

# Putting the Cycle Back into Business Cycle Analysis

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# Business Cycle Analysis and Modern Macroeconomics

Forces and mechanisms that drive economic fluctuations remain a debated subject.

Two theoretical approaches:

- ① BCs are primarily driven by persistent exogenous shocks
- ② BCs are mostly driven by forces internal to the economy which endogenously favor recurrent periods of boom and bust.

This paper argues that data favors the second theoretical approach.

## Appeals of the First Approach

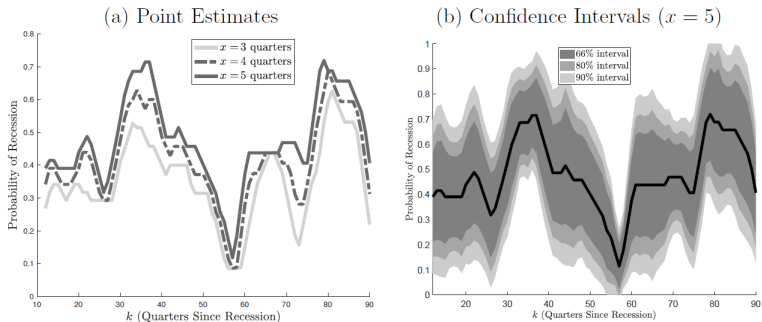
- ① Empirical estimation of DSGE models support the existence and the persistence of exogenous driving forces to explain the data.
- ② Since Granger (1966) and Sargent (1987), it has been argued that data are generally not supportive of strong internal boom-bust mechanisms.

# This Paper

- ① They examine spectral density properties of macro aggregates
  - a recurrent peak in several spectral densities at periodicities around 9 to 10 years
- ② They analyze necessary features to theoretically reproduce peaks in the spectral densities
  - strategic complementarities across agents
  - accumulation of a stock with decreasing returns
- ③ They estimate a NK model where persistent shocks and endogenous cyclical mechanisms compete to explain data
  - estimation favors endogenous mechanisms to match empirical spectral density
  - estimation suggests the existence of stochastic limit cycles

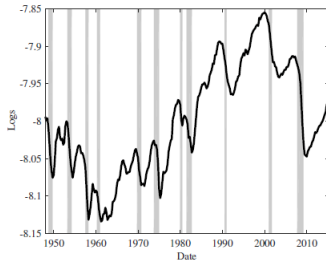
# Empirical Evidence - NBER Recessions

Figure 1: Conditional Probability of Being in a Recession

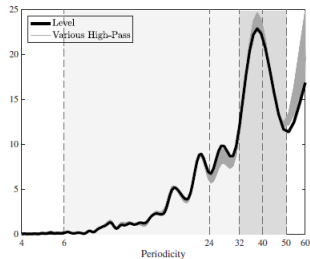
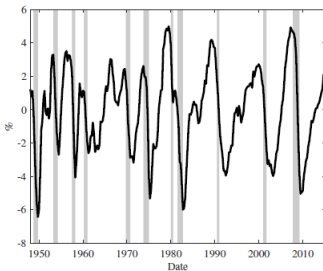


Notes: Panel (a) displays the fraction of time the economy was in a recession within an  $x$ -quarter window around time  $t + k$ , conditional on being in a recession at time  $t$ , where  $x$  is allowed to vary between 3 and 5 quarters. Panel (b) shows confidence intervals for the  $x = 5$  case. See Appendix B for the  $x = 3$  and  $x = 4$  cases, as well as for details of how these confidence intervals were constructed. The figure was constructed using NBER recession dates over the period 1946Q1-2017Q2.

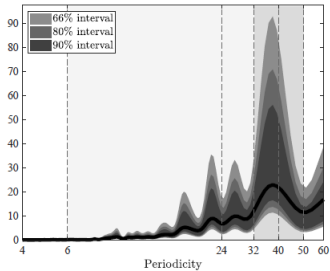
# Empirical Evidence (II) - Hours Worked per Capita



(c) High-Pass (60) Filtered Hours

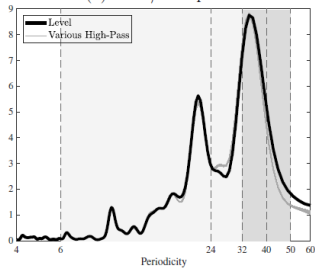


(d) Spectral Density: Confidence Bands

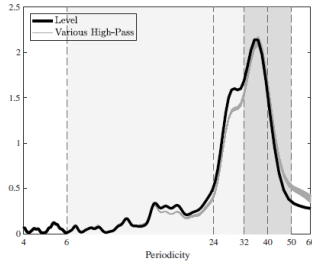


# Empirical Evidence (II) - Financial Variables

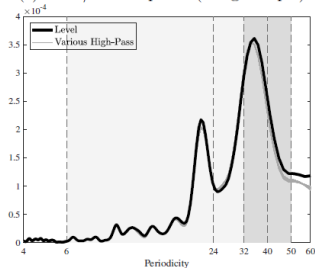
(a) BAA/FF Spread



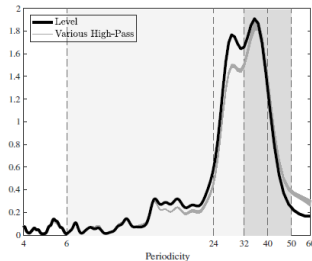
(b) National Financial Conditions Index



(c) AAA/T-bill Spread (Long Sample)

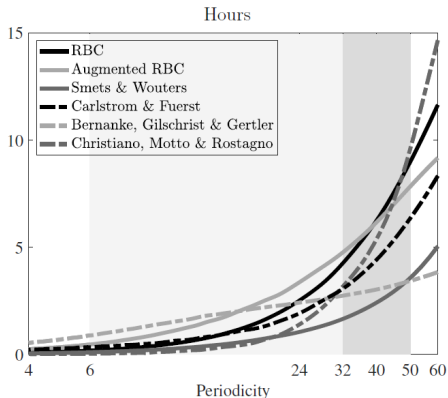


(d) NFCI Risk Sub-index



# Spectra Implied by Current DSGE Models

Figure 5: Spectral Densities of Hours in Some Standard Models



Notes: The figure displays the mean spectral density of hours over 1000 simulations of length 270. Models and parameters values are Cooley and Prescott [1995] for the standard RBC model, Fernandez-Villaverde [2016] for the augmented RBC (with variable capital utilization and investment specific technology shocks), Smets and Wouters [2007], Carlstrom and Fuerst [1997] and Christensen and Dib [2008] for a version of Bernanke, Gertler, and Gilchrist [1999] estimated on U.S. data and Christiano, Motto, and Rostagno [2014]. For better visual display, all the series have been standardized to have unit variance.



# Takeaway

- Probability to fall in a recession rises every 8-10 years
- Measures of labor activity exhibit significant spectral peaks at a periodicity of around 36-40 quarters
- Financial variables display analogous patterns
- Current theoretical models are unable to match previous facts

They provide a class of models able to reproduce cyclical outcomes based on equilibrium interactions internal to the model.

## A Class of Models

Consider an environment with  $N$  agents indexed by  $j$ .

Agent  $j$  makes decision  $e_{j,t}$  according to,

$$e_{j,t} = \alpha_1 X_{j,t} + \alpha_2 e_{j,t-1} + \alpha_3 q_t + \mu_t, \quad 0 < \alpha_2 < 1 \quad (1)$$

where  $X_t$  is a stock variable with the following law of motion,

$$X_{j,t+1} = (1 - \delta)X_{j,t} + e_{j,t}, \quad 0 < \delta < 1 \quad (2)$$

and  $\mu_t$  is an exogenous driving force. Finally,  $q_t$  is a aggregate market determined variable,

$$q_t = \alpha_4 \frac{1}{N} \sum_j e_{j,t} = \alpha_4 e_t \quad (3)$$

where  $\alpha_3 \alpha_4$  governs the degree of strategic complementarity (substitutability) in the economy.

## Spectrum of $e_t$

Invoke symmetry and solve for  $e_t$ ,

$$e_t = \left( \frac{\alpha_1 + \alpha_2}{1 - \alpha_3\alpha_4} + 1 - \delta \right) e_{t-1} - \frac{\alpha_2(1 - \delta)}{1 - \alpha_3\alpha_4} e_{t-2} + \frac{1 - (1 - \delta)L}{1 - \alpha_3\alpha_4} \mu_t$$

which implies the following spectral density

$$s_e(\omega) = s_\mu(\omega) \frac{[1 - (1 - \delta) \exp(i\omega)][1 - (1 - \delta) \exp(i\omega)]}{(1 - \alpha_3\alpha_4)^2} g(\omega)$$

where

- $g(\omega) \equiv [B(\exp(i\omega))B(\exp(i\omega))]^{-1}$
- $B(L) \equiv 1 - \left( \frac{\alpha_1 + \alpha_2}{1 - \alpha_3\alpha_4} + 1 - \delta \right) L + \frac{\alpha_2(1 - \delta)}{1 - \alpha_3\alpha_4} L^2$

## Necessary Conditions for Peak in the Spectral Density

Sargent (1979): necessary conditions is  $B(L)$  to have complex roots in  $L$ .

Assumption: parameters are such that if  $\alpha_3\alpha_4 = 0$ , then the eigenvalues are real, positive and smaller than 1.

Then, necessary conditions are

- $\alpha_3\alpha_4 > 0$ : strategic complementarity
- $\alpha_1 < 0$ : decreasing returns in  $X_{j,t}$

$$\begin{cases} e_{j,t} = \alpha_1 X_{j,t} + \alpha_2 e_{j,t-1} + \alpha_3 \alpha_4 e_t + \mu_t \\ X_{j,t+1} = (1 - \delta) X_{j,t} + e_{j,t} \end{cases}$$

# Technical Ingredients

In order to have a peak in the spectral density which is not driven by exogenous forces, they need

- Complex eigenvalues
  - Dynamics are represented by trigonometric functions
  - For an AR(2) process complex eigenvalues are a necessary condition for a peak in the spectral density (Sargent, 1979)
- Local instability surrounded by a stochastic limit cycle
  - Cyclical dynamics are perpetual
  - Main critique of limit cycle dynamics: predictability and regularity of the cycle
  - However, when a limit cycle is perturbed by unpredictable disturbances, size and period of the cycle changes permanently

In order to have a peak in the spectral density which is not driven by exogenous forces, they need

- Strategic complementarity
  - It is a well-known source of instability
- Decreasing return to scale in a stock variable
  - When combined with strategic complementarity, the economy exhibits periods of accumulation and dissipation.

# A New Keynesian Model

# Household

Household  $h$ 's preferences are given by

$$E_0 \sum_t \beta^t \zeta_{t-1} [U(C_{h,t} - \gamma C_{t-1}) + \nu(1 - e_{h,t})] \quad (4)$$

In addition to purchase consumption services  $C_{h,t}$  at price  $P_t$  and labor  $e_{h,t}$  at wage  $W_t$ , household  $h$  decides to purchase an amount  $I_t$  of durable consumption  $X_{h,t}$  at price  $P_t^X$ .

Law of motion of  $X_{h,t}$  is:

$$X_{h,t+1} = (1 - \delta)X_{h,t} + I_t$$

and household budget constraint is:

$$\underbrace{(1 + i_t)}_{\text{Deposit Rate}} Y_{h,t} \geq \underbrace{[e_t + (1 - e_t)\phi]}_{\text{Prob. of Repay}} \underbrace{(1 + r_t)}_{\text{Risky Rate}} \underbrace{(P_t C_{h,t} + P_t^X I_{h,t})}_{\text{Loan}}$$



Households have to pay in advance purchases of consumption services ( $C_{h,t}$ ) and durable goods ( $I_{h,t}$ ).

Banks finance household purchases at interest rate  $1 + r_t$  which satisfies the following zero-profit condition

$$1 + r_t = (1 + i_t) \frac{1 + (1 - e_t)\phi\Phi}{e_t + (1 - e_t)\phi}$$

where risk premium is

$$1 + r_t^p = \frac{1 + (1 - e_t)\phi\Phi}{e_t + (1 - e_t)\phi} \quad (5)$$

Intermediate firm  $k$  produces consumption services as follows,

$$C_{k,t} = s[X_{k,t} + \theta F(e_{k,t})], \quad s > 0$$

where  $\theta$  is exogenous productivity.

Moreover, the market for intermediate services is subject to sticky prices à la Calvo (1983).

Accordingly, final goods sector is competitive and combine  $k$ -specific consumption of services according to a Dixit-Stiglitz aggregator.

# Central Bank and Equilibrium

To close the model, central bank determine the risk-free rate  $i_t$  according to

$$1 + i_t = \Theta E_t[e_{t+1}^{\varphi_e}(1 + \pi_{t+1})]$$

Equilibrium is defined as

$$X_{t+1} = (1 - \delta)X_t + \psi\theta F(e_t) \tag{6}$$

$$\begin{aligned} & U'\{s[X_t + \theta F(e_t)] - \gamma s[X_{t-1} + \theta F(e_{t-1})]\} \\ &= (1 + (1 - e_t)\phi\Phi)\beta \frac{\zeta_t}{\zeta_{t-1}} \Theta \\ &\times E_t[\{s[X_t + \theta F(e_t)] - \gamma s[X_{t-1} + \theta F(e_{t-1})]\} e_{t+1}^{\varphi_e}] \end{aligned} \tag{7}$$