Indeterminacy, Aggregate Demand, and the Real Business Cycle*

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Abstract

We show that under indeterminacy aggregate demand shocks are able to explain not only aspects of actual fluctuations that standard RBC models predict fairly well, but also aspects of actual fluctuations that standard RBC models cannot explain, such as the hump-shaped, trend reverting impulse responses to transitory shocks found in US output (Cogley and Nason, AER, 1995); the large forecastable movements and comovements of output, consumption and hours (Rotemberg and Woodford, AER, 1996); and the fact that consumption appears to lead output and investment over the business cycle. Indeterminacy arises in our model due to capacity utilization and mild increasing returns to scale.

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1. Introduction

General equilibrium real business cycle models have been subject to a number of criticisms. A basic criticism is the heavy reliance of such models on technology shocks to explain business cycle facts (for example, Blanchard, 1989 and 1993; Cochrane, 1994; Evans, 1992; Gordon, 1993; Mankiw, 1989; Summers, 1986). Another is the lack of an endogenous amplification and propagation mechanism, which has resulted in the failure of standard RBC models to explain the large hump-shaped, trend-reverting output responses to transitory shocks (Cogley and Nason, 1995, Watson, 1993).[1] Third, real business cycle models have been criticized for failing to match the forecastable movements and comovements of basic macroeconomic variables observed in the data (see Rotemberg and Woodford, 1996). Finally, standard RBC models cannot explain why consumption appears to lead output and investment over the business cycle.

These problems could be related. From a Keynesian view point, demand shocks are thought to be important for generating business cycles because the slow adjustment in prices may cause resources (such as labor or capital) to be underutilized, making possible the expansion of output without significant increases in marginal costs in response to a higher aggregate demand. In contrast, resources in standard equilibrium business cycle models are fully utilized because prices adjust quickly to clear markets. Therefore, transitory demand shocks tend to generate a strong crowding-out effect, resulting in negative comovements among the components of aggregate demand and in having only a minimal impact on aggregate output and employment. Consequently, standard RBC models have relied on supply shocks to explain the business cycle.

Despite significant empirical evidence favoring demand shocks as the main source of the business cycle (e.g., see Blanchard, 1989 and 1993; Blanchard and Quah, 1989; Cochrane, 1994; and Wen, 2002), "it is not as easy as it seems to specify a consistent dynamic model in which consumption [demand] shocks generate business-cycle fluctuations" (Cochane, 1994). The assumption of sticky prices coupled with demand shocks may not be enough to account for the propagation mechanism through which shocks to real demand generate persistent and trend reverting output dynamics.[2]

[1] Much effort has been made recently to find ways to enrich the internal propagation mechanisms of RBC models driven by technology shocks. Prominent examples include Burnside and Eichenbaum (1996), Andolfatto (1996), Carlstrom and Fuerst (1997), Chang, Gomes and Schorfheide (2002), and Bernanke, Gertler, and Gilchrist (1999), among many others.

[2] Monetary shocks, on the other hand, do not appear to be quantitatively important for explaining the business cycle (e.g., see Cochrane, 1994). Models with nominal rigidities can have rich propagation mechanisms to transmit monetary shocks. But real shocks in these models apparently do not lead to hump-shaped output dynamics. For example, Dufourt (2000) shows that in a sticky price model monetary shocks can potentially resolve the Cogley-Nason (1995) and the Rotemberg-Woodford (1996) puzzles. But it is clear from his analysis that real shocks alone cannot do the job. Similar results can also be found in Christiano, Eichenbaum, and Evans (2001).

We show in this paper that a very simple general equilibrium model of indeterminacy (Wen, 1998) has the potential to capture the more "Keynesian" features of the demand-driven business cycle without abandoning the hypotheses of market clearing and flexible prices. In particular, we show that in such a model demand shocks *alone* can generate predictions that are broadly consistent with a rich array of seemingly unrelated empirical business cycle "anomalies" which the traditional RBC models fail to explain. These anomalies include the hump-shaped output dynamics, the large forecastable movements and comovements found in consumption, hours, investment and output, and the fact that consumption appears to lead output and investment over the business cycle.

There are two essential elements in the model that give demand shocks a primary role for explaining business cycles in general equilibrium. The first is variable capacity utilization. The second is the presence of a small and empirically plausible externality in production. Variable capacity utilization has the effect of magnifying the marginal product of labor in the short-run by enhancing the output elasticity of labor. Coupled with a mild production externality that is consistent with recent empirical estimates, it makes the model behave as if there were increasing returns to the labor input (or as if there were unutilized resources). This factor is crucial not only because it gives rise to a multiplier that mitigates the crowding-out effect in response to demand shocks, but also because it results in an endogenous propagation mechanism essential for explaining the characteristics of forecastable movements as well as hump-shaped, trend reverting time series observed in the data.

We examine three different types of aggregate demand shocks: shocks to consumption demand, shocks to government spending, and sunspot shocks to investors' animal spirits. We find that: a) Demand shocks to either consumption, government spending, or investors' animal spirits can each generate fluctuations in output, hours, and investment that are broadly consistent with the U.S. data and are comfortably comparable to predictions of standard RBC models under technology shocks. b) Serially correlated demand shocks to either consumption or government spending can generate hump-shaped impulse responses for output, investment and hours. c) Demand shocks to either consumption, government spending, or investor's animal spirits are able to induce large forecastable movements in consumption, investment, hours, and output that are broadly consistent with the findings of Rotemberg and Woodford (1996). d) When the main source of shocks is consumption demand, the model is able to generate consumption series that leads both output and investment over the business cycle.

Our paper is closely related to Schmitt-Grohe (2000). Schmitt-Grohe (2000) studies a two-sector RBC model with sector-specific external effects that produces indeterminacy and that tends to generate negative comovements between consumption and investment (see Benhabib and Farmer (1996)). She concludes that RBC models with sunspots cannot generate hump-shaped output dynamics and forecastable movements in consumption that are comparable to U.S. data. We obtain different results for two reasons. First, our model is a one-sector RBC model with variable capacity utilization. It allows for the possibility of multiple equilibria under small external effects, and with positive comovements in consumption and investment. Second, we show that serially correlated demand shocks are the key for generating hump-shaped output dynamics in models with indeterminacy. Schmitt-Grohe (2000) does not consider serially correlated demand shocks such as preference shocks and government spending shocks in her model, and therefore does not generate hump-shaped output dynamics. We suspect that a broad class of indeterminate RBC models including the two-sector model of Benhabib-Farmer (1996) can generate the correct hump-shaped output dynamics if serially correlated demand shocks are allowed.

The rest of the paper is organized as follows. Section 2 explains the model. Section 3 presents the predictions of the model with respect to conventional simple measures of the business cycle. Section 4 addresses the Cogley-Nason criticism of RBC models. Section 5 addresses the Rotemberg-Woodford criticism of RBC models. Section 6 addresses the puzzle that consumption appears to lead the business cycle, and section 7 concludes the paper.

2. The Model

This is the one-sector RBC model studied by Wen (1998), based on Benhabib and Farmer (1994). A representative agent in the model chooses sequences of consumption, hours, capacity utilization, and capital accumulation to solve

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(c_t - \Delta_t) - a \frac{n_t^{1+\gamma}}{1+\gamma} \right)$$

subject to

$$c_t + i_t + g_t = \Phi_t \left(e_t k_t\right)^{\alpha} n_t^{(1-\alpha)}, \tag{2.1}$$

$k_{t+1} = i_t + (1-\delta_t)k_t$;where Δ_t is a random shock to consumption that generates the urge to consume (see Baxter and King, 1991); g_t is a shock to government spending, representing a pure resource drain on the economy; $e \in [0,1]$ denotes capital utilization rate, and Φ is a measure of production externalities and is defined as a function of average aggregate output which individuals take as parametric:

$$\Phi = \left[(ek)^{\alpha} n^{1-\alpha}\right]^{\eta}, \quad \eta \geq 0. \tag{2.2}$$

When the externality parameter η is zero, the model reduces to a standard RBC model studied by Greenwood *et al.* (1988) and Burnside and Eichenbaum (1996). To have an interior solution for e in the steady state, we follow Greenwood *et al.* by assuming that the capital stock depreciates faster if it is used more intensively:

$$\delta_t = \lambda e_t^{\theta}, \quad \theta > 1; \tag{2.3}$$

which imposes a convex cost structure on capital utilization.[3]

[3]The externality model can also be cast as a monopolistic competition model with mild increasing returns to scale at the individual firm level. The final output sector is defined as

$$Y = \left[\int y(i)^{\lambda} di\right]^{\frac{1}{\lambda}},$$

To solve the model, we log-linearize the first order conditions around the steady state as in King *et al.* (1988). To study investors' "animal spirits" as a possible source of aggregate demand uncertainty, we arrange the system of linearized equations in a way such that investment rather than the Lagrangian multiplier appears in the state vector. Denoting S_t as the vector $(k_t, i_t)'$, disregarding the fundamental shock variables for a moment, the model can be reduced to the following system of linear difference equations (hat-variables denote percentage deviations from their steady state values):

$$\begin{aligned} \hat{S}_{t+1} &= W\hat{S}_t + R\Theta_{t+1}, \\ \hat{Z}_t &= H\hat{S}_t; \end{aligned}$$

where Θ_{t+1} is a 2×1 vector of one-step ahead forecasting errors given by

$$\Theta_{t+1} = \hat{S}_{t+1} - E_t\hat{S}_{t+1} = \begin{pmatrix} \hat{k}_{t+1} - E_t\hat{k}_{t+1} \\ \hat{\imath}_{t+1} - E_t\hat{\imath}_{t+1} \end{pmatrix},$$

satisfying

$$E_t\Theta_{t+1} = 0;$$

and $\hat{Z}_t$ is a vector of any other endogenous variables in the model. Notice that the first element in Θ_t is 0 since $\hat{k}_{t+1}$ is known at the beginning of period t. Define the second element in Θ_t as $v_{st} = \hat{\imath}_t - E_{t-1}\hat{\imath}_t$.

When the model has a unique equilibrium (i.e., one of the eigenvalues of W lies outside the unit circle), the optimal decision rule for investment does not depend on the forecasting error, v_{st}, since in that case $\hat{\imath}_t$ can be solved forward under the expectation operator E_t to eliminate any forecasting errors associated with future investment. Consequently, the optimal decision rules at t depend only on the current capital stock ($\hat{k}_t$). If both eigenvalues of W lie inside the unit circle, however, the model is indeterminate in the sense that any value of $\hat{\imath}_t$ is consistent with equilibrium given $\hat{k}_t$. Hence, the forecasting error v_{st} can play a

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and the intermediate commodity $y(i)$ is given by

$$y(i) = [e(i)k(i)]^a \, n(i)^b,$$

Then the aggregate production function in the model has the reduced form:

$$y = (ek)^a \, n^b.$$

The markup is defined as the logarithm of price-marginal cost ratio:

$$\mu = \ln \frac{1}{1 + (\lambda - 1)},$$

where $(\lambda - 1)$ is the inverse of the price elasticity of demand facing an intermediate goods producing firm. For the monopolistic competition model to be exactly equivalent to the externality model, we simply set $\lambda a = \alpha$, $\lambda b = 1 - \alpha$, and $a + b = 1 + \eta$. This implies that $\lambda = \frac{1}{a+b} = \frac{1}{1+\eta}$. The markup is therefore given by$\mu \simeq \eta$ Since the model requires only a very mild externality η in the order around 0.11 to generate indeterminacy, the markup (μ) required in the corresponding monopolistic version of the model is also very mild.

role in determining the equilibrium level of investment.[4] In particular, under indeterminacy the decision rule for investment at time t takes the special form:

$$\hat{\imath}_t = \omega_{21}\hat{k}_{t-1} + \omega_{22}\hat{\imath}_{t-1} + r_2 v_{st}, \tag{2.4}$$

where ω_{21}, ω_{22}, and r_2 are the second row elements in W and R respectively.

The condition, $E_t v_{st+1} = 0$, implies that rational agents do not make systematic errors in forecasting the future based on current information. Since v_{st} can reflect purely extraneous shocks, it can be interpreted as shocks to autonomous investment.[5] There are therefore three possible types of aggregate demand disturbances in the model: innovations to government spending ε_g, innovations to consumption demand ε_Δ, and innovations to autonomous investment v_s.

Following the existing literature, we calibrate our model by setting the time interval to be a quarter, the discount factor $\beta = 0.99$, the capital's share $\alpha = 0.3$, the inverse elasticity of labor supply $\gamma = 0$ (Hansen's (1988) indivisible labor), and we choose θ such that the rate of capital depreciation in the steady state is 10 percent a year (implying $\delta = 0.025$ in the steady state and $\theta \approx 1.4$). The steady state value of Δ is chosen so that the ratio, $\frac{\Delta}{c}$, is 0.1 in the steady state. Also, the steady state government spending to output ratio is set at $\frac{g}{y} = 0.2$ (consistent with post-war U.S. data).[6] The minimum degree of the externality η required for indeterminacy is 0.104. We calibrate η with a value of 0.11 so that the implied frequency of cycles in the model roughly matches that of the U.S. economy. This value of η implies a markup around 0.1 or a degree of aggregate returns to scale around 1.1, which, based on recent empirical studies (e.g., Basu and Fernald, 1997; and Burnside *et al.,* 1995) is in the empirically plausible range.[7] Notice that the aggregate labor demand curve is downward sloping when indeterminacy arises in the model, which is in sharp contrast to models with fixed capacity utilization.[8]

With variable capacity utilization, the *effective returns* to labor can exceed one even though the labor-output elasticity, $(1-\alpha)(1+\eta)$, is substantially less than one. To illustrate this, we derive a reduced-form aggregate production function evaluated at the optimal

[4]For more discussions on this issue, see Farmer (1999) and Farmer and Guo (1994).

[5]But v_s can also reflect innovations in the fundamentals. When this is the case, we say that sunspots are correlated with fundamental shocks.

[6]The endogenous propagation mechanism of the model does not depend on parameters involving the exogenous shock processes, and is not sensitive to the values of $\frac{\Delta}{c}$ and $\frac{g}{y}$.

[7]The basic business-cycle property or propagation mechanism of the model is well preserved for $\eta \in (0.106 - 0.4)$. Hence the qualitative results in this paper continue to hold for a wide range of values of η. But the qualitative results of the paper no longer hold when η is too close to the bifurcation point at $\eta = 0.104$, where the model's stable eigenvalues become negative rather than complex with positive real parts. Negative eigenvalues imply high-frequency cycles rather than smooth cycles. The degree of markup or externality required for indeterminacy can be reduced even further if the time discount factor β is larger or if the steady-state depreciation rate δ is higher. For example, when $\beta = 0.995$, the minimum value of η for indeterminacy reduces to 0.057 and complex eigenvalues arise for $\eta = 0.058$. For analytical conditions of indeterminacy linking η to other structural parameters, see Wen (1998).

[8]See Benhabib and Farmer (1994) and Farmer and Guo (1994). In our model, the slope of the aggregate labor demand curve (in log) is given by $(1-\alpha)(1+\eta) - 1$. A negative slope requires $\eta < \frac{\alpha}{1-\alpha} = 0.43$, when $\alpha = 0.3$.

rate of capacity utilization:[9]

$$y_t = q k_t^{\alpha(1+\eta)\tau_k} n_t^{(1-\alpha)(1+\eta)\tau_n}$$

where q is a constant and t_k and t_n are defined as

$$\tau_k = \frac{\theta - 1}{\theta - \alpha(1+\eta)}, \tau_n = \frac{\theta}{\theta - \alpha(1+\eta)}.$$

Stationarity requires that $\alpha(1+\eta) < 1$, hence we have $\tau_k < 1$ and $\tau_n > 1$, because $\theta > 1$. The reduced-form aggregate production function evaluated at the optimal capacity utilization rate effectively amplifies labor's elasticity of output, as if there were increasing returns to the labor, even though the true returns to labor, $(1-\alpha)(1+\eta)$, are less than one. For example, given $\alpha = 0.3, \beta = 0.99, \delta = 0.025$, $\eta = 0.11$, the true labor-output elasticity is $(1-\alpha)(1+\eta) \approx 0.78$, but the *effective* labor-output elasticity (taking into account optimal capacity utilization) is $(1-\alpha)(1+\eta)\tau_n \approx 1.02$.

We can also numerically compute multiplier effects in our model, to measure the impact of government or of autonomous investment shocks on output in the current and subsequent periods. For example, with the present calibrations, and assuming that the government shocks follow a stationary $AR(1)$ process with persistence coefficient of 0.9, at the maximum impact point the government-spending multiplier is 1.84. In contrast, in a standard RBC model, the government-spending multiplier is 0.14 (e.g., KPR, 1988). While the dynamics of indeterminacy around the steady state translate the initial impact of the shocks into persistent, serially correlated movements in capital and investment, the propagation mechanism depends on the multiplier effects of these state variables on output. If changes in the state variables have little effect on output, then the initial impact of a shock would be quickly damped, even if the state variables are highly serially correlated. This is exactly what happens in standard RBC models where the capital stock is highly serially correlated but output growth is not, because changes in the capital stock have very little multiplier effect on output.

3. Preliminary Evaluation

This section presents a preliminary evaluation of our model based on a small set of unconditional second moments commonly used in the literature for evaluating the empirical success of RBC models. The model's second moments depend on the variance of the sunspot variable, v_{st}, hence we use σ_s^2 as an equilibrium selection device in our simulations. In particular, in the cases when only fundamental shocks are considered, the variance of sunspots is set to zero.

We calibrate parameters pertaining to exogenous shocks following the existing literature. Specifically, we assume through out the paper that shocks to fundamentals follow stationary

[9]See Wen (1998) for details.

$AR(1)$ processes and that the sunspot shocks are $i.i.d.$:

$$\begin{aligned}
\log \Delta_t &= \rho_\Delta \log \Delta_{t-1} + \varepsilon_{\Delta t},\ \varepsilon_{\Delta t} \sim i.i.d(0, \sigma_\Delta^2); \\
\log g_t &= \rho_g \log g_t + \varepsilon_{gt},\ \varepsilon_{gt} \sim i.i.d(0, \sigma_g^2); \\
v_{st} &= \varepsilon_{st},\ \varepsilon_{st} \sim i.i.d(0, \sigma_s^2);
\end{aligned}$$

where innovations in fundamental shocks are orthogonal to each other and are orthogonal to sunspots, v_{st}. We choose $\rho_g = \rho_\Delta = 0.9$.[10] Since only the relative moments matter in our discussions, we do not calibrate the variances of the different shocks in the present section and we arbitrarily set $\sigma_g = \sigma_\Delta = \sigma_s = 1$.

The predicted second moments for growth rates and their empirical counterparts are reported in table 1. For comparison purpose, predictions from a standard RBC model of King, Plosser, and Rebelo (KPR, 1988) driven by permanent technology shocks are also reported in table 1. The same parameter values are used for the KPR model for any shared common parameters.

Table 1. Selective Moments for Growth Rates

	$\sigma_{\Delta x}/\sigma_{\Delta y}$			$cor(\Delta x_t, \Delta y_t)$				$cor(\Delta x_t, \Delta x_{t-1})$			
	Δc	Δi	Δn	Δc	Δi	Δn	Δp	Δy	Δc	Δi	Δn
U.S.	0.62	3.11	1.08	0.75	0.80	0.42	0.49	0.39	0.38	0.48	0.11
KPR_A	0.52	2.83	0.50	0.98	0.99	0.98	0.98	-0.005	0.09	-0.03	-0.04
ICM_Δ	0.50	4.90	0.99	-0.02	0.96	0.99	0.38	0.78	-0.05	0.56	0.78
ICM_g	0.03	4.90	0.99	0.38	0.96	0.99	0.38	0.78	0.94	0.56	0.78
ICM_s	0.02	4.65	0.99	0.62	1.00	0.99	0.61	0.10	0.59	0.10	0.10

The estimated U.S. sample moments can vary depending on the precise definition of the variables in question and the sample period used.[11] However, regardless of the definitions

[10] Given the stationarity assumption, the more persistent the shocks are, the better our model explains the U.S. data. The estimated ρ_g for the detrended U.S. real government expenditure (1960:1 - 1994:4) is 0.96, and the estimated ρ_Δ from the intertemporal Euler equation of consumption by Baxter and King (1991) is 0.97. Using these larger persistence parameter values for ρ_g and ρ_Δ in our model provides better matches between our model and the U.S. data with respect to all the business cycle facts considered in this paper. We choose to use the more conservative values of 0.9 simply to show that our model is robust to the values of the persistence parameters as long as they are large enough to capture the notion that demand shocks are highly persistent. For calibration exercises using larger values of ρ_g and ρ_Δ, see our working paper (Benhabib and Wen, 2000).

[11] The data used here are logged quarterly real fixed investment, real consumption, and aggregate weekly hours (household survey). Output is defied as the sum of investment, consumption and government spending. Productivity is defined as labor to output ratio. All data series are taken from CITIBASE (1960:1 - 1994:4).

of variables, the most robust features of the U.S. data (regarded as the defining features of the U.S. business cycle in the literature) are: (1) Consumption growth is less volatile than output growth, which in turn is less volatile than investment growth, and employment growth is about as volatile as output growth (top row first column in table 1).[12] (2) Changes in consumption, investment, employment, and productivity are all positively correlated with changes in output (top row second column). (3) The growth rates of output, consumption, investment, and hours are all positively serially correlated (top row third column).

The middle row of table 1 confirms that the standard RBC model is quite successful in matching the relative volatilities of consumption and investment growth with respect to output growth as well as the positive comovements between changes in consumption, investment, employment, productivity and changes in output (second row). But, the model fails dramatically on an important ground: the serial correlations in growth rate are essentially zero for output, consumption, investment, and hours (second row, third column). This failure has provided the ground for criticisms of RBC models by Cogley and Nason (1995) and Rotemberg and Woodford (1996). In addition, the KPR model generates employment growth that is too smooth relative to output growth ($\sigma_{\Delta n}/\sigma_{\Delta y}$ is 0.5 in the KPR model and it is 1.09 in the U.S. economy), and it generates a correlation between productivity and output growth that is too high ($cor(\Delta p, \Delta y)$ is 0.98 in the KPR model and it is 0.48 in the U.S. economy).

The bottom rows in table 1 presents predictions of the indeterminate capacity utilization model (ICM) driven by the three types of demand shocks respectively. It shows that all three versions of the model are comfortably comparable to the standard RBC model driven by technology shocks with regard to predictions on the relative volatility orders with respect to output growth (first column) and on the comovements of growth rates (second column). There are however a couple of exceptions. First, when the shocks are from consumption demand (ICM_Δ), the correlation between consumption growth and output growth is slightly negative. Second, under government spending shocks or sunspots shocks (ICM_g and ICM_s) the relative volatility of consumption growth is too small. However, the indeterminate model is quite successful in predicting the volatility of employment growth relative to output growth ($\sigma_{\Delta n}/\sigma_{\Delta y}$ is 0.99) and the correlations between productivity growth and output growth ($cor(\Delta p, \Delta y)$ is positive but substantially less than one). Most importantly, all versions of the model are capable of predicting the positive serial correlations in the growth rates of output, investment, and hours (third column). Overall, therefore, with regard to the conventional measures of business cycles, it is fair to say that the indeterminate RBC model driven solely by demand shocks does no worse than the standard RBC model driven by technology shocks.

4. Hump-Shaped Output Dynamics

Cogley and Nason (1995) point out that standard RBC models cannot explain two related stylized fact about U.S. output: its impulse responses to transitory demand shocks

[12] Also see Kydland and Prescott (1982) and Prescott (1986) on discussions regarding these statistics.

are hump-shaped and it exhibits substantial amount of serial correlation in growth rate. This section formally tests the capacity utilization model of indeterminacy in light of these criticisms.

4.1. Stylized Responses to Demand

Following Blanchard and Quah (1989) and Cogley and Nason (1995), we decompose U.S. aggregate output into two components, one pertaining to permanent shocks and the other pertaining to transitory shocks. The transitory component is interpreted by Blanchard and Quah as fluctuations due to aggregate demand shocks. We use the ratio of investment to output as the covariate in a bivariate VAR to carry out the Blanchard -Quah decomposition. Balanced growth in RBC models implies that the investment-to-output ratio is stationary. The demand shocks so identified have the natural interpretation of being disturbances that affect short-run aggregate savings, such as shocks to government spending, to consumers' preferences, or to firms' autonomous investment. To ensure that the transitory output dynamics identified reflect responses to demand disturbances, we also use the government expenditure-to-output ratio as the covariate in carrying out the Blanchard-Quah decomposition.[13]

Figure 1 shows impulse responses to demand from output (first row windows) and as well as the implied autocorrelation functions for output growth (second row windows). The first column windows are estimated using investment-to-output ratio as the covariate, and the second column windows are estimated using government spending-to-output ratio as the covariate. These impulse responses exhibit the familiar hump-shaped, trend-reverting dynamics very similar to those identified by Blanchard and Quah (1989).[14] The implied autocorrelation functions for growth rate (second row windows) also show a familiar pattern of featuring positive serial correlations for the first couple of lags and negative serial correlations afterwards.

[13]Output is defined here as the sum of U.S. real fixed investment, real total consumption, and real government expenditure. All data series are from CITIBASE (1960:1 - 1994:4). 2 lags are used in the VAR estimations and they capture the dynamics of the data quite well. More lags tend to produce coefficients with large standard errors. A linear time trend is included in the VARs to capture any possible time trends. In order to be consistent with our theoretical models, government expenditure is not included in output when government shocks are not under consideration. Adding government expenditure into the picture does not have significant effect on the identified impulse response functions of output, however.

[14]Hump-shaped impulse responses are also observed in other variables such as investment and hours. We focus on output dynamics in the present section, however. See our working paper (Benhabib and Wen, 2000) for analysis on dynamics of investment and hours.

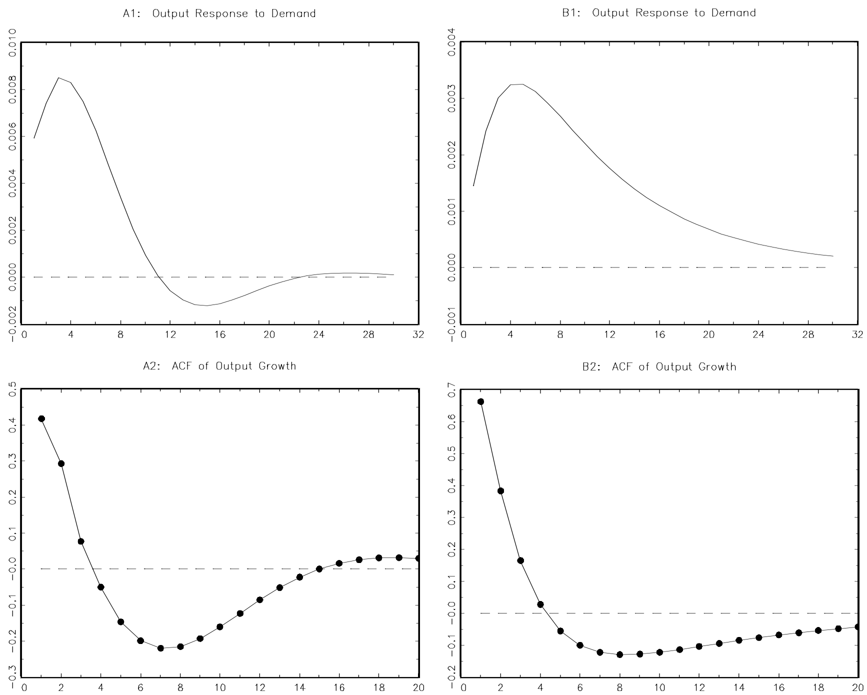

Fig. 1. Estimated responses of output to transitory shocks in the U.S. (windows A1 and B1). The botom 2 windows show the implied autocorrelations of output growth. Windows A1 and A2 are based on VARs using investment-to-output ratio as the covariate; Windows B1 nd B2 are based on VARs using government spending-to-output ratio as the covariate.

As pointed out by Cogley and Nason (1995), a fundamental weakness of the real-business-cycle paradigm as a convincing explanation of the business cycle is its failure to account for the salient output dynamics shown in figure 1. Under transitory but serially correlated shocks, standard RBC models generate monotonic impulse responses for output and near-zero serial correlations for output growth. This is illustrated in figure 2.[15]

[15]Figure 2 shows the impulse responses of output and the implied autocorrelation function of output growth in the King-Plosser-Rebelo (KPR 1988) model under $AR(1)$ technology shocks and $AR(1)$ consumption shocks respectively with autocorrelation coefficient of 0.9 in each case.

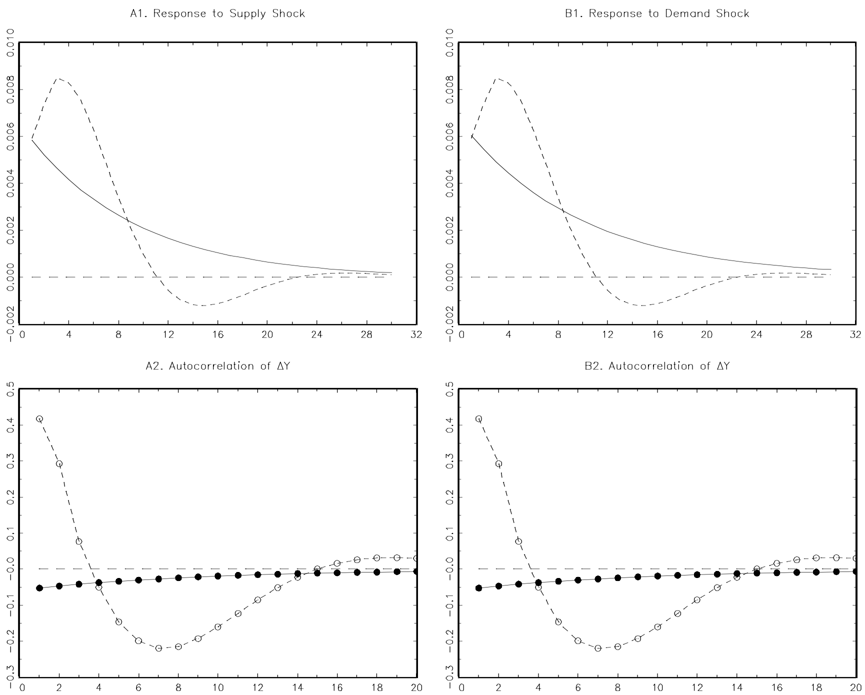

Fig. 2. Predicted impulse responses of output and autocorrelations of output growth in a standard RBC model (solid lines represent model, dashed lines represent U.S. data). Windows A1 and A2 are predictions under AR(1) technology shocks and windows B1 and B2 are predictions under AR(1) consumption shocks.

4.2. Predicted Responses to Demand

In order to highlight the dramatic effect of indeterminacy on the propagation mechanism of RBC models, we present impulse responses of the model to demand shocks with and without indeterminacy. In particular, we examine two versions of the model, one with $\eta = 0.1$ at which the steady state is locally a saddle, and one with $\eta = 0.11$ at which the steady state is a sink. Figure 3 shows the responses of output, consumption, investment and hours to a preference shock (solid lines) and a government spending shock (dashed lines) when $\eta = 0.1$. Figure 4 shows the impulse responses of the model when $\eta = 0.11$, where solid lines pertain to consumption shock, long dashed lines to government shock, and short dashed lines to sunspot shock.[16]

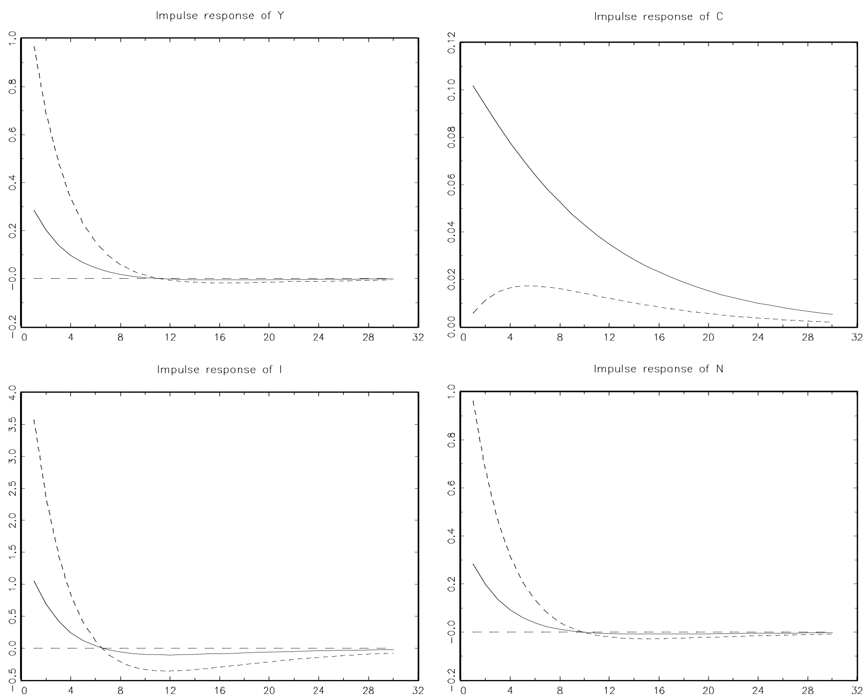

Fig. 3. Predicted impulse responses of output, consumption, investment and hours without indeterminacy ($\eta = 0.1$). Solid lines are responses to a consumption shock, dashed lines are responses to a government shock.

[16] When the model is indeterminate, analytical impulse response function to fundamental shocks is difficult to define without taking a stand on the initial values of the indeterminate variables (i.e., investment). We assume in figure 4 that the sunspot variable $v_{st} = 0$ when the model is subject to fundamental shocks.

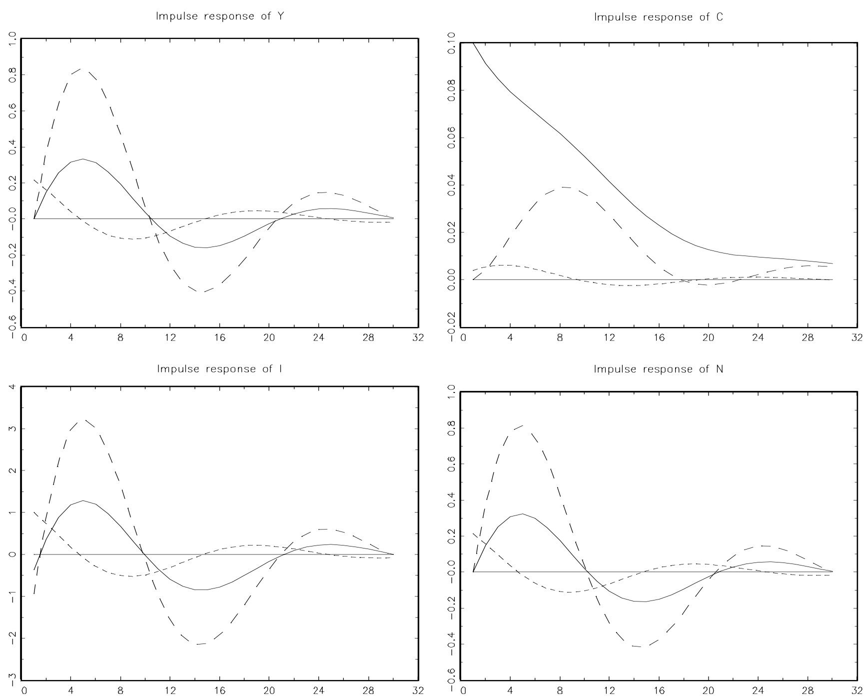

Fig. 4. Predicted impulse responses of output, consumption, investment and hours under indeterminacy ($\eta = 0.11$). Solid lines are responses to a consumption shock; long dashed lines are responses to a government shock; short dashed lines are responses to a sunspot shock.

By comparison, several features of figures 3 and 4 deserve particular mention. First, even without indeterminacy, both preference shocks and government spending shocks can each induce positive comovements among output, consumption, investment and hours (figure 3). This is consistent with the analysis of Baxter and King (1991) except that the required degree of the externality is substantially lower in our model than that in Baxter and King (1991), due to the variable capacity utilization in our model. Second, without indeterminacy, however, the impulse responses of output to demand shocks are monotonic, as opposed to hump-shaped, despite the presence of capacity utilization. Third, when the steady state becomes indeterminate (figure 4), the impulse responses exhibit dramatically different dynamics. In this case, even *i.i.d.* shocks to demand can induce highly persistent business-cycle like movements in output, hours and investment (short-dashed lines in figure 4), indicating that indeterminacy holds the key for the propagation mechanism of the model. Finally, with indeterminacy, the initial impulse responses of output become significantly hump-shaped when demand shocks are serially correlated (solid lines and long-dashed lines in figure 4).[17]

To test the impulse responses of the indeterminate model in a way that is comparable to the data, we also simulate the model with both a permanent shock and a transitory shock and then use the Blanchard-Quah method to identify a transitory component from the model-generated data. This is done simply to ensure consistency in methodology with our previous empirical data analysis. We assume that the permanent component in both the data and the model is caused by shifts in the total factor productivity in the production function and these shifts follow a random-walk process:

$$\log A_t = \log A_{t-1} + \varepsilon_{At}.$$

We also calibrate the persistent parameters for consumption shocks and government shocks respectively as $\rho_\Delta = 0.97$ (according to Baxter and King, 1991) and $\rho_g = 0.96$ (according to 1960:1-1994:4 quarterly U.S. real total government expenditure). We use the following procedure to obtain predicted impulse responses of output to demand shocks. We simulate the model with technology shocks and one particular type of demand shocks 300 times (the sample length of each simulation is the same as that of the U.S. data, namely, 140 quarters). Based on each simulation, we apply exactly the same method to the model-generated data as we did to the U.S. data to estimate and identify the impulse response function of output to demand shocks. Namely, we use bivariate VARs in output growth and an instrument variable (such as investment-to-output ratio or government spending-to-output ratio) to identify the effects of demand shocks on output under the Blanchard-Quah identification assumptions. For each type of demand shocks considered in our experiments, the standard deviations of innovations in permanent shocks and transitory shocks are always chosen so that the predicted output response functions match with the U.S. data as closely as possible, subject to the constraint that the predicted variance of output growth is in line with the data.[18] The results are shown in figure 5.

The solid line in window A_1 in Figure 5 represents the sample mean of the predicted impulse response function of output to consumption shocks (Δ_t) in our model, and the dashed lines are the one-standard-deviation band. Window A_2 represents the counterparts of the predicted autocorrelation function of output growth. These estimates are based on 300 simulations and are obtained by applying the Blanchard-Quah method to VARs consisting of output growth and investment-to-output ratio. Clearly, a comparison between figure 5 and figure 1 indicates that consumption demand shocks are capable of explaining the observed hump-shaped output dynamics identified in the data, because the predicted

[17]The fact that *i.i.d.* sunspots shocks cannot generate the initial hump in the impulse responses of output is pointed out by Schmitt-Grohe (2000).

[18]The variance of sunspots is set to zero in all simulations unless it is indicated otherwise. Also, we set the steady-state government spending-to-output ratio in the model to zero, $\frac{g}{y} = 0$, when consumption shocks are considered; and set the steady-state ratio, $\frac{\Delta}{c}$, to zero when government shocks are considered.

impulse response function and autocorrelation function under consumption shocks are not significantly different from those implied by the U.S. data (see the Q-test below). Similarly, when we try to identify the effects of government shocks on output using VARs consisting of output growth and government spending-to-output ratio, windows B_1 and B_2 show that government shocks in the model can also explain well the observed effects of government shocks on output identified in the U.S. data (comparing the second column windows in figure 5 to those in figure 1).

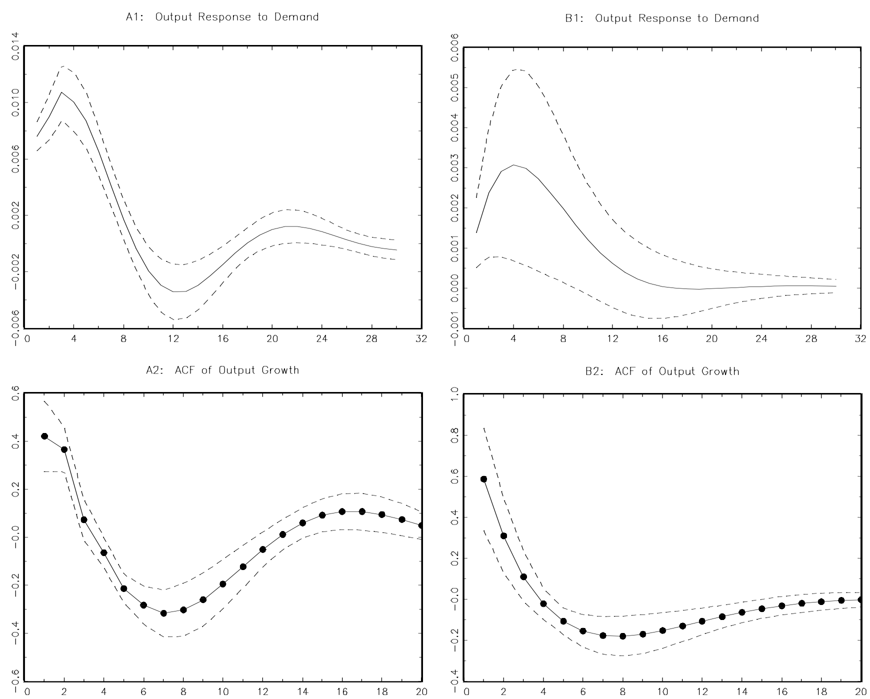

Fig. 5. Predicted impulse responses of output and autocorrelation functions of output growth. In each window the solid line is the mean and the dashed lines are the one-stadard-deviation band based on 300 simulations. Windows A1 and A2 are based on VARs using investment-to-output ratio as the covariate; Windows B1 nd B2 are based on VARs using government spending-to-output ratio as the covariate.

Thus, qualitatively speaking, demand shocks under indeterminacy can resolve the Cogley-Nason criticism on RBC models. Next, we test quantitatively the likelihood that the transitory component in U.S. output is generated by demand shocks in an indeterminate economy like ours. We test the model using a version of the Q-test proposed by Cogley and Nason (1995) by computing the generalized Q statistics, which are defined as:

$$Q = (\hat{c} - c)' \hat{V}_c^{-1} (\hat{c} - c).$$

The vector $\hat{c}$ represents the sample impulse response function of output or autocorrelation function of output growth implied by the U.S. data, and c represents the mean of N $(= 300)$ estimated impulse response functions or autocorrelation functions implied by the simulated time series of the model. That is, $c = \frac{1}{N} \sum_{i=1}^{N} c_i$. The covariance matrix, $\hat{V}_c$, is estimated by

$$\hat{V}_c = \frac{1}{N} \sum_{i=1}^{N} (c_i - c)(c_i - c)'.$$

The test statistic Q has approximate χ^2 distribution with degrees of freedom equal to the number of elements (lags) in c. Following Cogley and Nason (1995) as well as Schmitt-Grohe (2000), the number of lags chosen is 8, which gives a critical value of 20.1 at the 1% significance level and a critical value of 15.5 at the 5% significance level.

Table 2. Q Statistics

Permanent Shocks	Transitory Shocks	Q_{imp}	Q_{acf}
$A_t\|_{(\sigma_A=0.00004)}$	$\Delta_t\|_{(\sigma_\Delta=0.055)}$	8.34	13.5
$A_t\|_{(\sigma_A=0.000016)}$	$g_t\|_{(\sigma_g=0.017)}$	1.89	2.49
$g_t\|_{(\sigma_g=0.0001)}$	$\Delta_t\|_{(\sigma_\Delta=0.03)}, s_t\|_{(\sigma_s=0.02)}$	9.76	7.22

The Q statistics for impulse response function of output and autocorrelation function of output growth are reported in table 2. The first two columns indicate the type of shocks assumed in the model, as well as the standard deviations of these shocks. The last two columns report the Q statistics for each version of the model considered. The first row in the middle panel reports the Q statistics for the likelihood that the transitory component in U.S. output is caused by consumption shocks. Judged by both impulse response function (Q_{imp}) and autocorrelation function (Q_{acf}), the model passes the test at the 5% significance level. The second row in the middle panel reports the Q statistics for the likelihood that the transitory component in U.S. output identified using government expenditures is consistent with our model driven by government shocks. Judged by both impulse response function (Q_{imp}) and autocorrelation function (Q_{acf}), the model passes the test also at the 5% significance level. Thus, either type of demand shocks can explain the hump-shaped output dynamics in the U.S. identified using the Blanchard-Quah method. Furthermore, the required sizes of the standard deviations of the demand shocks in our model are also reasonable (for example, $\sigma_g = 0.017$ in the model and $\hat{\sigma}_g = 0.016$ in the data according to 1960:1-1994:4 quarterly real total government spending; $\sigma_\Delta = 0.055$

in the model and $\hat{\sigma}_\Delta = 0.06$ in the data according to the University of Michigan Index for Consumer Confidence, 1960:1-1994:4). Note that the required standard deviation of technology shocks in our model ($\sigma_A = 0.00002 \sim 0.00004$) is extremely small compared to what is commonly assumed in standard RBC models (e.g., $\sigma_A = 0.012 \sim 0.018$ according to Christiano and Eichenbaum, 1992). Taking into consideration of labor hoarding and capacity utilization, Burnside and Eichenbaum (1996) show that σ_A can be reduced further down to 0.007, which is still substantially larger than what is required in our indeterminate model. This is attributable to the externality assumed in our model. Such a small degree of externality in our model ($\eta = 0.11$) can have such a big reduction in the required variance of technology shocks is amazing, suggesting that the business-cycle component in the measured "Solow residual" obtained using a standard constant returns-to-scale production function may actually reflect demand shocks rather than technology shocks. This view is consistent with the analysis of Burnside, Eichenbaum and Rebelo (1993).

It needs to be emphasized that the goodness of fit of our model does not rely on technology shocks. Schmitt-Grohe (2000) argues that the success of indeterminate RBC models in matching the data relies often on assuming both technology shocks and sunspots shocks in the model. To prove that this is not the case for our model, we also investigate the possibility that the unit root property of the U.S. output may reflects permanent shocks to demand, such as to Government spending, rather than to technology.[19] For this reason, we simulate the model with demand shocks only, by allowing for a permanent shock to government spending, a transitory but serially correlated shock to consumption, and a transitory *i.i.d.* shock to investment expectations (sunspots). The Q-test statistics for impulse response function and autocorrelation function are reported in the last row of table 2. It shows that the model with demand shocks only has no problem explaining the hump-shaped output dynamics observed in the U.S. data.[20]

5. The Forecastable-Movement Puzzle

Another important characteristic of economic fluctuations that is difficult for the canonical real-business-cycle paradigm to explain is highlighted by Rotemberg and Woodford (1996).

[19] Unit root tests, for example, cannot reject the hypothesis that the U.S. government expenditure follows a random walk.

[20] We also investigated cases where sunspots are correlated (either positively or negatively) with innovations in fundamental shocks. The results indicate that correlations among different types of demand shocks do not affect the model's goodness of fit, as long as the relative standard deviations of different shocks can be adjusted accordingly. In addition, we have investigated the predictions of the model with respect to other variables such as investment and hours. The results show that the model can predict the shape of the impulse response functions of these variables, and it can pass the Q-test with respect to the autocorrelation function of growth rate of these variables simultaneously. However, the impulse response functions of these variables do not perform as well as that of output with respect to the Q-test. This is because the model tends to over predict the volatility of investment relative to output whereas we choose the variance of the shocks to only match output. In fact, the predicted impulse response function of investment and hours are the same as those in the data up to a constant. Hence the model can match the impulse response functions of hours and investment individually if the size of the variance of shocks are chosen appropriately. See our working paper (Benhabib and Wen, 2000) for more discussions on this.

They show that expected changes in U.S. output, hours, investment and consumption have striking patterns: 1) They are highly forecastable in the sense that the standard deviations of expected changes in these variables are about half as large as the standard deviations of actual changes in these variables; 2) they are strongly positively correlated with each other; 3) the relative volatilities of expected changes follow an order similar to the relative volatilities of actual changes, namely, in response to one percent increase in expected-output growth, the expected-consumption growth is substantially less than one percent and the expected-investment growth is substantially greater than one percent. Standard RBC models driven by permanent technology shocks are not able to predict these regularities. The theoretical counter parts in these models have very little forecastability, they are negatively correlated, and they follow an entirely different pattern of volatility orders from that of actual changes.

Table 3 presents estimated and predicted ratios of standard deviations between expected and actual k-quarter changes in output.[21] The first row shows that in post-war U.S. economy, changes in output are highly forecastable (numbers in parentheses are estimates reported by Rotemberg and Woodford (1996) using different data samples). Our data sample indicates that at least 36 percent of actual changes in output or more are forecastable, whereas Rotemberg and Woodford report that more than 55 percent of actual changes in output are forecastable. The second row, in contrast, shows that changes in output in the KPR model driven by permanent technology shocks have essentially zero forecastability. This is so because changes in output are essentially white noise processes in standard RBC models.

[21] The variables used in the VARs for computing the forecastable moments in the U.S. economy are output growth, investment-to-output ratio and total hours, $\{\Delta y_t, i_t - y_t, h_t\}$. Following Rotemberg and Woodford (1996), h is detrended by deterministic time trend in the VAR. The forecastable moments are computed in the same way as that in Rotemberg and Woodford (1996). We use investment-to-output ratio rather than consumption-to-output ratio in the VARs so as to be consistent with the VARs used in the previous section. The results are nevertheless very similar to those obtained by Rotemberg and Woodford (1996). Predicted moments for forecastable changes in theoretical models are computed based on linearized equilibrium decision rules derived from the models, as in Rotemberg and Woodford (1996).

Table 3. Relative Standard Deviations of Cumulative Changes in Output

	Horizon (in quarters)						
	1	2	4	8	12	24	∞
U.S. Economy							
$\Delta\hat{y}_t^k/\Delta y_t^k$	0.36	0.36	0.43	0.51	0.47	0.38	NA
	(0.57)	(0.60)	(0.68)	(0.78)	(0.72)	(0.56)	
KPR Model (A_t)							
$\Delta\hat{y}_t^k/\Delta y_t^k$	0.01	0.02	0.02	0.03	0.03	0.03	NA
ICM Model (Δ_t)							
$\Delta\hat{y}_t^k/\Delta y_t^k$	1.00	0.96	0.88	0.86	0.84	0.70	NA
ICM Model (g_t)							
$\Delta\hat{y}_t^k/\Delta y_t^k$	1.00	0.96	0.88	0.86	0.84	0.70	NA
ICM Model (s_t)							
$\Delta\hat{y}_t^k/\Delta y_t^k$	0.56	0.72	0.86	0.88	0.79	0.72	NA

Note: Δy_t^k denotes the change in the log of output from t to $t+k$; $\Delta\hat{y}_t^k$ denotes the expectation of this change based on information available at t.

Our indeterminate RBC model, however, has the potential to meet the Rotemberg-Woodford criticisms. The lower three rows in table 3, for example, show that the indeterminate capacity utilization model driven by each type of demand shock respectively predicts that changes in output are highly forecastable (and even more so than what is observed in the U.S. data). The best match is delivered by sunspot shocks (last row). The extremely high forecastability is attributable to the strongly periodic propagation mechanism of the model.[22]

Rotemberg and Woodford's (1996) main criticism of the real business cycle theory, however, is that it implies counterfactual comovements and counterfactual relative volatilities among forecastable changes in hours, consumption, investment, and output. This is shown in table 4 and table 5. Table 4 presents estimates and model predictions for the correlation between expected k-quarter changes in output and corresponding k-quarter changes in consumption, hours, and investment.[23] The top panel of table 4 shows that in the U.S. economy expected changes in hours and investment are highly positively correlated with expected changes in output for all forecasting horizons considered, while expected changes in consumption are positively correlated with that of output but substantially less so than

[22]Notice that consumption demand shocks and government spending shocks give exactly the same predictions for the variables under consideration.

[23]Following Rotemberg and Woodford (1996), we use the equation,

$$0.74\hat{c}_t + 0.26\hat{\imath}_t = \hat{y}_t,$$

to compute moments pertaining to consumption. Adding a government spending component into the equation does not change the results substantially.

hours and investment. The predictions of the KPR model are shown in the middle panel. It correctly predicts the highly positive correlations with respect to hours and investment series, but it fails dramatically on consumption. It generates perfectly negative correlations between expected changes in consumption and output.[24] The lower panels of table 4 presents predictions of the indeterminate model under each type of demand shocks respectively. It performs much better than the KPR model in all aspects. The improvement on the correlation between expected changes in consumption and output, for example, is particularly substantial regardless the source of demand shocks or the forecasting horizon considered. Namely, expected changes in consumption are predicted to be positively correlated with that of output for all horizons considered and the correlations are substantially less than one, as in the data (the only exception is the first quarter change under consumption shocks).[25]

[24]Rotemberg and Woodford (1996) found that the sign of predicted correlations in the KPR model depends sensitively on the parameters. For example, the correlation between expected-consumption growth and expected-output growth can be positive if the intertemporal elasticity of substitution parameter in the preference or the capital-output elasticity in the technology change. But the consequence is that the correlations of other variables (investment or hours) change sign from positive to negative. It is therefore not possible to generate positive correlations with output for all variables simultaneously with any sensible parameter choices in the KPR model.

[25]As before, consumption demand shocks (Δ_t) and government spending shocks (g_t) generate exactly the same predictions for investment and hours. In the short horizon, government spending shocks and sunspot shocks preform better than consumption demand shocks with respect to consumption series. It is clear from table 5 that all three versions of the model outperform the KPR model significantly with respect to all variables and all forecasting horizons considered. In contrast, using a two-sector indeterminate model with fixed rather than variable capacity utilization, Schimitt-Grohe shows that sunspots shocks predict strongly negative correlations between expected-consumption growth and expected-output growth. Hence, she concludes that indeterminate RBC models driven solely by sunspots do not overcome the shortcomings of standard RBC models in this regard.

Table 4. Correlations among Forecasted Changes

Horizon (in quarters)		1	2	4	8	12	24	∞
U.S. Economy								
	$\text{Cor}(\Delta\hat{c}_t^k, \Delta\hat{y}_t^k)$	0.336	0.388	0.335	0.324	0.392	0.488	0.498
	$\text{Cor}(\Delta\hat{n}_t^k, \Delta\hat{y}_t^k)$	0.884	0.933	0.952	0.968	0.972	0.967	0.965
	$\text{Cor}(\Delta\hat{\imath}_t^k, \Delta\hat{y}_t^k)$	0.895	0.924	0.937	0.932	0.912	0.842	0.825
KPR Model (A_t)								
	$\text{Cor}(\Delta\hat{c}_t^k, \Delta\hat{y}_t^k)$	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
	$\text{Cor}(\Delta\hat{n}_t^k, \Delta\hat{y}_t^k)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	$\text{Cor}(\Delta\hat{\imath}_t^k, \Delta\hat{y}_t^k)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ICM Model (Δ_t, g_t, s_t)								
(Δ_t)	$\text{Cor}(\Delta\hat{c}_t^k, \Delta\hat{y}_t^k)$	-0.094	0.029	0.265	0.557	0.627	0.526	0.561
	$\text{Cor}(\Delta\hat{n}_t^k, \Delta\hat{y}_t^k)$	0.9996	0.9995	0.9994	0.9994	0.9994	0.9990	0.9989
	$\text{Cor}(\Delta\hat{\imath}_t^k, \Delta\hat{y}_t^k)$	0.998	0.998	0.997	0.997	0.994	0.976	0.974
(g_t)	$\text{Cor}(\Delta\hat{c}_t^k, \Delta\hat{y}_t^k)$	0.381	0.372	0.306	0.199	0.296	0.434	0.408
	$\text{Cor}(\Delta\hat{n}_t^k, \Delta\hat{y}_t^k)$	0.9996	0.9995	0.9994	0.9994	0.9994	0.9990	0.9989
	$\text{Cor}(\Delta\hat{\imath}_t^k, \Delta\hat{y}_t^k)$	0.998	0.998	0.997	0.997	0.994	0.978	0.974
(s_t)	$\text{Cor}(\Delta\hat{c}_t^k, \Delta\hat{y}_t^k)$	0.033	0.117	0.267	0.385	0.328	0.383	0.370
	$\text{Cor}(\Delta\hat{n}_t^k, \Delta\hat{y}_t^k)$	0.9997	0.9997	0.9997	0.9995	0.9993	0.9995	0.9994
	$\text{Cor}(\Delta\hat{\imath}_t^k, \Delta\hat{y}_t^k)$	0.9999	0.9999	0.9999	0.9999	0.9998	0.9999	0.9998

Note: $\Delta\hat{x}_t^k$ denotes the expected change in x_t from t to $t+k$ based on information available at t.

Table 5 presents regression coefficients of the expected changes in consumption, hours, and investment on expected changes in output. As pointed out by Rotemberg and Woodford (1996), the regression coefficients are good measures of the relative volatilities of the various series, and they provide an economic way of discussing the movement of these variables over the business cycle. For example, they indicate the percentage by which a given variable can be expected to change when output is expected to increase by one percent. Table 5 shows that the elasticity (regression coefficient) of expected-consumption growth with respect to expected-output growth is positive but substantially less than one, while the corresponding elasticity of expected-investment growth is substantially greater than one for all forecasting horizons. This means that expected changes in consumption are very smooth while expected changes in investment are very volatile in response to expected changes in output. Expected-hours growth, on the other hand, responds nearly one for one to expected changes in output in the U.S. economy.

These salient features of the data are not captured by the KPR model. The middle panel in table 5 shows that the elasticity of expected-consumption growth with respect to expected-output growth is negative and substantially larger than one in absolute value. This leads to excessively volatile expected-investment growth (in the order of nearly 30!) in response to one percent expected changes in output. Such excessive volatility in expected growth is also observed in hours (the elasticity is nearly 8!).

The lower panels in table 5, in contrast, show a remarkable improvement on the KPR model in explaining the elasticities of forecastable changes in consumption, hours, and investment with respect to forecastable changes in output. The magnitudes of regression coefficients for each series considered under each type of demand shock are broadly consistent with the data for the indeterminate model. For example, for each type of demand shock and for most forecasting horizons, the elasticity of expected-consumption growth with respect to expected-output growth is positive and substantially less than one, while the expected growth of hours responds nearly one for one to expected-output growth. Quantitatively speaking, however, the indeterminate model tends to underestimate the elasticity of expected-consumption growth and overestimate the expected-investment growth.[26]

Table 5. Regression Coefficients among Forecasted Changes

Horizon (in quarters)		1	2	4	8	12	24	∞
U.S. Economy								
	$\Delta\hat{c}_t^k$ on $\Delta\hat{y}_t^k$	0.204	0.200	0.159	0.159	0.217	0.356	0.382
	$\Delta\hat{n}_t^k$ on $\Delta\hat{y}_t^k$	0.896	0.895	0.874	0.880	0.902	0.933	0.937
	$\Delta\hat{\imath}_t^k$ on $\Delta\hat{y}_t^k$	3.266	3.277	3.395	3.393	3.230	2.832	2.760
KPR Model (A_t)								
	$\Delta\hat{c}_t^k$ on $\Delta\hat{y}_t^k$	-6.770	-6.770	-6.770	-6.770	-6.770	-6.770	-6.770
	$\Delta\hat{n}_t^k$ on $\Delta\hat{y}_t^k$	7.770	7.770	7.770	7.770	7.770	7.770	7.770
	$\Delta\hat{\imath}_t^k$ on $\Delta\hat{y}_t^k$	29.595	29.595	29.595	29.595	29.595	29.595	29.595
ICM Model								
(Δ_t)	$\Delta\hat{c}_t^k$ on $\Delta\hat{y}_t^k$	-0.010	0.003	0.033	0.088	0.136	0.207	0.234
	$\Delta\hat{n}_t^k$ on $\Delta\hat{y}_t^k$	0.989	0.988	0.989	0.993	0.989	0.979	0.977
	$\Delta\hat{\imath}_t^k$ on $\Delta\hat{y}_t^k$	4.707	4.671	4.598	4.439	4.308	4.112	4.039
(g_t)	$\Delta\hat{c}_t^k$ on $\Delta\hat{y}_t^k$	0.010	0.011	0.010	0.007	0.010	0.019	0.020
	$\Delta\hat{n}_t^k$ on $\Delta\hat{y}_t^k$	0.989	0.988	0.989	0.993	0.989	0.979	0.977
	$\Delta\hat{\imath}_t^k$ on $\Delta\hat{y}_t^k$	4.707	4.671	4.598	4.439	4.308	4.112	4.039
(s_t)	$\Delta\hat{c}_t^k$ on $\Delta\hat{y}_t^k$	0.001	0.003	0.006	0.011	0.012	0.012	0.012
	$\Delta\hat{n}_t^k$ on $\Delta\hat{y}_t^k$	0.999	0.997	0.993	0.987	0.987	0.987	0.987
	$\Delta\hat{\imath}_t^k$ on $\Delta\hat{y}_t^k$	4.678	4.673	4.663	4.649	4.648	4.648	4.647

Note: $\Delta\hat{x}_t^k$ denotes the expected change in x_t from t to $t+k$ based on information available at t.

[26]Note that the effects of consumption demand shocks and government spending shocks on the dynamics of hours and investment are still exactly the same. But consumption shocks appear to give much better predictions than the other types of demand shocks on consumption elasticities for longer forecasting horizons.

Caveat: Much applied work in the RBC literature assumes that technology shocks have a transitory component (e.g., see Kydland and Prescott (1982), and King, Plosser, and Rebelo (1988)). This is so because it appears that in many aspects transitory technology shocks perform better than permanent technology shocks in explaining the U.S. data. Therefore, the Rotemberg-Woodford criticism of RBC models may apply only to the case of permanent technology shocks. Indeed, incorporating transitory technology shocks into the KPR model can substantially improve the model's performance regarding forecasted dynamics of the model. Table 6 shows that when technology follows a stationary AR(1) process, the model's performance in explaining the expected changes of various variables is substantially improved along all dimensions considered (the performance is now only slightly worse than that of the indeterminate model). For example, it does a pretty good job in accounting for the standard deviation of expected k-quarter changes relative to that of actual k-quarter changes. The correlation between expected-consumption growth and expected-output growth becomes much less negative and even turns positive for horizons beyond 4 quarters. The elasticities of expected changes in consumption, hours, and investment with respect to expected changes in output are also improved dramatically, especially for longer forecasting horizons (the bottom panel). However, transitory technology shocks do not help address the Cogley-Nason criticism (see Cogley and Nason, 1995). In addition, when the source of transitory shocks are from aggregate demand, the KPR model with permanent technology shocks performs just as poorly. This implies that our indeterminate RBC model still represents a significant progress over standard RBC models for explaining the business cycle. This is further illustrated by discussions in the next section, where we show that standard RBC models driven by technology shocks are not able to explain why consumption appears to lead output over the business cycle. Adding transitory technology shocks into these models only exacerbates the problem.

Table 6. Expected Changes under Transitory Technology Shocks (KPR)

	Horizon (in quarters)						
	1	2	4	8	12	24	∞
Ratio of Standard Deviations							
$\Delta\hat{y}_t^k/\Delta y_t^k$	0.23	0.32	0.43	0.55	0.61	0.69	NA
Correlations with $\Delta\hat{y}_t^k$							
$\Delta\hat{c}_t^k$	-0.391	-0.315	-0.158	0.132	0.348	0.693	0.778
$\Delta\hat{n}_t^k$	0.980	0.978	0.975	0.968	0.959	0.927	0.855
$\Delta\hat{i}_t^k$	0.987	0.986	0.984	0.980	0.975	0.959	0.925
Regression Coeff. on $\Delta\hat{y}_t^k$							
$\Delta\hat{c}_t^k$	-0.095	-0.076	-0.038	0.034	0.099	0.251	0.429
$\Delta\hat{n}_t^k$	1.095	1.076	1.038	0.966	0.901	0.749	0.571
$\Delta\hat{i}_t^k$	5.029	4.958	4.819	4.556	4.316	3.755	3.103

Note: Δy_t^k denotes the change in the log of output from t to $t+k$; $\Delta\hat{y}_t^k$ denotes the the expectation of this change based on information available at t.

6. Why does Consumption Lead the Business Cycle?

Standard RBC models driven by technology shocks predict that consumption lags both output and investment. Post-war U.S. data, however, reveal the opposite: at the business cycle frequency consumption leads output and investment. In what follows, we present the puzzle first, then we show that the puzzle can be resolved by our indeterminate RBC model.

6.1. The Puzzle

Applying the band-pass filter (Baxter and King, 1995) to post-war U.S. data (1960:1–1994:4),we found that consumption leads output by one quarter and leads investment by two quarters at business cycle frequencies. The cross correlations among these series at these frequencies are reported in table 7 (top panel). It shows that the strongest correlation between consumption and output occurs at lag $k=-1$, whereas the strongest correlation between consumption and investment occurs at lag $k=-2$, indicating that consumption leads output and investment.[27] Standard RBC models cannot explain these stylized facts. The middle panel in table 7 shows that the strongest correlation between consumption

[27] The data used here are U.S. quarterly real GDP, real total consumption and real business fixed investment (total fixed investment minus residential investment) from 1960:1 to 1994:4. The same lead-lag relationship holds when output is defined as the sum of consumption, business fixed investment, and government expenditure. Housing investment is excluded for reasons that will become clear later. The window size used in the band-pass filter is for frequency interval of 8 to 40 quarters per cycle and we used 12 truncation points at each end of a time series. Changing the window size to "6 to 32" quarters produces little difference in results.

and output in the KPR model occurs at $k = +1$and the strongest correlation between consumption and investment occurs at $k = +2$, indicating that consumption lags output and investment in the model.[28]

The reasons for the sharp discrepancy between data and standard RBC models are simple. To highlight the problem at stake, we assume that technology shocks are transitory. The motive for consumption smoothing in a utility based optimization model implies that consumption comove with the capital stock (permanent income). At the same time, output and investment comove with transitory income. The capital stock, however, strongly lags investment because it is a weighted sum of past investment:[29]

$$\begin{aligned} k_t &= (1-\delta)k_{t-1} + i_{t-1} \\ &= i_{t-1} + (1-\delta)i_{t-2} + (1-\delta)^2 i_{t-3} + ... \end{aligned}$$

Consequently, consumption (along with the capital stock) lags both output and investment in standard models.

Table 7. Correlations at Business Cycle Frequencies (8-40 quarters)

	$k=4$	$k=3$	$k=2$	$k=1$	$k=0$	k =-1	k =-2	k =-3	k =-4
U.S. Sample									
Cor(c_{t+k}, y_t)	-.020	.248	.519	.749	.899	.929	.853	.688	.468
Cor(c_{t+k}, i_t)	-.333	-.128	.119	.374	.600	.746	.805	.770	.652
KPR Model									
Cor(c_{t+k}, y_t)	.562	.751	.871	.884	.771	.534	.233	-.073	-.330
	(.114)	(.070)	(.034)	(.025)	(.021)	(.020)	(.049)	(.081)	(.101)
Cor(c_{t+k}, i_t)	.584	.747	.835	.812	.665	.402	.088	-.218	-.461
	(.109)	(.066)	(.041)	(.040)	(.036)	(.023)	(.039)	(.067)	(.082)

Note: Numbers in parentheses are standard errors based on 500 simulations.

If the technology-shock view is correct, it is then puzzling to observe consumption leading the business cycle in data. It is tempting to think that sluggish investment adjustment may

[28]The predictions of theoretical models shown in tables 7 and 8 are based on 500 simulations, each with length of 140 quarters (the US sample size). We pass each series generated from each round of simulation through the band-pass filter to isolate the business cycle components, and then compute the cross correlations. The model predictions shown in tables 7 and 8 are the means and standard errors of cross correlations based on the 500 simulations. The predictions of the KPR model are generated under stationary AR(1) technology shocks with first-order autocorrelation coefficient of 0.9. When technology shocks are permanent in the KPR model, consumption appears to lag investment by one quarter and coincide with output.

[29]The linear filter,

$$f(L) = 1 + (1-\delta)L + (1-\delta)^2 L^2 + ... = \frac{1}{(1-(1-\delta)L)},$$

is a backward phase shifter. E.g., see Harvey (1993, section 6.6) on the phase effect of linear filters.

hold the key for explaining the puzzle. The idea is that if investment responds to technology shocks with a lag, it would then appear to lag output. This, however, does not necessarily result in consumption leading output. When investment is slow to respond to technology shocks, consumption would be forced to absorb the impact of technology shocks. Although this helps break the link between consumption and the capital stock at the impact period (namely, by preventing consumption from complete smoothing), it does not resolve the puzzle because consumption would then appear to coincide with output, rather than lead output.[30]

An alternative explanation is that business cycles are caused mainly by consumption demand shocks rather than by technology shocks. Consumption demand shocks, however, may also not generate a leading consumption series. If responses of output to consumption shocks do not display a delayed multiplier effect, output would appear to coincide with consumption rather than lagging consumption. Therefore, both the consumption shocks and a multiplier-accelerator like endogenous propagation mechanism seem essential to explain the lead-lag pattern of the business cycle. Recall that output in the U.S. economy has a hump-shaped impulse response pattern with respect to demand shocks. When output responses to shocks are hump-shaped, output may appear to lag consumption if the main source of shocks comes from consumption demand. A potential problem is that consumption shocks may generate countercyclical movements in investment due to crowding out. Our indeterminate RBC model, however, solves not only the hump-shaped response problem, but also the crowding-out problem.

6.2. Calibrated Analysis

Using the same calibrated parameters as in the previous sections for the indeterminate capacity utilization model, table 8 presents the predicted correlations between consumption and output under consumption demand shocks for various leads and lags, as well as the correlations between consumption and investment for various leads and lags (standard errors in parentheses). The version of the model driven by consumption demand shocks alone (Δ_t) is presented in the top panel. It indicates that consumption leads output by one quarter (at k = -1) and leads investment by two quarters (at k = -2). The bottom panel presents the version of the model when there are both consumption demand shocks and sunspot shocks (the relative standard deviations of the two types of shocks are chosen so that the model passes the Q-tests as shown in table 2). It shows that adding more sources of shocks does not alter the qualitative predictions of the model.[31]

[30] By the same token, permanent technology shocks does not make consumption lead output. Our analysis shows that consumption appears to coincides with output and lag investment in the KPR model when technology shocks are permanent.

[31] The same results also hold if we add technology or government spending shocks into the model. What is crucial for generating the correct lead-lag relationship between consumption and output is that consumption demand shocks dominate other types of shocks. For example, the versions of the model considered in table 2 (first row and third row) are consistent with the lead-lag relationship.

Table 8. Predicted Correlations at Business Cycle Frequency (8-40 quarters)

	$k=4$	$k=3$	$k=2$	$k=1$	$k=0$	$k=$-1	$k=$-2	$k=$-3	$k=$-4
Δ_t									
Cor(c_{t+k}, y_t)	.004	.136	.307	.494	.660	.751	.741	.620	.398
	(.213)	(.209)	(.185)	(.142)	(.089)	(.054)	(.063)	(.082)	(.086)
Cor(c_{t+k}, i_t)	-.015	.088	.235	.409	.578	.687	.704	.610	.409
	(.216)	(.216)	(.198)	(.160)	(.109)	(.071)	(.075)	(.096)	(.104)
Δ_t, s_t									
Cor(c_{t+k}, y_t)	.028	.151	.308	.477	.624	.699	.681	.561	.351
	(.205)	(.205)	(.190)	(.157)	(.114)	(.083)	(.083)	(.098)	(.107)
Cor(c_{t+k}, i_t)	.011	.107	.239	.394	.542	.632	.639	.545	.356
	(.207)	(.211)	(.202)	(.175)	(.135)	(.101)	(.097)	(.112)	(.122)

Note: Numbers in parentheses are standard errors based on 500 simulations.

6.3. Caveat

Aggregate investment in the U.S. is often defined as the sum of residential investment and non-residential investment. Aggregate investment so defined appears to coincide with consumption rather than lagging consumption. This is so simply because residential investment strongly leads output and business investment. The intriguing question, therefore, is why residential investment leads the business cycle? We think the answer may be that residential housing is, at least in part, a durable consumption good, not a capital good. Hence, the question is akin to the same puzzle addressed in this section.

7. Conclusion

Technology changes are arguably the single most important source of long-term economic growth. The recent literature, however, has questioned the notion that technology changes are *also* the main source of economic fluctuations. In this paper we show that equilibrium business cycle theories need not to rely on technology shocks to explain economic fluctuations. In an indeterminate RBC model with capacity utilization and mild increasing returns to scale, demand shocks can play a pivotal role in explaining actual economic fluctuations. Our analysis thus brings the real business cycle theory into closer conformity not only with the predictions of the Keynesian theory, but also with the actual data.

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