Learning from Prices: Amplification and Business Fluctuations*

Ryan Chahrour†
Boston College

Gaetano Gaballo‡
Banque de France
Paris School of Economics
CEPR

March 29, 2018

Abstract

We provide a new theory of expectations-driven business cycles based on learning from prices. Upon observing a price change, households are unsure about its cause: a price increase caused by lower productivity may be misinterpreted as an improvement in local conditions, leading households to consume more. As a result, unobserved productivity shocks generate positive price-quantity comovement. The feedback of beliefs into prices can be so strong that even arbitrarily-small surprises in productivity lead to substantial fluctuations. Augmented with public information, the model generates a rich mix of supply- and demand-driven fluctuations, even though productivity is the only source of aggregate randomness. Our model qualitatively matches several business cycle facts that are a challenge for more sophisticated DSGE models.

Keywords: expectations, animal spirits, business cycles, incomplete information.

JEL Classification: D82, D83, E3.

---

*The authors thank Jess Benhabib, Mehmet Ekmekci, Roger Guesnerie, Christian Hellwig, Jianjun Miao, Patrick Pintus, Kristoffer Nimark, Richard Tresch, Robert Ulbricht, Rosen Valchev, Laura Veldkamp, Venky Venkateswaran, Xavier Vives, and seminar participants at the Toulouse School of Economics, Cornell University, Barcelona GSE Summer Forum, Society for Economic Dynamics, and Boston Green Line Macro conferences for valuable suggestions and comments. Laura Veronika Gáti provided excellent research assistance. The research leading to these results has received financial support from the European Research Council under the European Community's Seventh Framework Program FP7/2007-2013 grant agreement No.263790. An earlier version of this paper circulated under the title "On the Nature and Stability of Sentiments". The views expressed in this paper do not necessarily reflect the opinion of the Banque de France or the Eurosystem.

†Department of Economics, Boston College, Chestnut Hill, MA 02467, U.S.A. Email: ryan.chahrour@bc.edu

‡Monetary Policy Division, Banque de France, 31 rue Croix des Petits Champs, 75005 Paris. Email: gaetano.gaballo@banque-france.fr

1 Introduction

We propose a new mechanism—based on learning from prices—that delivers expectations-driven economic fluctuations without relying on any source of extrinsic noise. We show that when households learn from the prices of the goods they consume, higher prices can lead households to become unduly optimistic about their economic prospects. Initial optimism causes households to demand more goods, further increasing prices beyond their full-information level. The self-reinforcing nature of this feedback loop leads to equilibria in which even small shocks to supply may drive large changes in beliefs, inducing the type of aggregate comovement typically associated with demand shocks. We develop our learning from prices mechanism within a stylized macroeconomic model, and show that it has several promising features for explaining business cycles.

Our model economy consists of a continuum of islands, each inhabited by competitive households and producers. Producers employ local labor and a homogenous global factor—capital—to produce a local consumption good using a decreasing returns technology. Households buy the local consumption good, supply labor elastically, and save in money. Islands are informationally isolated, connected only by frictionless markets for capital and money.

There are two sources of randomness in the economy: one local and one global. The local disturbance shifts the velocity of money within an island, and has the effect of an island-specific wealth shock. The global shock drives aggregate variation in the productivity of producers. The equilibrium price for each local good thus reflects (i) local demand conditions, because of decreasing returns; (ii) aggregate productivity, because productivity shocks affect production opportunities; and (iii) the aggregate price of capital, because capital is an input in production.

The key friction in our environment is that only a fraction of households observe the local shock when they choose consumption; the remaining households must infer local conditions from the prices they see on the market. These uninformed households are uncertain whether a rising price indicates improving local conditions or falling aggregate productivity. They therefore attribute a part of of every observed price change to local conditions. Because of this, a price increase driven by lower aggregate productivity is interpreted on every island as a positive local shock, increasing the demand for each island's local good. Higher total demand for final goods, however, leads to higher demand for the global capital good, raising its price, which is reflected in yet higher final good prices. When this feedback is strong enough, higher prices spur aggregate demand, amplifying capital price volatility and making households' equilibrium inference worse.

The microfoundation of our signal structure as a *price* is crucial to our mechanism. First, the fact that information comes from market prices, rather than from exogenously specified signals, allows aggregate demand to be upward sloping in the aggregate price level, thereby engendering positive price-quantity comovement. Second, the feedback of the aggregate capital price into final good prices allows an initial impulse, such as a small surprise[1] in aggregate productivity, to be reinforced and to ultimately produce large fluctuations in beliefs.

When the feedback of actions into beliefs is strong enough, the economy exhibits sizable aggregate fluctuations, even in the limit of arbitrarily small aggregate productivity shocks. Fluctuations occur in the limit because, as aggregate shocks decrease in variance, local price signals better reflect local conditions, increasing the weight that households place on their price observations. Approaching the limit of no aggregate shocks, inference weights can increase fast enough to fully offset the falling variance of aggregate shocks, leading to a positive finite variance of beliefs in the limit.

To an econometrician, the fluctuations emerging at the limit of no aggregate shocks would appear to be driven by something akin to "sentiment." Indeed, at that limit, our equilibria have the same stochastic properties as the sentiment equilibria characterized in Benhabib et al. (2015). Yet, they are not sentiment equilibria in the sense intended by those authors. Instead, in our model, belief fluctuations emerge as a case of extreme sensitivity to fundamental shocks: whether agents are optimistic or pessimistic is entirely pinned down by fundamentals and, we show, *cannot* be influenced by extrinsic noise. From a theoretical point of view, this result demonstrates that the equilibria in Benhabib et al. (2015) are fragile to the introduction of an exogenous aggregate component in market signals.

While the limiting cases are of theoretical importance, we also demonstrate that the learning from prices mechanism can be of practical importance for understanding business cycles.

[1] We use the word "surprise" to mean "an unanticipated shock".

To do this, we first show that the model delivers fluctuations with Keynesian features under far broader conditions than those needed for the limit results. In particular, it yields positive price-quantity comovement whenever there are enough uninformed agents and aggregate productivity shocks are sufficiently small, restrictions encompassing cases with both equilibrium multiplicity and uniqueness. Constraining ourselves to work within the parameter region where a unique equilibrium exists, we then explore the potential of our model to explain business cycle facts.

For matching qualitative features of the data, one challenge is that any model driven by a single productivity shock must imply perfect correlation between prices, quantities, and productivity, while the data support only weak correlation. To better align model predictions with observed business cycles, we allow a portion of productivity to be anticipated by agents in the form of public news. More public information might be expected to mitigate the expectational errors of agents, dampening demand-side effects. On the contrary, a smaller contribution of surprise productivity to the price signal leads agents to place more weight on prices when forming their inference. *Smaller* surprise shocks to productivity thus drive *stronger* price-quantity comovement, while anticipated productivity shocks lead to standard supply-driven comovements.

With distinct transmission mechanisms for the two components of productivity, our model generates a rich mix of supply- and demand-driven fluctuations, even though productivity is the only source of aggregate randomness. This version of the model can match the qualitative pattern of conditional and unconditional moments seen in the data, including modest price-quantity comovements, contractionary labor responses to positive technology shocks, and a low correlation of total productivity with output and inflation. Without learning from prices, productivity shocks would move aggregate prices and quantities in opposite directions, so that matching these facts would require some combination of price-setting frictions, aggregate demand shocks, or exogenous coordination of beliefs on an extrinsic shock. Thus, although too stylized for a full-fledged quantitative analysis, our model qualitatively matches several basic facts that are a challenge for more sophisticated DSGE models.

We conclude with several extensions that demonstrate the robustness of the basic insight. First, we show that our analysis easily generalizes to the introduction of noisy private signals about local conditions. Second, we present an alternative economy with different foundations for the labor market, and show that it delivers the same qualitative results as the baseline economy. Third, we show that while higher prices do indeed spur total demand, the model need not imply the existence of a positive price-quantity relationship at the good level. Finally, we show the equilibria we emphasize are stable in the sense of being locally-learnable.

Connection with literature. This paper is the first to demonstrate that learning from prices might play a central role in explaining business cycle comovements. Nevertheless, endogenous signal structures have previously appeared in macroeconomic contexts, starting with Lucas (1972). More recent examples include Amador and Weill (2010) and Venkateswaran (2013). Gaballo (2017) is also predicated on a learning through prices mechanism, but his results are both formally and qualitatively different. Formally, his paper shows how small dispersion of fundamentals may generate under-reaction of beliefs to aggregate shocks, whereas here we focus on the amplification of small aggregate shocks. In his application, Gaballo (2017) uses the endogenous information rigidity to explain aggregate price rigidity; here, we use endogenous informational amplification to explain business cycle fluctuations.

Agents make correlated errors in our economy, a theme of the recent noise-shock and sentiment literatures (Lorenzoni, 2009; Angeletos and La'O, 2013; Benhabib et al., 2015). Our approach here is distinct because (i) signals have market microfoundations (ii) all shocks are fundamental, and (iii) there is no role for extrinsic noise. Moreover, the inference problem solved by agents in our economy is entirely static, with different transmission for observed and unobserved *current* productivity shocks. In contrast, many papers in literature on news and noise (Barsky et al., 2015; Chahrour and Jurado, 2017a) focus on the impact of anticipated *future* productivity shocks. Our paper also answers a criticism of that literature, emphasized by Angeletos et al. (2014) and Angeletos et al. (2016), that productivity is only weakly correlated with business cycle variables at all horizons. Our model is consistent with this observation because the unanticipated and surprise components of productivity are transmitted very differently, leading to weak correlation between total productivity and other variables.

Our focus on the informative role of prices echoes a long tradition in finance, starting with Grossman and Stiglitz (1976, 1980). The potential of this mechanism to deliver price amplification and/or multiple equilibria has been documented by many authors, including

Burguet and Vives (2000) and Barlevy and Veronesi (2000) and more recently by Albagli et al. (2014), Manzano and Vives (2011), and Vives (2014).[2] Unlike these papers, which usually include noise traders or exogenous shocks to information, every shock in our model is fundamental. And, we are the first to show the potential for extreme amplification in limit cases.

Amplification is also a common theme in macroeconomic theory: there is often too little variation in observed fundamentals for standard models to justify the size of observed business cycles. Historically, amplification has often been introduced via strong complementarities, notably through production externalities that, in extreme cases, can support sunspot fluctuations (see Azariadis, 1981; Cass and Shell, 1983; Cooper and John, 1988; Manuelli and Peck, 1992; and Benhabib and Farmer, 1994, among others). Recent literature has proposed several financial frictions to help account for the large macroeconomic effects of modest shocks (Kiyotaki and Moore, 1997; Bernanke et al., 1999; Brunnermeier and Sannikov, 2014). Many of these mechanisms share with our paper the potential to generate an upward sloping demand curve *in some market.* Nonetheless, it is often difficult to find empirical support for calibrations of these models that deliver the strongest amplification (Basu and Fernald, 1997; Dmitriev and Hoddenbagh, 2017). In our model, upward sloping aggregate demand and strong amplification arise for very different reasons and under a broad range of parameters.

Finally, recent work by Bergemann and Morris (2013) characterizes the full set of incomplete-information equilibria in a large class of coordination games. Related work by Bergemann et al. (2015) studies the exogenous information structures that give rise to maximal aggregate volatility, and the extrema they find are typically achieved when the price signal delivers sentiment-like fluctuations in our economy. A final implication of our analysis is that the addition of a small amount of *aggregate* noise in the signal—in this case, captured by the effect of productivity on the price signal—can sustain additional equilibria that do not arise under full information. A previous literature has demonstrated cases in which adding *idiosyncratic* noise to signals can either eliminate (Morris and Shin, 1998) or generate (Gaballo, 2017) additional equilibria.

[2]The literature on price revelation in auction markets following Milgrom (1981) also features a dual informational/allocative role for prices. For recent examples, see Rostek and Weretka (2012); Lauermann et al. (2012); Atakan and Ekmekci (2014).

2 A microfounded model

In this section, we present an RBC model with the aim of providing a simple and transparent intuition for our main mechanism. Our economy gives full microfoundations for the information structure that generates imperfect learning. In particular, all shocks are fundamental in nature and all signals are derived as endogenous outcomes of competitive markets.

2.1 Preferences and technology

The economy consists of a continuum of islands, indexed by $i \in [0,1]$, each inhabited by a continuum of price-taking households and producers. Producers on island i produce a local consumption variety by employing local labor and a globally-traded productive input, capital. Households, in turn, make saving/consumption choices and supply labor in local markets.

A household $j \in [0,1]$ living on island i enjoys utility,

$$\sum_{t=0}^{\infty} \beta^t \big(\log C_{ij,t} - V(N_{ij,t}) + \chi e^{-\mu_i - \sigma_\mu^2/2} M_{ij,t} \big), \tag{1}$$

and faces the budget constraint,

$$M_{ij,t} + P_{i,t} C_{ij,t} = Q_t Z + W_{i,t} N_{ij,t} + \Pi_{i,t} + M_{ij,t-1}. \tag{2}$$

In the above problem, $C_{ij,t}$ denotes household (i,j)'s consumption of the local consumption good i, which is purchased at price $P_{i,t}$. Household (i,j)'s supply of labor is given by $N_{ij,t}$ and is remunerated at local wage rate $W_{i,t}$. For future reference, we define island level consumption, $C_{i,t} \equiv \int C_{ij,t} dj$, and labor, $N_{i,t} \equiv \int N_{ij,t} dj$. The function $V(\cdot)$ governs the disutility of labor. $\Pi_{i,t}$ captures any profits earned by firms located on island i.

The household is endowed each period with a fixed quantity of a capital good, Z.[3] The good trades freely across islands at a common price Q_t and depreciates fully at the end of the period. Finally, households choose each period their money holding, $M_{ij,t}$, which yields stochastic utility gains in the form of services from money.[4] These gains are i.i.d. across islands

[3]While we call the fixed input good capital, our mechanism will work as long as there is a common input whose aggregate supply is imperfectly elastic within the period. Nothing qualitative would change if capital depreciated, and agents could invest to produce new capital.

[4]The money in the utility function term can also be interpreted as effort spent shopping. For example, see

and (for simplicity) are assumed to be permanent across time and are distributed according to $\mu_i \sim N(0, \sigma_\mu^2)$. The presence of money in the utility function greatly simplifies our analysis (even considering an arbitrarily small χ, see later discussion) as variation in utility gains exogenously pin down fluctuations in in future wealth prospects. The money trades freely across islands and its aggregate supply is fixed, so that $\int M_{ij,t} d(i,j) = M$ in every period.

Each island is also inhabited by a representative firm that maximizes profits,

$$\Pi_{i,t} = P_{i,t}C_{i,t} - W_{i.t}N_{i,t} - Q_t Z_{i,t}, \tag{3}$$

by choosing $N_{i,t}$ and $Z_{i,t}$, taking prices as given.[5] The capital good is combined with island-specific labor to produce the final good, $C_{i,t}$, according to the technology,

$$C_{i,t} = \left(N_{i,t}^{\phi} \left(e^{\tilde{\zeta}} Z_{i,t} \right)^{1-\phi} \right)^{\alpha}, \tag{4}$$

where $\phi \in (0,1)$ is the labor share of revenue excluding profits in the economy, $\alpha \in (0,1)$ measures the return to scale in production, and aggregate productivity $\tilde{\zeta}$ is drawn by nature from a distribution $\tilde{\zeta} \sim N(0, \sigma_{\tilde{\zeta}}^2)$.

Period $t = 0$ is divided into three stages.

1. In the first stage, before any shock realizes, households fix their wage for the coming period.

2. In the second stage, shocks realize and a fraction $\kappa \in (0,1)$ of households on each island receives perfect information about their local shock, μ_i.

3. In the third stage, households make consumption choices based on their private information about μ_i and the observed market price of their local consumption goods; firms produce contingent on realized productivity and the prices of their input and output goods; and all markets clear.

Let us comment on two important assumptions that we make. First, our assumption that a fraction of households knows local fundamentals is reminiscent of Grossman and Stiglitz

Amador and Weill (2010) (in particular, footnote 7, page 871).

[5] Note that capital demanded by firms on island i, $Z_{i,t}$, need not be equal to Z, the quantity of capital supplied by households from the same island.

(1980). This assumption, together with decreasing returns to scale (see later discussion), ensures that consumption prices contain local information. The fraction κ of informed consumer is crucial because, as emphasized by Hellwig (1980), prices cannot reveal information unless that information is already available, perhaps noisily, to some agents in the economy.

Second, our assumption of wage rigidity ensures that a higher desire to consume today translates into higher actual consumption. This could not be true if, instead, wages perfectly comoved with consumers' marginal utility, in which case increased desire to consume would be perfectly offset by a rising price for the consumption good. Wage rigidity is not necessary for this result, however: what is crucial is that the first order conditions for consumption and labor supply hold with different information sets.

In Section 5.2, we relax this informational wage rigidity and show that our mechanism works in the same way. In that setting, families are divided into workers and consumers, *à la* Lucas (1980), which enables us to isolate the role information plays in each market. In the appendix, we assume that (only) labor market choices are made under perfect information, implying local prices are informative about local conditions even when $\kappa = 0$. The microfoundation in Chahrour and Gaballo (2017) achieves similar results under flexible prices.

Subsequent periods unfold exactly as period $t = 0$, but since all uncertainty is resolved by the end of the initial period, equilibrium prices and quantities are constant from period $t = 1$ onwards. In particular, to make the household inference problem non-trivial, we assume that it does not observe in real-time the realized value of their resources – labor earnings plus profits plus the price of capital – when making consumption choices.[6] This assumption prevents households from using information about the price of capital, which would reveal aggregate productivity. The price of capital is instead revealed at the end of the period, so that households enter subsequent periods without any uncertainty about aggregate productivity.

The formal definition of equilibrium is given by the following.

Definition 1. *For a given realization of* $\{\mu_i\}_0^1$ *and* $\tilde{\zeta}$*, a rational expectations equilibrium is a collection of prices* $\{P_{i,t}, W_{i,t}, Q_t\}$ *and quantities* $\{W_{ij,t}, N_{ij,t}, C_{ij,t}, Z_{ij,t}\}$ *for each* i, j *and* t *such that household and firms choices are optimal given the prices they observe, and all markets clear.*

[6] As we will see, optimality only requires knowing the steady state value of the resources and the realization of the local shock.

In later sections, we generalize our baseline structure. In Section 4, we study the case where households also receive public information about productivity. This extension proves important in reproducing several salient facts about the business cycle. In Section 5, we demonstrate the robustness of the mechanism by exploring several extensions of the baseline model.[7]

2.2 Equilibrium with learning from prices

Given an initial fixed period-0 wage, the first order conditions of the household and firm are:

$$
\begin{aligned}
E\left[C_{ij,t}^{-1}|\mathfrak{I}_{ij,0}\right] &= E\left[\Lambda_{ij,t}P_{i,t}|\mathfrak{I}_{ij,0}\right], && (5)\\
E[\Lambda_{ij,t}|\mathfrak{I}_{ij,0}], &= E[\beta\Lambda_{ij,t+1} + \chi e^{-\mu_i-\sigma_\mu^2/2}|\mathfrak{I}_{ij,0}], && (6)\\
Q_t &= \alpha(1-\phi)P_{i,t}N_{i,t}^{\phi}Z_{i,t}^{\alpha(1-\phi)-1}e^{\tilde{\zeta}\alpha(1-\phi)}, && (7)\\
W_{i,t} &= \phi\alpha P_{i,t}N_{i,t}^{\alpha\phi-1}(e^{\tilde{\zeta}}Z_{i,t})^{\alpha(1-\phi)}. && (8)
\end{aligned}
$$

where $\mathfrak{I}_{ij,0}$ is the information set of household (i,j), $\Lambda_{ij,t}$ is the Lagrange multiplier on the budget constraint in equation (2), and $W_{i,0}$ is fixed at its steady state $V'(\bar{N}_{i,t})$ where $\bar{N}_{i,t}$ is steady state working hours. In particular, $\mathfrak{I}_{ij,0} = \{P_{i,0}\}$ if household (i,j) if of the uninformed type and $\mathfrak{I}_{ij,0} = \{\mu_i, P_{i,0}\}$ otherwise. It is useful to remark right away that, since each household faces only two types of shocks, access to the second, larger, information set will aways allow agents to take the full-information optimal action.

By iterating forward (6) we find

$$\Lambda_{ij,0} = E[e^{-\mu_i}|\mathfrak{I}_{ij}]e^{-\sigma_\mu^2/2}\frac{\beta}{1-\beta}\chi, \tag{9}$$

[7] Earlier drafts of this paper showed that our mechanism could also arise on the supply side of the economy, more like Lucas (1972). In that version, we assume firms are uncertain about the value of an intermediate input, which is produced with local and global factors. This lead demand-driven fluctuations to occur in the market for intermediate inputs rather than in the final market. Our choice to place the main friction on the side households is consistent, however, with the recent evidence of Chahrour and Ulbricht (2017) that information frictions on the part of households, rather than firms, are needed to match aggregate data.

which can be written as an exact log-linear relationship,[8]

$$\lambda_{ij,0} = -E[\mu_i | \mathfrak{I}_{ij}]. \tag{10}$$

Thus, the local shock μ_i creates cross-sectional heterogeneity in the opportunity cost of consumption. Notice that if the μ_i were not permanent, then equation (10) would still follow as the log-linear *approximation* of the corresponding version of (9), without affecting any of our results. In the passage from (5) to (9), it is now clear that the main role of money in the utility function is to exogenously pin down future Lagrangian multipliers, which helps to simplify our analysis. The parameter χ can be made arbitrarily small without changing anything that follows.

We focus our analysis on the inference of agents in the initial period, suppressing time subscripts in what follows. The remaining equilibrium conditions of the economy can be also written exactly in terms of log-deviations from the stochastic steady state. These conditions are given by:

$$c_i = \mu_i^e - p_i \tag{11}$$

$$w_i = p_i - (1 - \alpha\phi)n_i + \alpha(1 - \phi)(z_i + \tilde{\zeta}) \tag{12}$$

$$q = p_i + \alpha\phi n_i + (\alpha(1 - \phi) - 1)z_i + \alpha(1 - \phi)\tilde{\zeta} \tag{13}$$

$$c_i = \alpha\phi n_i + \alpha(1 - \phi)(z_i + \tilde{\zeta}) \tag{14}$$

where $\mu_i^e \equiv \int E[\mu_i | \mathfrak{I}_{ij}]dj = \kappa\mu_i + (1 - \kappa)E[\mu_i | p_i]$ is the average within-island expectation of the local shock and $w_i = 0$ because of our timing assumption. For given realizations of the shocks and the distribution of household expectations, the system of equations above pins down equilibrium allocations and prices. A rational expectations equilibrium is therefore characterized by allocations and prices that jointly satisfy (11) through (14) and optimality in households' expectation formation.

In the appendix, we solve the linear system (11)-(14). The equilibrium price of local consumption is a function of local demand conditions and the cost of the capital, adjusted for

[8] We denote $x \equiv \log(X/\bar{X})$ for any level variable X whose stochastic steady state (which includes deterministic Jansen terms) is denoted by $\bar{X}$. For future reference, define also $x \equiv \int x_i di$ as the aggregate analogue of any idiosyncratic variable x_i.

productivity:

$$p_i = (1-\alpha)\mu_i^e + \alpha(1-\phi)(q - \tilde{\zeta}). \tag{15}$$

Notice that local conditions are reflected in the local price only to the extent that the production function has exhibits decreasing returns.

Crucially, the price signal captured in equation (15) depends on the price of capital, q, which in turn depends on average demand conditions in the economy. Specifically, we show in the appendix that

$$q = \bar{\mu} \tag{16}$$

where $\bar{\mu} \equiv \int \mu_i^e di = (1-\kappa)\int E[\mu_i|p_i]di$ is the average belief across all agents regarding their own μ_i. After rescaling and removing terms known to all agents on island i, the price signal in (15) is informationally equivalent to

$$s_i = \gamma\mu_i + (1-\gamma)\left(\int E[\mu_i|s_i]di - \zeta\right). \tag{17}$$

where

$$\gamma \equiv \frac{(1-\alpha)\kappa}{(1-\alpha)\kappa + \alpha(1-\phi)(1-\kappa)}, \tag{18}$$

and $\zeta \equiv \tilde{\zeta}/(1-\kappa)$ is distributed according to $N(0, \sigma_\zeta^2)$ with $\sigma_\zeta^2 \equiv \sigma_{\tilde{\zeta}}^2/(1-\kappa)^2$.

The signal structure implied by equation (17) captures the endogenous feedback effect of inference *from* prices back *into* prices, and it is on this structure that we focus our subsequent analysis. Before proceeding to an analytical characterization, it is helpful to spell out the economic intuition behind the inference problem being solved by households. From equation (17), it is clear that an increase in price can be triggered by local factors—that is, by an increase in μ_i—in which case the household desires to increase their consumption of the local variety. Yet, the same increase in price could be driven by aggregate factors, either an increase in the price of capital or a decrease in aggregate productivity, that are not related to local conditions, in which case the household wishes to reduce consumption.

In this context, a household's optimal response to a price change depends on the reason that the price has changed. Yet, uninformed households cannot directly observe why prices are changing, and they attribute a part of of every observed price change to local conditions. Because of this, a price increase driven by a fall in aggregate productivity is interpreted *on every island* as a positive local shock. This common mistake triggers an increase ceteris paribus in demand for each island's local good. Higher total demand for final goods, however, leads to higher demand for the inelastically supplied capital good, raising its price, which is reflected in yet higher final good prices. If this feedback is strong enough, higher prices can spur aggregate demand, thereby amplifying capital price volatility and making households' equilibrium inference worse.

3 Amplification through learning

In this section, we analyze the signal extraction problem created by the information structure microfounded above, and summarized by equation (17). We show how to solve the households' inference problem, highlighting the strategic interaction engendered by the endogeneity of the price signal. In particular, we demonstrate that informational feedback can generate amplification of fundamental shocks, which in some cases is strong enough to deliver non-trivial responses to vanishingly small shocks. We also show that in our model Sentiment equilibria *à la* Benhabib et al. (2015) do not exist, i.e. fluctuations in average expectation are uniquely determined by aggregate productivity. The focus in this section is on the inference of uninformed households. To keep things self-contained, references to average expectations in this section concern the average expectation *among the uninformed.*

Best individual weight function

The key feature of the signal extraction problem is that the precision of the signal depends on the nature of average actions across the population and, therefore, on the average reaction of other households to their own price signals. A rational expectations equilibrium is a situation in which the individual reaction to the signal is consistent with its actual precision, i.e., is an optimal response to the average reaction of others.

Since we assume that all stochastic elements are normal, the optimal forecasting strategy is linear. As a consequence, the individual expectation is linear in s_i and can be written as

$$E[\mu_i|s_i] = a_i\left(\gamma\mu_i + (1-\gamma)\left(\int E[\mu_i|s_i]di - \zeta\right)\right), \tag{19}$$

where a_i is the coefficient, determined prior to the realization of shocks, that measures the strength of the reaction of household i's beliefs to the signal she will receive. Since the signal is *ex ante* identical for all households, each uses a similar strategy, and we can recover the average expectation by integrating across the population:

$$\int E[\mu_i|s_i]di = a\,(1-\gamma)\left(\int E[\mu_i|s_i]di - \zeta\right), \tag{20}$$

with $a \equiv \int a_i di$ denoting the average weight applied to the signal. Solving the expression above for the average expectation yields

$$\int E[\mu_i|s_i]di = -\frac{a\,(1-\gamma)}{1-a\,(1-\gamma)}\zeta, \tag{21}$$

which is a nonlinear function of the average weight, a. Importantly, this function features a singularity at the point $1/(1-\gamma)$. When $a < 1/(1-\gamma)$, the average expectation comoves with the productivity shock and the opposite holds when $a > 1/(1-\gamma)$.

The variance of the average expectation is given by

$$\sigma_E^2(a) = \left(\frac{a\,(1-\gamma)}{1-a\,(1-\gamma)}\right)^2\sigma^2, \tag{22}$$

where $\sigma_E^2 \equiv var(\int E[\mu_i|s_i]di)/\sigma_\mu^2$ and $\sigma^2 \equiv \sigma_\zeta^2/\sigma_\mu^2$ are the variances of the average expectation and the aggregate shock, respectively, once each is normalized by the variance of the idiosyncratic fundamental.

Substituting the average expectation in (21) into the price signal described in equation (17), we get an expression for the local signal exclusively in terms of exogenous shocks:

$$s_i = \gamma\mu_i + \frac{\gamma-1}{1-a\,(1-\gamma)}\zeta, \tag{23}$$

whose precision with regard to μ_i is given by

$$\tau(a) = \left(\frac{\gamma\,(1-a\,(1-\gamma))}{(1-\gamma)\sigma}\right)^2. \tag{24}$$

We are now ready to compute the household's optimal inference, taking the average weight of other households as given. We seek an a_i such that $\mathrm{E}[s_i(\mu_i - a_i s_i)] = 0$, i.e., the covariance between the signal and forecast error is zero in expectation. This condition implies that information is used optimally. The best individual weight is given by

$$a_i(a) = \frac{1}{\gamma}\left(\frac{\tau(a)}{1+\tau(a)}\right). \tag{25}$$

Given the linear-quadratic environment, we can interpret $a_i(a)$ in a game-theoretic fashion as an individual's best reply to the profile of others' actions summarized by the sufficient statistic a. To be precise, every a_i is associated with one and only one contingent strategy that describes the conditional expectation $E[\mu_i|s_i] = a_i s_i$ of household i, where s_i identifies a set of states of the world indistinguishable to household i.

Equilibria

Given that agents face an information structure with the same stochastic properties, a rational expectations equilibrium must be symmetric. This last requirement completes our notion of equilibrium, which is formally stated below.

Definition 2. *A rational expectations equilibrium is characterized by a profile of households' expectations* $\{E[\mu_i|s_i]\}$ *such that* $E[\mu_i|s_i] = \hat{a}s_i$ *with* $a_i(\hat{a}) = \hat{a}$*, for each* $i \in (0,1)$*.*

Our game-theoretic interpretation of the optimal coefficient makes clear the equivalence between a rational expectations equilibrium and a Nash equilibrium: No one has any individual incentive to deviate when everybody else conforms to the equilibrium prescriptions.

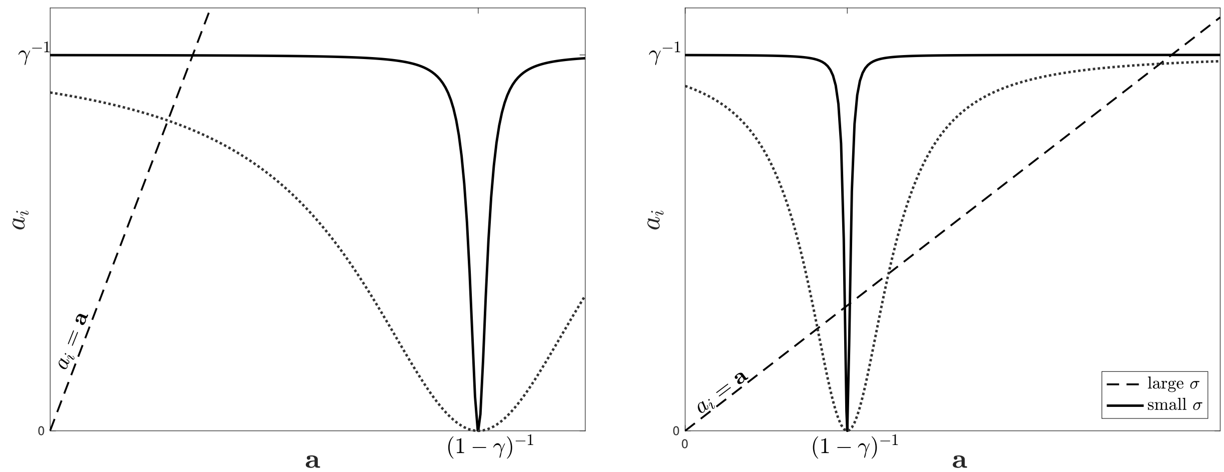
An equilibrium of the model is a fixed point of the individual best-weight mapping given by equation (25). In practice, there are as many equilibria as intersections between $a_i(a)$ and the bisector. The fixed-point relation delivers a cubic equation, which may have one or three real roots. The following proposition characterizes these equilibrium points.

Proposition 1. *For* $\gamma \geq 1/2$*, there always exists a unique REE equilibrium for* $\hat{a} = a_u \in (0, \gamma^{-1})$*.*

For $\gamma < 1/2$*, there always exists a low REE equilibrium for* $\hat{a} = a_- \in (0, (1-\gamma)^{-1})$*. In addition, there exists a threshold* $\bar{\sigma}^2$ *such that, for any* $\sigma^2 \in (0, \bar{\sigma}^2)$*, a middle and a high REE equilibrium also exist for* $\hat{a} = a_\circ$ *and* $\hat{a} = a_+$*, respectively, both lying in the range* $((1-\gamma)^{-1}, \gamma^{-1})$*.*

Proof. Given in Appendix A2. ∎

Proposition 1 states that when the aggregate component receives relatively high weight in the signal, the model may exhibit multiplicity. In particular, there are three equilibria



(a) $\gamma = 0.75$

(b) $\gamma = 0.25$

Figure 1: The figure illustrates four properties of $a_i(a)$ for given γ and σ: (i) $a_i(0) > 0$; (ii) $a_i'(a) < 0$ for $a \in (0, (1-\gamma)^{-1})$, and $a_i((1-\gamma)^{-1}) = 0$; (iii) $a_i'(a) > 0$ for for $a \in ((1-\gamma)^{-1}, \gamma^{-1})$ and $\lim_{a\to\infty} = \gamma^{-1}$; (iv) $\partial a_i(a)/\partial\sigma \geq 0$.

whenever $\gamma < 1/2$ and the variance of the productivity shock is small enough; otherwise, a unique equilibrium exists. While an analytical characterization of these equilibria is possible, the expressions are rather complicated. Nevertheless, the relevant properties can be grasped from the reaction functions plotted in Figure 1 (see figure caption).

The slope of the $a_i(a)$ curve at the intersection with the bisector determines the nature of the strategic incentives underlying each equilibrium. Equilibria a_u and a_- are characterized by *substitutability in information*, as the optimal individual weight is decreasing in the average weight, i.e., $a_i'(\hat{a}) < 0$.[9] In contrast, the equilibria $a_\circ$ and a_+ are characterized by *complementarity in information* since $a_i'(\hat{a}) > 0$. In fact, as soon as $a > (1-\gamma)^{-1}$, the higher the a the higher the precision of the signal regarding μ_i, which further pushes up the optimal weight. The emergence of complementarity explains the upward-sloping part of the best-weight function and is key for the existence of multiple equilibria.

While complementarity is essential for generating multiple equilibria, it is neither necessary nor sufficient to imply a strong informational multiplier. To see this, define $\Gamma(\hat{a}) \equiv \sigma_E^2(\hat{a})/\sigma^2$ as the volatility of beliefs relative to the volatility of the shock ζ for some equilibrium point

[9]See equation (43) in appendix A2.

$\hat{a}$. We will say that the economy exhibits *amplifying* informational feedback whenever a fall in the volatility of the exogenous shock leads to an increase in $\Gamma(\hat{a})$, i.e., $\partial\Gamma(\hat{a})/\partial\sigma < 0$, and *dampening* feedback otherwise. The following proposition classifies the equilibria in Proposition 1 according to the type of feedback they generate.

Proposition 2. *The equilibria a_u, a_-, and $a_\circ$ all exhibit amplifying feedback, while the equilibrium a_+ exhibits dampening feedback.*

Proof. Given in Appendix A2. ∎

The characterization of informational feedbacks as either amplifying or dampening depends on whether the equilibrium value of a gets closer to $(1-\gamma)^{-1}$ as σ shrinks. From Figure 1, it is clear that $a_u, a_\circ$, and a_- feature amplifying feedback, whereas a_+ features dampening feedback. Nevertheless, the feedback effects in $a_\circ$ and a_- are distinct from that in a_u for reasons we discuss in the following section.

3.1 Limit equilibria: Amplification without Sentiments

Here we show that learning from prices can generate amplification strong enough that the economy sustains sizable aggregate fluctuations in the limit $\sigma^2 \to 0$. Fluctuations in the limit case result from households' correlated errors regarding local conditions, echoing a central theme in the recent literature on sentiments. Yet, the correlated errors generated in our economy are distinct from the sentiments highlighted by Benhabib et al. (2015) and Acharya et al. (2017), for the shock driving them is determinant in every equilibrium. Indeed, we show in the final proposition of this section that extrinsic sentiments à la Benhabib et al. (2015) are fragile to the introduction of *any* fluctuations in aggregate fundamentals.

The first proposition of this section characterizes equilibria in the limit case that aggregate fundamentals are arbitrarily small.

Proposition 3. *In the limit $\sigma^2 \to 0$,*

i. the unique equilibrium (for $\gamma \geq 1/2$) and the high equilibrium (for $\gamma < 1/2$) converge to a point with no aggregate volatility:

$$\lim_{\sigma^2 \to 0} a_{u,+} = \min\left(\frac{1}{\gamma}, \frac{1}{1-\gamma}\right) \qquad \lim_{\sigma^2 \to 0} \sigma_E^2(a_{u,+}) = 0. \tag{26}$$

ii. the low and middle equilibria (for $\gamma < 1/2$) converge to the same point and exhibit non-trivial aggregate volatility:

$$\lim_{\sigma^2 \to 0} a_{\circ,-} = (1-\gamma)^{-1} \qquad \lim_{\sigma^2 \to 0} \sigma_E^2(a_{\circ,-}) = \frac{\gamma(1-2\gamma)}{(1-\gamma)^2}. \tag{27}$$

Proof. Given in Appendix A2. ∎

Part (i) of Proposition 3 points to cases in which amplification is not strong enough to result in non-trivial fluctuations infinitesimal fundamental shocks. Interestingly, this occurs for the unique equilibrium case, even though we have just shown that it exhibits amplifying feedback; in this case, responses to aggregate shocks grow with shrinking σ, but not fast enough to offset the falling size of those shocks. Figure 2, which plots the variance of aggregate beliefs as a function σ, captures these patterns. In particular, both the "unique" and "high" lines converge to zero as σ^{-1} goes to infinity, although the former is non-monotonic well away from the limit.

In contrast, Part (ii) of Proposition 3 points to cases — the middle and low equilibria — where responses to fundamental shocks grow fast enough to completely offset the shrinking size of shocks; in the limit the product of the two converges to deliver a positive, finite variance of beliefs. Figure 2 also captures these patterns, showing the variance of beliefs in both equilibria converging to the same strictly positive value.

Surprisingly, the limiting cases of these two equilibria have the same stochastic properties as the extrinsic sentiment equilibria described by Benhabib et al. (2015). In our economy, however, equilibria *à la* Benhabib et al. (2015) do not exist. In our case fluctuations are driven by infinitesimally-small fundamental shocks, whose realization is able to coordinate sizable fluctuations in agents' expectations via their effects on the endogenous price signal; in other words, whether agents are optimistic or pessimistic is entirely pinned down by the realization of fundamentals, even at the limit.

A natural question, given the results in Proposition 3, is whether errors driven by extrinsic shocks can coexist with the fundamental-driven fluctuations in aggregate beliefs captured by our model. The next proposition demonstrates that, in fact, *extrinsic* sentiments are always crowded-out by common shocks to productivity.

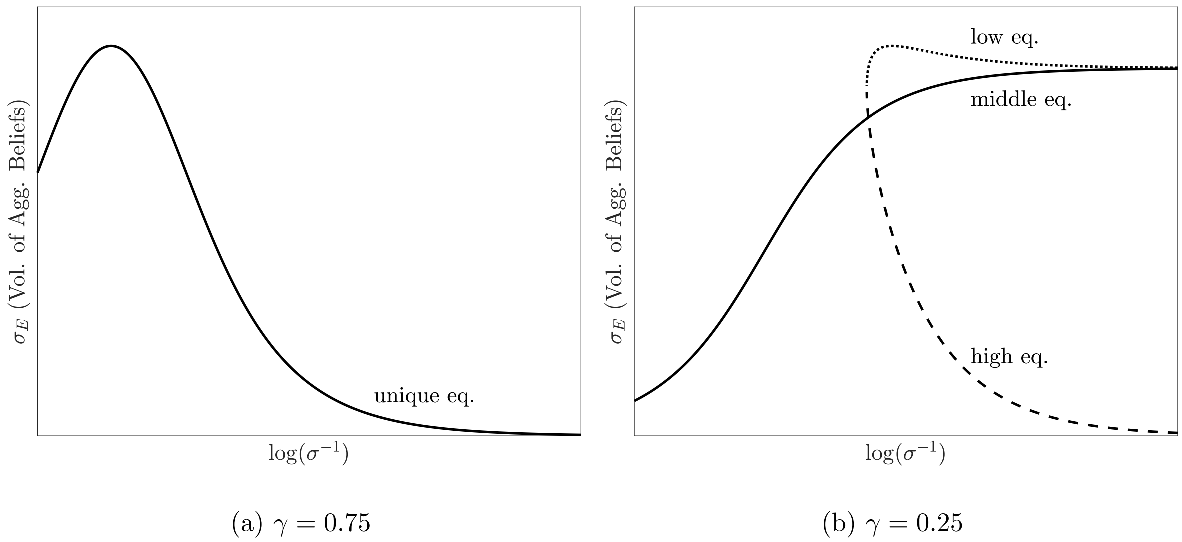


Figure 2: Belief volatility approaching the limit.

Proposition 4. *Suppose that*

$$\int E[\mu_i|s_i]di = \phi_\zeta \zeta + \phi_\varepsilon \varepsilon,$$

where ϕ_ε is the equilibrium effect of an extrinsic sentiment shock, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, not related to fundamentals. Then, $\phi_\varepsilon = 0$ for any $\sigma^2 > 0$.

Proof. Given in Appendix A2. ∎

The fundamental shock always dominates the extrinsic shock because its fundamental nature gives it two channels — one endogenous and one exogenous — through which it influences people's information and, therefore, their actions. Intuitively, conjecture that the average action reflects a response to both fundamental and extrinsic shocks. In equilibrium, agents respond to the average expectation, and therefore proportionally to the conjectured endogenous coefficients for each shock. But agents also respond to the exogenous component of the fundamental that appears in the price signal. Thus, any equilibrium must depend somewhat more-than-conjectured on the fundamental relative to the extrinsic shock. This guess and update procedure cannot converge unless the weight on the extrinsic shock is exactly zero.

This logic highlights the fragility of the extrinsic version of sentiments, which are coordinated by endogenous signal structures. For, any shock which tends to coordinate actions for exogenous reasons will also benefit from the self-reinforcing nature of learning, thereby absorbing the role of belief shock for itself. Indeed, in the appendix we show that the same outcome arises if *local* shocks μ_i have any common component.

4 Business cycle fluctuations

In this section, we explore the implications of the learning-from-prices mechanism for the business cycle comovement of our economy. We show that many qualitative features of the business cycle can be explained by our model in which productivity is the only aggregate shock, and agents learn from prices. Moreover, we show that matching business cycles patterns does not require parameterizing the economy to have multiple equilibrium. Rather the most realistic comovements emerge from a version of the model with a *unique equilibrium* and small but non-trivial surprises in productivity.

Before proceeding, we briefly review several stylized facts about the business cycle that drive our exploration. These facts are summarized by Table 1: (1) output, inflation, and hours comove; (2) total factor productivity and hours are negatively correlated; (3) inflation is only weakly correlated with output;[10] and (4) aggregate productivity is only weakly correlated with any endogenous aggregate variable.

Without learning from prices, productivity shocks would move aggregate prices and quantities in opposite directions, so that matching these facts would require some combination of price-setting frictions, aggregate demand shocks, or exogenous coordination of beliefs on an extrinsic shock. Our setting can be extended, however, to allow productivity shocks to generate the appearance of both supply- and demand-driven fluctuations, thereby bringing the model closely in line with the set of business cycle facts summarized above.

Our aim here is not to give a full quantitative account of the business cycle, but rather to demonstrate that our theoretical mechanism challenges assumptions about how productivity shocks can be related to the business cycle. In addition to offering new foundation for economic fluctuations, our findings suggest it may be worthwhile to revisit the conclusions

[10]Because our model is static, it cannot distinguish between the level of prices and inflation. If we substituted the detrended GDP deflator for inflation in this table, then output and inflation would have a slight negative — rather than slightly positive — correlation. We show below that our model can match a weak positive or a weak negative correlation between output and prices.

Table 1: Business Cycle Comovements

	GDP	**hours**	**inflation**	**TFP**
$\rho(\text{GDP}, x)$	1.00	0.86	0.18	-0.06
$\rho(\text{TFP}, x)$	-0.06	-0.36	-0.24	1.00

Note: Data are real per-capita gross domestic product, real per-capita hours in the non-farm business sector, GDP deflator growth, and capacity utilization adjusted TFP described by Basu, Fernald, and Kimball (2006) and maintained by John Fernald at www.frbsf.org. All data are in log-levels, HP-detrended using the longest available sample and smoothing parameter $\lambda = 1600$. Date range: 1960Q1 to 2012Q4.

of the quantitative macroeconomic literature which has, with very few exceptions, assumed away the possibility of learning from prices.

Equilibrium with public news

We begin by extending our framework to include public information in the form of news on productivity. We assume that the productivity shock is composed of two independently distributed components

$$\zeta = \zeta^n + \zeta^s;$$

with $\zeta^n \sim (N, \sigma^2_{\zeta^n})$, $\zeta^s \sim (N, \sigma^2_{\zeta^s})$ and $\sigma^2_{\zeta^n}+\sigma^2_{\zeta^s} = \sigma^2_{\zeta}$. The first term, ζ^n, is a "news"component; it corresponds to the forecastable component of productivity, and is commonly known to all agents before their consumption choices are made. Conversely, ζ^s is the "surprise" component; it is unknown to uninformed households and they seek to forecast it using their observation of prices.[11] For future reference, let $\sigma^2_n \equiv \sigma^2_{\zeta^n}/\sigma^2_{\mu}$, and $\sigma^2_s \equiv \sigma^2_{\zeta^s}/\sigma^2_{\mu}$ be the normalized variances of the forecasted and surprise components of productivity respectively.

The decomposition of productivity into a forecastable and surprise component plays two roles in this section. First, it allows us to isolate the effects of learning through prices, as the forecasted component of productivity will transmit in the economy as a usual supply-side shock. Second, by combining the responses of the economy to forecasted and surprise productivity shocks, we can generate the rich cross-correlation structure seen in the data.

Only modest modifications are necessary to characterize equilibrium in this general case. Households use the forecasted component to refine the information contained in the price signal by "partialing-out" the known portion of productivity. In particular, we can rewrite

[11] Chahrour and Jurado (2017a) show that this information structure is equivalent to assuming that agents observe a noisy aggregate signal, $s = \zeta + \vartheta$.

households' expectations as

$$E[\mu_i|s_i] = a_i(s_i + (1-\gamma)\zeta^n), \tag{28}$$

where $s_i + (1-\gamma)\zeta^n$ represents a new signal embodying the information available to the individual household, after she has controlled for the effect of ζ^n. It follows that the equilibrium values $\{a_u, a_-, a_\circ, a_+\}$ and the conditions for their existence are isomorphic to the ones in the baseline economy once σ_s^2 takes the place of σ^2.

An immediate implication is that increasing the fraction of productivity that is forecastable actually pushes the economy towards a situation of high information multipliers and, when $\gamma < 1/2$, towards the region of equilibrium multiplicity. For the low equilibrium, this implies an *increase* in the variance of the average expectation of households. This result demonstrates that the mechanism of Section 3 is robust to increasing the information sets of households; so long as any aggregate component remains unknown, agents endogenously coordinate their errors though the pricing system.

Fact 1: Supply shocks generate demand-driven fluctuations

Our key observation, from the standpoint of generating realistic business cycles, is that all the equilibria of our model can generate business cycle fluctuations with demand-side features; that is, final good prices, total output, the price of capital, and total employment all positively comove in response to the surprise component of productivity. This happens because, as aggregate volatility falls, the informational value of the price signal rises, leading agents' beliefs about their local conditions to respond more strongly to it. Stronger aggregate effects on beliefs eventually lead the informational channel of prices to dominate, so that consumption increases in response to higher prices. In this way, learning from prices provides a new mechanism for generating expectations-driven demand shocks in an economy hit only by fundamental shocks to productivity.

This consequence of endogenous information for business cycle comovements can be seen intuitively by analyzing the aggregate demand and aggregate supply schedules in our economy. Using the aggrgeate version of equations (11) - (14), we can express aggregate demand and supply as

$$AD \;:\; c = \bar{\mu} - p, \tag{29}$$

$$AS \;:\; c = \frac{1}{1-\alpha\phi}(\alpha\phi p + \alpha(1-\phi)(1-\kappa)\zeta). \tag{30}$$

When aggregate conditions have no endogenous effect on households' beliefs, this relationship implies a standard downward-sloping aggregate demand relation. However, this changes once we account for the equilibrium feedback of prices into households' inference.

To derive equilibrium aggregate demand and supply relations, in the appendix we compute the dependence of the average belief $\bar{\mu}$ on prices average prices and the known news shock ζ^n:

$$\bar{\mu} = (1-\kappa)\int E[\mu_i|p_i]di = \varphi(a)(p + \alpha(1-\phi)(1-\kappa)\zeta^n),$$

with $\varphi(a) \equiv \frac{a(1-\kappa)}{(1-\alpha)\kappa+\alpha(1-\phi)(1-\kappa)+a(1-\alpha)(1-\kappa)}$. Substituting this expression into the expression for aggregate demand expression (29) yields

$$c = (\varphi(a) - 1)p + \varphi(a)\alpha(1-\phi)(1-\kappa)\zeta^n \tag{31}$$

Notice that *both* aggregate demand and aggregate supply are shifted by the forecasted productivity shock, ζ^n, while the surprise component, ζ^s, shifts only aggregate supply. This is natural since, in our environment, the surprise productivity shock can only influence households' actions through its effect on prices.

Using equation (31), it is straightforward to demonstrate the following.

Proposition 5. *For σ_s^2 sufficiently small, equilibria $a_-, a_\circ$ exhibit comovement of aggregate output, employment, the price level, and the price of capital in response to surprise productivity shocks. For σ_s^2 sufficiently small, also a_+, a_u do provided $\kappa < \alpha$.*

Proof. The results follows from continuity of the best-response function, and the observation that

$$\lim_{\sigma_s \to 0} \varphi(\hat{a}) = \frac{1}{1-\alpha\phi} > 1 \tag{32}$$

for $\hat{a} \in \{a_-, a_\circ\}$, and

$$\lim_{\sigma_s \to 0} \varphi(\hat{a}) = \frac{1-\kappa}{1-\alpha} > 1 \tag{33}$$

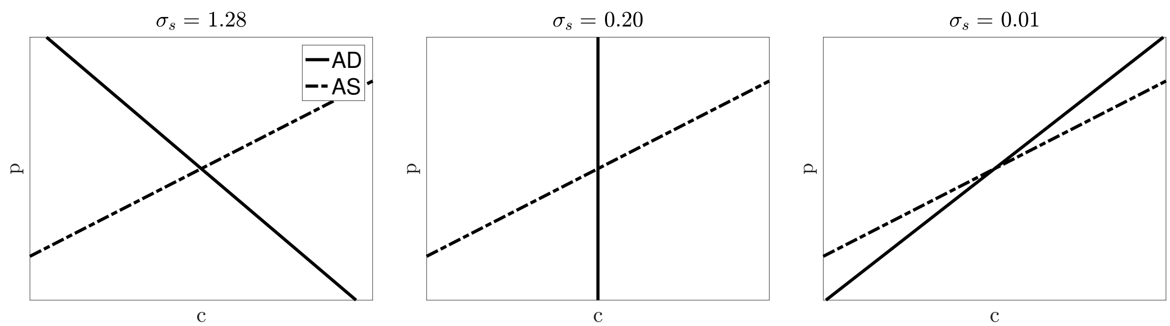
for $\hat{a} \in \{a_+, a_u\}$ provided $\kappa < \alpha$. ∎

Crucially, the relation in (31) implies that aggregate demand is upward sloping for any $\varphi(a)$ larger than unity. In this case, price and quantity will move together in response to shifts of either aggregate demand or aggregate supply! Moreover, as the relative variance σ_s decreases, this will be true *for all equilibria* in the economy provided that $\kappa < \alpha$. Even the unique and high equilibria, which display no fluctuations in response to surprise shocks in the limit $\sigma_n \to 0$, exhibit (conditional) comovements in prices and quantities away from that limit, *as if* the economy were hit by a common demand shock.

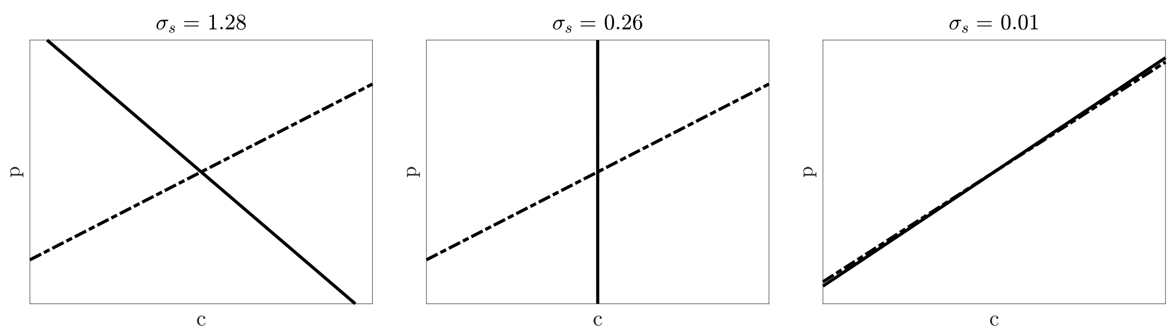
To assess the likely implications of learning from prices for aggregate comovements in practice, we perform a very simple calibration exercise. In particular, we fix the the degree of decreasing returns $\alpha = 0.9$, and the select the parameter ϕ so that the stead-state labor share of total income is 60%. We then consider two values for the share of informed households, $\kappa^h = 0.80$ and $\kappa^l = 0.20$ respectively. These parameters correspond to values for the reduced-form parameter $\gamma = 0.57$ and $\gamma = 0.08$ respectively, implying a unique equilibrium in the first case and (potentially) multiple equilibria in the second.

Figure 3 plots aggregate supply and demand relations for different values of the relative volatility, σ_s, in the two cases; when the economy exhibits multiplicity, we consider the low equilibrium. In both cases, as σ_s shrinks, the slope of aggregate demand turns clockwise until it becomes upward sloping. In particular, the upward slope in aggregate demand exceeds the slope of aggregate supply as shown by the two panels in the last column. In both equilibria, when the variance of productivity shocks is sufficiently small, outward shifts in supply move prices and quantities in the same direction.

Therefore, aggregate demand in the low equilibrium behaves in a manner that qualitatively resembles its behavior in the unique equilibrium. The peculiarity of the low equilibrium is that, in the limit of σ_s approaching zero, supply and demand overlie each other. The last panel of Figure 3 therefore provides an easy intuition for the extremely large informational multiplier implied by our sentiment-like equilibria, as even small shifts in aggregate supply imply large changes in the equilibrium quantity of consumption.



(a) Unique Equilibrium ($\kappa = 0.80$)



(b) Low Equilibrium ($\kappa = 0.20$)

Figure 3: Aggregate supply and demand in the microfounded model.

Fact 2: Contractionary technology

One robust — and from the perspective of an RBC model, surprising — fact about business cycles is that hours typically fall on impact in response to improvements in aggregate technology, while aggregate productivity is only weakly associated with output at any horizon.[12] Basu et al. (2006) document the first fact in detail, and shows that it can be rationalized in the context of a sticky price model. Our model offers an alternative account.

To see that hours can fall in response to technology improvements, recall that aggregate labor supply is equal to the average expectation in the economy. By equation (21), we have

$$n = (1-\kappa)\int E[\mu_i|p_i]di = -(1-\kappa)\frac{a(1-\gamma)}{1-a(1-\gamma)}\zeta^s. \tag{34}$$

Thus, unanticipated positive technology shocks lead to a decrease in hours whenever $a < (1-\gamma)^{-1}$, which is always true of the low and unique equilibria.[13] The intuition is straightforward: an increase in the price seen by a household could be caused by improving local conditions or by falling aggregate productivity and agents become overly optimistic precisely when (the unobserved part of) productivity is falling. In contrast, forecasted productivity shocks have no effect on labor in our economy, implying that the correlation between hours and total productivity is always both negative and imperfect.

While the correlation of hours with productivity is unambiguous in the model, the output-productivity relationship is slightly more subtle. Once again, it turns out that this relationship hinges on the strength of the learning from prices channel. In particular, higher productivity causes output contractions in the unique and low equilibria whenever information effects are sufficiently *strong*, that is whenever, on average, the surprise component relative to the anticipated one is *small* enough (i.e. σ_s is small enough) that aggregate demand slopes upward as depicted in Figure 3. The weak contemporaneous relationship between (total) productivity and output is also consistent with the findings of Basu et al. (2006), who show a small impact response of output to identified TFP shocks.

[12]The disconnect between productivity and endogenous variables has been emphasized by several authors, including Angeletos et al. (2014), Angeletos et al. (2016), Chahrour and Ulbricht (2017), and Chahrour and Jurado (2017b).

[13]Conversely, it never holds for the high and middle equilibria.

Facts 3 and 4: Weakening price-quantity correlation

While Proposition 5 shows that surprise shocks induce positive comovement among business cycle variables, the opposite is true in the case of shocks to productivity that are forecasted. Since forecasted productivity shocks affect both supply and demand, it is helpful to solve for equilibrium consumption and price as a function of shocks and the equilibrium inference coefficient:

$$p = -\alpha(1-\phi)(1-\kappa)\zeta + (1-\alpha\phi)\psi(1-\kappa)\zeta^s \tag{35}$$

$$c = \alpha(1-\phi)(1-\kappa)\zeta + \alpha\phi\psi(1-\kappa)\zeta^s, \tag{36}$$

where $\psi \equiv \frac{\alpha(1-\phi)}{\varphi(a)^{-1}-1+\alpha\phi}$. From equation (36) it is immediate that forecasted technology shocks always expand output, while equation (34) implies zero impact on labor supply. Moreover, comparing (35) and (36), it is clear that the news component, ζ^n, will move prices and quantities in opposite directions, generating the comovement more typically associated with a supply shock. Thus, overall comovements — the degree to which labor and prices are procyclical, as well as the correlation of total factor productivity with all endogenous variables — will depend on the balance of forecastable and surprise productivity, as well as the overall size of these shocks relative to local conditions.

Such different transmission of productivity components allows our model to escape a common criticism to the literature on news and noise that productivity is only weakly correlated with business cycle variables at all horizons (see for example Angeletos et al. (2014)). Our model is consistent with this observation because the unanticipated and surprise components of productivity are transmitted very differently, leading to weak correlation between total productivity and other variables. Notice that this is the case even if productivity has small surprise components. In fact, it is exactly when current productivity is largely anticipated that our mechanism becomes more important.

Business cycles: All facts together in the unique equilibrium

Putting together the observations above, it is plain that our model can qualitatively match our set of business cycle facts one at a time, but can it match them simultaneously? It

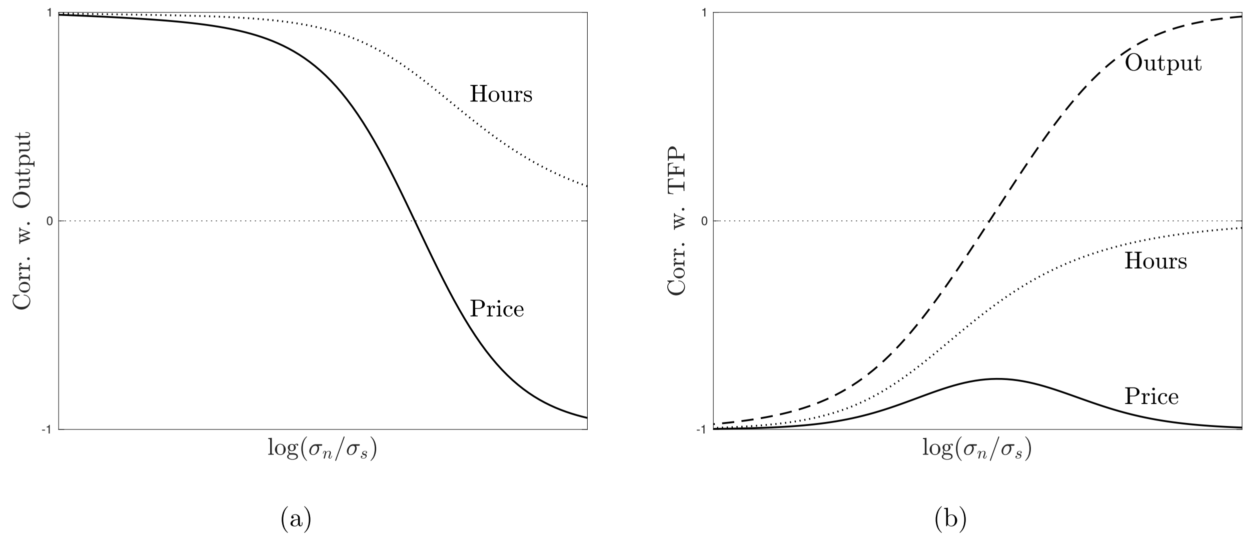


Figure 4: Correlations in the economy with both anticipated and unanticipated technology, with $\kappa = 0.80$ and $\sigma = 0.10$.

turns out the answer is "yes". The key degree of freedom, and the only one we exploit here, is the decomposition of productivity into its news and surprise components. Because the economy responds differently to these components, we can combine the demand-like effects of surprise productivity shocks with the supply effects of forecasted productivity shocks, delivering comovements between minus one and one.

Figure 4 plots the correlations of output, prices, hours, and productivity as function of the ratio σ_n/σ_s in the unique equilibrium economy, fixing $\kappa = 0.80$ and $\sigma_s^2 + \sigma_n^2 = 0.1^2$. When only a small fraction of productivity is forecastable, comovements are driven by the strong information effects inherent in surprise shocks, leading to strongly positive price-quantity comovements, perfectly contractionary productivity shocks, and a price level that is very strongly negatively correlated with TFP. Conversely, in the extreme of perfectly forecasted productivity, the economy appears to be driven by pure supply shocks, with perfect negative correlation of output and prices. In the intermediate range of this ratio, however, these forces offset each other, leading to correlations that qualitatively match all of the implications in Table 1: hours are strongly procyclical, prices are procyclical but less strongly, hours are negatively correlated with productivity while output is only weakly correlated with it, and prices are imperfectly negatively correlated with productivity. While simple, the model does

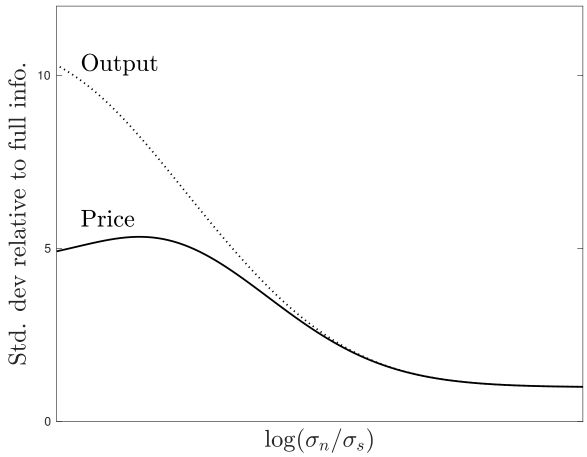


Figure 5: Aggregate volatility with $\kappa = 0.80$ and $\sigma = 0.10$.

a remarkable job matching the stylized facts with which we began.

Finally, Figure 5 plots the overall degree of amplification in the unique equilibrium economy as a function of the fraction of shocks that are forecasted. As suggested by equation (36), the overall size of the response to surprise shocks is substantially larger than to forecasted productivity shocks, such that in the extreme of perfectly unforecastable technology, output is roughly ten times as volatile as it is under full information. Prices are also amplified, and are up to about five times as volatile as they are under full information. In the intermediate range that best matches the various business cycle moments in Table 1, the figure shows overall output volatility that is roughly three times that implied by the model under full information. In short, even when the economy has a unique equilibrium, the model delivers substantial amplification of aggregate productivity shocks.

5 Extensions and Robustness

This section presents several extensions to the basic setup, showing that the insights of the main mechanism are robust to various modeling details. In Section 5.1, we allow households to observe additional private information about local conditions and show that our results do not rely on excluding exogenous sources of information. To emphasize that wage rigidity is not essential for our story, Section 5.2 considers a version of the model without wage rigidity and show that it has similar implications. Section 5.3 allows for the disaggregation of goods at the island level to demonstrate that the existence of upward-sloping aggregate demand in our model does not require the existence of upward-sloping demand at the good level. Finally, to address concerns about the plausibility of learning from prices equilibria, Section 5.4 studies the issue of stability under adaptive learning for the various equilibria of the baseline model.

5.1 Signal extraction problem with private signals

Here we show that the signal extraction problem, and corresponding equilibria, are not qualitatively affected by the availability of a private signal about the local shock. Instead, the addition of private information maps into our analysis of Section 3 as an increase in the relative variance of aggregate shocks.

Let us assume that a household $j \in (0,1)$ in island i has a private signal

$$\omega_{ij} = \mu_i + \eta_{ij} \tag{37}$$

where $\eta_{ij} \sim N(0, \sigma_\eta)$ is identically and independently distributed across households and islands. In this case, households form expectations according to

$$E[\mu_i|s_i, \omega_{ij}] \quad = \quad a\left(\gamma\epsilon_i + (1-\gamma)\left(\int E[\mu_i|s_i, \omega_{ij}]di - \zeta\right)\right) + b\left(\mu_i + \eta_{ij}\right),$$

where b measures the weight given to the additional private signal. Averaging out the relation above and solving for the aggregate expectation gives

$$\int E[\mu_i|p_i, \omega_{ij}]di \quad = \quad -\frac{a(1-\gamma)}{1-a(1-\gamma)}\zeta,$$

which is identical to (21). However, now we need two optimality restrictions to determine a and b. These are

$$E[p_i(\mu_i - E[\mu_i|p_i, \omega_{ij}])] = 0 \quad \Rightarrow \quad \gamma\sigma_\mu - a\left(\gamma^2\sigma_\mu + \frac{(1-\gamma)^2}{(1-a(1-\gamma))^2}\sigma_\zeta\right) - b\gamma\sigma_\mu = 0,$$
$$E[\omega_{ij}(\mu_i - E[\mu_i|p_i, \omega_{ij}])] = 0 \quad \Rightarrow \quad \sigma_\mu - a\gamma\sigma_\epsilon - b(\sigma_\mu + \sigma_\eta) = 0,$$

which identify the equilibrium a and b such that each piece of information is orthogonal with the forecast error. Solving the system for a, we get a fix point equation written as

$$a = \frac{\gamma}{\gamma^2 + \frac{(1-\gamma)^2}{(1-a(1-\gamma))^2}\frac{\sigma_\mu+\sigma_\eta}{\sigma_\eta}\frac{\sigma_\zeta}{\sigma_\mu}}. \tag{38}$$

For $\sigma_\eta \to \infty$, the right-hand side of the relation above matches (25). In particular, it follows that a lower σ_η in (38) is equivalent to considering a larger σ_ζ in (25). The analysis of the baseline model thus applies directly to this generalization, and small amounts of exogenous private information do not qualitatively change any of our earlier results.

5.2 Model without wage rigidity

Wage rigidity provides an easy microfoundation for our key equations. Here we provide an alternative with flexible wages to demonstrate our results do not require this particular model specification.

The utility function of the household is modified as follows:

$$\beta^t \left(\log C_{ij,t} - \frac{N_{ij,t}^{1+\delta}}{1+\delta} + \chi e^{-\mu_i - \sigma_\mu^2/2} M_{ij,t} \right),$$

subject to the same budget constraint (2). The problem of the representative firm on island i also remains the same.

As in Lucas (1980), we assume now that an household is constituted by two family members, a household and a worker. Although both share the same payoff, they cannot communicate among each other and they act independently (to bear in mind: although they share pay-offs the equilibrium concept that we use remains Nash). The timing is as follows: In the first stage, the shocks realize. In a second stage, all workers receive information about the local shock whereas only a fraction $\kappa \in (0,1)$ of consumers receive it. In the third stage, makes consumption or labor choices conditional on the information received and the market prices observed.

Therefore, we have now a labor supply equation. Workers provide work according to:

$$N_{ij,0}^{\delta} = \Lambda_{ij,0} W_{i,0}.$$

Since (9) and (10) still hold, we can rewrite the optimal labor supply in terms of log-linear deviations from the stochastic steady state as $\delta n_{ij,0} = -\mu_i + w_{i,0}$. Aggregating across agents j we get that

$$\delta n_{i,0} = -\mu_i + w_{i,0}.$$

In appendix A3 we show that this version of the model yields with minor changes the same setting that we analyse here.

In this version of the model one has to prevent consumer's learning from the labor market splitting the household in different information types. In fact, if the consumer could observe the wage, then she would be able to infer the local shock as she observe two signal to infer two shocks. One could then complicate the model allowing for two local shocks, making the observation of two signals no longer sufficient to perfectly infer three shocks. However, it turns out that in order to have real effects - by only relying on our learning friction - expectations in the first-order-conditions for labor and consumption must be taken with different information sets. This is the essential ingredient that our assumption of wage rigidity is buying, which however can be obtained by other information assumptions.

5.3 Upward-sloping demand?

One possible objection to the realism of our mechanism is the implication that the consumption of island-specific good C_i is rising in its price, i.e., that local consumption goods appear to be Giffen goods. Such behavior at the good level is not an essential aspect of our story. The most natural way to avoid this complication is to presume that, *within* islands, quantity-choosing firms produce a continuum of goods indexed by (i, k), which are then aggregated at the island-level by a standard Dixit-Stiglitz aggregator, $C_i = \left(\int C_{ik}^{1-\frac{1}{\theta}}\right)^{\frac{1}{1-\frac{1}{\theta}}} dk$ with $\theta > 1$.

Suppose now that each (i, k) producer is hit with an idiosyncratic, mean-zero productivity shock, υ_{ik}. In this case, the price of good c_{ik} in logs turns out to be

$$p_{ik} = (1-\alpha)\mu_i^e + \alpha(1-\phi)(q - \tilde{\zeta} - \upsilon_{ik}).$$

Demand for good c_{ik} is governed by the standard formula

$$c_{ik} = -\theta(p_{ik} - p_i) + c_i,$$

which reflects a substitution effect governed by the standard elasticity parameter *at the good level:* An econometrician studying good-level prices would find no evidence that the typical good is Giffen. Nevertheless, the total price level on island i,

$$p_i = \int p_{ik} dk = (1-\alpha)\mu_i^e + \alpha(1-\phi)(q-\tilde{\zeta}),$$

is both (i) identical to its value in the baseline economy, and (ii) reflects the optimal (even) weighting of the signals p_{ik} that households use in equilibrium to infer their local shock: Subsequent analysis of the island-level and aggregate economy is not affected.

5.4 Stability analysis

Here, we examine the issue of out-of-equilibrium convergence, that is, whether or not an equilibrium is a rest point of a process of revision of beliefs in a repeated version of the static economy. We suppose that agents behave like econometricians. At time t they set a weight $a_{i,t}$ that is estimated from the sample distribution of observables collected from past repetitions of the economy.

Agents learn about the optimal weight according to an optimal adaptive learning scheme:

$$a_{i,t} = a_{i,t-1} + \gamma_t \; S_{i,t-1}^{-1} \; p_{i,t} \left(\mu_{i,t} - a_{i,t-1} p_{i,t}\right) \tag{39}$$

$$S_{i,t} = S_{i,t-1} + \gamma_{t+1} \left(p_{i,t}^2 - S_{i,t-1}\right), \tag{40}$$

where γ_t is a decreasing gain with $\sum \gamma_t = \infty$ and $\sum \gamma_t^2 = 0$, and matrix $S_{i,t}$ is the estimated variance of the signal. A rational expectations equilibrium $\hat{a}$ is a locally learnable equilibrium if and only if there exists a neighborhood $\digamma(\hat{a})$ of $\hat{a}$ such that, given an initial estimate $a_{i,0} \in \digamma(\hat{a})$, then $\lim_{t\to\infty} a_{i,t} \overset{a.s}{=} \hat{a}$; it is a globally learnable equilibrium if convergence happens for any $a_{i,0} \in \mathbb{R}$.

The asymptotic behavior of statistical learning algorithms can be analyzed by stochastic approximation techniques (for details, refer to Marcet and Sargent, 1989a,b and Evans and Honkapohja, 2001). Below we show that the relevant condition for stability is $a_i'(a) < 1$, which can easily checked by inspection of Figure 1.

It turns out that the unique equilibrium is globally learnable, that is, no matter the initial estimate, revisions will lead agents to coordinate on the equilibrium. In case of multiplicity, the high and low equilibrium are locally learnable, whereas the middle equilibrium is not.

Hence the middle equilibrium works as a frontier between the basins of attraction of the two equilibria.

To check local learnability of the rational expectations equilibrium, suppose we are already close to the resting point of the system. That is, consider the case $\int \lim_{t\to\infty} a_{i,t}\, di = \hat{a}$, where $\hat{a}$ is one of the equilibrium points $\{a_-, a_\circ, a_+\}$, and so

$$\lim_{t\to\infty} S_{i,t} = \sigma_s^2(\hat{a}) = \gamma^2\sigma_\mu^2 + \frac{(1-\gamma)^2}{(1-\hat{a}(1-\gamma))^2}\sigma_\zeta^2. \tag{41}$$

According to stochastic approximation theory, we can write the associated ODE governing the stability around the equilibria as

$$\begin{aligned}
\frac{\mathrm{d}a}{\mathrm{d}t} &= \int \lim_{t\to\infty} \mathrm{E}\left[S_{i,t-1}^{-1} p_{i,t}\left(\mu_{i,t} - a_{i,t-1}p_{i,t}\right)\right] di \\
&= \sigma_s^2(\hat{a})^{-1} \int \mathrm{E}\left[p_{i,t}\left(\mu_{i,t} - a_{i,t-1}p_{i,t}\right)\right] di \\
&= \sigma_s^2(\hat{a})^{-1}\left(\gamma\sigma_\mu^2 - a_{i,t-1}\left(\gamma^2\sigma_\mu^2 + \frac{(1-\gamma)^2}{(1-a_{t-1}(1-\gamma))^2}\sigma_\zeta^2\right)\right) \\
&= a_i(a) - a. \qquad (42)
\end{aligned}$$

For asymptotic local stability to hold, the Jacobian of the differential equation in (42) must be less than zero at the conjectured equilibrium. The derivative of $a_i(a)$ with respect to a is given by:

$$a_i'(a) = -\frac{2\gamma(1-\gamma)^3(1-(1-\gamma)a)\sigma^2}{\left((1-\gamma)^2\sigma^2 + (1-(1-\gamma)a)^2\gamma^2\right)^2}, \tag{43}$$

which is positive whenever $a > (1-\gamma)^{-1}$. Then, necessarily, $a_i'(a_\circ) > 1$, $a_i'(a_+) \in (0,1)$, $a_i'(a_-) < 0$ and $a_i'(a_u) < 0$. This proves that the low and unique equilibrium are respectively locally and globally learnable.

6 Conclusion

Learning from prices has played an important role in our understanding of financial markets since at least Grossman and Stiglitz (1980). Yet, learning from prices appeared even earlier in the macroeconomics literature, including in the seminal paper of Lucas (1972). Nevertheless, that channel gradually disappeared from models of the business cycle, in large part because people concluded that fundamental shocks would be almost completely revealed before incomplete knowledge about them could influence relatively slow-moving macroeconomic aggregates.

In this paper we have shown that, even if aggregate shocks are *nearly* common knowledge, learning from prices may still play a crucial role driving fluctuations in beliefs. In fact, the feedback mechanism we described is strongest precisely when the aggregate shock is almost, but not-quite-fully, revealed. Endogenous information structures can deliver strong multipliers on small common disturbances, and thus offer a foundation for coordinated, expectations-driven economic fluctuations. Such fluctuations are completely consistent with rational expectations. Moreover, the key feature of our theory is also a feature of reality: agents observe and draw inference from prices that are, themselves, influenced by aggregate conditions.

Applied to an economy driven only by productivity shocks, we have shown that this mechanism captures several salient features of business cycles, including weak-but-positive price quantity correlation and the contractionary labor effects of positive productivity shocks. Our approach is consistent both with the evidence that productivity and endogenous outcomes are weakly correlated and with the prior of many macroeconomists that productivity plays a central role in explaining business cycles. Our results suggest that the relationship between supply and demand shocks is more subtle than typically assumed in the empirical literature, and future work may wish to take in account the implications of price-based learning.

Appendix

A1 Derivations

Log-linear equilibrium. The system of aggregate log-linear relations is written as

$$\begin{bmatrix} c \\ q \\ n \\ p \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & \alpha\phi & 1 \\ \frac{1}{\alpha\phi} & 0 & 0 & 0 \\ 0 & 0 & 1-\alpha\phi & 0 \end{bmatrix} \begin{bmatrix} c \\ q \\ n \\ p \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \alpha\left(1-\phi\right) \\ 0 & -\frac{\alpha(1-\phi)}{\alpha\phi} \\ 0 & -\alpha\left(1-\phi\right) \end{bmatrix} \begin{bmatrix} \bar{\mu} \\ \widetilde{\zeta} \end{bmatrix}$$

whose solution is

$$\begin{bmatrix} c \\ q \\ n \\ p \end{bmatrix} = \begin{bmatrix} \alpha\phi & \alpha\left(1-\phi\right) \\ 1 & 0 \\ 1 & 0 \\ 1-\alpha\phi & -\alpha\left(1-\phi\right) \end{bmatrix} \begin{bmatrix} \bar{\mu} \\ \widetilde{\zeta} \end{bmatrix}.$$

On island i we have

$$\begin{bmatrix} c_i \\ z_i \\ n_i \\ p_i \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & \frac{\alpha\phi}{1-\alpha(1-\phi)} & \frac{1}{1-\alpha(1-\phi)} \\ \frac{1}{\alpha\phi} & -\frac{\alpha(1-\phi)}{\alpha\phi} & 0 & 0 \\ 0 & -\alpha\left(1-\phi\right) & 1-\alpha\phi & 0 \end{bmatrix} \begin{bmatrix} c_i \\ z_i \\ n_i \\ p_i \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\alpha(1-\phi)}{1-\alpha(1-\phi)} & -\frac{1}{1-\alpha(1-\phi)} \\ 0 & -\frac{\alpha(1-\phi)}{\alpha\phi} & 0 \\ 0 & -\alpha\left(1-\phi\right) & 0 \end{bmatrix} \begin{bmatrix} \mu_i^e \\ \widetilde{\zeta} \\ q \end{bmatrix}$$

whose solution is

$$\begin{bmatrix} c_i \\ z_i \\ n_i \\ p_i \end{bmatrix} = \begin{bmatrix} \alpha & \alpha\left(1-\phi\right) & -\alpha\left(1-\phi\right) \\ 1 & 0 & -1 \\ 1 & 0 & 0 \\ 1-\alpha & -\alpha\left(1-\phi\right) & \alpha\left(1-\phi\right) \end{bmatrix} \begin{bmatrix} \mu_i^e \\ \widetilde{\zeta} \\ q \end{bmatrix}.$$

Finally, consider first the following identity at the household level

$$M_{ij,0} + P_{i,0}\left(C_{ij,0} - C_{i,0}\right) = Q_t\left(Z - Z_{i,0}\right) + M$$

obtained by adding and subtracting P_iC_i to the right-hand side of (2) and noting $P_iC_i = Q_tZ_{i,t} + W_{i,t}N_{i,t} + \Pi_i$ according to (3). In particular, note that here we have assumed $N_{ij,t} = N_{i,t}$, which has no effect on island-level aggregates. At the level of each island then an increase (resp. a decrease) in the stock of the saving asset obtains when the island employ less (resp. more) capital then it is endowed with according to

$$\int M_{ij,0}dj = Q_t\left(Z - Z_{i,0}\right) + M,$$

whereas differences in savings at the individual level comes from differences in consumption according to

$$M_{ij,0} + P_{i,0}\left(C_{ij,0} - C_{i,0}\right) = \int M_{ij,0}dj$$

after using the previous relation.

Derivation of the price signal. The market price can be rewritten as

$$p_i = (1-\alpha)\Big(\kappa\mu_i + (1-\kappa)E[\mu_i|p_i]\Big) + \alpha(1-\phi)\left((1-\kappa)\int E[\mu_i|p_i]di - \tilde{\zeta}\right).$$

Given that the expectation $E_i[\mu_i|p_i]$ is common knowledge among households on island-i, the market price is informationally equivalent to

$$s_i \equiv \frac{1}{(1-\alpha)\kappa+\alpha(1-\phi)(1-\kappa)}(p_i-(1-\alpha)(1-\kappa)E[\mu_i|p_i]) =$$
$$= \frac{1-\kappa}{(1-\alpha)\kappa+\alpha(1-\phi)(1-\kappa)}\left((1-\alpha)\frac{\kappa}{1-\kappa}\mu_i+\alpha(1-\phi)\left(\int E[\mu_i|p_i]di-\frac{\tilde{\zeta}}{1-\kappa}\right)\right)$$

which removes the common knowledge term and rescales so that signal weights sum to one.

Defining
$$\gamma \equiv \frac{(1-\alpha)\kappa}{(1-\alpha)\kappa+\alpha(1-\phi)(1-\kappa)} \in (0,1)$$
we can rewrite the signal received by uninformed households on island i as

$$s_i = \gamma\mu_i + (1-\gamma)\left(\int E[\mu_i|s_i]di-\zeta\right).$$

where obviously $E[\mu_i|p_i]=E[\mu_i|s_i]$ and $\zeta \equiv \tilde{\zeta}/(1-\kappa)$ is distributed according to $N(0,\sigma_\zeta^2)$ and $\sigma_\zeta^2 \equiv \sigma_{\tilde{\zeta}}^2/(1-\kappa)^2$. The signal structure implied by this final equation captures the endogenous feedback effect of inference *from* prices back *into* prices, and it is on this structure that we focus our analysis in the text.

A2 Proofs of Propositions

Proof of Proposition 1. To prove uniqueness for $\gamma \geq 1/2$, observe that the function $a_i(a)$ is continuous, bounded above by γ^{-1}, and monotonically decreasing in the range $(-\infty,(1-\gamma)^{-1})$. From $\gamma \geq 1/2$, we have $(1-\gamma)^{-1} > \gamma^{-1}$. Thus $a_i(a)$ intersects the 45-degree line a single time.

To prove the existence of a_-, notice that $\lim_{a\to-\infty} a_i = \gamma^{-1}$ and $a_i((1-\gamma)^{-1}) = 0$. By continuity, an equilibrium $a_- \in (0,(1-\gamma)^{-1})$ must always exist. Moreover a_- must be monotonically decreasing in σ^2 as a_i is monotonically decreasing in σ^2.

We now assess the conditions under which additional equilibria may also exist. Because $\lim_{a\to\infty} a_i = \gamma^{-1}$, the existence of a second equilibrium (crossing the 45-degree line in Figure 1) implies the existence of a third. Thus, we must determine whether the difference $a_i(a)-a$ is positive anywhere in the range $a > (1-\gamma)^{-1}$. Such a difference is positive if and only if

$$\Phi(\sigma) \equiv \gamma(1-a(1-\gamma))^2(1-\gamma a) - a(1-\gamma)^2\sigma^2 > 0, \tag{A1}$$

which requires $a < \gamma^{-1}$ as a necessary condition. Therefore, if two other equilibria exist they must lie in $((1-\gamma)^{-1},\gamma^{-1})$. Fixing $a \in ((1-\gamma)^{-1},\gamma^{-1})$, $\lim_{\sigma\to0}\Phi(\sigma)$ is positive, implying that there always exists a threshold $\bar{\sigma}$ such that two equilibria $a_+, a_\circ \in ((1-\gamma)^{-1},\gamma^{-1})$ exist with $a_+ \geq a_\circ$ for $\sigma^2 \in (0,\bar{\sigma}^2)$. ∎

Proof of Proposition 2. Notice that $\partial\Gamma/\partial a > 0$ if and only if $\gamma < \min\{(1-\gamma)^{-1},\gamma^{-1}\}$. The left-hand side of the fixed-point expression in (25) is downward-sloping in a and falling in σ, implying that the fixed-point intersection a_u and a_- must increase as σ falls. Similarly, $a_\circ$

falls and a_+ grows as σ falls, which implies amplifying feedback for the former and dampening feedback for the latter. ∎

Proof of Proposition 3. To prove the limiting statement for $\gamma \geq 1/2$, consider any point $a_\delta = \frac{1-\delta}{1-\gamma}$ such that $\delta > 0$. We then have

$$a_i(a_\delta) = \frac{\gamma\delta^2}{\gamma^2\delta^2 + \sigma^2(1-\gamma)^2.} \tag{A2}$$

Since $\lim_{\sigma^2 \to 0} a_i(a_\delta) = \frac{1}{\gamma}$ for any δ, the unique equilibrium must converge to the same point. That the variance of this equilibrium approaches zero follows from equation (21).

To prove the limiting statement for $\gamma < 1/2$, recall the monotonicity of $a_i(a)$ on the range $(0, (1-\gamma)^{-1})$. Following the logic of Proposition 1, for any point a_δ in that range, $\lim_{\sigma^2 \to 0} a_i(a_\delta) = \gamma^{-1}$, while $a_i((1-\gamma)^{-1}) = 0$. Thus, the intersection defining a_- must approach $(1-\gamma)^{-1}$. An analogous argument for the point just to the right of $(1-\gamma)^{-1}$ establishes that a_- converges to the same value. Finally, the bounded monotonic behavior of $a_i(a)$ establishes that $\lim_{\sigma^2 \to 0} a_+ = \gamma^{-1}$ for the high equilibrium.

That the output variance of the high equilibrium in the limit $\sigma \to 0$ is zero follows from equation (22). The limiting variance of the two other limit equilibria can be established by noticing that (25) implies

$$\frac{\sigma^2}{(1-a(1-\gamma))^2} = \frac{\gamma(1-a\gamma)}{(1-\gamma)} \tag{A3}$$

which, substituted into (22), gives (27) for $a \to (1-\gamma)^{-1}$. ∎

Proof of Proposition 4. Suppose not, i.e. suppose that

$$\int E[\mu_i|s_i]di = \phi_\zeta \zeta + \phi_\varepsilon \varepsilon,$$

where ϕ_ε is the equilibrium effect of an extrinsic sentiment shock, ε, not related to fundamentals. Then, the price signal is equivalent to

$$s_i = \gamma\mu_i + (1-\gamma)\left((\phi_\zeta + 1)\zeta + \phi_\varepsilon\varepsilon\right)$$

Using the conjectured weights a_i, we have

$$\int a_i s_i di = a(1-\gamma)(\phi_\zeta + 1)\zeta + a(1-\gamma)\phi_\varepsilon\varepsilon$$

implying that

$$\begin{aligned} \phi_\zeta &= a(1-\gamma)(\phi_\zeta + 1) \\ \phi_\varepsilon &= a(1-\gamma)\phi_\varepsilon \end{aligned}$$

which cannot both be true unless $\phi_\varepsilon = 0$. Notice that, differently from the case with multiple sources of signals studied by Benhabib et al. (2015) (section 2.8 page 565), in our case an aggregate shock (our productivity shock) shows up directly in the signal, which ensures determinacy of the average expectation. This is equivalent to say that the analysis in Benhabib et al. (2015) is not robust to the introduction of correlation (no matter how small) in the v_{jt} shocks appearing in their endogenous signals. ∎

A3 Extensions

Correlation in island-specific shocks

We now consider a version of the model in which local shocks are correlated—that is $\mu_i = \mu + \epsilon_i$ where $\mu \sim N\left(0, \sigma_\mu^2\right)$—and there are no productivity shocks. Notice that previously, productivity shocks acted as noise in the signal, since households were only interested in the forecast of μ_i. Now, the aggregate term μ represents a common objective in the signal extraction problem of households.

Following the derivation of (17), the signal derived from the observed price is

$$s_i = \gamma(\mu + \epsilon_i) + (1-\gamma)\int E[\mu + \epsilon_i | s_i] di, \tag{A4}$$

which no longer embeds a productivity shock. Nonetheless, correlated fundamentals generate confusion between the idiosyncratic and common components of the signal. As before, the individual expectation of a household of type i is formed according to the linear rule $E[\mu + \epsilon_i | s_i] = a_i s_i$. Hence, the signal embeds the average expectation, which again causes the precision of the signal to depend on the average weight a. Following the analysis of the earlier section, the realization of the price signal can be rewritten as

$$s_i = \gamma\epsilon_i + \frac{\gamma}{1 - a(1-\gamma)}\mu, \tag{A5}$$

where a represents the average weight placed on the signal by other households. The variance of the average expectation is given by

$$\sigma_E^2(a) = \left(\frac{\gamma a}{1 - a(1-\gamma)}\right)^2 \sigma^2, \tag{A6}$$

which is slightly different from (22). The household's best response function is now given by

$$a_i(a) = \frac{1}{\gamma}\left(\frac{(1 - a(1-\gamma))^2 + (1 - a(1-\gamma))\,\sigma^2}{(1 - a(1-\gamma))^2 + \sigma^2}\right). \tag{A7}$$

While the best-response function in equation (A7) is slightly different than that of equation (25) for the case with productivity shocks, we can prove that the characterization of the limit equilibria is identical.

Proposition 6. *In the limit $\sigma_\mu^2 \to 0$, the equilibria of the economy converge to the same* *points as the baseline economy:*

$$\lim_{\sigma_\mu^2 \to 0} a_e^\mu = \lim_{\sigma^2 \to 0} a_e \qquad \lim_{\sigma_\mu^2 \to 0} \sigma^2(a_e^\mu) = \lim_{\sigma^2 \to 0} \sigma^2(a_e) \qquad \text{for } e \in \{u, -, \circ, +\} \tag{A8}$$

Proof. We can prove that a sentiment-free equilibrium with no aggregate variance exists for $a = \gamma^{-1}$ by simple substitution in (A7). The limiting variance of the other limit equilibrium at the singularity $a \to (1-\gamma)^{-1}$ can be established by noticing that (A7) implies that

$$\frac{\sigma^2}{(1-a(1-\gamma))^2} = \frac{1-a\gamma}{a\gamma} + \frac{1-a(1-\gamma)}{a\gamma}\frac{\sigma^2}{(1-a(1-\gamma))^2},$$

which gives

$$\frac{\sigma^2}{(1-a(1-\gamma))^2} = -\frac{1-a\gamma}{1-a}.$$

Substituted into (A6), this gives (27) for $a \to (1-\gamma)^{-1}$. ∎

More generally, it is possible to show that Propositions 1 through 3 follow identically, and their proofs proceed in parallel with only the obvious algebraic substitutions.

Flexible wages

The system of aggregate log-linear relations consist of 5 equations in 5 variables. This is written as

$$\begin{bmatrix} c \\ q \\ n \\ p \\ w \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & \alpha\phi & 1 & 0 \\ \frac{1}{\alpha\phi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-\alpha\phi & 0 & 1 \\ 0 & 0 & \delta & 0 & 0 \end{bmatrix} \begin{bmatrix} c \\ q \\ n \\ p \\ w \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \alpha(1-\phi) \\ 0 & -\frac{\alpha(1-\phi)}{\alpha\phi} \\ 0 & -\alpha(1-\phi) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{\mu} \\ \tilde{\zeta} \end{bmatrix}$$

whose solution is

$$\begin{bmatrix} c \\ q \\ n \\ p \\ w \end{bmatrix} = \begin{bmatrix} \alpha\frac{\phi}{1+\delta} & \alpha(1-\phi) \\ 1 & 0 \\ \frac{1}{1+\delta} & 0 \\ \frac{1}{1+\delta}(\delta+1-\alpha\phi) & -\alpha(1-\phi) \\ \frac{\delta}{\delta+1} & 0 \end{bmatrix} \begin{bmatrix} \bar{\mu} \\ \tilde{\zeta} \end{bmatrix}.$$

On island i we have

$$\begin{bmatrix} c_i \\ z_i \\ n_i \\ p_i \\ w_i \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & \frac{\alpha\phi}{1-\alpha(1-\phi)} & \frac{1}{1-\alpha(1-\phi)} & 0 \\ \frac{1}{\alpha\phi} & -\frac{\alpha(1-\phi)}{\alpha\phi} & 0 & 0 & 0 \\ 0 & -\alpha(1-\phi) & 1-\alpha\phi & 0 & 1 \\ 0 & 0 & \delta & 0 & 0 \end{bmatrix} \begin{bmatrix} c_i \\ z_i \\ n_i \\ p_i \\ w_i \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\alpha(1-\phi)}{1-\alpha(1-\phi)} & -\frac{1}{1-\alpha(1-\phi)} \\ 0 & 0 & -\frac{\alpha(1-\phi)}{\alpha\phi} & 0 \\ 0 & 0 & -\alpha(1-\phi) & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mu_i \\ \mu_i^e \\ \tilde{\zeta} \\ q \end{bmatrix}$$

whose solution is

$$\begin{bmatrix} c_i \\ z_i \\ n_i \\ p_i \\ w_i \end{bmatrix} = \begin{bmatrix} -\alpha\frac{\phi}{\delta+1} & \frac{\alpha}{1+\delta}\left(1+\delta\left(1-\phi\right)\right) & \alpha\left(1-\phi\right) & -\alpha\left(1-\phi\right) \\ 0 & 1 & 0 & -1 \\ -\frac{1}{1+\delta} & \frac{1}{1+\delta} & 0 & 0 \\ \alpha\frac{\phi}{1+\delta} & \frac{1}{1+\delta}\left(1-\alpha+\delta\left(1-\alpha\left(1-\phi\right)\right)\right) & -\alpha\left(1-\phi\right) & \alpha\left(1-\phi\right) \\ \frac{1}{1+\delta} & \frac{\delta}{1+\delta} & 0 & 0 \end{bmatrix} \begin{bmatrix} \mu_i \\ \mu_i^e \\ \widetilde{\zeta} \\ q \end{bmatrix}.$$

Individual savings are calculated as before.

Derivation of the price signal. The market price can be now rewritten as

$$p_i = \underbrace{\frac{\alpha\phi}{1+\delta}}_{A_1}\mu_i + \underbrace{\frac{1-\alpha+\delta(1-\alpha+\alpha\phi)}{1+\delta}}_{A_2}\Big(\kappa\mu_i + (1-\kappa)E[\mu_i|p_i]\Big) + \underbrace{\alpha(1-\phi)}_{A_3}\left((1-\kappa)\int E[\mu_i|p_i]di - \tilde{\zeta}\right).$$

Given that the expectation $E_i[\mu_i|p_i]$ is common knowledge among households on island-i, the market price is informationally equivalent to

$$\begin{aligned} s_i &\equiv \frac{1}{A_1 + A_2\kappa + A_3(1-\kappa)}(p_i - A_2(1-\kappa)E[\mu_i|p_i]) = \\ &= \frac{1}{A_1 + A_2\kappa + A_3(1-\kappa)}\left((A_1 + A_2\kappa)\mu_i + A_3(1-\kappa)\left(\int E[\mu_i|p_i]di - \frac{\tilde{\zeta}}{1-\kappa}\right)\right) \end{aligned}$$

which removes the common knowledge term and rescales so that signal weights sum to one.

Defining

$$\gamma \equiv \frac{A_1 + A_2\kappa}{A_1 + A_2\kappa + A_3(1-\kappa)} \in (0,1)$$

we can rewrite the signal received by uninformed households on island i,

$$s_i = \gamma\mu_i + (1-\gamma)\left(\int E[\mu_i|s_i]di - \zeta\right),$$

as in the text. Then our analysis of the signal extraction problem stays unaffected and our business cycle analysis remains qualitatively the same (only the mapping from γ to parameters changes).

References

Acharya, S., J. Benhabib, and Z. Huo (2017): "The Anatomy of Sentiment-Driven Fluctuations," Working Paper 23136, National Bureau of Economic Research.

Albagli, E., C. Hellwig, and A. Tsyvinski (2014): "Dynamic Dispersed Information and the Credit Spread Puzzle," NBER Working Papers 19788, National Bureau of Economic Research, Inc.

Amador, M. and P.-O. Weill (2010): "Learning From Prices: Public Communication and Welfare," *The Journal of Political Economy*, 118, pp. 866–907.

Angeletos, G.-M., F. Collard, and H. Dellas (2014): "Quantifying Confidence," Working Paper 20807, National Bureau of Economic Research.

——— (2016): "An Anatomy of the Business Cycle," Working Paper.

Angeletos, G.-M. and J. La'O (2013): "Sentiments," *Econometrica*, 81, 739–779.

Atakan, A. E. and M. Ekmekci (2014): "Auctions, Actions, and the Failure of Information Aggregation," *American Economic Review*, 104, 2014–48.

Azariadis, C. (1981): "Self-fulfilling prophecies," *Journal of Economic Theory*, 25, 380 – 396.

Barlevy, G. and P. Veronesi (2000): "Information Acquisition in Financial Markets," *The Review of Economic Studies*, 67, 79–90.

Barsky, R. B., S. Basu, and K. Lee (2015): "Whither News Shocks?" *NBER Macroeconomics Annual*, 29, 225–264.

Basu, S. and J. G. Fernald (1997): "Returns to Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy*, 105, 249–283.

Basu, S., J. G. Fernald, and M. S. Kimball (2006): "Are Technology Improvements Contractionary?" *The American Economic Review*, 96, pp. 1418–1448.

Benhabib, J. and R. E. Farmer (1994): "Indeterminacy and Increasing Returns," *Journal of Economic Theory*, 63, 19 – 41.

Benhabib, J., P. Wang, and Y. Wen (2015): "Sentiments and Aggregate Demand Fluctuations," *Econometrica*, 83, 549–585.

Bergemann, D., T. Heumann, and S. Morris (2015): "Information and volatility," *Journal of Economic Theory*, 158, Part B, 427 – 465, symposium on Information, Coordination, and Market Frictions.

Bergemann, D. and S. Morris (2013): "Robust predictions in games with incomplete information," *Econometrica*, 81, 1251–1308.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The Financial Accelerator in a Quantitative Business Cycle Framework," Elsevier, vol. 1, Part 3 of *Handbook of Macroeconomics*, chap. 21, 1341 – 1393.

Brunnermeier, M. K. and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104, 379–421.

Burguet, R. and X. Vives (2000): "Social Learning and Costly Information Acquisition," *Economic Theory*, 15, 185–205.

Cass, D. and K. Shell (1983): "Do Sunspots Matter?" *Journal of Political Economy*, 91, pp. 193–227.

Chahrour, R. and G. Gaballo (2017): "Learning from Prices: Amplification and Business Fluctuations," Working Paper Series 2053, European Central Bank.

Chahrour, R. and K. Jurado (2017a): "News or Noise? The Missing Link," *American Economic Review*, forthcoming.

——— (2017b): "Recoverability," Working paper.

Chahrour, R. and R. Ulbricht (2017): "Information-driven Business Cycles: A Primal Approach," Boston College Working Paper 925.

Cooper, R. and A. John (1988): "Coordinating Coordination Failures in Keynesian Models," *The Quarterly Journal of Economics*, 103, pp. 441–463.

Dmitriev, M. and J. Hoddenbagh (2017): "The financial accelerator and the optimal state-dependent contract," *Review of Economic Dynamics*, 24, 43–65.

Evans, G. W. and S. Honkapohja (2001): *Learning and Expectations in Macroeconomics*, Princeton University Press.

Gaballo, G. (2017): "Price Dispersion, Private Uncertainty, and Endogenous Nominal Rigidities," *The Review of Economic Studies*, rdx043.

Grossman, S. J. and J. E. Stiglitz (1976): "Information and Competitive Price Systems," *The American Economic Review*, 66, 246–253.

——— (1980): "On the Impossibility of Informationally Efficient Markets," *The American Economic Review*, 70, pp. 393–408.

Hellwig, M. F. (1980): "On the aggregation of information in competitive markets," *Journal of economic theory*, 22, 477–498.

Kiyotaki, N. and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105.

Lauermann, S., W. Merzyn, and G. Virág (2012): "Learning and Price Discovery in a Search Model," Working Paper.

Lorenzoni, G. (2009): "A Theory of Demand Shocks," *American Economic Review*, 99.

Lucas, Robert E., J. (1975): "An Equilibrium Model of the Business Cycle," *The Journal of Political Economy*, 83, 1113–1144.

Lucas, R. E. (1972): "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4, 103 – 124.

——— (1980): "Equilibrium in a Pure Currency Economy," *Economic Inquiry*, 18, 203–220.

Manuelli, R. and J. Peck (1992): "Sunspot-like effects of random endowments," *Journal of Economic Dynamics and Control*, 16, 193 – 206.

Manzano, C. and X. Vives (2011): "Public and private learning from prices, strategic substitutability and complementarity, and equilibrium multiplicity," *Journal of Mathematical Economics*, 47, 346–369.

Marcet, A. and T. J. Sargent (1989a): "Convergence of Least-Squares Learning in Environments with Hidden State Variables and Private Information," *Journal of Political Economy*, 97, pp. 1306–1322.

——— (1989b): "Convergence of Least Squares Learning Mechanisms in Self-referential Linear Stochastic Models," *Journal of Economic Theory*, 48, 337 – 368.

Milgrom, P. R. (1981): "Rational Expectations, Information Acquisition, and Competitive Bidding," *Econometrica*, 49, 921–943.

Morris, S. and H. S. Shin (1998): "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *The American Economic Review*, 88, pp. 587–597.

Rostek, M. and M. Weretka (2012): "Price Inference in Small Markets," *Econometrica*, 80, 687–711.

Venkateswaran, V. (2013): "Heterogeneous Information and Labor Market Fluctuations," Manuscript.

Vives, X. (2014): "On The Possibility Of Informationally Efficient Markets," *Journal of the European Economic Association*, 12, 1200–1239.