Dynamic Stochastic General Equilibrium Models

Dr. Andrea Beccarini M.Sc. Willi Mutschler

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The log-linearization procedure:

One starts from the Taylor-series approximation of the first order of the function $f(x_t, y_t)$ around the non-stochastic *steady-state* values x and y:

$$f(x_t, y_t) \approx f(x, y) + f'_x(x, y)(x_t - x) + f'_y(x, y)(y_t - y)$$

Having defined $x_t \approx xe^{\hat{x}_t}$ it follows $\hat{x}_t = log(x_t) - log(x) \approx \frac{x_t - x}{x}$ substituting it in the Taylor-series approximation yields:

$$f(x_t, y_t) \approx f(x, y) + xf'_x(x, y)\hat{x}_t + yf'_y(x, y)\ \hat{y}_t$$

It follows for example:

- $x_t y_t = xye^{\hat{x}_t + \hat{y}_t} \approx xy(1 + \hat{x}_t + \hat{y}_t)$
- $(x_t - y_t)^\theta \approx (x - y)^\theta + \theta(x - y)^{\theta - 1}(x\hat{x}_t - y\hat{y}_t)$

finally recall $\hat{x}_t \hat{y}_t \approx 0$

The linearized model:

Equation of consumption

The linearization of the Euler equation of consumption is the following:

$$\hat{C}_t = \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}E_t\hat{C}_{t+1} - \frac{1-h}{(1+h)\sigma_c}\left(\hat{R}_t - E_t\hat{\pi_{t+1}}\right) + \\ + \frac{1-h}{(1+h)\sigma_c}\left(\hat{\varepsilon}_t^B - E_t\hat{\varepsilon}_{t+1}^B\right)$$

By the external habit, present consumption depends on both the expected consumption of the next period and on consumption of the last period.

For $h = 0$ one obtains the usual Euler equation.

The elasticity of consumption with respect to interest rates depends both on the intertemporal elasticity of substitution σ_c and on the degree of the habit h.

A higher h reduces the effect on consumption of a variation of interest rate holding σ_c.

From the maximization problem of households, combine

$$\lambda_t \; = \varepsilon_t^B (C_t^\tau - H_t)^{-\sigma_c} \text{ with } E_t \left[\beta \lambda_{t+1} \frac{R_t}{P_{t+1}} \right] = \frac{\lambda_t}{P_t}$$

$$E_t \left[\beta \varepsilon_{t+1}^B (C_{t+1} - hC_t)^{-\sigma_c} \frac{R_t}{P_{t+1}} \right] \approx$$

$$E_t \left[\begin{array}{c} \beta \varepsilon^B (C - hC)^{-\sigma_c} \frac{R}{P} + \beta \varepsilon^B (C - hC)^{-\sigma_c} \frac{R}{P} \hat{\varepsilon}_{t+1}^B + \\ -\sigma_c \beta \varepsilon^B (C - hC)^{-\sigma_c - 1} (C \hat{C}_{t+1} - hC \hat{C}_t) \frac{R}{P} + \\ +\beta \varepsilon^B (C - hC)^{-\sigma_c} \frac{R}{P} \hat{R}_t - \beta \varepsilon^B (C - hC)^{-\sigma_c} \frac{R}{P} \hat{P}_{t+1} \end{array} \right] =$$

$$E_t \left[\begin{array}{c} \beta \varepsilon^B (C - hC)^{-\sigma_c} \frac{R}{P} \cdot \\ \cdot (1 + \hat{\varepsilon}_{t+1}^B - \sigma_c (C - hC)^{-1} (C \hat{C}_{t+1} - hC \hat{C}_t) + \hat{R}_t - \hat{P}_{t+1}) \end{array} \right]$$

$$\varepsilon_t^B(C_t - hC_{t-1})^{-\sigma_c} P_t^{-1} \approx$$

$$\varepsilon^B(C - hC)^{-\sigma_c} P^{-1} + \varepsilon^B(C - hC)^{-\sigma_c} P^{-1} \hat{\varepsilon}_t^B +$$
$$-\sigma_c \varepsilon^B (C - hC)^{-\sigma_c - 1} P^{-1}(C\hat{C}_t - hC\hat{C_{t-1}}) - \varepsilon^B(C - hC)^{-\sigma_c} P^{-1}\hat{P}_t =$$

$$\varepsilon^B(C - hC)^{-\sigma_c} P^{-1}(1 + \hat{\varepsilon}_t^B - \sigma_c(C - hC)^{-1}(C\hat{C}_t - hC\hat{C_{t-1}}) - \hat{P}_t)$$

and considering that in the *steady-state* it holds that

$$\beta R \left[\varepsilon^B (C - hC)^{-\sigma_c}\right] = \varepsilon^B (C - hC)^{-\sigma_c} \tfrac{P}{P} \Leftrightarrow \beta = \tfrac{1}{R}$$

Putting both sides together yields:
$$R_t + E_t \hat{\varepsilon}_{t+1}^B - \tfrac{\sigma_c}{1-h}\left(E_t(\hat{C}_{t+1}) - h\hat{C}_t\right) = \hat{\varepsilon_t} - \tfrac{\sigma_c}{1-h}\left(\hat{C}_t - h\hat{C_{t_-1}}\right) + E_t \hat{\Pi_{t+1}}$$

from which one obtains the explicit expression for $\hat{C}_t$

The linearized model:
Equation of investment

The linearization of the investment equation yields

$$\hat{I}_t = \frac{1}{1+\beta} \hat{I_{t-1}} + \frac{\beta}{1+\beta} E_t \hat{I_{t+1}} + \frac{1}{S''(1)(1+\beta)} \hat{Q}_t + \frac{\beta E_t \hat{\varepsilon}^I_{t+i} - \hat{\varepsilon}^I_t}{1+\beta}$$

The dependency of the optimal investment of past values introduce the dynamic in this equation.

A positive shock of the adjustment costs function reduces investment and hence it can be considered a negative investment shock.

Recall the optimal condition for investment:

$$
\begin{aligned}
\frac{\partial L^K}{\partial I_t} &= E_t \left[\begin{array}{c} -\beta^t \lambda_t - \beta^t \lambda_t Q_t \left(-1 + S\left(\frac{\varepsilon_t^I I_t}{I_{t-1}} \right) + I_t \cdot S'\left(\frac{\varepsilon_t^I I_t}{I_{t-1}} \right) \cdot \frac{\varepsilon_t^I}{I_{t-1}} \right) + \\ -\beta^{t+1} \lambda_{t+1} Q_{t+1} \left(I_{t+1} \cdot S'\left(\frac{\varepsilon_{t+1}^I I_{t+1}}{I_t} \right) \frac{-\varepsilon_{t+1}^I I_{t+1}}{I_t^2} \right) \end{array} \right] \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & Q_t S\prime \left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right)\frac{\varepsilon_t^I I_t}{I_{t-1}} - \beta E_t \left[\begin{array}{c} Q_{t+1}\frac{\lambda_{t+1}}{\lambda_t} S\prime \left(\frac{\varepsilon_{t+1}^I I_{t+1}}{I_t}\right) \\ \frac{\varepsilon_{t+1}^I I_{t+1}}{I_t}\frac{I_{t+1}}{I_t} \end{array} \right] + 1 = \\
= \quad & Q_t \left(1 - S\left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right)\right) \qquad (1)
\end{aligned}
$$

In the *steady-state* it holds:

$$
Q\underbrace{S'\left(\frac{\varepsilon^I I}{I}\right)}_{=0}\frac{\varepsilon^I I}{I} - \beta\left[Q\frac{\lambda}{\lambda}\underbrace{S'\left(\frac{\varepsilon^I I}{I}\right)}_{=0}\frac{\varepsilon^I I}{I}\frac{I}{I}\right] + 1 = Q\left(1 - \underbrace{S\left(\frac{\varepsilon^I I}{I}\right)}_{=0}\right) \Leftrightarrow
$$

$$
Q = 1,
$$

since in the *steady-state* $\varepsilon^I = 1$ and $S(1) = S'(1) = 0$.

The approximation of the single terms yields

$$S'\left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right)\frac{\varepsilon_t^I I_t}{I_{t-1}} \approx S'(1)\frac{\varepsilon^I I}{I} + + \left[S''(1)\frac{I}{I}\frac{\varepsilon^I I}{I} + S'(1)\frac{I}{I}\right]\left(\varepsilon_t^I - \varepsilon^I\right) +$$

$$+\left[S''(1)\frac{\varepsilon^I}{I}\frac{\varepsilon^I I}{I} + S'(1)\frac{\varepsilon^I}{I}\right](I_t - I) +$$

$$+\left[S''(1)\frac{-\varepsilon^I I}{I^2}\frac{\varepsilon^I I}{I} + S'(1)\frac{-\varepsilon^I I}{I^2}\right](I_{t-1} - I)$$

$$= S''(1)\left[\hat{I}_t - \hat{I}_{t-1} + \hat{\varepsilon}_t^I\right]$$

Note that since $S'(1) = 0$ one does not need the approximation of Q_t, Q_{t+1}, λ_t and λ.

$$
\begin{aligned}
&\beta E_t \frac{Q_{t+1}\lambda_{t+1}}{Q_t\lambda_t}\left[S'\left(\frac{\varepsilon^I_{t+1} I_{t+1}}{I_t}\right)\frac{\varepsilon^I_{t+1} I^2_{t+1}}{I^2_t}\right] \approx \\
&\beta\frac{Q\lambda}{Q\lambda}S'(1)\frac{\varepsilon^I I^2}{I^2} + \beta\frac{Q\lambda}{Q\lambda}\left[S''(1)\frac{\varepsilon^I}{I}\frac{\varepsilon^I I^2}{I^2} + 2S'(1)\frac{\varepsilon^I I}{I^2}\right](E_t I_{t+1} - I) \\
&\quad + \beta\frac{Q\lambda}{Q\lambda}\left[S''(1)\frac{-\varepsilon^I I}{I^2}\frac{\varepsilon^I I^2}{I^2} - 2S'(1)\frac{\varepsilon^I I}{I^3}\right](I_t - I) + \\
&\quad + \beta\frac{Q\lambda}{Q\lambda}\left[S''(1)\frac{I}{I}\frac{\varepsilon^I I^2}{I^2} + S'(1)\frac{I^2}{I^2}\right]\left(E_t\varepsilon^I_{t+1} - \varepsilon^I\right) = \\
&\qquad = \beta S''(1)\left(E_t\hat{I}_{t+1} - \hat{I}_t + E_t\hat{\varepsilon}^I_{t+1}\right)
\end{aligned}
$$

since $Q = 1:$

$$\frac{1}{Q_t} \approx \frac{1}{Q} - \frac{1}{Q^2}\left(Q_t - Q\right) = 1 - \hat{Q}_t$$

$$S\left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right) \approx S(1) + S'(1)\left(\frac{I}{I}(\varepsilon_t^I - \varepsilon) + \frac{\varepsilon^I}{I}(I_t - I) - \frac{\varepsilon^I I}{I^2}(I_{t-1} - I)\right) = 0 \tag{2}$$

and hence it follows

$$
\begin{aligned}
& S''(1)\left[\hat{I}_t - \hat{I}_{t-1} + \hat{\varepsilon}^I_t\right] - \beta S''(1)\left[E_t \hat{I}_{t+1} - \hat{I}_t + E_t \hat{\varepsilon}^I_{t+1}\right] \\
\approx \quad & 1 - 1 + \hat{Q}_t \\
\Leftrightarrow \quad & \hat{I}_t = \frac{1}{1+\beta} I_{t}\hat{}_{-1} + \frac{\beta}{1+\beta} E_t I_{t}\hat{}_{+1} + \\
& + \frac{1}{S''(1)(1+\beta)} \hat{Q}_t + \frac{\beta E_t \hat{\varepsilon}^I_{t+i} - \hat{\varepsilon}^I_t}{1+\beta} \qquad (3)
\end{aligned}
$$

The linearization of the Q-equation yields:

$$\hat{Q}_t = -(\hat{R}_t - E_t \hat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+r^k} E_t \hat{Q}_{t+1} + \frac{r^k}{1-\tau+r^k} E_t \hat{r}^k_{t+1} + \eta^Q_t$$

whereby because of the steady-state condition it holds, that
$\beta = 1/(1-\tau+r^k)$.

An equity premium shock is added in order to take into account the effect of changes of the external refinancing premium over the value of the capital stock: η^Q_t which is n.i.d..

Starting from $Q_t = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left[Q_{t+1}(1-\tau) + z_{t+1} r^k_{t+1} - \Psi(z_{t+1}) \right]$

In the *steady-state*: $Q = 1$, $z = 1$ and $\psi(1) = 0$. It follows:

$$Q = \beta \frac{\lambda}{\lambda} \left[Q(1-\tau) + z r^k - \psi(z) \right] \tag{4}$$

$$1 = \beta(1 - \tau + r^k) \tag{5}$$

The linearization of the left side yields $Q_t \approx 1 + \hat{Q}_t$.

The right-hand side becomes:

$$
\begin{aligned}
E_t\beta\frac{\lambda_{t+1}}{\lambda_t}\left[Q_{t+1}(1-\tau)+z_{t+1}r^k_{t+1}-\Psi(z_{t+1})\right] \approx \beta(1-\tau+r^k)+ \\
+\beta\frac{1}{\lambda}\underbrace{\left[Q(1-\tau)+zr^k-\psi(z)\right]}_{=\beta^{-1}}(E_t\lambda_{t+1}-\lambda)+ \\
-\beta\frac{\lambda}{\lambda^2}\underbrace{\left[Q(1-\tau)+zr^k-\psi(z)\right]}_{=\beta^{-1}}(\lambda_t-\lambda)+ \\
+\beta\frac{\lambda}{\lambda}(1-\tau)(E_tQ_{t+1}-Q)+\beta\frac{\lambda}{\lambda}z(E_tr^k_{t+1}-r)+ \\
\beta\frac{\lambda}{\lambda}\left(r^k-\psi'(z)\right)(z_{t+1}-z)= \\
=1+E_t\hat{\lambda}_{t+1}-\hat{\lambda}_t+\beta(1-\tau)E_t\hat{Q}_{t+1}+\beta E_tr^k\hat{r}_{t+1}
\end{aligned}
$$

whereby one has exploited that $r^k=\psi'(z)$.

From the Euler equation it follows: $E_t\lambda_{t+1} - \lambda_t = -(\hat{R}_t - \hat{\pi}_{t+1})$.

Putting these conditions together yields:

$$\hat{Q}_t = -(\hat{R}_t - \hat{\pi}_{t+1}) + +\beta(1-\tau)E_t\hat{Q}_{t+1} + \beta E_t r^k \hat{r}_{t+1}$$

whereby $\beta = 1/(1 - \tau + r^k)$.

The linearization of the equation of the capital accumulation yields:

$$\hat{K}_t = (1-\tau)\hat{K}_{t-1} + \tau \hat{I}_{t-1}$$

Starting from $K_t = K_{t-1}(1-\tau) + I_t - I_t S\left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right)$.

In the *steady-state* it holds $K = (1-\tau)K + I - I\underbrace{S(1)}_{=0} \Leftrightarrow I = \tau K$.

The approximation of the left side yields $K_t = K + (K_t - K) \approx K + \hat{K}_t K$.

For the right side, one obtains :

$$K_{t-1}(1-\tau) + I_t - I_t S\left(\frac{\varepsilon_t^I I_t}{I_{t-1}}\right) \approx$$
$$(1-\tau)K + I + (1-\tau)(K_{t-1} - K) + (I_t - I)$$

and considering the *steady-state* relationships:

$$\hat{K}_t K = (1-\tau)K\hat{K}_{t-1} + \hat{I}_t I = (1-\tau)K\hat{K}_{t-1} + \hat{I}_t \tau K \qquad (6)$$

$$\hat{K}_t = (1-\tau)\hat{K}_{t-1} + \tau\hat{I}_{t-1}$$

The linearized model:

The Phillips curve

Through the partial indexation it follows that the linearized equation depends not only on the present marginal costs and on the expectations of the future inflation but also on the past inflation rate.

$$
\begin{aligned}
\hat{\pi}_t &= \frac{\beta}{1+\beta\gamma_p} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \hat{\pi}_{t-1} + \\
&+ \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p} \underbrace{\left(\alpha \hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\varepsilon}_t^a + \eta_t^p\right)}_{\hat{mc}_t}
\end{aligned}
$$

For $\gamma_p = 0$ the usual forward-looking Phillips curve

Starting from

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \frac{\lambda_{t+i}}{\lambda_t} y_{t+i}^j \left[\frac{\widetilde{p}_t}{P_t} \left(\frac{(P_{t+i-1}/P_{t-1})^{\gamma_p}}{P_{t+i}/P_t} \right) - (1+\lambda_{p,t+i}) \frac{MC_{t+i}}{P_{t+i}} \right] = 0$$

denote $\varphi_t = \log(\widetilde{p}_t)$, $p_t = \log(P_t)$, $mc_t = \log(MC_t/P_t)$ and $\mu = \log(1+\lambda_p)$, then it follows :

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \frac{\lambda_{t+i}}{\lambda_t} y_{t+i}^j \left[e^{\varphi_t - p_{t+i} + \gamma_p (p_{t+i-1} - p_{t-1})} - e^{\mu + mc_{t+i}} \right] = 0$$

In the *steady-state* it holds $\varphi_t = p_t = p$. It follows:

$$e^0 - e^{\mu + mc} = 0 \Leftrightarrow mc = -\mu$$

The linearization yields (having noted that the internal bracket is null in the *steady-state)*.

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \frac{\lambda}{\lambda} y \left[e^0(\varphi_t - p) + e^0(-1)(p_{t+i} - p) \right.$$
$$\left. + e^0 \gamma_p (p_{t+i-1} - p) - e^0 \gamma_p (p_{t-1} - p) - e^0 (mc_{t+i} - mc) \right] = 0 \quad (7)$$

$$E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \varphi_t = E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \left[\underbrace{-mc}_{=\mu} + mc_{t+i} + p_{t+i} - \gamma_p (p_{t+i-1} - p_{t-1}) \right] \quad (8)$$

$$\frac{1}{1-\beta\xi_p} \varphi_t = \frac{1}{1-\beta\xi_p} \mu + E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \left[mc_{t+i} + p_{t+i} - \gamma_p (p_{t+i-1} - p_{t-1}) \right] \quad (9)$$

$$\varphi_t = \mu + (1 - \beta\xi_p) E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \left[mc_{t+i} + p_{t+i} - \gamma_p (p_{t+i-1} - p_{t-1}) \right]$$

$$\hat{\varphi}_t = \varphi_t - p = (1 - \beta\xi_p) E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \left[\hat{mc}_{t+i} + \hat{p}_{t+i} - \gamma_p (p_{t+i-1} - p_{t-1}) \right] \tag{10}$$

$$\hat{\varphi}_t - \beta\xi_p E_t \hat{\varphi}_{t+1} = (1 - \beta\xi_p)(\hat{mc}_t + \hat{p}_t) \tag{11}$$

The linearization of the equation of motion of the price level yields:

$$(P_t)^{\frac{-1}{\lambda_{p,t}}} = \xi_p \left[\left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} P_{t-1} \right]^{\frac{-1}{\lambda_{p,t}}} + \left(1 - \xi_p\right) \widetilde{p}_t^{\,\frac{-1}{\lambda_{p,t}}}$$

$$1 = \xi_p \left[\Pi_{t-1}^{\gamma_p} \Pi_t^{-1} \right]^{-1/\lambda_{p,t}} + (1 - \xi_p) \left(\frac{\widetilde{p}_t}{P_t} \right)^{-1/\lambda_{p,t}}$$

$$\begin{aligned} 0 \quad = \quad & -\frac{1}{\lambda_{p,t}} \xi_p \left[\Pi^{\gamma_p} \Pi^{-1} \right]^{-\frac{1}{\lambda_{p,t}}-1} \left(\gamma_p \pi^{\gamma_p - 1} (\Pi_{t-1} - \Pi) - \Pi^{-2} (\Pi_t - \Pi) \right) \\ & -\frac{1}{\lambda_{p,t}} (1 - \xi_p) \frac{P}{P}^{-\frac{1}{\lambda_{p,t}}-1} \frac{1}{P} \left((\widetilde{p}_t - P) - (P_t - P) \right) \end{aligned} \qquad (12)$$

$$\hat{\varphi}_t - \hat{p}_t = \frac{\xi_p}{1 - \xi_p} \left(\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} \right)$$

The substitution in eq. (10) yields the new-keynsian Phillips curve

$$\begin{aligned}\hat{\pi}_t &= \frac{\beta}{1+\beta\gamma_p} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \hat{\pi}_{t-1} + \\ &\quad \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p} \underbrace{\left(\alpha \hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\varepsilon}_t^a + \eta_t^p\right)}_{\hat{mc}_t}\end{aligned}$$

Similarly one obtains the linearized equation of motion for the real wage:

$$\begin{aligned}\hat{w}_t &= \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \\ &\quad \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} - \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)\left(1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right)\xi_w}\cdot \\ &\quad \cdot\left(\hat{w}_t - \sigma_l\hat{L}_t - \frac{\sigma_c}{1-h}\left(\hat{C}_t - h\hat{C}_{t-1}\right) - \hat{\varepsilon}_t^L - \eta_t^{\hat{w}}\right) \qquad (13)\end{aligned}$$

The real wage is a function of expected and past real wages and the expected, current and past inflation rate if $\gamma_w > 0$.

There is a negative effect of the deviation of the actual real wage from the wage that would prevail in a flexible labour market.

The linearized demand of labour is:

$$\hat{L}_t = -\hat{w}_t + (1 + \frac{\psi'(1)}{\psi''(1)})\hat{r}_t^k + \hat{K}_{t-1}$$

Starting from: $\frac{W_t L_t}{r_t^k z_t K_{t-1}} = \frac{1-\alpha}{\alpha}$.
In the steady-state it holds $\frac{WL}{r^k zK} = \frac{1-\alpha}{\alpha}$, whereby $z = 1$.

substracting from the logarithmic transformation of (**??**) the logarithmic *steady-state:*

$$\begin{aligned} \log(L_t) - \log(L) &= \log(r_t^k) - \log(r^k) - (\log(W_t) - \\ &\quad + log(P_t) - (\log(W) - \log(P_t))) + \\ &\quad + \log(z_t) - \log(z) + \log(K_{t-1} - \log(K)) \end{aligned} \tag{14}$$

and hence: $\hat{L}_t = \hat{r}_t^k - \hat{w}_t + \hat{z}_t + \hat{K}_{t-1}$

$$r_t^k = \Psi'(z_t) \Longrightarrow r^k + r^k \hat{r}_t^k = \Psi'(z) + \Psi''(z) z_t \hat{z}_t \approx \Psi'(z_t)$$
$$r^k = \Psi'(z),\ z = 1 \Longrightarrow \hat{r}_t^k \Psi'(z) = \Psi''(1)\hat{z}_t$$
$$\hat{z}_t = \frac{\Psi'(1)}{\Psi''(1)} \hat{r}_t^k$$

It follows: $\hat{L}_t = -\hat{w}_t + \left(1 + \frac{\Psi'(1)}{\Psi''(1)}\right) \hat{r}_t^k + \hat{K}_{t-1}$

This works like the derivation of the equation of inflation.

Given a capital stock, the demand of labour negatively depends on real wages.

The market equilibrium:

The good market is in equilibrium if the production is equal to the public expense G_t, the household demand of consumption and investment plus related costs.

$$Y_t = C_t + G_t + I_t + \psi(z_t)K_{t-1}$$

The capital market is in equilibrium if the demand (of firms of the intermediate sector) of capital goods equals the supply of households.

The labour market is in equilibrium if the demand of labour of firms equals the supply of households at wages set by households.

The good market equilibrium in steady-state deviations is:

$$
\begin{aligned}
\hat{Y}_t &= (1-\tau k_y - g_y)\hat{C}_t + \tau k_y \hat{I}_t + g_y \varepsilon_t^G = \\
&= \phi\hat{\varepsilon}_t^a + \phi\alpha\hat{K}_{t-1} + \phi\alpha\frac{\psi'(1)}{\psi''(1)}\hat{r}_t^k + \phi(1-\alpha)\hat{L}_t
\end{aligned}
$$

where k_y and g_y denote the steady-state proportion of capital and public spending on output.

$1+\phi$ is the proportion of fix costs of the production.

The shock of the public spending ε_t^G follows a AR(1) process.

The monetary policy:

The monetary policy follows a generalized Taylor-Rule:

$$\begin{aligned}\hat{R}_t &= \rho \hat{R}_{t-1} + (1-\rho)\left[\bar{\pi}_t + r_\pi \left(\hat{\pi}_{t-1} - \bar{\pi}_t\right) + r_Y \hat{Y}_t\right] + \\ &\quad + r_{\Delta_\pi}\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + r_{\Delta_Y}\left(\hat{Y}_t - \hat{Y}_{t-1}\right) + \eta_t^R \qquad (15)\end{aligned}$$

- The Taylor rule is obtained by minimizing the central bank loss function subject to the Phillips curve and the IS curve.
- The central bank loss function is generally set to be equal to a weighted sum of the squared difference between the actual and the target inflation and the squared output gap.
- The output gap $\hat{Y}_t$ is the difference between the actual and the potential output.
- The potential output is the production that theoretically the economy performs under perfect flexibility of prices and wages.

- The central bank reacts to deviations of both the lagged inflation with respect to the target inflation (normalized to 0) and of the lagged output gap.
- The monetary policy is subject to two different kinds of shock: a shock of the target inflation process, $\bar{\pi}_t = \rho_\pi \bar{\pi}_{t-1} + \eta_t^\pi$, and shock of the interest rate: η_t^R.
- The central smooths the interest rates by setting the present rate as a weighted average of the past interest rate and the optimal rate as a function of the above quantities.