Numerical Methods Bootcamp

Monday Assignment
Value Function Iteration

In this assignment you are asked to solve and simulate the stochastic growth model using value function iteration on a discretised state space. You will be provided with an incomplete Matlab script which you need to understand, and then program the blanks.

The model is summarized by the Bellman equation

$$v(k,z) = \max_{k'}\{u(zk^\alpha + (1-\delta)k - k') + \beta \sum_{z'} v(k',z')P(z'|z)\}$$

with associate decision rule $k' = g(k,z)$. Productivity, z, can only take two values; "good" and "bad", and we will discretise the domain for capital, k, such that we will be analysing an entirely discrete problem. The values of the parameters; the values for z; and the grid for k are all provided in the code. All graphs in this document are generated setting $N = 1,000$.[1] To complete today's assignment you should carry out the following steps:

(i) Iterate on the Bellman equation until[2]

$$\|v_n(k,z) - v_{n-1}(k,z)\| < 1(-6).$$

(ii) Plot your decision rules in the (k,k')-plane, and check that your grid was wide enough to cover the ergodic set. The results are illustrated in Figure 1.

(iii) Simulate the model for $t = 1,2,\ldots T$ periods, with $T = 10,000$. In particular, make 10,000 draws from a uniform distribution on $[0,1]$ using the random seed `1979`. Call this $T \times 1$ vector e. Let Z_t be a variable taking on values in $\{1,2\}$. Whenever $e_t < P(Z_t, 1)$, then $Z_{t+1} = 1$, otherwise $Z_{t+1} = 2$. P refers to the transition matrix for z. Plot your results for the *last* 100 periods. Figure 2 illustrates what your results should look like.

(iv) Calculate the simulated series for productivity, z, output, $y = zk^\alpha$, investments $i = k' - (1-\delta)k$, and consumption, $c = y - i$.[3] Calculate their means. Take logarithms and calculate the correlation matrix. Calculate the relative standard deviation of the logarithm of each variable to output.

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[1] As you experiment with the code, it is useful to run your code with $N = 50$ or so. Otherwise you will spend too much time waiting for the iteration to converge.

[2] $1(-6)$ is scientific notation for 0.000001. In matlab you can declare such small numbers by `1e-6`.

[3] I would discourage you from naming any variable "i" in matlab, as it can be interpreted as an irrational number.

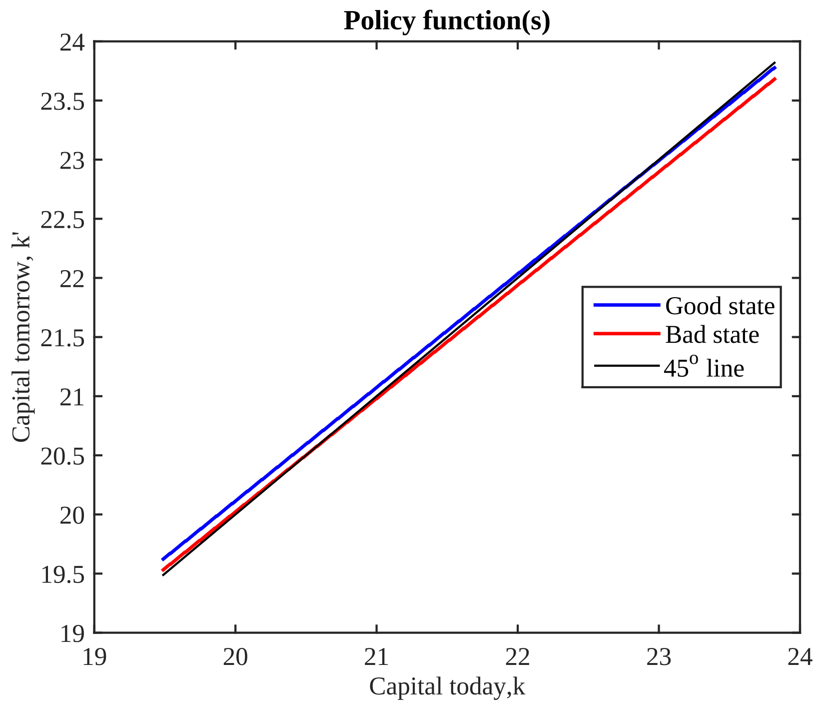

FIGURE 1. Policy functions.

(v) Calculate the transition matrix T for (k, z). This matrix will be $2N \times 2N$, and it will be sparse. Find the long run distribution as the eigenvector associated with a unit eigenvalue normalised to sum to one. Compare this long run distribution with its simulated counterpart in part (iv). If your calculations are correct, your results should replicate the yellow and orange lines in Figure 3.[4]

(vi) Set $\gamma = 1$ and $\delta = 1$. Under these conditions we know that $k' = z\alpha\beta k^{\alpha}$. Does your code confirm this result? (It should)

[4]The orange line is somewhat hidden behind the yellow, as they coincide very well.

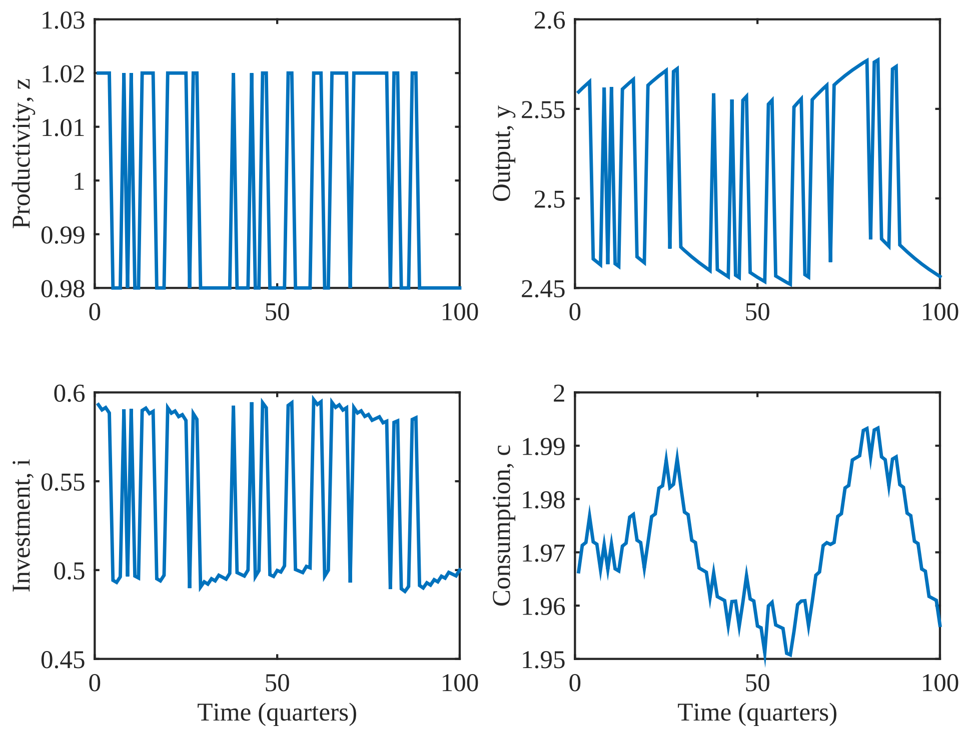

FIGURE 2. Simulated variables.

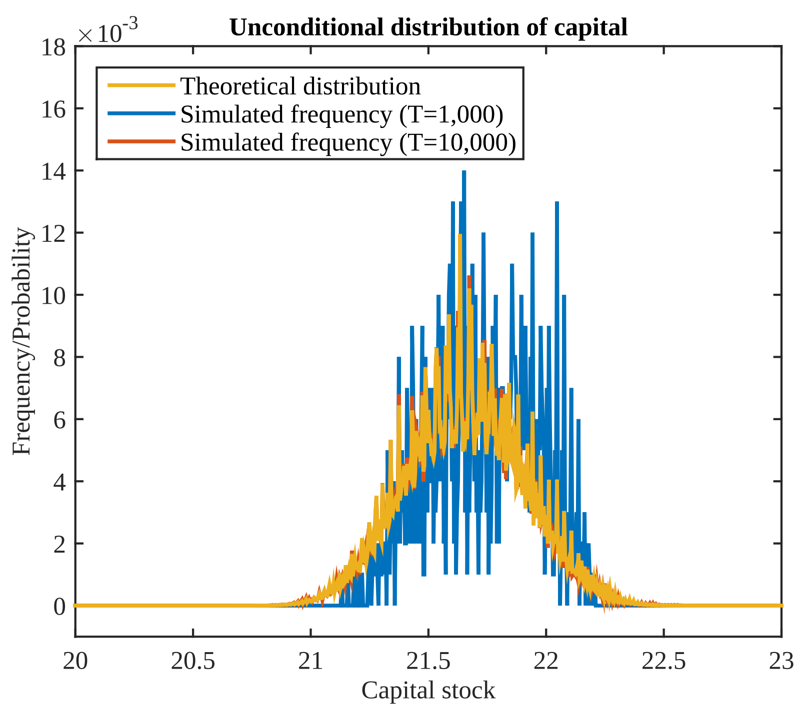

FIGURE 3. Probability distributions.