Financial Cycles with Heterogeneous Intermediaries

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Abstract

This paper develops a dynamic macroeconomic model with heterogeneous financial intermediaries and endogenous entry. It features time-varying endogenous macroeconomic risk that arises from the risk-shifting behaviour of the cross-section of financial intermediaries. We show that when interest rates are high, a decrease in interest rates stimulates investment and increases financial stability. In contrast, when interest rates are low, further stimulus can increase aggregate risk while inducing a fall in the risk premium. In this case, there is a trade-off between stimulating the economy and financial stability.

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1 Introduction

The global financial crisis of 2007-2008 has called into question our modelling of the role of financial intermediaries in the economy. The financial sector, far from being a veil, plays a key role in the transmission and amplification of shocks and in driving fluctuations in aggregate risk. The precise mechanisms by which this happens are still debated. In particular, understanding the underlying forces driving endogenous systemic risk, the concentration of risk in some balance sheets and the interactions between monetary policy and financial stability are key issues. A long tradition of scholars such as Fisher (1933), Minsky (1977) and Kindleberger (1978) argued that financial sector expansions and contractions are important drivers of fluctuations in economic activity and financial stability. Kaminsky and Reinhart (1999), Reinhart and Rogoff (2009*b*) and Gourinchas and Obstfeld (2012) among others show that financial crises tend to be preceded by a rapid expansion of credit. Schularick and Taylor (2012) study the long run dynamics of money, credit and output over the period 1870-2008 and find that financial crises tend to be "booms gone bust". Financial cycles have been analysed in the literature typically through the lenses of models featuring one representative financial intermediary subject to capital market frictions. In contrast, we emphasise the importance of heterogeneity in risk taking across financial intermediaries in driving aggregate outcomes.

Changes in market shares due to increased risk taking by some intermediaries play a large role in the risk build-up phase of a crisis. For Sweden, Englund (2016) explains how between 1985 and 1990 the rate of increase of lending by financial institutions jumped to 16% due in part to deregulation. There were rapid shifts in market shares: between 1985 and 1988 the lending shares of Sparbanken Sverige and of Gota increased from 20.8 to 22.1% and from 7.9% to 8.9% of all bank lending respectively, while more conservative players held back. There was a significant correlation between the rate of credit expansion and the subsequent credit losses in the crisis, leading to bailouts. For Spain, Santos (2017) emphasizes how between 2002 and 2009, the regional banks (cajas) leveraged a lot to invest in the real estate sector, their combined balance sheet reaching 40% of Spanish GDP in 2009. Some (Bancaja) more than tripled their balance sheet while more "conservative" ones (Catalunya Caixa) doubled it. They ended up all being nationalized in the crisis. In Germany, as described by Hellwig (2018), Landesbanken and local savings banks whose borrowing was guaranteed by German Lander and municipalities until 2005 took the opportunity to gorge on cheap funds increasing their debt by around €250bn over the period 2001 to 2005. As the market for Asset Backed Commercial Paper (ABCP) doubled in size between 2000 and 2006 the share of Landesbanken in that market grew from 6% to 8%. In fact, the weaker was the stand-alone credit rating of the Landesbank, the larger the subsequent increase in risk taking. Deutsche Bank leveraged up to quadruple the size of its balance sheet from about €0.5 trillion in early 1990s to about €2 trillion in 2008 as a RoE of 25% was regularly targeted by the bank CEO. German taxpayers ended up paying about €70 billion to support their financial institutions. In the US, as noted by Korinek and Nowak (2017), large risk taking by some financial intermediaries also played a big role in the mortgage boom. Countrywide increased its size to capture more than 20% of the US market in 2006 and had to be rescued in 2008. Wilmarth (2013) mentions the high risk culture of the too-big-to-fail Citigroup as a possible explanation behind the massive expansion of its balance sheet during the boom years. Citigroup nearly doubled the share of its subprime mortgage business from 10% in 2005 to 19% in 2007. During the period 2007 to the spring of 2010, Citigroup recorded more than $130 billion in credit losses and write-downs. It received its first government bailout in October 2008 (it was bailed out 3 times in total).

Accounting for such heterogeneity in risk taking behaviour and its macroeconomic implications is important. A large literature has recognized the centrality of financial frictions such as collateral, net worth or Value-at-Risk (VaR) constraints for representative firms and intermediaries[1]. But that literature has not allowed for heterogeneity in risk taking; it has focused on the transmission and the amplification of shocks rather than the endogenous risk build up phase of the financial cycle and the concentration of risk in some balance sheets. We build a novel framework with a continuum of

[1]See in particular, but not only, Bernanke and Gertler (1989), Kiyotaki and Moore (1997)), Lorenzoni (2008), Mendoza (2010), Gertler and Kiyotaki (2015), Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2012), Farhi and Werning (2016), Aoki, Benigno and Kiyotaki (2016) who use collateral constraints; Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013) where an intermediary cannot raise more than a fixed amount of equity; Jon Danielsson, Hyun Song Shin and Jean-Pierre Zigrand (2010), Adrian and Shin (2010), Bruno and Shin (2015*b*), Coimbra (2016) and Acharya et al. (2017) where an intermediary faces Value-at-Risk constraints.

financial intermediaries heterogeneous in their VaR constraints[2] and a moral hazard friction due to limited liability and government guarantees which generate risk-shifting. Heterogeneous VaRs may reflect different risk attitudes by the boards of financial intermediaries or different implementations of regulatory constraints across institutions and supervisors.

In our model, the dynamics of the distribution of leverage across intermediaries is a key determinant of financial stability. When high risk-taking intermediaries are dominant, they increase the price of risky assets and concentrate most of the aggregate risk on their balance sheets. The leverage distribution across intermediaries is positively skewed and financial stability is lower, as a large fraction of assets are in the hands of intermediaries with a high probability of default. This tends to happen when financing costs are low, perhaps due to deregulation, a savings glut, expansionary monetary policy or when volatility is low. We link financial stability explicitly to the risk of intermediary default. This is unlike most of the previous literature where systemic risk is mapped into the probability that a financing constraint binds in the future. Credit booms generated by low costs of funds will then be associated with worsened financial stability and lower risk premia. This is consistent with the evidence reported in Krishnamurthy and Muir (2017) that spreads tend to be low before crises.[3] Our model also generates booms driven by high expected productivity which do not increase financial instability, unlike those driven by lower costs of funds (or low volatility). Risk-shifting and different VaR constraints across financial intermediaries jointly generate heterogeneous willingness to pay for risky assets and a link between aggregate risk taking and the distribution of leverage[4]. We provide therefore a different and complementary view of financial fragility from Gennaioli, Shleifer and Vishny (2012). For these authors, excess risk taking comes from neglecting that some improbable risk materializes. In our model, it

[2]See Adrian and Shin (2014) for microfoundations of VaR constraints.

[3]These authors note that standard models such as Gertler and Kiyotaki (2015), He and Krishnamurthy (2013) or Brunnermeier and Sannikov (2014) "will not match the pre-crisis spread evidence. In the(se) models, a prolonged period in which fragility and leverage rises will also be coupled with an increase in spreads and risk premia. That is, the logic of these models is that asset prices are forward looking and will reflect the increased risk of a crisis as fragility grows".

[4]Allen and Gale (2000) have shown that current and future credit expansion can increase risk-shifting and create bubbles in asset markets. Nuño and Thomas (2017) show that the presence of risk-shifting creates a link between asset prices and bank leverage.

is the presence of limited liability that leads bankers to optimally ignore downside risk within the default region, while government guarantees insure depositors[5].

Our framework generates an endogenous non-linearity in the trade-off between monetary policy (which affects the funding costs of intermediaries) and financial stability[6]. When the level of interest rates is high, a fall in interest rates leads an increase in leverage (*intensive margin*) and to entry of less risk-taking intermediaries into the market for risky projects (*extensive margin*). The average intermediary is then less risky, so a fall in the cost of funds has the effect of improving financial stability and expanding the capital stock. There is no trade-off in this case between stimulating the economy and financial stability. However, when interest rates are very low, a further decrease benefits the most leveraged risk-taking intermediaries and competition drives out the more prudent ones. Stimulating the economy also shifts the distribution of assets towards the more risk-taking intermediaries, which have a higher default risk and increases aggregate risk-shifting. There is a trade-off between increasing investment and financial stability. This non-monotonicity constitutes a substantial difference from the existing literature and is a robust mechanism coming from the interplay of the two margins. It provides a novel way to model the risk-taking channel of monetary policy analysed in Borio and Zhu (2012), Challe, Mojon and Ragot (2013)[7], Angeloni, Faia and Lo Duca (2015), Bruno and Shin (2015*a*) and Acharya and Plantin (2016).[8]

[5]Baron and Xiong (2017) show that, more broadly, creditors of banks do not price the risk taken by bankers during credit expansions. Deposit guarantees have also the effect of ruling out bank runs in our framework. For models focusing on runs see Diamond and Dybvig (1983), Gertler and Kiyotaki (2015) and Angeloni and Faia (2013)). Kareken and Wallace (1978) point out that an important side effect of deposit insurance is excessive risk taking.

[6]Our model is about the behaviour of the real interest rate so the connection with monetary policy is only partial. Any change in regulation that affects funding costs would have similar implications. So would higher savings rates or large capital inflows. An extension of the model featuring nominal variables is left for future work.

[7]Challe, Mojon and Ragot (2013) describe a two-period model with heterogeneous intermediaries and limited liability which, like ours, features a link between interest rates and systemic risk. They focus on portfolio choice and heterogeneity in equity of intermediaries while we emphasize aggregate uncertainty and differences in risk taking. Unlike them, we embed the financial sector in a DSGE model.

[8]Recent empirical evidence on the risk-taking channel of monetary policy has been provided by Dell'Ariccia, Laeven and Suarez (2017) on US data, Jimenez et al. (2014) and Morais et al. (2019), exploiting registry data on millions of loans of the Spanish and Mexican Central Banks respectively. Importantly, using detailed Turkish data, Baskaya et al. (2017) highlight the importance of bank heterogeneity for credit creation and the transmission of global financing cost shocks. Coimbra and Rey

A few papers have, like us, put their emphasis on the boom phase of the financial cycle. In Martinez-Miera and Suarez (2014), bankers determine their exposure to systemic shocks by trading-off the risk-shifting gains due to limited liability with the value of preserving their capital after a crisis. Malherbe (2015) and Gersbach and Rochet (2017) present models with excessive credit during economic booms as increased lending by an individual bank exerts a negative externality on all other banks. Martinez-Miera and Repullo (2017) analyse "search for yield" in an environment with safe and risky entrepreneurs and banks (with no equity) facing a moral hazard friction a la Holmstrom and Tirole (1997). In their model, riskier entrepreneurs endogenously borrow from monitoring banks while safer entrepreneurs borrow from non-monitoring banks (called shadow banks).[9] Another small set of papers have analysed financial sectors with heterogeneous agents. Geanakoplos (2010) studies leverage cycles driven by wealth reallocations between optimists and pessimists. Fostel and Geanakoplos (2012) also emphasize financial frictions and heterogeneity in the beliefs of investors to generate fluctuations in asset prices. Boissay, Collard and Smets (2016) feature intermediaries heterogeneous in their intermediation skills. In their set up, low ability intermediaries become active in boom times and adverse selection plays an important role in credit collapses.[10]

The recent paper of Korinek and Nowak (2017) is the most closely related to our work while using a very different modelling approach. Like us, the authors emphasize that heterogeneity in the financial sector and compositional effects drive the dynamics of aggregate risk. They use evolutionary dynamics tools however to characterize the dynamics of the distribution of the wealth of bankers: good shocks raise the fraction of wealth controlled by high risk takers and therefore increase aggregate risk taking in an incomplete market environment. The converse holds for bad shocks. They do not study

(2018) show that in a cross section of countries, credit creation tends to me more elastic to decreases in funding costs when the leverage distribution of the banking system is more skewed. For a model of the deposit channel of monetary policy see Drechsler, Savov and Schnabl (2017).

[9] Begenau and Landvoigt (2017) present a quantitative model of banks and shadow banks to analyse optimal banking regulation.

[10] Their modelling strategy and ours are however very different and so are the implications of the two models. In particular in their set-up there is a backward bending demand curve for loans; not in ours. They also do not model monetary policy, nor the cross-section of banks leverage, which is a key variable for us.

the interaction between low rates (monetary policy) and financial stability. On the empirical side, our work relates to recent work by Koijen and Yogo (2019) who develop and test models where heterogeneity across institutional investors is an important driver of asset pricing.

Our model of financial intermediation has several advantages. First, it embeds an endogenous risk-taking channel in general equilibrium and therefore allows to study the usual expansionary effect of monetary policy jointly with its effect on financial stability. Second, it is able to generate periods of low risk premium which coincide with periods of high endogenous macroeconomic risk. These periods also correspond to high levels of investment and inflated asset prices due to stronger risk-shifting motives. The model also generates credit booms driven by high expected productivity, which do not increase financial fragility. Third, the model opens the door to a vast array of empirical tests based on microeconomic data on banks, shadow banks, asset managers, etc... Indeed, the heterogeneity can in principle be matched to data on the leverage behavior of financial intermediaries or business lines within them.[11]

Section 2 of the paper describes the model. Section 3 presents the main results in partial equilibrium to build intuition. Section 4 shows the general equilibrium results and the response to monetary policy and productivity shocks. Section 5 looks at some illustrative empirical evidence for the cross-sectional implications of the model. The effect of large negative productivity shocks and the case of financial crises with costly intermediary default is analyzed in section 6. Section 7 concludes.

2 The Model

The general equilibrium model is composed of a representative risk-averse household who faces an intertemporal consumption saving decision, a continuum of risk-neutral, heterogeneous financial intermediaries, and a stylized central bank and government. There is aggregate uncertainty, in the form of productivity and monetary policy shocks. Given the heterogeneity in bank balance sheets that the model features, this will lead

[11]Our model attempts to perform in macro-finance something similar to what Melitz (2003) has done in international trade by relating aggregate outcomes to underlying microeconomic heterogeneity. We are not aware of any other paper in the macro-finance literature that pursues a similar aim.

to heterogeneity in default risk in the intermediation sector.

2.1 Households and the production sector

The representative household has an infinite horizon and consumes a final good C_t^H. She finances her purchases using labour income W_t and returns from a savings portfolio. We assume that the household has a fixed labour supply and does not invest directly in the capital stock K_t.[12] She can either save using a one-to-one storage technology S_t^H and/or deposit D_t^H with financial intermediaries at interest rate r_t^D. The return on deposits $R_t^D \equiv 1 + r_t^D$ is risk-free and guaranteed by the government. Intermediaries use deposits, along with inside equity ω_{it}, to invest in capital and storage. In Section 4 we will introduce monetary policy as a source of wholesale funding. Monetary policy will therefore affect the weighted average cost of funds for intermediaries.

The production function combines labour and capital in a typical Cobb-Douglas function. Since labour supply is fixed, we normalize it to 1. Output Y_t is produced according to the following technology:

$$Y_t = Z_t K_{t-1}^{\theta} L_t^{1-\theta} \tag{1}$$

where Z_t represents total factor productivity and θ the capital share of output. Given $L_t = 1$ in equilibrium, firm maximization implies that wages $W_t = (1-\theta) Z_t K_{t-1}^{\theta-1}$. We will introduce some idiosyncratic risk to financial intermediation, so the return on a unit of capital will be intermediary specific $R_{it}^K = \theta Z_{it} K_{t-1}^{\theta-1} + (1-\delta)$ (more on this later).

[12]Given households are risk-averse and intermediaries are risk neutral (and engage in risk-shifting), relaxing the assumption households cannot invest directly would make no difference to their portfolio in equilibrium unless all intermediaries are constrained. There are also little hedging properties in the asset, since the correlation of the shock to returns with wage income is positive. In the numerical exercises it is never the case that all intermediaries are constrained as some choose not to leverage, so to simplify notation and clarify the household problem, we assume directly that only intermediaries can invest in the risky capital stock.

The household program can be written as follows:

$$\max_{\{C_t, S_t^H, D_t^H\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^H) \qquad \text{s.t.} \tag{2}$$

$$C_t^H + D_t^H + S_t^H = R_t^D D_{t-1}^H + S_{t-1}^H + W_t - T_t \qquad \forall_t \tag{3}$$

where β is the subjective discount factor and $u(.)$ the period utility function. T_t are lump sum taxes and S_t^H are savings invested in the one-to-one storage technology. Note that the return on deposits is risk-free despite the possibility of intermediary default. The reason is that deposits are guaranteed by the government, which may need to raise taxes T_t in the event intermediaries cannot cover their liabilities. Households understand that the higher the leverage of intermediaries, the more likely it is for them to be taxed in the future. However, they do not internalize this in their individual portfolio decisions since each household cannot by itself change aggregate deposits nor the expectation of future taxes.

The return on storage is also risk-free, which implies that households will be indifferent between deposits and storage if and only if $R_t^D = 1$. Therefore, they will not save in the form of deposits if $R_t^D < 1$ and will not invest in storage if $R_t^D > 1$. In equilibrium, the deposit rate will be bounded from below by the unity return on storage, implying that $R_t^D \geq 1$. In the case $R_t^D = 1$, the deposit quantity will be determined by financial intermediary demand, with the remaining household savings being allocated to storage.

2.2 Financial intermediaries

The financial sector is composed of two-period financial intermediaries which fund themselves through inside equity and household deposits[13]. They use these funds to invest in the aggregate risky capital stock and/or in the riskless one-to-one storage technology. Intermediaries are risk neutral agents who maximize expected second period consumption subject to a Value-at-Risk constraint. They also benefit from limited

[13]We will extend the funding options to include wholesale funding, whose cost is influenced by monetary policy, in section 4. The economy in our benchmark case does not feature an interbank market or other funding possibilities. We relax this assumption and allow for interbank market in Appendix E. Qualitative results are unchanged.

liability. To capture the diversity of risk attitudes among financial intermediaries, we assume that they are heterogeneous in α^i, the maximal probability their return on equity is negative according to their Value-at-Risk (VaR) constraint. α^i is exogenously given and is the key parameter in the VaR constraint. This probability varies across intermediaries and is continuously distributed according to the measure $G(\alpha^i)$ with $\alpha^i \in [\underline{\alpha}, \overline{\alpha}]$.

The balance sheet of intermediary i at the end of period t is as follows:

Assets	Liabilities
k_{it}	ω_{it}
s_{it}	d_{it}

where k_{it} are the shares of the aggregate capital stock held by intermediary i, s_{it} the amount of storage held, d_{it} the deposit amount contracted at interest rate r_t^D, and ω_{it} the inside equity. At the beginning of the next period, aggregate and idiosyncratic shocks are revealed and the net cash flow $\pi_{i,t+1}$ is:

$$\pi_{i,t+1} = R^K_{i,t+1} k_{it} + s_{it} - R^D_t d_{it} \tag{4}$$

Intermediary return on capital $R^K_{i,t+1}$ is risky and depends on the ex-post productivity of the capital held by the intermediary. It features an idiosyncratic and an aggregate productivity component. With probability ζ, the intermediary is hit by a negative idiosyncratic shock and its capital fails to produce anything, although it still recovers undepreciated capital at $t+1$. With probability $(1-\zeta)$ it is not hit by the negative idiosyncratic shock.[14] We can then describe idiosyncratic returns $R^K_{i,t+1}$ as follows:

$$R^K_{i,t+1} = \begin{cases} 1-\delta & \text{with probability} \quad \zeta \\ \theta \tilde{Z}_{t+1} K_t^{\theta-1} + (1-\delta) & \text{with probability } 1-\zeta \end{cases} \tag{5}$$

where $\tilde{Z}_t$ is the aggregate component and can be interpreted as the productivity of capital conditional on no idiosyncratic shock. ζ is a measure of idiosyncratic risk. The

[14] We can think of ζ as an operational risk shock. It is mainly introduced for computational purposes in order to ensure that the lowest (positive) probabilities of default of leveraged intermediaries are never numerically indistinguishable from zero.

aggregate component follows a simple AR(1) process in logs

$$\log \tilde{Z}_{t+1} = (1-\rho^z)\mu_z + \rho^z \log \tilde{Z}_t + \varepsilon^z_{t+1} \tag{6}$$

$$\varepsilon^z_{t+1} \sim N(0, \sigma_z) \tag{7}$$

ε^z_t is the shock to the log of exogenous productivity (conditional on no idiosyncratic shock) with persistence ρ^z and standard deviation σ_z. μ_z is a scaling parameter such that $E(Z) = E(\tilde{Z}(1-\zeta)) = 1$. Let $F(\epsilon^z_t)$ be the cumulative distribution function (cdf) of $\exp(\epsilon^z_t)$, a notation which will be convenient later. Expected return on capital will be equal across intermediaries and we define $E\left[R^K_{t+1}\right] \equiv E\left[R^K_{i,t+1}\right]$. Differences in willingness to pay for shares of the capital stock will however arise in the presence of heterogeneous default risk and limited liability, generating an intermediary-specific option value of default.

2.2.1 Value-at-Risk constraint

Financial intermediaries are assumed to be constrained by a Value-at-Risk condition (VaR). This condition imposes that intermediary i invests in such a way that the probability its return on equity is negative must be smaller than an exogenous intermediary-specific parameter α^i.[15] The VaR constraint for intermediary i can then be written as:

$$\Pr(\pi_{i,t+1} < \omega_{it}) \leq \alpha^i \tag{8}$$

The probability that the net cash flow is smaller than starting equity ω_{it} must be less or equal than α^i. This constraint follows the spirit of the Basel Agreements, which aim at limiting downside risk and preserving an equity cushion. Furthermore, Value-at-Risk techniques are used by banks and other financial intermediaries (for example asset managers) to manage risk internally. When binding, it also has the property of generating procyclical leverage, which can be observed in the data for some

[15] Alternatively we could posit that the threshold is at a calibrated non-zero return on equity. There is a mapping between the distribution $G(\alpha^i)$ and such a threshold, so for any value we could find a $\tilde{G}(\alpha^i)$ that would make the two specifications equivalent given expected returns. We decide to use the current one as it reduces the parameter space.

intermediaries as described in Geanakoplos (2011) and Adrian and Shin (2014) when equity is measured at book value. Using a panel of European and US commercial and investment banks Kalemli-Özcan, Sorensen and Yesiltas (2012) also provide evidence of procyclical leverage while emphasizing cross-sectional variations across types of intermediaries.

Heterogeneity in the parameter of the *VaR* constraint can be rationalized in different ways. It could be understood as reflecting different risk management practices or differentiated implementation of regulatory requirements by different supervisors. For example, the Basel Committee undertook a review of the consistency of risk weights used when calculating how much capital a sample of banks put aside for precisely defined portfolio. When given a diversified test portfolio the banks surveyed produced a wide range of results in terms of modelled *VaR* and gave answers ranging from 13 million to 33 million euros in terms of capital requirement with a median of about 18 million (see Basel Committee on Banking Supervision (2013) p.52). Some of the differences are due to different models used, some to different discretionary requirements by supervisors and some to different risk appetites, as "Basel standards deliberately allow banks and supervisors some flexibility in measuring risks in order to accommodate for differences in risk appetite and local practices" (p.7). As a matter of fact, Figure 15 shows that leverage is highly heterogeneous in the cross-section of financial intermediaries.

2.2.2 Intermediary investment problem

We assume that the risk neutral intermediaries live for two periods, receive a constant endowment of equity $\omega_{it} = \omega$ in the first and consume their net worth in the second. This assumption of constant equity is a simplifying assumption but we find that book value equity is indeed very sticky in the data. We show in Figure 11 the almost one-for-one correlation between changes in the size of debt and assets at book value, for a very broad sample of banks using Bankscope data. Figure 11 also shows the stickiness of book value equity relative to assets and debt. Balance sheet expansions and contractions tend to be done through changes in debt and not through movements in equity. Krishnamurthy and Vissing-Jorgensen (2015) present remarkable evidence on the time series of bank long-term assets, short-term debt and equity as a percentage of GDP for the US. We replicate their findings and show in Figure 12 a strong correlation between long-term assets and short term debt (0.994) and a far smaller one between equity and assets (0.283). In addition, if we detrend the series, the correlations are, respectively, 0.972 and -0.02174 so still very high for assets and debt but virtually zero between equity and assets. Furthermore the magnitudes of long term assets and short term debt are comparable throughout, highlighting the central role of leverage in funding investment in the economy. The macro-finance literature often focuses on the dynamics of net worth, assuming a representative agent (see e.g. Gertler and Kiyotaki (2015)), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2014), Jermann and Quadrini (2012)) and abstracting from the cross sectional differences in intermediaries. We take a complementary approach. To highlight the novel nature of our mechanism, we instead assume constant equity, thus abstracting from the net worth channel and putting a sharp focus on the effects generated by the heterogeneous dynamics of leverage in the cross-section.

When the net cash flow $\pi_{i,t}$ is positive, it is consumed by financial intermediary i and we denote its consumption by c_{it}.[16] When the net cash flow is negative, $c_{it} = 0$ and the intermediary defaults. Government steps in to repay depositors as it upholds deposit insurance. This is a pure transfer, funded by a lump sum tax on households. Hence, in our model, households are forward-looking and do intertemporal optimization while most of the action in the intermediation sector comes from heterogenous leverage and risk taking in the cross-section. This two-period modeling choice is made for simplicity and allows us to focus on the role of different leverage responses across financial intermediaries[17].

Each intermediary has to decide whether it participates or not in the market for risky assets or invests in the storage technology (*participating intermediary* versus *non-participating intermediary*) and, conditionally on participating, whether it uses deposits to lever up (*risky intermediary*) or just invests its own equity (*safe intermediary*). Note that this label of *risky* or *safe* is based on the possibility (or not) of defaulting on lenders, not in terms of the volatility of their return on assets or equity. These will only be risk free for *non-participating* ones, which invest only in storage. In Appendix E, we

[16]When intermediary j is inactive, then $c_{jt} = \omega$ as the return of the storage technology is one.

[17]Other papers in the literature have used related assumptions, for example exogenous death of intermediaries in Gertler and Kiyotaki (2015).

show that an alternative model where intermediaries can choose to lend to each other as an outside option has very similar implications.[18]

Intermediaries are assumed to be (constrained) risk-neutral price takers, operating in a competitive environment. Each maximizes consumption over the next period by picking k_{it} (investment in risky assets) and s_{it} (investment in the storage technology), under its *VaR* constraint, while taking interest rates on deposits R_t^D and asset return distributions $R_{t+1}^K(\varepsilon)$ as given. The program of each intermediary i is given by:

$$
\begin{aligned}
V_{it} = & \max \ \mathbb{E}_t(c_{i,t+1}) && (9)\\
\text{s.t.} \ & \Pr(\pi_{i,t+1} < \omega_{it}) \leq \alpha^i && (10)\\
& k_{it} + s_{it} = \omega_{it} + d_{it} && (11)\\
& c_{i,t+1} = \max\left(0, \pi_{i,t+1}\right) && (12)\\
& \pi_{i,t+1} = R_{i,t+1}^K k_{it} + s_{it} - R_t^D d_{it}
\end{aligned}
$$

where α^i is the *VaR* threshold (the maximum probability of not being able to repay stakeholders fully) and $\pi_{i,t+1}$ the net cash flow.

Intermediaries can also choose to stay out of capital markets and not participate. In this case, they have the outside option of investing all their equity in the storage technology and collect it at the beginning of the next period. The value function of a non-participating intermediary investing in the outside option is:

$$V_{it}^O = V^O = \omega \tag{13}$$

2.2.3 Limited liability

The presence of limited liability truncates the profit function at zero, generating an option value of default that intermediaries can exploit. For a given expected value of returns, a higher variance increases the option value of default as intermediaries benefit from the upside but do not suffer from the downside. For a given choice of k_{it} and d_{it}

[18] In Appendix E, we consider a standard centralized market for intermediary borrowing. For a model of financial stability issues arising from banking networks see Aldasoro, Gatti and Faia (2017).

we have that:

$$\mathbb{E}_t\left[\max(0,\pi_{i,t+1})\right] \geq \mathbb{E}_t\left[\pi_{i,t+1}\right] \tag{14}$$

with the inequality being strict whenever the probability of default is strictly positive. Deposit insurance transfers t_t^i happen when the net cash flow is negative and are given by:

$$t_{t+1}^i = \max\left(0, -\pi_{t+1}\right) \tag{15}$$

The max operator selects the appropriate case depending on whether intermediary i can repay its liabilities or not. If it can, then deposits repayments are lower than return on assets and deposit insurance transfers are zero. Total intermediary consumption C_t^I and aggregate transfers/taxes T_t are given by integrating over the mass of intermediaries:

$$C_t^I = \int c_{it}\; dG(\alpha^i) \tag{16}$$

$$T_t = \int t_t^i\; dG(\alpha^i) \tag{17}$$

For now we assume default is costless in the sense that there is no deadweight loss when the government is required to pay deposit insurance. In section 6, we will drop the assumption of costless default by having a more general setup that allows for a lower return on assets held by defaulting intermediaries.

2.3 Investment strategies and financial market equilibrium

Financial intermediaries are price takers, therefore the decision of each one depends only on the expected return on assets (taking into account limited liability) and the cost of liabilities. Since the mass of each intermediary is zero, individual balance sheet size does not affect returns on the aggregate capital stock. Intermediary i will be a *participating intermediary* in the market for risky assets whenever $V_{it} \geq V^O$. This condition determines entry and exit into the market for risky capital endogenously.

There is however another important endogenous decision. Intermediaries which participate in the market for risky assets have to choose whether to lever up and, if they do, by how much. We will refer to the decision to lever up or not, i.e. to enter the market for deposits as the *extensive margin*. We will refer to the decision regarding how much to lever up as the *intensive margin*. Financial intermediaries which lever up are *risky* intermediaries. Financial intermediaries which participate in the market for risky capital but do not lever up are *safe* intermediaries.

Proposition 2.1 *When $\mathbb{E}[R^K_{t+1}] \geq 1$, participating intermediary i will either lever up to its VaR constraint or not raise deposits at all.*

Proof: See Appendix B.

Proposition 2.1 states that if the return to risky capital is higher in expectation than the return on the storage technology then whenever an intermediary decides to lever up, it will do so up to its *VaR* constraint and will not invest in storage. Hence all *risky* intermediaries will be operating at their constraint.

When expected return on risky capital is smaller than return on storage: $\mathbb{E}[R^K_{t+1}] < 1$, it might still be the case that capital is preferred to storage in equilibrium by some intermediaries due to limited liability. We would then have equilibria in which some intermediaries invest in storage and possibly some of the most risk-taking ones leverage up a lot taking advantage of the option value of default. In what follows we focus on cases where $\mathbb{E}[R^K_{t+1}] \geq 1$ which is always the case in our simulations.

2.3.1 Intensive margin and endogenous leverage

Let $Z^e_{t+1} \equiv \mathbb{E}_t(\tilde{Z}_{t+1})$, an expectation known at t. For a participating intermediary i deciding to lever up, the *VaR* condition will bind (see Proposition 2.1):

$$\Pr\left[\pi^i_{t+1} \leq \omega\right] = \alpha^i \tag{18}$$

Hence, after some straightforward algebra, we obtain the following:

$$\zeta + (1-\zeta)\Pr\left[e^{\varepsilon^z_{t+1}} \leq \frac{r^D_t + \delta - \frac{\omega}{k_{it}} r^D_t}{\theta Z^e_{t+1} K^{\theta-1}_t}\right] = \alpha^i \tag{19}$$

The leverage λ_{it} of an active intermediary is given by:

$$\lambda_{it} \equiv \frac{k_{it}}{\omega} = \frac{r_t^D}{r_t^D - \theta Z_{t+1}^e K_t^{\theta-1} F^{-1}\left(\frac{\alpha^i-\zeta}{1-\zeta}\right) + \delta} \tag{20}$$

where we defined leverage as assets over equity and F^{-1} as the inverse cdf of the technology shock $e^{\varepsilon_{t+1}^z}$ evaluated at probability $\frac{\alpha^i-\zeta}{1-\zeta}$. Note that intermediaries with $\alpha^i < \zeta$ will never participate.

Let $r_t^{\alpha^i} \equiv \theta Z_{t+1}^e K_t^{\theta-1} F^{-1}\left(\frac{\alpha^i-\zeta}{1-\zeta}\right) - \delta$ be the ex-post return on capital for which the return on equity of *risky* intermediary α^i is zero. The expression above can then be simply written as:

$$\lambda_{it} = \frac{r_t^D}{r_t^D - r_t^{\alpha^i}} \tag{21}$$

This expression for leverage is only true when the constraint is binding for *risky* intermediary α^i. In equilibrium, decreasing marginal returns to K ensure that the denominator is always positive. Otherwise the constraint would not be binding and *risky* intermediaries would increase K, which in turn would reduce $r_t^{\alpha^i}$.

Proposition 2.2 *For a participating intermediary i, the leverage λ_{it} has the following properties: it is increasing in α^i, decreasing in the cost of funds r_t^D and increasing in expected marginal productivity of capital $\theta Z_{t+1}^e K_t^{\theta-1}$. Furthermore, $\frac{\partial^2 \lambda_{it}}{\partial (r_t^D)^2} > 0$ and $\frac{\partial^2 \lambda_{it}}{\partial r_t^D \partial \alpha^i} < 0$.*

Proof: Immediate from Equation (20) and given the monotonicity of the cdf and the shape of $F^{-1}()$.

Proposition 2.2 implies that, from the perspective of a participating individual intermediary (i.e. absent general equilibrium effects on K_t), leverage will be decreasing in the cost of funds r_t^D. For a given balance sheet size, decreasing the cost of liabilities increases expected *net* cash flows and thus decreases the probability of distress. From 2.1, intermediaries would then choose to increase leverage until their probability of distress hits the VaR constraint. Furthermore, when interest rates are low the probability of default is lower *ceteris paribus*. In that region, the pdf is flatter therefore increases in leverage translate into small increases in probability of distress. This means that intermediaries can increase leverage by sizable amounts until they hit the *VaR* constraint. So the lower r_t^D, the stronger the intensive margin effect. Similarly for high α (looser constraints) leverage can be increased a lot before the constraint is hit. Therefore the leverage of the most risk-taking intermediaries will react more to interest rate changes.

Generally, intermediary leverage will also be decreasing in the volatility of the productivity shocks σ_z. This will be true whenever $F^{-1}\left(\frac{\alpha^i-\zeta}{1-\zeta}\right)$ is increasing in σ_z, implying realistically that the probability of a negative return on equity is (ceteris paribus) increasing in the volatility of returns.

2.3.2 Extensive margin and endogenous leverage

We now focus on the *extensive margin*, that is to say whether intermediaries who participate in risky capital markets choose to lever up using deposits or not.[19]

Let V^L denote the value function of *risky* intermediaries who decide to lever up using deposits and V^N the value function of the *safe* ones who only invest at most their equity in the risky capital stock. We denote by $\mathbb{E}_t^i$ the expectation of a financial intermediary taking into account limited liability (expectation truncated at zero).

$$V_{it}^L = \mathbb{E}_t^i[R_{i,t+1}^K k_{it} - R_t^D d_{it}] \tag{22}$$

$$V_{it}^N = \mathbb{E}_t[R_{i,t+1}^K] k_{it}^N + \omega - k_{it}^N \tag{23}$$

with $k_{it}^N \in \{0, \omega\}$. Since there is no risk of defaulting on deposits if you have none, there is no option value of default for non-levered intermediaries. This N group could potentially also include intermediaries who invest only a fraction of their equity in the capital stock. Given our choice of *VaR* constraint, *safe* intermediaries will either invest ω in the capital stock or not at all.[20]

We can then use the condition $V_{it}^L = V_{it}^N$ to find the cut-off value $\alpha_t^L = \alpha_t^j$ for

[19] Intermediaries can also decide not to invest in risky capital markets and instead to use the storage technology. If they do so, then their value function is $V^O = \omega$ given the unit return to storage and linear utility.

[20] Note that the *VaR* condition of a *safe* intermediary can be written as $\Pr\left(e^{\varepsilon_{t+1}} < \frac{\delta K^{1-\alpha}}{\theta Z_{t+1}^e}\right) \leq \frac{\alpha^i-\zeta}{1-\zeta}$. Since this is not a function of k_{it}, the inequality will either be true and the intermediary will invest up to ω, or it won't and he cannot invest any amount in the capital stock without violating it. Note also that the inequality is always false for $\alpha^i < \zeta$.

which intermediary j is indifferent between leveraging up or not. Above α_t^L (looser *VaR* constraints), all intermediaries will be levered up to their respective constraints and do not invest in storage as shown in Proposition 2.1. For any levered intermediary i, we have:

$$\mathbb{E}_t^i \left[k_{it} R_{i,t+1}^K - R_t^D d_{it} \right] \geq \omega \; \mathbb{E}_t \left[R_{t+1}^K \right] \tag{24}$$

where the left hand side is the expected payoff on the assets of intermediary i and the right hand side is the expected payoff when it invests only its equity ω in capital markets. Using the balance sheet equation $k_{it} = d_{it} + \omega$, we can substitute for deposits, which leads to the following condition:

$$\mathbb{E}_t^i \left[k_{it} \left(R_{i,t+1}^K - R_t^D \right) + R_t^D \omega \right] \geq \omega \; \mathbb{E}_t \left[R_{t+1}^K \right] \tag{25}$$

For the marginal intermediary j, equation (25) holds with equality:

$$\mathbb{E}_t^j \left[k_{jt} \left(R_{j,t+1}^K - R_t^D \right) + R_t^D \omega \right] = \omega \; \mathbb{E}_t \left[R_{t+1}^K \right] \tag{26}$$

Since all *risky* intermediaries will be at the constraint, we can combine equation (26) with equation (20) evaluated at the marginal intermediary (whose *VaR* parameter is α_t^L). Moreover, $\mathbb{E}_t \left[R_{t+1}^K \right]$ is a function of Z_{t+1}^e and K_t but is independent of i. Therefore equation (26) and equation (20) jointly define an implicit function of the threshold *VaR* parameter α_t^L $(= \alpha^j)$ with variables (r_t^D, Z_{t+1}^e, K_t).

Hence we have the following result:

Proposition 2.3 *There exists a cut-off value α_t^L in the distribution of VaR parameters such that all intermediaries with VaR constraints looser than the cut-off will borrow to leverage up to their constraint. All intermediaries with VaR constraints tighter than the cut-off will choose to not leverage. Equations (26) and (20) define an implicit function of the threshold $\alpha_t^L = A(r_t^D, Z_{t+1}^e, K_t)$.*

2.3.3 Financial market equilibrium and deposit demand curve

To close the financial market equilibrium, we need to use the market clearing condition. The aggregate capital stock of the economy is equal to the total investment in risky projects by all intermediaries.

$$K_t = \int_{\underline{\alpha}}^{\overline{\alpha}} k_{it} \; dG(\alpha^i) \tag{27}$$

This integral can be divided into capital held by *risky* levered intermediaries (above α_t^L) and capital held by *safe* intermediaries who do not lever up but invest all their equity in the capital stock (between α_t^N and α_t^L). Below α_t^N all intermediaries invest all their equity in storage.

For *safe* intermediaries who invest all their equity in capital shares, the VaR constraint is given by $\zeta + (1-\zeta)F\left(\frac{\delta K_t^{1-\theta}}{\theta Z_{t+1}^e}\right) \leq \alpha^i$. We can pin down α_t^N by looking at the marginal *safe* intermediary for whom the constraint binds exactly.

$$\alpha_t^N = \zeta + (1-\zeta)F\left(\frac{\delta K_t^{1-\theta}}{\theta Z_{t+1}^e}\right) \tag{28}$$

In equilibrium, the market clearing condition for K can then be written as:

$$K_t = \int_{\alpha_t^L}^{\overline{\alpha}} k_{it} \; dG(\alpha^i) + \left[G(\alpha_t^L) - G(\alpha_t^N)\right]\omega \tag{29}$$

Where k_{it} is given by the asset purchases of *risky* intermediaries described in equation (20). Along with the expression for α_t^N in equation (28), the market clearing equation (29) defines an implicit function of $(\alpha_t^L, r_t^D, Z_{t+1}^e, K_t)$. Since Z_{t+1}^e is determined at t by state variables and intermediaries are price takers, the financial market clearing function together with the implicit function $\alpha_t^L = A(r_t^D, Z_{t+1}^e, K_t)$ pin down the aggregate capital stock K_t and the marginal levered intermediary α_t^L, for a given deposit rate r_t^D and expected productivity Z_{t+1}^e.

Together they determine the *aggregate demand curve for deposits* as a function of deposit rates and expected productivity. By pinning down (α_L, K), they also determine the entire distribution of leverage in the financial sector for a given (r_t^D, Z_{t+1}^e). In general equilibrium, the deposit rate r_t^D will be determined in conjunction with the *aggregate deposit supply curve* coming from the recursive household problem described in section 4.

2.4 Measuring Financial Stability

The model establishes an important relation between funding costs and the cross-sectional distribution of risk taking by financial intermediaries. Financial stability is a multidimensional object depending on time-varying distributions of leverage and risk taking which are functions of present and future states. For expositional purposes, we summarize this object into a few simple but relevant measures of financial instability in order to track its evolution.

Our baseline measure M^1 is *the probability that in the next period all leveraged intermediaries will be in distress*, defined as the inability to repay in full their stakeholders (deposits and equity). This has a very direct link with the Value-at-Risk constraint, as for each levered intermediary the probability of distress will be simply the parameter α^i. Given aggregate shocks by definition affect all intermediaries, then $M_t^1 = \alpha_t^L$. If the least risk-taking leveraged intermediary is in distress, so must all the intermediaries with higher leverage.[21] In the model, a rise in α_t^L (meaning that the marginal entrant has a looser Value-at-Risk constraint) is then a fall in financial stability according to M^1. The baseline measure has the advantage of not only describing the risk of the whole sector but also of tracking the marginal investor in financial markets an important concept in leverage cycles, as highlighted by Geanakoplos (2011).

The model features significant risk-shifting behavior, as levered financial intermediaries take advantage of limited liability and the option value of default. Moreover, the riskier the intermediary, the larger will be their option value of default. To have a sense of aggregate distortions to investment caused by risk-shifting, we calculate a *Weighted Option Value of Default* by weighing each intermediary's option value of default by their total assets. This measure M^2 can therefore be interpreted as the average option value of default per unit of capital in the economy.

[21] More precisely, in the presence of idiosyncratic shocks M^1 would be an affine transformation of α_t^L, with $M_t^1 = \zeta + (1-\zeta)\alpha_t^L$. Given this transformation is time-invariant, for simplicity we set $M^1 = \alpha_t^L$ even in the case with idiosyncratic shocks.

In the following sections we will use measures M^1 and M^2 to track the dynamics of financial stability in response to monetary and productivity shocks.[22]

3 Partial equilibrium results

To provide a better illustration of the financial sector mechanics in the model, we first show a set of partial equilibrium results taking as given the deposit rate, before moving on to general equilibrium in section 4 where the household problem will close the model. From now on we study the properties of the model using numerical simulations.[23]

We begin by analysing the distribution of intermediary leverage conditional on the deposit rates r_t^D and on expected productivity Z_{t+1}^e. In Figure 1, we show an example of the cross-sectional distribution of leverage for three different values of the deposit rate. The calibration of the model is discussed in more detail in section 4.

In the three cases, the area below each line[24] is proportional to the aggregate capital stock $K_t = \int k_{it}\ dG(\alpha^i)$. The vertical line showing a drop in leverage marks the cut-off and identifies the marginal levered intermediary α_t^L. To the left of the cut-off α_t^L, intermediaries are not levered, which corresponds to the more conservative VaR constraints. They are the *safe* intermediaries. To the right of the cut-off, leverage and balance sheet size k_{it} increase with α_t^i. That is, the more risk-taking is the intermediary, the larger will be its balance sheet for a given r_t^D and Z_{t+1}^e. Those are *risky* intermediaries.

The graph illustrates how the *intensive* and *extensive* margins affect leverage and the aggregate capital stock as the deposit interest rate changes. For the three cases displayed, as deposit rates fall, the *intensive* margin for the most risky intermediaries is always increasing. That is, for each such intermediary that is levered up, the balance sheet grows when the cost of funds falls. For a given balance sheet size, a lower rate would reduce the probability of default as it reduces the amount that needs to be

[22] Given that we can describe the whole cross-sectional distribution of leverage and intermediary risk we can also use a range of potential alternative measures. We highlight this point by providing 3 other measures in Appendix C.

[23] We performed many different calibrations but only report a few. Results (available upon request) are qualitatively robust across simulations.

[24] Assuming a uniform distribution for $G(\alpha^i)$ as in the baseline calibration. The details of the numerical method to solve the model are given in Appendix A.

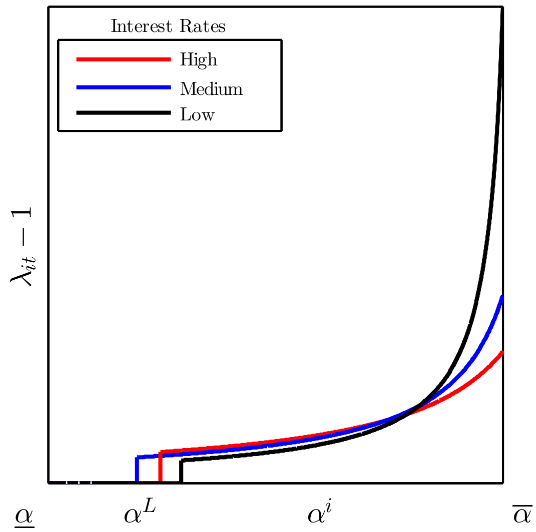

Figure 1: *Cross-sectional distribution of leverage* λ_{it} *as a function of the VaR parameter* α^i

repaid next period. This relaxes the VaR constraint, so intermediaries at the top of the distribution expand their balance sheet up to the new limit and grow in size.

Perhaps less intuitively, the effect for intermediaries in the middle of the distribution and on the *extensive* margin is ambiguous. One would expect that a fall in interest rate would lead to entry by more risk averse intermediaries. This is what happens when one goes from a high level of interest rate to a medium level of interest rate (the cut-off moves to the left). But this is no longer the case when one moves from a medium level of interest rate to a low level of interest rate: the cut-off moves to the right. Depending on the level of interest rates, a fall in interest rates can lead to more or fewer intermediaries choosing to lever up. We explain below this strong non-linearity of the effect of interest rates on financial stability and the leverage of intermediaries in the middle of the distribution.

3.1 Non-linear trade-off between increased output and financial stability

Following a fall in deposit rates, the riskier intermediaries expand their asset holdings raising the aggregate capital stock. This lowers the return on risky asset holdings due to decreasing returns to (aggregate) capital. As seen in the graph above, there are very interesting asymmetries depending on the *level* of the interest rate.

When the interest rate level is high, the lower cost of liabilities reduces the probability of default for a given balance sheet size. Hence all intermediaries with a risky business model are able to lever more (*intensive* margin). In this case, there are also positive returns for the (previously) marginal intermediary due to the now lower cost of leverage. More intermediaries can lever up and enter the market for deposits (*extensive* margin), reducing the cut-off α^L. The financial system then becomes less risky since newly entered intermediaries have a stricter VaR constraint. There is therefore no trade-off between using lower interest rates to stimulate investment and financial stability.

When the interest rate level is low, the *intensive* margin effect of a decrease in the interest rate is strong (see Proposition 2.2), leverage and investment are high and the curvature of the production function leads to a decrease in expected asset returns which is large enough to price out of the market the most risk averse of the previously levered intermediaries. The sign of the effect on α^L depends on whether the fall in asset returns is stronger than the fall in the cost of liabilities. In the case of initially low interest rates, a further fall (in those rates) leads to fewer intermediaries choosing to lever up. The intermediaries remaining are larger and more risk taking on average. There is therefore a clear trade-off between an expansionary monetary policy (that lowers funding costs) and financial stability.

In order to gain some intuition, we can look at two polar cases. In the first, aggregate capital is infinitely elastic and return distributions $R^K_{t+1}(\varepsilon)$ are fixed. In this case, a decrease in the cost of funding can only lead to entry as the (previously) marginal intermediary will now make positive profits. The cut-off falls and there is no trade-off. In the second example, aggregate capital is fixed and returns adjust to clear the market[25]. If a fall in the cost of funding allows more leverage from the more risk-taking

[25] In this case, the price of capital will adjust as it is no longer pinned down by the investment

intermediaries, then it must be that the (previously) marginal intermediary no longer holds capital and returns fall enough to price him out. In this case, there will always be a trade-off. In intermediate cases, the strength of the intensive margin effect is important as it determines the extent to which returns fall due to decreasing returns in the aggregate capital stock. The stronger is this effect (i.e. the more leverage increases following a fall in interest rates or the more *interest-elastic* the intermediaries are), the more likely a trade-off will be present. As stated in Proposition 2.2, leverage increases faster as the interest rate falls (conditional on being levered). This means the *intensive* margin effect is particularly strong when interest rates are low. Proposition 2.2 also states that leverage reacts more, the more risk-taking is the intermediary implying additionally that the most risk-taking intermediaries grow faster. This leads to additional skewness in the cross-sectional distribution of leverage.

Hence, as shown in Figure 1, when interest rates fall from high to medium to low, balance sheets become more heterogeneous in size and the difference between the most leveraged and the least leveraged intermediary rises. We highlight the following properties of our model:

1) Heterogeneity, skewness of leverage and aggregate investment

In Figure 2, the left panel plots the cut-off α_t^L as a function of deposit rates r_t^D for three different productivity levels, while the middle panel does the same for the aggregate capital stock K_t. K_t is monotonically decreasing with r_t^D. As expected, the lower is the interest rate, the higher will be aggregate investment and we have a standard deposit demand curve. However, the change in *financial structure* underlying the smooth response in the capital stock is non-monotonic. As we can see from the left panel, the cut-off α_t^L first decreases when we go from high interest rates to lower ones and then goes up sharply as we approach zero. There is a change in the composition of intermediaries. Less risk-taking intermediaries reduce their exposure and decrease asset holdings as they are priced out by more risk-taking institutions due to decreasing returns to capital. The latter use low interest rates to increase their leverage significantly.

technology. For recent macroeconomic models in which extensive and intensive margin have interesting interactions (albeit in very different contexts) see Martin and Ventura (2015) and Bergin and Corsetti (2015).

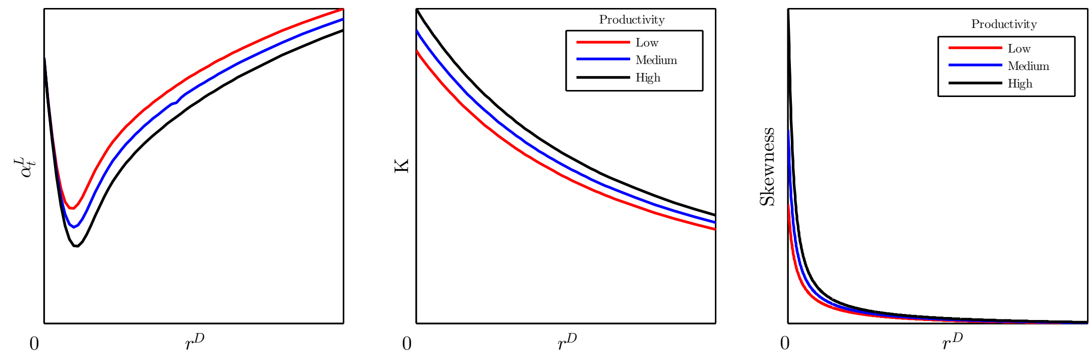

Figure 2: *Cut-off level α_t^L and aggregate capital stock as a function of deposit rates r_t^D*

The lower is the interest rate, the more heterogeneous is leverage across intermediaries. Since the intensive margin of high α^i intermediaries responds more than low α^i, when interest rates are low there is an increased concentration of assets in the most risk-taking intermediaries. Also, a fall in the extensive margin is more likely at low rates, which amplifies this effect. In the right panel of Figure 2 we show the cross-sectional skewness of leverage is a decreasing function of the interest rate. The concentration of assets in riskier intermediaries generates more risk-shifting in aggregate. Hence, similar aggregate investment outcomes can be supported by different underlying financial structures with very different implications for financial stability.

2) Trade-off between financial stability and economic activity

When interest rates are high, a fall in interest rates leads to entry by less risk-taking intermediaries (a fall in the cut-off α_t^L) into levered markets. But when interest rates are low, a fall in interest rates leads to a rise in the cut-off α_t^L, which means the least risk-taking intermediaries reduce their exposure to the risky asset through deleveraging, while the more risk-taking intermediaries increase their balance sheet size and leverage.

We illustrate this point in our partial equilibrium setting by doing a 100 basis points monetary expansion for different target rates. As we will see in section 4, these results carry on to the general equilibrium setting. For this experiment, we assume a very simple monetary policy rule:

$$R_t = R_{t-1}^{\nu} \bar{R}^{1-\nu} \varepsilon_t^R \tag{30}$$

where $R_t = 1 + r_t^D$ is the return on deposit or the cost of leverage for intermediaries. ε_t^R is a monetary policy shock and ν is the persistence of the shock, calibrated[26] to 0.24. $\bar{R}$ is the long-run level of interest rates therefore each of the lines above is calibrated to a different $\bar{R}$. For simplicity, in this simple partial equilibrium exercise, we assume that the monetary authority can directly affect the deposit rate. We relax this assumption in section 4 and show how it can be mapped into this exercise.

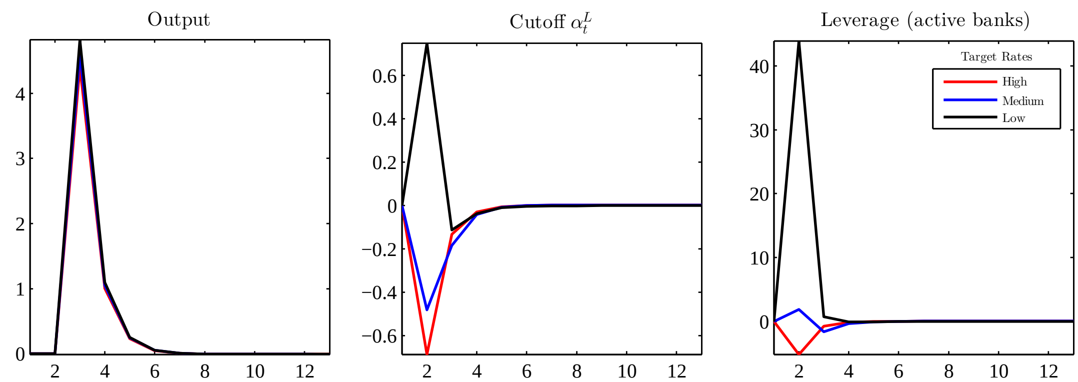

Figure 3: *Partial equilibrium IRF to a 100 basis points fall in deposit rates. Scale in percentage point deviations from the baseline*

Results can be seen in Figure 3, plotted as percentage changes from their respective values at target rates $\bar{R}$.[27] The time period corresponds to one year and the state of the economy when the shock hits is the one corresponding to the target rates. In the left graph we see that the rise in output is relatively insensitive to the level of the target interest rate. The behaviour of the cut-off α_t^L is, however, very differentiated. When the target rates are high, there is a negative effect of a monetary expansion on the cut-off.

[26] Annualized value as estimated by Curdia et al. (2015)

[27] Note that there is no truly dynamic aspect in the partial equilibrium model and it can be seen as a sequence of static problems. The general equilibrium model of section 4 will feature a fully dynamic household problem which affects the banking problem, since the household inter-temporal maximization will determine the deposit supply curve and the equilibrium level of deposit rates.

That means that less risk-taking intermediaries enter risky markets and the average probability of intermediary default falls. In this case, there is no trade-off between financial stability and monetary expansion. This is definitely not the case when target interest rates are low. In that case, average leverage of active banks increases massively by 43% and the cut-off also rises. The large increase in leverage by very risk-taking intermediaries then prices out the less risk-taking ones at the margin, raising the average probability of default among levered intermediaries. This large effect on leverage is a combination of both the intensive margin effect, and a composition effect due to exit of the most risk averse intermediaries. For intermediate levels, we see that this effect is muted, with leverage increasing only slightly and financial stability improving (cutoff going down).

Hence, according to our baseline measure $M^1 = \alpha^L$ of financial stability, there is a trade-off between financial stability and monetary policy when interest rates are low, but not when they are high.

The level of the interest rate matters since it affects the sensitivity of the intensive margin to changes in the cost of funds. The fact that risk-taking intermediaries are able to lever more during a monetary expansion can increase the capital stock while pricing out of the market less risk-taking intermediaries. This means that the financial sector becomes less stable, with risky assets concentrated in very large, more risk-taking financial institutions. Hence, there is also potentially large mispricing of risk since the riskier intermediaries are those who engage the most in risk-shifting (measured in the aggregate by M2). Other measures of financial stability presented in Appendix C also highlight the presence of an important trade-off which occurs only at low levels of the interest rate.

We note all the effects described above regarding the dispersion and the cyclicality of leverage, financial stability and aggregate risk-shifting can occur even in the absence of monetary policy shocks. The cyclicality of the savings behaviour or of capital flows and their effect on equilibrium deposit rates will also lead to cyclical movements in leverage and investment. To understand this more fully, we now close the general equilibrium model by adding the intertemporally optimizing household sector to determine the deposit rate endogenously.

4 General Equilibrium

In this section, we solve the model in general equilibrium by joining the household and intermediary problems. We show that the financial sector equilibrium can be easily integrated in a standard dynamic stochastic general equilibrium framework, with monetary policy and productivity shocks. We introduce costly default in section 6.

4.1 Monetary policy as a change in the cost of external funds

In this section we allow intermediaries to fund themselves through wholesale funding l_{it}. We assume that the monetary authority can control the rate of wholesale funding relative to deposits, by providing funds at a spread γ_t from deposits.[28]. Wholesale funding is remunerated at rate $R_t^L = 1 + r_t^L$ and we denote the deposit rate r_t^D as before. We assume that:

$$R_t^L = R_t^D(1 - \gamma_t) \tag{31}$$

Monetary policy is exogenous, akin to a funding subsidy γ_t which follows a simple AR(1) process in logs.

$$\log \gamma_t = (1 - \rho_\gamma)\mu_\gamma + \rho_\gamma \log \gamma_{t-1} + \varepsilon_t^\gamma \tag{32}$$

$$\varepsilon_t^\gamma \sim N(0, \sigma_\gamma) \tag{33}$$

where μ_γ is the central bank target subsidy, ρ_γ the subsidy's persistence and ε^γ are monetary policy shocks with σ_γ standard deviation.

If the central bank were to provide unlimited funds to intermediaries at this rate, they would leverage using only wholesale funding. We assume that wholesale funding is given in a fixed proportion χ of other liabilites, which in this case are simply deposits.

[28]The monetary authority is assumed to be a deep-pocketed institution which can always fund wholesale funding. Like deposits, wholesale funds are always repayed (by bailout if necessary). To avoid dealing with the monetary authority's internal asset management, we assume that the cost of fund is a deadweight loss (or gain).

Total wholesale funding for intermediary i is then:

$$l_{it} = \chi d_{it} \tag{34}$$

The balance sheet of an intermediary i is then:

Assets	Liabilities
k_{it}	ω
s_{it}	d_{it}
	l_{it}

Given our assumptions, we can then define R_t^F as the total cost of a unit of funding and f_{it} as total external funds of bank i.

$$R_t^F = \frac{1 + \chi(1 - \gamma_t)}{1 + \chi} R_t^D \tag{35}$$

$$f_{it} = (1 + \chi)d_{it} \tag{36}$$

The balance sheet can be rewritten as follows:

Assets	Liabilities
k_{it}	ω
s_{it}	f_{it}

With external funds being remunerated at rate R_t^F. We obtain the same banking problem as before, replacing deposits by total funds f_{it} and the deposit rate by the unit cost of funds R_t^F. We can solve as before by mapping f_{it} and R_t^F easily into deposits d_{it} and their rate R_t^D. By moving γ_t the central bank will be able to change R_t^F as long as changes in equilibrium R_t^D do not offset perfectly the changes in the spread on the total cost of funding.

4.2 Solving the dynamic model

The financial sector equilibrium determines investment given funding costs R_t^F and expected productivity Z_{t+1}^e. We can then solve for the aggregate capital stock K and cut-off α_t^L as a function of R_t^F and expected productivity Z_{t+1}^e.

$$K = K^*(R^F, Z^e) \tag{37}$$

$$\alpha^L = \alpha^{L,*}(R^F, Z^e) \tag{38}$$

By integrating balance sheet equations, we obtain an expression for total funds F_t and deposit supply D_t:

$$F_t = \int_{\alpha_t^L}^{\overline{\alpha}} (k_{it} - \omega) \; dG(\alpha^i) \tag{39}$$

$$D_t = \int_{\alpha_t^L}^{\overline{\alpha}} d_{it} \; dG(\alpha^i) = \frac{F_t}{1+\chi} \tag{40}$$

where $F_t = \int f_{it} \; dG(\alpha^i)$ are total liabilities held by *leveraged* intermediaries and D_t is the aggregate deposit demand. Market clearing in the deposit market requires supply and demand to be equal.

$$D_t^H = D_t \tag{41}$$

Goods market clearing requires that output is used in consumption of intermediaries and households, investment and the accumulation of storage. The investment good is the consumption good and there are no capital or investment adjustment costs[29]. Aggregate investment I_t is given by the law of motion of the capital stock $K_t = (1-\delta)K_{t-1} + I_t$. The resource constraint of the economy is as follows:

$$S_{t-1}^H + S_{t-1}^I + Y_t = C_t^H + C_t^I + S_t^H + S_t^I + I_t + T_t^L \tag{42}$$

where $C_t^I = \int c_{it} \; dG(\alpha^i)$ and $T_t^L = \int l_{it} \; dG(\alpha^i) - R_{t-1}^L \int l_{i,t-1} \; dG(\alpha^i)$ is the net wholesale funding. S_t^H are the holdings of storage held by households and $S_t^I = \int s_{it} \; dG(\alpha^i)$ are aggregate storage holdings held by financial intermediaries at t.

Definition 2: Equilibrium.

[29] We also do not constrain investment to be necessarily positive.

Let $\mathcal{S} = \{D_{t-1}, S^H_{t-1}, S^I_{t-1}, K_{t-1}, Z_{t-1}, \gamma_{t-1}, \varepsilon^z_t, \varepsilon^\gamma_t\}^\infty_{t=0}$ be the vector of state variables and shocks. Given a sequence of rates $\{r^D_t\}^\infty_{t=0}$, monetary policy rule and financial market rules $K(\mathcal{S}), \alpha^L(\mathcal{S}), S(\mathcal{S})$, let us define the optimal decisions of the representative household as $C^H(\mathcal{S}), D^H(\mathcal{S}), S^H(\mathcal{S})$.

An equilibrium is a sequence of rates $\{r^D_t\}^\infty_{t=0}$, and policy rules $C^H(\mathcal{S})$, $D^H(\mathcal{S})$, $S^H(\mathcal{S})$, $S^I(\mathcal{S})$, $K(\mathcal{S}), \alpha^L(\mathcal{S})$, such that:

- $C(\mathcal{S}), D^H(\mathcal{S}), S^H(\mathcal{S}), S^I(\mathcal{S}), K(\mathcal{S}), \alpha^L_t(\mathcal{S})$ are optimal given $\{r^D_t\}^\infty_{t=0}$
- Asset and goods markets clear at every period t

In equilibrium, we need to find a deposit rate which, conditional on exogenous variables and the financial sector equilibrium, is consistent with the household problem. We proceed by iterating on r^D_t, imposing the financial market equilibrium results. For a given deposit rate r^D_t, we find the law of motion for household wealth and consumption, use the Euler equation errors to update the deposit rate and repeat until convergence. A more detailed explanation of the algorithm used for our global solution method can be seen in Appendix A.

4.3 Calibration

To solve the model numerically, we need to specify the period utility function, the shape of the distribution of the *VaR* probabilities and calibrate the remaining parameters. Given the interaction between extensive and intensive margin effects, the mass of intermediaries in a given section of the distribution could have an important role in determining which of the two effects dominates. To highlight that the results described are not a consequence of this distribution, we assume that $G(\alpha^i)$ is uniform between $[0, \overline{\alpha}]$. For the utility function, we assume a standard CRRA representation.

$$u(C) = \frac{C^{1-\psi} - 1}{1 - \psi} \tag{43}$$

The calibration can be seen in Table 1. For the utility function parameters, risk aversion ψ, the subjective discount factor β, the TFP parameters ρ^z and σ_z we use

Table 1: Calibration of selected parameters

Parameter	Value	Description
ψ	4	Risk aversion parameter
β	0.96	Subjective discount factor
ρ^z	0.9	AR(1) parameter for TFP
σ_z	0.03	Standard deviation of TFP shock
μ_γ	0.023	Target spread over deposit rates
ρ_γ	0.816	Spread persistence
σ_γ	0.0128	Standard deviation of spread
$\frac{\chi}{1+\chi}$	0.41	Wholesale funding percentage
θ	0.35	Capital share of output
δ	0.1	Depreciation rate
ω	0.697	Equity of intermediaries
$\overline{\alpha}$	0.4961	Upper bound of distribution $G(\alpha^i)$
ζ	0.01	Idiosyncratic unproductive capital probability

standard values from the literature. Similarly for θ, the capital share of output, and for δ the depreciation rate of the capital stock. To calibrate the monetary policy parameters, we calculate the subsidy as the difference between the Effective Fed funds Rate and $1/\beta$, the long-run deposit rate. We then fit an AR(1) process to get the parameters used.

The wholesale funding percentage used to calibrate χ was calculated from the time series mean of the cross-sectional asset-weighted average in Bankscope data[30] for the period 1993-2015. For the purpose of this calibration, wholesale funding was assumed to be all non-deposit liabilities of each financial intermediary.

We calibrate $\overline{\alpha}$ to match the probability of default of the median *risky* intermediary when deposit rates are at steady-state. Using FDIC data on failed banks, we find that the median age of failed banks in the US was around 20.5 years. The full sample distribution of ages at failure can be seen in Figure 18. We then calibrate $\overline{\alpha}$ to match a default probability 5% for the median intermediary when $R_t^D = 1/\beta$. This also implies a default probability for the riskiest intermediary $\overline{\alpha}$ of 22%. Turan G. Bali, Stephen J. Brown and Mustafa O. Caglayan (2014) report that the median lifespan of a hedge

[30]Bankscope contains a large panel of financial intermediaries' balance sheet data. See Appendix D.

fund is slightly less than 5 years, a value close to what our calibration implies for $\overline{\alpha}$.

ω is chosen to fit leverage at steady-state. Some of the intermediaries are leveraged and others are not, so we cannot use only Bankscope data (which contains mostly leveraged banks) to calibrate leverage. According to the "broad measure" of Other Financial Institutions (OFIs) in the Global Shadow Banking Report (Financial Stability Board (2015)), non-levered intermediaries hold about 137 trillions of assets while banking assets are around 135 trillion. We use these figures to calculate an asset-weighted average of leverage of 7.3, which is reached by combining the Bankscope asset-weighted average leverage of 13.5 for 2015 and assuming a leverage of 1 for the OFIs. We target our calibration of ω so that the median *risky* intermediary matches this value.

The size of the equity endowment ω and the volatility of aggregate shocks σ_z will also contribute to determine the financial sector reaction to changes in deposit rates. For that reason, we also conducted some comparative statics on both σ_z and ω to see how the model changes with those parameter calibrations. There is very little effect on the first moments of real variables such as output and consumption but there are important changes on equilibrium leverage and financial stability when we vary ω and/or σ_z. In general, the easier it is for riskier intermediaries to absorb the market, then less stable will financial markets be. Increases in ω and decreases in σ_z both *worsen* financial stability. Low volatility of the fundamental shocks σ_z will lead to lower financial stability since riskier intermediaries will find it easier to capture the market. More details can be found in Appendix F.

The value of $\overline{\alpha}$ and the shape of its distribution will also matter for financial stability. Increasing $\overline{\alpha}$ leads to a less financially stable financial sector. We leave for future work to perform a (technically challenging) estimation of the model where the distributions of α or ω could potentially be backed out from the data and focus here on understanding the qualitative implications of the model.

4.4 Monetary policy shocks

We now look at the impact of a positive subsidy shock, which we will refer to as an expansionary monetary policy shock or a decrease in the cost of funds. In Figure

4 we see the impact of a 100 basis points to the subsidy[31] in three different scenarios to illustrate the non-linear effects of monetary policy on financial stability. Impulse response functions are expressed as deviations from the respective scenario in the absence of the shock. This monetary policy loosening decreases the funding rate of the banks by 8 bp as can be seen in the left panel of Figure 5. Scenario 1 (blue line) features a low initial capital stock (corresponding to high equilibrium levels of the interest rate). Scenario 2 (red line) is for a larger capital stock (corresponding to a low level of equilibrium interest rate). Scenario 3 (black line) is at the risky steady-state[32]. As in Coeurdacier, Rey and Winant (2011) we define the risky steady-state as the steady-state in which there are no shocks but economic agents take into account the full stochastic structure of the model when they optimize (unlike in the deterministic steady-state where they expect no shocks).

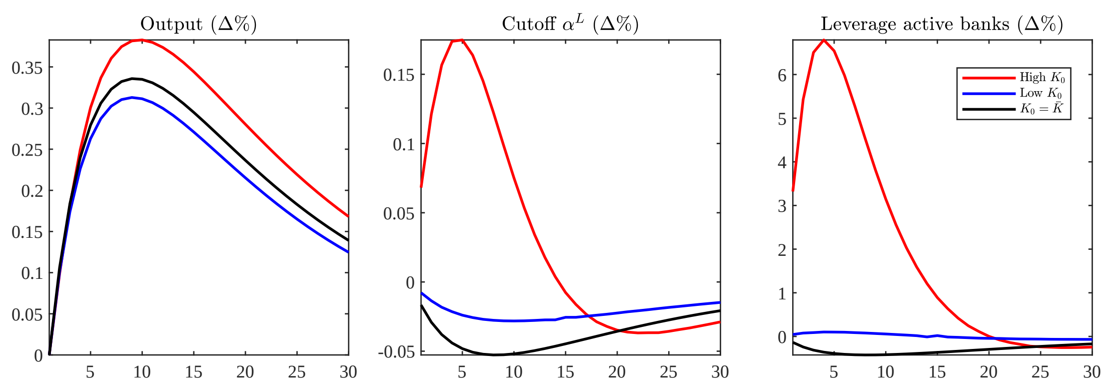

Figure 4: *Monetary policy shock of 100 basis points to* γ_t

We can easily relate the general equilibrium results to the partial equilibrium intuitions developed above. In the case of a low initial capital stock (associated with a high equilibrium funding rate), a positive monetary policy shock expands output, increases aggregate leverage and at the same time it reduces the cut-off α^L, due to the

[31]Note that this translates into a lower reduction in the total cost of funds (see Figure 5). This is due to the fact that the cost of funds is a composite of deposits and wholesale funds, but also due to endogenous movements in the deposit rate.

[32]These three scenarios were chosen to illustrate the parallel with the partial equilibrium setting, since the solution of the model is such that there is, ceteris paribus, a negative correlation between the initial capital stock and the funding rate.

entry of less risk-taking intermediaries in deposit markets. We are in the "no trade-off zone of monetary policy" where a decrease in the interest rate increases investment and financial stability. In the case of a high initial capital stock (associated to a low funding cost for intermediaries), an expansionary shock has a larger positive effect on output and leverage but this time intermediaries at the margin choose not to lever up. In contrast, the most risk-taking intermediaries leverage significantly and financial stability is affected negatively.

This is a very different trade-off from the traditional Phillips curve which has been the benchmark model driving monetary policy analysis for many years. Aggregate economic variables such as consumption, wealth or capital behave smoothly as evidence in Figure 13, but the underlying change in financial structure supporting these macroeconomic outcomes can become less stable depending on the level of the interest rate.

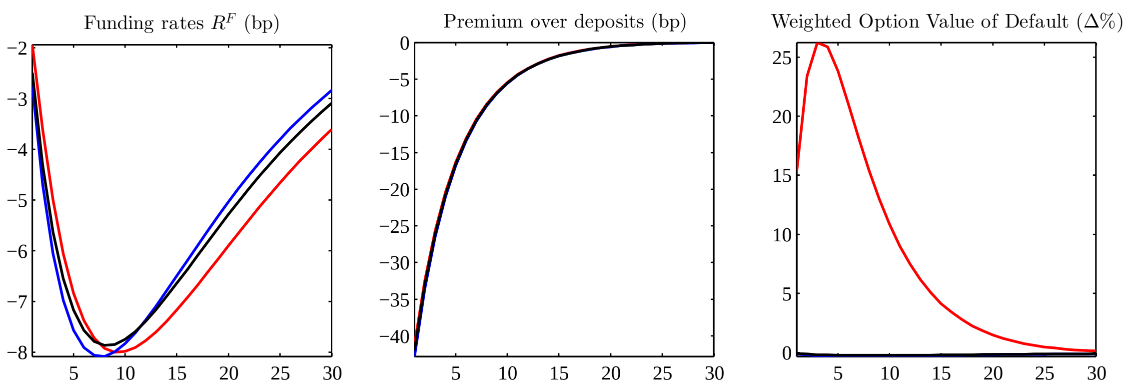

Figure 5: *Monetary policy shock of 100 basis points to* γ_t*: Financial variables*

As seen in Figure 5, the Weighted Option Value of Default also increases drastically with a monetary policy loosening when interest rates are low. The option value of default is defined as the difference between expected profits under limited liability and the (untruncated) cash flow. The larger this difference, the bigger the distortions coming from the presence of the limited liability and the worse for financial stability. Since the option value of default is intermediary-specific due to the heterogeneity of balance sheets, we construct an asset-weighted mean to illustrate the aggregate effect. When the interest rate is lower, the decrease in the cost of funds generates a very large increase due to the exit of safer intermediaries but also to the increase in leverage skewness in the cross-section. The impulse response functions for the alternative risk measures of Figure 17 in Appendix C also illustrate the presence of a strong trade-off when interest rates are low. Finally, the premium over deposits goes down as monetary policy expands since the demand for deposits goes up and the expected return to risky capital goes down due to decreasing returns.

4.5 Productivity driven leverage

Cycles in leverage can be driven by movements in the cost of funds, but also by changes in expected productivity. When leverage is driven by an increase in productivity then the ensuing leverage growth does not come at the cost of financial stability. There is a fundamental difference between a credit boom driven by a shift in supply (i.e. cheaper access to funds) and a boom driven by demand for credit (i.e. better investment opportunities). Productivity shocks in our framework are an example of the latter as they forecast larger productivity in the future. In general equilibrium supply and demand of credit are interdependent so this distinction is simply to clarify the intuition and relate it to the original shock leading to credit growth.

We now look at a shock to productivity. In Figure 6 we see the impact of a one standard deviation positive productivity shock in the same 3 scenarios as the previous section.

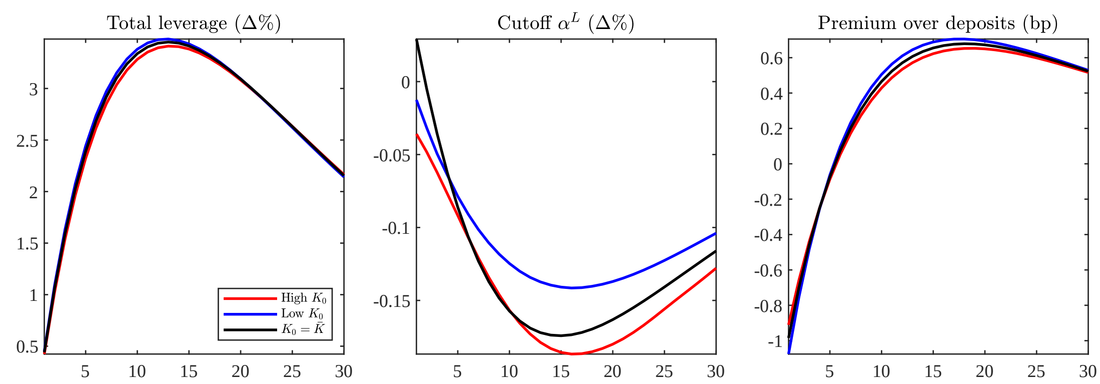

Figure 6: *Shock to exogenous productivity*

The effects are similar irrespective of the position in the state space and the level of interest rates at the time of the shock. Total leverage goes up due to increased investment opportunities and is hump-shaped, as can be seen in the first panel. The hump-shape is due to the initial pressure of credit demand which requires higher deposit rates to clear the market. After impact household wealth accumulates and deposit rates start to fall, leading to a hump-shaped response of credit and investment as the positive productivity shock fades out.

This effect can be also seen on the premium over deposits (right panel). On impact, there is a larger rise of deposit rates than expected returns, despite the better investment opportunities coming from higher expected productivity. The effect on the premium is however very small (1bp decrease on impact), only a small fraction of the effect seen after a monetary policy shock (40bp). In the middle panel, we also see that financial stability overall slightly improves in all scenarios, apart from a short-lived marginal uptick on impact in the middle scenario. Again these are small effects, indicating that productivity driven leverage booms are not a concern for financial stability in the same way that credit supply driven ones are. As Krishnamurthy and Muir (2017) show, credit booms accompanied by the tightening of spreads can predict financial crises, while those without such a tightening do not. We are able to rationalize this fact through the cross-sectional composition of the financial sector and the difference between productivity driven and credit supply driven leverage.

5 Empirical evidence on the cross-section of intermediary balance sheets

We do not present here a test of our model but a number of important new stylized facts on the cross sectional distribution of intermediaries balance sheets over the cycle. To the best of our knowledge, these facts were not reported previously as the literature did not feature intermediary heterogeneity. We use balance sheet data of financial intermediaries from Bankscope (see Appendix D) to compute leverage at the intermediary level. Leverage is defined as the ratio of assets over equity at book value.

Fact 1: Heterogeneous leverage dynamics and correlation with the Fed

funds rate. In Figure 7 we show the time series of leverage weighted by intermediary assets for different quantiles of the distribution and for the aggregate together with the CPI-deflated Fed Funds rate. There is strong heterogeneity within the financial sector in terms of time variation of leverage and its correlation with the interest rate. Up to 2007, the correlation between the top 1% and the real effective Fed Funds rate is -0.49 and it is -0.31 for the average leverage.[33] The more leveraged intermediaries increase leverage sharply as interest rates fall to low levels in the early 2000s[34]. This large increase in leverage is neither apparent in the median nor the bottom 1% of the distribution. In the pre-crisis period we find strong evidence of heterogeneity in the cross-section of leverage. The correlation between the median quantile with the top 1% is strongly negative (-0.81) and the correlation between the bottom 1% and the top 1% is -0.1.

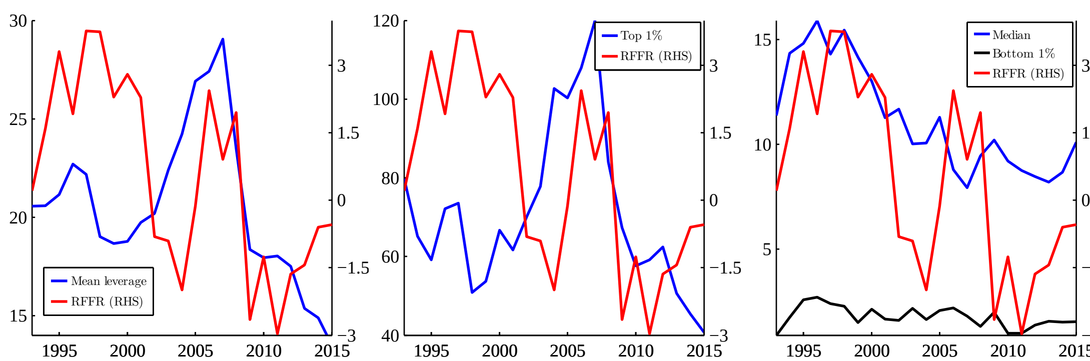

Figure 7: *Mean and selected quantiles of asset-weighted leverage of intermediaries (blue, LHS scale) and real (CPI-deflated) Effective Fed Funds Rate (red, RHS scale, pp).*

The model predicts heterogeneous leverage dynamics in the cross-section, even as aggregate leverage is monotonically decreasing with the interest rate. In the top quantiles, leverage and interest rates are negatively related. In intermediate quantiles,

[33] Results are very similar if we use instead nominal rates, as can be seen in Figure 15. The correlation between the top 1% and the nominal effective Fed Funds rate is -0.26 and it is -0.11 for the average leverage.

[34] Note that in 2006-07 the Fed Funds rate increases. However, during that period, the maturity of liabilities of the large banks decreased: about a quarter of the liabilities of broker dealer balance sheet was overnight repos at the eve of the financial crisis, thereby keeping funding costs very low.

some intermediaries may stop leveraging as interest rate goes down so the correlation sign may flip. For intermediaries that remain levered, Proposition 2.2 indicates that the covariance between interest rates and leverage should be larger for the most risk-taking (and most levered) intermediaries. The post-2008 period is of course very special with large state interventions and changes in regulation as well as unconventional monetary policy at the Zero Lower Bound (ZLB), all elements which are absent for our model.

Fact 2: Skewness of leverage and Fed Funds rate

We use the same data as before to compute the time series of the skewness of leverage. In Figure 8, we present the time series of asset-weighted skewness in parallel with the movements of the Effective Fed Funds Rate both in real terms (left panel) and in nominal terms (right panel). There is a strong negative correlation between the Effective Fed Funds Rate and skewness (correlation of -0.45 up to 2007 for the real rate and of -0.58 for the nominal rate), with a large spike in skewness when interest rates fell strongly in the early 2000s.[35]

The model is consistent with this fact. In the right panel of Figure 2, we show the shape of cross-sectional skewness as a function of r_t^D for three different levels of productivity. It is apparent that the direct impact of productivity on skewness, although positive, seems second-order relative to the impact of interest rates. Low levels of the interest rate are associated with an increased skewness of leverage: risk gets concentrated in the (endogenously) larger, more risk-taking players. This is a very distinctive implication, which is borne out in the data. These results are striking and very encouraging for the mechanism of the model. We are not aware of any paper studying the distribution and skewness of leverage and linking it to the interest rate.

Fact 3: Exposure to macroeconomic risk, leverage and returns

We analyse the leverage of financial intermediaries in the run up to the crisis and look at its correlation with returns and the exposure to aggregate risk (measured by the world market beta). The left panel of Figure 9 shows a positive correlation (0.59)

[35] Admittedly, the situation after 2008 when monetary policy is at the ZLB is (as explained above) quite unusual with large state interventions in the banking sector and changes in regulation, including leverage caps. We also computed skewness using only US bank data, results were very similar and are available on request.

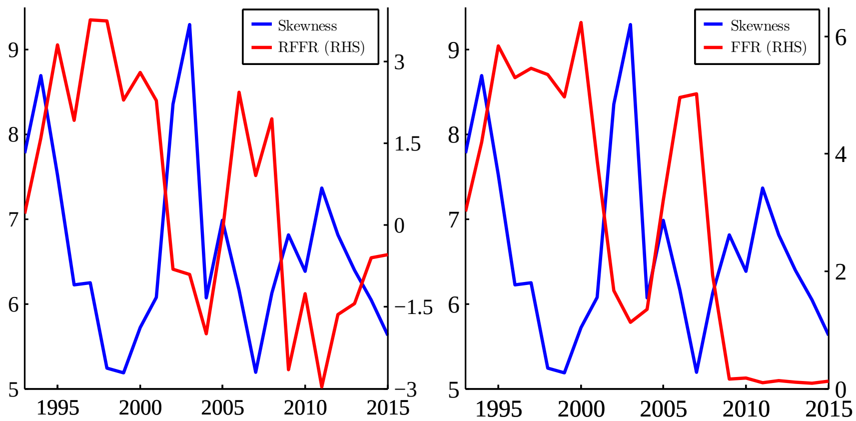

Figure 8: Time series of asset-weighted skewness of intermediary leverage (blue, LHS scale) and the real Effective Fed Funds Rate (left panel) and the nominal effective Fed Funds rate (right panel) in pp (red, RHS scale).

between pre-crisis betas and leverage, while the one on the right also shows a positive correlation (0.32) between pre-crisis leverage and returns, confirming the results of Miranda-Agrippino and Rey (2015), who in addition show that the higher beta banks tended to do worse in the crisis as they were more exposed to aggregate risk.

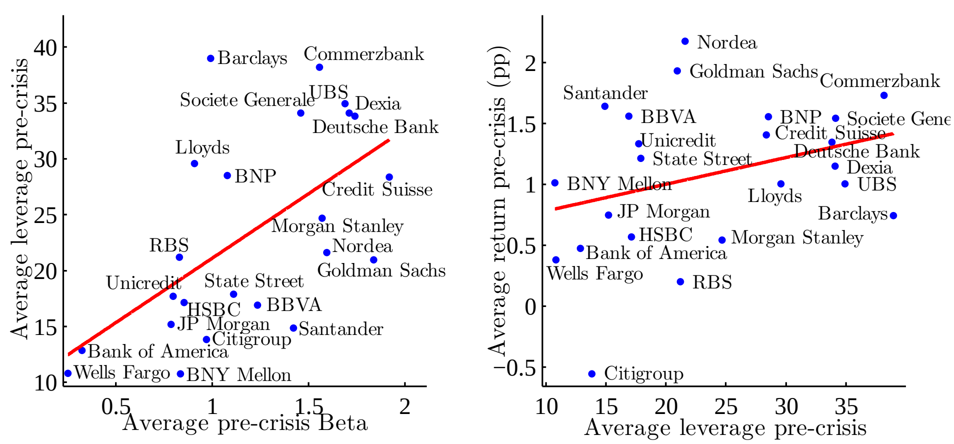

Figure 9: *Pre-crisis leverage, world market betas and returns for Systemically Important Financial Institutions (SIFIs)*

The model is consistent with this pattern: it implies that the intermediaries which become endogenously larger and more leveraged make higher profits in good states of the world but are more exposed to aggregate risk.

Although none of this constitutes a formal test of the model, we view these facts as supporting the relevance of the main mechanism of our framework. This underlines the importance of looking at cross-sectional dynamics of the balance sheets of financial intermediaries in order to understand macroeconomic developments.

6 Costly intermediary default

We now consider the case of costly intermediary default. As in the previous section, leveraged intermediaries in risky financial markets will default on depositors if the realisation of the productivity shock is low enough. This requires intervention by the government to pay for deposit insurance, which is now less benign than previously assumed as there is a deadweight loss[36].

To include a cost of intermediary bailouts we assume that capital held by defaulting intermediaries suffers a proportional productivity loss $\overline{\Delta}$ relative to the productivity of capital held by non-defaulting intermediaries. This loss can arise from (real) bankruptcy costs or some degree of inalienability in investment projects. The main assumption is that these costs are proportional to the output of the respective capital shares. Let μ_t^d be the share of capital held by defaulting intermediaries. We can define an aggregate productivity loss $\Delta_t = \mu_t^d \overline{\Delta}$ which is an increasing function in the share of capital held by defaulting intermediaries. Note that the productivity of capital held by healthy intermediaries is unaffected at t, so the impact on aggregate productivity is coming only from cross-sectional differences between defaulting and non-defaulting intermediaries.

We also consider the possibility that this disruption spreads to the entire financial market in the following periods by affecting productivity of *all* intermediaries in future periods by Δ_t. The loss of aggregative productivity is then intermediary-specific during default, but it can affect the whole economy moving forward (the allocative process of the whole economy is impaired). When it happens we call this the *crisis* state. We

[36] As before, deposit guarantees will be financed by lump sum taxation of households. The welfare analysis of our setup is left for future work.

model the persistence of the crisis state through a Poisson process, with a constant probability p of exiting the crisis at each period. Depending on the process, variable ξ_t takes the value of one if the crisis carries on to the next period or zero if it does not. Our specification nests both the case of costless default ($\overline{\Delta} = 0$) and the case where there is no disruption of financial markets in subsequent periods ($p = 1$). We have:

$$\mu_t^d = \frac{\int k_{it} \ \mathbb{1}_{(\pi^i<0)} \ dG(\alpha^i)}{K_t} \tag{44}$$

$$\Delta_t = \xi_{t-1} \max(\mu_{t-1}^d \ \overline{\Delta}, \Delta_{t-1}) \tag{45}$$

$$\tag{46}$$

where the indicator function takes the value of 1 if intermediaries of type i default or 0 if not. If there are also defaults during a crisis state, then the max operator ensures that the largest penalty applies going forward. Whenever the economy is in crisis, productivity for all financial intermediaries is scaled down by a factor μ_t^d proportional to the percentage of total capital held by defaulting intermediaries. ξ_{t-1} is known to agents when they make their investment decisions at period $t-1$, so the uncertainty on the returns on their capital investment is only on the realization of the exogenous productivity process[37]. This timing assumption allows us to keep tractability as the main difference in the financial sector block is that now $Z_{t+1}^e = (1 - \Delta_t) Z_t^{\rho^Z}$. Since both Δ_t and Z_t are state variables, we can still solve for the financial sector equilibrium as before.

This set up is tractable and allows us to parameterize crises of different severity and length. Reinhart and Rogoff (2009*a*) present a classic description of the characteristics of crises across history, and evidence that crises associated with banking crises are more severe. Borio et al. (2016) and Laeven and Valencia (2012) present empirical evidence showing that there can be substantial and long lasting productivity drops after financial crises. To calibrate these parameters we refer to the database of Laeven and Valencia (2012), setting $p = 0.5$ to target an average crisis length of 2 years as in the data, and

[37] There is still uncertainty on asset returns if the intermediary defaults but this is not considered in the intermediary problem due to limited liability truncating the profit functions at zero in those states.

$\overline{\Delta} = 0.115$ implying a maximal efficiency loss of 11.5% per year[38].

6.1 Productivity shocks and financial crises

In this section we study the impact of a financial crisis on the path of the economy, following a large productivity shock. Figure 10 shows the impact of a large productivity shock in 3 possible scenarios[39].

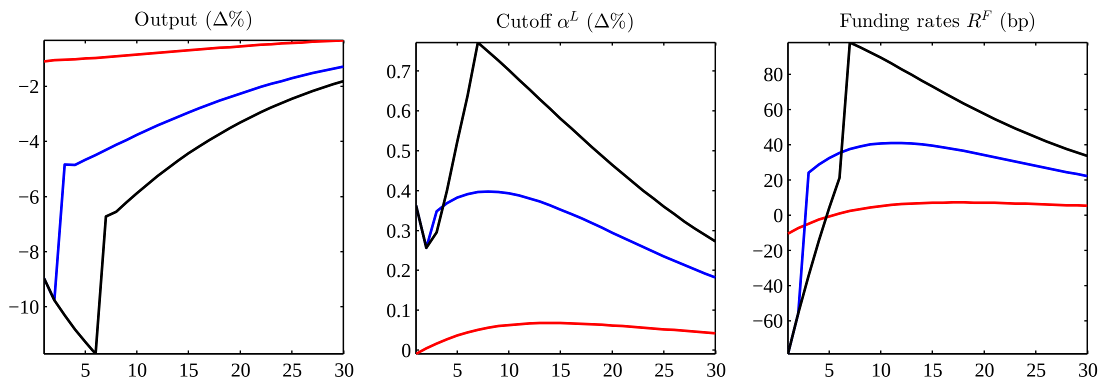

Figure 10: *Large shock to exogenous productivity*

In scenario 1 (red line) the economy at the risky steady-state is hit at period t by the largest possible shock that does not trigger any defaults. In scenarios 2 (blue line) and 3 (black line) the economy is hit with the smallest shock such that all *levered* intermediaries default. The difference between scenarios 2 and 3 is in the length of the crisis. Scenario 2 is the short crisis scenario, where the crisis only carries on to the next period, $\xi_1 = 1$. Scenario 3 is the "unlucky" scenario, where the crisis carries on for an additional 5 periods: $\xi_s = 1$ for $t = 1$ to $t = 6$. The length of the crisis is unknown beforehand to the agents in the economy, although as mentioned before they observe the value of ξ_t when they make their investment decisions at t. Not surprisingly, when the crisis hits there is a large decline in output. As expected productivity is low, only the intermediaries with the looser *VaR* constraints can operate. There is a strong fall

[38]In the database of Laeven and Valencia (2012), the average cumulative output loss is 23% over the length of the crisis, which is on average two years.

[39]Impulse response functions expressed in basis points deviations for rates or otherwise in percent deviations from the risky-steady state

in deposit demand due to the low expected productivity, which severely tightens the constraint.

In equilibrium the fall in deposit demand generates a fall in funding costs due to decreased deposit rates. For the cases with defaults, one can observe a small initial decrease in the cut-off after an initial jump. This is because on impact the economy jumps to the trade-off region. As interest rates start rising from that point onward, the economy travels through the U-shape with the cut-off falling initially and then increasing as interest rates rise.

The length of the crisis also has very interesting dynamic effects on wealth. Given that households expect to exit the crisis state with probability p, when exit fails to materialize in Scenario 3 they are running down their wealth and their consumption dips down (see Figure 14). As wealth falls, deposit rates and funding costs (see Figure 10) grow as it becomes more costly for the household to save and fund bank leverage. When eventually the economy exits the crisis state, household wealth is low and demand for leverage jumps, leading to a jump in funding rates to compensate households for decreased consumption today. This leads also to a higher risk premium as expected return to capital jumps up. Total leverage and investment, which had seen severe contractions start to go up again (see Figure 14). This effect is also present with a short crisis, but is particularly stark for the longer crisis.

7 Conclusion

This paper develops a novel framework for modeling a financial sector with heterogeneous financial intermediaries and aggregate risk. The heterogeneity in the *VaR* constraints coupled with limited liability generates endogenous time variation in leverage, risk-shifting and financial stability. The interaction between the intensive and the extensive margins of investment creates a rich set of non-linear dynamics where the level of interest rates plays a key role. When interest rates are high, a monetary expansion increases both the intensive margin and the extensive margin. The monetary authority is able to stimulate the economy, while at the same time increasing financial stability. When interest rates are already low, a further reduction can lead to large increases in leverage by the most risk-taking institutions, pricing out previously active intermediaries, due to decreasing aggregate returns to capital. Importantly, the intermediaries which decrease their balance sheet size have lower probabilities of default than those that remain levered, leading to an increase in systemic risk. Our model, unlike the existing literature, generates a trade-off between economic activity and financial stability depending on the level of the interest rate. During booms driven by low funding costs and increased credit supply, risk premia are low as there is a lot of risk-shifting by the most risk-taking intermediaries in the economy. Booms driven by positive productivity shocks do not lead to an increase in financial instability nor to such low levels of risk premia.

Because our framework has heterogeneity at its heart, it allows us to make use of cross-sectional data on intermediary balance sheets. We derive novel implications linking the times series of the skewness of leverage and monetary policy which are strikingly borne out in the data. We believe we are the first paper to link changes in the cross-sectional distribution of leverage, macroeconomic developments and fluctuations in financial stability. We show that similar macroeconomic outcomes can be supported by very different underlying financial structures. This has important implications for the transmission of monetary policy and the sensitivity of the economy to interest rate movements.

A major advantage of our framework is that our financial block is easy to embed in a standard dynamic stochastic general equilibrium framework. We plan to extend our model to environments with sticky prices and a more complex portfolio choice on the bank side as well as to study boom and bust cycles in emerging markets. We also plan to apply it to explain the dynamics of the real estate market, using detailed data, as well as the endogenous dynamics of the VIX. The model could also be calibrated to fit a distribution of financial intermediaries characteristics. One could in practice back out the distribution of α^i from leverage data and allow for a distribution of intermediary-specific equity ω^i. That said, allowing for time variation in equity would require the introduction of an additional state-variable in the financial sector problem which would make the solution more computationally intensive.[40] We leave these issues and the welfare implications of our model for future research.

[40] And having together time-varying and intermediary-specific equity would require an infinitely dimensional state-space without additional assumptions.

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Figures

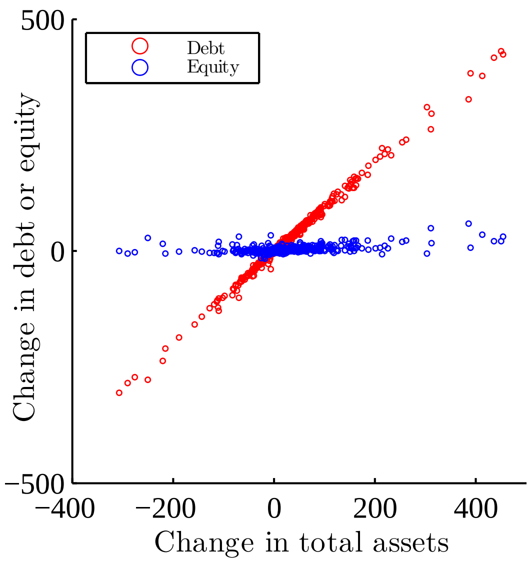

Figure 11: *Yearly changes in total asset against yearly changes in equity or debt from 1993 to 2015. Billions of USD. Source: Bankscope*

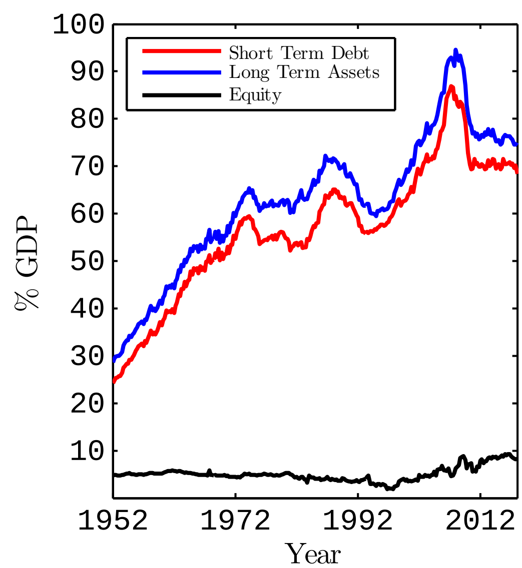

Figure 12: *Bank short-term debt, long-term assets and equity as a percentage of US GDP. Data constructed as in Krishnamurthy and Vissing-Jorgensen (2015).*

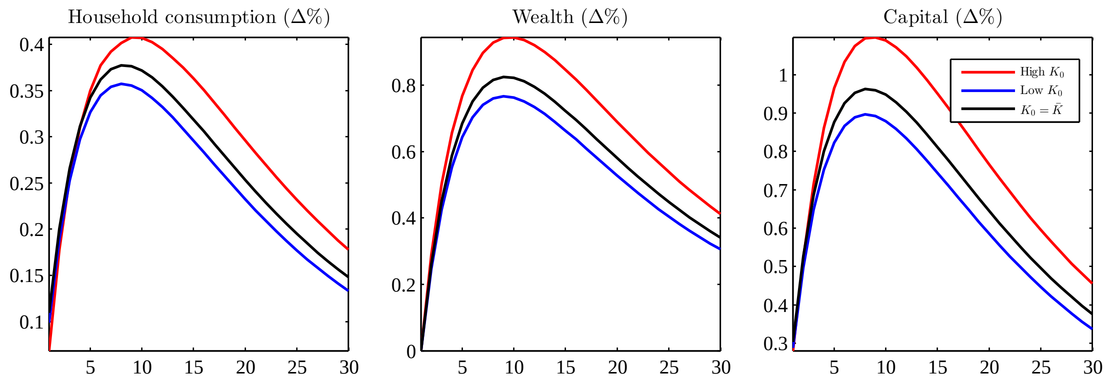

Figure 13: *Monetary policy shock of 100 basis points to γ_t: Real variables*

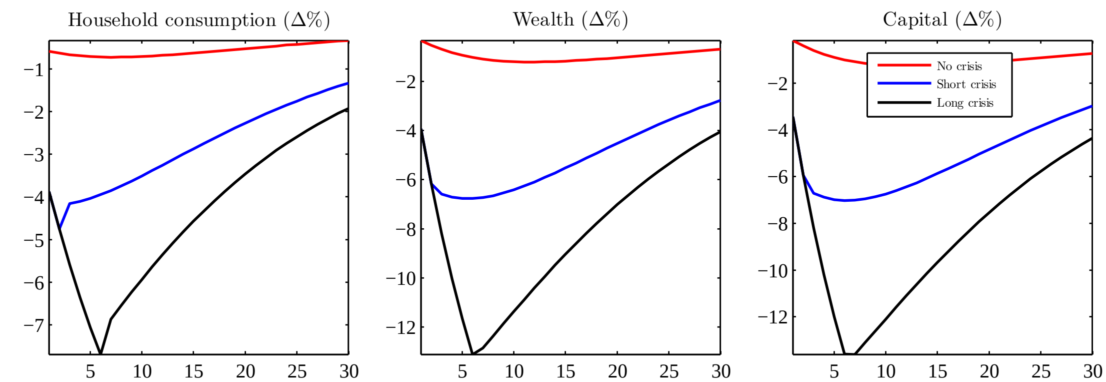

Figure 14: *Large shock to exogenous productivity: Real variables*

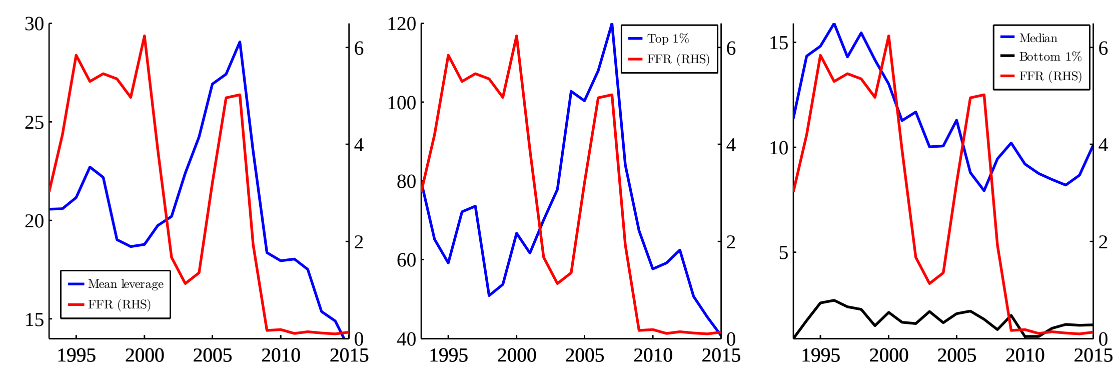

Figure 15: *Mean and selected quantiles of asset-weighted leverage of intermediaries (blue, LHS scale) and the Effective Fed Funds Rate (red, RHS scale, pp).*

Appendix A. Numerical solution method

For online publication

The solution method is composed of two main blocks. The first block solves the partial equilibrium problem for a grid of points for variables r^F and Z^e. We discretize the state space using 100 nodes for Z^e and 200 for r^F. Given funding costs r^F and expected productivity Z^e we can solve jointly for equations (25) and (27), plugging in equation (20) in the latter. We also use the property that levered intermediaries never invest in storage. This gives us policy functions $K^*(r^F, Z^e)$ and $\alpha^{L,*}(r^F, Z^e)$.

The second block is the recursive one. First we define the household savings problem as a function of disposable wealth Ω_t, productivity $\tilde{Z}_t$, efficiency adjustment Δ_t and monetary policy γ_t.

$$\Omega_t = (1-\theta)Y_t - T_t + D^H_{t-1} + S^H_{t-1}$$

The procedure entails the following steps

1. Discretize the state space $\mathcal{S}$ for the variables $(\Omega, Z, \Delta, \gamma)$. The process for Z and γ are approximated using a Tauchen and Hussey (1991) quadrature procedure with 11 and 7 nodes respectively. The state space for the variable Ω is discretized using 500 nodes and we use 10 for Δ.

2. Iterate on prices r^D and policy function $C^*(\mathcal{S})$ starting with an initial guess $r^D(\mathcal{S})$ for deposit prices and $C^*(\mathcal{S})$. For every point $\mathcal{S}_j \in \mathcal{S}$:

   (a) Using the state vector and r^D_j, calculate r^F_j and Z^e_j.

   (b) Solve for (K_j, α^L_j) using $K^*(r^F_j, Z^e_j)$ and $\alpha^{L,*}(r^F_j, Z^e_j)$. Back out deposit supply D_j from the balance sheet equations.

   (c) Plug D_j in the budget constraint of the agent. Together with $C_j = C^*(\mathcal{S}_j)$ this pins down S^H_j.

   (d) Calculate expectations of $(\mathcal{S}'|\mathcal{S})$ and update deposit prices and policy functions using the optimality conditions and numerical integration.

   (e) Check for convergence. If $||(r'_j - r_j)|| + ||(C^*_j)' - C^*_j||$ is smaller than a threshold value stop. Else, go back to (a) and repeat.

To numerically integrate intermediary variables, Gauss-Legendre quadrature using 51 points is used. To calculate expectations of future net disposable wealth, we also need to calculate taxes conditional on future shocks. For a given productivity draw $Z'|Z_j$ we identify the threshold intermediary for which no bailout is needed: $(R^K k_i - R^D d_i) = \omega$. We can then calculate the amount T_t of taxes required by numerical integration.

Appendix B. Proof of Proposition 2.1

For online publication

When $\mathbb{E}[R^K_{i,t+1}] \geq 1$, participating intermediary i will either lever up to its Value-at-Risk constraint: $d_{it} = \overline{d}^i_t$, or not raise deposits at all : $d_{it} = 0$.

Given the option value of default and the condition $\mathbb{E}[R^K_{i,t+1}] \geq 1$, participating intermediaries will not invest in storage. The Value-at-Risk constraint bounds the maximum level of leverage of intermediary i, therefore $d_{it} \in [0, \overline{d}^i_t]$. The expected profits of intermediary i as a function of deposits are:

$$\pi^i_t(d_{it}) = (1-\zeta)\int_{\varepsilon^i_t(d_{it})}^{\infty} \left[R^K_{t+1}(\omega + d^i_t) - R^D_t d^i_t\right] dF(\varepsilon) \tag{47}$$

where ε^i_t is the max of 0 (the lower bound of the support for ε) and the shock for which profits are zero).

$$\varepsilon^i_t(d^i_t) = \max\left(0, \frac{\frac{R^D_t d^i_t}{\omega + d^i_t} - 1 + \delta}{\theta Z^{\rho^Z} K_t^{\theta-1}}\right) \tag{48}$$

Taking derivatives:

$$\frac{\partial \pi^i_t}{\partial d^i_t} = (1-\zeta)\int_{\varepsilon^i_t(d^i_t)}^{\infty} \left(R^K_{t+1}(\varepsilon) - R^D_t\right) dF(\varepsilon) - \pi^i_t(\varepsilon^i)\frac{\partial \varepsilon^i}{\partial d^i_t} \tag{49}$$

Lemma 1 *Given equations (47) and (48), then* $\pi^i_t(\varepsilon^i_t)\frac{\partial \varepsilon^i}{\partial d^i_t} = 0$

For any $d_{it} \geq \frac{\omega(1-\delta)}{R^D_t - 1 + \delta}$, then $\pi^i_t(\varepsilon^i_t) = 0$ by definition of ε^i_t. For $d^i_t < \frac{\omega(1-\delta)}{R^D_t - 1 + \delta}$, then $e^i = 0$ and $\frac{\partial \varepsilon^i}{\partial d^i_t} = 0$ due to the max operator.

We have as first and second derivatives:

$$\frac{\partial \pi_t^i}{\partial d_{it}} = (1-\zeta)\int_{\varepsilon_t^i(d_t^i)}^{\infty} \left(R_{t+1}^K(\varepsilon) - R_t^D\right) dF(\varepsilon)$$
$$\frac{\partial^2 \pi_t^i}{\partial d_{i,t}^2} = -(1-\zeta)\left[R_{t+1}^K(\varepsilon(d_{it})) - R_t^D\right]\frac{\partial \varepsilon_t^i}{\partial d_{it}} \tag{50}$$

Given the monotonicity of $R_{t+1}^K(\varepsilon)$, then $\forall \tilde{d}$ such that $\left.\frac{\partial \pi_t^i}{\partial d_{it}}\right|_{\tilde{d}} = 0$, it follows that $R_{t+1}^K(\varepsilon_t^i(\tilde{d})) - R_t^D < 0$ or all elements in the integral are non-negative and it cannot be zero. Since $\frac{\partial \varepsilon_t^i}{\partial d_{it}} > 0$, then $\left.\frac{\partial^2 \pi_t^i}{\partial d_{it}^2}\right|_{\tilde{d}} > 0$ by equation (50). If $\tilde{d}$ exists, it must be a minimum and we therefore conclude that the maximum must be at the bounds: $d_{it} = \arg\max\left(\pi_t(0), \pi_t^i(\overline{d}_t^i)\right)$.

Appendix C. Alternative Measures of Financial Stability

For online publication

We present three alternative measures of financial stability. M^3 is the *asset-weighted mean of active* α^i. We have that $M_t^3 = \int_{\alpha_t^L}^{\overline{\alpha}} \alpha^i \frac{k_{it}}{K_t^L} dG(\alpha^i)$, where K_t^L is the total asset holdings of leveraged intermediaries. This measure has the advantage of not only capturing the extensive margin effect but also capturing the effect of skewness on aggregate financial stability. That is, a financial sector with the same cut-off α_t^L but with a more skewed distribution of leverage will on aggregate be more risky, as a larger share of the capital would be held by more risk-taking intermediaries.

We also explore a fourth measure of financial stability M^4: the *probability that a fraction* κ *of the capital* K_t^L *is held by distressed intermediaries in the next period.* M_t^4 is the solution to the equation $\int_{M_t^4}^{\overline{\alpha}} k_{it} dG(\alpha^i) = \kappa K_t^L$. This measure would be equivalent to the baseline measure M^1 if we set $\kappa = 1$, so it can be seen as a generalization of the first measure. Setting this fraction to a lower value captures some of the skewness effects mentioned. We implement this measure with a fraction arbitrarily set at $\kappa = 0.5$, so the probability that half of the capital is held by distressed intermediaries in the next period.

Finally we also calculate a fifth measure, M^5: *the expected share of capital held by defaulting intermediaries at* $t+1$. This measure relates to the costly default described in Section 6. If the deadweight loss is proportional to the share of capital held by defaulting intermediaries, then this measure gives us a sense of the expected efficiency costs of decreasing financial stability. Figure 16 shows theses measures as a function of the interest rate in partial equilibrium. All of them show a significant adverse effect of an interest rate decrease on financial stability when interest rates are low. In contrast financial stability does not worsen with a decrease in the interest rate when the level of the interest rate is high.

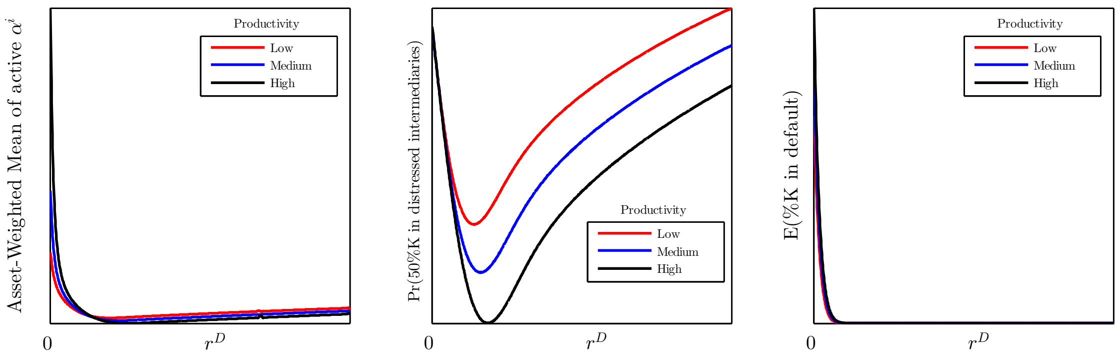

Figure 16: *Alternative measures of financial stability and interest rate*

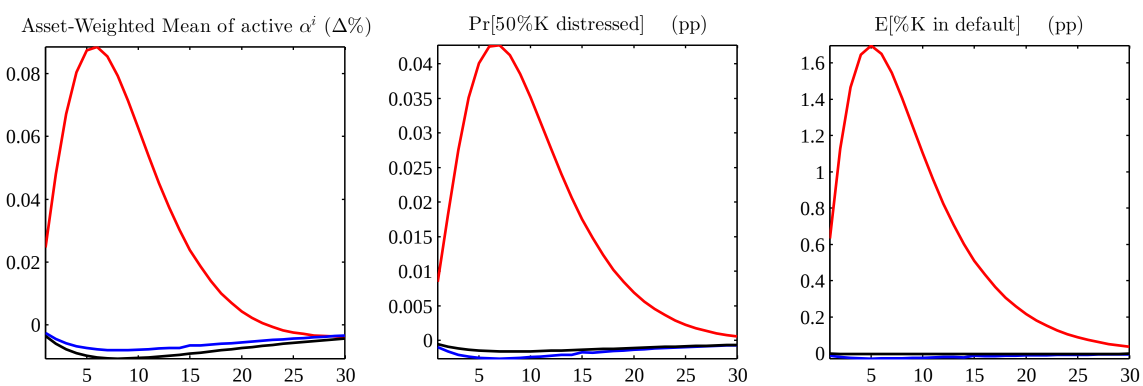

Figure 17: *Monetary policy shock of 100 basis points to γ_t: Alternative measures*

Appendix D. Data Description

For online publication

Bank balance sheet data uses annual data from the Bankscope database. Bank return data are from Datastream. Market returns were calculated using the MSCI World Index data available from Bloomberg. The Effective Federal Funds Rate and the CPI are from the Federal Reserve Economic Data.

The leverage ratio is defined as the ratio of total assets to total equity, here defined as common equity. We drop negative equity from the dataset, and institutions with assets worth less than 1 million USD. We also remove institutions that have leverage larger than 1000 at least once across the sample.

For the leverage series, we compute both unweighted and weighted averages of the leverage ratio for each quarter. For the weighted average we use total assets as weights. We checked using total equity as weights and results are qualitatively unchanged. We also compute the 1st and 99th percentiles of both unweighted and weighted leverage.

For the skewness of leverage, we compute the cross-sectional standard deviation and third moment of the leverage ratio for every period. And then compute the cross-sectional sample skewness using a simple approach laid out below.

$$
\begin{aligned}
m_t(3) &= \frac{\sum_{i=1}^{N} (x_{it} - \bar{x}_t)^3}{N} \\
s_t &= \sqrt{\frac{\sum_{i=1}^{N} (x_{it} - \bar{x}_t)^2}{N}} \\
S_t &= \frac{m_t(3)}{(s_t)^3}
\end{aligned}
$$

where x_{it} is the leverage ratio of bank i in period t, $\bar{x}_t$ is the period-specific cross-sectional mean of leverage, S_t is the sample cross-sectional in period t, s_t is the period-specific sample cross-sectional variance and $m_t(3)$ the period-specific sample third central moment of the cross-section. In the leverage figures shown, we define x_{it} as asset-weighted leverage and then use the above formulas. We also ran the same exercise using unweighted leverage or equity-weighted leverage with similar results. For robustness we also used the small sample variant $s_t = \sqrt{\frac{\sum_{i=1}^{N} (x_{it} - \bar{x}_t)^2}{N-1}}$ and there was no qualitative difference.

Year	Assets		Equity		Leverage			#Obs
	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.	Skewness	
1993	12419	39962	737	1930	14.71	13.77	10.67	276
1994	13725	44263	825	2179	14.81	10.95	5.47	329
1995	14638	42323	898	2200	14.33	11.16	5.73	349
1996	14278	35918	912	2109	14.82	16.26	9.48	364
1997	16778	42793	1073	2469	14.57	16.96	10.44	369
1998	18661	47154	1290	3158	13.71	10.69	5.67	392
1999	21478	64999	1526	4632	14.30	13.26	7.95	437
2000	24176	77576	1768	5599	14.04	15.11	10.79	456
2001	27648	87629	2079	6843	14.00	17.40	11.10	446
2002	28693	91765	2224	7328	14.09	25.57	13.34	458
2003	32012	103752	2414	8030	14.52	28.92	11.23	454
2004	44022	149190	3160	10731	14.58	17.96	7.65	444
2005	46877	152941	3287	10794	16.28	27.52	7.77	468
2006	56112	193439	3697	12586	15.69	24.76	8.28	422
2007	71684	245968	4458	14634	15.23	19.00	5.78	396
2008	67652	258317	4359	16032	15.59	27.13	13.73	374
2009	63755	240442	4872	17701	14.54	17.88	6.21	401
2010	66402	232329	5191	17920	12.90	11.93	4.77	448
2011	64599	235938	5318	18441	12.96	20.23	13.19	461
2012	65011	238222	5570	19317	12.51	21.56	13.29	467
2013	69559	249110	6468	22352	10.69	7.47	3.18	469
2014	70429	245651	6734	23223	10.58	7.71	4.31	475
2015	67587	228692	6923	23808	10.43	7.31	3.49	439

Table 2: Descriptive cross-sectional statistics by period (unweighted).

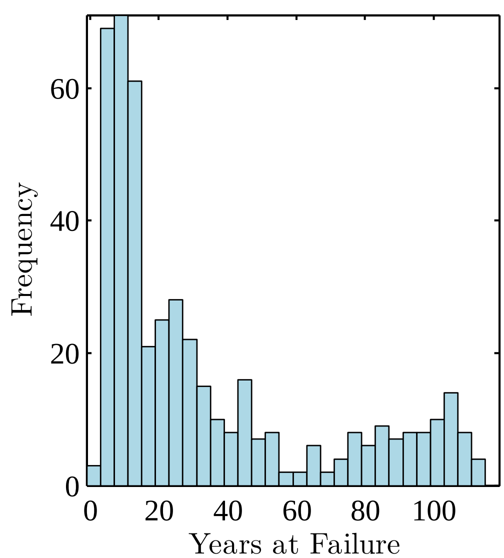

Figure 18: *Histogram of age of banks at closing date (in years). Data for failures in the US since October, 2000. Source: FDIC.*

Appendix E. Interbank market

For online publication

In this appendix, we present a version of the baseline model where intermediaries can supply funds to each other through deposits. The main difference in the financial intermediary problem, is that inactive intermediaries will optimally choose to deposit their net worth, thus supplying funds to leveraged banks. These deposits are also guaranteed by the government and therefore the same asset as household deposits from the point of view of the borrowing bank.

Whenever $R_t^D > 1$, storage is dominated by deposits and will never be used. Inactive intermediaries will also optimally prefer to hold deposits over shares of the capital stock. Since intermediaries are risk-neutral, the presence of an option value of default implies that in equilibrium $R_t^D > E(R_{t+1}^K)$. Since inactive intermediaries will not be able to exploit the option value of default, they strictly prefer deposits over shares of the capital stock, implying $\alpha_t^N = \alpha^L$ for all t. The balance sheet of an intermediary i that chooses to lend its net worth is then:

Assets	Liabilities
$-d_{it}$	ω_{it}

where to maintain consistency in notation, deposits held as assets are noted as negative d_{it}. The intermediary program is as before:

$$
\begin{aligned}
V_{it} = \max \ & \mathbb{E}_t(c_{i,t+1}) && (51)\\
\text{s.t.} \ & \Pr(\pi_{i,t+1} < \omega_{it}) \leq \alpha^i && (52)\\
& k_{it} + s_{it} = \omega_{it} + d_{it} && (53)\\
& c_{i,t+1} = \max\left(0, \pi_{i,t+1}\right) && (54)\\
& \pi_{i,t+1} = R_{i,t+1}^K k_{it} + s_{it} - R_t^D d_{it}
\end{aligned}
$$

Since borrowing to deposit is revenue neutral, it follows that Proposition 2.1 again holds in this case. Each intermediary will choose to leverage up to its VaR constraint or not raise deposits at all. Writing the value functions under this case we have

$$V_{it}^{L} = \mathbb{E}_t^i[R_{i,t+1}^K k_{it} - R_t^D d_{it}] \tag{55}$$

$$V_{it}^{N} = R_t^D \omega \tag{56}$$

$$V_{it}^{O} = \omega \tag{57}$$

The deposit market clearing equation is as before:

$$D_t = \int d_{it} dG(\alpha^i) = D_t^H \tag{58}$$

With the difference that now D_t is the *net* borrowing from the financial sector as a whole. The market clearing is $D_t = D_t^H$, where D_t^H are total household deposits. We also define $D_t^L = \int_{\alpha_t^L}^{\overline{\alpha}} d_{it} dG(\alpha^i)$ as the total deposit liabilities in levered intermediaries. Equation (40) then becomes:

$$F_t = \frac{D_t^L}{1+\chi} \tag{59}$$

The rest of the equations of the model are exactly the same, but underlying them are a few key differences. All capital is now held by levered intermediaries, which implies that no fraction of the capital stock is ever free from potential distress at $t+1$. Moreover, the extensive margin now also affects the deposit supply. The more intermediaries drop out from levered markets, the larger is aggregate deposit supply (ceteris paribus). Partial equilibrium results are very similar to the ones without the interbank market, as can be seen in Figure 19. Note that for the aggregate capital stock supply curve, the two models are almost indistinguishable.

The main difference in partial equilibrium is that for a given interest rate, the cut-off is now lower. Non-active intermediaries no longer invest directly in the capital stock. Had leverage and the cut-off remained the same the capital stock would be smaller and returns higher. This leads to both higher leverage from intermediaries above the cut-off (intensive margin) and a lower cut-off (extensive margin).

In general equilibrium, the main results are extremely similar to our baseline model as can be seen in figures 21 and 22. The main difference seems to be in the behaviour of

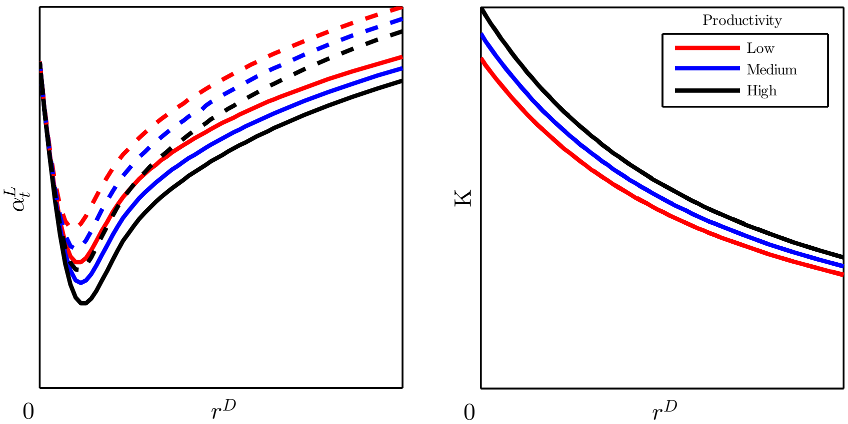

Figure 19: *Cut-off level α_t^L and aggregate capital stock as a function of deposit rates r_t^D in the model with an interbank market (full lines). For comparison, the baseline model is also plotted (dotted lines).*

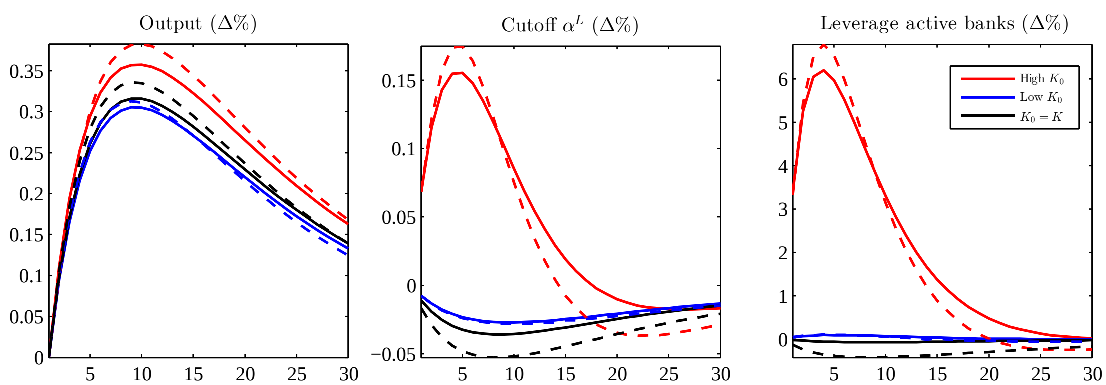

Figure 20: *Monetary policy shock of 100 basis points to γ_t in the model with an interbank market (full lines). For comparison, the baseline model is also plotted (dotted lines).*

the cut-off where the baseline model seems to have additional amplification, particularly away from the steady-state.

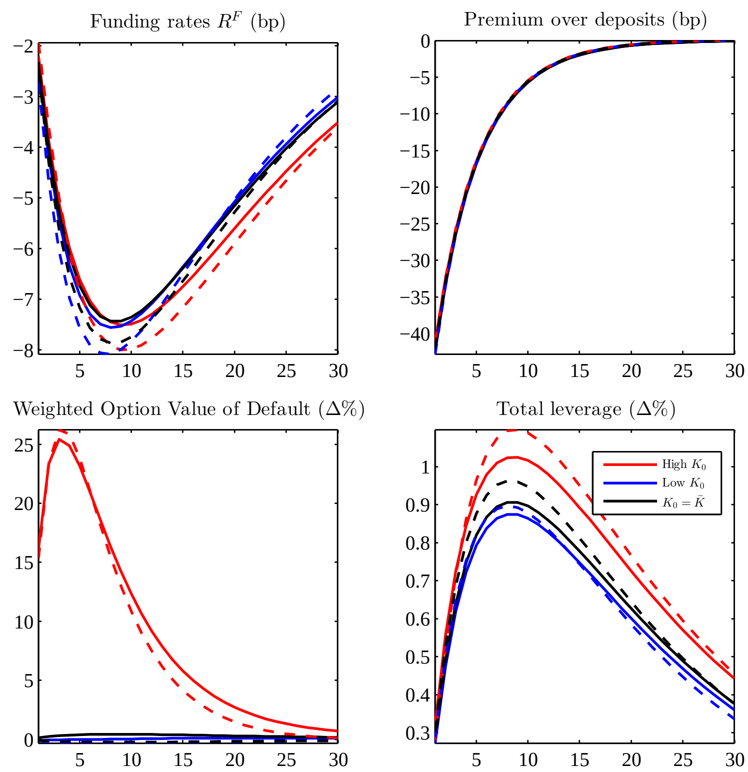

Figure 21: *Monetary policy shock of 100 basis points to γ_t: Financial variables*

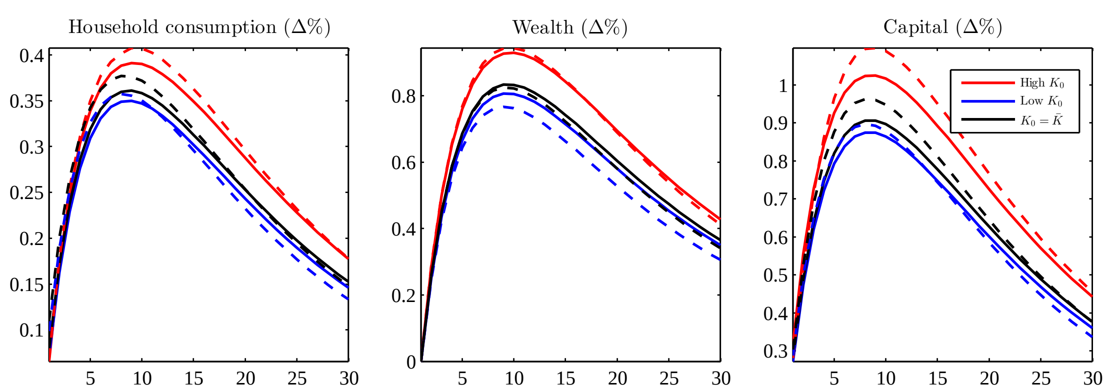

Figure 22: *Monetary policy shock of 100 basis points to γ_t: Real variables*

Appendix F. Comparative statics on size of equity and volatility

For online publication

Here we explore the role of volatility and net worth in the financial block of the model. We perform two exercises. In the first one we change the parameter σ_z, governing the exogenous volatility of the TFP process. As we can see in figure 23, the main change is in the composition of the financial sector. When volatility is higher, the VaR is tighter and therefore the intensive margin is reduced. Leverage from active intermediaries is lower, which leads to both lower capital stock and cut-off α^L. As it turns out, the lower is volatility, the easier it is for more risk-taking intermediaries to capture more of the market due to the loosening of VaR constraints.

We also look at the effect of changing the parameter ω, the endowment of net worth received by intermediaries. As can be seen in Figure 24, the effect is almost purely compositional with almost no effect on the total amount of capital (differences too marginal to show up in the graph). Given that the right hand side of equation (20) is independent of ω, then changing net worth is just allowing the more risk-taking intermediaries to acquire more assets (given aggregate variables). As with lower volatility, the higher ω is the easier it is for more risk-taking intermediaries to capture a larger share of the market.

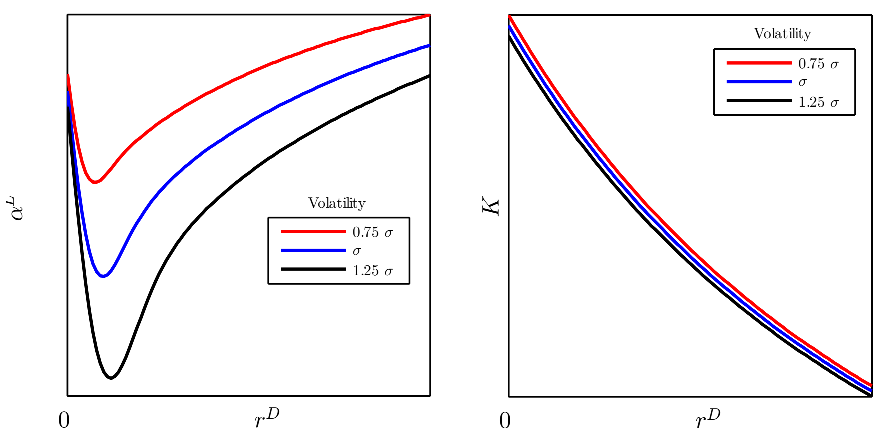

Figure 23: Comparative statics on volatility and interest rates for the financial sector block

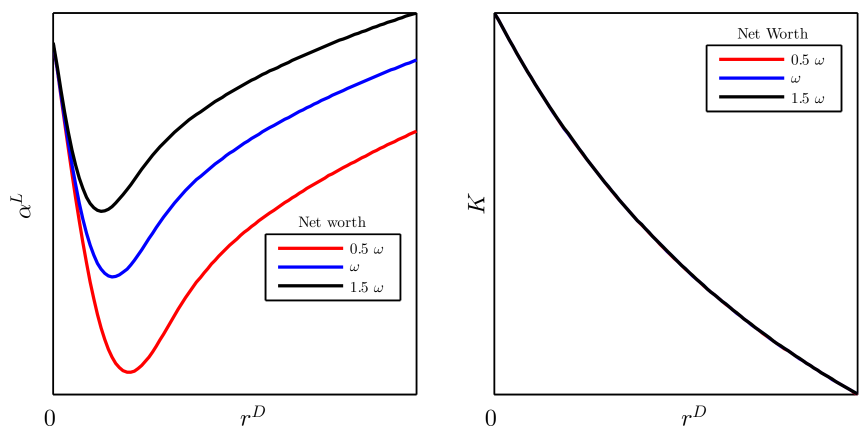

Figure 24: Comparative statics on net worth and interest rates for the financial sector block