There Is No Largest Prime Number

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Outline

1. Motivation

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1.1 The Basic Problem That We Studied

What Are Prime Numbers? I

Definition: Prime number

A *prime number* is a number that has exactly two divisors.

What Are Prime Numbers? II

Example:

- 2 is prime (two divisors: 1 and 2).
- 3 is prime (two divisors: 1 and 3).
- 4 is not prime (**three** divisors: 1, 2, and 4).

There Is No Largest Prime Number

Theorem: Prime numbers

There is no largest prime number.

There Is No Largest Prime Number

Proof:

1. Suppose p were the largest prime number.

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2. Let *q* be the product of the first *p* numbers.

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3. Then $q + 1$ is not divisible by any of them.

There Is No Largest Prime Number

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2. Let *q* be the product of the first *p* numbers.

3. Then $q + 1$ is not divisible by any of them.

4. But $q + 1$ is greater than 1, thus divisible by some prime number not in the first *p* numbers.

The proof used *reductio ad absurdum*.

What's Still To Do?

- Answered Questions
  - How many primes are there?
- Open Questions
  - Is every even number the sum of two primes?

An Algorithm For Finding Prime Numbers. I

An Algorithm For Finding Prime Numbers. II

FindPrimeNumbers

```
Input: an integer n > 1.

Let A be an array of Boolean values, indexed by
    integers 2 to n,
initially all set to true.

for i = 2, 3, 4, ..., not exceeding $\sqrt n$:
  if A[i] is true:
    for j = i2, i2+i, i2+2i, i2+3i, ..., not
        exceeding n:
      A[j] := false.

Output: all i such that A[i] is true.
```

It's me, Euclid

Figure: *It's me, Euclid [1]*

References I

[1] URL: `https: //upload.wikimedia.org/wikipedia/commons/3/30/Euklid- von-Alexandria_1.jpg` (visited on 10/22/2018).

[2] URL: `https://en.wikipedia.org/wiki/Sieve_of_Eratosthenes` (visited on 10/29/2018).

[3] Noam Chomsky. *Syntactic Structures*. The Hague: Mouton, 1957.

[4] William Labov. *Sociolinguistic Patterns*. Philadelphia: University of Pennsylvania Press, 1972.