

Booms and Busts with dispersed information

Kenza Benhima [‡]
UNIVERSITY OF LAUSANNE
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Abstract

Dispersed information can generate booms and busts in economic activity. Boom-bust dynamics appear when firms are initially over-optimistic about demand due to a noisy private news. Consequently, they overproduce, which generates a boom and depresses their mark-ups. Because the news is private, firms cannot relate these low mark-ups to aggregate optimism. As low mark-ups can also signal low demand, this overturns their expectations, generating a bust. We emphasize a novel role for imperfect common knowledge: dispersed information makes firms ignorant about their competitors' actions, which makes them confuse high noise-driven supply with low fundamental demand.

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*HEC, Department of Economics, University of Lausanne, 1015 Lausanne, Switzerland, kenza.benhima@unil.ch, +41 21 692 36 92.

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1 Boom-bust episodes are a recurring feature in economic history. During boom periods,
 2 new projects employ large resources, which are then followed by deep slumps in investment
 3 and economics activity. Economists since Pigou (1927) have conjectured that these business
 4 cycles may be the consequence of "waves of optimism and pessimism". This paper provides
 5 a model for this phenomenon based on imperfect common knowledge. Unlike in the litera-
 6 ture before, booms are not followed by reversals to trend, but by subsequent possibly deep
 7 recessions.

8 Our model focuses on the difficulties faced by firms to correctly forecast the state of
 9 demand when deciding on their output level. We illustrate the mechanism in a standard
 10 two-period Dixit-Stiglitz model with imperfect competition. Busts originate in the preceding
 11 booms as the result of an initial over-optimistic private news about demand. When a positive
 12 aggregate noise shock occurs, i.e. when the news is on average *excessively optimistic* about
 13 the state of demand, firms over-produce, which generates a boom in the first period. This
 14 depresses prices, hence lowers profits and mark-ups. Because the original news was private,
 15 firms cannot relate these low mark-ups to the over-optimistic news. Since low mark-ups can
 16 also be driven by low demand, this makes firms' new expectations *excessively pessimistic*,
 17 generating a bust in the second period. Importantly, expectations do not simply revert to
 18 the true value of demand but *undershoot* it.

19 There is suggestive evidence that recessions are preceded by excessive optimism about
 20 demand. Between 2000 and 2006, the number of vacant houses all year round rose by 20%
 21 while construction spending increased by 45%, feeding the real estate bubble that lead to the
 22 subprime crisis. About the 1953 recession, the Council of Economic Advisors (1954) states:
 23 "Production and sales gradually fell out of balance in the early months of 1953. [...] The
 24 reason was partly that, while demand was high, business firms had apparently expected it
 25 to be higher still."

26 In the model, information is dispersed and agents learn from endogenous signals. This
 27 approach relates to imperfect information models with dispersed information, which date

back to Lucas (1972) and Frydman and Phelps (1984).¹ But while in the literature the focus has been on the welfare consequences of the endogenous nature of signals, here we study how they can create booms and busts patterns through expectations. First, dispersed information makes firms ignorant about their competitors' actions, and thus about aggregate supply. This implication of dispersed information is novel and crucial for the mechanism. Second, the nature of the endogenous signal matters: high demand affects the mark-ups positively, while high supply affects them negatively, so a low mark-up due to a noise-driven high aggregate supply can be confused with a sign of low demand.

We illustrate this in a model that is in most respects very standard, except for the timing. Labor is hired before quantities can be sold, so quantities are determined before prices. This can be thought about as a medium run horizon, where firms invest in capacities and prices are flexible. As a result, prices are observed only with a lag, leaving room for the boom-bust dynamics to develop in two steps: the boom driven by a positive news; the bust driven by a negative mark-up signal.

The presence of strategic substitutability between firms is key in generating booms and busts. In that case, a shock that affects aggregate supply (the noise shock) has a negative effect on firms' outcomes (prices), which then misleadingly signal low demand. Monopolistic competition, concave utility, fixed inputs are some realistic sources of strategic substitutability, and a high degree of strategic substitutability between firms arises with standard calibrations. This is also consistent with the industry evidence in Hoberg and Phillips (2010). They show that boom-bust dynamics are more likely to arise in competitive industries.

Importantly, our framework also allows for strategic complementarities through "trade linkages", and we are able to assess the role of these trade linkages. Indeed, as in Angeletos and La'O (2010), we have a nested CES utility function that allows both for a low elasticity of substitution between sectors and a high elasticity of substitution between goods of a given sector. While the high elasticity between goods generates competition and gives rise to

¹In particular, Hellwig and Venkateswaran (2009), Graham and Wright (2010), Gaballo (2013) and Amador and Weill (2012) study models where agents receive endogenous signals. Townsend (1983), Sargent (1991) and Pearlman and Sargent (2005) are earlier contributions.

strategic substitutability between firms, the low elasticity between sectors generates trade linkages and gives rise to strategic complementarity between sectors. In our context, for a given sectoral supply, a higher aggregate supply typically increases the mark-up because of strategic complementarity between sectors, so the mark-up could turn into a positive signal on demand. However, this happens only if the initial signal received by firms is strongly correlated across firms within a sector. Indeed, only in that case can the firms correctly assess the sectoral supply, and interpret the mark-up as a positive signal on demand. Therefore, what determines the relevant strategic substitutability parameter is the share of idiosyncratic and sectoral noise in the initial signal. If it is dominated by idiosyncratic noise, then strategic substitutability prevails and booms and busts still arise. While we do not settle this issue in the paper, rational inattention and large firm-level volatility relative to sectoral volatility provide arguments that idiosyncratic noise might be in fact large relative to sectoral noise.

Our approach provides several other insights. First, the less frequent noise shocks are, the more severe are the boom-bust cycles. This is because firms believe more easily that negative signals arise from actual low demand when noise shock are less likely. Second, while in our benchmark model the fundamental demand shock is a preference shock, other demand shocks, like productivity shocks in downstream production, can play the same role. Third, our mechanism is robust to adding other public and private signals. Fourth, in a dynamic extension with sticky information à la Mankiw and Reis (2002), the magnitude of busts is decreasing, but their length is increasing, with the strength of information frictions. Finally, temporary aggregate demand shocks which firms mistakenly interpret as a permanent shock can play the role of the initial aggregate noise shock. In that case, the dynamics start with an increase in credit, which is consistent with several boom-bust episodes.

Close to our approach, the news and noise shocks literature relates optimism and pessimism waves to aggregate signals about current or future productivity.² Depressions nevertheless do not breed into past exuberance. Waves of optimism fade out progressively as

²See, among others: Beaudry and Portier (2006), Jaimovich and Rebelo (2009), La'O and Angeletos (2011) Blanchard et al. (2013) and Lorenzoni (2009).

agents learn about the true state. Exceptions are Beaudry and Portier (2004), Christiano et al. (2010) and Lambertini et al. (2013), where busts arise due to the cumulated economic imbalances when a positive news is revealed to be false. In our setup, on the opposite, busts are driven by the agents' expectational errors.

Section 1 presents the benchmark model, a standard Dixit-Stiglitz monetary model with imperfect competition. To convey the intuition of the mechanism, we first present a simplified version of the model in Section 2. Section 3 then presents the implications of the benchmark model. In Section 4, we examine some extensions of the model. Section 5 concludes.

1 The benchmark model

We consider a two-period general equilibrium monetary model with imperfect competition à la Dixit-Stiglitz. There is a continuum of sectors indexed by $n \in [0, 1]$. In each sector n , there is a continuum of firms indexed by $i \in [0, 1]$ who each produce a differentiated good using labor. One representative household supplies labor on a competitive labor market and consumes the differentiated goods from all sectors. Aggregate demand is affected by a preference shock, which is not observed by firms.

1.1 Preferences and technology

There is a representative household with the following utility function:

$$U = U_1 + \beta U_2 \tag{1}$$

where $0 < \beta < 1$ is the discount factor and U_t is period- t utility:

$$U_t = \Psi \frac{Q_t^{1-\gamma}}{1-\gamma} - \frac{L_t^{1+\eta}}{1+\eta} \tag{2}$$

$Q = \left(\int_0^1 (Q_n)^{1-\epsilon} dn \right)^{\frac{1}{1-\epsilon}}$ is the consumption basket. Q_n represents the bundle of goods from sector $n \in [0, 1]$, with $Q_n = \left(\int_0^1 (Q_{in})^{1-\rho} di \right)^{\frac{1}{1-\rho}}$. Q_{in} is the quantity of good $i \in [0, 1]$ from sector n that is consumed. This nested C.E.S. utility function enables us to distinguish between $1/\rho \in [1, \infty)$, the elasticity of substitution between goods within a sector, from $1/\epsilon \in [0, \infty)$, the elasticity of substitution between sectors. $1/\gamma > 0$ is the elasticity of intertemporal substitution and $1/\eta > 0$ is the Frisch elasticity of labor supply.

L is the labor bundle $L = \left(\int_0^1 (L_n)^{1+\chi} dn \right)^{\frac{1}{1+\chi}}$, where L_n is the labor provided in sector n . $\chi \in [0, \infty)$ is used to introduce the possibility of imperfect mobility between sector. When $\chi = 0$, then $L = \int_0^1 L_n dn$, so the household is indifferent between working in the different sectors. When $\chi > 0$, hours worked in different sectors are not perfect substitutes for the worker. As in Horvath (2000), the motivation for this specification is the desire to capture some degree of sector-specificity to labor while not deviating from the representative household assumption.

Finally, Ψ determines the preference of the household for consumption relative to leisure.

Money is the numéraire. The consumer maximizes her utility subject to the following budget constraint, expressed in nominal terms:

$$\int_0^1 \left(\int_0^1 P_{int} Q_{int} di \right) dn + M_t + B_t = W_t L_t + \int_0^1 \left(\int_0^1 \Pi_{int} di \right) dn + M_{t-1} + r_{t-1} B_{t-1} + T_t \quad (3)$$

where P_{int} is the nominal price of good i in sector n , T_t are the nominal transfers from the government, M_t are money holdings, B_t are bond holdings, r_{t-1} is the nominal return on bond holdings, $W_t L_t$ is the nominal labor income and Π_{int} are the nominal profits distributed to the household by firm i from sector n .

Money is created by the government and supplied to the household through transfers T_t , following $M_t - M_{t-1} = T_t$. Bonds are in zero supply, so $B_t = 0$ in equilibrium.

Finally, the household faces a cash-in-advance constraint, $\int_0^1 \left(\int_0^1 P_{int} Q_{int} di \right) dn \leq M_{t-1} + T_t$. Because money yields no interest, this constraint holds with equality. Solving for the price index and combining with the government budget constraint, we obtain the quantity

equation:

$$P_t Q_t = M_t \quad (4)$$

where $P = \left(\int_0^1 (P_n)^{\frac{-(1-\epsilon)}{\epsilon}} dn \right)^{\frac{-\epsilon}{1-\epsilon}}$ is the general price index, and $P_n = \left(\int_0^1 (P_{in})^{\frac{-(1-\rho)}{\rho}} di \right)^{\frac{-\rho}{1-\rho}}$ is the price index in sector n .

The production function of each firm $i \in [0, 1]$ in sector n involves labor with a constant return to scale technology:

$$Q_{int} = L_{int} \quad (5)$$

Labor L_{int} is hired on a sector-specific labor market at rate W_{nt} . The government sets a constant subsidy on labor τ to correct for monopolistic competition. This subsidy is financed through a lump sum tax on firms $T_t^f = \tau \int_0^1 (W_{nt} \int_0^1 L_{int} di) dn$. The firm's profits are therefore:

$$\Pi_{int} = P_{int} Q_{int} - (1 - \tau) W_{nt} L_{int} - T_t^f \quad (6)$$

We assume that the firms behave monopolistically, while taking sectoral and aggregate variables as given, as is typically assumed.

1.2 Shocks, timing and information

At the beginning of period 1, the economy is hit by a shock on the preference parameter Ψ . This represents a permanent “demand shock” for the differentiated goods. We assume that $\psi = \log(\Psi)$ is normally distributed with mean zero and standard error σ_ψ : $\psi \sim \mathcal{N}(0, \sigma_\psi)$. We assume that ψ is directly observed by households, but not by firms.

The money supply is set by the government to $M_t = \exp(m_t)$, where $m_t \sim \mathcal{N}(0, \sigma_m)$ is a monetary shock. We assume that the household receives money transfers at the beginning of period, so she observes m_t directly but the firms do not.

At this stage, we can define Ω_{int} , the set of signals available to firm i of sector n when making its time- t production decision. At the beginning of period 1, the firm receives an exogenous signal about ψ that incorporates an aggregate, a sectoral, and an idiosyncratic

145 error:

$$\psi_{in} = \psi + \theta + \Lambda_n + \lambda_{in} \quad (7)$$

146 where $\theta \sim \mathcal{N}(0, \sigma_\theta)$ is an aggregate noise shock, $\Lambda_n \sim \mathcal{N}(0, \sigma_\Lambda)$ is a sectoral noise shock
 147 and $\lambda_{in} \sim \mathcal{N}(0, \sigma_\lambda)$ is an idiosyncratic noise shock. All shocks are i.i.d. so the law of large
 148 numbers implies that $\int_0^1 \lambda_{in} di = 0$ and $\int_0^1 \Lambda_n dn = 0$.

149 The other signals observed by firms when making their production decisions depend on
 150 the market timing. First, in each period, the labor market opens before the goods market.
 151 Second, transactions are made in terms of money. As a result, labor hirings and nominal
 152 wages are determined first, before firms can observe nominal prices. This has two important
 153 consequences. On the one hand, it makes quantities predetermined with regards to nominal
 154 prices, so quantities are contingent only on the signal ψ_{in} and on the nominal wage W_{nt} .
 155 On the other hand, nominal prices P_{int} incorporate new information that the firms can use
 156 when setting their next period's supply.

157 Therefore, the information set of firm i in sector n at the beginning of period 1 is $\Omega_{in1} =$
 158 $\{\psi_{in}, W_{n1}\}$. At the beginning of period 2, firms have observed the price of their good during
 159 period 1, so $\Omega_{in2} = \{\psi_{in}, W_{n1}, P_{in1}, W_{n2}\}$. We denote by $E_{int}(y)$ the expected value of y
 160 conditional on Ω_{int} .

161 Importantly, we assume that the aggregate supply is not part of their information set.
 162 The idea behind this restrictive information structure is that firms pay attention to their
 163 local interactions and limited attention to public releases of aggregate information.

164 Finally, we assume that, besides ψ and m_t , the household observes θ , Λ_n and λ_{in} .³

165 1.3 Definition of the equilibrium

166 We define an equilibrium as follows:

167 **Definition 1 (Equilibrium)** *An equilibrium is a sequence of nominal prices, nominal wages,*
 168 *money holdings, and production levels such that, in each period $t = 1, 2$: (i) the household*

³This is without loss of generality given that the household observes all quantities and prices in all markets, and therefore can infer θ , Λ_i and λ_{ij} using her information on ψ and m_t .

169 sets its demand for goods, money and labor supply to maximize her utility (1) subject to her
 170 budget constraint (3) and the cash-in-advance constraint (4), given full information about
 171 the shocks hitting the economy and given the nominal prices and wages; (ii) each firm i from
 172 sector n sets its supply Q_{int} monopolistically to maximize its profits (6) subject to the demand
 173 schedule for good i for sector n and given its information set Ω_{int} ; (iii) the money market,
 174 labor market and the good markets clear.

175 1.4 Characterization of the equilibrium

176 For expositional purposes, we neglect constant terms, but we derive them in the online
 177 appendix. The consumer's maximization program yields the following demand for good i of
 178 sector n , in logs:

$$q_{int} - q_{nt} = -\frac{1}{\rho} [p_{int} - p_{nt}] \quad (8)$$

179 where lower-case letters denote the log value of the variable. The elasticity of substitution
 180 between goods $1/\rho$ governs the equilibrium response of the individual price p_{in} to sectoral
 181 output q_n . Similarly, the demand for goods of sector n is given by:

$$q_{nt} - q_t = -\frac{1}{\epsilon} [p_{nt} - p_t] \quad (9)$$

182 The elasticity of substitution between sectors $1/\epsilon$ governs the equilibrium response of the
 183 sectoral price p_n to aggregate output q and sectoral output q_n .

184 The consumer's labor supply optimization and the production functions yield the follow-
 185 ing:

$$w_{nt} - p_t = \sigma q_t + \chi(q_{nt} - q_t) - \psi \quad (10)$$

186 with $\sigma = \eta + \gamma$. σ is the elasticity of the aggregate wage to aggregate supply q . χ is the
 187 elasticity of the sectoral wage to the sectoral supply.

The money market clears, so $P_t Q_t = \exp(m_t)$. In logs, this gives:

$$p_t + q_t = m_t \quad (11)$$

Firm i in sector n maximizes (6) over Q_{int} subject to (5) and (8), and takes sectoral production Q_{nt} and aggregate production Q_t as given. Optimal supply must then be such that the mark-up P_{int}/W_{nt} is equal to $(1 - \tau)/(1 - \rho)$ in expectations: $E_{int}(P_{int}/W_{nt}) = (1 - \tau)/(1 - \rho)$. Since shocks are log-normal, and since the government sets $\tau = \rho$ to undo monopolistic competition, this equation can be written in logs:

$$E_{int}(p_{int} - w_{nt}) = 0 \quad (12)$$

Equilibrium prices, quantities and wages in period 1 and 2 are characterized by Equations (8)-(12) given the realization of shocks ψ , θ , $(\Lambda_n)_{n \in \{1, \dots, N\}}$, $(\lambda_{in})_{(i,n) \in [0,1] \times \{1, \dots, N\}}$, m_1 and m_2 , with $q = \int_0^1 (\int_0^1 q_{in} di) dn$, $q_n = \int_0^1 q_{in} di$, $p = \int_0^1 (\int_0^1 p_{in} di) dn$ and $p_n = \int_0^1 p_{in} di$.

1.5 Reduced-form model

The model can be reduced even further by focusing on quantity-setting and signals.

The quantity-setting equation It is useful to define the normalized supply $\hat{q}_{int} = \sigma q_{int}$. By using the individual and sectoral demand equations (8) and (9), along with the labor supply equation (10) and the individual supply equation (12), the optimal - normalized - individual supply can be written as follows:

$$\hat{q}_{int} = (1 + \kappa_b + \kappa_w) E_{int}(\psi) - \kappa_b E_{int}(\hat{q}_t) - \kappa_w E_{int}(\hat{q}_{nt}) \quad (13)$$

where $\kappa_b = \sigma/\rho - (\chi + \epsilon)/\rho$ and $\kappa_w = (\chi + \epsilon)/\rho - 1$. In order to decide its optimal supply \hat{q}_{in} , firm i has three variables to infer: the fundamental shock ψ , but also the aggregate and sectoral supplies \hat{q} and \hat{q}_n . The parameter κ_b describes the best response of the individual

firm to its expectation of the aggregate output while κ_w is the best response to its expectation of sectoral output. In other words, κ_b is the level of strategic substitutability between sectors while κ_w is the level of strategic substitutability between firms within a sector.

Endogenous signals Regarding signals, at the beginning of period 1 and 2, firms from sector n observe the nominal wage w_n , which can be written as follows by using equations (10) and (11):

$$w_{nt} = \left(1 - \frac{1}{\sigma}\right) \hat{q}_t + \frac{\chi}{\sigma} (\hat{q}_{nt} - \hat{q}_t) - \psi + m_t \quad (14)$$

At the end of period 1, and relevant for period 2's decisions, firms observe their nominal price p_{int} which, together with the nominal wage, can be combined to determine the mark-up:

$$p_{in1} - w_{n1} = \frac{1}{1 + \kappa_b + \kappa_w} \{ (1 + \kappa_b + \kappa_w) \psi - \kappa_b \hat{q}_1 - \kappa_w \hat{q}_{n1} - q_{in1} \} \quad (15)$$

where we have used equations (8)-(11). The mark-up is affected directly and positively by the fundamental shock ψ since a demand shock lowers the real wage, but it is also affected by the aggregate and sectoral supply.

Equations (14) and (15), along with the exogenous signal ψ_{in} , are enough to characterize the information sets in both periods. Together with the quantity-setting equation (13), they can fully characterize the equilibrium output.

The role of monopolistic competition and trade linkages The way the mark-up reacts to aggregate and sectoral supply will be crucial to determine the type of information it will reveal to firms. These reactions are tightly linked to the strategic substitutability parameters κ_b and κ_w . It is useful to focus on strategic substitutability between sectors, κ_b , and on “aggregate substitutability”, defined as $\kappa_a = \kappa_b + \kappa_w$. κ_a describes how the mark-up moves when sectoral and aggregate demand move by the same amount. It has the simple expression $\kappa_a = \sigma/\rho - 1$.

Here, the distinction between ρ , which determines “monopolistic competition”, from

ϵ , which determines trade linkages, is crucial. ρ is typically below one, which generates positive mark-ups, while ϵ is close to or above one, which reflects complementarity between sectors and so-called “trade linkages”. More competition between firms (lower ρ) generates more aggregate strategic substitutability (a higher κ_a), while stronger trade linkages between sectors (higher ϵ) generate more complementarity between sectors (a lower κ_b).

1.6 Perfect information outcome

Before solving the model with imperfect information, consider the perfect information outcome. If firms were all able to observe ψ directly, then they would set \hat{q}_{int} at ψ :

$$\hat{q}_{int} = \psi \tag{16}$$

This can be shown by acknowledging that firms have common knowledge so they set their quantities in the same way, so that $\hat{q}_{int} = \hat{q}_{nt} = \hat{q}_t$. Replacing in (13), we find (16), which is the first-best response. Under perfect information, the mark-up is zero: $p_{int} - w_{nt} = 0$.

2 Booms and busts in a simple case

The main mechanism of the model comes from the fact that firms can observe the first-period mark-up $p_{in1} - w_{n1}$ only at the end of period 1. To highlight the mechanisms at play and obtain analytical solutions, we first consider a set of restrictions that, together, will constitute a useful simplification of the model. We next define boom-bust patterns, and then study the equilibrium in order to characterize the conditions under which they arise.

2.1 Some useful restrictions

First, we simplify the set of endogenous signals used by the firms. They include the nominal wage, and after the first period, the mark-up, as described in Equations (14) and (15). But notice that while the nominal wage is made noisy by the monetary shock m_t , the

mark-up, being a real price, is not. Because we want to focus on the role of the mark-up as a signal in our benchmark model, it is useful to assume that w_t is not used by firms as a source of information. This is the case if the variance of the monetary shock σ_m is infinitely large, making any nominal variable uninformative on real shocks. We therefore make the following assumption:

Assumption 1 (Infinite monetary noise) $\sigma_m \rightarrow +\infty$.

Under this assumption, the set of relevant signals reduces to $\Omega_{in1} = \{\psi_{in}\}$ in period 1. In period 2, firms have observed the price of their good during period 1, so $\Omega_{in2} = \{\psi_i, p_{in1} - w_{1n}\}$.

Second, it is possible to abstract from the sectoral dimension by assuming that there is no sectoral noise ($\sigma_\Lambda = 0$). Because there is no sector-specific information, firms behave in the same way on average across sectors. Therefore, $p_t = p_{nt}$ and $q_{nt} = q_t$ for all $n \in [0, 1]$. This case boils down to a one-sector economy. We thus make the following assumption:

Assumption 2 (No sector-specific noise) $\sigma_\Lambda = 0$.

Under this assumption, we suppress the sectoral index n as the sectoral dimension of the economy is irrelevant. The exogenous signal, the mark-up and the individual supply become:

$$\psi_i = \psi + \theta + \lambda_i \quad (17)$$

$$p_{i1} - w_1 = \frac{1}{1 + \kappa_a} \{(1 + \kappa_a)\psi - \kappa_a \hat{q}_1 - \hat{q}_{i1}\} \quad (18)$$

$$\hat{q}_{it} = (1 + \kappa_a)E_{it}(\psi) - \kappa_a E_{it}(\hat{q}_t) \quad (19)$$

Both the effect of aggregate supply on the mark-up, and the best response of individual quantities to the expectation of aggregate supply, are now governed by $\kappa_a = \kappa_b + \kappa_w = \sigma / \rho - 1$, the degree of aggregate substitutability. Because noise cancels out at the sectoral level, the sectoral supply comoves perfectly with the aggregate supply, so κ_a combines the effect of both the aggregate and sectoral supply on the individual mark-up. Similarly, when firms set their quantities, if they expect a high aggregate supply, they also expect an equally high

supply in their sector, so they have to take into account strategic substitutability both at the sector level and between sectors, which is summarized in κ_a . Notice that Equations (18) and (19) could also describe a one-sector economy with no trade linkages.

Aggregate strategic substitutability ($\kappa_a > 0$) arises under the following condition:

Condition 1 (Aggregate strategic substitutability - $\kappa_a > 0$) $\sigma > \rho$.

As we will see, this condition is strongly satisfied with a standard calibration. Importantly, the effect of \hat{q}_1 on the mark-up is tied to κ_a , and it is negative under Condition 1.

The optimal supply equation (19), with $\Omega_{i1} = \{\psi_i\}$ and $\Omega_{i2} = \{\psi_i, p_{i1} - w_1\}$, where ψ_i and $p_{i1} - w_1$ are described by (17) and (18), can fully characterize the equilibrium output.

2.2 Definition of the boom-bust pattern

A boom-bust episode is not simply characterized by a decrease in output following an increase in output. Output has to be lower than its initial value. Boom-bust episodes are thus defined as follows:

Definition 2 (Boom-bust) *Consider q_1 and q_2 , the respective values of output in period 1 and 2. A boom-bust occurs when $q_1 > 0$ and $q_2 < 0$.*

2.3 Equilibrium production

Under Condition 1, the firms' mark-up, which is observed by firms at the end of period 1, is positively affected by the fundamental shock and negatively by the noise shock. An initially positive noise shock then adversely affects firms' expectations on the fundamental shock, thus generating boom-bust dynamics.

First-period production As firms receive the signal $\psi_i = \psi + \theta + \lambda_i$ at the beginning of period 1, they extract information from this signal according to the following formula:

$$E_{i1}(\psi) = k_\psi \psi_i = k_\psi (\psi + \theta + \lambda_i) \quad (20)$$

294 where k_ψ is the standard Bayesian weight $k_\psi = \sigma_\psi^2 / (\sigma_\psi^2 + \sigma_\theta^2 + \sigma_\lambda^2)$.

295 Firm i 's supply follows (19). We establish the following (see proof in the Appendix):

$$\hat{q}_{i1} = K_\psi \psi_i \quad (21)$$

296 where $K_\psi = (1 + \kappa_a) \sigma_\psi^2 / [(1 + \kappa_a)(\sigma_\psi^2 + \sigma_\theta^2) + \sigma_\lambda^2]$.

297 At the aggregate level, firms produce the following quantities:

$$\hat{q}_1 = K_\psi (\psi + \theta) \quad (22)$$

298 Since $0 < K_\psi < 1$, the aggregate supply under imperfect information, as compared with
 299 the equilibrium supply under perfect information (16), reacts less to the fundamental shock
 300 ψ , because information is noisy. On the opposite, aggregate supply reacts positively to the
 301 aggregate noise shock θ , because firms cannot distinguish it from the fundamental.⁴

New signal The new signal received by firms is their mark-up, as described in (18). The firm can use $p_{i1} - w_1$ to extract information on ψ by combining it with its other signal ψ_i . The firm knows individual supply $\hat{q}_{i1} = K_\psi \psi_i$, but ignores \hat{q}_1 because of dispersed information. Therefore, it can “filter” the mark-up from the influence of \hat{q}_{i1} but not from the influence of \hat{q}_1 . The “filtered” mark-up writes as follows:

$$p_{i1} - w_1 + \frac{1}{(1 + \kappa_a)} \hat{q}_{i1} = \left(1 - \frac{\kappa_a}{(1 + \kappa_a)} K_\psi \right) \psi - \frac{\kappa_a}{(1 + \kappa_a)} K_\psi \theta$$

302 The fundamental shock ψ has a positive effect on the filtered mark-up. Indeed, under
 303 imperfect information, aggregate supply does not fully respond to the demand shock, so
 304 aggregate demand is in excess of supply, which increases the mark-up. On the opposite, the
 305 noise shock θ has a negative effect on the filtered mark-up under Condition 1, that is, under

⁴Moreover, if there is aggregate strategic substitutability ($\kappa_a > 0$), we have $K_\psi > k_\psi$, which means that agents over-react to their private signal ψ_i . This implication of strategic substitutability is in line with the literature on imperfect common knowledge (see for example Angeletos and Pavan (2007)). Here, imperfect common knowledge plays an additional role, which is to produce confusion between demand and supply.

strategic substitutability. In this case, aggregate supply is in excess of actual demand, which depresses the mark-up. Condition 1 means that supply does not generate its own demand through a “market potential” effect. Therefore, as a result of strategic substitutability, a positive shock on θ makes the filtered mark-up a *negative* signal of ψ .

Imperfect common knowledge is crucial here. If firms received the same information, they would be able to infer \hat{q}_1 even without observing it directly. They could then filter their mark-up from the influence of others’ supply. In short, even if firms observed low mark-ups following a positive noise shock $\theta > 0$, they would be able to put these mark-ups in perspective with the high aggregate supply \hat{q}_1 . As a result, low mark-ups would not be perceived as a negative signal on ψ , but simply as the result of high supply. In the online appendix, we illustrate this point by considering the case where the initial signal is public, and showing that it does not lead to a boom-bust.

This yields the following lemma:

Lemma 1 *The information set available at the beginning of period 2 $\Omega_{i2} = \{\psi_i, p_{i1} - w_1\}$ is equivalent to two independent signals of ψ , s and x_i , defined as follows:*

$$s = \psi - \omega_\theta \theta$$

$$x_i = \psi + \omega_\lambda \lambda_i$$

with $\omega_\theta = \kappa_a K_\psi / [(1 + \kappa_a) - \kappa_a K_\psi]$ and $\omega_\lambda = \omega_\theta / (1 + \omega_\theta)$.

Under Condition 1, $\omega_\theta > 0$. Besides, ω_θ is increasing in κ_a .

Proof. s is obtained simply by normalizing the filtered mark-up. x_i is obtained by combining s with ψ_i : $x_i = (\omega_\theta \psi_i + s) / (1 + \omega_\theta) = \psi + \omega_\theta \lambda_i / (1 + \omega_\theta)$. As x_i and s are independent linear combinations of ψ_i and $p_{i1} - w_1$, the information set $\{x_i, s\}$ is equivalent to $\{\psi_i, p_{i1} - w_1\}$. ■

s is simply the normalized filtered mark-up. Together with ψ_i , it can be used to extract another signal, x_i , that is independent of θ .

Second period Using the above discussion, we establish the following:

Proposition 1 *Suppose Assumptions 1 and 2 hold. Following a positive noise shock θ , output experiences a boom-bust if and only if Condition 1 is satisfied.*

Consider first expectations. As firms receive two independent signals of ψ , solving for $E_{i2}(\psi)$ is straightforward (see proof in the Appendix):

$$E_{i2}(\psi) = f_x x_i + f_s s = (f_x + f_s)\psi - f_s \omega_\theta \theta + f_x \omega_\lambda \lambda_i \quad (23)$$

with $0 < f_x, f_s < 1$ and $k_\psi < f_x + f_s < 1$. f_s is decreasing in κ_a .

As $f_x + f_s > k_\psi$, after a fundamental shock ψ , the forecast of ψ becomes closer to the fundamental in the second period as firms gather more information. Conversely, the effect of a noise shock θ on the forecast of ψ turns from positive in the first period ($k_\psi > 0$) to negative in the second period ($-f_s \omega_\theta < 0$). This happens because, as a result of an excessive aggregate supply, firms observe lower mark-ups than expected. They revise their forecasts of ψ downwards, because low mark-ups can also signal a low ψ , that is, low demand. Equation (23) states that this updating more than reverses the initial positive forecast.

Importantly, θ has a negative effect on period 2's expectations, regardless of the parameters that affect the signals' precision (σ_ψ , σ_θ and σ_λ). At first sight, it seems that the new expectation of ψ should be of the form $E_{i2} = \alpha_0 \psi_i + \alpha_1 s$, with α_0 and α_1 positive, so that the effect of θ would be described by $\alpha_0 - \omega_\theta \alpha_1$, and should therefore depend on the signals' precisions. However, this reasoning does not take into account the fact that ψ_i and s are not independent signals. In that context, the mark-up signal s plays a dual role. First, s is used to correct the initial signal ψ_i from the influence of θ by constructing $x_i = (\omega_\theta \psi_i + s)/(1 + \omega_\theta)$, which is independent from θ . Namely, by putting in perspective the initial positive signal received in the first period, with the new negative mark-up signal received in the second period, firms recognize that they were overoptimistic in the first period. This brings back expectations to zero. But s constitutes also a new, negative signal on demand, that firms incorporate in their expectations. This worsens expectations, which then become negative.

The effect of θ on output derives naturally from its effect on expectations. In period 2,

as in period 1, the supply by firm i follows Equation (19). We establish the following (see proof in the online appendix):

$$\hat{q}_{i2} = F_x x_i + F_s s \quad (24)$$

with $0 < F_x, F_s < 1$ and $K_\psi < F_x + F_s < 1$. At the aggregate level, firms produce the following quantities:

$$\hat{q}_2 = [F_x + F_s]\psi - F_s \omega_\theta \theta \quad (25)$$

In period 2, as $F_x + F_s > K_\psi$, following a fundamental shock ψ , output gets closer to its first-best value. On the opposite, the effect of the aggregate noise shock θ on aggregate supply becomes negative through the public signal s .

Whereas the effect of θ on the second-period output is clearly negative when $\kappa_a > 0$, the marginal impact of the strategic substitutability parameter κ_a is not straightforward. While κ_a has a positive effect on the reaction of the filtered mark-up s to noise ω_θ , it has a negative impact on the weight f_s firms put on s . Indeed, as s becomes more reactive to θ , it becomes a poorer signal of ψ , so firms rely less on it to infer ψ . Besides, with strategic substitutability, firms under-react to s because it is a public signal (identical for all firms). However, in the limit case where σ_θ^2 goes to zero, the first effect dominates, as suggested by the following corollary (see proof in the Appendix):

Corollary 1 *As σ_θ^2 goes to zero, $-F_s \omega_\theta$ goes to $-\kappa_a \sigma_\psi^2 / (\sigma_\psi^2 + \sigma_\lambda^2)$.*

This implies that, following a noise-driven boom, expectations can be arbitrarily low as aggregate noise shocks are less likely and strategic substitutability is strong.

3 Booms and busts in the benchmark model

In this section, we solve the full version of the benchmark model, by relaxing Assumptions 1 and 2, which means that nominal wages are now part of the firms' information set, and sector-specific noise is allowed. We first relax Assumption 2 only, and show that including sectoral noise allows a richer discussion on the parameters that generate booms and busts.

In particular, the level of strategic substitutability between sectors is relevant, but only to the extent that there is heterogeneous information between sectors. We then relax both assumptions to explore numerically the role of parameters for the occurrence and size of booms-busts in the general case.

3.1 Sectoral and idiosyncratic noise

Here we relax Assumption 2 to allow for sectoral noise. Then, the general quantity-setting and mark-up equations (13) and (15) hold. As Assumption 1 still holds, firms use only the exogenous signal ψ_{in} and the mark-up as described in (15) as sources of information.

No idiosyncratic noise Consider the polar case without idiosyncratic noise ($\sigma_\lambda = 0$). In that case, firms behave in the same way within a given sector. Therefore, $q_{in} = q_n$ for all $i \in [0, 1]$ and $n \in [0, 1]$. We can then discard the i index, because the firm level is not relevant anymore. As a result, Equations (13) and (15) boil down to

$$\hat{q}_{nt} = \left(1 + \frac{\kappa_b}{1 + \kappa_w}\right) E_{nt}(\psi) - \frac{\kappa_b}{1 + \kappa_w} E_{nt}(\hat{q}_t) \quad (26)$$

$$p_{n1} - w_{n1} = \frac{1}{1 + \frac{\kappa_b}{1 + \kappa_w}} \left\{ \left(1 + \frac{\kappa_b}{1 + \kappa_w}\right) \psi - \frac{\kappa_b}{1 + \kappa_w} \hat{q}_1 - \hat{q}_{n1} \right\} \quad (27)$$

Equations (26) and (27) are similar to (19) and (18), except that κ_a is replaced with $\frac{\kappa_b}{1 + \kappa_w}$. Now, the coefficient governing strategic substitutability and the reaction of the mark-up to aggregate supply depends on the parameter of strategic substitutability between sectors κ_b .

If information is sector-specific, then \hat{q}_{n1} is common knowledge within sector n , but \hat{q}_1 is not. Firms in sector n can then clean their mark-up from the influence of \hat{q}_{n1} , but not from the influence of \hat{q}_1 . Whether a positive noise shock, by increasing \hat{q}_1 , turns the mark-up into a positive or a negative signal on demand, depends therefore on the sign of κ_b . The degree of strategic substitutability between sectors κ_b thus determines the structure of the information revealed by the mark-up.

The condition for strategic substitutability between sectors ($\kappa_b > 0$) is defined as follows:

Condition 2 (Strategic substitutability between sectors - $\kappa_b > 0$) $\sigma - \chi > \epsilon$.

By analogy, the following proposition ties the emergence of boom-bust dynamics to the strategic substitutability between sectors:

Proposition 2 *Suppose Assumption 1 holds and $\sigma_\lambda = 0$. Following a positive noise shock θ , output experiences a boom-bust if and only if Condition 2 is satisfied.*

In the quantitative analysis, we will see that Condition 2 is typically not satisfied, so there is typically strategic complementarity between sectors, and boom-busts thus do not emerge for $\sigma_\lambda = 0$. With this polar case, we can see that whether information is dispersed at the sectoral or idiosyncratic level is crucial as it determines the parameter of strategic substitutability that is relevant for the emergence of boom-busts.

General case In the more general case, where individual signals have a sectoral and an idiosyncratic noise ($\sigma_\lambda > 0$ and $\sigma_\Lambda > 0$), both parameters of strategic substitutability κ_a and κ_b matter, with a relative weight that depends on the relative size of the sectoral and idiosyncratic noise. This condition is summarized as follows:

Condition 3 (Weighted strategic substitutability) $\kappa_a[(1+\kappa_a)-\kappa_a K_\psi]\sigma_\lambda^2 + \kappa_b[(1+\kappa_a)-\kappa_b K_\psi]\sigma_\Lambda^2 > 0$.

Notice that when $\sigma_\lambda = 0$, Condition 3 boils down to Condition 2, and when $\sigma_\Lambda = 0$, it boils down to Condition 1. The following Proposition holds (see proof in the Appendix):

Proposition 3 *Suppose Assumption 1 holds. Following a positive noise shock θ , expectations experience a boom-bust if and only if Condition 3 is satisfied.*

In the most likely case where Condition 1 holds ($\kappa_a > 0$) and Condition 2 does not ($\kappa_b < 0$), boom and busts will tend to arise if σ_λ is large relative to σ_Λ . The numerical analysis confirms these insights in the general case.

3.2 Numerical analysis

We now relax Assumption 1 to allow firms to use nominal wages as a source of information and perform a numerical analysis. We first discuss the relevant set of parameters to explore and set the benchmark calibration. Starting from our benchmark calibration, we explore the ranges of parameters for which booms and busts appear and are sizeable.

Our analysis confirms the insights of Proposition 3, namely, the crucial role played by κ_a and κ_b , and by the ratio $\sigma_\lambda/\sigma_\Lambda$. Booms and busts do not arise for all admissible parameters but do arise for some plausible parameter sets.

3.2.1 Parameter ranges and benchmark values

The benchmark calibration is described in Table 1. Regarding the preference parameters, Proposition 3 suggests that they matter through the reduced-form parameters of aggregate and sectoral substitutability, $\kappa_a = \sigma/\rho - 1$ and $\kappa_b = \sigma/\rho - (\chi + \epsilon)/\rho$.

First, consider $\sigma = \gamma + \eta$. In the New Keynesian literature, σ is related to the degree of “real rigidities”, that is, the elasticity of the marginal cost to the output gap, which is highly debated. A higher σ (lower real rigidities), lead to more substitutability (higher κ_a and κ_b). Richer models find estimates of σ that vary between 0.3 and 3.⁵ In our benchmark calibration, we set $\sigma = 1$, but we will also consider lower and larger values. While σ represents the elasticity of the aggregate marginal costs to aggregate output, χ is the elasticity of the sectoral marginal cost to sectoral production. In our benchmark calibration, we set $\chi = 0$, its minimum value. A larger χ would imply more “sector-specificity” of labor input and hence more complementarity between sectors (a lower κ_b). We will also consider Horvath (2000)’s estimate, which corresponds to $\chi = 1$.

Since we relax Assumption 1, σ and χ matter directly as well as they determine how the aggregate and sectoral supply affect the wage signal (see Equation (14)). In our benchmark

⁵For instance, this value is equal to 0.3 in Nakamura and Steinsson (2010), 0.34 in Smets and Wouters (2007) and 3 in Galí (2009). Kryvtsov and Midrigan (2013) show that the behavior of inventories is compatible with a low level of real rigidities, hence a high σ . See Woodford (2003) for a discussion on this parameter.

calibration where $\sigma = 1$ and $\chi = 0$, these effects are shut down. In that case, the wage simply plays the role of an additional exogenous signal: $w_{nt} = w_t = -\psi + m_t$.

Consider now the elasticities of substitution $1/\rho$ and $1/\epsilon$. $1/\rho$ governs the degree of imperfect competition, while $1/\epsilon$ governs “trade linkages”. Micro studies report values for the elasticity of substitution between goods (within a sector) that are consistently of the order of 6-7, so we set $1/\rho$ to 7.⁶ As a result, $\kappa_a = 6$, which strongly satisfies Condition 1. If we assume, as is common, that goods produced in different sectors are complementary, then $1/\epsilon \leq 1$. If σ is not too much above 1, this implies that Condition 2 is typically not satisfied. In the literature, $1/\epsilon$ has been estimated between 0 and slightly above 1.⁷ We set $1/\epsilon = .85$ as a benchmark value, which implies that $\kappa_b = -1.2$, and will consider a wide range of values between 0 and 2.

Because our model is too simple to be brought to the data, our benchmark variance parameters are set arbitrarily: $\sigma_\psi = 0.01$, $\sigma_\theta = 0.001$, $\sigma_m = 0.01$, $\sigma_\lambda = 0.01$ and $\sigma_\Lambda = 0.001$. This calibration is prone to the appearance of large booms and busts, because σ_θ is relatively small and σ_λ is large relative to σ_Λ , as suggested by Corollary 1 and Proposition 3. Starting from this benchmark, we will explore a broad range of these parameters to determine their specific role.

3.2.2 Role of κ_a , κ_b and $\sigma_\lambda/\sigma_\Lambda$

We first focus on the role the reduced-form parameters of aggregate and sectoral strategic substitutability κ_a and κ_b and on the relative size of idiosyncratic and sectoral noise $\sigma_\lambda/\sigma_\Lambda$. These parameters, as highlighted in Proposition 3, are key to determine the occurrence of booms and busts. We impose for this exercise that the structure of the wage signal remains unchanged and keeps the exogenous form $w_{nt} = w_t = -\psi + m_t$. This ad hoc assumption helps us isolate the role of strategic substitutability from the role of the wage signal and will be relaxed later. Note however that it can be interpreted as a case where σ and χ are kept fixed at their benchmark level and changes in κ_a and κ_b are driven by $1/\rho$ and $1/\epsilon$ only.

⁶See for instance Broda and Weinstein (2006) and Imbs and Mejean (2009).

⁷See for instance Comin and Mestieri (2015) and Atalay (2017).

Panel (a) (Panel (b)) in Figure 1 represents, for each κ_a , the minimal value of κ_b for booms and busts in expectations (in output) to appear, depending on the ratio $\sigma_\lambda/\sigma_\Lambda$, while setting σ_ψ , σ_θ , σ_m and $\sigma_\lambda + \sigma_\Lambda$ at their benchmark values. For values of κ_a and κ_b that are above the x-axis (κ_a and κ_b positive), booms and busts always arise. Below the x-axis (only κ_a is positive), booms and busts arise only for some values of $\sigma_\lambda/\sigma_\Lambda$. Namely, a more negative κ_b (more complementarity between sectors), necessitates either a larger κ_a (more aggregate substitutability) or a higher $\sigma_\lambda/\sigma_\Lambda$. The insights of Proposition 3 thus generalize to the case where firms have a wage signal.

By comparing panels a) and b), conditions for booms and busts in output appear more restrictive than conditions for booms and busts in expectations. This means that there are situations where a reversal in expectations about the fundamental appears without a reversal in quantities. This is particularly true for large κ_a , that is, for strong strategic substitutability. Indeed, as stressed earlier, with strong substitutability, firms tend to under-react to the mark-up signal, because it is a public signal.

Discussion We consider the benchmark calibration and alternative calibrations in Figure 1. According to Panel (b), with the benchmark calibration, booms and busts in output appear if σ_λ is at least twice as large as σ_Λ . Decreasing σ relative to the benchmark decreases both κ_a and κ_b (we move south-west on the graphs) and requires larger values of $\sigma_\lambda/\sigma_\Lambda$ for booms and busts to appear. Decreasing $1/\rho$ while keeping ϵ/ρ constant (i.e. decreasing $1/\rho$ and $1/\epsilon$ proportionally) has the same effect as decreasing σ , as κ_a and κ_b both increase. Increasing χ or decreasing $1/\epsilon$ makes κ_b more negative (we move south on the graph), which requires a larger $\sigma_\lambda/\sigma_\Lambda$ as well.

We can then illustrate in what dimensions of the parameters the model can have trouble generating booms and busts: for high levels of real rigidities (low σ), for strong trade linkages (low $1/\epsilon$) and high sector-specificity of inputs (high χ). Conservative calibrations can then rule out booms and busts. In particular, as $1/\epsilon$ goes to zero, κ_b goes to $-\infty$, which rules out booms and busts even for very high $\sigma_\lambda/\sigma_\Lambda$. But booms and busts appear independently

of that ratio for more liberal, yet plausible calibrations yielding $\kappa_b > 0$ (e.g. $\sigma = 1.5$ or $1/\rho = 9$).⁸ Besides, in the medium run, which is our horizon of analysis, labor is more mobile and goods are more substitutable, so χ is on the lower end while $1/\epsilon$ is on the higher end of the spectrum, making booms and busts more prone to appear.

Even for conservative calibrations, booms and busts can appear for high enough idiosyncratic noise relative to sectoral noise $\sigma_\lambda/\sigma_\Lambda$. We argue that idiosyncratic noise is large relative to sectoral noise. One argument is based on rational inattention. If ψ_{in} comes from a public signal on ψ , $\psi + \theta$, that is processed with a cost by firms, as in Sims (2003), then the noise should be purely idiosyncratic ($\sigma_\Lambda = 0$). Another argument is based on the high empirical ratio of idiosyncratic to sectoral volatility. Indeed, suppose that ψ_{in} comes from endogenous signals gathered by firms. Then the noise has both an idiosyncratic and a sectoral component, that come from other fundamental idiosyncratic and sectoral shocks. There is evidence that idiosyncratic shocks are at least of one order of magnitude larger than sectoral shocks.⁹

3.2.3 Role of the wage signal

Now we study more specifically the role of the wage signal, by relaxing our ad hoc assumption that $w_{nt} = w_t = -\psi + m_t$ and instead use Equation (14), which gives the structure of the wage signal that is consistent with the parameters. σ affects the informational content of the wage, but it is unclear how. To answer this question, consider Figure 2. Panels (a) and (b) show how $1/\rho$ (keeping ϵ/ρ constant) and σ affect the equilibrium output. While these parameters both increase κ_a and κ_b , only σ affects the wage signal. To understand the specific role of σ through the wage signal, we must focus on the blue curves, along which σ varies, but κ_a and κ_b are kept constant by adjusting $1/\rho$. Neither Q_1 nor Q_2 is affected in a significant way. Given that the monetary noise m makes the wage a poor signal of ψ , the structure of that signal matters little.

Similarly, panels (c) and (d) show how χ and $1/\epsilon$ affect the equilibrium output. While

⁸In the online appendix, we show how Figure 1 is affected by changing other variance parameters: σ_θ , σ_m or $\sigma_\lambda + \sigma_\Lambda$, the total size of non-aggregate noise.

⁹See for instance Mackowiak and Wiederholt (2009).

these parameters have similar effects on κ_b , only χ affects the wage signal. If χ is positive, the wage signal w_{nt} will be affected by the sector-specific noise Λ_n . By focusing on the blue curves, along which χ varies, but κ_b is kept constant by adjusting $1/\epsilon$, we can understand the specific role of χ . As for σ , the equilibrium is not affected significantly.¹⁰

3.2.4 Magnitude of booms and busts

When booms and busts appear, they can be sizeable. For instance, in the benchmark calibration, as a response to $\theta = 10\%$, input increases by 7.5% above the steady state in the first period and falls by 6.5% below the steady state in the second period, which implies a total fall of 14%. Figure 5 illustrates when booms and busts are sizeable. Panels (a) and (b) show how output reacts to $\theta = 10\%$ in the first and second period, for different values of $\sigma_\lambda/\sigma_\Lambda$ and $1/\rho$, holding ϵ/ρ constant. As $1/\rho$ increases, both κ_a and κ_b increase, which makes the mark-up signal react more negatively to θ . Initially, a larger $1/\rho$ decreases the second-period output, but for large values of $1/\rho$, the response of output is mitigated as the mark-up becomes a poorer signal of ψ . Since κ_a is larger than κ_b , the magnitude of booms and busts typically increases with the relative size of idiosyncratic noise.

Figure 5 represents the effect of other variance parameters as well. Consistently with Corollary 1, the lower σ_θ , the larger the boom and the steeper the bust. On the opposite, the larger σ_m , the stronger the boom-bust. In the limit where σ_m goes to infinity, the wage signal conveys no information and the outcomes are those of the version of the model where Assumption 1 holds. On the opposite, when σ_m goes to zero, the wage signal reveals ψ perfectly and no boom-bust appears. Finally, we consider the role of non-aggregate noise $\sigma_\lambda + \sigma_\Lambda$. Interestingly, booms and busts are stronger for intermediate levels. For low non-aggregate noise, firms are more prone to recognize the role of the aggregate noise in driving the negative mark-up. Indeed, in that case, it is unlikely that the initial optimistic signal would be due to noise other than the aggregate. For high non-aggregate noise, firms are less

¹⁰In the online appendix, we show that with a higher σ_m , the role of σ and χ is even more negligible, as the wage becomes an even poorer signal. But with a lower σ_m , σ and χ still have little effect. Indeed, the wage gives then sufficient information for firms not to be confused about θ in the first place. If anything, a higher χ makes booms and busts stronger.

prone to rely on their signal to decide quantities in the first period, so mark-ups react little,
driving only a modest bust.

Discussion We have shown that as σ_θ gets smaller, a fixed-size shock on θ leads to a larger decline in output. But then a typical shock is also smaller. In the online appendix, we show that θ , when scaled by σ_θ , can only generate up to a 0.08-0.09% decline in output. This is 10 times smaller than the scaled effect of ψ , which is close to 1%. While this is not negligible, it suggests that we need exceptionally large shocks to obtain large booms and busts.

This result is obtained under the assumption of a normal distribution of θ . With this type of distribution, the “typical” size of the shock is tightly linked to the informativeness of the mark-up signal, which constrains the size of booms and busts. Alternative distributions could potentially yield larger booms and busts. For instance, if θ had a fixed size but occurred with a probability p , then as p becomes smaller, the mark-up signal would become more reliable, leading to stronger booms and busts. Booms and busts can then be thought of as “Black Swan” types of events. Moreover, firms can have distorted beliefs. In particular, if the aggregate noise shock is believed to be less frequent than it really is, then its contribution to output variations can be much larger. A more complex model can also feature amplification mechanisms, like a financial accelerator, that can worsen the busts.

4 Discussion and extensions

We address several limitations of the benchmark model. First, we discuss which other categories of shocks can generate booms and busts. We then examine alternative information structures. One extension shows how the model can be consistent with credit-fuelled booms and busts. Lastly, we show how our mechanism extends to a more dynamic model. The results are summarized here, but more details can be found in the online appendix.

4.1 Alternative shocks

We extend the model to accommodate other shocks. First, we show that our preference shock can be interpreted as a shock to the productivity of the final good sector. By contrast, productivity shocks in the differentiated goods sector does not generate booms and busts. Booms and busts can thus arise from optimism about productivity if it concerns downstream production (i.e. production that is closer to the final demand). In the online appendix, we additionally show that government spending shocks play the same role as our preference shock, but we do not report the results here.

Shocks to the productivity of the final good sector A final good sector produces the final consumption good by using differentiated intermediate inputs. The intermediate goods sector serves the final good sector, so productivity shocks in the final good sector play the role of demand shocks for the intermediate goods sector. As a result, noisy signals about the final good productivity can generate booms and busts in the intermediate good sector.

Shocks to the productivity of the intermediate goods By contrast, productivity shocks in the intermediate goods sector do not generate booms and busts. We assume that the production of the differentiated goods is affected by a common productivity shock. Firms do not observe this shock, but only a noisy signal of it. One can think of imperfect information within the firm, where the manager receives noisy signals about the firm's productivity. This is not an unrealistic assumption, as marginal costs are notoriously difficult to measure (as opposed to the average cost). Whenever firms receive a positive signal about their productivity, they increase production in the first period, whether the signal is driven by actual productivity or by noise. As a result, both the fundamental and the noise shock generate an increase in supply, which has to be accommodated by a decline in the price level. Therefore, the fundamental and noise shocks affect the endogenous signal of the firms in the same way. A positive noise shock can then be confused only with a positive fundamental shock, and therefore cannot generate booms and busts.

4.2 Alternative information structure

Our model shows that firms must receive a *private* signal with an *aggregate* error for booms and busts to arise. What if firms received other private signals that do not have an aggregate error? Or received other signals with aggregate noise, but that are not private?

We first assume that, in addition to ψ_i , firms observed a purely public signal $\psi_e = \psi + e$ at the beginning of period 1, where $e \sim \mathcal{N}(0, \sigma_e)$. We then assume instead that, in addition to ψ_i , firms received a purely private signal $\psi_{ui} = \psi + u_i$ at the beginning of period 1, where $u_i \sim \mathcal{N}(0, \sigma_u)$. We solve these two cases analytically and show that the main message of the model is not affected. In a nutshell, these signals make the forecasts of ψ more precise, and limit the impact of θ , but do not radically change the effect of θ . In particular, the structure of the filtered mark-up is unchanged and plays the same role as before.

4.3 Credit and endogenous initial signal

We account for the typical surge in credit that characterizes booms and busts. To do so, we consider a small open economy with a fixed world interest rate r , and introduce a traded good X , in fixed supply \bar{X} in the country but in infinite supply from the rest of the world. Households can exchange good X with the rest of the world and save or borrow vis-à-vis the rest of the world. Strategic substitutability still affects the nontradable sector where firms produce differentiated goods as in Section 1. We also endogeneize noise shocks by introducing an initial period 0, where temporary aggregate and idiosyncratic demand shocks can appear. These shocks generate noise because firms cannot distinguish them from the permanent demand shock. An aggregate temporary demand shock will then make households borrow and in the same time mislead firms about the true value of the permanent shock.

4.4 More dynamics

A caveat to our analysis is its static nature. We consider a dynamic extension of the benchmark model in order to study how the boom-bust pattern generalizes to a more stan-

dard dynamic framework. In particular, how long do booms and busts last? Besides, is the dynamic oscillatory, one boom generating a bust, then the bust generating a boom, etc.?

In order to map the model to standard DSGE models, we introduce sticky information à la Mankiw and Reis (2002). We also add noise to the observed mark-up for information not to be trivially revealed after a few periods. In this context, excessive optimism does not generate oscillations. It generates a temporary boom followed by a prolonged recession: optimism is reversed quickly but pessimism is long-lasting. More information frictions (either more sticky information or more noisy mark-ups) make the bust milder but more protracted.

However, booms do not last more than one period. To generate more protracted booms, a more complex dynamic model should introduce delays in capacity building. Search-and-matching frictions for instance, by delaying hiring, can introduce this kind of delays. An extension of the model with capital, where capital needs time to build, could also introduce more delays and hence generate a more protracted boom.

5 Conclusion

One key contribution of this paper is to show that the source of information is important. Future empirical research should then document what variables agents observe and extract information from. Moreover, we show that the way information is dispersed is important to determine the relevant parameter of strategic substitutability or complementarity. In that respect, the importance of sector-specific information as opposed to firm-specific information is a crucial empirical question, as it determines the relevance of trade linkages as opposed to monopolistic competition.

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Table 1: Benchmark calibration for the numerical analysis

Parameter	Value
Preference parameters	
$\sigma = \gamma + \eta$	1
χ	0
$1/\rho$	7
$1/\epsilon$.85
Variance parameters	
σ_ψ	0.01
σ_θ	0.001
σ_Λ	0.001
σ_λ	0.01
σ_m	0.01

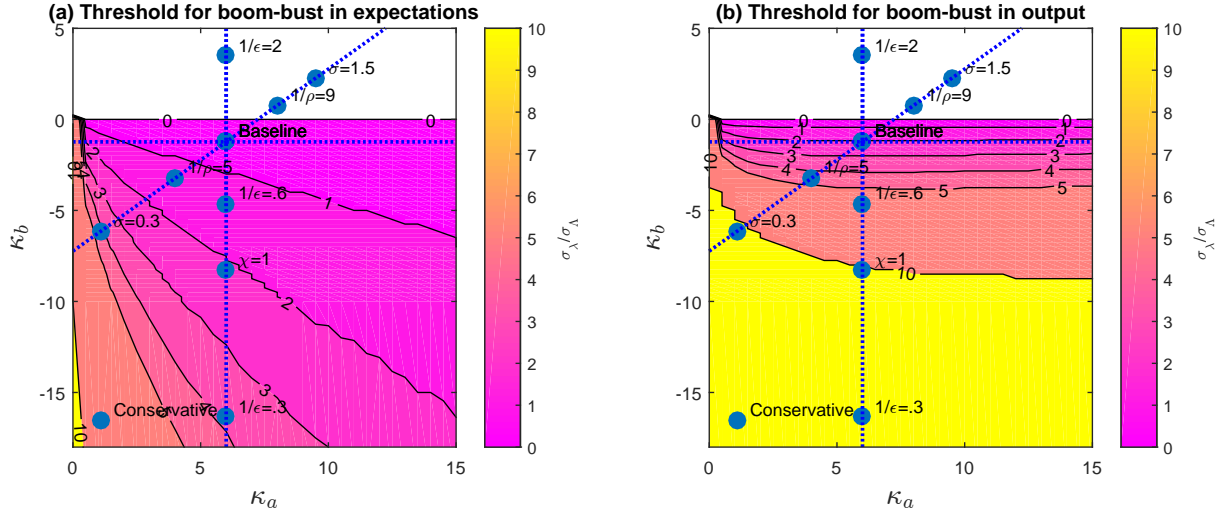


Figure 1: Thresholds for boom-bust in expectations and output

Each curve represents threshold values for κ_b for a given ratio of $\sigma_\lambda/\sigma_\Lambda$. In panel (a), points that are at the north-east of the curves feature booms and busts in expectations. In panel (b), points that are at the north-east of the curves feature booms and busts in output. σ_λ and σ_Λ are set so that $\sigma_\Lambda + \sigma_\lambda = 0.011$, which is the value in the baseline calibration. σ_ψ , σ_θ and σ_m are set as in the benchmark calibration (see Table 1).

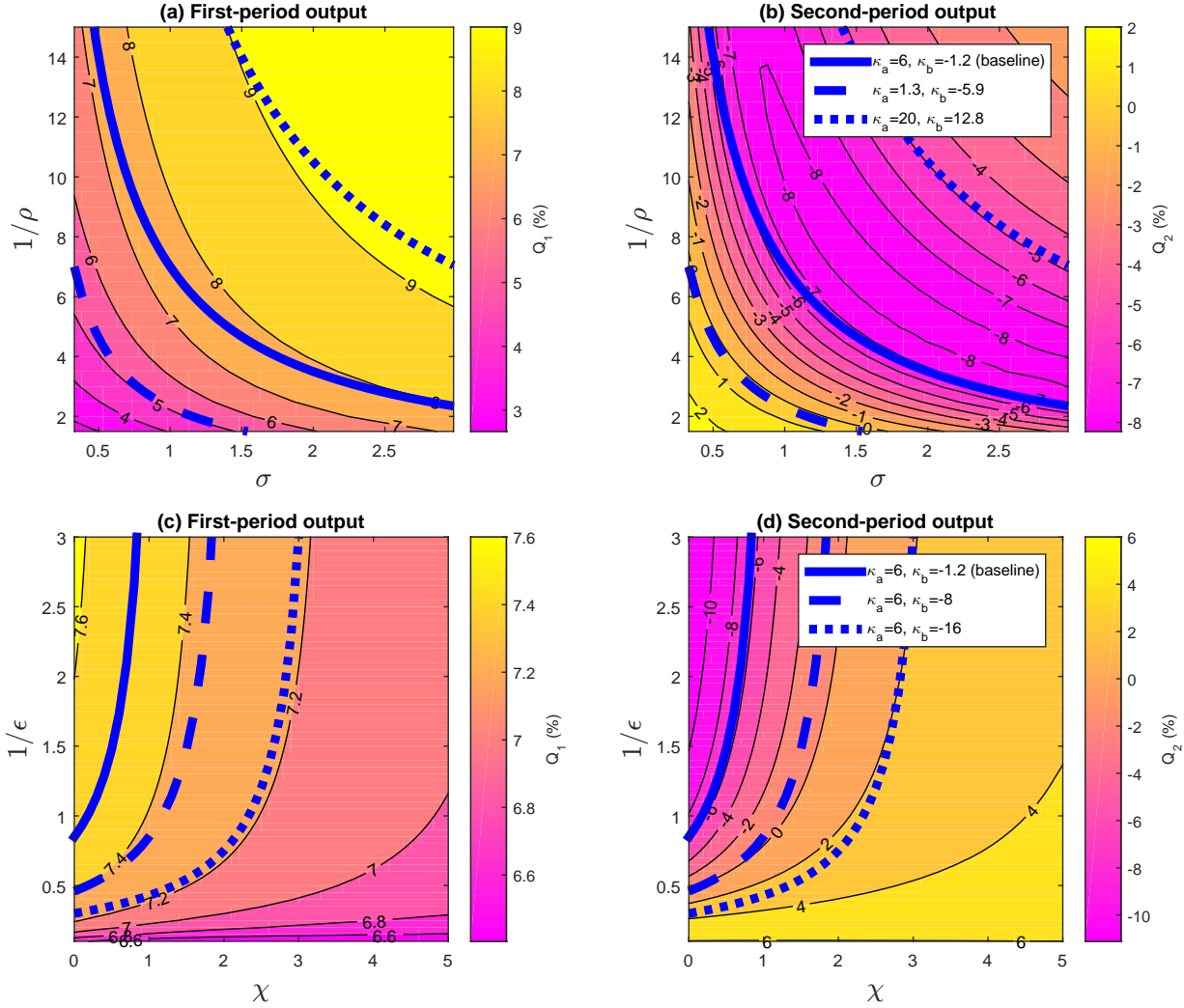


Figure 2: Role of the wage signal

We represent the effects of $\theta = 0.1$. σ_λ , σ_Λ , σ_ψ , σ_θ and σ_m are set as in the benchmark calibration (see Table 1). In panels (a) and (b), χ and ρ/ϵ are set as in the benchmark calibration ($1/\epsilon$ varies proportionally to $1/\rho$ so that ϵ/ρ is kept constant at $7/.85$). In panels (c) and (d), σ and $1/\rho$ are set as in the benchmark calibration.

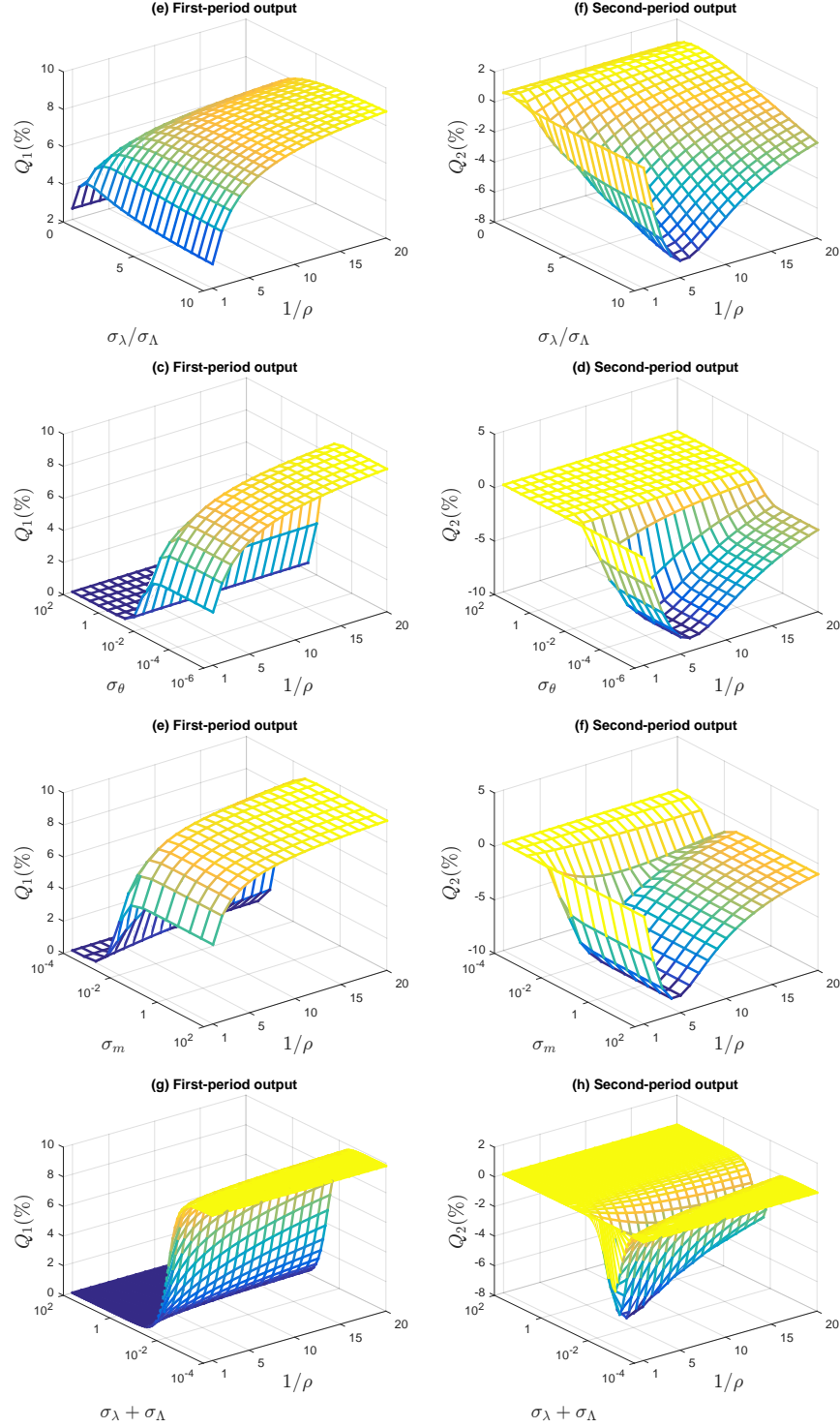


Figure 3: Role of variance parameters

We represent the effects of $\theta = 0.1$. Other coefficients are set as in the benchmark calibration (see Table 1). In all panels, $1/\epsilon$ varies proportionally to $1/\rho$ so that ϵ/ρ is kept constant at $7/85$ as in the benchmark calibration. In panels (a) and (b), σ_λ and σ_Λ are set so that $\sigma_\Lambda + \sigma_\lambda = 0.011$, which is the value in the benchmark calibration. In panels (g) and (h), σ_λ and σ_Λ are set so that $\sigma_\lambda/\sigma_\Lambda = 10$ as in the benchmark calibration.