Financial Market Analysis (FMAx)
Module 2

Bond Pricing

The Relevance to You

You might be…

- You may be an investor.
- Your institution may be an investor.
    - You may be managing a portfolio of foreign assets in a sovereign wealth fund or in a central bank.
- Your institution may be in charge of issuing sovereign bonds.
- Your institution is a financial regulator.

Defining a Bond – 1

A bond is a type of fixed income security. Its promise is to deliver known future cash flows.

- Investor (bondholder) lends money (principal amount) to issuer for a defined period of time, at a variable or fixed interest rate
- In return, bondholder is promised
  - Periodic coupon payments (most of the times paid semiannually); and/or
  - The bond's principal (maturity value/par value/face value) at maturity.

Defining a Bond – 2

Some bond have embedded options.

- **Callable Bond**:
  The issuer can repurchase bond at a specific price before maturity.

- **Putable Bond**:
  Bondholder can sell the issue back to the issuer at par value on designated dates (bond with a put option). Bondholder can change the maturity of the bond.

Review of Central Concepts: Present Value – 1

The Present Value is…

- The value calculated today of a series of expected cash flows discounted at a given interest rate.
- Always less than or equal to the future value, because money has interest-earning potential: time value of money.

$PV \leq FV$

Review of Central Concepts: Present Value – 2

Net Present Value (NPV) of an Investment:

Is the difference between the present value of the cash flows and the initial value of the investment.

$$NPV = \sum_{t=1}^{T} \frac{CF_t}{(1+i)^t} - CF_0 \qquad (2.1)$$

CF_0 = initial value of the investment;

CF_t = cash flow received at time t.

Review of Central Concepts: Rate of Return

Net Present Value (NPV) of an Investment:

- The internal rate of return (IRR) is that particular interest rate that makes NPV equal to zero.
- IRR ensures that the present value of the cash flows is equal to the initial value of the investment:

$$\sum_{t=1}^{T} \frac{CF_t}{(1+i)^t} - CF_0 = 0$$

$$\sum_{t=1}^{T} \frac{CF_t}{(1+i)^t} = CF_0 \qquad (2.2)$$

The Bond Pricing Formula – 1

Bond Price is equal to **Present Value** of all the cash flows you receive if you hold to maturity.

$$P = \sum_{s=1}^{nT} \frac{C}{\left(1+\frac{y}{n}\right)^{s}} + \frac{M}{\left(1+\frac{y}{n}\right)^{nT}} \qquad (2.3)$$

P: bond price;

C: coupon payment (assumed constant);

n: number of coupon payments per year;

T: number of years to maturity;

y: interest rate used to discount the cash flows; yield-to-maturity (YTM) or market required yield;

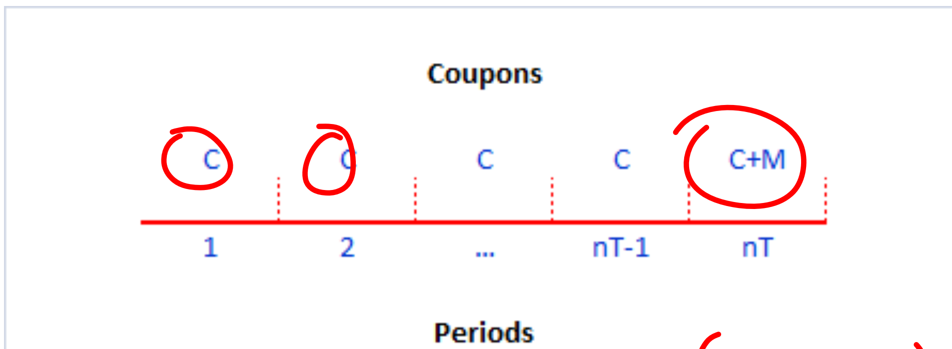
M: par value (or face value, or maturity value) of the bond.

The Bond Pricing Formula – 2

$C \quad T \quad m \qquad m \times T$

Consider a bond paying coupons with frequency **n**.

- Cash flows: coupon **C** paid with frequency n up to year **T**, plus the principal amount **M**, paid at **T**.



$$P = \frac{C}{\left(1+\frac{y}{m}\right)^{1}} + \frac{C}{\left(1+\frac{y}{m}\right)^{2}} + \cdots + \frac{(C+M)}{\left(1+\frac{y}{m}\right)^{mT}}$$

The Bond Pricing Formula – 3

Price equal to **Present Value** of these cash flows:

$$P = \frac{C}{\left(1+\frac{y}{n}\right)} + \frac{C}{\left(1+\frac{y}{n}\right)^2} + \cdots + \frac{C}{\left(1+\frac{y}{n}\right)^{n(T-1)}} + \frac{C+M}{\left(1+\frac{y}{n}\right)^{nT}}$$

$$= \sum_{s=1}^{nT} \frac{C}{\left(1+\frac{y}{n}\right)^s} + \frac{M}{\left(1+\frac{y}{n}\right)^{nT}}$$

$\uparrow y \rightarrow \downarrow P$

$P =$

$$\boxed{= \frac{C}{\left(\frac{y}{n}\right)}\left[1 - \frac{1}{\left(1+\frac{y}{n}\right)^{nT}}\right]} + \frac{M}{\left(1+\frac{y}{n}\right)^{nT}} \qquad (2.4)$$

The Bond Pricing Formula – 4

$$\underbrace{P = \sum_{s=1}^{nT} \frac{C}{\left(1+\frac{y}{n}\right)^s} + \frac{M}{\left(1+\frac{y}{n}\right)^{nT}}}$$

Note three special cases:

- A zero-coupon bond: $$P = \frac{M}{\left(1+\dfrac{y}{n}\right)^{nT}}$$

  $C = 0$

- An annuity (coupons only): $\rightarrow$ $$P = \frac{C}{\left(\dfrac{y}{n}\right)}\left[1 - \frac{1}{\left(1+\dfrac{y}{n}\right)^{nT}}\right] = \sum_{s=1}^{nT} \frac{C}{\left(1+\frac{y}{n}\right)^s}$$

  $M = 0$

- A perpetuity (coupons only, infinite maturity): $$P = \frac{C}{\left(\dfrac{y}{n}\right)}$$

  $T \to \infty$

The Bond Pricing Formula – 5

The bond pricing formula tells us…

- The higher the coupon rate, the higher the coupon payments, and the higher the bond price.

- The higher the YTM, the lower the bond price.
  The higher the YTM, the more you discount the bond cash flows; this ultimately lowers the bond price.

Yield Measures:
The Yield-to-Maturity – 1

The most important bond yield measure is the yield-to-maturity (YTM).

The YTM is the internal rate of return (IRR) of a bond investment.

- The IRR is the interest rate which ensures that the net present value (NPV) of an investment is equal to zero.
- Hence, the YTM is the interest rate which ensures that the NPV of a bond investment is equal to zero.

Yield Measures:
The Yield-to-Maturity – 2

Let us compute the present value of the bond cash flows assuming that the annual YTM is equal to 10%. The market bond price is US$ 655.9.

Recall the pricing formula (2.4):

$$P = \sum_{s=1}^{nT} \frac{C}{\left(1+\frac{y}{n}\right)^{s}} + \frac{M}{\left(1+\frac{y}{n}\right)^{nT}} = \frac{C}{\left(\frac{y}{n}\right)}\left[1 - \frac{1}{\left(1+\frac{y}{n}\right)^{nT}}\right] + \frac{M}{\left(1+\frac{y}{n}\right)^{nT}}$$

Yield Measures: The Yield-to-Maturity – 3

C denotes the coupon payment and is calculated as follows:

$$C = \frac{c * M}{n} \qquad (2.5)$$

$n = 2$ $\qquad C = \frac{cM}{2}$

c = the coupon rate
M = the face value of the bond
n = the number of intra-year coupon payments: with semiannual coupon payments, n=2.

Yield Measures:
The Yield-to-Maturity – 4

We can use (2.4) and (2.5) to compute the bond price, assuming that the annual YTM is 10%:

$$P = \sum_{s=1}^{2*15} \frac{\left(\frac{0.07*1000}{2}\right)}{\left(1+\frac{0.10}{2}\right)^{s}} + \frac{1000}{\left(1+\frac{0.10}{2}\right)^{2*15}} = \frac{35}{\frac{0.10}{2}} * \left[1 - \frac{1}{\left(1+\frac{0.10}{2}\right)^{30}}\right] + \frac{1000}{\left(1+\frac{0.10}{2}\right)^{30}}$$

$$= \underline{769.4} > 655.9$$

The computed price is higher than the market bond price: 10% cannot be the YTM.

Yield Measures:
The Yield-to-Maturity – 5

Let us compute the bond price assuming that the annual YTM is 12%:

$$P=\sum_{s=1}^{2*15}\frac{\left(\frac{0.07*1000}{2}\right)}{\left(1+\frac{0.12}{2}\right)^{s}}+\frac{1000}{\left(1+\frac{0.12}{2}\right)^{2*15}}=\frac{35}{\frac{0.12}{2}}*\left[1-\frac{1}{\left(1+\frac{0.12}{2}\right)^{30}}\right]+\frac{1000}{\left(1+\frac{0.12}{2}\right)^{30}}$$

$$=655.9$$

The annual YTM is 12%, as the computed bond price is equal to the actual market price (US$655.9).

Yield Measures:
The Yield-to-Maturity – 6

Finally, let us recall:

$$P = \frac{C}{\left(1+\frac{y}{n}\right)} + \frac{C}{\left(1+\frac{y}{n}\right)^{2}} + \ldots + \frac{C}{\left(1+\frac{y}{n}\right)^{n(T-1)}} + \frac{C+M}{\left(1+\frac{y}{n}\right)^{nT}}$$

C and M are known.

- If you know P, you can derive the YTM (y).
- If you know the YTM, you can derive P.

The YTM is an alternative way to express the bond price.

Yield Measures: Bond-Equivalent Yield

The Bond Equivalent Yield:

In the previous example, the annual YTM was 12%, hence the semiannual YTM was 6%.

More generally, the semiannual YTM is annualized using simple interest techniques:

- To get the annual YTM, you have to multiply the semiannual YTM by 2.

$$\boxed{YTM} = \frac{YTM}{2} \times 2$$

Yield Measures: Effective Yield

The effective annual yield-to-maturity Y however, takes into account the compound interest as well.

$$Y = \left(1 + \frac{YTM}{2}\right)^2 - 1 \Rightarrow (1+Y) = \left(1 + \frac{YTM}{2}\right)^2$$

$$Y = (1 + 0.06)^2 - 1 = 0.1236 = \boxed{12.36\%}$$

$$Y = 12.36\% > YTM = 12\%$$

Yield Measures: The Current Yield

Another bond yield measure is the **current yield**.

- The current yield is calculated as the annual coupon payment divided by the bond price;
- For example, the current yield for an 18-year, 6% coupon bond selling for US$700.89 per US$1,000 face value is…

$$CY = \frac{60}{700.89} = 0.0856 = 8.56\%$$

Yield Measures: The Yield-to-Call

Callable bond: the issuer can repurchase bond at a specific price before maturity.

- YTM is calculated assuming that the bond will be held until maturity.
- How to calculate the yield of a bond which is callable?
- We need to calculate the **yield-to-call** (YTC).

Yield Measures:
The Yield-to-Put and Yield-to-Worst

Yield-to-put (YTP):

It is the yield on a putable bond. Price of putable bond is like the price of a standard coupon bond, but with a modified maturity (T^*), and where the maturity value M is the put price (PP).

$$P_{PB} = \frac{C}{\left(1+\frac{y}{n}\right)} + \frac{C}{\left(1+\frac{y}{n}\right)^2} + \ldots + \frac{C}{\left(1+\frac{y}{n}\right)^{2T^*-1}} + \frac{C+PP}{\left(1+\frac{y}{n}\right)^{2T^*}}$$

Yield-to-worst (YTW):

It is the smallest of the possible yield measures that can be computed for a given bond issue.

Yield Measures: Bond Investment Risks

Interest Rate Risk:

Risk that the bond value changes following a change in interest rates.

- An increase in the YTM after purchase of the bond leads to a capital loss.

Reinvestment Risk:

Risk of not being able to reinvest all future coupon payments at the initial YTM.

- Zeros have no reinvestment risk.

Default Risk:

Risk that the bond issuer is unable to make the required payments to the bondholder.

Bond Pricing
Example 1 – Coupon Bond

Applying the formula:

$$P=\sum_{s=1}^{nT}\frac{C}{\left(1+\frac{y}{n}\right)^{s}}+\frac{M}{\left(1+\frac{y}{n}\right)^{nT}}=\frac{C}{\left(\frac{y}{n}\right)}\left[1-\frac{1}{\left(1+\frac{y}{n}\right)^{nT}}\right]+\frac{M}{\left(1+\frac{y}{n}\right)^{nT}}$$

$$=\sum_{s=1}^{40}\frac{45}{\left(1+\frac{0.06}{2}\right)^{s}}+\frac{1000}{\left(1+\frac{0.06}{2}\right)^{40}}=\frac{45}{\left(\frac{0.06}{2}\right)}\left[1-\frac{1}{\left(1+\frac{0.06}{2}\right)^{40}}\right]+\frac{1000}{\left(1+\frac{0.06}{2}\right)^{40}}$$

$$=1{,}040.2+306.5=\underline{1{,}346.7}$$

Alternatively, you could use Excel:

- Write out the cash flows over time and discount them back to the present, or...
- Use the PRICE function
- Repeat the exercise, assuming that the bond pays 9% annual coupons.

Bond Pricing
Example 2 – Zero-Coupon Bond

Consider now **zero-coupon bonds** ("zeros").

- Zero-coupon bonds pay no coupons.
- With no coupons, the pricing formula (2.4) simplifies to:

$$P_Z = \frac{M}{\left(1+\frac{y}{n}\right)^{nT}}$$

Bond Pricing
Example 2 – Zero-Coupon Bond

For example, consider a 20-year zero-coupon bond with a face value of US$1,000.

- Assume that the market required yield (YTM) is 6%.
- What is the price of the bond?

40 = 2 × 20

$$P_Z = \frac{1000}{\left(1+\frac{0.06}{2}\right)^{40}} = 306.6$$

Premium, Par, and Discount Bonds – 1

YTM	Coupon rate	Maturity value	Bond price
6%	9%	US$1,000	US$1,346.7
7%	9%	US$1,000	US$1,213.6
8%	9%	US$1,000	US$1,099
9%	9%	US$1,000	US$1,000
10%	9%	US$1,000	US$914.2

Premium, Par, and Discount Bonds – 2

- If coupon rate = YTM ➔ bond price = maturity value

  The bond is trading **at par**.

- If coupon rate > YTM ➔ bond price > maturity value

  The bond is trading **at a premium**.

- If coupon rate < YTM ➔ bond price < maturity value

  The bond is trading **at a discount**.

Premium, Par, and Discount Bonds – 3

If coupon rate < YTM ➔ bond price < maturity value; the bond is trading at a **discount**.

Investors purchasing the bond receive interest payments which are **below** the required market return.

- They are prepared to hold the bond only if they can purchase the bond at a discount, only if the bond price is below the par value.

Premium, Par, and Discount Bonds – 4

If coupon rate > YTM ➔ bond price > maturity value: bond is trading at a **premium**.

Investors purchasing the bond receive interest payments which are **above** the required market return.

- Prepared to pay a premium to hold that bond, will get more interest income than if lending at current market rate.

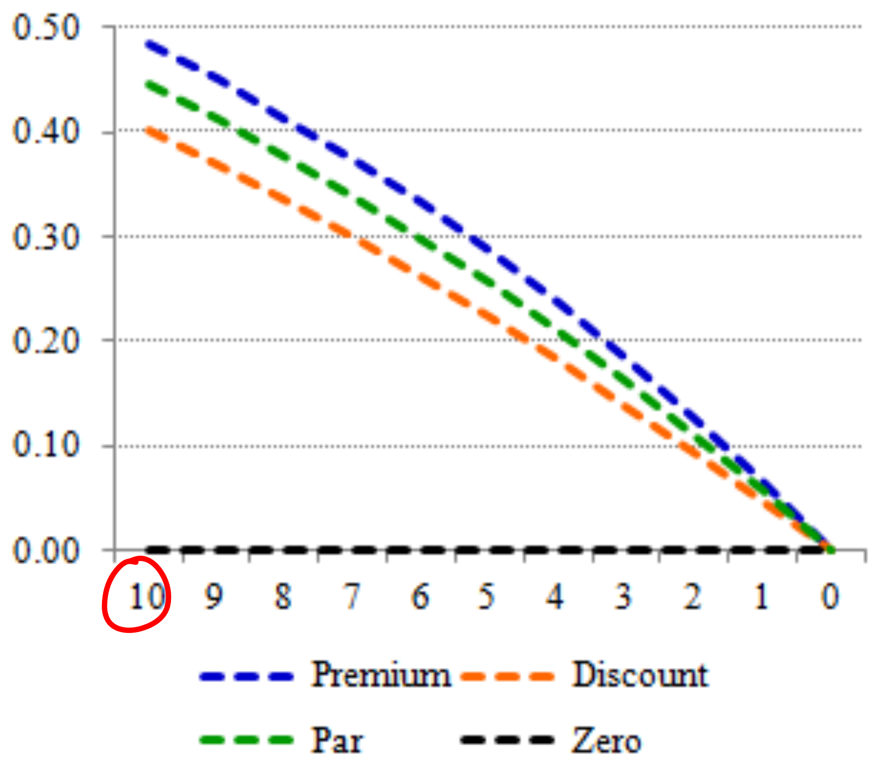
Premium, Par, and Discount Bonds – 5

The only difference is:

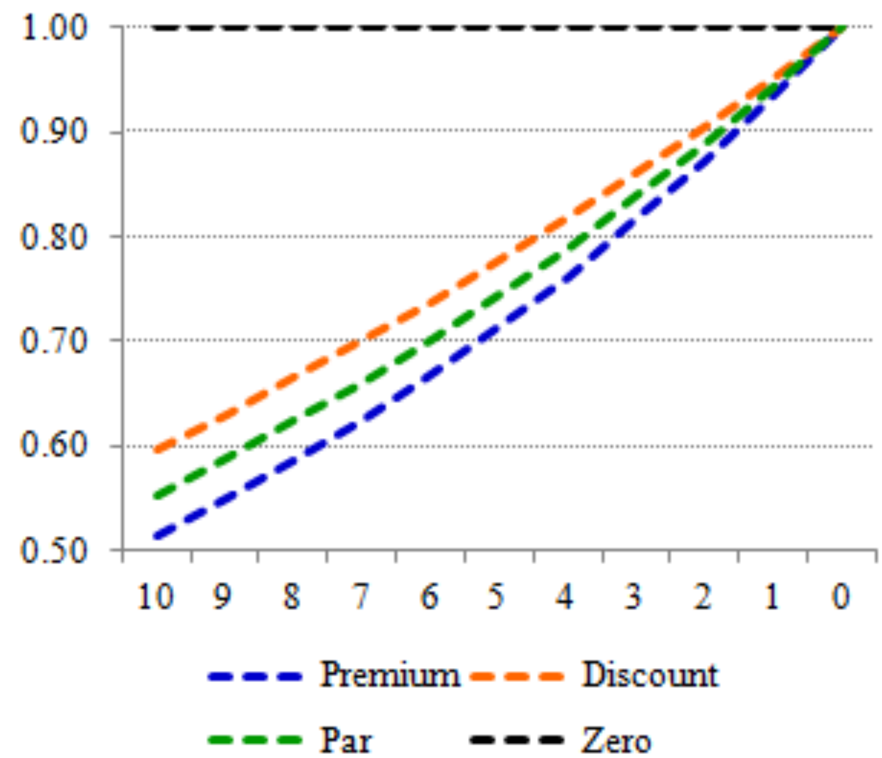
- Compared to discount and par, **premium** bonds get more return from the coupons through time (see next slide).
- Compared to par and premium, **discount** bonds get more return from the principal through time (capital gains).
- Zeros get **all of their return** from capital gains.

Premium, Par, and Discount Bonds – 6

Ratio between discounted coupon payments and bond price through time.



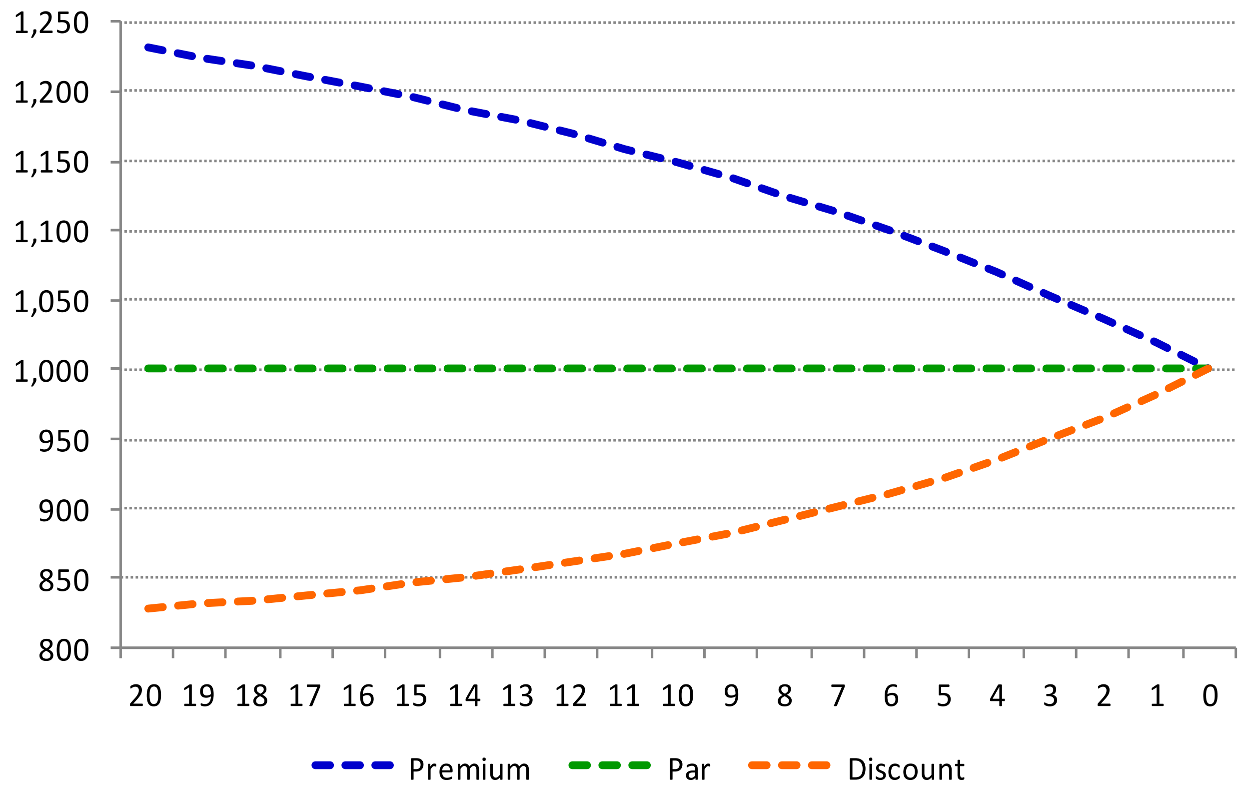
Ratio between discounted principal and bond price through time.



The Time Path of the Bond Price – 1

Consider the following question:

How does the bond price change as we approach maturity?



The Time Path of the Bond Price – 2

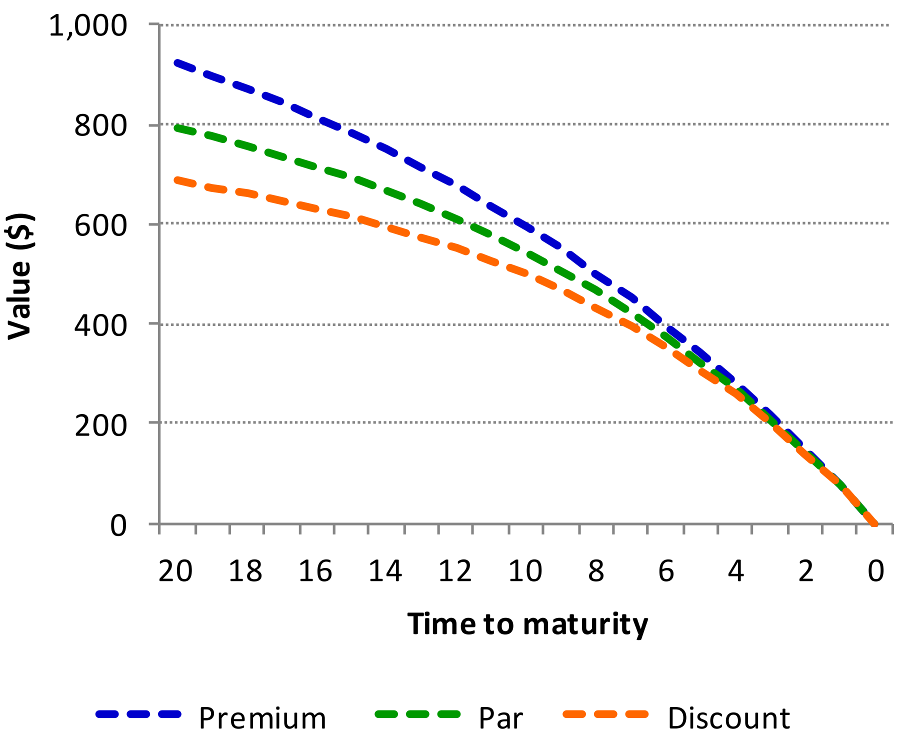
Let us remember…

$$P = \sum_{s=1}^{nT} \frac{C}{\left(1+\frac{y}{n}\right)^{s}} + \frac{M}{\left(1+\frac{y}{n}\right)^{nT}}$$

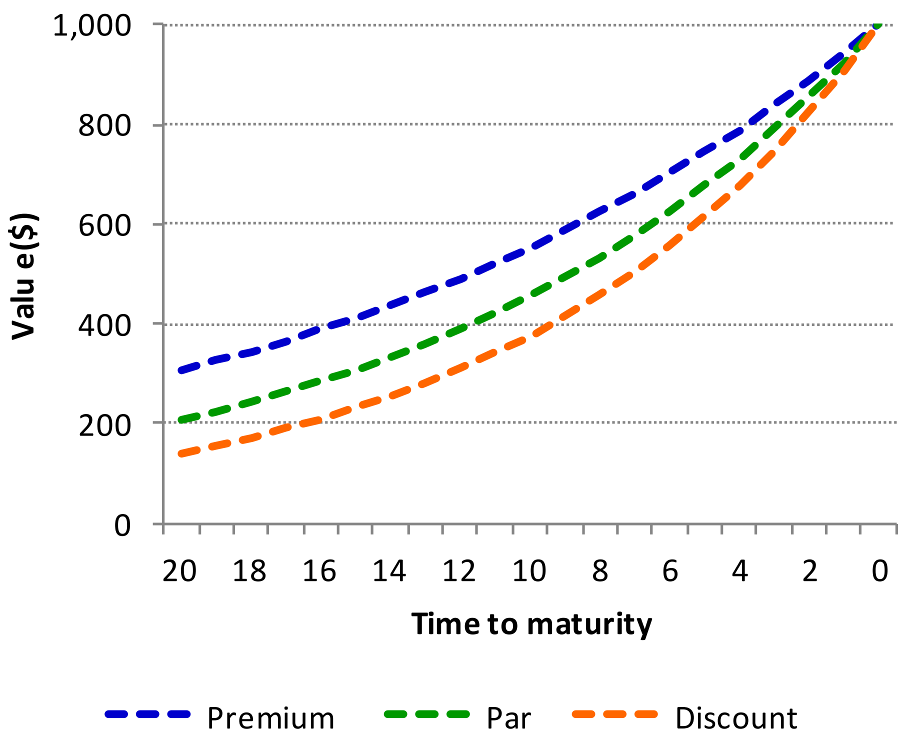
$\boxed{P \to M}$

The Time Path of the Bond Price – 3

Time path of discounted coupon payments:



Time path of discounted face value:



Pricing and Bond Coupon Dates – 1

So far, we considered only pricing bonds at issuance. However, most transactions involve purchases in the secondary market, with **existing bonds**.

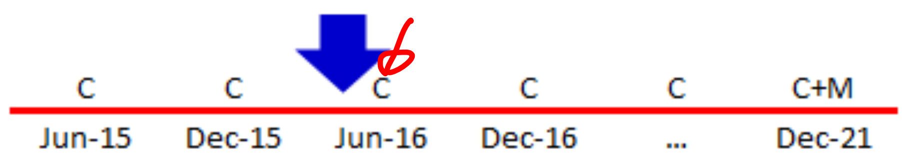
- These generally take place in between coupon dates
- How to price them?

Pricing and Bond Coupon Dates – 2

- Price should include compensation that the buyer must give to the seller for the portion of the next coupon that the seller has earned but will not receive.
- This is called **accrued interest**

For example…

- Suppose coupon payments are at end-June and end-December, and you are buying the bond at end-May 2016.
- Hence, the seller has "accrued" 5 out of the 6 months of the end-June coupon payment, and therefore is entitled to receive 5/6 of the coupon.



$\frac{5}{6} \cdot C$

Pricing and Bond Coupon Dates – 3

Accrued interest (AI) depends on the number of days between the last coupon payment and the date when the bond is traded as a portion of the number of days in the coupon period.

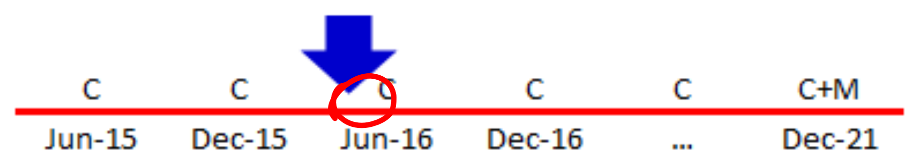
$$AI = C^* \left(\frac{\text{Number of days between last coupon payment and trading date}}{\text{Number of days in coupon period}} \right) = C^* w \qquad (2.6)$$

- In the previous example, w=5/6
- Also, the day/month convention will matter.

Pricing and Bond Coupon Dates – 4

In considering, "How to price a bond that is traded in between coupon dates?"

- We always have to discount cash flows back to the present
- The only complication is that, because of where you are in the timeline, you have to consider fractions of coupon periods (in this example, semesters) for discounting the cash flows.



$$P_B^D = \sum_{s=0}^{nT-1} \frac{C}{\left(1+\frac{y}{n}\right)^{s+(1-w)}} + \frac{M}{\left(1+\frac{y}{n}\right)^{nT-1+(1-w)}}$$

Pricing and Bond Coupon Dates – 5

In the financial press, a bond price is typically quoted net of AI; this price is called clean (or flat) price.

The actual dirty (or invoice) price that a buyer pays for the bond is equal to the clean price plus AI

$$P^D = P^C + AI$$

When settlement date and coupon date are the same, dirty and clean prices are equal.

Pricing and Bond Coupon Dates – 6

Step 1 - Accrued Interest:

$$AI = \frac{0.0675 * 100}{2} * \left(\frac{135}{365} * 2\right) = 2.50$$

Step 2 - Dirty Price:

$$P_B^D = \sum_{s=0}^{T-1} \frac{C}{(1+y)^{s+(1-w)}} + \frac{M}{(1+y)^{T-1+(1-w)}}$$

$$P_B^D = \sum_{s=0}^{38-1} \frac{10}{\left(1+\frac{0.0515}{2}\right)^{s+0.26}} + \frac{100}{\left(1+\frac{0.0515}{2}\right)^{38-1+0.26}} = 121.52$$

Step 3 - Clean Price:

$$P_B^C = P_B^D - AI = 121.52 - 2.50 = 119.02$$

A Yield for a Bond Portfolio – 1

Step 1: We first calculate the semiannual coupon payments for each bond, where M is the maturity value, c is the coupon rate and n the frequency of coupon payments.

$$C = \frac{c * M}{n}$$

$$C = \frac{c * M}{2}$$

Bond	Coupon rate	Maturity (years)	Maturity value ($)	Semiannual coupon payment	Price ($)	YTM
A	7.0%	5	10,000,000	**350,000**		6.0%
B	10.5%	7	20,000,000	**1,050,000**		10.5%
C	6.0%	3	30,000,000	**900,000**		8.5%
Total						

A Yield for a Bond Portfolio – 2

Step 2: We calculate the price of each bond in the portfolio using the pricing formula.

$$P = \frac{nC}{y}\left[1-\left(1+\frac{y}{n}\right)^{-nT}\right]+M\left(1+\frac{y}{n}\right)^{-nT}$$

Bond	Coupon rate	Maturity (years)	Maturity value ($)	Semiannual coupon payment	Price ($)	YTM
A	7.0%	5	10,000,000	350,000	**10,426,510**	6.0%
B	10.5%	7	20,000,000	1,050,000	**20,000,000**	10.5%
C	6.0%	3	30,000,000	900,000	**28,050,098**	8.5%
Total					**58,476,608**	

A Yield for a Bond Portfolio – 3

Step 3: Calculate the cash flows stemming from all bonds in the portfolio.

- The **portfolio YTM** will be the rate at which the discounted value of all cash flows is equal to the market value of the portfolio (the sum of prices).
- The portfolio YTM is 8.97% (obtained with Goal Seek).
- You can verify that the portfolio YTM is **not** the average of the YTMs of the individual bonds.

!!!

A Yield for a Bond Portfolio – 4

Period	Bond A cash flow	Bond B cash flow	Bond C cash flow	Portfolio market value	Present value of period cash flow
1	350,000	1,050,000	900,000	2,300,000	2,201,224
2	350,000	1,050,000	900,000	2,300,000	2,106,691
3	350,000	1,050,000	900,000	2,300,000	2,016,217
4	350,000	1,050,000	900,000	2,300,000	1,929,629
5	350,000	1,050,000	900,000	2,300,000	1,846,759
6	350,000	1,050,000	30,900,000	32,300,000	24,821,118
7	350,000	1,050,000		1,400,000	1,029,635
8	350,000	1,050,000		1,400,000	985,416
9	350,000	1,050,000		1,400,000	943,097
10	10,350,000	1,050,000		11,400,000	7,349,699
11		1,050,000		1,050,000	647,874
12		1,050,000		1,050,000	620,050
13		1,050,000		1,050,000	593,422
14		21,050,000		21,050,000	11,385,777
Total					**58,476,608**
YTM for the portfolio	**8.97%**				

Total Return Analysis Concepts – 1

Holding period of the bond is called **investment horizon**.

- We need to perform total return analysis (TRA).
- To find the Total Return, we need to compute the Total Future Value (TFV): the value of the bond investment at the end of the investment horizon.

Total Return Analysis Concepts – 2

Information we need to calculate TFV:

- The investment horizon.
- The reinvestment rate of coupon payments (which need not be constant or equal to YTM).
- The market required yield at the end of the investment horizon (which may not be equal to the rate today).

Total Return Analysis Concepts – 3

TFV is the sum of two components:

1. The future value of the coupon payments plus the interest-on-interest income at the end of the investment horizon.

2. The projected price of the bond at the end of the investment horizon.

Thus TRA computes the return that would be achieved by buying the bond today and getting the TFV at the end of the investment horizon.

Total Return Analysis Mechanics – 1

Investment horizon 4YS

Start End Maturity

1. Future value of coupon payments plus interest-on-interest income

S

\+ 2. Projected price of the bond

P

= TFV of the bond investment

Total Return Analysis Mechanics – 2

RR

Computing TFV (semi-annual coupon payments)

Step 1: Find the future value of the coupon payments plus the interest-on-interest income at the end of the horizon (S), and the purchase price of the bond (P):

$$S = C\left(1+\frac{rr}{2}\right)^{2t-1} + C\left(1+\frac{rr}{2}\right)^{2t-2} + \;---\; + C\left(1+\frac{rr}{2}\right) + C$$

$$S = \frac{C}{\left(\frac{rr}{2}\right)}\left[\left(1+\frac{rr}{2}\right)^{2t} - 1\right] = \frac{2C}{rr}\left[\left(1+\frac{rr}{2}\right)^{2t} - 1\right] \qquad (2.7)$$

$$P = \sum_{s=1}^{2T} \frac{C}{\left(1+\frac{y}{2}\right)^{s}} + \frac{M}{\left(1+\frac{y}{2}\right)^{2T}}$$

C is the semiannual coupon payment;

t is the investment horizon (number of years);

rr is the reinvestment rate.

1 2 3 2t-1 2t

C C C C C

$$C\left(1+\frac{RR}{2}\right)^{2t-1} + C\cdot\left(1+\frac{RR}{2}\right)^{2t-2} + \cdots + C\left(1+\frac{RR}{2}\right)^{1} + C$$

Total Return Analysis Mechanics – 3

Step 2: Find the projected sale price of the bond at the end of the investment horizon.

(1)

$$B = \sum_{s=1}^{2m} \frac{C}{\left(1+\frac{y}{2}\right)^{s}} + \frac{M}{\left(1+\frac{y}{2}\right)^{2m}} \qquad (2.8)$$

$m < T$

m = the number of years left before maturity at the end of the investment horizon (m<T)

y = is the end-of-horizon market required yield.

Total Return Analysis Mechanics – 4

Step 3: Add up (2.7) and (2.8).

$$S = \frac{C}{\left(\frac{rr}{2}\right)}\left[\left(1+\frac{rr}{2}\right)^{2t} - 1\right] = \frac{2C}{rr}\left[\left(1+\frac{rr}{2}\right)^{2t} - 1\right]$$

$$B = \sum_{s=1}^{2m} \frac{C}{\left(1+\frac{y}{2}\right)^{s}} + \frac{M}{\left(1+\frac{y}{2}\right)^{2m}}$$

$$TFV = S + B = \frac{2C}{rr}\left[\left(1+\frac{rr}{2}\right)^{2t} - 1\right] + \sum_{s=1}^{2m} \frac{C}{\left(1+\frac{y}{2}\right)^{s}} + \frac{M}{\left(1+\frac{y}{2}\right)^{2m}} \qquad (2.9)$$

(2.9) denotes TR: return achieved by an investment that purchases the bond today at its market price and receives **TFV** at the end of the investment horizon (hence similar to the return on a zero).

Total Return Analysis
An Example – 1

The Future Value:

$$S = \frac{C}{\left(\frac{rr}{2}\right)}\left[\left(1+\frac{rr}{2}\right)^{2t}-1\right] = \frac{40}{0.03}\left[\left(1.03\right)^{6}-1\right] = 258.74$$

$$B = \frac{C}{\left(\frac{y}{2}\right)}\left[1-\frac{1}{\left(1+\frac{y}{2}\right)^{2m}}\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{2m}} = 1{,}098.50$$

TFV = 258.74 + 1,098.50 = 1,357.24

Total Return Analysis
An Example – 2

The Purchase Price:

$$P = \sum_{s=1}^{2T} \frac{C}{\left(1+\frac{y}{2}\right)^{s}} + \frac{M}{\left(1+\frac{y}{2}\right)^{2T}}$$

$$= \sum_{t=1}^{40} \frac{\frac{0.08 * 1000}{2}}{\left(1+\frac{0.10}{2}\right)^{t}} + \frac{1000}{\left(1+\frac{0.10}{2}\right)^{40}}$$

$$= \sum_{t=1}^{40} \frac{40}{(1+0.05)^{t}} + \frac{1000}{(1+0.05)^{40}} = 828.4$$

Total Return Analysis
An Example – 3

$$TFV = PP\left(1+\frac{r}{2}\right)^6$$

TR of the Bond Investment:

$$\frac{r}{2} = \left(\frac{TFV}{PP}\right)^{1/6} - 1$$

Semi-annual rate of return

$$\left(\frac{\text{TFV}}{\text{Purchase price}}\right)^{\frac{1}{\text{year}*2}}-1=\left(\frac{1,357.24}{828.4}\right)^{\frac{1}{6}}-1=0.0858=8.58\%$$

Annualized rate of return

$$(\text{Semi-annual rate of return}*2)=0.0858*2=0.1715=17.15\%$$

Effective annualized rate of return

$$\left[(1+\text{Semi-annual rate of return})^2-1\right]=(1.0858)^2-1=0.1789=17.89\%$$

Module Wrap-Up – 1

The price of a bond is equal to the present value of its cash flows:

$$P = \sum_{s=1}^{nT} \frac{C}{\left(1+\frac{y}{n}\right)^{s}} + \frac{M}{\left(1+\frac{y}{n}\right)^{nT}} = \frac{C}{\left(\frac{y}{n}\right)}\left[1 - \frac{1}{\left(1+\frac{y}{n}\right)^{nT}}\right] + \frac{M}{\left(1+\frac{y}{n}\right)^{nT}}$$

P = bond price

C = coupon payment

M = face value/par value/maturity value

y = semiannual yield to maturity

T = number of years to maturity

n = number of coupon payments per year.

Module Wrap-Up – 2

The YTM is another way to express the bond price.

YTM

1. **Par** bond: coupon rate = y.
2. **Discount** bond: coupon rate < y.
3. **Premium** bond: coupon rate > y.

~~For the same issuing institution, same market, and same maturity, all three types of bonds will have the same YTM.~~

Module Wrap-Up – 3

When buying a bond in between coupon payments:

- Clean price + AI = Sum of Discounted Cash Flows = Dirty Price
- Accrued Interest (AI) = Coupon × (# days from last coupon to settlement date) / (# days in coupon period).

Note: If buying a bond with the intention of selling before maturity, **total return** includes coupon payments and their reinvestment (interest-on-interest), plus the sale price at the end of the horizon (expected capital gains).