Putting the Cycle Back into Business Cycle Analysis

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Business Cycle Analysis and Modern Macroeconomics

Forces and mechanisms that drive economic fluctuations remain a debated subject.

Two theoretical approaches:

1. BCs are primarily driven by persistent exogenous shocks
2. BCs are mostly driven by forces internal to the economy which endogenously favor recurrent periods of boom and bust.

This paper argues that data favors the second theoretical approach.

Appeals of the First Approach

1. Empirical estimation of DSGE models support the existence and the persistence of exogenous driving forces to explain the data.

2. Since Granger (1966) and Sargent (1987), it has been argued that data are generally not supportive of strong internal boom-bust mechanisms.

This Paper

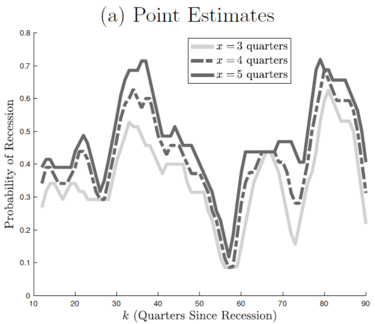
1. They examine spectral density properties of macro aggregates
   - a recurrent peak in several spectral densities at periodicities around 9 to 10 years

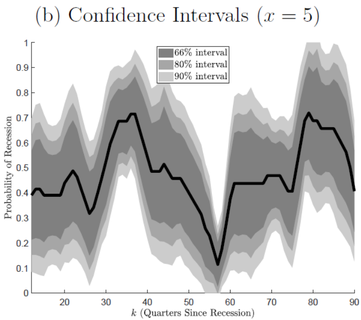
2. They analyze necessary features to theoretically reproduce peaks in the spectral densities
   - strategic complementaries across agents
   - accumulation of a stock with decreasing returns

3. They estimate a NK model where persistent shocks and endogenous cyclical mechanisms compete to explain data
   - estimation favors endogenous mechanisms to match empirical spectral density
   - estimation suggests the existence of stochastic limit cycles

Empirical Evidence - NBER Recessions

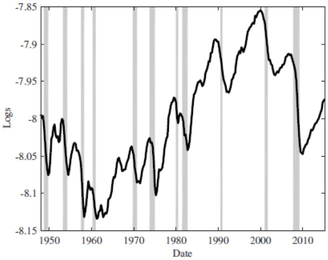
Figure 1: Conditional Probability of Being in a Recession

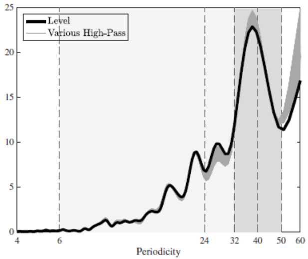




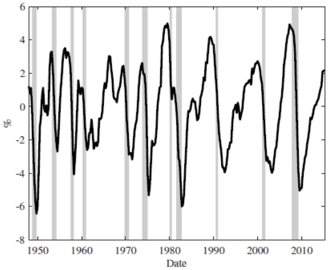
Notes: Panel (a) displays the fraction of time the economy was in a recession within an x-quarter window around time $t + k$, conditional on being in a recession at time t, where x is allowed to vary between 3 and 5 quarters. Panel (b) shows confidence intervals for the $x = 5$ case. See Appendix B for the $x = 3$ and $x = 4$ cases, as well as for details of how these confidence intervals were constructed. The figure was constructed using NBER recession dates over the period 1946Q1-2017Q2.

Empirical Evidence (II) - Hours Worked per Capita

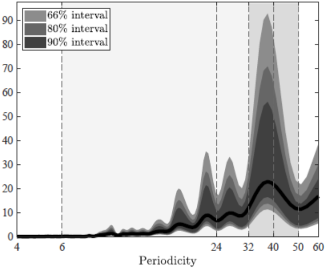




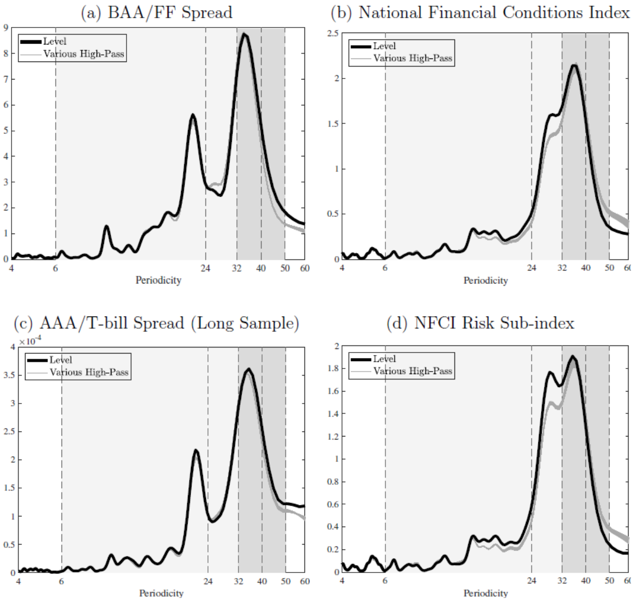
(c) High-Pass (60) Filtered Hours



(d) Spectral Density: Confidence Bands

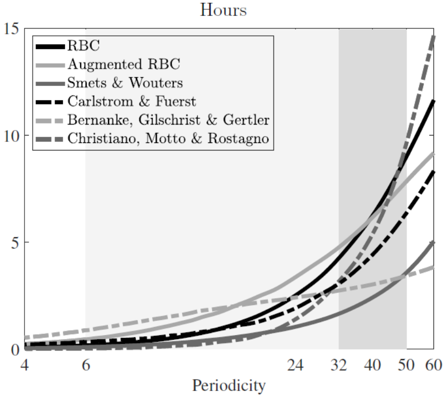


Empirical Evidence (II) - Financial Variables



Spectra Implied by Current DSGE Models

Figure 5: Spectral Densities of Hours in Some Standard Models



Notes: The figure displays the mean spectral density of hours over 1000 simulations of length 270. Models and parameters values are Cooley and Prescott [1995] for the standard RBC model, Fernandez-Villaverde [2016] for the augmented RBC (with variable capital utilization and investment specific technology shocks), Smets and Wouters [2007], Carlstrom and Fuerst [1997] and Christensen and Dib [2008] for a version of Bernanke, Gertler, and Gilchrist [1999] estimated on U.S. data and Christiano, Motto, and Rostagno [2014]. For better visual display, all the series have been standardized to have unit variance.

Takeaway

- Probability to fall in a recession rises every 8-10 years
- Measures of labor activity exhibit significant spectral peaks at a periodicity of around 36-40 quarters
- Financial variables display analogous patters
- Current theoretical models are unable to match previous facts

They provide a class of models able to reproduce cyclical outcomes based on equilibrium interactions internal to the model.

A Class of Models

Consider an environment with N agents indexed by j.

Agent j makes decision $e_{j,t}$ according to,

$$e_{j,t} = \alpha_1 X_{j,t} + \alpha_2 e_{j,t-1} + \alpha_3 q_t + \mu_t, \quad 0 < \alpha_2 < 1 \tag{1}$$

where is X_t is a stock variable with the following law of motion,

$$X_{j,t+1} = (1-\delta) X_{j,t} + e_{j,t}, \quad 0 < \delta < 1 \tag{2}$$

and μ_t is an exogenous driving force. Finally, q_t is a aggregate market determined variable,

$$q_t = \alpha_4 \frac{1}{N} \sum_j e_{j,t} = \alpha_4 e_t \tag{3}$$

where $\alpha_3 \alpha_4$ governs the degree of strategic complementarity (substitutability) in the economy.

Spectrum of e_t

Invoke symmetry and solve for e_t,

$$e_t = \left(\frac{\alpha_1 + \alpha_2}{1 - \alpha_3\alpha_4} + 1 - \delta\right) e_{t-1} - \frac{\alpha_2(1-\delta)}{1-\alpha_3\alpha_4} e_{t-2} + \frac{1-(1-\delta)L}{1-\alpha_3\alpha_4}\mu_t$$

which implies the following spectral density

$$s_e(\omega) = s_\mu(\omega)\frac{[1-(1-\delta)\exp(i\omega)][1-(1-\delta)\exp(i\omega)]}{(1-\alpha_3\alpha_4)^2} g(\omega)$$

where

- $g(\omega) \equiv [B(\exp(i\omega))B(\exp(i\omega))]^{-1}$
- $B(L) \equiv 1 - \left(\frac{\alpha_1+\alpha_2}{1-\alpha_3\alpha_4} + 1 - \delta\right)L + \frac{\alpha_2(1-\delta)}{1-\alpha_3\alpha_4}L^2$

Necessary Conditions for Peak in the Spectral Density

Sargent (1979): necessary conditions is $B(L)$ to have complex roots in L.

Assumption: parameters are such that if $\alpha_3\alpha_4 = 0$, then the eigenvalues are real, positive and smaller than 1.

Then, necessary conditions are

- $\alpha_3\alpha_4 > 0$: strategic complementarity
- $\alpha_1 < 0$: decreasing returns in $X_{j,t}$

$$\begin{cases} e_{j,t} = \alpha_1 X_{j,t} + \alpha_2 e_{j,t-1} + \alpha_3\alpha_4 e_t + \mu_t \\ X_{j,t+1} = (1-\delta)X_{j,t} + e_{j,t} \end{cases}$$

Technical Ingredients

In order to have a peak in the spectral density which is not driven by exogenous forces, they need

- Complex eigenvalues
  - Dynamics are represented by trigonometric functions
  - For an AR(2) process complex eigenvalues are a necessary condition for a peak in the spectral density (Sargent, 1979)

- Local instability surrounded by a stochastic limit cycle
  - Cyclical dynamics are perpetual
  - Main critique of limit cycle dynamics: predictability and regularity of the cycle
  - However, when a limit cycle is perturbed by unpredictable disturbances, size and period of the cycle changes permanently

Economic Ingredients

In order to have a peak in the spectral density which is not driven by exogenous forces, they need

- Strategic complementarity
  - It is a well-known source of instability

- Decreasing return in a stock variable
  - When combined with strategic complementarity, the economy exhibits periods of accumulation and dissipation.

A New Keynesian Model

Augment a standard NK Model with

- External habit formation
- Intra-temporal budget constraint on the household side
- Positive and endogenous probability of bankruptcy
- Lender cannot fully recover its investment after bankruptcy

Household

Household h's preferences are given by

$$E_0 \sum_t \beta^t \zeta_{t-1}[U(C_{h,t} - \gamma C_{t-1}) + \nu(1 - e_{h,t})] \tag{4}$$

In addition to purchase consumption services $C_{h,t}$ at price P_t and labor $e_{h,t}$ at wage W_t, household h decides to purchase an amount I_t of durable consumption $X_{h,t}$ at price P_t^X.

Law of motion of $X_{h,t}$ is:

$$X_{h,t+1} = (1-\delta)X_{h,t} + I_t$$

and household budget constraint is:

$$\underbrace{(1+i_t)}_{\text{Deposit Rate}} \; Y_{h,t} \geq \underbrace{[e_t + (1-e_t)\phi]}_{\text{Prob. of Repay}} \underbrace{(1+r_t)}_{\text{Risky Rate}} \underbrace{(P_t C_{h,t} + P_t^X I_{h,t})}_{\text{Loan}}$$

Banks

Households have to pay in advance purchases of consumption services ($C_{h,t}$) and durable goods ($I_{h,t}$).

Banks finance household purchases at interest rate $1 + r_t$ which satisfies the following zero-profit condition

$$1 + r_t = (1 + i_t)\frac{1 + (1 - e_t)\phi\Phi}{e_t + (1 - e_t)\phi}$$

where risk premium is

$$1 + r_t^p = \frac{1 + (1 - e_t)\phi\Phi}{e_t + (1 - e_t)\phi} \tag{5}$$

Firms

Intermediate firm k produces consumption services as follows,

$$C_{k,t} = s[X_{k,t} + \theta F(e_{k,t})], \quad s > 0$$

where θ is exogenous productivity.

Moreover, the market for intermediate services is subject to sticky prices à la Calvo (1983).

Accordingly, final goods sector is competitive and combine k-specific consumption of services according to a Dixit-Stiglitz aggregator.

Central Bank and Equilibrium

To close the model, central bank determine the risk-free rate i_t according to

$$1 + i_t = \Theta E_t[e_{t+1}^{\varphi_e}(1 + \pi_{t+1})]$$

Equilibrium is defined as

$$X_{t+1} = (1 - \delta)X_t + \psi\theta F(e_t) \tag{6}$$

$$\begin{aligned} & U'\{s[X_t + \theta F(e_t)] - \gamma s[X_{t-1} + \theta F(e_{t-1})]\} \\ = & (1 + (1 - e_t)\phi\Phi)\beta\frac{\zeta_t}{\zeta_{t-1}}\Theta \\ \times & E_t\left[\{s[X_t + \theta F(e_t)] - \gamma s[X_{t-1} + \theta F(e_{t-1})]\}e_{t+1}^{\varphi_e}\right] \end{aligned} \tag{7}$$

Economic Intuition

- Suppose to be in a recession
- Unemployment is high, so bankruptcy rate
- Risk premium is large and household consumption is low
- Capital X_t is keep decreasing during this phase
- The lower X_t the larger the marginal utility of the household
- MU will eventually be so high to be larger than the loan rate
- Household increases borrowings, increasing the demand
- Unemployment thus decrease, so does the bankruptcy rate
- Risk premium decreases, consumption and X_t increase
- Capital X_t is keep increasing during this phase
- The higher X_t the lower the marginal utility of the household
- MU will eventually be so low to be lower than the loan rate
- ...

Estimation of the Model

Exogenous parameters:

- $\delta = 0.05$
- $\alpha = 2/3$
- Θ such that steady state unemployment rate is 0.0583

They estimate the remaining parameters (10) of the model via spectral density matching.

Estimation Results - Endogenous Parameters

Table 7: Estimated Parameter Values

		(a) Non-Linear RP	(b) Linear RP	(c) No Friction	(d) Canonical
ω	CRRA parameter	0.2997	0.2408	0.2408*	0.2408*
		(0.0200)	(0.0423)	—	—
γ	Habit	0.5335	0.5876	0.9405	0.5876*
		(0.0031)	(0.0512)	(0.0836)	—
ψ	One minus initial dep.	0.4000	0.2994	0.2994*	0*
		(0.0028)	(0.0644)	—	—
φ_e	Taylor rule	0.0421	0.0467	0.0467*	0.0467*
		(0.0028)	(0.0057)	—	—
ϕ	Debt backing	0.8668	0.8827	1*	1*
		(0.0067)	(0.0074)	—	—
Φ	Recovery cost	0.0421	0.0458	0*	0*
		(0.0029)	(0.0067)	—	—
ϱ_2	Risk prem. (2nd order)	0.0167	—	—	—
		(0.0008)	—	—	—
ϱ_3	Risk prem. (3rd order)	0.5929	—	—	—
		(0.0575)	—	—	—
ρ	Autocorrelation	-0.0000	0.1387	0.8541	0.8609
		(0.0000)	(0.0799)	(0.1118)	(0.1077)
σ	Innovation s.d.	0.00016	0.00027	0.00148	0.00148
		(0.00003)	(0.00010)	(0.00076)	(0.00075)
$s.d.(\mu)$	Implied uncond. s.d.	0.00016	0.00027	0.00285	0.00292

Notes: Table displays the estimated parameters of the model for each of the four estimation scenarios with standard errors in parentheses. * indicates calibrated values. Estimates for the non-linear RP model imply $\varrho_1 = -0.1624$, $\phi'' = -3.1$ and $\phi''' = -227.1$, while for the linear RP model we have $\varrho_1 = -0.1506$. In the bottom row, we report the unconditional standard deviation of the shock process μ_t implied by the point estimates for ρ and σ.

Estimation Results - Eigenvalues

Solving the model with a less restrictive solution method allows to obtain a parameterization that supports limit cycle and local instability

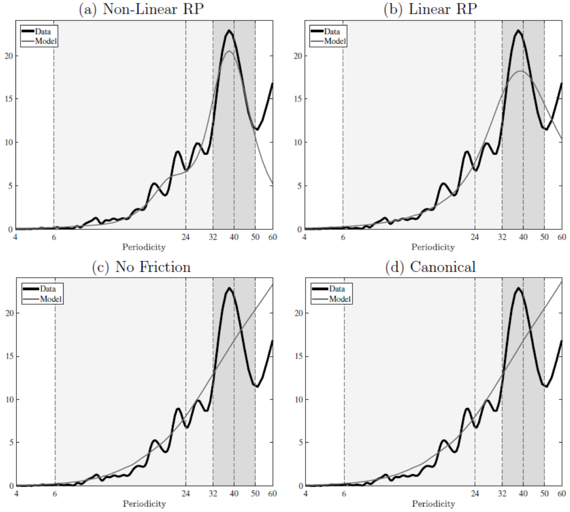
Table 8: Eigenvalues at the Steady State

	Non-Linear RP	Linear RP	No Friction	Canonical
$\lambda_{11}, \lambda_{12}$	$1.1032 \pm 0.2164i$	$0.9258 \pm 0.1372i$	0.5617, 0.9488	0.5202
$\lvert\lambda_{11}\rvert$	1.1242	0.9359	0.5617	0.5202
$\lvert\lambda_{12}\rvert$	1.1242	0.9359	0.9488	—

Notes: Table reports the eigenvalues of the first-order approximation to the solved model around the non-stochastic steady state. Note that the solved canonical model has only one dimension (X is no longer a relevant state variable) and therefore has only one eigenvalue.

Estimation Results - Spectral Density

Figure 7: Fit of Hours Spectral Density



Notes: This figure compares the estimate of the spectral density of U.S. Non-Farm Business hours per capita with the ones obtained from our four estimated models.

Conclusions

Why do market economies experience business cycle?

- Persistent outside disturbance
- Internal forces that endogenously favor cyclical outcomes

This paper:

- Macro variables display predictable cyclical dynamics
- Strategic complementarity is key to match empirical properties of spectral density
- Models do not need large and persistent shocks to match observable features in the data