Journal of Monetary Economics 60 (2013) 789–805

Contents lists available at ScienceDirect

Journal of Monetary Economics

journal homepage: www.elsevier.com/locate/jme

Financial constraints, endogenous markups, and self-fulfilling equilibria

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ARTICLE INFO

Article history:
Received 7 May 2012
Received in revised form
10 June 2013
Accepted 14 June 2013
Available online 29 June 2013

Keywords:
Financial constraints
Endogenous markups
Self-fulfilling equilibria
Indeterminacy

ABSTRACT

Self-fulfilling equilibria and indeterminacy can easily arise in a simple financial accelerator model with reasonable parameter calibrations and without increasing returns in production. A key feature for generating indeterminacy in our model is the countercyclical markup due to the procyclical loan-to-output ratio. We illustrate, via simulations, that our financial accelerator model can generate rich business cycle dynamics, including hump-shaped output in response to demand shocks as well as autocorrelation in output growth rates.

1. Introduction

Since the seminal work of Bernanke and Gertler (1990)[1] the financial accelerator model with collateral constraints has been widely used to explain the amplification of shocks to business cycles Since firms and businesses face borrowing constraints to finance their working capital that depend on the collateral value of their assets and output, any downturn or negative shock that depresses the value of their collateral will curtail their ability to finance investments and increase their operating costs. This, in turn, will amplify the downturn. Conversely, any positive shock that appreciates the value of a firm's collateral will decrease the cost of external finance, increase profitability, and amplify the effect of the initial shock. This mechanism, however, suggests the possibility of self-fulfilling multiple equilibria: Optimistic expectations of higher output may well lead to increased lending to financially constrained firms. Even though our model has no increasing returns in production, the relaxation of the borrowing constraint implies that unit marginal costs can increase with output as firms compete for more labor and capital. In such a case, markups can become countercyclical and factor returns can increase sufficiently so that the expectation of higher output can become self-fulfilling. The purpose of this paper is to show that multiple equilibria and indeterminacy can easily arise in a simple financial accelerator model with realistic parameter calibrations, and that the model can reasonably match some of the quantitative features of economic data.

Our paper is related to several other papers on financial constraints and business cycle fluctuations that ascribe a significant part of such fluctuations to financial shocks. Examples include the work of Jermann and Quadrini (2012), Liu et al. (2013), and Gertler et al. (2012), among many others. But where do financial shocks come from? A growing literature links

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[1] See also Bernanke and Gertler (1995), Bernanke et al. (1996, 1999), and Kiyotaki and Moore (1997).

0304-3932/$ - see front matter
http://dx.doi.org/10.1016/j.jmoneco.2013.06.004

financial constraints to asset bubbles as a source of financial shocks. For example, Farhi and Tirole (2012), Miao and Wang (2012), Wang and Wen (2012), Miao et al. (2012), and Martin and Ventura (2012) study asset bubbles in economies with borrowing constraints. Those authors show that the growth and burst of asset bubbles can generate endogenous fluctuations in borrowing limits which result in booms and busts in the real economy. A shortcoming of such asset bubble models is that the bubbleless steady state is a sink, so such models cannot explain the recurrent fluctuations in the borrowing limits unless bubbles arrive exogenously.

Our paper is also related to the recent paper of Liu and Wang (forthcoming). They show that financial constraints can generate indeterminacy through an endogenous TFP channel as a result of resource reallocation across firms. In their model, firms differ in productivity, and in the absence of credit constraints, only the most productive firms survive while the unproductive firms with high costs perish. Some unproductive firms, however, continue to produce as the more productive firms are financially constrained by the value of their assets. An expected increase in aggregate output increases the value of the assets of all firms, and relaxes their borrowing constraints. This relaxation in the borrowing constraints allows more productive firms to expand production. This in turn pushes up the factor prices and increases the cost of production for the unproductive firms. As some of the unproductive firms stop producing the resource reallocation towards more productive firms generates endogenous increasing return to scale. Liu and Wang (forthcoming) show that their model is isomorphic to the Benhabib-Farmer (1994) model after aggregation.

Our model provides an alternative and complementary mechanism for indeterminacy to Liu and Wang (forthcoming). While they emphasize reallocation effects of financial constraints, our focus is on an endogenous markup channel. For this purpose, borrowing constraints are introduced into an otherwise standard Dixit–Stiglitz monopolistic competition model. Firms rent capital and hire labor in the competitive markets to produce differentiated intermediate goods. The firms, however, may default on their promise or contract to repay their debt. Therefore, firms face borrowing constraints when financing their working capital, determined by the fraction of firm revenues and assets that the creditors can recover, minus some fixed collection costs. This constrains the output as well as the unit marginal costs of firms. Given the fixed collection costs, however, if households expect a higher equilibrium output, they will be willing to increase their lending to firms, even if the marginal costs of firms rise and their markups decline as they compete for additional labor and capital.[2] At the new equilibrium, both output and factor returns will be higher. Despite the income effects on labor supply, the increase in wages associated with lower markups will allow employment and output to increase, so the optimistic expectations of higher output will be fulfilled.

The next section describes the baseline model.[3] Section 2.6 provides the main results characterizing the parameter ranges where equilibrium is indeterminate, as well as examples and graphical illustrations. Section 2.7 discusses the calibration of the parameters of our model based on estimates of these parameters in the literature. Section 3 offers extensions that relax the fixed cost component of the borrowing constraint, eliminates capacity utilization, and provides examples of indeterminacy under these extensions. Section 4 introduces sunspots into the discrete-time version of the model, calibrates it to match the moments of US data, and generates impulse responses to technology and sunspot shocks. Section 5 concludes. Proofs of Propositions 1 and 3 are contained in the online supplementary appendix.

2. A baseline model

Our baseline model includes fixed costs in the borrowing constraint and capacity utilization. This section shows conditions under which indeterminancy can arise.

2.1. Firms

Final goods producers: To illustrate the driving features of our model, it is best to start with a simple benchmark model of monopolistic competition and borrowing constraints for intermediate goods producers. The production side is a standard Dixit and Stiglitz model of monopolistic competition. There is a competitive final goods producer that combines a continuum of intermediate goods $Y_t(i)$ of unit mass to produce final goods Y_t according to the technology

$$Y_t = \left[\int Y_t^{(\sigma-1)/\sigma}(i)\, di\right]^{\sigma/(\sigma-1)}, \tag{1}$$

where $\sigma \geq 1$. The final goods producer solves

$$\max_{y_t(i)} \left[\int Y_t^{(\sigma-1)/\sigma}(i)\, di\right]^{\sigma/(\sigma-1)} - \int P_t(i) Y_t(i)\, di. \tag{2}$$

[2] The countercyclical markup is consistent with data (see Rotemberg and Woodford, 1999).

[3] Proofs of propositions presented in the next section can be found in the online appendix.

where $P_t(i)$ the price of the ith type of intermediate goods. The first-order conditions lead to the following inverse-demand functions for intermediate goods:

$$P_t(i) = Y_t^{-1/\sigma}(i) Y_t^{1/\sigma}, \tag{3}$$

where the aggregate price index is

$$1 = \left[\int P_t^{1-\sigma}(i)\, di \right]^{1/(1-\sigma)}. \tag{4}$$

Intermediate goods producers: The technology for producing intermediate goods is given by

$$Y_t(i) = AK_t^{\alpha}(i) N_t^{1-\alpha}(i), \tag{5}$$

where $A > 0$, $0 < \alpha < 1$. Symmetry is assumed: the technology for producing intermediate goods is the same for all i. The profit for the ith intermediate good producer is

$$\Pi_t(i) = P_t(i) Y_t(i) - w_t N_t(i) - r_t K_t(i). \tag{6}$$

Denote by $\phi_t = 1/A(r_t/\alpha)^{\alpha}(w_t/(1-\alpha))^{1-\alpha}$ the unit cost for the intermediate goods firms. Then their profit is

$$\Pi_t(i) = P_t(i) Y_t(i) - \phi_t Y_t(i). \tag{7}$$

Financial constraints: Unlike the final goods producer, producers of intermediate goods are assumed to face financial constraints due to limited enforcement. In our simple benchmark model, it is assumed that in the beginning of each period, the ith intermediate goods firm decides to rent capital $K_t(i)$ from the households and hire labor $N_t(i)$. The firm promises to pay $w_t N_t(i) + r_t K_t(i) \equiv b_t(i)$ at the end of the period. In this sense, the households are effectively providing credit to finance the firm's working capital. However, the firm may default on its contract or promise. It is assumed that if the firm does not pay its debt $b_t(i)$, the households can recover a fraction $\xi < 1$ of the firm's revenue $P_t(i) Y_t(i)$ by incurring a liquidation cost f.[4] One possibility is that the firm must pay the labor wages as production takes place and that creditors can always redeem the physical capital, but the interest on borrowing may not be fully recoverable. So if the household can recover $\xi P_t(i) Y_t(i) - f$, they will lend to the firm only if $\xi P_t(i) Y_t(i) - f$ can at least cover the wage bill plus principal and interest. Knowing that the household cannot recover more than $\xi P_t(i) Y_t(i) - f$, the firm will have no incentive to repay more than $\xi P_t(i) Y_t(i) - f$. The incentive-compatibility constraint for the firm then is

$$P_t(i) Y_t(i) - [w_t N_t(i) + r_t K_t(i)] \geq P_t(i) Y_t(i) - [\xi P_t(i) Y_t(i) - f], \tag{8}$$

or

$$w_t N_t(i) + r_t K_t(i) \leq \xi P_t(i) Y_t(i) - f. \tag{9}$$

After substituting $P_t(i)$ from Eq. (3) into Eq. (8), the profit maximization for the ith firm becomes

$$\max_{Y_t(i)} Y_t^{1-1/\sigma}(i) Y_t^{1/\sigma} - \phi_t Y_t(i), \tag{10}$$

subject to

$$\phi_t Y_t(i) + f \leq \xi Y_t^{1-1/\sigma}(i) Y_t^{1/\sigma}. \tag{11}$$

Given w_t, r_t, final output Y_t, and the borrowing constraint (11), the feasible choices of $Y_t(i)$ are represented by the shaded area in Fig. 1.[5]

Denote by $\mu_t(i)$ the Lagrangian multiplier of constraint (11). The first-order conditions for the profit maximization are

$$r_t K_t(i) = \alpha \phi_t Y_t(i), \tag{12}$$

$$w_t N_t = (1-\alpha) \phi_t Y_t(i), \tag{13}$$

and

$$\left(1 - \frac{1}{\sigma}\right) P_t(i) - \phi_t + \mu_t(i) \left[\xi \left(1 - \frac{1}{\sigma}\right) P_t(i) - \phi_t \right] = 0, \tag{14}$$

[4] To calibrate $\xi < 1$, one should note that outstanding credit market debt for domestic non-financial business corporate and non-corporate sectors in the US in 2012 stood at 12 trillion, or about 77% of GDP. See the Federal Reserve Flow of Funds Report, June 07, 2012, table D.3 in particular, at http://www.federalreserve.gov/releases/z1/Current/z1.pdf.

[5] In an alternative formulation, the firm also borrows enough to directly purchase its capital stock in addition to its needs for operating costs: $w_t N_t(i) + (1 + r_t) K_t(i)$. For simplicity, assume that lenders can always recover the capital stock in case of default but only a fraction of the output; the constraint becomes $w_t N_t(i) + (1 + r_t) K_t(i) \leq \xi P_t(i) Y_t(i) + K_t(i) - f$. After canceling $K_t(i)$ from both sides, one again obtains the constraint (9). In this case, however, debt would exceed GDP, since it would include the borrowed capital stock. Of course, some part of the capital stock may represent business equity that also yields a competitive return of r_t, rather than debt. In such a case, however, equity returns and principal may be subordinated to debt, but for simplicity, the present model abstracts from these considerations.

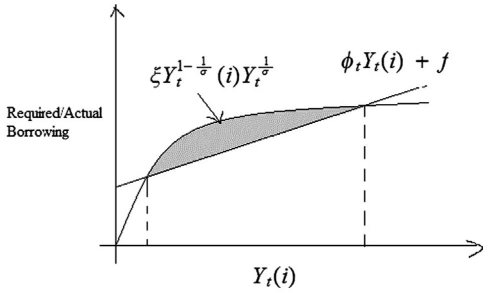

Fig. 1. The credit constraints and feasible output choice.

with the slackness condition

$$[\xi Y_t^{1-1/\sigma}(i)Y_t^{1/\sigma} - \phi_t Y_t(i) - f]\mu_t(i) = 0. \tag{15}$$

2.2. Households

Let us now turn to the intertemporal optimization problem faced by a representative household. To facilitate the stability analysis of a steady state, our model is set in continuous time. The instantaneous utility of the representative household is given by

$$\log C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}, \tag{16}$$

where C is consumption, N is labor supply, and $\chi \geq 0$. Taking the market interest rate r_t and wage w_t as given, the representative household maximizes

$$\int_0^\infty \left[\log C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}\right] e^{-\rho t}\, dt, \tag{17}$$

subject to

$$\dot{K}_t = r_t e_t K_t - \delta(e_t)K_t + w_t N_t - C_t + \Pi_t, \tag{18}$$

where K_t is the capital stock and K_0 is given. Endogenous capacity utilization is modeled along the lines of Greenwood et al. (1988). For simplicity, assume that households choose the capacity utilization rate e_t. As a result, the total effective capital available for production is $e_t K_t$. A higher e_t implies that the capital is more intensively utilized, at the cost of faster depreciation, so that $\delta(e_t)$ is a convex increasing function. The parameter ρ represents the discount rate, and Π_t is the total profit of all firms.

At this point, note that the indeterminacy results that follow will hold even in the absence of variable capacity utilization, but including it in our model improves the calibration results in Section 2.7.

The first-order conditions for the household's optimization problem are given by

$$\frac{\dot{C}_t}{C_t} = r_t e_t - \rho - \delta(e_t), \tag{19}$$

$$r_t = \delta'(e_t), \tag{20}$$

and

$$\psi N_t^\chi = \frac{1}{C_t} w_t. \tag{21}$$

2.3. Equilibrium

The equilibrium in the economy is a collection of prices $\{w_t, r_t, P_t(i)\}$ and quantities $\{K_t(i), N_t(i), Y_t(i), Y_t, K_t, N_t, e_t, \Pi_t\}$ such that (a) given the prices and the aggregate Π_t, the households choose K_t, e_t, and N_t to maximize their utility; (b) given $P_t(i)$, the final goods firms choose $\{Y_t(i)\}$ to maximize their profits defined in (2); (c) given w_t, r_t, and the financial constraint (11), the intermediate goods producers maximize their profit by choosing $K_t(i)$ and $N_t(i)$; and (d) all markets clear. Since firms are symmetric, $K_t(i) = e_t K_t$, $N_t(i) = N_t$, $P_t(i) = 1$, $Y_t(i) = Y_t$, and $\Pi_t(i) = \Pi_t = Y_t - w_t N_t - r_t e_t K_t$. The budget constraint becomes

$$\dot{K}_t = Y_t - C_t - \delta(e_t)K_t. \tag{22}$$

The wage w_t and the interest rate r_t, respectively, are

$$w_t = (1-\alpha)\phi_t \frac{Y_t}{N_t}, \tag{23}$$

and

$$r_t = \alpha\phi_t \frac{Y_t}{e_t K_t}. \tag{24}$$

Eq. (14) becomes

$$\left(1-\frac{1}{\sigma}\right)-\phi_t + \mu_t\left(\xi\left(1-\frac{1}{\sigma}\right)-\phi_t\right) = 0. \tag{25}$$

These calculations yield the following lemma regarding the financial constraint (11).

Lemma 1. *If $\xi(1-1/\sigma) < \phi_t < 1-1/\sigma$, then the financial constraint binds, that is*

$$\phi_t Y_t(i) + f = \xi Y_t^{1-1/\sigma}(i) Y_t^{1/\sigma}. \tag{26}$$

Based on fact that $Y_t(i) = Y_t$, one obtains

$$\phi_t = \xi - \frac{f}{Y_t}. \tag{27}$$

The intuition for Lemma 1 is as follows. The firms' profit function is $\Pi_t(i) = Y_t^{1-1/\sigma}(i)Y_t^{1/\sigma} - \phi_t Y_t(i)$, and if the profit for the marginal unit evaluated at equilibrium is $(1-1/\sigma)-\phi_t > 0$, the firms would have the incentive to increase their output. The revenue that households can recover in case of default exceeds the cost on the marginal unit if $\phi_t \le \xi(1-1/\sigma)$. It then suggests that the original output level cannot be optimal because firms would be able to borrow and produce more to increase their production and their profits. If $\xi(1-1/\sigma) < \phi_t < 1-1/\sigma$, however, firms would not be able to increase their production since the borrowing constraint binds: an additional unit of output would allows the firms to borrow only an additional $\xi(1-1/\sigma)$, which is not enough to cover the marginal unit production cost ϕ_t.[6]

Our focus will be on the parameters that make financial constraint (11) always binding in equilibrium. To summarize, the following system of equations fully characterize the equilibrium:

$$\frac{\dot{C}_t}{C_t} = \phi_t \frac{\alpha Y_t}{K_t} - \rho - \delta(e_t), \tag{28}$$

$$\dot{K}_t = Y_t - \delta(e_t)K_t - C_t, \tag{29}$$

$$\psi N_t^{\gamma} = \frac{1}{C_t}\phi_t \frac{(1-\alpha)Y_t}{N_t}, \tag{30}$$

$$Y_t = A(e_t K_t)^{\alpha} N_t^{1-\alpha}, \tag{31}$$

$$\phi_t \frac{\alpha Y_t}{e_t K_t} = \delta'(e_t) \tag{32}$$

$$\phi_t = \xi - \frac{f}{Y_t}, \tag{33}$$

[6] Formally,

$$\left(1-\frac{1}{\sigma}\right)-\phi_t = -\mu_t\left(\xi\left(1-\frac{1}{\sigma}\right)-\phi_t\right)$$

$$0 > \frac{1}{-\mu_t} = \frac{\left(\xi\left(1-\frac{1}{\sigma}\right)-\phi_t\right)}{\left(1-\frac{1}{\sigma}\right)-\phi_t}$$

$$sign\left(\left(1-\frac{1}{\sigma}\right)-\phi_t\right) = -sign\left(\xi\left(1-\frac{1}{\sigma}\right)-\phi_t\right)$$

$$\xi\left(1-\frac{1}{\sigma}\right)-\phi_t < \left(1-\frac{1}{\sigma}\right)-\phi_t \text{ if } \xi < 1$$

$$\xi\left(1-\frac{1}{\sigma}\right) < \phi_t < \left(1-\frac{1}{\sigma}\right).$$

subject to the constraint $\xi(1-1/\sigma) < \phi_t < 1-1/\sigma$. Following Greenwood et al. (1988), let the depreciation function be given by

$$\delta(e_t) = \delta_0 \frac{e_t^{1+\nu}}{1+\nu}. \tag{34}$$

It follows that

$$\phi_t \frac{\alpha Y_t}{e_t K_t} = \delta'(e_t) = \delta_0 e_t^{\nu}. \tag{35}$$

2.4. Steady state

Let us first solve for the deterministic steady state. Denote by X the steady-state value of X_t. Unfortunately, the model does not have a full analytical solution for the steady state with the fixed financial cost. The following paragraph describes the major steps for solving for steady state $\{Y, K, N, e, c, \phi, r, w\}$. This entails expressing the other variables as a function of the steady-state ϕ recursively.

Normalize A to 1. Using the first-order condition $\phi\alpha Y/K = \delta_0 e^{\nu+1}$ from Eq. (35), one obtains

$$\delta(e) = \frac{1}{1+\nu}\phi\frac{\alpha Y}{K}.$$

Eq. (28) then implies $(\nu/(1+\nu))\phi\alpha Y/K = \rho$, so the ratio of capital to output is

$$\frac{K}{Y} = \frac{\nu}{1+\nu}\frac{\phi\alpha}{\rho}.$$

Eq. (35) then implies

$$\delta(e) = \delta_0 \frac{e^{1+\nu}}{1+\nu} = \frac{1}{1+\nu}\phi\frac{\alpha Y}{K} = \frac{\rho}{\nu},$$

or

$$e = \left[\frac{(1+\nu)\rho}{\nu\delta_0}\right]^{1/(1+\nu)}.$$

Set $\delta_0 = (1+\nu)\rho/\nu$ to normalize e to 1. To solve for N, one first uses Eq. (29) to obtain the ratio of consumption to output:

$$\frac{C}{Y} = 1-\delta(e)\frac{K}{Y} = 1-\frac{\rho}{\nu}\frac{\nu}{1+\nu}\frac{\phi\alpha}{\rho} = 1-\frac{\phi\alpha}{1+\nu},$$

where the second line uses the fact $\delta(e) = \rho/\nu$ and $K/Y = (\nu/(1+\nu))\phi\alpha/\rho$. Then, use Eq. (30) to express

$$N = \left[\phi\frac{(1-\alpha)}{\frac{C}{Y}}\frac{1}{\psi}\right]^{1/(1+\chi)} = \left[\phi\frac{(1-\alpha)}{1-\frac{\phi\alpha}{1+\nu}}\frac{1}{\psi}\right]^{1/(1+\chi)}.$$

Given N, K and $e=1$, one can then obtain output using Eq. (31) as

$$Y = K^{\alpha}N^{1-\alpha} = \left(\frac{\nu}{1+\nu}\frac{\phi\alpha}{\rho}Y\right)^{\alpha}\left[\phi\frac{(1-\alpha)}{1-\frac{\phi\alpha}{1+\nu}}\frac{1}{\psi}\right]^{(1-\alpha)/(1+\chi)},$$

or

$$Y = \left(\frac{\nu}{1+\nu}\frac{\phi\alpha}{\rho}\right)^{\alpha/(1-\alpha)}\left[\phi\frac{(1-\alpha)}{1-\frac{\phi\alpha}{1+\nu}}\frac{1}{\psi}\right]^{1/(1+\chi)} \equiv Y(\phi).$$

Finally, from the definition of $\phi = \xi - f/Y$, one has

$$f = (\xi-\phi)Y(\phi) \equiv \Psi(\phi), \tag{36}$$

which determines the steady-state value of ϕ. In what follows, it is convenient to treat the steady-state value of marginal cost ϕ as a parameter and allow f to adjust. For the existence of a steady state, however, it is necessary to assume $\xi(1-1/\sigma) < \phi < 1-1/\sigma$. Define $\overline{\Psi} = \max_{0 \le \phi \le \xi}\Psi(\phi)$. Notice also that

$$\Psi(\xi) = \Psi(0) = 0. \tag{37}$$

Lemma 2. *If $0 < f < \overline{\Psi}$, then Eq. (36) has at least two solutions such that*

$$\Psi(\phi) - f = 0. \tag{38}$$

Lemma 3. $0<f<\Psi(\xi(1-1/\sigma))$ *there is a steady-state* ϕ *such that* $\xi(1-1/\sigma)<\phi<\xi$.[7]

Proof. Since $\Psi(\xi)-f<0$ and $\Psi(\xi(1-1/\sigma))-f>0$, by the intermediate value theorem there is a steady-state ϕ that lies between $\xi(1-1/\sigma)$ and ξ such that $\Psi(\phi)-f=0$. □

2.5. Log-linearization

After obtaining the steady state (Y,K,N,C,ϕ), the next step is to log-linearize the system of equations around the steady-state values. Denote by $\hat{X}_t$ the percentage deviation of variable X_t from its steady-state value X_t; that is, $\hat{X}_t=\log X_t-\log X$. Let $\dot{c}_t=d(\log C_t-\log C)/dt$ and $\dot{k}_t=d(\log K_t-\log K)/dt$. Then the log-linearized system of equations is

$$\dot{c}_t=\rho[\hat{Y}_t-\hat{K}_t+\hat{\phi}_t] \tag{39}$$

$$\dot{k}_t=\frac{(1+\nu)\delta}{\alpha\phi}(\hat{Y}_t-\hat{K}_t)-\left(\frac{(1+\nu)\delta}{\alpha\phi}-\delta\right)(\hat{C}_t-\hat{K}_t)-\delta(\hat{Y}_t+\hat{\phi}_t-\hat{K}_t) \tag{40}$$

$$\chi\hat{N}_t=\hat{\phi}_t+\hat{Y}_t-\hat{N}_t-\hat{C}_t \tag{41}$$

$$\hat{Y}_t=\alpha(\hat{K}_t+\hat{e}_t)+(1-\alpha)\hat{N}_t \tag{42}$$

$$\hat{e}_t=\frac{1}{1+\nu}(\hat{\phi}_t+\hat{Y}_t-\hat{K}_t) \tag{43}$$

$$\hat{\phi}_t=\frac{f/Y}{\xi-f/Y}\hat{Y}_t\equiv\gamma\hat{Y}_t \tag{44}$$

where $\delta=\rho/\nu$. Note that γ can also be defined by the steady-state value of ϕ as $\gamma=(\xi-\phi)/\phi$. Eliminating $\hat{N}_t$ from Eq. (41), $\hat{\phi}_t$ from Eq. (44), and $\hat{e}_t$ from Eq. (43) allows us to obtain the expression for $\hat{Y}_t$ in terms of capital and consumption. First, substitute $\hat{e}_t$ out of the production function to obtain

$$\hat{Y}_t=\frac{1}{1+\nu-(1+\gamma)\alpha}[(1+\nu)(1-\alpha)\hat{N}_t+\alpha\nu\hat{K}_t]\equiv\omega_1\hat{N}_t+\omega_2\hat{K}_t \tag{45}$$

where $\omega_1=(1+\nu)(1-\alpha)/(1+\nu-(1+\gamma)\alpha)$ and $\omega_2=(\alpha\nu/(1+\nu-(1+\gamma)\alpha)$. It is easy to check that $\omega_1+\omega_2>1$ if $\gamma>0$. Finally, it remains to substitute out $\hat{N}_t$. Combining the labor-demand and labor-supply curves yields

$$\hat{Y}_t=\lambda_1\hat{K}_t+\lambda_2\hat{C}_t, \tag{46}$$

where $\lambda_1=\omega_2(1+\chi)/(\chi+1-(1+\gamma)\omega_1), \lambda_2=-\omega_1/(\chi+1-(1+\gamma)\omega_1)$.

Using the factor $\hat{\phi}_t=\gamma\hat{Y}_t$ from (44), the log-linearized Euler condition becomes

$$\dot{c}_t=\rho[(1+\gamma)(\lambda_1\hat{K}_t+\lambda_2\hat{C}_t)-\hat{K}_t]. \tag{47}$$

Then Eq. (40) yields

$$\dot{k}_t=\left\{\frac{(1+\nu)\delta}{\alpha\phi}\lambda_1-\delta(1+\gamma)\lambda_1\right\}\hat{K}_t+\left\{\frac{(1+\nu)\delta}{\alpha\phi}(\lambda_2-1)+\delta-\delta(1+\gamma)\lambda_2\right\}\hat{C}_t \tag{48}$$

In a matrix form, one can write

$$\begin{bmatrix}\dot{k}_t\\ \dot{c}_t\end{bmatrix}=J\begin{bmatrix}\hat{K}_t\\ \hat{C}_t\end{bmatrix} \tag{49}$$

where

$$J=\delta\begin{bmatrix}\frac{(1+\nu)}{\alpha\phi}\lambda_1-(1+\gamma)\lambda_1 & \frac{(1+\nu)}{\alpha\phi}(\lambda_2-1)+1-(1+\gamma)\lambda_2\\ \nu[(1+\gamma)\lambda_1-1] & \nu(1+\gamma)\lambda_2\end{bmatrix}. \tag{50}$$

Notice that the above calculations used the factor $\rho=\delta\nu$ to substitute δ out in J.

2.6. Dynamics around the steady state

The local dynamics around the steady state are determined by the roots of J. The trace of the J is

$$\text{Trace}(J)=\delta\left[\frac{(1+\nu)}{\alpha\phi}\lambda_1-(1+\gamma)\lambda_1+\nu(1+\gamma)\lambda_2\right], \tag{51}$$

[7] Section 2.7 discusses the conditions that give a steady state with $\xi(1-1/\sigma)<\phi<(1-1/\sigma)$.

and the determinant of J is

$$\det(J) = \left\{ [(1+\gamma)\lambda_1 - 1 + \lambda_2]\left(\frac{1+\nu}{\alpha\phi} - 1\right) - \gamma\lambda_2 \right\}\delta^2\nu. \tag{52}$$

The roots of J, denoted x_1 and x_2, satisfy the following constraints:

$$x_1 + x_2 = \mathrm{Trace}(J), \tag{53}$$

and

$$x_1 x_2 = \det(J). \tag{54}$$

If $\det(J) > 0$ and Trace $(J) < 0$, then the roots x_1 and x_2 will both be negative, and the model will have local indeterminacy around the steady state. Since given other parameters the trace and determinant are functions of γ and ϕ, our analysis will first examine the possibility of indeterminacy in the parameter space of γ and ϕ. The mapping between (γ, ϕ) and (f, ξ) will then be used to establish the possibility of indeterminacy supported by the deep parameters of the model.

Proposition 1. *Let γ and ϕ satisfy the following two constraints:*

$$\gamma > \frac{(1+\nu)(1+\chi)}{\alpha(1+\chi) + (1+\nu)(1-\alpha)} - 1 \equiv \gamma_{\min}, \tag{55}$$

and

$$\gamma < \min\left(\frac{1+\nu-\alpha}{\alpha}, \frac{\left(\frac{1+\nu}{\phi} - \alpha\right)(1+\chi) - (1+\nu)(1-\alpha)}{\alpha(1+\chi) + (1+\nu)(1-\alpha)}, \frac{(1-\alpha)(1+\chi)}{(1+\nu)(1-\alpha)\frac{1}{1+\nu-\alpha\phi} + (1+\chi)\alpha} \right) \equiv \gamma_{\max}. \tag{56}$$

Then

$$\mathrm{Trace}(J) < 0, \quad \det(J) > 0 \tag{57}$$

Proof. See Appendix A.1.

To gain intuition for self-fulfilling expectations of higher output and higher factor rewards, let us first focus on labor-demand and labor-supply curves incorporating the equilibrium effects of the borrowing constraint on marginal costs and markups. The labor-demand curve is given by

$$\hat{w}_t = (1+\gamma)\hat{Y}_t - \hat{N}_t = \frac{\alpha\nu}{1+\nu-(1+\gamma)\alpha}\hat{K}_t + \left[\frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} - 1\right]\hat{N}_t, \tag{58}$$

and the labor-supply curve in the economy is

$$\hat{w}_t = \hat{C}_t + \chi\hat{N}_t. \tag{59}$$

The slope of the labor-demand curve is positive and steeper than that of the labor-supply curve under the condition $(1+\gamma) > (1+\nu)(1+\chi)/(\alpha(1+\chi) + (1+\nu)(1-\alpha))$ of the Proposition above. The indeterminacy result then parallels the results in Benhabib and Farmer (1994) and Wen (1998). However, unlike their works, our model has no increasing returns in the production technology. Instead, indeterminacy arises from the borrowing constraints and their indirect effects on marginal costs through wages and the rental rate on capital. If households expect a higher equilibrium output, they will be willing to increase their lending to firms. Given positive fixed collection costs f, an expected increase in output levels relaxes the borrowing constraint disproportionately more so that the unit marginal costs of firms, $\phi_t = \xi - f/Y_t$, can rise and markups can fall. This implies that as firms compete for inputs, factor rewards will also increase with Y_t. The labor-demand curve incorporating these general-equilibrium effects on marginal costs will then be positively sloped and steeper than the labor-supply curve. Normally, higher output levels increase the demand for leisure, so barring inferiority in preferences, the higher demand for labor will be contained by the income effect on labor supply. However, if labor demand slopes up more steeply than labor supply, employment will increase robustly as the labor-supply curve shifts to the left with income effects. The rise in hours as well as the accumulation of capital will raise output, so that the optimistic output expectations of households will be self-fulfilling.

Before turning to calibrations, we formally state the indeterminacy result of the paper. Define the set

$$\Omega = \{(\gamma, \phi) |\ \text{constraints (55) and (56) are satisfied}\}.$$

Proposition 2. *For $(\gamma, \phi) \in \Omega$, $\phi < 1 - 1/\sigma$ and $\gamma = (\xi - \phi)/\phi < 1/(\sigma - 1)$ (that is $\xi(1 - 1/\sigma) < \phi$), construct the set $\Theta = \{(\xi, f)|$ such that $\gamma = (\xi - \phi)/\phi$, $f = (\xi - \phi)Y(\phi)$, and $(\gamma, \phi) \in \Omega\}$. For $(\xi, f) \in \Theta$, the model is indeterminate.*

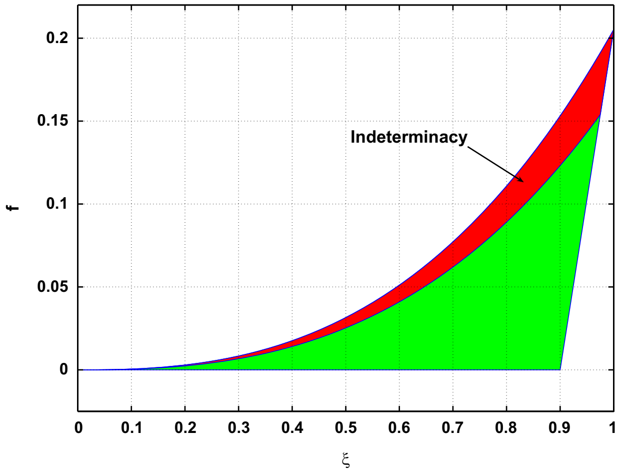

Fig. 2. Parameter spaces for indeterminacy. The shaded areas (the red areas together with the green areas) are the feasible ξ and f. The upper shaded areas with red color yield indeterminacy around the steady state. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

Proof. For $(\xi, f) \in \Theta$, by construction one can find a solution for ϕ and γ such that

$$\gamma = \frac{\xi - \phi}{\phi}; \quad f = (\xi - \phi)Y(\phi). \tag{60}$$

And since $(\gamma, \phi) \in \Omega$, one has Trace < 0, det $(J) > 0$ by Proposition 1. So the model is indeterminate around the steady state. □

2.7. Calibrations

Our calibration uses some standard parameter values in the literature. The quarterly discount rate is set to $\rho = 0.01$, so the quarterly discount factor is $\beta = 1/(1+\rho) = 0.99$. Labor supply is fully elastic, so $\chi = 0$. The factor share of capital is $\alpha = \frac{1}{3}$. The steady-state depreciation rate is given by Eq. (34), where the normalized steady-state utilization rate is $e = 1$. This gives $\delta_0 = 0.04333$. Under the assumption that the lifetime of new equipment averages 30 quarters, or that $\delta(e) = 0.0333$, one can solve for $\nu = 0.3$. The markup in our model is the inverse of the steady-state marginal cost $1/\phi - 1$. The steady-state markup is set to 12%, which implies a steady-state marginal cost $\phi = \xi - f/Y(\phi) = 0.88$. This also represents the steady-state ratio of non-financial private business debt to GDP. As noted earlier, this ratio in the data is a little lower at 0.77 and excludes all private debt issued by financial institutions, mostly to consumers, as well as state and federal debt. (See footnote 4 or the Federal Reserve Flow of Funds Report, June 07, 2012, table D.3 in particular, at http://www.federalreserve.gov/releases/z1/Current/z1.pdf.) Note that choosing the steady-state markup and therefore ϕ constrains the parameters ξ and f through Eq. (36): $\phi = \xi - f/Y(\phi)$. Furthermore, the steady-state marginal cost ϕ together with parameters ξ and σ must satisfy the inequality constraints of Proposition 2, $\xi(1 - 1/\sigma) < \phi < 1 - 1/\sigma$. In models with constant markups, the steady-state Dixit–Stiglitz elasticity σ is usually calibrated to immediately obtain a markup of 10–15%, and often σ is set to 10 (see for example Dotsey and King, 2005 or Sbordone, 2008, p. 20). Our calibration sets $\sigma = 10$ as well, even though σ does not exactly determine ϕ in our model but constrains it. Finally, one can calibrate the financial constraint parameters to assure that the markup is indeed 12%. If $\xi = 0.9768$, then Eq. (36) implies a steady-state value of the fixed liquidation cost $f = 0.1908$ in case of a default. Fig. 2 illustrates the regions of indeterminacy in the ξ–f plane. Finally, for the above calibration, one can easily check that the constraints $\xi(1 - 1/\sigma) < \phi < 1 - 1/\sigma$ are satisfied, so by Proposition 2 the steady state is indeed indeterminate, with the value of output at this steady state equal to 1.9711.

Note that for our calibrated parameters, the implied liquidation costs amount to 12% of the firm's total sale revenue.[8] For 88 firms that reorganized during 1982–1993, Alderson and Betker (1995) find that the liquidation costs measured as the percent of loss in going-concern value is large. For example, only 25% of firms have liquidation costs less than 12.8%. The mean and median liquidation costs among all the firms are 36.5% and 34.7%, respectively. Using a very comprehensive sample of corporate bankruptcies, Bris et al. (2006) find the direct expenses alone account for 8.1% of pre-bankruptcy assets for firms who filed for bankruptcy in the United States under Chapter 7 of the U.S. Bankruptcy Code, and 16.9% for firms who filed for bankruptcy under Chapter 11[9] (see Bris et al., 2006; table X, p. 1281). Similarly for international data Thorburn (2000) reports that direct expenses on average account for 19.1% (with medium 13.2%) of the market value of assets in

[8] The liquidation costs include the fixed cost f and the loss in output $(1-\xi)Y$. So in the steady state, they account for the fraction $f/Y + (1-\xi)$ of total output (sales revenue).

[9] Chapter 7 expenses mainly include expenses on debtor's attorney, accountant, and trustee. Chapter 11 expenses mainly include debtor expenses and unsecured creditors' expenses.

Sweden. So the implied liquidation costs in our model are in line with the data. In addition, both Thorburn (2000) and Bris et al. (2006) find that the expenses-to-assets ratio declines significantly with firm's scale, suggesting an important fixed cost component in the direct liquidation expenses.

Fig. 2 illustrates the combinations of f and ξ that yield indeterminacy with the other parameters set to $\nu = 0.3$, $\alpha = \frac{1}{3}$, $\rho = 0.01$, $\chi = 0$. The feasible parameter values for f and ξ are graphed in these two shaded areas. Consider the borrowing constraint $f = (\xi - \phi)Y(\phi)$. For a given ξ, there exist a minimum f and a maximum f consistent with the steady-state equilibrium such that $((\sigma-1)/\sigma)\xi < \phi < (\sigma-1)/\sigma$. Notice that if $f = 0$ and $\phi = \xi$, as long as $\xi < (\sigma-1)/\sigma$, the condition $((\sigma-1)/\sigma)\xi < \phi < (\sigma-1)/\sigma$ is automatically satisfied. This implies that for $\xi < (\sigma-1)\sigma$ the minimum f is zero. But if $\xi \geq (\sigma-1)/\sigma$, then $f=0$ (hence $\phi = \xi$) is no longer consistent with the equilibrium. If f is too small, then ϕ will be larger than $(\sigma-1)/\sigma$. Since $f = (\xi - \phi)Y(\phi)$ is decreasing in ϕ, the lower bound for f is $f_{\min}(\xi) = (\xi - (\sigma-1)/\sigma)Y((\sigma-1)/\sigma)$ for $\xi \geq (\sigma-1)/\sigma$. We can write it as $f_{\min}(\xi) = \max((\xi - (\sigma-1)/\sigma)Y((\sigma-1)/\sigma), 0)$ for $0 < \xi \leq 1$. On the other hand, if f is too large, then the marginal cost will fall below $((\sigma-1)/\sigma)\xi$. Now maximizing f over $((\sigma-1)/\sigma)\xi < \phi < (\sigma-1)/\sigma$, the upper bound for f for a given ξ is $f_{\max}(\xi) = (1/\sigma)\xi Y(((\sigma-1)/\sigma)\xi)$. For these feasible parameters, if f is greater than some cut-off level, then the implied γ will be bigger than $\gamma_{\min}$. It turns out that the condition $\gamma < \gamma_{\max}$ is automatically satisfied. The cut-off f can be determined by

$$f_{cut}(\xi) = \max\left\{ \frac{\gamma_{\min}}{1+\gamma_{\min}} \xi Y\left(\frac{\xi}{1+\gamma_{\min}} \right), \quad f_{\min}(\xi) \right\}.$$

For any f such that $f_{cut}(\xi) \leq f < f_{\max}(\xi)$, it will be the case that $\gamma > \gamma_{\min}$, so the model is locally indeterminate around the steady state. In Fig. 2, the indeterminacy region is shown in red.

3. Discussion and extensions of the model

The following section examines the individual components of the model to see which ones generate indeterminacy. The assumptions of fixed liquidation costs and variable capacity utilization can be relaxed and still result in indeterminacy.

3.1. The role of fixed costs

It is not the fixed liquidation costs per se that generate indeterminacy. It is the procyclical leverage generated by fixed liquidity costs that is the source of indeterminacy. Note that with fixed costs the debt-to-GDP ratio $b_t/Y_t = \xi - f/Y_t$ is procyclical. In what follows, we construct an example in which the firm's borrowing constraint is

$$\phi_t Y_t(i) \leq \xi\left(\frac{Y_t}{Y}\right) Y_t^{1-1/\sigma}(i) Y_t^{1/\sigma}, \tag{61}$$

where $\xi_t = \xi(Y_t/Y) < (\sigma-1)/\sigma$ is an increasing function of Y_t/Y with $\xi(1) = \phi < (\sigma-1)/\sigma$ and $\xi'(1) = \gamma\phi$. In this case, the marginal cost is $\phi_t = \xi_t$. The condition $\xi_t(\sigma-1)/\sigma < \phi_t < (\sigma-1)/\sigma$ is automatically satisfied, so the borrowing constraint is binding. The equilibrium can be characterized by a system of nonlinear equations similar to Eqs. (28)– (33), except that Eq. (33) is now replaced by $\phi_t = \xi_t = \xi(Y_t/Y)$. The log-linearized system of equations, however, is exactly the same as Eqs. (39)–(44). So one can directly invoke Proposition 2 to show indeterminacy if

$$\gamma_{\min}(\phi) < \gamma < \gamma_{\max}(\phi) \tag{62}$$

for $0 < \phi < (\sigma-1)/\sigma$. The more general function $\xi(Y_t/Y)$ that replaces the fixed liquidation cost eliminates the constraint relating γ and ϕ in the benchmark model and provides more flexibility in generating a range of parameters such that indeterminacy holds.

3.2. The role of capacity utilization

With the more general function $\xi(Y_t/Y)$, one can now show that endogenous capacity utilization is not essential for indeterminacy even though it makes indeterminacy possible for a wider range of parameters, as demonstrated by Wen (1998). The equilibrium is characterized by

$$\frac{\dot{C}_t}{C_t} = \phi_t \frac{\alpha Y_t}{K_t} - \rho - \delta, \tag{63}$$

$$\dot{K}_t = Y_t - \delta(e_t)K_t - C_t, \tag{64}$$

$$\psi N_t^{\chi} = \frac{1}{C_t} \phi_t \frac{(1-\alpha)Y_t}{N_t}, \tag{65}$$

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha}, \tag{66}$$

$$\phi_t = \xi_t = \xi\left(\frac{Y_t}{Y}\right). \tag{67}$$

The local dynamics around the steady state are

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} \frac{\rho+\delta}{\alpha\phi}\lambda_1 - \delta & \frac{\rho+\delta}{\alpha\phi}\lambda_2 - \frac{\rho+\delta}{\alpha\phi} + \delta \\ (\rho+\delta)[(1+\gamma)\lambda_1 - 1] & (\rho+\delta)(1+\gamma)\lambda_2 \end{bmatrix} \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix} \tag{68}$$

where $\lambda_1 = \alpha(1+\chi)/(\chi+1-(1+\gamma)(1-\alpha))$ and $\lambda_2 = -(1-\alpha)/(\chi+1-(1+\gamma)(1-\alpha))$.[10]

Proposition 3. *The model is indeterminate if*

$$\frac{(1+\chi)}{1-\alpha} - 1 < \gamma < \frac{1}{1-\alpha}\frac{\frac{1}{\phi} - \frac{\delta}{\rho+\delta}}{1 - \frac{\delta}{\rho+\delta}}(1+\chi) - 1, \tag{69}$$

and

$$\gamma < \frac{(1-\alpha)(1+\chi)}{(1+\alpha\chi) + (1-\alpha)\frac{\delta}{\frac{\rho+\delta}{\alpha\phi} - \delta}}. \tag{70}$$

Proof. See Appendix A.2.

Example 1. Suppose $\chi = 0$, $\sigma = 10, \rho = 0.01$, $\alpha = \frac{1}{3}$, $\sigma = 10$, and $\delta = 0.0333$. Assume that $\xi(1) = 0.88$ and $\xi'(1) = 0.88\gamma$. This implies $\phi = 0.88$ in the steady state. The model is indeterminate if $0.5 < \gamma < 0.5582$.

3.3. Credit constraints and firms' savings

In our model, credit constraints give firms incentives to build up a large amount of savings from retained earnings. This would allow them to eventually overcome the borrowing constraints. The possibility that the constrained agents will ultimately accumulate enough wealth to become fully self-financing is well-known in the extensive literature using models of credit constraints. To explain why firms nonetheless hold debt subject to credit constraints (see footnote 4), the literature has introduced several additional features into models of firm borrowing.[11] A standard approach is to assume that the borrower has a finite lifetime (see e.g., Bernanke et al., 1999; Song et al., 2011, among many others). Another common approach is to assume that the borrowers discount the future more than the lenders do (see e.g., Kiyotaki and Moore, 1997; Liu et al., 2013). A large corporate finance literature assumes that there are tax benefits of debt. Finally, as pointed by Jensen (1986), firms typically prefer debt financing because of the agency problem between the managers and the shareholders. The ideas are that the managers may use retained earnings to benefit their own private interests, for example by having large expense accounts and management perks. Arellano et al. (2012) have recently incorporated the ideas of Jensen (1986) into their model to prevent firms from becoming fully self-financing.

Our approach slightly modifies that of Arellano et al. (2012) to model the agency problem introduced by Jensen (1986). Our focus is on the conditions under which firms have no incentive to save. For this purpose, a discrete-time model illustrates the intuition. When the time interval between two periods t and $t+1$ approaches zero, the discrete model will converge its counterpart in continuous time.

Denote the value of a firm with retained earning $S_t(i)$ in period t as $V_t(S_t(i))$. The firm then solves

$$V_t(S_t(i)) = \max_{Y_t(i), S_{t+1}(i)} Y_t^{1-1/\sigma}(i)Y_t - \phi_t Y_t(i) + S_t(i) - \frac{S_{t+1}(i)}{(1-\tau)R_{ft}} + \beta E_t \frac{C_t}{C_{t+1}} V_{t+1}(S_{t+1}(i)), \tag{71}$$

with a credit constraint to be specified below. Assume that the return on the retained earning is the risk free rate, and satisfies $1 = R_{ft}\beta E_t C_t / C_{t+1}$. The parameter $1 > \tau > 0$ captures the agency problem between the managers and the shareholders as in Jensen (1986) if the managers can use a fraction τ of their retained earnings for their own non-verifiable private benefit. In addition, assume that the firm makes factor payments after the production is completed (maybe to assure that households do not walk away after receiving their factor payments). Since firms' savings (retained earnings) are liquid, they can also easily divert them if they default. Specifically, assume that the firm can divert a fraction $1-\omega$ of its retained earnings in default, as in Gertler et al. (2012). Instead of assuming a fixed liquidity cost, assume as before that the creditors can re-claim a fraction $\xi(Y_t/Y)$ of the firm's total sale revenue. As discussed above, the general function $\xi(Y_t/Y)$ allows more flexibility. The borrowing constraint (61) then changes to

$$\phi_t Y_t(i) \le \xi\left(\frac{Y_t}{Y}\right) Y_t^{1-1/\sigma}(i) Y_t^{1/\sigma} + \omega S_t(i), \tag{72}$$

[10] With minor modifications and reinterpretations, it is possible to transform our model so that its local dynamics around the steady states associated with (68) are isomorphic to the dynamics in Wen (1998).

[11] See Quadrini (2011) for a review of the recent literature.

Finally assume that $S_{t+1}(i) \geq 0$. Our interest is in obtaining parameters such that in the steady state $S_{t+1}(i) = 0$. Denote μ_t as the Lagrangian multiplier of the constraint (72). In a symmetric equilibrium, one has

$$\mu_t = \frac{(1-\frac{1}{\sigma})-\phi_t}{\phi_t-\xi(1-\frac{1}{\sigma})}, \tag{73}$$

and

$$\frac{\partial V_t(S_t(i))}{\partial S_t(i)} = 1+\mu_t\omega = 1+\omega\frac{(1-\frac{1}{\sigma})-\phi_t}{\phi_t-\xi(1-\frac{1}{\sigma})}, \tag{74}$$

The intuition for this equation is as follows. Here $\omega((1-1/\sigma)-\phi_t)/(\phi_t-\xi(1-1/\sigma))$ is external financial premium. Because of the borrowing constraint ($\mu_t > 0$), the firm values a dollar of internal saving more than a dollar: one dollar allows the firm to relax its credit constraint by ω dollars to produce an additional $1/(\phi_t-\xi(1-1/\sigma))$ units output. The marginal profit per unit of output is $(1-1/\sigma)-\phi_t$. The firm will have no incentive to save if, and only if,

$$\frac{1}{1-\tau} > \beta R_{ft} E_t \frac{C_t}{C_{t+1}} \frac{\partial V_{t+1}(S_{t+1}(i))}{\partial S_{t+1}(i)} = \beta R_{ft} E_t \frac{C_t}{C_{t+1}}\left[1+\omega\frac{(1-\frac{1}{\sigma})-\phi_{t+1}}{\phi_{t+1}-\xi(1-\frac{1}{\sigma})}\right], \tag{75}$$

So in the steady state with $S=0$, one has $\phi = \xi$ by Eq. (72). Then the above restriction implies

$$1 > (1-\tau)\left[1+\omega\frac{(1-\frac{1}{\sigma})-\phi}{\phi-\phi(1-\frac{1}{\sigma})}\right]. \tag{76}$$

Clearly either a sufficiently large τ or a sufficiently small ω will satisfy the above constraint so that $S_{t+1}(i) = 0$ in the steady state. For small fluctuations around the steady state, one will have $S_{t+1}(i) = 0$ and the equilibrium condition will then be the same as the one in the benchmark model.

Alternatively, one can assume that firms face stochastic death as in the model of Bernanke et al. (1999). Suppose a firm has a probability θ of exit and a measure θ of new firms enter in each period, so the number of firms is stationary. Our focus is on the conditions under which all firms have no incentive to save. This then requires

$$1 > \beta(1-\theta) R_{ft} E_t \frac{C_t}{C_{t+1}}\left[1+\omega\frac{(1-\frac{1}{\sigma})-\phi_{t+1}}{\phi_{t+1}-\xi(1-\frac{1}{\sigma})}\right]. \tag{77}$$

In the steady state, this becomes

$$1 > (1-\theta)\left[1+\omega\frac{(1-\frac{1}{\sigma})-\phi}{\phi-\phi(1-\frac{1}{\sigma})}\right].$$

Typically, the estimated external financial premium is small. For example, De Graeve (2008) estimates it to be 1.30% per annum, and Liu et al. (2013) estimate it to be about 3.60% per annum. Then if θ or τ exceeds 1% when the model is calibrated quarterly, firms will have no incentive to save. Note that one can make the external financial premium arbitrarily small to fit these estimated values by letting ϕ approach $1-1/\sigma$ even with $\omega = 1$. The model is still indeterminate if condition (62) is satisfied.

4. Simulations and impulse responses

Let us now use our calibrated model to compute the implied moments and cross moments of consumption, investment, hours, and output, and to study the impulse responses of our model. First, our model is written in discrete time and solved by log-linearizing the equations that characterize the equilibrium around the steady state. Our parameterization is the one used and discussed in Section 2.7. The basic parameters are $\beta = 1/(1+\rho) = 0.99$, $\alpha = \frac{1}{3}$, $\delta = 0.033$, $\sigma = 10$, and $\nu = 0.3$. Set $\xi = 0.9768$, $f = 0.1908$, and fix the productivity level to $A = 1$. These parameter values imply steady-state values $\phi = 0.88$ and $\gamma = (f/Y)/((\xi-f)/Y) = 0.11$. As discussed in Section 2.7, these parameters are not only in accordance with various estimates in the literature, but also put the model squarely within the indeterminacy range with a continuum of equilibria. To coordinate the expectations of agents, our model relies on sunspot shocks. Our analysis below studies whether the various moments and impulse responses generated by our model with sunspots can match the data.

Let us begin without productivity shocks. The model's solution takes the form

$$\begin{pmatrix} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{pmatrix} = M \begin{pmatrix} \hat{K}_t \\ \hat{C}_t \end{pmatrix} + \begin{pmatrix} 0 \\ \varepsilon_{t+1} \end{pmatrix}, \tag{78}$$

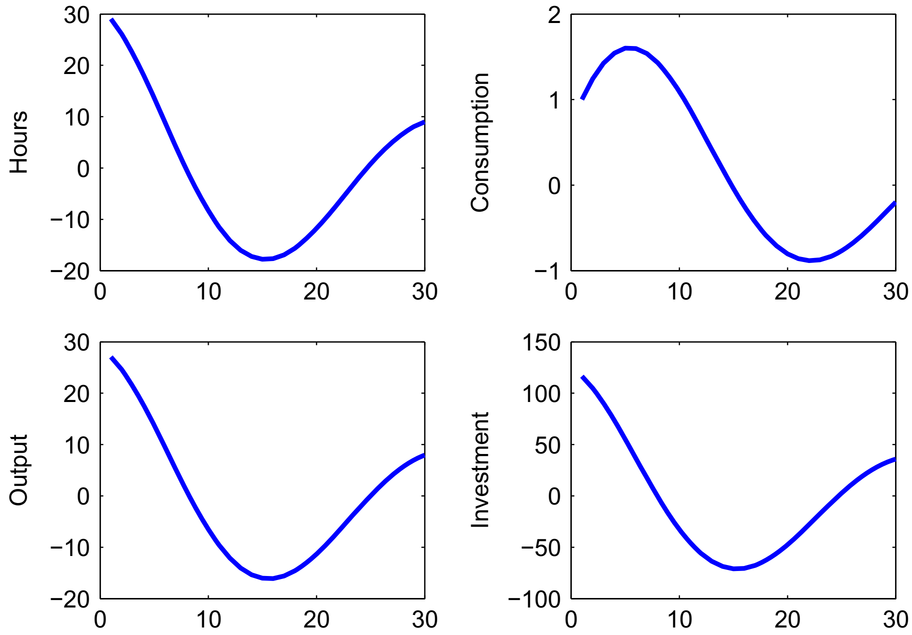

Fig. 3. Impulse responses to a consumption (sunspot) shock.

where M is a two-by-two matrix and $\varepsilon_{t+1} = \hat{C}_{t+1} - E_t\hat{C}_{t+1}$ is the zero-mean i.i.d. sunspot shock. Since the model is linearized, one can compute the relative moments and cross moments analytically.[12] The remaining variables can be written as functions of $\hat{K}_t$ and $\hat{C}_t$:

$$\begin{pmatrix} \hat{Y}_t \\ \hat{I}_t \\ \hat{N}_t \\ \hat{e}_t \end{pmatrix} = H \begin{pmatrix} \hat{K}_t \\ \hat{C}_t \end{pmatrix}, \tag{79}$$

where H is a four-by-two matrix.

Fig. 3 shows the impulse responses of output, investment, consumption, and hours to an unexpected 1% increase in the initial consumption level induced by the agent's optimistic expectations about future income, under the calibration of parameters given above.

These impulse-response functions resemble those obtained in the models with increasing returns to scale. Fig. 3 also shows that output, investment, consumption, and hours commove. The impulse responses also demonstrate that labor is slightly more volatile than output, an important feature of the data that the standard RBC model has difficulty explaining with a TFP shock. The impulse responses also exhibit cycles in output, investment, consumption, and hours, so the model has the potential to explain the boom-bust patterns often observed in data. However, as in the models with increasing returns to scale, the extremely large impact of autonomous consumption on output and investment seems empirically unjustified. In the impact period, a 1% increase in consumption leads to a 27% increase in output and a 116% increase in investment.

These volatile responses of output and investment can be understood by studying the effect of consumption on labor. Equating the labor demand (58) and labor supply (59), one obtains

$$\hat{N}_t = \frac{1}{\frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} - 1 - \chi}\hat{C}_t. \tag{80}$$

where $(1+\gamma)(1+\nu)(1-\alpha)/(1+\nu-(1+\gamma)\alpha-1)$ is the slope of the labor-demand curve and χ is the slope of the labor-supply curve. When these two slopes are close, a 1% increase in autonomous consumption increase can lead to huge increases in labor and hence output. Denote s as the ratio of steady-state investment to income. Then from the resource constraint,

$$s\hat{I}_t + (1-s)\hat{C}_t = \hat{Y}_t, \tag{81}$$

[12] Of course if the model can match the moments and cross moments relative to output, one can then choose the standard deviation of the sunspot shock, or later below the standard deviation of the sunspot and productivity shocks, so as to match the absolute volatilities of output and other variables to that of the data.

Table 1
Sample and model moments.

Var	US sample			Model		
	σ_X/σ_Y	$corr(X,Y)$	$corr(X_t,X_{t-1})$	σ_X/σ_Y	$corr(X,Y)$	$corr(X_t,X_{t-1})$
Y	1.00	1.00	0.87	1.00	1.00	0.91
N	1.01	0.88	0.92	1.08	0.99	0.91
C	0.52	0.83	0.90	0.07	0.48	0.97
I	3.33	0.92	0.92	4.31	0.99	0.91
ϕ	0.32	0.16	0.70	0.11	1.00	0.91

Note: Variables (Y, N, C, I, ϕ) denote output, labor (in hours), consumption, investment and marginal cost, respectively. The marginal cost in the data can be computed via $\phi = labor\ share/(1-\alpha)$. σ_X/σ_Y is the standard deviation of variable X relative to output, $corr(X,Y)$ computes the correlation between X and output, and $corr(X_t, X_{t-1})$ computes the first-order autocorrelation of X_t.

Table 2
Moments with correlated TFP and sunspot shocks.

Var	Model with correlated shocks			The RBC model		
	σ_X/σ_Y	$corr(X,Y)$	$corr(X_t,X_{t-1})$	σ_X/σ_Y	$corr(X,Y)$	$corr(X_t,X_{t-1})$
Y	1.00	1.00	0.98	1.00	1.00	0.95
N	0.96	0.95	0.99	0.53	0.73	0.90
C	0.37	0.55	0.99	0.62	0.86	0.99
I	3.38	0.96	0.98	2.65	0.88	0.91
ϕ	0.11	1.00	0.98	0	NA	NA

The RBC model refers to $f=0$, so $\gamma = 0$, and $\phi = \xi$ is a constant. We select the parameter values such that the two models have the same steady state. For the RBC model, we use TFP shocks with $\rho_a = 0.98$ as the only driving force.

so it is clear that the combination of smooth consumption and volatile income will make investment even more volatile as $s \ll 1$. In the current calibration, $s = 0.23$. So the response of investment upon impact will be about 4.4 times that of output.

Table 1 reports some basic moments of the linearized model assuming that sunspots are the only driving force. All moments for the model are calculated analytically. The table shows that all variables are positively correlated with output. The correlations between them are also highly persistent. By our construction, the sunspots are i.i.d., so the persistence of the variables is not due to the persistence of exogenous shocks, but comes from the internal propagation mechanism of the model. Table 1 confirms that labor is slightly more volatile than output. The relative volatility of labor is 1.10 in the data and 1.08 in the model, while the relative volatility of labor is 0.53 in the real business cycle model (see Table 2).

To better match the relative volatilities of consumption and output, a TFP shock can be added to the model. Assume that the technology level in the economy follows an AR(1) process

$$\hat{A}_{t+1} = \rho_a \hat{A}_t + \sigma_a \varepsilon_{at+1}. \tag{82}$$

Following Benhabib and Wen (2004), assume that the sunspots shocks and technology shocks are correlated. Following King and Rebelo (1999), assume that $\rho_a = 0.98$. The technology shock ε_{at} and sunspot shocks ε_t are assumed to be perfectly correlated, and the relative volatility of sunspot to technology shocks is set to $\sigma_s/\sigma_\varepsilon = 1.5$. This brings the relative volatility of consumption closer to data. The moments obtained using correlated TFP shocks and sunspots shocks are in Table 2.

Hump-shaped output dynamics: The above simulation exercises show that our model with indeterminacy is similar in its ability to the RBC model to match some key moments of the data. The simulation exercise that follows illustrates how our indeterminacy model can also predict some aspects of actual fluctuations that standard RBC models cannot explain, such as the hump-shaped, trend-reverting impulse response of output to transitory demand shocks and the substantial serial correlation in output growth rates in the data (see Cogley and Nason, 1995). Since there is some significant empirical evidence favoring demand shocks as a main source of business cycles (e.g., see Blanchard and Quah, 1989; Watson, 1993; Cogley and Nason, 1995; Benhabib and Wen, 2004), it is important to examine whether demand shocks can generate persistent business cycles. Our simulations include two types of demand shocks as in Benhabib and Wen (2004): government spending shocks and preference shocks. With preference shocks, the period-by-period utility function is now given by $U = \exp(\Delta_t)\log C_t - \psi N_t^{1+\chi}/(1+\chi)$. Assume that the preference shocks Δ_t follow an AR(1) process, namely $\Delta_t = \rho_\Delta \Delta_{t-1} + \varepsilon_{\Delta t}$. With government spending G_t in period t, the resource constraint changes to $\dot{K}_t = Y_t - \delta(e_t)K_t - C_t - G_t$. Assume that $\log(G_t) = \rho_g \log(G_{t-1}) + \varepsilon_{gt}$. The autocorrelations are set to $\rho_g = \rho_\Delta = 0.90$ as in Benhabib and Wen (2004). To highlight the effect of indeterminacy on the propagation mechanism of RBC models, Fig. 4 graphs the impulse responses to a persistent government spending shock with and without indeterminacy. Fig. 5 graphs the impulse response of the model to a persistent preference shock. For the model without indeterminacy, f is set to zero and ξ is reset to 0.88 so that the

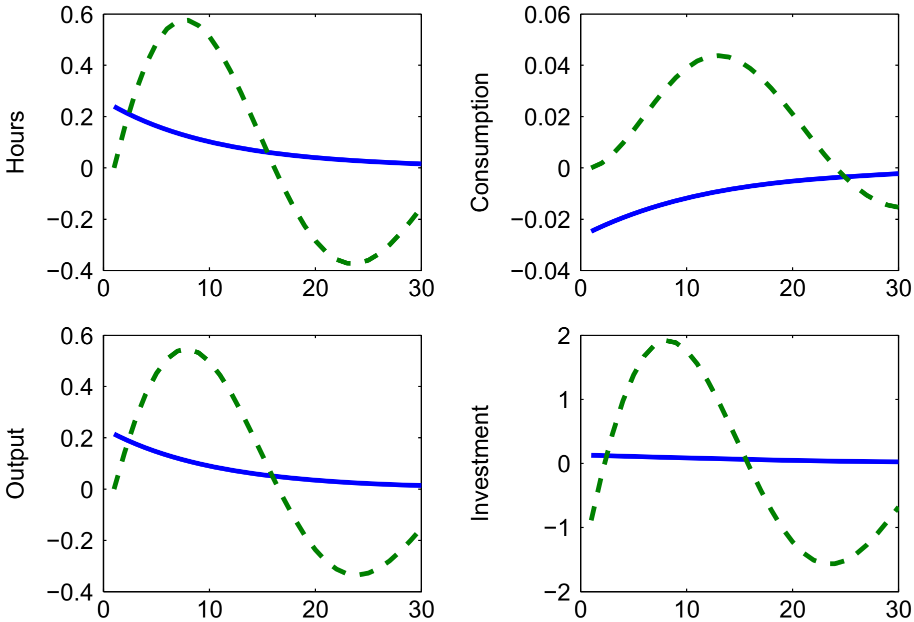

Fig. 4. Impulse responses to a government spending shock. Solid lines are responses under determinacy ($f=0$) and dashed lines are responses under indeterminacy.

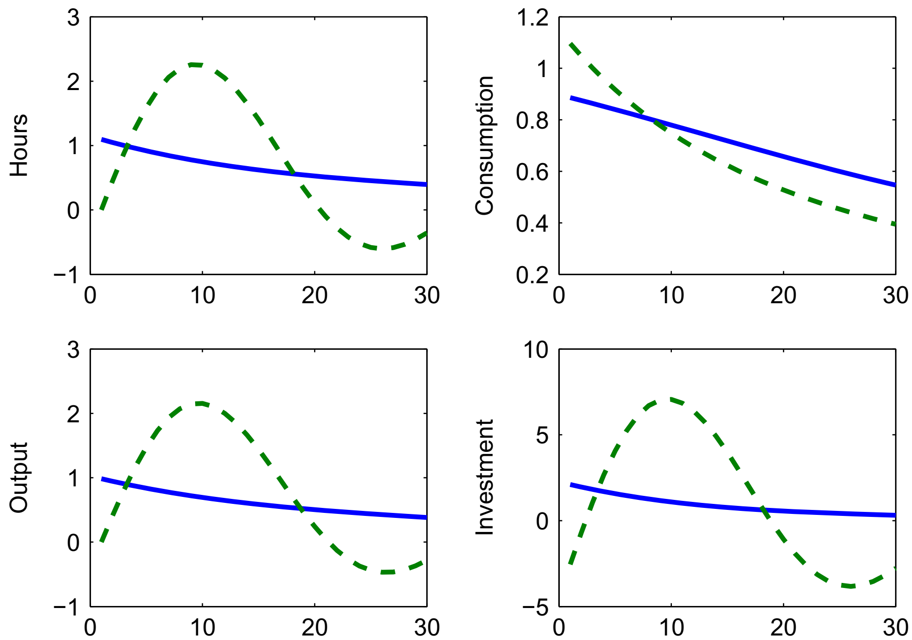

Fig. 5. Impulse responses to a preference shock. Solid lines are responses under determinacy ($f=0$) and dashed lines are responses under indeterminacy.

models with and without indeterminacy have the same steady state. The steady-state government-spending-to-GDP ratio is set to 0.2 as in Benhabib and Wen (2004).

Several features of Fig. 4 deserve particular mention. First, in the case of $f=0$, the marginal cost $\phi_t=\xi$ is a constant. Hence the impulse responses of our model with financial constraints resemble those of a standard RBC model. Figs. 4 and 5 show that the RBC model without indeterminacy has difficulty in generating realistic business cycle fluctuations. For the determinate case, Fig. 4 shows that consumption and investment move against each other after a positive government spending shock. An increase in government spending generates a negative wealth effect, which reduces both consumption and leisure. The decrease in leisure leads to an increase in output, and an increase in output together with a decrease in consumption imply that investment has to increase. Second, even though the model generates comovement without indeterminacy under persistent preference shocks, the responses of output to such demand shocks are monotonic. Neither government spending shocks nor preference shocks can generate the hump-shaped output dynamics observed in the data. And these monotonic and persistent output responses to demand shocks mostly come from the persistence of shocks, not from an inner propagation mechanism of the model. If the persistence of the shocks is reduced, the persistence of output responses will be reduced accordingly. When the model is indeterminate, the responses of output to both the government spending shocks and the preference shocks are dramatically changed. For the indeterminate case, Figs. 4 and 5 clearly show persistent and hump-shaped responses of output to both shocks. In addition, these persistent responses of output are not due to the persistence in shocks. As Fig. 3 has already demonstrated, the model with indeterminacy can generate persistent fluctuations even under i.i.d. shocks. Figs. 4 and 5 again highlight the similarity of our indeterminacy model to models with increasing returns to scale, so it can explain other puzzles. For example, Benhabib and Wen (2004) demonstrate that their indeterminacy model based on increasing returns to scale can explain the forecastable movement puzzle pointed out by Rotemberg and Woodford (1996). It is easy to show that our indeterminacy model can also replicate the highly forecastable comovements observed in changes in output, hours, investment, and consumption highlighted by Rotemberg and Woodford (1996). To avoid repetition, such a simulation exercise is not presented here; instead, readers can refer to Benhabib and Wen (2004). In brief, these simulation exercises illustrate the ability of our indeterminacy model to replicate rich business cycle dynamics observed in the data.

5. Conclusion

Borrowing or collateral constraints can be a source of self-fulfilling fluctuations in economies that have no increasing returns to scale in production. Expectations of higher output can relax borrowing constraints, and firms can expand their output by bidding up factor prices and eliciting a labor supply response that allows the initial expectations to be fulfilled. The parameter ranges and markups that allow self-fulfilling expectations to occur are within realistic ranges and compatible with US macroeconomic data. Simulating our data, we obtain moments and impulse responses that match the US macroeconomic data reasonably well.

Acknowledgments

The authors thank the editor and referee for helpful comments. Pengfei Wang acknowledges financial support from Hong Kong Research Grant Council (Project 645811).

Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org.10.1016/j.jmoneco.2013.06.004.

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