# Financial Market Analysis (FMAx)
# Module 0

## Pre-Course Basics

## Financial Mathematics

# The Time Value of Money – 1

**Claim:** "A dollar today is more valuable than a dollar received in the future"

**Why?**

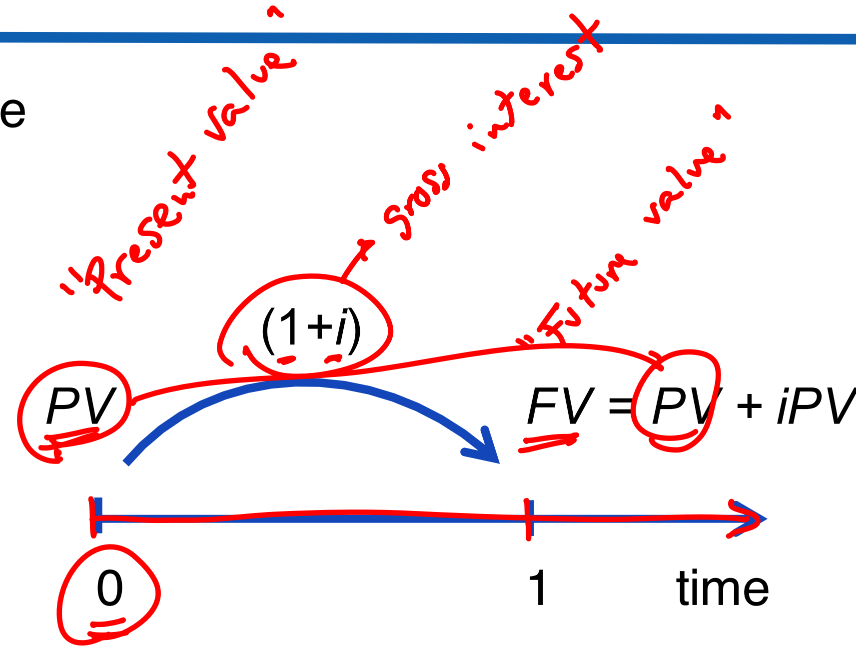
- People have "time preference"
- Inflation erodes the value of money over time
- There is uncertainty about the future

# The Time Value of Money – 2

Interest rates are the **price of money** over time

Interest Rates reflect:

- People's time preference
- Expectations about inflation
- Uncertainty about future events




$$\boxed{FV = PV \times (1+i)}$$

# The Time Value of Money – 3

| Interest Rates reflect | Nominal Interest Rates contain | $i$ |
|---|---|---|
| People's "time preference" | Real interest rate (a real remuneration or return) | $r$ |
| Erosion in the value of money | Expectations about inflation | $\pi^e$ |
| Uncertainty about future events | Risk premium | $\phi$ |

$$\underbrace{\boxed{(1+i)}}_{\text{nominal}} = \boxed{(1+r)\times(1+\pi^e)\times(1+\phi)}$$

# Simple and Compound Interest

Interest earned on an instrument depends on…

- **Interest Rate**
  Usually expressed per year
- **Payment frequency**
  Interest might be paid out periodically before maturity, so that it can be reinvested
- **Maturity**
  Time (i.e., expressed in years $t$)

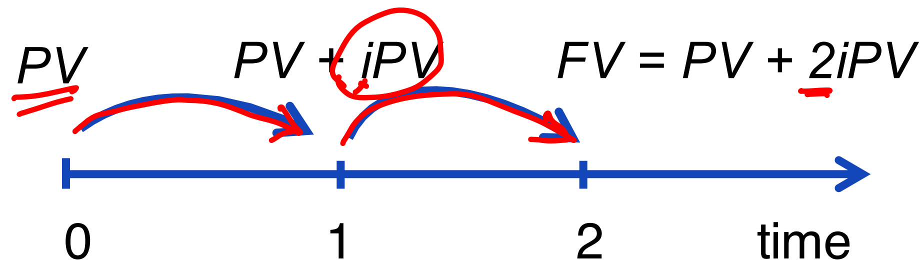
**Simple Interest** – interest does not earn interest

- Short-term instruments ($t < 1$ year) quoted on simple interest basis

**Compound Interest** – interest-on-interest

## Simple Interest

Interest **does not earn** interest




$$FV = PV(1+ti)$$

# An Example: Simple Interest – 1

Deposit $100 at 8 percent for 92 days, what is the end balance in the account?

8%

100 → 102.02 → Int $2.02

0 — 92 days

**Solution:**

- Interest rate is annual, but maturity is 92 days
- Daily interest rate 0.08/365 = $\boxed{0.00021918}$
- At end of 92 days, will receive
- Total proceeds (future value) under simple interest

$$100\times\left(1+.08\times\frac{92}{365}\right)=102.02$$

$$\boxed{\mathrm{FV} = \mathrm{PV}\times\left(1+i\times\frac{days}{365}\right)}$$

FV i days FV

# An Example: Simple Interest – 2

This can also be rearranged to solve for the interest rate

- If I invest 100 today and receive 102.02 in 92 days, what is the simple interest rate?

$$i = \left(\frac{102.02}{100} - 1\right) \times \boxed{\frac{365}{92}} = 0.08 \rightarrow 8\%$$

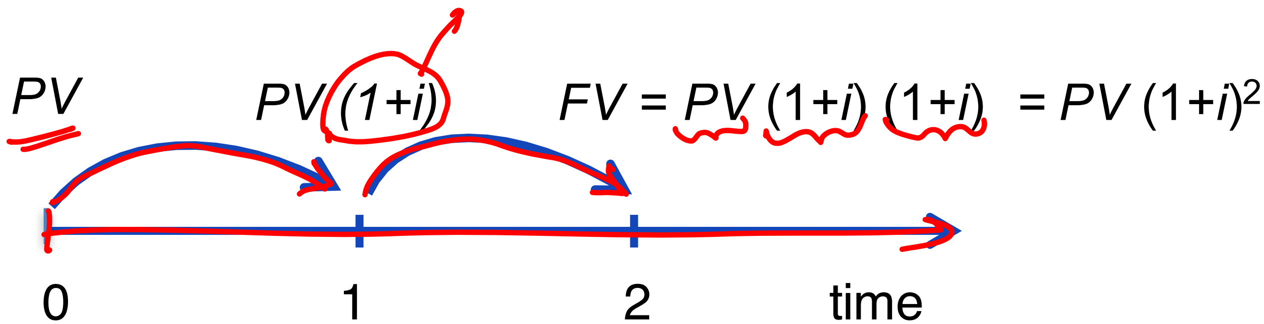
$i$

- In standard notation

$$i = \left(\frac{FV}{PV} - 1\right) \times \frac{365}{days}$$

# Compound Interest

Interest **earns** interest




$$\boxed{FV = PV \times (1+i)^t}$$

# Intra-Year Compounding

Compounding can be intra-year: quarterly (n=4), monthly (n=12), daily (n=365)...

$$FV = PV \times \underbrace{\left(1+\frac{i}{n}\right)}_{\text{Gross rate}}^{n\times t}$$

frequency

n: # of years

Agree

What happens as n tends to infinity? (called continuous compounding)

$$FV = \lim_{n\to\infty} \underbrace{PV \times \left(1+\frac{i}{n}\right)}^{n\times t} = PV \times \exp^{i\times t}$$

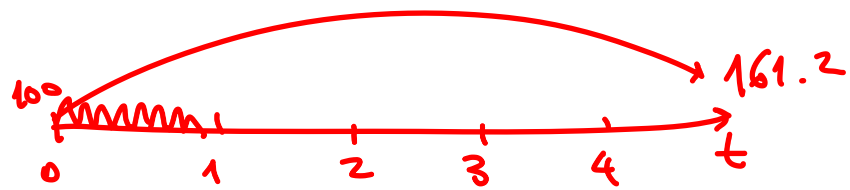
frequency ↑ → Int ↑

$\exp \approx 2.72$

# An Example: Compound Interest

You have invested $100 for 4 years at an annual rate of 12 percent, compounded monthly. What is the future value of the investment at the end of the fourth year?

- Number of payments per year, $n = 12$
- Number of years, $t = 4$
- Monthly interest rate, $i = 0.12/12 = 0.01$




$$FV = 100 \times \left(1 + \frac{0.12}{\underbrace{12}_{\text{monthy interest}}}\right)^{(4\times 12)} = 161.2$$

# Nominal and Effective Interest Rates

**Nominal interest rates** usually quoted in terms of annual compounding

- Also known as Annual Percent Rate (APR)

**Effective interest rate** is the actual return earned

- Also known as effective yield (*y*) or Effective Annual Rate (EAR)

$$1 + EAR = \left(1 + \frac{i}{n}\right)^{n}$$

With intra-year compounding: EAR > APR

## An Example:
## Nominal and Effective Interest

Compute the effective rate of an investment that offers an interest of 12 percent compounded monthly

Nominal interest rate (APR) $\boxed{i = 12\%} = 0.12$

Effective Annual Rate

$$EAR = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 0.1268 = \boxed{12.68\%}$$

With intra-annual compounding, effective interest rates > nominal rates

# An Illustration:
# Nominal and Effective Rates with Intra-Year Compounding

**Nominal Rate = 12.00 %**

| Compounding Frequency | Effective Rate |
|---|---|
| Annual | 12.00% |
| Semi-Annual | 12.36% |
| Quarterly | 12.55% |
| Monthly | 12.68% |
| Daily | 12.75% |

# Net Present Value:
# A Fundamental Concept in Finance and Economics

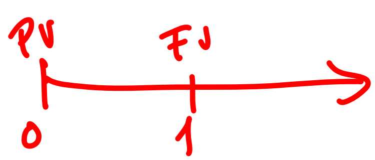
To compare different streams of cash flows…

**Present Value:** How much the future cash flows are worth to the investor today.

- Recall, in a single period case:

$$\boxed{FV = PV \times (1+i)}$$

- Can be rearranged to compute the present value:

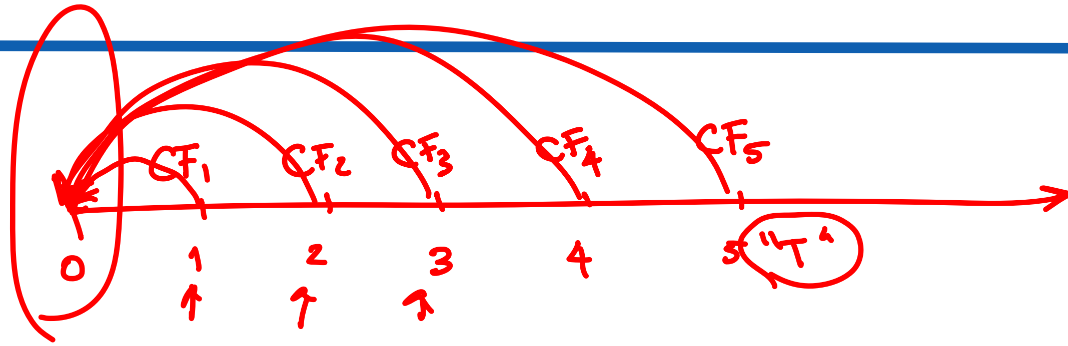



$$\boxed{PV = \frac{FV}{(1+i)}} = \frac{1}{1+i} FV$$

Discount factor < 1

# Net Present Value

**In multi-period case:**




**Present value:**

$$PV = \sum_{t=1}^{T} \frac{CF_t}{(1+i)^t}$$

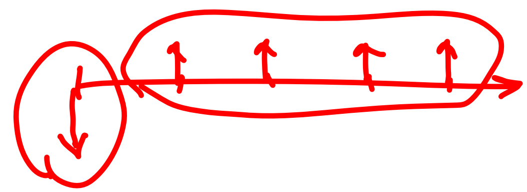
**If *CF's* constant:**

$$PV = \frac{CF}{i} \times \left(1 - \frac{1}{(1+i)^T}\right)$$

- In other words, future cash flows are **"discounted"** to the present at the rate *i*

# Comparing Investment Alternatives

Investment alternatives can be assessed by comparing their **Net Present Values (NPV)**



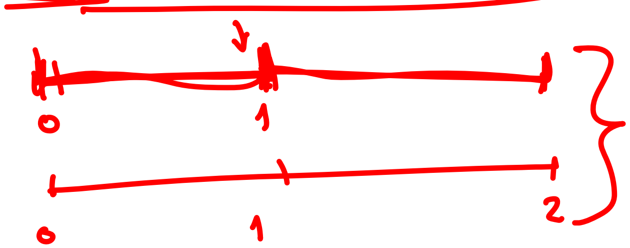
- NPV is the sum of the PV of all cash flows

- The NPV critically depends on the interest rate
  A higher interest rate will decrease the PV of the future cash flows

$i \uparrow \rightarrow PV \downarrow$

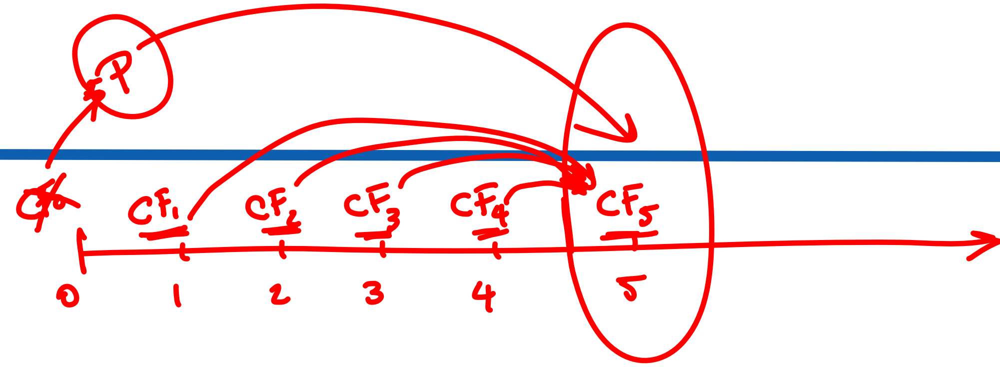
The investment with the higher NPV is the most profitable for the chosen interest rate

same T



# Future Value

**In multi-period case:**




**Future value:**

$$FV = \underbrace{\sum_{t=1}^{T} CF_t \times (1+i)^{T-t}}$$

**Under constant cash flows:**

$$\boxed{FV = CF \times \left( \frac{(1+i)^T}{i} - \frac{1}{i} \right)}$$

# An Example
# Computing Future Value

What is the future value of a payment of $5,000 today and a stream of 15 consecutive yearly revenues of $500 each if you use an interest rate of 5 percent?

P = $5000 $\quad$ $500

$$FV = CF_0 \times (1+i)^T + \underbrace{CF \times \left( \frac{(1+i)^T}{i} - \frac{1}{i} \right)}$$

$$FV = \underbrace{-\$5{,}000 \times (1+0.05)^{15}} + \underbrace{\$500 \times \left( \frac{(1+0.05)^{15}}{0.05} - \frac{1}{0.05} \right)} = \$395$$

# A Fundamental Concept:
# The Internal Rate of Return (IRR)

We have seen that:

$$NPV = \underbrace{-CF_0 + \sum_{t=1}^{T} \frac{CF_t}{(1+i)^t}}$$

Computing NPV requires knowledge of all cash flows and the interest rate

But we can also ask what' she interest rate that satisfies:

$$\boxed{\underbrace{-CF_0 + \sum_{t=1}^{T} \frac{CF_t}{(1+\boxed{IRR})^t}} = 0}$$

# Intuition Behind the Internal Rate of Return (IRR)

**Claim:** "The IRR is the opportunity cost of the associated investment alternative"

Take note that…

- The IRR is the average interest rate for which the present value of the discounted CFs equals the initial "payment" NPV = Ø
- The IRR also equates the *FV* of the *CF*s and the initial "payment"