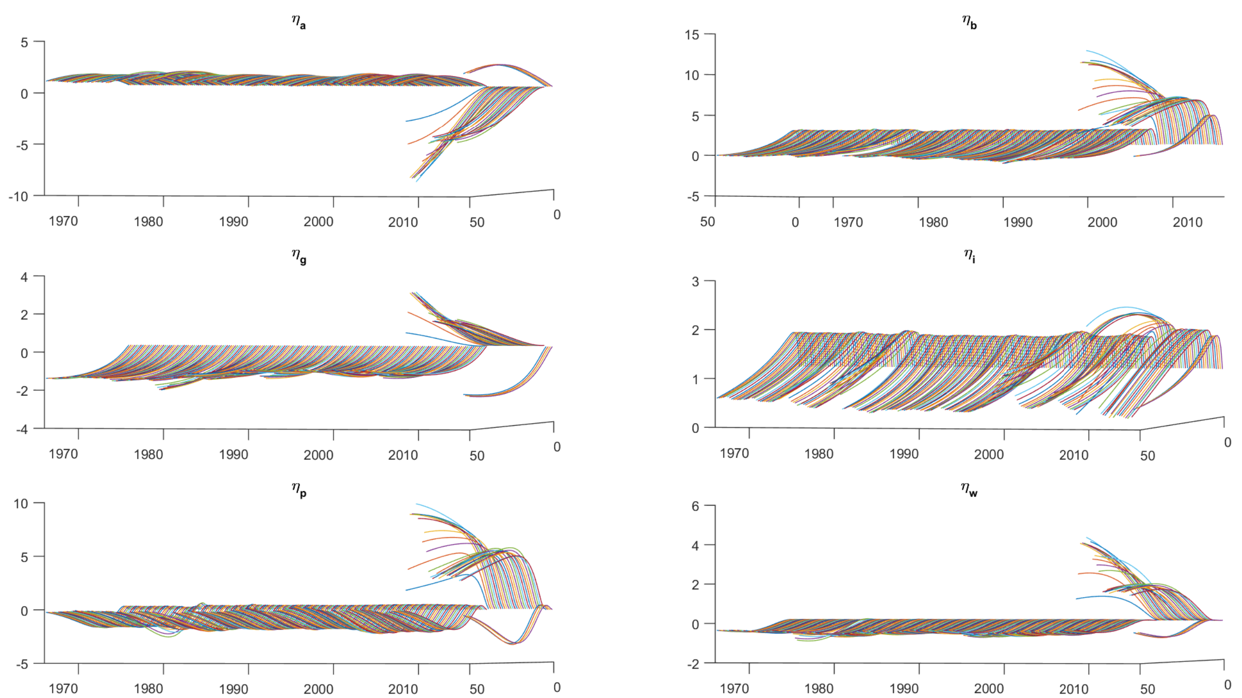


Implied time variation in IRFs under learning: AR(1) case

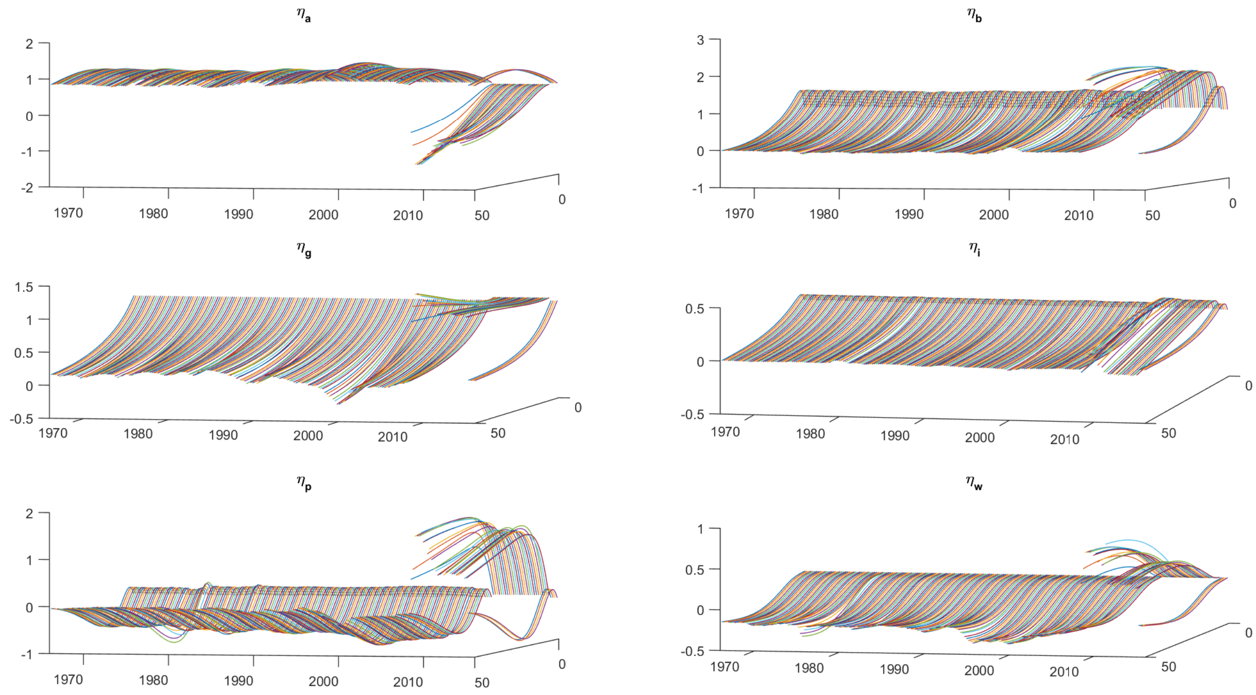
Consumption:



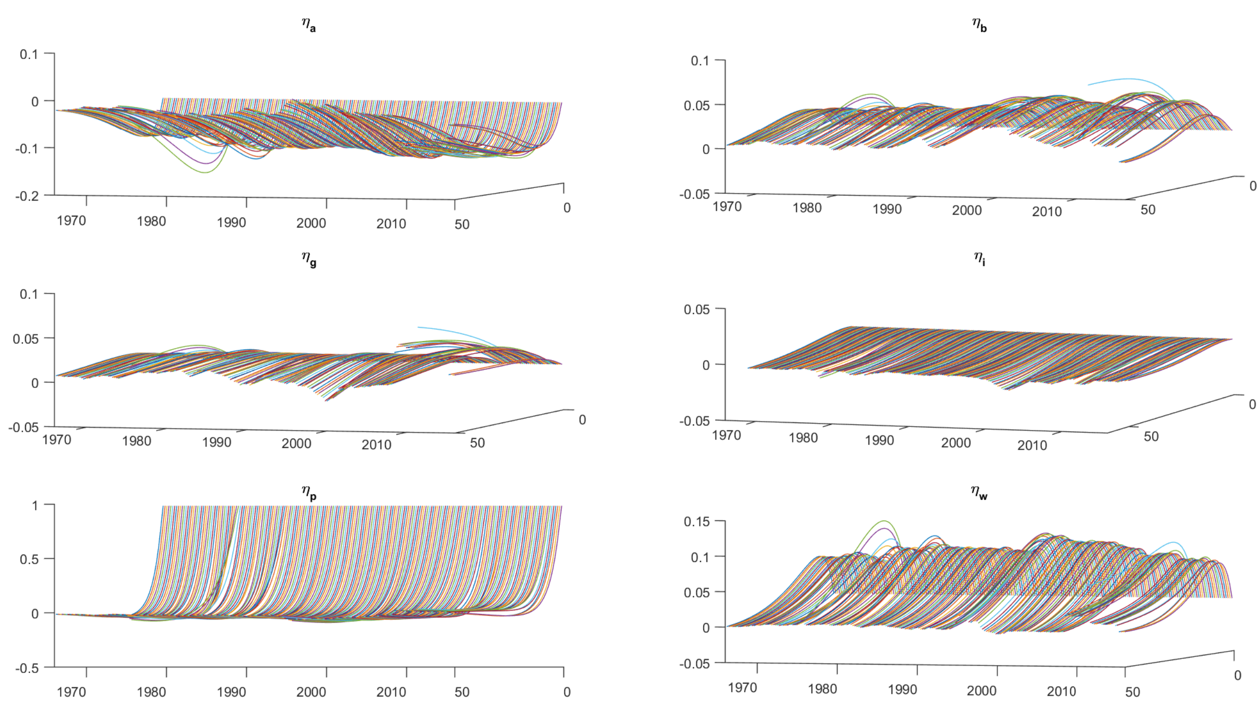
Investment:



Output:

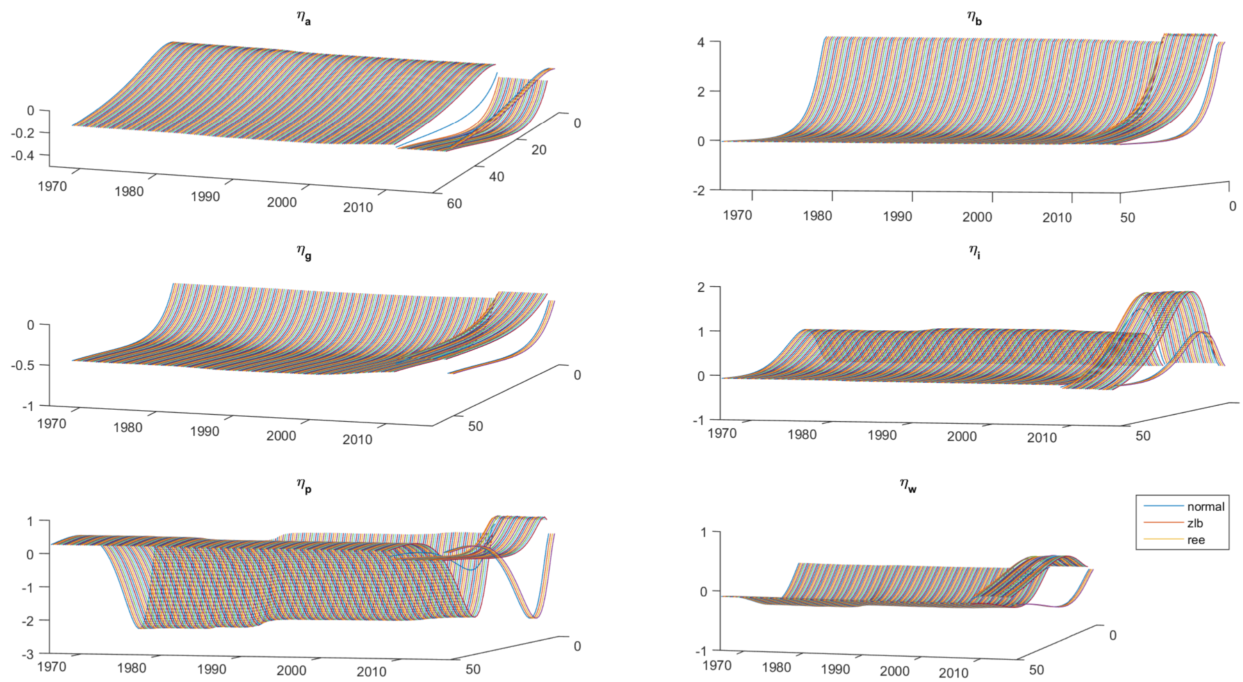


Inflation:

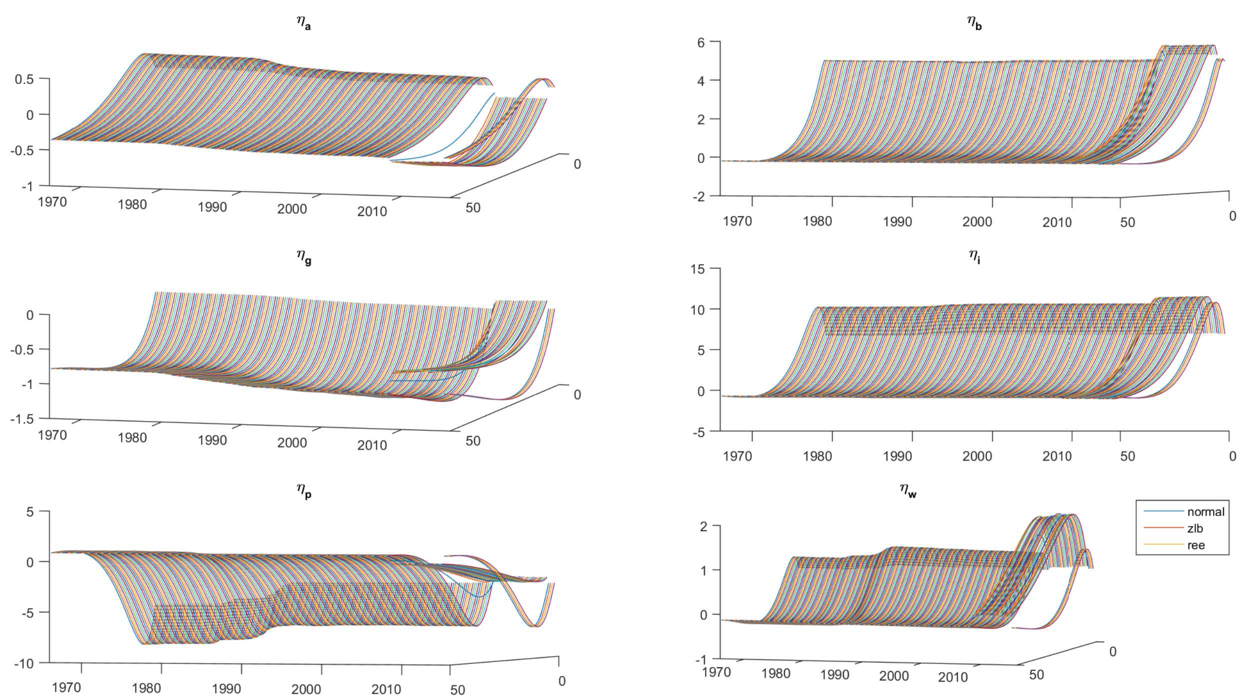


Implied time variation in IRFs under learning: MSV case

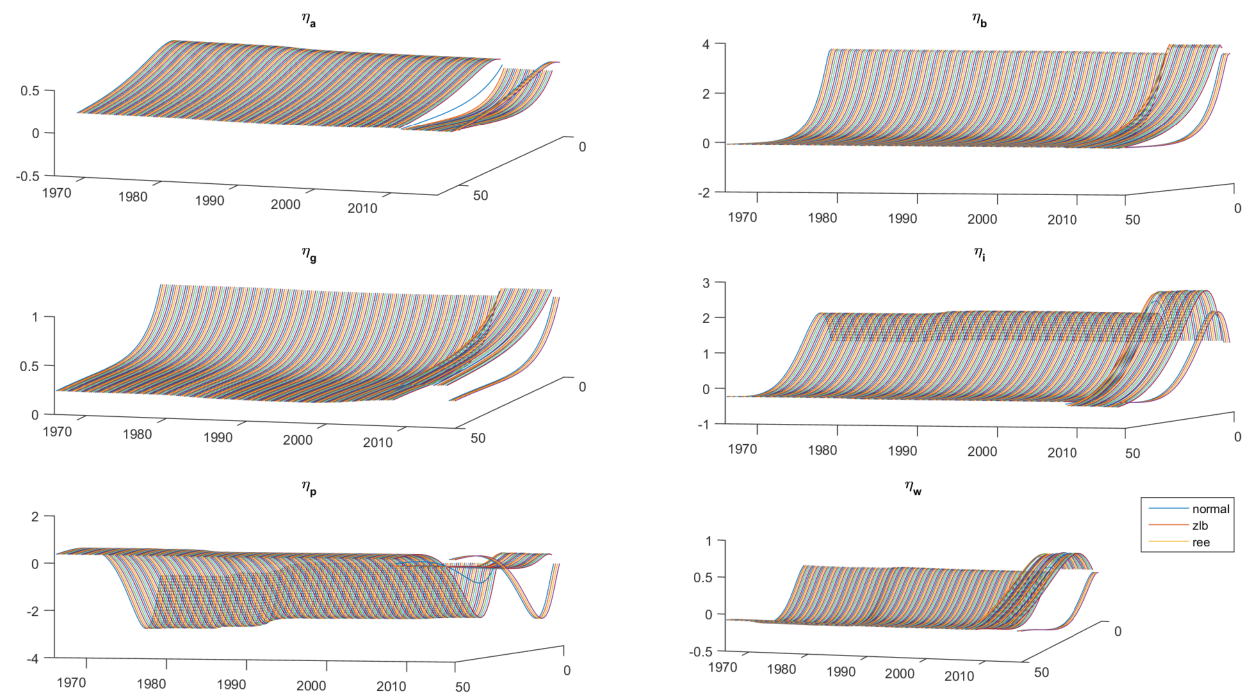
Consumption:



Investment:



Output:



Inflation: