Dynamic Stochastic General Equilibrium Models

Dr. Andrea Beccarini, M.Sc Willi Mutschler

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Households:

capital accumulation and investment decision

- Households own the stock of capital of the economy and rent it at the rate r_t^k at the firms of the intermediate sector.
- Investment in period $t-1$ increases the stock of capital in period t.
- Households choose the utilization rate z_t of their stock of capital.
- The equation of motion of capital is:

$$K_t = K_{t-1}(1-\tau) + I_t \cdot \left[1 - S\left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right)\right] \tag{1}$$

where τ is the depreciation rate.

- The function $S(\cdot)$ introduces in the model adjustment costs for investment that depend on its rate of growth.

- Without these adjustment costs the model predicts strong oscillations of capital.

- If the growth rate of investment is constant as in the steady-state, then $S(1) = 0$.

- One assumes that in the neighborhood of the steady-state $S'(1) = 0$. Hence, adjustment costs depend only on the second derivative.

- The function $S(\cdot)$ is subject to a shock ε_t^I that follows an AR(1) process.

Define $\beta^t \lambda_t Q_t$ the Lagrange-multiplier for eq (1), the maximization problem with respect to K_t, I_t and z_t can be specified as follows:

$$L^K = E_0 \sum_{t=0}^{\infty} \beta^t \left[\underbrace{U\left(C_t^\tau,\ I_t^\tau,\ M_t^\tau\right)}_{\text{Objective function}} - \lambda_t \underbrace{\left(I_t + (\Psi(z_t) - r_t^k z_t) K_{t-1} + \ldots\right)}_{\text{Budget constraint}} \right.$$
$$\left. - \lambda_t Q_t \underbrace{\left(K_t - K_{t-1}(1-\tau) - I_t + I_t \cdot S\left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right)\right)}_{\text{Capital accumulation}} \right] \quad (2)$$

The first-order conditions are:

$$\frac{\partial L^K}{\partial z_t} = E_t\left[-\beta^t \lambda_t \left(\Psi\prime(z_t) - r_t^k\right) K_{t-1}\right] = 0$$
$$\Leftrightarrow r_t^k = \Psi\prime(z_t) \tag{3}$$

- The capital utilization rate is set so that the revenue r_t^k of the marginal utilization equals the marginal costs $\Psi'(z_t)$.

$$\frac{\partial L^K}{\partial K_t} = E_t \left[\begin{array}{c} \beta^{t+1}\lambda_{t+1}\left(r^k_{t+1}z_{t+1} - \Psi(z_{t+1})\right) - \beta^t\lambda_t Q_t + \\ +\beta^{t+1}\lambda_{t+1}Q_{t+1}(1-\tau) \end{array} \right] = 0 \qquad (4)$$

$$\Leftrightarrow Q_t = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t}\left(Q_{t+1}(1-\tau) + z_{t+1}r^k_{t+1} - \Psi(z_{t+1})\right)\right] \qquad (5)$$

- The real value of the stock of capital today $\lambda_t Q_t$ is equal to the expected value of the not depreciated stock of capital of the next period $(1-\tau)\lambda_{t+1}Q_{t+1}$ and to the expected revenue of the future utilization $z_{t+1}r^k_{t+1}$ minus the related costs $\Psi(z_{t+1})$.
- The future value is also discounted by β.

$$\frac{\partial L^K}{\partial I_t} = E_t \Bigg[-\beta^t \lambda_t - \beta^t \lambda_t Q_t \left(-1 + S\left(\frac{\varepsilon_t^I I_t}{I_{t-1}} \right) + I_t \cdot S'\left(\frac{\varepsilon_t^I I_t}{I_{t-1}} \right) \cdot \frac{\varepsilon_t^I}{I_{t-1}} \right) \tag{6}$$

$$-\beta^{t+1} \lambda_{t+1} Q_{t+1} \left(I_{t+1} \cdot S'\left(\frac{\varepsilon_{t+1}^I I_{t+1}}{I_t} \right) \frac{-\varepsilon_{t+1}^I I_{t+1}}{I_t^2} \right) \Bigg] = 0$$

- The costs of the marginal investment (included the adjustment costs) must be equal to the expected marginal revenue of investment.

- The Lagrange multiplier $\lambda_t Q_t$ is the marginal value of capital.
- Since λ_t equals the marginal utility of consumption, Q_t can be interpreted as the ratio of the marginal value of capital due to the increase of the capital stock over the marginal opportunity cost: $Q_t = \frac{\lambda_t Q_t}{\lambda_t}$. This is the definition of the marginal Tobin-Q.
- at the steady-state, holds $Q = 1$, marginal revenues and costs compensate each other. If $Q > 1$ there are incentives to increase the capital accumulation through investment. If $Q < 1$ the reverse is true.