Bootstrap

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As long as $t \geq H$, then the dependent variable of interest Y_t can be represented by H equations where H is the forecast horizon.

$$Y_t = B_0 U_t + \varepsilon_t^0 \tag{1}$$

$$Y_t = B_1 U_{t-1} + \varepsilon_{t-1}^1 \tag{2}$$

$$Y_t = B_2 U_{t-2} + \varepsilon_{t-2}^2 \tag{3}$$

$$\vdots$$

$$Y_t = B_{H-1} U_{t-H+1} + \varepsilon_{t-H+1}^{H-1} \tag{4}$$

$$Y_t = B_H U_{t-H} + \varepsilon_{t-H}^H \tag{5}$$

Now, lag Equation 1 and isolate over U_{t-1} as follows,

$$U_{t-1} = \frac{Y_{t-1} - \varepsilon_{t-1}^0}{B_0} \tag{6}$$

Substitute Equation 6 into Equation 2 as follows,

$$Y_t = \frac{B_1}{B_0} Y_{t-1} - \frac{B_1}{B_0} \varepsilon_{t-1}^0 + \varepsilon_{t-1}^1 \tag{7}$$

Now there are two possible avenues:

1. Lag twice Equation 1, isolate over U_{t-2}, substitute into Equation 3 and obtain

$$Y_t = \frac{B_2}{B_0} Y_{t-2} - \frac{B_2}{B_0} \varepsilon_{t-2}^0 + \varepsilon_{t-2}^2 \tag{8}$$

As a general result we obtain,

$$Y_t = \frac{B_h}{B_0} Y_{t-h} - \frac{B_h}{B_0} \varepsilon_{t-h}^0 + \varepsilon_{t-h}^h \tag{9}$$

2. Lat Equation 2, isolate over U_{t-2}, substitute into Equation 3 and obtain

$$Y_t = \frac{B_2}{B_1} Y_{t-1} - \frac{B_2}{B_1} \varepsilon^1_{t-2} + \varepsilon^2_{t-2} \tag{10}$$

As a general result we obtain,

$$Y_t = \frac{B_h}{B_{h-1}} Y_{t-1} - \frac{B_h}{B_{h-1}} \varepsilon^{h-1}_{t-h} + \varepsilon^h_{t-h} \tag{11}$$