Financial Market Analysis (FMAx)
Module 7

Introduction to Risk Management

The Relevance to You

You might be…

- An investor (reserve management, sovereign wealth fund, bank, insurance company).
- A policymaker interested in financial intermediaries' exposures and portfolio losses (central bank, banking sector, pension funds, insurance companies).

The Relevance to You

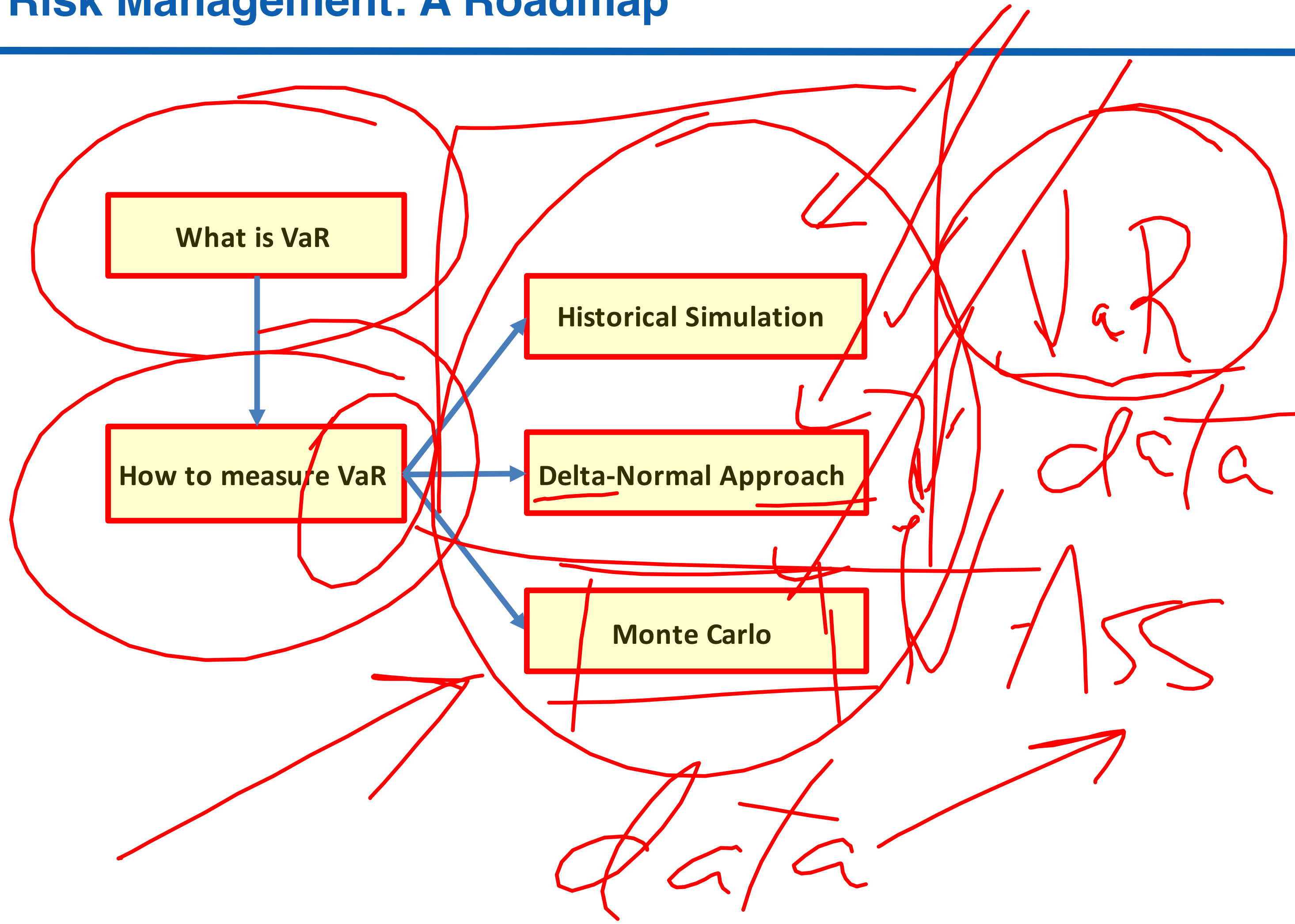
Historical background…

- JPMorgan (90s):
  - 4:15 Report
  - RiskMetrics: Technical document

risk exp

96

Introduction to Risk Management: A Roadmap



Value at Risk (VaR)

$VaR = [\text{loss} \mid \text{Prob}]$

Characteristics of…

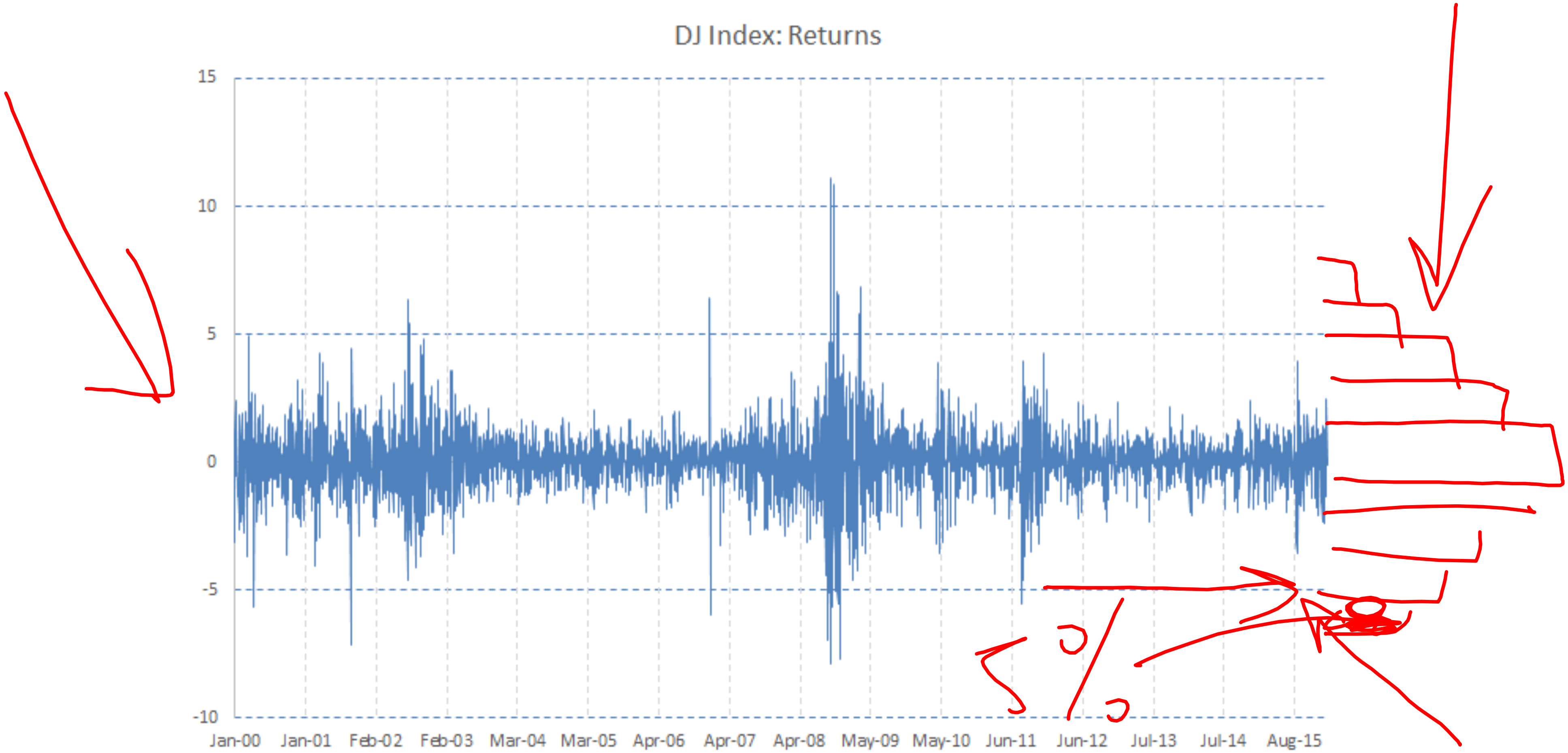
- Tries to estimate the level of possible losses over a given time period with a certain probability.
- VaR summarizes the expected maximum loss over a time horizon within a given confidence interval.
  - For example, the $\boxed{\text{95\% VaR loss}}$ is the amount of loss that will be exceeded only 5% of the time.

-3% $\quad \frac{1}{20} \approx \frac{5}{100} \quad \frac{95}{100}$

Value at Risk (VaR): How much can I lose? – 1

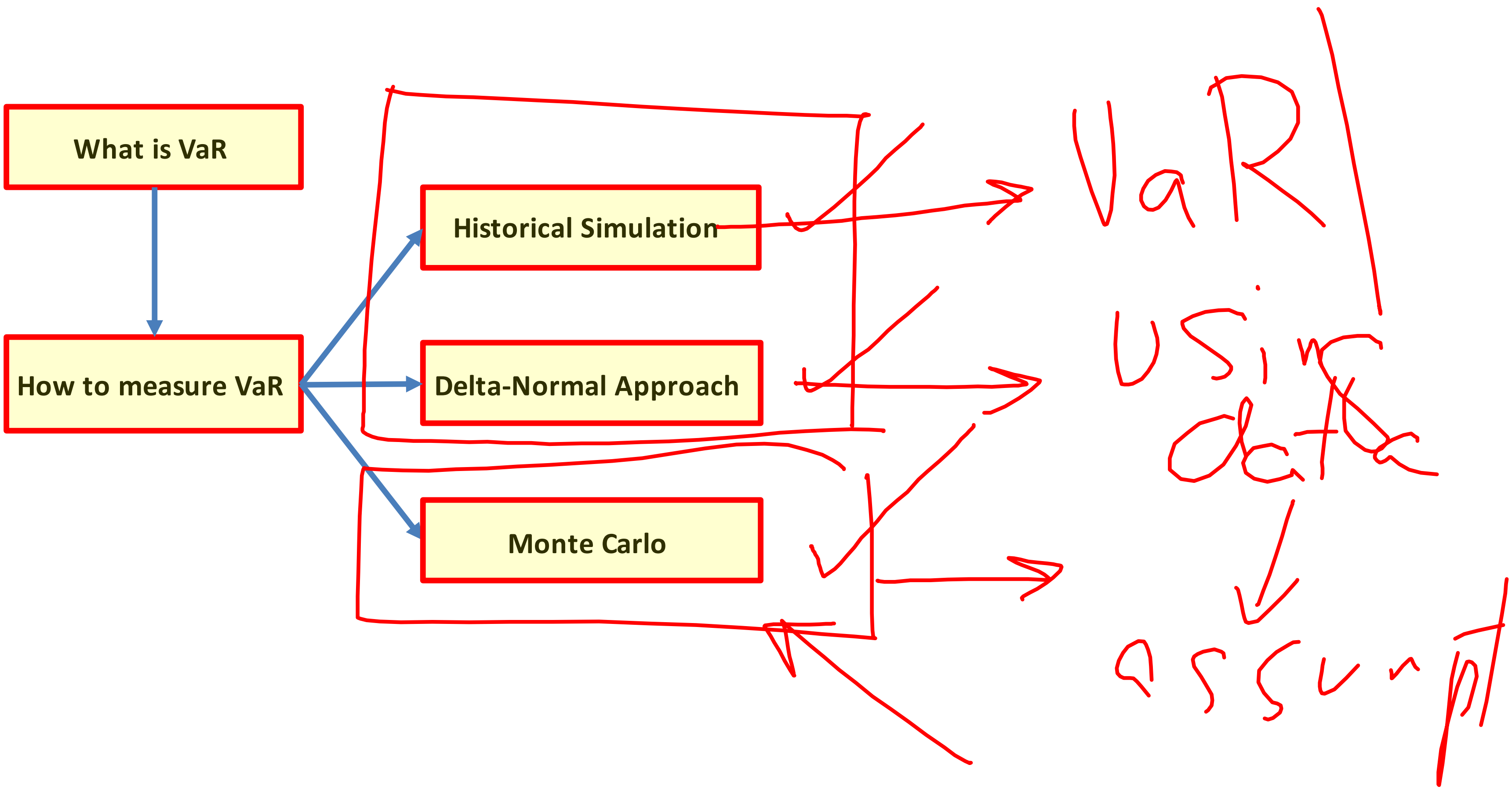
Source: Bloomberg

Value at Risk (VaR): How much can I lose? – 2



Source: Bloomberg

What we cover next…

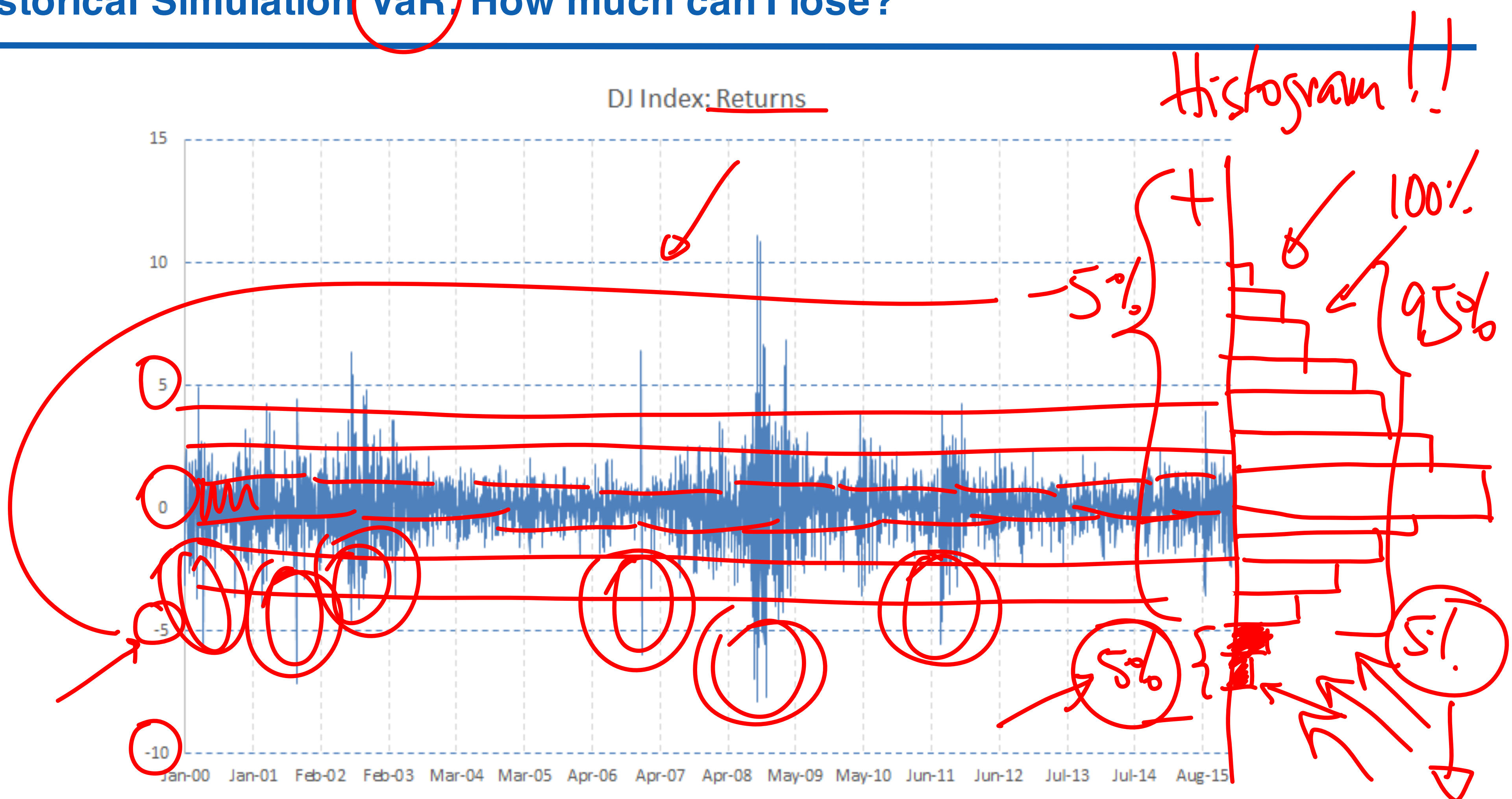


Historical Simulation

Characteristics of…

- Utilizes historical data.
- Generates histograms.
  - Examines historical daily returns.
  - Provides statistics (mean, minimum, maximum, standard deviation)
- Calculates VaR.

Historical Simulation VaR: How much can I lose?



Source: Bloomberg

Value at Risk (VaR)

The Advantages of VaR:

- It is intuitive.
- It states potential loss in terms of amounts (US$) rather than percentage returns.

VaR d 5% 95% $\frac{5}{100} = \frac{1}{20}$

Choice of time horizon:

- Probability of loss is greater with longer time horizons.
- Typical choices: day, week, month

-2% 20 days

Different Time Horizons

Up to now, we have assumed that the horizon is 1 day, but what about longer horizons?

1 → 5

We can extend the horizon ("holding period") to N days in a simple way (assuming each day is an independent observation).

$V(r_1+r_2+r_3) = \underbrace{V(r_1)+V(r_2)+V(r_3)}_{3\cdot\sigma^2}$

1, 5, 20

$$\sigma^2 = \sigma_D^{\ 2} N \qquad\qquad \sigma = \sigma_D \sqrt{N}$$

$\sigma_M^2 = \sigma_D^2 \cdot 20 \longrightarrow \sigma_M = \sigma_D \cdot \sqrt{20}$

Pros/Cons of Historical Approach

options

- No dependence on any distributional assumptions; No need to estimate volatilities or correlations.
- Accommodates fat tails. # obs tails
- Accommodates assets whose payoff are non-linear.
- Assumes that the past is a good and reliable representation of the future.

DATA → Histogram

VaR = losses / Prob

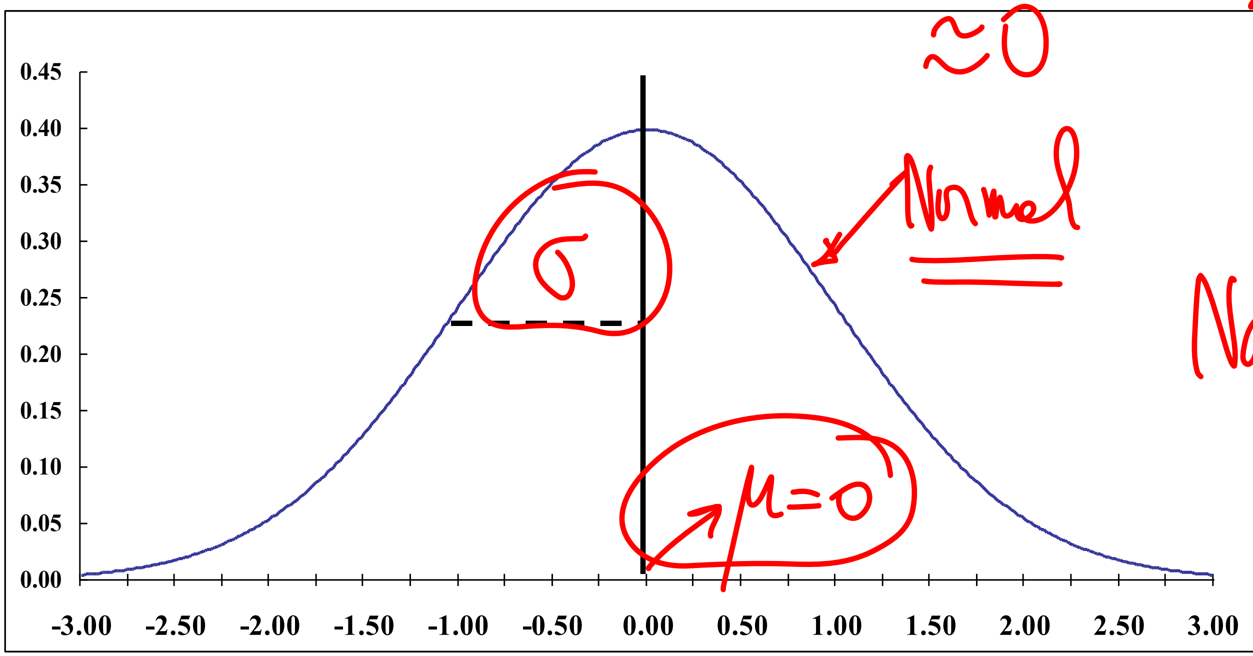
Delta-Normal Method
An Analytical Framework – 1

Assume that asset returns are normally distributed.

$N(\mu, \sigma^2)$

Their behavior can be fully described in terms of mean and standard deviation.

$\simeq 0$ $\Rightarrow$ VaR = losses / Prob



Normal distribution

$N(\mu, \sigma^2)$

Delta-Normal Method
An Analytical Framework – 2

Why this method vs historical simulation?

- Advantage… It is very easy to compute.
- Has limitations with comparison to historical simulation.
  The assumption about normality could be inaccurate.
  Specially with asymmetric returns

σ^2

data

assumption

fat tails

~~VaR~~

~~Normal~~

VaR with Normally Distributed Returns – 1

1.65
2.33

How far below the mean return is the threshold depends on how many standard deviations the threshold is below the mean return.

VaR

Threshold L*	μ-1.0σ	μ-1.65σ	μ-2σ	μ-2.33σ	μ-3σ
Pr(loss>L*)	0.16	0.05	0.023	0.01	0.001

5%

1%

There is nothing sacred about those multiples of sigma--we choose them because they map easily onto the normal probabilities.

VaR with Normally Distributed Returns – 2

$\mu = 0$

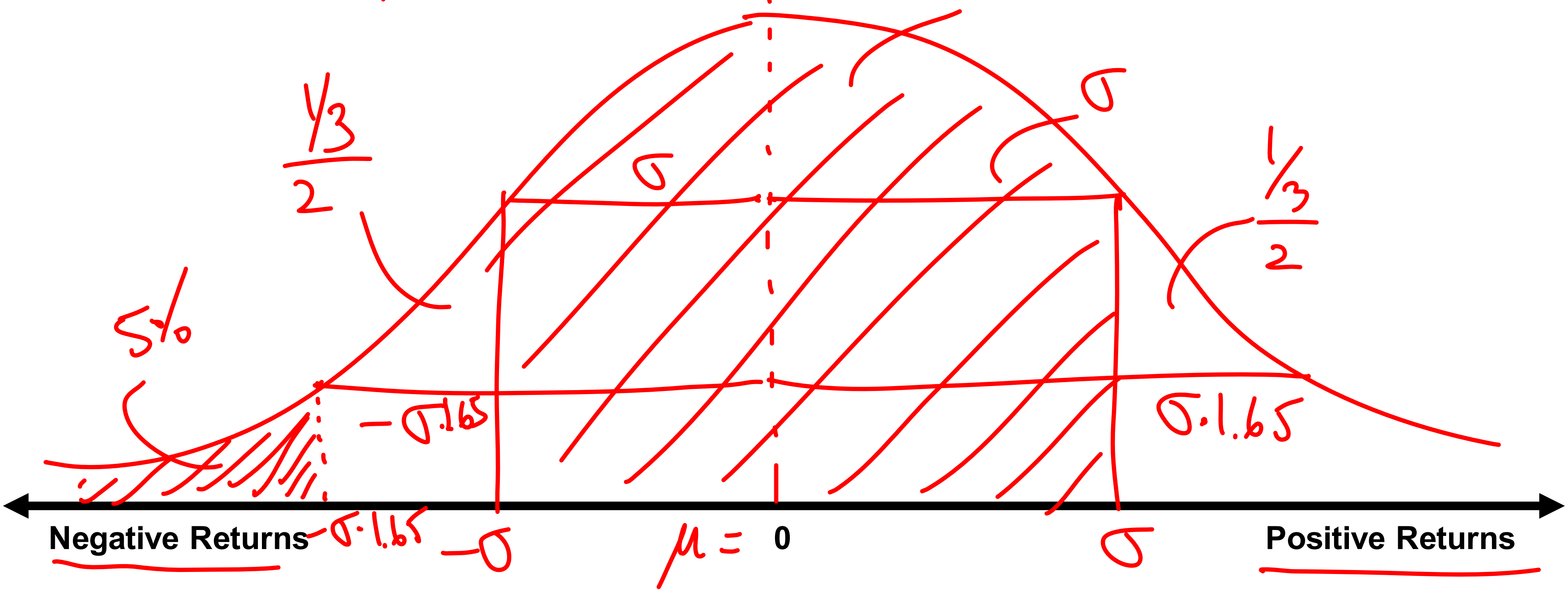
$VaR = -\sigma \cdot 1.65 = -1.65 \cdot \$$

1%

5% → −1.65 σ move

95% → −1.65 at most with 95% prob.

67% data $\frac{2}{3}$



An Example with One Asset

1.65

Suppose we want to compute the 95% VaR.

5% 1/20

Normal Tables

The critical threshold is 1.65 standard deviations below the mean.

$$0.0001-1.65 \cdot 0.00295=-0.00477 = 0.477\%$$

$1.65 \cdot \sigma$

μ

$$\text{VaR} = 0.00477 \cdot 200m = 0.95m$$

\$ \$

200m

1m once 5% 1/20

Portfolio VaR

Correlations of returns

When we have more than one asset in our portfolio, we can exploit the gains from diversification.

- There are **gains from diversification** whenever the VaR for the portfolio does not exceed the sum of the stand-alone VaRs.
- The VaR for the portfolio equals the sum of the stand-alone VaRs if and only if the securities are perfectly correlated.

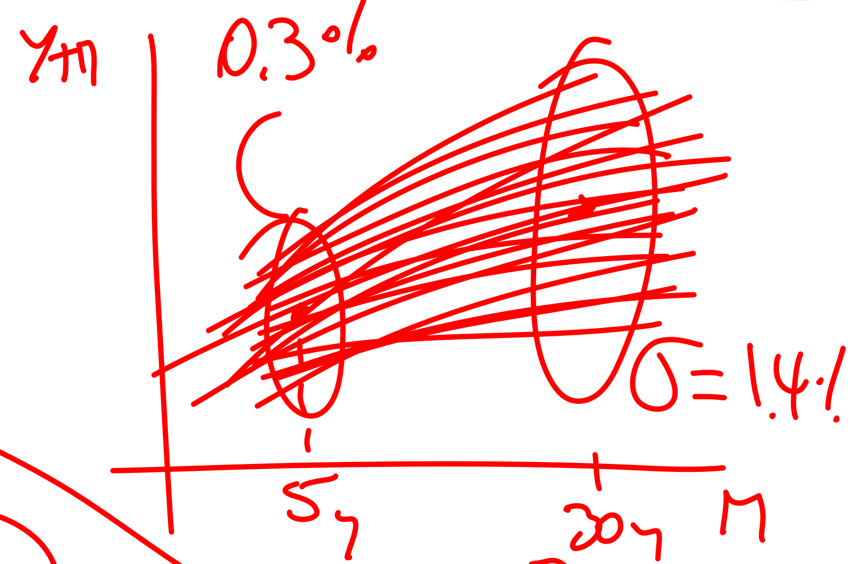
$$VaR_p \leq \sum_{j=1}^{N} VaR_j \qquad \rho_{12} : Corr\, 1,2$$

An Example of Portfolio VaR

On-Page Text

Consider two securities:

- 30-year zero-coupon U.S. Treasury bond
- 5-year zero-coupon U.S. Treasury bond



For example…

Invest US$100 million in the 30-year bond

- Daily return volatility (std dev) $\sigma_1 = 1.409\%$

1.65 . $

Invest US$200 million in the 5-year bond

- Daily return volatility (std dev) $\sigma_2 = 0.295\%$

1.65 $

Stand Alone VaRs versus Portfolio and Diversification

On-Page Text

Take the following…

95% confidence level

1.65

30 year zero VaR:

1.65 * 0.01409 * 100m = $\boxed{\$2,325,000}$

σ_{30} $\$$

5 year zero VaR:

1.65 * 0.00295 * 200m = $\boxed{\$974,000}$

$VaR_1 + VaR_2 =$

$\rho_{1,2}$

$\rho = 1$

Sum of individual VaRs = US$ 3.299m

VaR of the Portfolio

V(rp) V(r2) corr(r1, r2)

vol. of prt.

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$$

-1, 0, 1

st. dev.

$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{(\quad)+(\quad)+(\quad)}$

$\sqrt{V(r_1)}\cdot\sqrt{V(r_2)}$

Portfolio Variance:

(100*0.01409)^2 + (200*0.00295)^2 + 2(100*0.01409)(200*0.00295) * 0.88 = \$3.797m

$\rho = 0.88$

< 1

Portfolio Standard Deviation: σ_p = \$1.948m

Portfolio VaR = 1.65 * 1.948m = $\boxed{\$3.214m}$ 3.299m

VaR of the Portfolio - 1

2 assets

$Cov(r_1, r_2) = \sigma_1 \sigma_2 \rho_{12}$

$r_p = w_1 \cdot r_1 + w_2 r_2 \qquad / \cdot V(\ )$

$V(r_p) = V(w_1 r_1 + w_2 r_2)$

$= V(w_1 r_1) + V(w_2 r_2) + 2\,Cov(w_1 r_1, w_2 r_2)$

$= w_1^2 V(r_1) + w_2^2 V(r_2) + 2\, w_1 w_2 \cdot \rho_{12} \sigma_1 \sigma_2$

$\sigma_p^2 = w_1^2 \sigma^2 + w_2^2 \sigma_2^2 + 2\, w_1 w_2 \rho_{12} \sigma_1 \sigma_2 \qquad / \sqrt{\ }$

95% $\underbrace{\boxed{1.65\sqrt{\sigma_p^2}}}_{VaR_{95\%}} = 1.65\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2 w_1 w_2 \rho_{12}\sigma_1\sigma_2}$

VaR of the Portfolio - 2

N assets $\quad \underset{1\times N}{w} \quad \underset{N\times 1}{R}$

$$\underset{1\times 1}{r_p} = \underset{1\times n}{w} \cdot \underset{n\times 1}{R}$$

$V[\,]$

$n\times n \quad \sigma_s \quad n\times n \text{ corr} \quad n\times n\, \sigma_s$

$$V(r_p) = V(wR) = w\,V(R)\,w'$$

$$\Sigma_n = \text{var-cov} = \sigma \cdot [C] \cdot \sigma$$

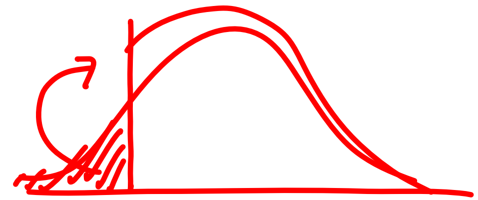
$$\sigma_p = \sqrt{w\sigma C \sigma w'}$$

1.65

$$1.65\,\sigma_p = \sqrt{1.65 \cdot w \cdot \sigma \cdot C \cdot 1.65 \cdot \sigma \cdot w'}$$

$$VaR_p = \sqrt{\underset{1\times n}{\overline{VaR}} \cdot C \cdot \underset{n\times 1}{\overline{VaR}}'} \Rightarrow \boxed{VaR_p = \sum_{j=1}^{n} VcR_j} \quad C = I$$

Monte Carlo Simulation Approach



Compute returns for the assets in the portfolio using computer generated random numbers.

Normal

Random numbers are generated by assuming a distribution for the asset returns.

The joint distribution for the random number generation is chosen to match the expected values, variances, and co-variances of the asset returns in the portfolio.

Similar to the Delta Normal approach, if portfolios have assets whose payoff are linear.

$\sigma, \rho, \mu \Longrightarrow MC$

Drawing the Monte Carlo Approach

{σ, C}

Advantages of the Monte Carlo Approach

Include the following:

- Allow for non-linear payoffs in the portfolio (options).
- Accommodates any distribution of risk factors.
- Allows the calculation of VaR confidence intervals.

{Ptf f(NER)

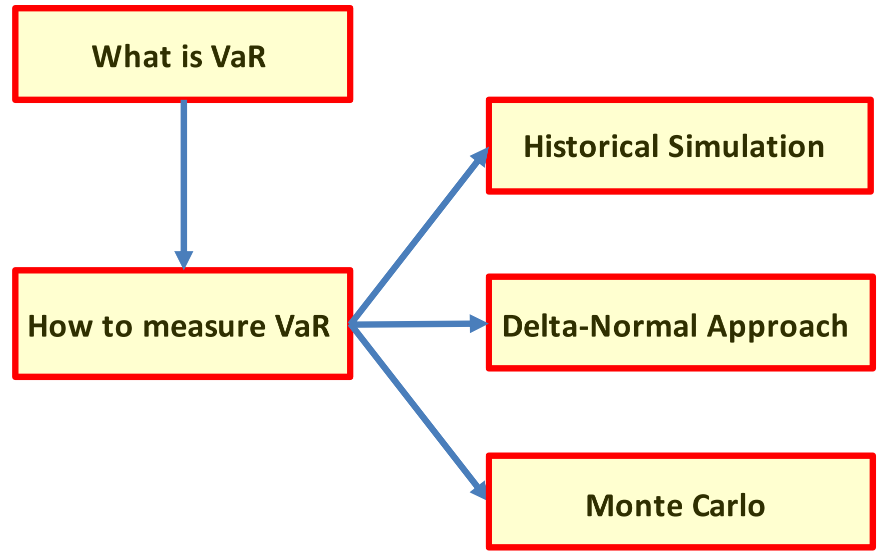
NER
GDP
stock act

{10,000 runs}

Summary: Introduction to Risk Management

Risk, market risk and VaR

- Excel exercises
  - Calculating returns
  - Histogram
  - Standard deviation



Refinements of VaR:
1. Backtesting

Is interested in answering…

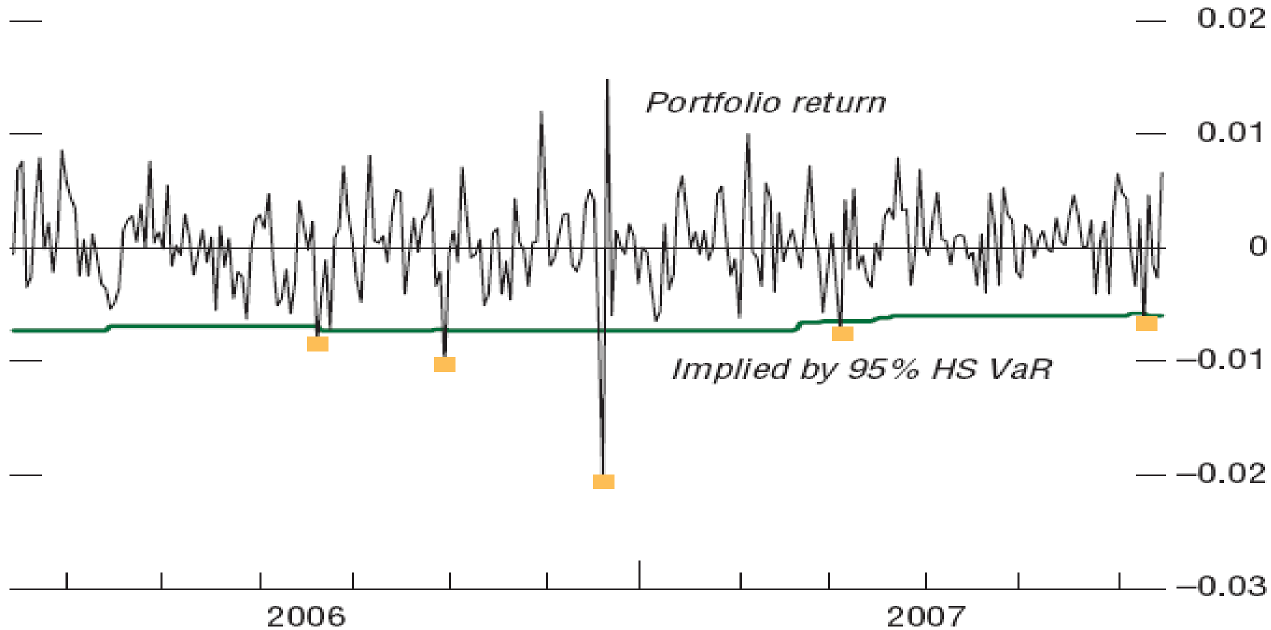
Did the investment perform as VaR had predicted?

Compares the calculated VaRs with the actual related returns.

- With a 95% VaR bound, expect 5% of losses greater than the bound. Approximately 12 days out of 250 trading days: “exceptions”.
- If actual number of exceptions is “significantly” higher than 5%, model may be inaccurate.

Backtesting – 1

Figure 2.3. Backtesting Results: Broad Portfolio, June 2006 to June 2007

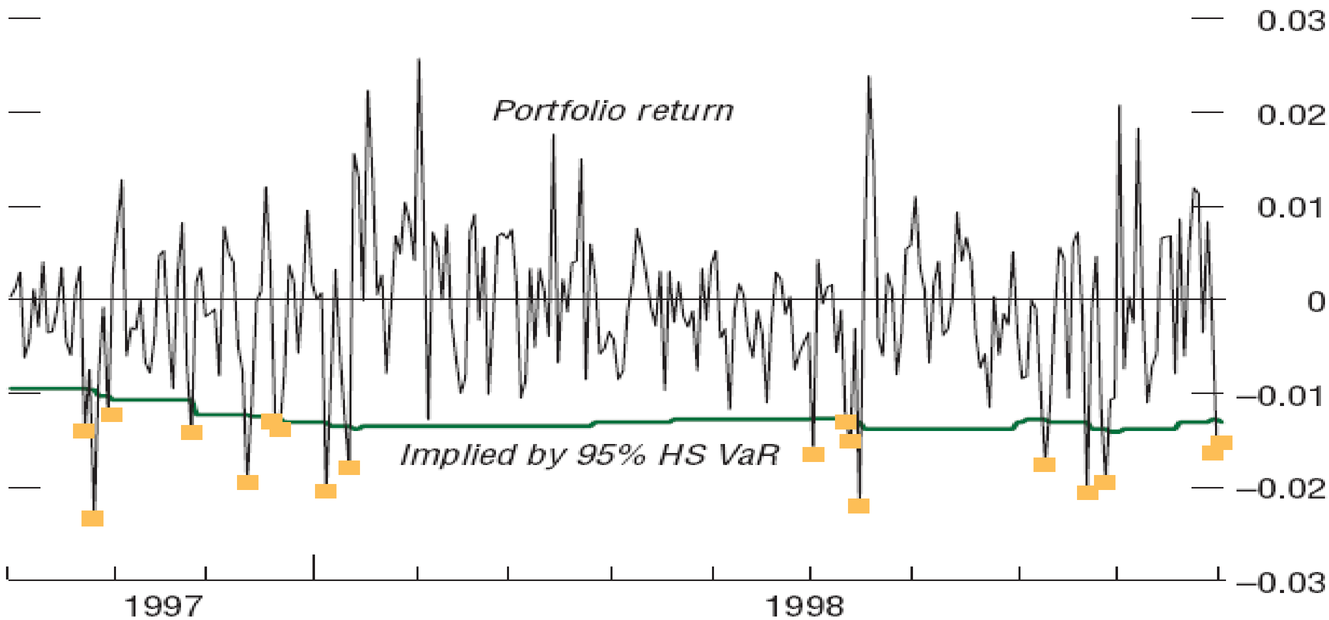


Sources: Bloomberg L.P.; and IMF staff estimates.
Note: HS VaR = historical simulation of value-at-risk. Yellow squares indicate VaR violations.

Source: IMF Global Financial Stability Report, September 2007

Backtesting – 2

Figure 2.2. Backtesting Results: Broad Portfolio, October 1997 to October 1998



Sources: Bloomberg L.P.; and IMF staff estimates.
Note: HS VaR = historical simulation of value-at-risk. Yellow squares indicate VaR violations.

Source: IMF GFSR September 2007

Refinements of VaR:
2. Basel Committee and VaR - Guidelines

1. VaR computed daily, holding period is 10 days.

2. The confidence interval is 99 percent
   - Banks are required to hold capital in proportion to the losses that can be expected to occur more often than once every 100 periods.

3. At least 1 year of data to calculate parameters.

4. Parameter estimates updated at least once every month.

5. Capital provision is the greater of…
   - Previous day's VaR.
   - Average of the daily VaR for the preceding 60 business days multiplied by a factor based on backtesting results.

calm period σ small → low VaR

Stressed Value-at-Risk Measure Introduced After the Crisis

- "Losses in most banks' trading books during the financial crisis have been significantly higher than the minimum capital requirements under the former Pillar 1 market risk rules."

- "The Committee requires banks to calculate a **stressed value-at-risk** taking into account **a one-year observation period relating to significant losses**, which must be calculated in addition to the value-at-risk based on the most recent one-year observation period."

BIS, Revisions to the Basel II Market Risk Framework, 2011

Stressed Value-at-Risk Measure

- Replicate a VaR calculation generated on bank's current portfolio if relevant market factors were experiencing a period of stress.

  • example: single asset
  • portfolio ← $\boxed{\text{mkt factors}}$

- Based on 10-day, 99th percentile VaR measure, with model inputs calibrated to historical data from a continuous 12-month period of **significant financial stress**

- "The stressed VaR should be calculated at least weekly".

BIS, Revisions to the Basel II Market Risk Framework, 2011.

Stressed Value-at-Risk Measure

Replicates a VaR calculation that would be generated if the relevant market factors were experiencing a period of stress.

The capital requirement c:

≥ 3

$$c = \max\{VaR_{t-1};\ m_c \cdot VaR_{avg}\} + \max\{sVaR_{t-1};\ m_s \cdot sVaR_{avg}\}$$

VaR_{t-1} = VaR yesterday

VaR_{avg} = average VaR over a 60 day period

$sVaR_{t-1}$ = stressed VaR last available day

$sVaR_{avg}$ = average VaR over a 60 day stessed period

m_c = multiplication factor of 3 (minmum)

m_s = multiplication factor of 3 (minmum)

Refinements of VaR:
3. Expected Shortfall (ES)

VaR tells us that a loss larger than "$X" will occur one day in 20 (with 95% confidence), but does not tell us how big the losses that exceeds "$X" will be.

- (VaR does not indicate what happens in extreme market events.)

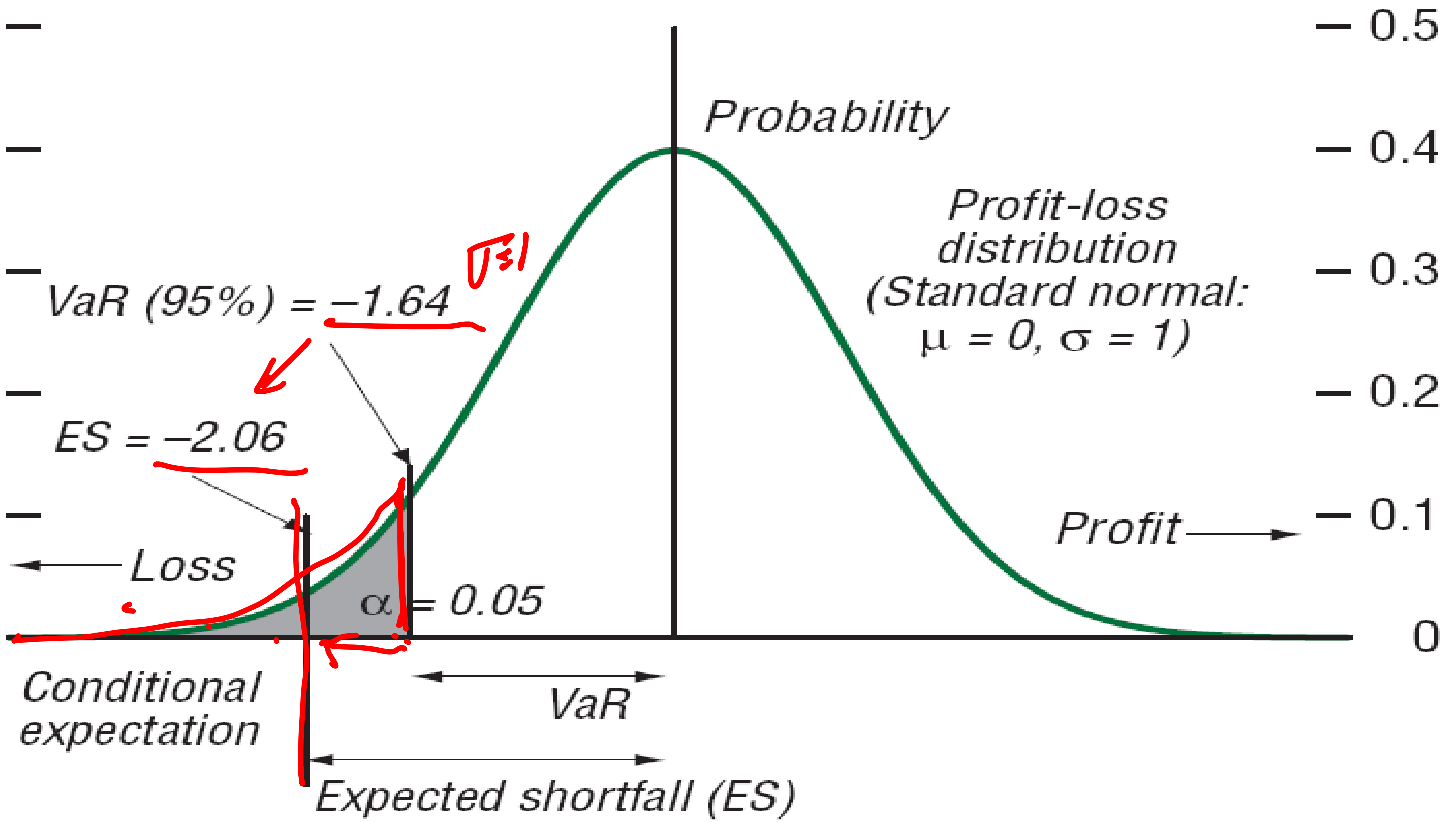
Assume the loss exceeds VaR… exceptions

Expected Shortfall –

1. Knowing the cut-off of what will happen *C* percent of the time (VaR),
2. Provides the average size of the loss when it exceeds the cut-off value.

How much could be lost if losses exceed VaR?

Expected Shortfall - E(-X I -X≤-VaR)



Source: GFSR, September 2007

BIS Committee -
Move from Value-at-Risk (VaR) to Expected Shortfall (ES)

Weaknesses identified with using VaR for determining regulatory capital requirements: **inability to capture "tail risk**".

Proposed in May 2012 to replace VaR with ES.

- 'ES measures the riskiness of a position by considering both the size and the likelihood of losses above a certain confidence level.'

Agreed to use a 97.5% ES for the internal models-based approach.

Has also used that approach to calibrate capital requirements under the revised market risk standardized approach.

BIS Committee -
Move from Value-at-Risk (VaR) to Expected Shortfall (ES)

Recognizes that basing regulatory capital on both current VaR and stressed VaR calculations may be unnecessarily duplicative.

- Proposed framework will simplify the capital framework by moving to a single ES calculation calibrated to a period of significant financial stress.

Expected shortfall for the bank's portfolio:

- Uses set of risk factors that explain at least 75% of the variation of the full ES model.
- Is calibrated to the most severe 12-month period of stress available over the observation horizon.

SES