# Financial Market Analysis (FMAx)
# Module 6

"Asset Allocation and Diversification"

## Preamble:
## Why should you care about portfolio allocation?

- You might be an investor.
- Your institution might be an investor.
- As a policymaker, you'll be interested in investor behavior.
- Your country may be interested in what drives international investment.

## Preamble:
## In this module we will…

1. Review statistical concepts related to return and risk.

2. Emphasize the importance of correlation.

3. Explore how to choose an “optimal portfolio”.
   - Of 2 assets
   - Of $n > 2$ assets

4. Do the same for an international portfolio.

# An Introduction

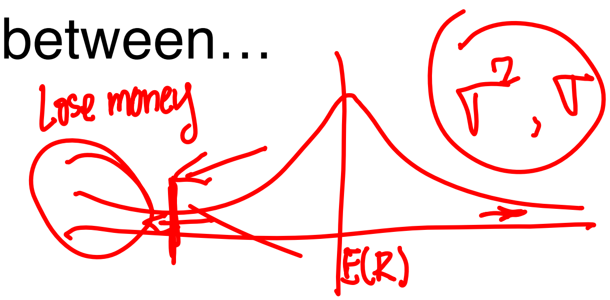
## - Concepts -

**Portfolio Theory - Harry Markowitz (1952)**

Investments are compared in terms of trade-off between…

(-) ▪ **Risk** (variance)

(+) ▪ **Return** (expected reward)




An investor who cares only about risk and return, will always prefer…

- Highest mean return for given amount of risk; and
- Lowest risk given the mean return.

# An Introduction

## - Concepts -

**Insights from H. Markowitz:**

- Diversification does not rely on returns being uncorrelated, but rather on having them be imperfectly correlated.

- Risk reduction from diversification is limited by the extent to which returns are correlated.

Less correl → more benefits
More " → less " } from diversification

## An Introduction

### - Review of Statistics -

**Expected Returns:**

Using historical data:

$$E(r) = r_g = \left[(1+r_1)(1+r_2)(1+r_3)\ldots.(1+r_T)\right]^{\frac{1}{T}} - 1$$

$$E(r) = r_a = \frac{r_1 + r_2 + r_3 + \ldots. + r_T}{T}$$

# An Introduction

## - Review of Statistics -

**Risk: (Variance or Standard Deviation of Returns)**

Using historical data:

$$Var(r) = r_g = \frac{(r_1 - E(r))^2 + (r_2 - E(r))^2 + \ldots (r_T - E(r))^2}{T}$$

# An Introduction

## - Review of Statistics -

**<u>Correlation</u>: (Degree of co movement between two variables.)**

Correlation coefficient ranges from:

**+ 1** (perfect co-movement)

**0** (variables are independent)

**- 1** (perfect negative correlation)

$$Cov\left(r_A, r_B\right) = \frac{\sum_{t=1}^{T}\left(r_{A,t} - E\left(r_A\right)\right)\left(r_{B,t} - E\left(r_B\right)\right)}{T}$$

$$\rho_{AB} = \frac{Cov\left(r_A, r_B\right)}{\sigma_A \sigma_B}$$

# Introducing Two Risky Assets

Define Portfolio ***P*** containing two risky assets (portfolios):
Bonds (***D***) and Equity (***E***), Weights $w_D$ and $w_E$

Expected rate of return:

$$E(r_P) = w_D E(r_D) + w_E E(r_E)$$

Correlation coefficient $\rho_{DE}$:

$$\rho_{DE} = \frac{Cov(r_D, r_E)}{\sigma_D \sigma_E}$$

# Introducing Two Risky Assets

Risk of ***P***:

$$Var(P) = \sigma_P^2 = E[r_P - E(r_P)]^2$$

$$= w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E \rho_{DE}\sigma_D\sigma_E$$

Question: What is the largest possible value for $\rho_{DE}$?

When $\rho_{DE}$=1 (perfect correlation):

$$Var(P) = \sigma_P^2 = w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E \sigma_D\sigma_E$$

$$= (w_D\sigma_D + w_E\sigma_E)^2$$

$$\Rightarrow \sigma_P = w_D\sigma_D + w_E\sigma_E$$

# Introducing Two Risky Assets

Unless correlation between ***D*** and ***E*** is perfect:

$$\sigma_P < w_D\sigma_D + w_E\sigma_E$$

Note that:

- Expected return of ***P*** is unaffected by the correlations
- We often call the difference between the weighted average of individual $\sigma$'s and $\sigma_p$ the "gains from diversification".
- Hedge asset: negative correlation with other assets ($\rho_{DE} < 0$)

# Introducing Two Risky Assets

In the extreme case of a perfect hedge ($\rho_{DE}$ = -1):

$$\sigma_p^2 = (w_D\sigma_D - w_E\sigma_E)^2$$

$$\sigma_p = abs(w_D\sigma_D - w_E\sigma_E)$$

Can construct a **zero risk portfolio**:

$$\sigma_p = w_D\sigma_D - w_E\sigma_E = 0, \quad w_D + w_E = 1$$

$$\Rightarrow w_D = \frac{\sigma_E}{\sigma_D + \sigma_E}, w_E = \frac{\sigma_D}{\sigma_D + \sigma_E} = 1 - w_D$$

# Graphical Representation
## - Two-Assets Portfolio -

Example:

$$E(r_D) = 2\%,\ E(r_E) = 6\%,\ \sigma_D = 5\%,\ \sigma_E = 10\%,\ \rho_{DE} = 0.2$$

Return and Risk:

$$E(r_p) = 0.02w_D + 0.06w_E$$

$$\sigma_p^2 = (0.05)^2 w_D^2 + (0.10)^2 w_E^2 + 2 \cdot 0.05 \cdot 0.10 \cdot 0.2 \cdot w_D w_E$$

$$= 0.025w_D^2 + 0.01w_E^2 + 0.002w_D w_E$$

# Graphical Representation

## - Two-Assets Portfolio -

**<u>Minimizing Risk</u>:**

$$Min \quad \sigma_p^2 = Min \quad w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E \rho \sigma_D \sigma_E$$

$$s.t.: w_D + w_E = 1, \text{or } w_E = 1 - w_D$$

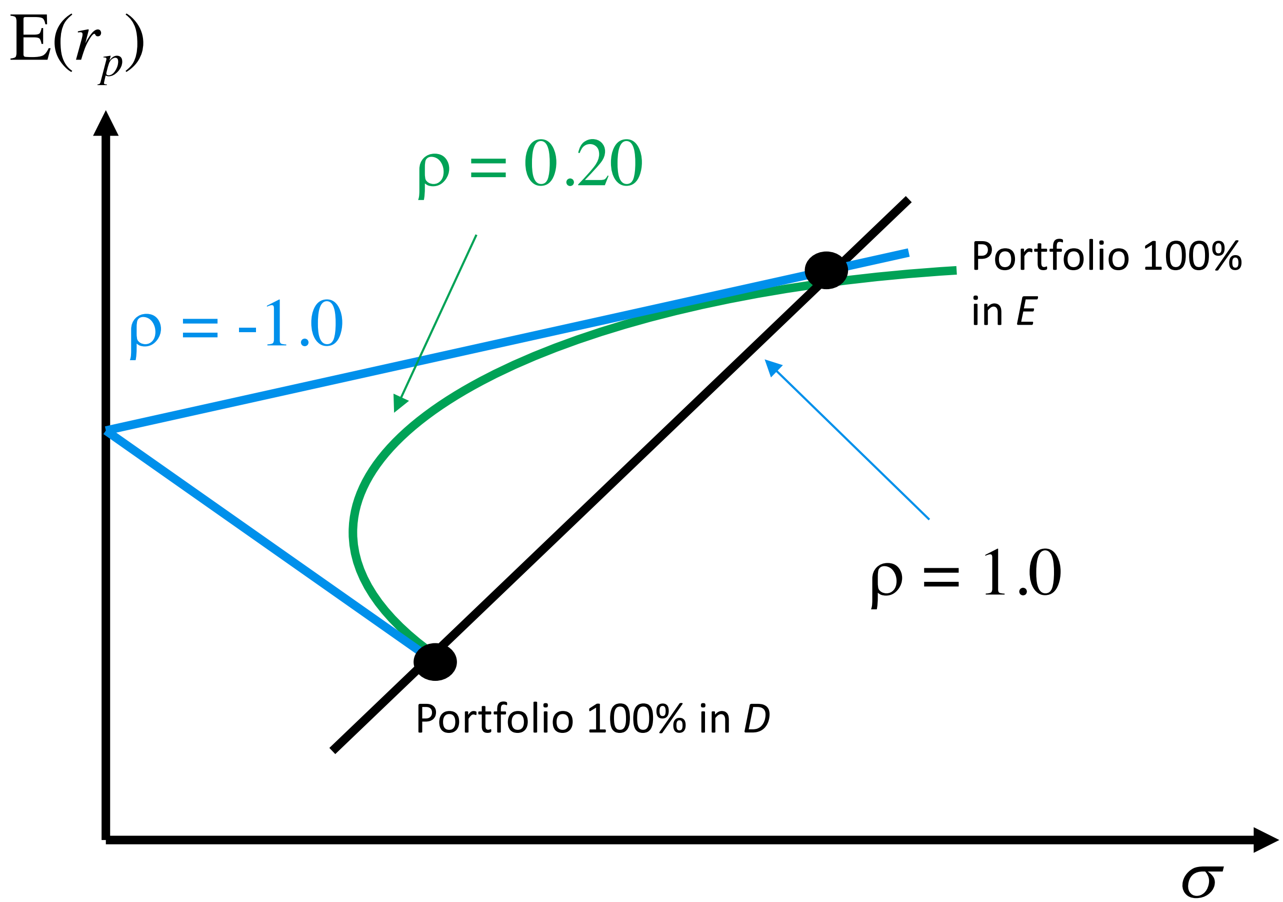
Solving the above for $w_D$, $w_E$

$$w_{\min}(D) = \frac{\sigma_E^2 - \rho_{DE}\sigma_D\sigma_E}{\sigma_E^2 + \sigma_D^2 - 2\rho_{DE}\sigma_D\sigma_E}$$

$$w_{\min}(E) = 1 - w_{\min}(D)$$

## Graphical Representation

**- Two-Assets Portfolio -**



# Graphical Representation

## - Two-Assets Portfolio -

**<u>Benefits from Diversification</u>:**

- Come from imperfect correlation between returns.
- The smaller $\rho$, the greater the benefits from diversification.
- If $\rho = 1$, no risk reduction is possible.
- Adding extra assets with lower correlation with the existing ones decreases total risk of the portfolio.
- Diversification can eliminate some, but not all risk.

# Comparing Different Portfolios
## - Two-Assets -

**The Risk-Free Asset:**

**IF** it is possible to borrow/lend at the risk-free rate ***$r_f$***,

**THEN** the portfolio selection problem is to maximize the excess *return* over the risk-free rate, for a given amount of risk:

$$Max \quad \frac{\overbrace{E(r_p)-r_f}^{\text{excess ret}}}{\sigma_p} = \frac{w_D\left(E(r_D)-r_f\right)+w_E\left(E(r_E)-r_f\right)}{\sqrt{\underbrace{w_D^2\sigma_D^2+w_E^2\sigma_E^2+2w_Dw_E\rho\sigma_D\sigma_E}_{\text{var p}}}}$$

risk

$$s.t.: w_D + w_E = 1$$

# Comparing Different Portfolios
## - Two-Assets -

**<u>Our Numerical Example</u>:**

- Assume $r_f$ =0.9%
- Define **A** as the minimum-variance portfolio obtained earlier
  ($w_D$ = 0.84, $w_E$ = 0.16)
- "**Capital allocation line**" (**CAL**): combinations of **A** and the risk-free asset.
- Slope of **CAL** = "reward-to-variability", or **Sharpe ratio**:

$$S_A = \frac{E(r_A) - r_f}{\sigma_A} = \frac{\overset{E(r_A)}{2.57} - \overset{r_f}{0.9}}{\underset{\sigma_A}{}\;4.78} = \overset{S_A}{0.350}$$

# Comparing Different Portfolios
## - Two-Assets -

**<u>Consider an alternative portfolio B:</u>**

$W_D = 0.65$, $W_E = 0.35$

Is it better than portfolio **A**?

Yes, $S_B > S_A$

$$S_B = \frac{E(r_B) - r_f}{\sigma_B} = \frac{5.2 - 0.9}{3.4} = 0.478$$

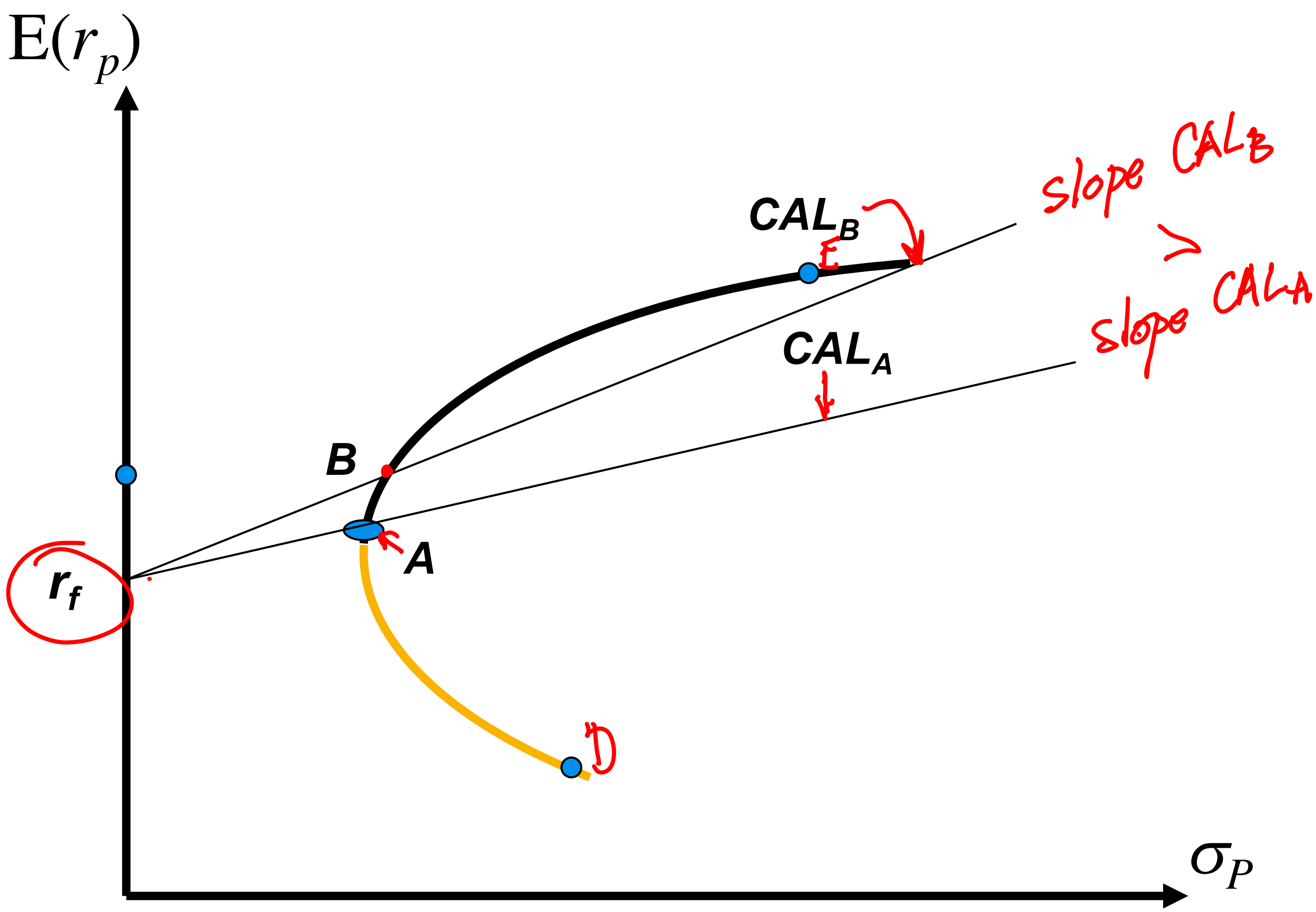
$E(r_B)$ $\sigma_B$ $S_B$

The optimal portfolio will be such that the reward-to-variability ratio is maximized (depends on $r_f$):

2 assets
$N = 2$

$$\underset{w_i}{Max}\, S_p = \frac{E(r_p) - r_f}{\sigma_p} \; s.t. \sum_{i=1}^{N} w_i = 1$$

## Comparing Different Portfolios
### - Two-Assets -



# Selecting the Best Portfolio
## - Two-Assets -

**The Optimization Problem:**

$$Sp \quad Max \quad \frac{E(r_p) - r_f}{\sigma_p} = \frac{w_D\left(E(r_D) - r_f\right) + w_E\left(E(r_E) - r_f\right)}{\sqrt{w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2w_D w_E \rho \sigma_D \sigma_E}}$$

$$\frac{dSp}{dw_D} = 0 \qquad s.t.: w_D + w_E = 1$$

After some algebra, and using $w_E = 1 - w_D$:

$$w_D^* = \frac{\left(E(r_D) - r_f\right)\sigma_E^2 - \left(E(r_E) - r_f\right)\text{cov}(r_D, r_E)}{\left(E(r_D) - r_f\right)\sigma_E^2 + \left(E(r_E) - r_f\right)\sigma_D^2 - \left(E(r_D) - r_f + E(r_E) - r_f\right)\text{cov}(r_D, r_E)}$$

# Selecting the Best Portfolio

## - Two-Assets -

**Expressing the Numerator in Matrix Notation:** $S^{-1}$ Excess Rt

**(E1)**

$$\begin{bmatrix} w_D^* \\ w_E^* \end{bmatrix} = \frac{\underset{2\times 2}{\begin{bmatrix} \sigma_E^2 & -\mathrm{cov}(r_D,r_E) \\ -\mathrm{cov}(r_D,r_E) & \sigma_D^2 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} E(r_D)-r_f \\ E(r_E)-r_f \end{bmatrix}}}{\left[\left(E(r_D)-r_f\right)\sigma_E^2 + \left(E(r_E)-r_f\right)\sigma_D^2 - \left(E(r_D)-r_f + E(r_E)-r_f\right)\mathrm{cov}(r_D,r_E)\right]}$$

$z_1 + z_2$

Let's call the numerator of (**E1**) a column vector ***z,*** which will be:

$2\times 1$

**(E2)**

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \sigma_E^2 & -\mathrm{cov}(r_D,r_E) \\ -\mathrm{cov}(r_D,r_E) & \sigma_D^2 \end{bmatrix} \begin{bmatrix} E(r_D)-r_f \\ E(r_E)-r_f \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \underset{2\times 2}{S^{-1}} \underset{2\times 1}{\left(R-R_f\right)}$$

# Selecting the Best Portfolio

## - Two-Assets -

You can verify that the denominator of (**E1**) is equal to the sum of the ***z***'s.

Therefore, the solution to the weights of the optimal portfolio ***P**** is:

$z$

**(E3)**
$$\underset{2\times 1}{\begin{bmatrix} w_D^* \\ w_E^* \end{bmatrix}} = \frac{\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}}{z_1 + z_2}$$

# Selecting the Best Portfolio
## - Two-Assets -

**<u>Example</u>:**

$$w_D^* = 0.34, w_E^* = 0.66$$

$$E(r_{p*}) = 4.7\%, \sigma_{p*} = 7.2\%, \boxed{S_{p*} = 0.524} > S_B > S_A$$

This is the "tangency portfolio" ***P****: no other portfolio achieves a higher reward-to-variability (**Sharpe ratio**) with respect to the risk-free rate.

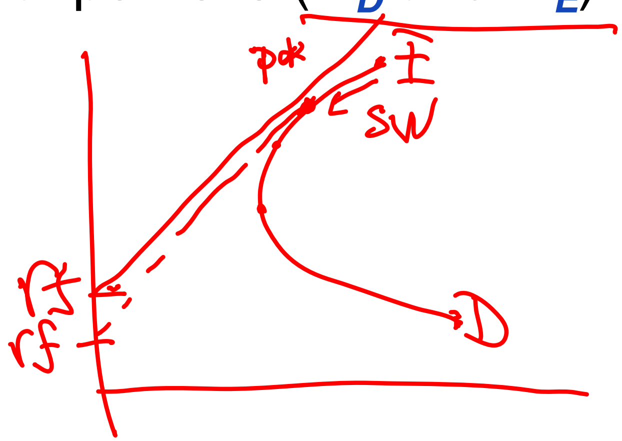
# Selecting the Best Portfolio

## - Two-Assets -

**<u>Questions</u>:**

If the risk-free rate declines (monetary loosening), what happens to…

- the composition of the optimal portfolio ($w_D$ and $w_E$)?
- its mean return?
- its risk?
- its **Sharpe ratio**?



# Selecting the Best Portfolio
## - Two-Assets -

**<u>Answers</u>:**

If the risk-free rate declines (monetary loosening), then…

- Optimal portfolio ***P**** shifts toward the Southwest: $\uparrow w_D$ and $\downarrow w_E$
- Both mean return and risk decline
- **Sharpe ratio** increases

# Recap of Optimal Portfolio
## - Two-Assets -

**Main Ideas**:

- Investor chooses the combination of ***D***, ***E*** that maximizes reward-to-variability relative to the risk-free rate.
- This reward to variability = **Sharpe ratio** = slope of the ***CAL***. 3 ways → opt portf
- Two methods… Excess Ret
  - Algebraic (matrix solution) $\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = S^{-1}\left(R - R_f\right)$ $\begin{bmatrix} w_D^* \\ w_E^* \end{bmatrix} = \dfrac{\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}}{z_1 + z_2}$ sum of z's
  - Numerical (using Solver) S → max S subject $w_D + w_E = 1$ $w_E = 1 - w_D$
- Recall: benefits to diversification depend on (imperfect) correlation between ***D*** and ***E***.

# Recap of Optimal Portfolio

## - Two-Assets -

**Questions**:

If the correlation between ***D*** and ***E*** increases, what happens to…

- the composition of the optimal portfolio ($w_D$ and $w_E$)?
- its mean return?
- its risk?

# Recap of Optimal Portfolio
## - Two-Assets -

**Answers**:

If the correlation between ***D*** and ***E*** increases, then:

- the optimal portfolio ***P**** shifts to the Northeast: $\downarrow w_D$ and $\uparrow w_E$
- (in this case, it means even shorting ***D***)
- Both mean return and risk increase
- **Sharpe ratio** increases slightly (from 0.450 to 0.454)

# Completing the Investor Decision
## - Two-Assets -

**What next?**

We have (1) the tangency portfolio ***P**** and (2) the riskless asset.
But each investor must now decide how much to invest in each.

- Will depend on personal preference (risk aversion or tolerance).

$P^*$

$C$

$rf$

This can be expressed by a utility function:

scaling factor

risk aversion

satisfaction enjoyment

$$U = E(r_C) - 0.5A\sigma_C{}^2$$

(+) return

(−) risk

where ***A*** = degree of risk aversion;
if ***A*** = 0, risk-neutral

# Completing the Investor Decision
## - Two-Assets -

**<u>Individual Investor Choice</u>:**

Maximize utility subject to the tangency portfolio and the riskless asset, to obtain the proportions to be invested in each ($w_{P^*}$ and $w_{rf}$).

The resulting portfolio will be called ***C***.

$$MaxU = E(r_C) - 0.5A\sigma_C^{\ 2}$$

$$= w_{P^*}E\left(r_{P^*}\right) + r_f\left(1 - w_{P^*}\right) - 0.5Aw_{P^*}^2\sigma_{P^*}^2$$

In our example, assuming risk aversion ***A*** = 9.4, solve for $w_{P^*}$:

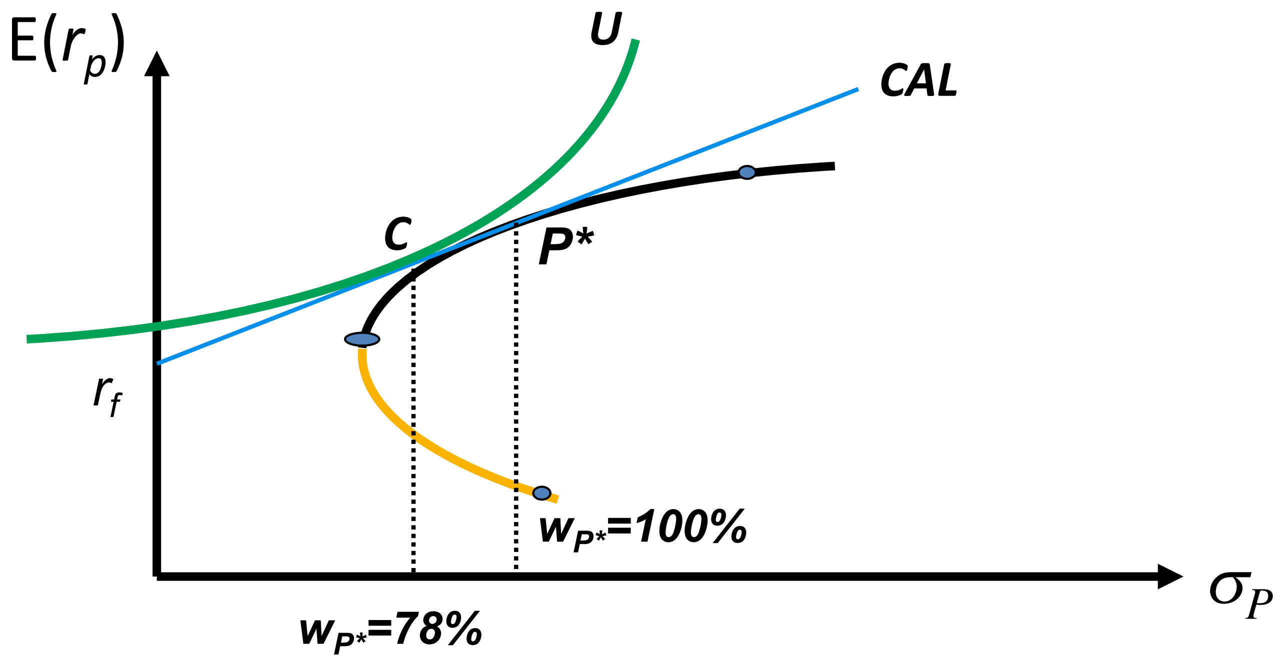
$$\frac{dU}{dw_{P^*}} = 0$$

$$w_{P^*} = \frac{0.0466 - 0.009}{2 \times 0.5 \times 9.4 \times 0.0051} = 78\%$$

in P*

22% in rf

## Completing the Investor Decision
### - Two-Assets -



# Completing the Investor Decision
## - Two-Assets -

**Separation Property:**

- Technical information guides the decision to choose the optimal portfolio of risky assets ***P****:
    - Mean returns, relative to risk-free rate
    - Volatilities
    - Correlation
- The choice of ultimate investor position (how much in ***P****, how much in $r_f$) depends on individual preferences (***A***).
- The two decisions are *separate.*

# Generalizing to n Assets
## - Part 1 -

Maximize reward-to-variability, subject to all weights summing to 1.

The solution, $w_i^*$, is the same matrix as before, now generalized to N assets:

***N***: # risky assets

***R***: The column vector: expected returns

$r_f$: risk-free rate

***w***: The column vector: portfolio shares

***S***: ***N***x***N*** variance-covariance matrix

$$\begin{cases} \underbrace{z}_{(N,1)} = \underbrace{S^{-1}}_{(N,N)} \underbrace{[R - r_f]}_{(N,1)} \\ w_i^* = \dfrac{z_i}{\sum z} \end{cases}$$

# Generalizing to n Assets

## - Part 1 -

**<u>Finding the optimal portfolio, step-by-step</u>:**

#1 - Construct an **Excess Return** (**ER**) matrix for the N assets.

$$ER = \underbrace{\begin{bmatrix} r_{A,1} - \bar{r}_A & r_{B,1} - \bar{r}_B & \cdots & r_{N,1} - \bar{r}_N \\ r_{A,2} - \bar{r}_A & r_{B,2} - \bar{r}_B & \cdots & r_{N,2} - \bar{r}_N \\ \vdots & \vdots & \ddots & \vdots \\ r_{A,T} - \bar{r}_A & r_{B,T} - \bar{r}_B & \cdots & r_{N,T} - \bar{r}_N \end{bmatrix}}_{\text{(T rows, N columns)}}$$

***T***: # observations

***N***: # risky assets

# Generalizing to n Assets

## - Part 1 -

**<u>Finding the optimal portfolio, step-by-step</u>:**

#2 - Multiply ***ER*** by its transpose, divide by the number of time observations, to obtain the **Variance-Covariance Matrix** (***S***).

$$S = \underbrace{\underset{(N,T)}{ER^T} \times \underset{(T,N)}{ER}}_{(N,N)} / T = \underbrace{\begin{bmatrix} \sigma_{rA}^2 & Cov(r_A, r_B) & \cdots & Cov(r_A, r_N) \\ Cov(r_A, r_B) & \sigma_{rB}^2 & \cdots & Cov(r_B, r_N) \\ \vdots & \vdots & \ddots & \vdots \\ Cov(r_A, r_N) & Cov(r_B, r_N) & \cdots & \sigma_{rN}^2 \end{bmatrix}}_{\text{(N rows, N columns)}}$$

# Generalizing to n Assets

## - Part 1 -

**Finding the optimal portfolio, step-by-step:**

#3 - Find the inverse of ***S*** and multiply it by the difference between the mean returns and the risk-free rate (***R*** **-** $r_f$). This gives us the ***z*** vector.

#4 - The optimal portfolio weights $w_i^*$ will be equal to the ***z*** vector divided by the sum of ***z***'s.

$$\begin{cases} \underbrace{z}_{(N,1)} = \underbrace{S^{-1}}_{(N,N)} \underbrace{[R - r_f]}_{(N,1)} \\ \\ w_i^* = \dfrac{z_i}{\sum z} \end{cases}$$

# Generalizing to n Assets

## - Part 2 -

With the optimal portfolio obtained (***P****), compute the (1) expected or mean return, (2) variance, and (3) standard deviation.

Expected or Mean Return: $$\underbrace{E(r_{P*})}_{(1,1)} = \underbrace{W_{P*}^{T}}_{(1,N)} \underbrace{R}_{(N,1)}$$

Variance: $$\underbrace{\sigma_{P*}^{2}}_{(1,1)} = \underbrace{W_{P*}^{T}}_{(1,N)} \underbrace{\Sigma}_{(N,N)} \underbrace{W_{P*}}_{(N,1)}$$

Standard Deviation $$\underbrace{\sigma_{P*}}_{(1,1)} = \sqrt{\underbrace{\sigma_{P*}^{2}}_{(1,1)}}$$

# Building the Entire Frontier
## - n Assets -

- You have computed the optimal portfolio ***P****, which holds for a certain level of the risk-free rate.
- Vary $r_f$ and compute a second optimal portfolio ***P*****.
  - *Can choose any arbitrary* $r_f$
- The frontier can be generated as a series of linear combinations of ***P**** and ***P*****.

# Building the Entire Frontier
## - n Assets -

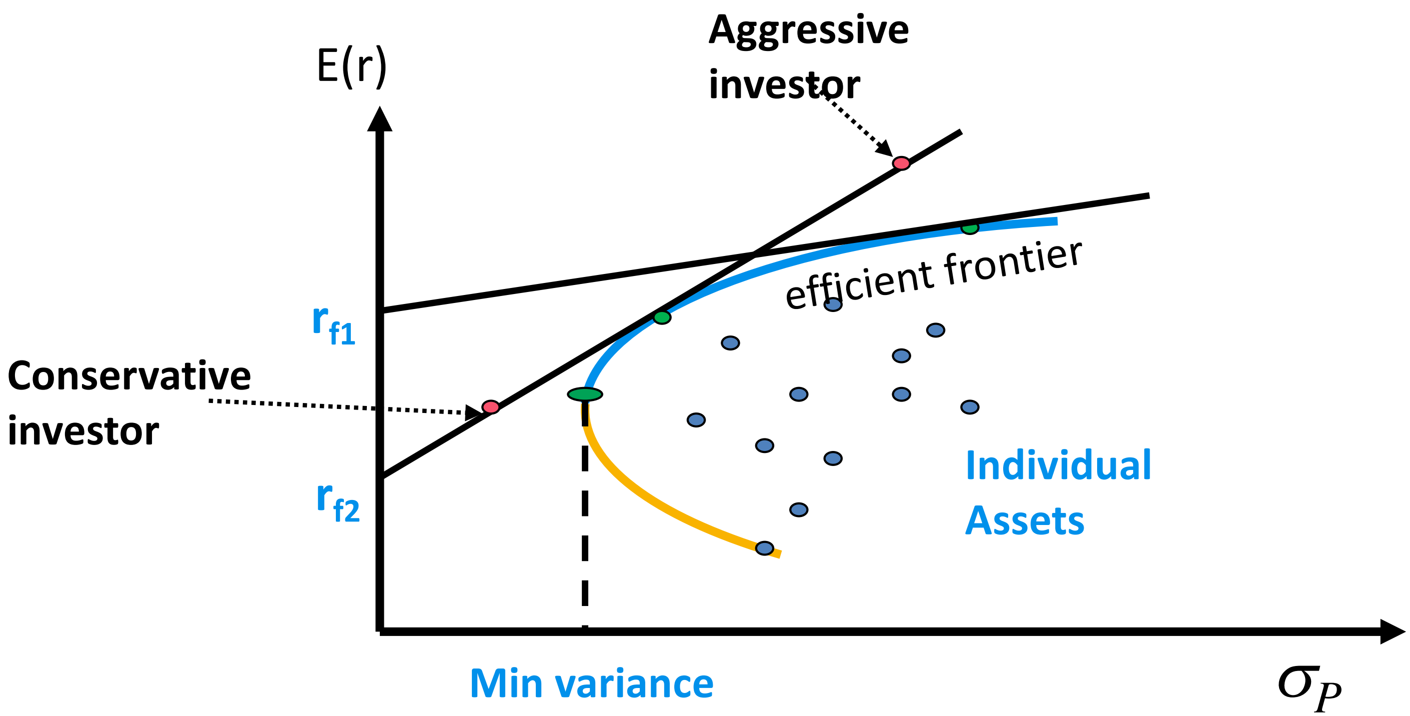
The mean return and standard deviation of each combination of ***P**** and ***P***** can then be computed:

**Mean Return:** $$E(r_{P*,P**}) = \alpha E(r_{P*}) + (1-\alpha)E(r_{P**})$$

**Standard Deviation**: $$\sigma_{EP} = \sqrt{\alpha^2\sigma_{P*}^2 + (1-\alpha)^2\sigma_{P**}^2 + 2\alpha(1-\alpha)Cov(P^*,P^{**})}$$

**Covariance** between two market portfolios: $$\underbrace{Cov(P^*,P^{**})}_{(1,1)} = \underbrace{W_{P*}^T}_{(1,N)} \underbrace{S}_{(N,N)} \underbrace{W_{P**}}_{(N,1)}$$

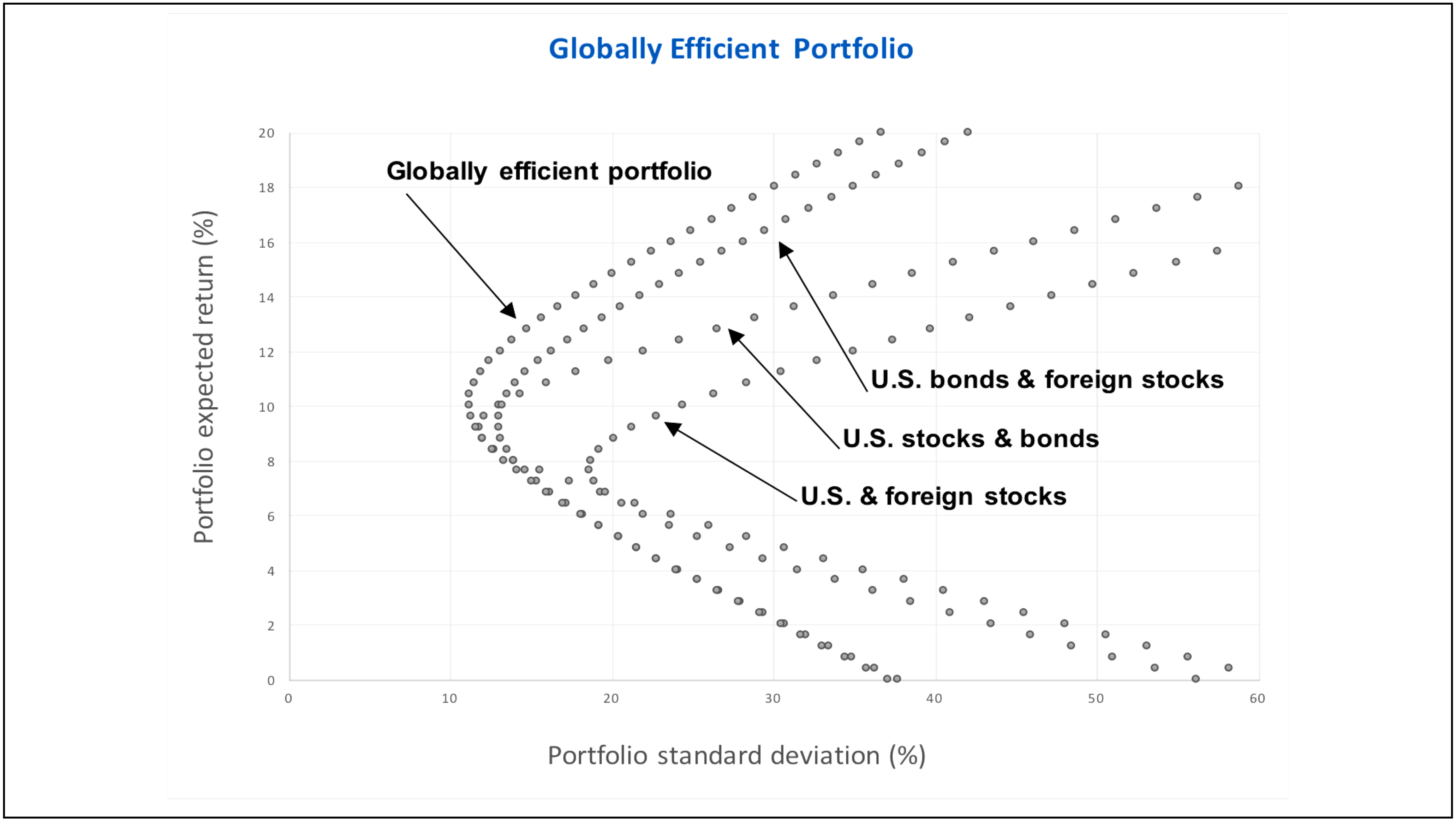
# Building the Entire Frontier
## - n Assets -



# International Diversification
## - Part 1 -

The **main principles** related to diversification within the domestic market continue to apply when diversifying internationally:

- Including additional (international) assets that are imperfectly correlated with domestic assets will produce gains (risk reduction).
- But not all risk can be eliminated.
- Currency fluctuations introduce an additional element (Part 2).



Globally Efficient Portfolio
Globally efficient portfolio
U.S. bonds & foreign stocks
U.S. stocks & bonds
U.S. & foreign stocks
Portfolio expected return (%)
Portfolio standard deviation (%)
0
2
4
6
8
10
12
14
16
18
20
0
10
20
30
40
50
60

# International Diversification
## - Part 2 -

Investing across international borders introduces additional risk-return elements coming from currency fluctuations.

What should an investor do?

1. Nothing.
2. Hedge their foreign currency exposure (in different ways).

# International Diversification
## - Part 2 -

**Example: A Japanese investor wants to buy U.S. stock [Apple, Inc.].**

Must buy dollars today at the spot exchange rate ($S_0$, ¥/$ ) in order to buy the shares at today's price ($P_{A,0}$).

Today's (Nov 30, 2015) cost (in ¥) is therefore:

- $P_{A,0} \times S_0$
- $P_{A,0}$ = **$118.88**, $S_0$ = **123.26 ¥/$**

Cash flow in one year: if unhedged, can sell the shares and exchange US$ for ¥ at the spot rates:

- $P_{A,1} \times S_1$

# International Diversification
## - Part 2 -

**Example: A Japanese investor wants to buy U.S. stock [Apple, Inc.].**

Suppose Apple's stock price rises by 5% in one year:

- $P_{A,1}$ = \$118.88 x (1+ 0.05) = **\$124.82**

Regarding the exchange rate, suppose there are two possible scenarios (6% depreciation or appreciation):

- $S_{1,d}$ = 123.26 x (1 + 0.06) = **130.66 ¥/\$**
- $S_{1,a}$ = 123.26 x (1 - 0.06) = **115.86 ¥/\$**

# International Diversification
## - Part 2 -

**Unhedged Return:**

$$r_U = \left( \frac{P_{A,1} \times S_1 - P_{A,0} \times S_0}{P_{A,0} \times S_0} \right)$$

$$r_U \approx \left( r_A + r_{FX} \right)$$

$r_A$ : return on US stock in \$

$r_{FX}$ : % change in the spot exchange rate (1\$=$S$ Yen)

$$r_{FX} = \frac{S_1 - S_0}{S_0}$$

$$r_{U,d} \left( \frac{124.82 \times 130.66 - 118.88 \times 123.26}{118.88 \times 123.26} \right) = 11.3\% \qquad r_{U,a} = ?$$

# International Diversification

## - Part 2 -

**Full currency hedging:**

Each investment in a foreign stock is fully hedged by a forward position; the investor agrees today to sell at the 1-year forward rate.

$$F_{1,0} = \mathbf{121.23\ ¥/\$}$$

Cash flow (per share) one year from now:

$$CF_{1,d} = (118.88 \times 121.23) + (124.82 - 118.88) \times 130.66 = \mathbf{¥15{,}188}$$

$$CF_{1,a} = (118.82 \times 121.23) + (124.82 - 118.88) \times 115.86 = \mathbf{¥15{,}101}$$

# International Diversification
## - Part 2 -

**Hedged Return:**

$$r_H = \frac{\overbrace{\left[P_{A,0} \times F_{1,0} + (P_{A,1} - P_{A,0}) \times S_1\right]}^{\text{Value in Japanese Yen of the American Stock at time 1}} - \overbrace{P_{A,0} \times S_0}^{\text{Initial Investment}}}{\underbrace{P_{A,0} \times S_0}_{\text{Initial Investment}}}$$

$$r_H - r_U = \frac{F_{1,0} - S_1}{S_0} \approx \text{Forward return}$$

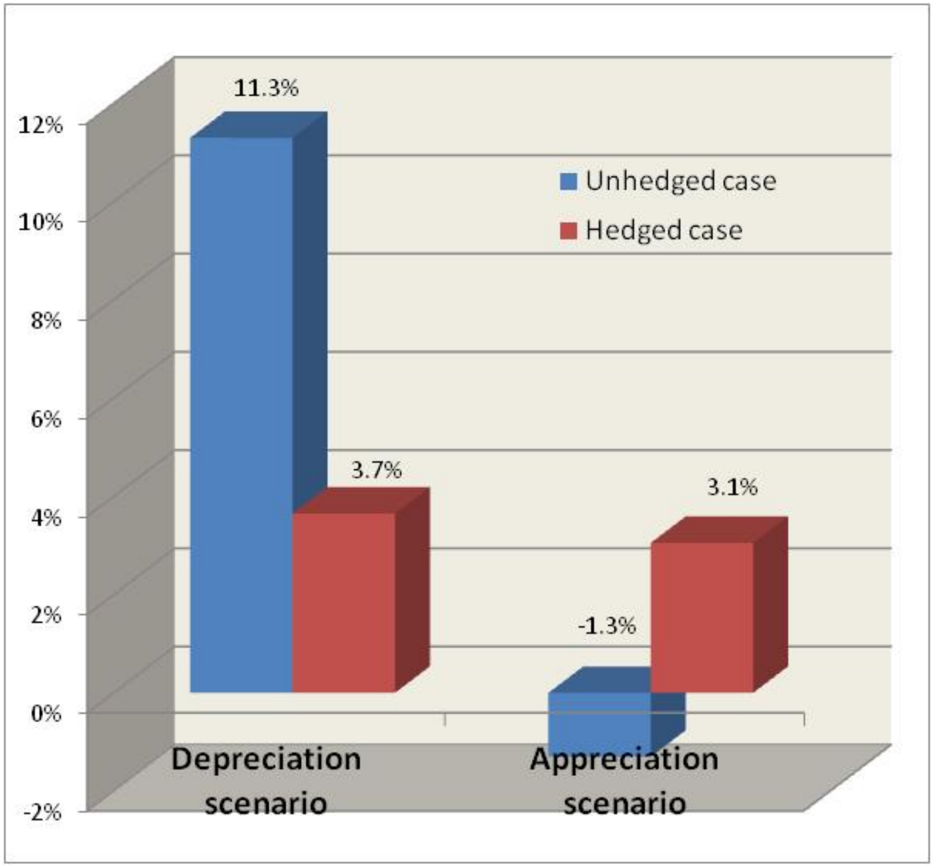
$$r_{H,d} = \left(\frac{(118.88 \times 121.23) + (124.82 - 118.88) \times 130.66 - (118.88 \times 123.26)}{118.88 \times 123.26}\right) = 3.7\%$$

$$r_{H,a} = ?$$

# International Diversification
## - Part 2 -

**<u>Two Strategy Comparison</u>:**

- As expected, much less variation under hedging (although exchange rate risk is not eliminated entirely).
- Hedging misses out on potentially large returns.
- Risk vs Reward yet again.
- Alternative hedging strategies: using Markowitz approach, incorporating $\rho_{s,p}$



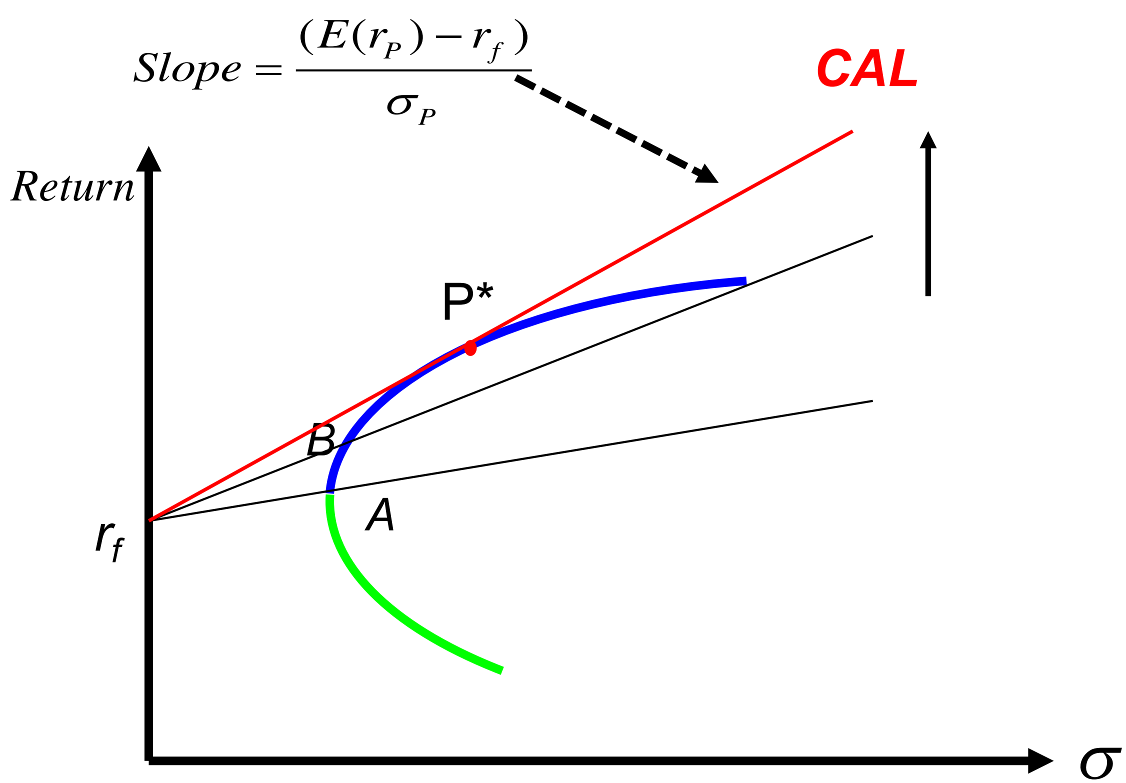
# Wrap-Up

**Portfolio allocation main messages:**

- Diversification benefits come from **imperfect** correlation ($\rho$<1)
  - Var(portfolio) < Weighted average of Var(individual assets)
- If $r_f$ is available, then portfolio choice maximizes excess return (over $r_f$) relative to the additional risk: **P***
  - In other words, maximizes the **Sharpe ratio** or **Reward-to-variability** (slope of **CAL**)
- **Finally,** investor chooses a combination of **P*** and $r_f$ according to risk attitude
- **Separation Property**

# Wrap-Up

**Portfolio allocation problem, in a nutshell:**




By choosing another portfolio (B vs. A), the CAL becomes steeper.

↓

The Sharpe Ratio of the portfolio increases.

↓

This continues until the CAL is tangent to the investment opportunity set.

↓

**The optimal portfolio P* maximizes the Sharpe Ratio**