Numerical Methods Bootcamp

Wednesday Assignment
Irreversible investment

In this assignment you are asked to solve model (1) in Christiano and Fisher (2000). That is, you are asked to solve a stochastic growth model with irreversible investments using functional approximations and a nonlinear equations solver.

This is a nontrivial problem, and you will therefore be provided with ample help. In particular, you will be provided with the entire code which solves the model *without* the irreversible constraint, and your job will be to modify the code to take this constraint into account. I would advice you to run the provided code first, understand it, and then start modifying to incorporate the constraint.[1]

Once you have solved the model you are supposed to

(i) Plot the log of investment (i.e. $\ln(k') - \ln(1-\delta) - \ln(k)$) with the log of capital on the x-axis. See Figure 1.

(ii) Calculate the long run distribution of capital holdings. See Figure 2.

(iii) Calculate an impulse response in which the economy suddenly experiences a negative productivity shock. See Figure 3.

In order to solve the model you will need two policy functions *in addition* to those from the unconstrained case: The two Lagrange multiplier functions that are related to the irreversibility constraint at the two productivity states. Notice that the future values of these functions will also enter into your first order condition according to

$$u'(c_t) - \mu_t = \beta E_t[(1 + z_{t+1}f'(k_{t+1}) - \delta)(u'(c_{t+1}) - \mu_{t+1}(1-\delta)).$$

However, since this is a concave problem you can find $\tilde{c}_t$ as the solution to

$$u'(\tilde{c}_t) = \beta E_t[(1 + z_{t+1}f'(k_{t+1}) - \delta)(u'(c_{t+1}) - \mu_{t+1}(1-\delta)),$$

and recover c_t as

$$c_t = f(k_t) + (1-\delta)k_t - \max\{\tilde{k}_{t+1}, (1-\delta)k_t\},$$

and finally "back out" the multiplier as

$$\mu_t = u'(c_t) - \beta E_t[(1 + z_{t+1}f'(k_{t+1}) - \delta)(u'(c_{t+1}) - \mu_{t+1}(1-\delta)).$$

[1] I would suggest that you do this piecewise. I.e. that you first try to understand how the problem is set up and solved. That you then try to understand how the transition matrix/long-run distribution is calculated. And lastly how impulse responses are done.

Below are the comparisons between the model with and without the irreversibility constraint. All graphs below are generated using $N = 100$, but as usual I would recommend you to start with a coarser grid, say $N = 10$.

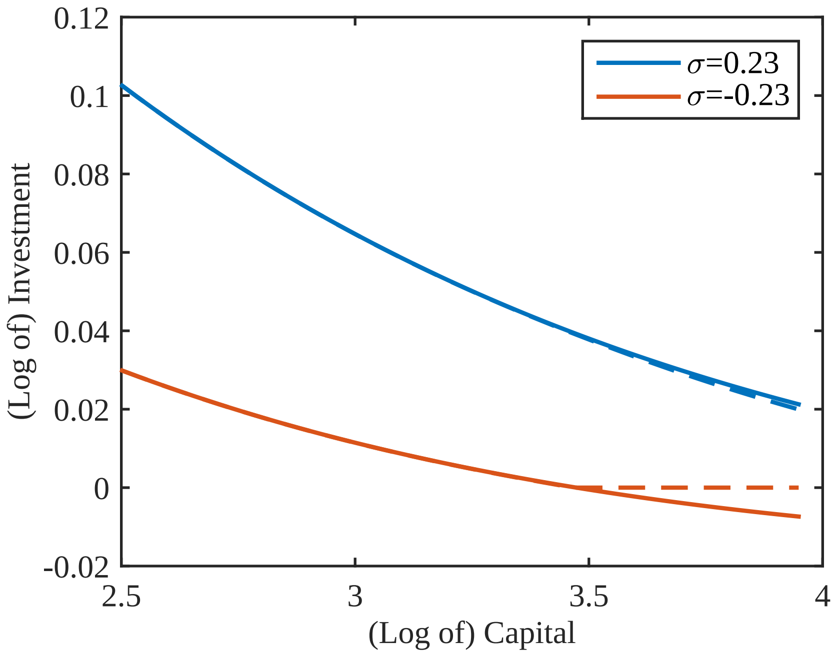

FIGURE 1. Policy functions. The dashed lines illustrates the case with an occasionally binding constraint on investment.

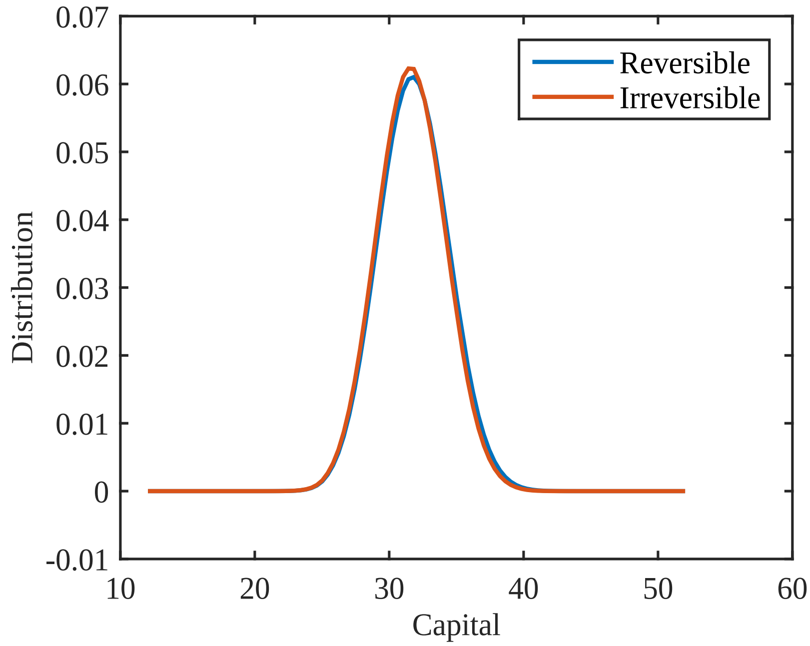

FIGURE 2. Long-run distributions.

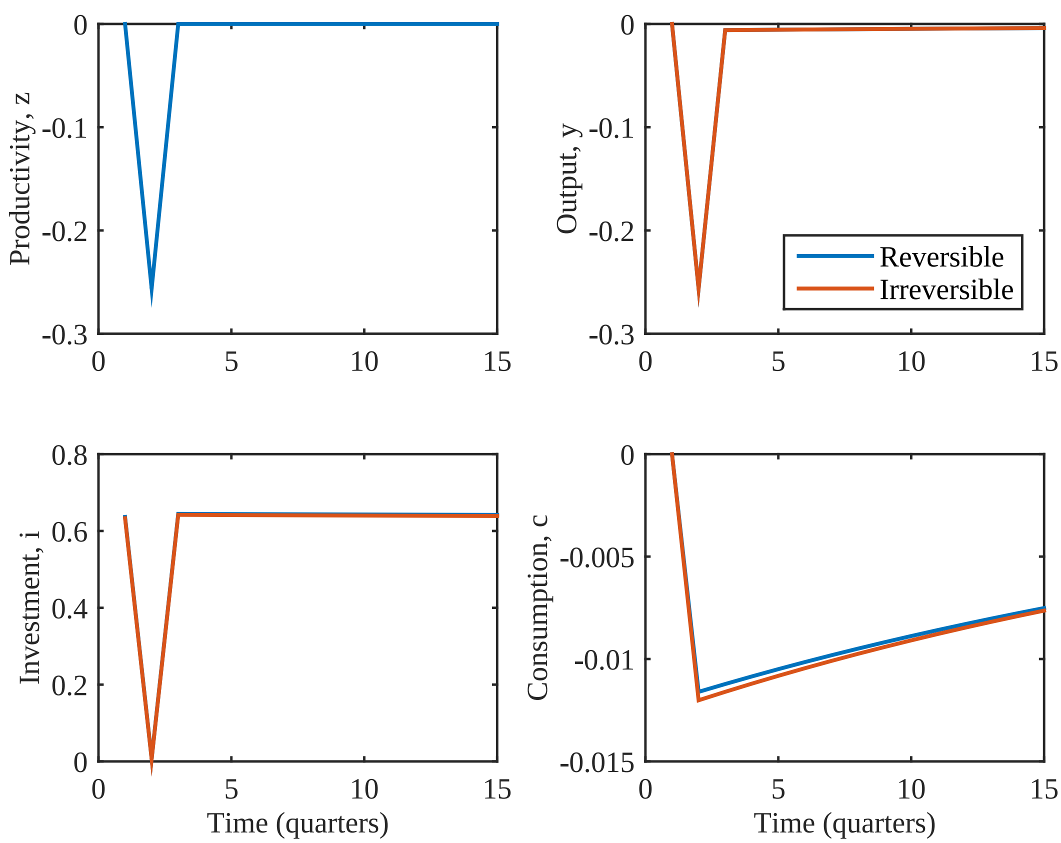

FIGURE 3. Impulse response functions. The red lines illustrates the case with an occasionally binding constraint on investment.