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CoVaR[†]

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We propose a measure of systemic risk, ΔCoVaR, defined as the change in the value at risk of the financial system conditional on an institution being under distress relative to its median state. Our estimates show that characteristics such as leverage, size, maturity mismatch, and asset price booms significantly predict ΔCoVaR. We also provide out-of-sample forecasts of a countercyclical, forward-looking measure of systemic risk, and show that the 2006:IV value of this measure would have predicted more than one-third of realized ΔCoVaR during the 2007–2009 financial crisis. (*JEL* C58, E32, G01, G12, G17, G20, G32)

In times of financial crisis, losses spread across financial institutions, threatening the financial system as a whole.[1] The spreading of distress gives rise to systemic risk: the risk that the capacity of the entire financial system is impaired, with potentially adverse consequences for the real economy. Spillovers across institutions can occur directly due to direct contractual links and heightened counterparty credit risk or indirectly through price effects and liquidity spirals. As a result of these spillovers, the measured comovement of institutions' assets and liabilities tends to rise above and beyond levels purely justified by fundamentals. Systemic risk measures gauge the increase in tail comovement that can arise due to the spreading of financial distress across institutions.

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[1]Examples include the 1987 equity market crash, which was started by portfolio hedging of pension funds and led to substantial losses of investment banks; the 1998 crisis, which was started by losses of hedge funds and spilled over to the trading floors of commercial and investment banks; and the 2007–2009 crisis, which spread from structured investment vehicles to commercial banks and on to investment banks and hedge funds. See, e.g., Brady (1988); Rubin et al. (1999); Brunnermeier (2009); and Adrian and Shin (2010b).

The most common measure of risk used by financial institutions—the value at risk (VaR)—focuses on the risk of an individual institution *in isolation*. For example, the $q\%$–VaR^i is the maximum loss of institution i at the $q\%$–confidence level.[2] However, a single institution's risk measure does not necessarily reflect its connection to overall *systemic* risk. Some institutions are *individually systemic*—they are so interconnected and large that they can generate negative risk spillover effects on others. Similarly, several smaller institutions may be *systemic as a herd*. In addition to the cross-sectional dimension, systemic risk also has a time-series dimension. Systemic risks typically build in times of low asset price volatility, and materialize during crises. A good systemic risk measure should capture this build-up. High-frequency risk measures that rely mostly on contemporaneous asset price movements are potentially misleading.

In this paper, we propose a new reduced-form measure of systemic risk, $\Delta CoVaR$, that captures the (cross-sectional) tail-dependency between the whole financial system and a particular institution. For the time-series dimension of systemic risk, we estimate the forward-looking *forward*–$\Delta CoVaR$ which allows one to observe the build-up of systemic risk that typically occurs in tranquil times. We obtain this forward measure by projecting the $\Delta CoVaR$ on lagged institutional characteristics (in particular size, leverage, and maturity mismatch) and conditioning variables (in particular market volatility and fixed income spreads).

To emphasize the systemic nature of our risk measure, we add to existing risk measures the prefix "*Co*," for conditional. We focus primarily on $CoVaR$, where institution i's $CoVaR$ relative to the system is defined as the VaR of the whole financial sector conditional on institution i being in a particular state. Our main risk measure, $\Delta CoVaR$, is the difference between the $CoVaR$ conditional on the distress of an institution and the $CoVaR$ conditional on the median state of that institution. $\Delta CoVaR$ measures the component of systemic risk that comoves with the distress of a particular institution.[3] $\Delta CoVaR$ is a statistical tail-dependency measure, and so is best viewed as a useful reduced-form analytical tool capturing (tail) comovements.

The systemic risk measure associated with institution i, $\Delta CoVaR^i$, differs from that institution's own risk measure, VaR^i. Figure 1 shows this for large US financial institutions. Hence, it is not sufficient to regulate financial institutions based only on institutions' risk in isolation: regulators would overlook excessive risk-taking along the systemic risk dimension.

$\Delta CoVaR$ is directional. Reversing the conditioning shifts the focus to the question of how much a particular institution's risk increases given that the whole financial system is in distress. This is useful for detecting which institutions are most at risk should a financial crisis occur (as opposed to which institution's distress is most dangerous to the system). Applying the $\Delta CoVaR$ concept to measure the directional tail-dependence of pairs of institutions allows one to map links across the whole network of financial institutions.

[2] See Kupiec (2002) and Jorion (2007) for detailed overviews.

[3] Under many distributional assumptions (such as the assumption that shocks are conditionally Gaussian), the VaR of an institution is proportional to the variance of the institution, and the $\Delta CoVaR$ of an institution is proportional to the covariance of the financial system and the individual institution.

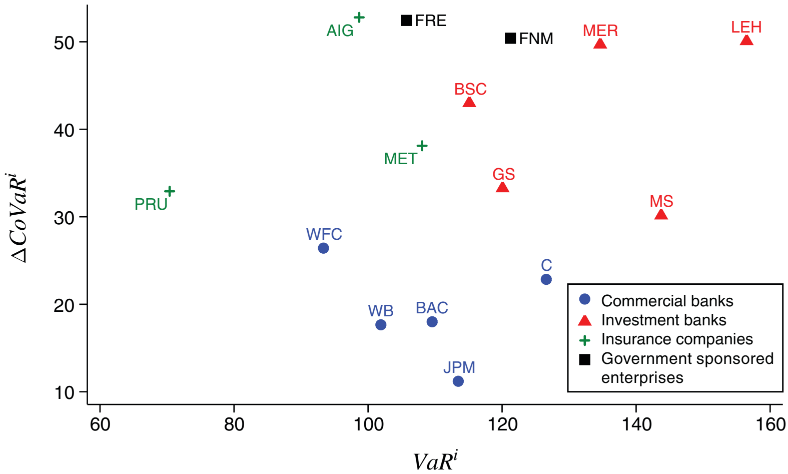

FIGURE 1. *VaR* AND $\Delta CoVaR$

Notes: The scatter-plot shows the weak correlation between institutions' risk in isolation, measured by VaR^i(x-axis), and institutions' systemic risk, measured by $\Delta CoVaR^i$ (y-axis). The VaR^i and $\Delta CoVaR^i$ are unconditional 99 percent measures estimated as of 2006:IV and are reported in quarterly percent returns for merger adjusted entities. $\Delta CoVaR^i$ is the difference between the financial system's *VaR* conditional on firm *i*'s distress and the financial system's *VaR* conditional on firm *i*'s median state. The institutions' names are listed in Appendix IV.

There are many possible ways to estimate $\Delta CoVaR$. In this paper, we primarily use quantile regressions, which are appealing for their simplicity. Since we want to capture all forms of risk, including the risk of adverse asset price movements, and funding liquidity risk, our estimates of $\Delta CoVaR$ are based on weekly equity returns of all publicly traded financial institutions. However, $\Delta CoVaR$ can also be estimated using methods such as generalized autoregressive conditional heteroskedasticity $(GARCH)$ models, as we show in Appendix II.

We calculate $\Delta CoVaR$ using weekly data from 1971:I to 2013:II for all publicly traded US commercial banks, broker-dealers, insurance companies, and real estate companies. We also verify for financial firms that are listed since 1926 that a longer estimation window does not materially alter the systemic risk estimates. To capture the evolution of tail risk dependence over time, we first model the variation of $\Delta CoVaR$ as a function of state variables. These state variables include the slope of the yield curve, the aggregate credit spread, and realized equity market volatility. In a second step, we use panel regressions and relate these time-varying $\Delta CoVaR$s—in a predictive, Granger causal sense—to measures of each institution's characteristics such as maturity mismatch, leverage, size, and asset valuation. We find relationships that are in line with theoretical predictions: higher leverage, more maturity mismatch, larger size, and higher asset valuations forecast higher $\Delta CoVaR$s across financial institutions.

Systemic risk monitoring should be based on forward-looking risk measures. We propose such a forward-looking measure, the *forward*-$\Delta CoVaR$. This *forward*-$\Delta CoVaR$ has countercyclical features, reflecting the build-up of systemic risk in good times, and the realization of systemic risk in crises. Crucially, the countercyclicality of our forward measure is a *result*, not an *assumption*. Econometrically, we construct the *forward*-$\Delta CoVaR$ by regressing time-varying $\Delta CoVaR$s on lagged institutional characteristics and common risk factors. We estimate *forward*-$\Delta CoVaR$ out-of-sample. Consistent with the "volatility paradox"—the notion that low volatility environments breed systemic risk—the *forward*-$\Delta CoVaR$ is negatively correlated with the contemporaneous $\Delta CoVaR$. We also demonstrate that the *forward*-$\Delta CoVaR$ has out of sample predictive power for realized $\Delta CoVaR$ in tail events. In particular, the *forward*-$\Delta CoVaR$ estimated using data until the end of 2006 predicts a substantial fraction of the cross-sectional dispersion in realized $\Delta CoVaR$ during the financial crisis of 2007–2008. The *forward*-$\Delta CoVaR$ can thus be used to monitor the build-up of systemic risk in real time. It remains, however, a reduced-form measure, and so does not *causally* allocate the source of systemic risk to different financial institutions.

Outline.—The remainder of the paper is organized in five sections. We first present a review of the related literature. Then, in Section II, we present the methodology, define $\Delta CoVaR$, and discuss its properties. In Section III, we outline the estimation method via quantile regressions. We allow for time variation in the $\Delta CoVaR$s by modeling them as a function of state variables and present estimates of these time-varying $\Delta CoVaR$s. Section IV then introduces the *forward*-$\Delta CoVaR$, illustrates its countercyclicality, and demonstrates that institutional characteristics such as size, leverage, and maturity mismatch can predict $\Delta CoVaR$ in the cross section of institutions. We conclude in Section V.

I. Literature Review

Our *corisk measure* is motivated by theoretical research on externalities across financial institutions that give rise to amplifying liquidity spirals and persistent distortions. It also relates closely to recent econometric work on contagion and spillover effects. $\Delta CoVaR$ captures the conditional tail-dependency in a noncausal sense.

A. *Theoretical Background on Systemic Risk*

Spillovers in the form of externalities arise when individual institutions take potential fire-sale prices as given, even though fire-sale prices are determined jointly by all institutions. In an incomplete markets setting, this pecuniary externality leads to an outcome that is not even constrained Pareto efficient. This result was derived in a banking context in Bhattacharya and Gale (1987), a general equilibrium incomplete markets setting by Stiglitz (1982) and Geanakoplos and Polemarchakis (1986), and within an international model in Brunnermeier and Sannikov (2015). Prices can also affect borrowing constraints. These externality effects are studied within an international finance context by Caballero and Krishnamurthy (2004), and are most recently shown in Lorenzoni (2008); Acharya (2009); Stein (2009); and Korinek (2010). Fire sale price discounts are large when market liquidity is low. Funding liquidity of institutions are subject to runs. Runs also lead to externalities. The margin/haircut spiral and precautionary hoarding behavior, outlined in Brunnermeier and Pedersen (2009) and Adrian and Boyarchenko (2012), lead financial institutions to shed assets at fire-sale prices. Adrian and Shin (2010a); Gorton and Metrick (2010); and Adrian, Etula, and Muir (2014) provide empirical evidence for the margin/haircut spiral. Borio (2004) is an early contribution that discusses a policy framework to address margin/haircut spirals and procyclicality. While liquidity hoarding might be microprudent from a single bank's perspective, it need not be macroprudent (due to the fallacy of composition). Finally, network effects can also lead to spillovers, as emphasized by Allen, Babus, and Carletti (2012).

Procyclicality occurs because financial institutions endogenously take excessive risk when volatility is low—a phenomenon that Brunnermeier and Sannikov (2014) termed the "volatility paradox."

B. *Other Systemic Risk Measures*

$\Delta CoVaR$, of course, is not the only systemic risk measure. Huang, Zhou, and Zhu (2012) develop a systemic risk indicator that measures the price of insurance against systemic financial distress from credit default swap (CDS) prices. Acharya et al. (2010) focus on high-frequency marginal expected shortfall as a systemic risk measure. Like our *Exposure-*$\Delta CoVaR$—to be defined later—they switch the conditioning and address the question of which institutions are most exposed to a financial crisis as opposed to the component of systemic risk associated with a particular institution. Importantly, their analysis focuses on a cross-sectional comparison of financial institutions and does not address the problem of procyclicality that arises from contemporaneous risk measurement. In other words, they do not address the stylized fact that risk builds up in the background during boom phases characterized by low volatility and materializes only in crisis times. Brownlees and Engle (2015) and Acharya, Engle, and Richardson (2012) develop the closely related *SRISK* measure which calculates capital shortfall of individual institutions conditional on market stress. Billio et al. (2012) propose a systemic risk measure that relies on Granger causality among firms. Giglio (2014) uses a nonparametric approach to derive bounds of systemic risk from CDS prices. A number of recent papers have extended the $\Delta CoVaR$ method and applied it to additional financial sectors. For example, Adams, Füss, and Gropp (2014) study risk spillovers among financial sectors; Wong and Fong (2010) estimate $\Delta CoVaR$ for Asia-Pacific sovereign CDS, and Fong et al. (2011) estimate ΔCoVaR for Asia-Pacific banks; Gauthier, Lehar, and Souissi (2012) estimate systemic risk exposures for the Canadian banking system; Hautsch, Schaumburg, and Schienle (2015) apply *CoVaR* to measure financial network systemic risk. Another important strand of the literature, initiated by Lehar (2005) and Gray, Merton, and Bodie (2007a), uses contingent claims analysis to measure systemic risk. Gray, Merton, and Bodie (2007b) develop a policy framework based on the contingent claims. Segoviano and Goodhart (2009) use a related approach to measure risk in the global banking system.

C. *The Econometrics of Tail Risk and Contagion*

The $\Delta CoVaR$ measure is also related to the literature on volatility models and tail risk. In a seminal contribution, Engle and Manganelli (2004) develop *CAViaR*, which uses quantile regressions in combination with a *GARCH* model to capture the time-varying tail behavior of asset returns. White, Kim, and Manganelli (2015) study a multivariate extension of *CAViaR*, which can be used to generate a dynamic version of *CoVaR*. Brownlees and Engle (2015) propose methodologies to estimate systemic risk measures using *GARCH* models.

The $\Delta CoVaR$ measure can additionally be related to an earlier literature on contagion and volatility spillovers (see Claessens and Forbes 2001 for an overview). The most common method to test for volatility spillovers is to estimate multivariate *GARCH* processes. Another approach is to use multivariate extreme value theory. Hartmann, Straetmans, and de Vries (2004) develop a contagion measure that focuses on extreme events. Danielsson and de Vries (2000) argue that extreme value theory works well only for very low quantiles.

Since an earlier version of this paper was circulated in 2008, a literature on alternative estimation approaches for *CoVaR* has emerged. *CoVaR* is estimated using multivariate *GARCH* by Girardi and Ergün (2013) (see Appendix II). Mainik and Schaanning (2012) and Oh and Patton (2013) use copulas. Bayesian inference for *CoVaR* estimation is proposed by Bernardi, Gayraud, and Petrella (2013). Bernardi, Maruotti, and Petrella (2013) and Cao (2013) make distributional assumptions about shocks and employ maximum likelihood estimators.

II. *CoVaR* Methodology

A. *Definition of* $\Delta CoVaR$

Recall that VaR_q^i is implicitly defined as the $q\%$ quantile, i.e.,

$$\Pr\left(X^i \leq VaR_q^i\right) = q\%,$$

where X^i is the (return) loss of institution i for which the VaR_q^i is defined. Defined like this, VaR_q^i is typically a positive number when $q > 50$, in line with the commonly used sign convention. Hence, greater risk corresponds to a higher VaR_q^i. We describe X^i as the "return loss."

DEFINITION 1: *We denote by* $CoVaR_q^{j|C(X^i)}$ *the VaR of institution j (or the financial system) conditional on some event* $C(X^i)$ *of institution i. That is,* $CoVaR_q^{j|C(X^i)}$ *is implicitly defined by the q%-quantile of the conditional probability distribution:*

$$\Pr\left(X^j \,|\, C\left(X^i\right) \leq CoVaR_q^{j|C(X^i)}\right) = q\%.$$

We denote the part of j's *systemic risk that can be attributed to i by*

$$\Delta CoVaR_q^{j|i} = \mathrm{Co}VaR_q^{j|X^i=VaR_q^i} - CoVaR_q^{j|X^i=VaR_{50}^i},$$

and in dollar terms

$$\Delta^{\$}CoVaR_q^{j|i} = {}^{\$}\mathrm{Size}^i \cdot \Delta CoVaR_q^{j|i}.$$

In our benchmark specification, j will be the financial system (i.e., portfolio consisting of all financial institutions in our universe).

Conditioning.—To obtain *CoVaR* we typically condition on an event C that is equally likely across institutions. Usually C is institution i's loss being at or above its

VaR_q^i level, which, by definition, occurs with likelihood $(1 - q)\%$. Importantly, this implies that the likelihood of the conditioning event is independent of the riskiness of i's business model. If we were to condition on a particular return level (instead of a quantile), then more conservative (i.e., less risky) institutions could have a higher *CoVaR* simply because the conditioning event would be a more extreme event for less risky institutions.

Δ*CoVaR*.—Captures the change in *CoVaR* as one shifts the conditioning event from the median return of institution i to the adverse VaR_q^i (with equality). Δ*CoVaR* measures the "tail-dependency" between two random return variables. Note that, for jointly normally distributed random variables, Δ*CoVaR* is related to the correlation coefficient, while *CoVaR* corresponds to a conditional variance. Conditioning by itself reduces the variance, while conditioning on adverse events increases expected return losses.

$\Delta^{\$}$*CoVaR*.—Captures the change in dollar amounts as one shifts the conditioning event. Two measures therefore take the size of institution i into account, allowing us to compare across differently sized institutions. For the purpose of this paper we quantify size by the market equity of the institution. Financial regulators (and in an earlier draft of our paper we) use total assets for both the return and the size definition.[4]

CoES.—One attractive feature of *CoVaR* is that it can be easily adapted for other "corisk-measures." An example of this is the coexpected shortfall, *CoES*. Expected shortfall, the expected loss conditional on a *VaR* event, has a number of advantages relative to *VaR*, and these considerations extend to *CoES*.[5] $CoES_q^{j|i}$ may be defined as the expected loss for institution j conditional on its losses exceeding $CoVaR_q^{j|i}$, and $\Delta CoES_q^{j|i}$ analogously is just $CoES_q^{j|i} - CoES_{50}^{j|i}$.

B. *The Economics of Systemic Risk*

Systemic risk has a time-series and a cross-sectional dimension. In the time-series, financial institutions endogenously take excessive risk when contemporaneously measured volatility is low, giving rise to a "volatility paradox" (Brunnermeier and Sannikov 2014). Contemporaneous measures are not suited to capture this build-up. In Section IV, we construct a *forward-*Δ*CoVaR* that avoids the "procyclicality pitfall" by estimating the relationship between current firm characteristics and future tail dependency, as proxied by $\Delta CoVaR_{q,t}^{j|i}$.

The cross-sectional component of systemic risk relates to the spillovers that amplify initial adverse shocks. The contemporaneous $\Delta CoVaR^i$ measures tail dependency and captures both spillover and common exposure effects. It

[4] For multistrategy institutions and funds, it might make sense to calculate the Δ*CoVaR* for each strategy s separately and obtain $\Delta^{\$}CoVaR_q^{j|i} = \sum_s Size^{s,i} \cdot \Delta CoVaR_q^{j|s}$. This ensures that mergers and carve-outs of strategies do not impact their overall measure, and also improves the cross-sectional comparison.

[5] In particular, the *VaR* is not subadditive and does not take distributional aspects within the tail into account. However, these concerns are mostly theoretical in nature as the exact distribution within the tails is difficult to estimate given the limited number of tail observations.

captures the association between an institution's stress event and overall risk in the financial system. The spillovers can be direct, through contractual links among financial institutions. Indirect spillover effects, however, are quantitatively more important. Selling off assets can lead to mark-to-market losses for all market participants with similar exposures. Moreover, the increase in volatility might tighten margins and haircuts, forcing other market participants to delever. This can lead to crowded trades which increase the price impact even further (see Brunnermeier and Pedersen 2009). Many of these spillovers are externalities. That is, when taking on the initial position with low market liquidity funded with short-term liabilities—i.e., with high liquidity mismatch—individual market participants do not internalize the subsequent individually optimal response in times of crises that impose (pecuniary) externalities on others. As a consequence, initial risk-taking is often excessive in the run-up phase, which generates the first component of systemic risk.

C. *Tail Dependency versus Causality*

$\Delta CoVaR_q^{j|i}$ is a statistical tail-dependency measure and does not necessarily correctly capture externalities or spillover effects, for several reasons. First, the externalities are typically not fully observable in equilibrium, since other institutions might reposition themselves in order to reduce the impact of the externalities. Second, $\Delta CoVaR_q^{j|i}$ also captures common exposure to exogenous aggregate macroeconomic risk factors.

More generally, causal statements can only be made within a specific model. Here, we consider for illustrative purposes a simple stylized financial system that can be split into two groups, institutions of type i and of type j. There are two latent independent risk factors, ΔZ^i and ΔZ^j. We conjecture that institutions of type i are directly exposed to the sector-specific shock ΔZ^i, and indirectly exposed to ΔZ^j via spillover effects. The assumed data-generating process of returns for type i institutions $-X_{t+1}^i = \Delta N_{t+1}^i / N_t^i$ is

$$(1) \qquad -X_{t+1}^i = \overline{\mu}^i(\cdot) + \overline{\sigma}^{ii}(\cdot)\,\Delta Z_{t+1}^i + \overline{\sigma}^{ij}(\cdot)\,\Delta Z_{t+1}^j,$$

where the short-hand notation $(\cdot)$ indicates that the (geometric) drift and volatility loadings are functions of the following state variables $\left(M_t, L_t^i, L_t^j, N_t^i, N_t^j\right)$: the state of the macroeconomy, M_t; the leverage and liquidity mismatch of type i institutions, L_t^i, and of type j institutions, L_t^j; as well as the net worth levels N_t^i and N_t^j. Leverage L_t^i is a choice variable and presumably, for i-type institutions, increases the loading to the own latent risk factor ΔZ_{t+1}^i. One would also presume that the exposure of i type institutions to ΔZ_{t+1}^j due to spillovers, $\overline{\sigma}^{ij}(\cdot)$, is increasing in own leverage, L_t^i, and others leverage, L_t^j.

Analogously, for institutions of type j, we propose the following data-generating process:

$$(2) \qquad -X_{t+1}^j = \overline{\mu}^j(\cdot) + \overline{\sigma}^{jj}(\cdot)\,\Delta Z_{t+1}^j + \overline{\sigma}^{ji}(\cdot)\,\Delta Z_{t+1}^i.$$

As the two latent shock processes ΔZ_{t+1}^{i} and ΔZ_{t+1}^{j} are unobservable, the empirical analysis starts with the following two reduced-form equations:[6]

$$-X_{t+1}^{i} = \mu^{i}(\cdot) - \sigma^{ij}(\cdot)\, X_{t+1}^{j} + \sigma^{ii}(\cdot)\, \Delta Z_{t+1}^{i}, \tag{3}$$

$$-X_{t+1}^{j} = \mu^{j}(\cdot) - \sigma^{ji}(\cdot)\, X_{t+1}^{i} + \sigma^{jj}(\cdot)\, \Delta Z_{t+1}^{j}. \tag{4}$$

Consider an adverse shock $\Delta Z_{t+1}^{i} < 0$. This shock lowers $-X_{t+1}^{i}$ by $\sigma_t^{ii}\Delta Z_{t+1}^{i}$. First round spillover effects also reduce others' return $-\Delta X_{t+1}^{j}$ by $\sigma_t^{ji}\sigma_t^{ii}\Delta Z_{t+1}^{i}$. Lower $-\Delta X_{t+1}^{j}$, in turn, lowers $-\Delta X_{t+1}^{i}$ by $\sigma_t^{ij}\sigma_t^{ji}\sigma_t^{ii}\Delta Z_{t+1}^{i}$ due to second round spillover effects. The argument goes on through third, fourth, and *n*th round effects. When a fixed point is ultimately reached, we obtain the volatility loadings of the initially proposed data-generating process $\bar{\sigma}_t^{ii} = \sum_{n=0}^{\infty}(\sigma_t^{ij}\sigma_t^{ji})^n\sigma_t^{ii} = \frac{\sigma_t^{ii}}{1-\sigma_t^{ij}\sigma_t^{ji}}$. Similarly, we obtain $\bar{\sigma}_t^{ij} = \sum_{n=0}^{\infty}(\sigma_t^{ij}\sigma_t^{ji})^n\sigma_t^{ij}\sigma_t^{jj} = \frac{\sigma_t^{ij}\sigma_t^{jj}}{1-\sigma_t^{ij}\sigma_t^{ji}}$. Analogously, by replacing *i* with *j* and vice versa, we obtain $\bar{\sigma}_t^{jj}$ and $\bar{\sigma}_t^{ji}$. This reasoning allows one to link reduced-form σs to primitive $\bar{\sigma}$s.

Gaussian Case.—An explicit formula can be derived for the special case in which all innovations ΔZ_{t+1}^{i} and ΔZ_{t+1}^{j} are jointly Gaussian distributed. In this case,

$$\Delta CoVaR_{q,t}^{j|i} = \Delta VaR_{q_t}^{i} \cdot \beta_t^{ij} \tag{5}$$

$$= -\left(\Phi^{-1}(q)\right)^2 \frac{Cov_t\left[X_{t+1}^{i}, X_{t+1}^{j}\right]}{\Delta VaR_{q,t}^{i}} = -\Phi^{-1}(q)\sigma_t^{j}\rho_t^{ij}, \tag{6}$$

where $\beta_t^{ij} = \frac{Cov_t\left[X_{t+1}^{i}, X_{t+1}^{j}\right]}{Var_t\left[X_{t+1}^{i}\right]} = \frac{\bar{\sigma}_t^{ii}\bar{\sigma}_t^{ji} + \bar{\sigma}_t^{ij}\bar{\sigma}_t^{jj}}{\bar{\sigma}_t^{ii}\bar{\sigma}_t^{ii} + \bar{\sigma}_t^{ij}\bar{\sigma}_t^{ij}}$ is the ordinary least squares (OLS) regression coefficient of reduced-form equation (5). Note that in the Gaussian case the OLS and median quantile regression coefficient are the same. $\Phi(\cdot)$ is the standard Gaussian CDF, σ_t^{j} is the standard deviation of N_{t+1}^{j}/N_t^{j}, and ρ_t^{ij} is the correlation coefficient between N_{t+1}^{i}/N_t^{i} and N_{t+1}^{j}/N_t^{j}. The Gaussian setting results in a "neat" analytical solution, but its tail properties are less desirable than those of more general distributional specifications.

D. *CoVaR*, *Exposure-CoVaR*, *Network-CoVaR*

The superscripts *j* or *i* can refer to individual institutions or a set of institutions. $\Delta CoVaR_q^{j|i}$ is directional. That is, $\Delta CoVaR_q^{system|i}$ of the system conditional on institution *i* is not necessarily equal to $\Delta CoVaR_q^{i|system}$ of institution *i* conditional on the financial system being in crisis. The conditioning radically changes the interpretation of the systemic risk measure. In this paper we consider primarily the direction

[6] The location scale model outlined in Appendix I falls in this category, with $\mu^{j}(M_t)$, $\sigma^{ji} = const.$, $\sigma^{jj}(M_t, X_{t+1}^{i})$, and the error term distributed i.i.d. with zero mean and unit variance. Another difference relative to this model is losses in return space (not net worth in return space) as the dependent variable.

of $\Delta CoVaR_q^{system|i}$, which quantifies the incremental change in systemic risk when institution i is in distress relative to its median state. Specifically,

$$\Delta CoVaR_q^{system|i} \;=\; CoVaR_q^{system|X^i=VaR_q^i} \;-\; CoVaR_q^{system|X^i=VaR_{50}^i}.$$

Exposure-$\Delta CoVaR$.—For risk management questions, it is useful to compute the reverse conditioning. We can compute $CoVaR^{j|system}$, which reveals the institutions that are most at risk should a financial crisis occur. $\Delta CoVaR^{j|system}$, which we label *Exposure-$\Delta CoVaR$*, reports institution j's increase in value at risk in the event of a financial crisis. In other words, the *Exposure-$\Delta CoVaR$* is a measure of an individual institution's exposure to system-wide distress, and is similar to the stress tests performed by individual institutions and regulators.

The importance of the direction of the conditioning is best illustrated with the following example. Consider a financial institution, such as a venture capital firm, with returns subject to substantial idiosyncratic noise. If the financial system overall is in significant distress, then this institution is also likely to face difficulties, so its *Exposure-$\Delta CoVaR$* is high. At the same time, conditioning on this particular institution being in distress does not materially impact the probability that the wider financial system is in distress (due to the large idiosyncratic component of the returns), and so $\Delta CoVaR$ is low. In this example the *Exposure-$\Delta CoVaR$* would send the wrong signal about systemicity, were it to be mistakenly viewed as such an indicator.

Network-$\Delta CoVaR$.—Finally, whenever both j and i in $CoVaR^{j|i}$ refer to individual institutions (rather than a set of institutions), we talk of *Network-$\Delta CoVaR$*. In this case we can study tail-dependency across the whole network of financial institutions.

To simplify notation we sometimes drop the subscript q when it is not necessary to specify the confidence level of the risk measures. Also, for the benchmark $\Delta CoVaR^{system|i}$ we often write only $\Delta CoVaR^i$. Later, we will also introduce a time-varying systemic risk measure and add a subscript t to denote time $\Delta CoVaR_{q,t}^{system|i}$.

E. *Properties of $\Delta CoVaR$*

Clone Property.—Our $\Delta CoVaR$ definition satisfies the desired property that, after splitting one large *individually systemic* institution into n smaller clones, the $CoVaR$ of the large institution (in return space) is exactly the same as the $CoVaR$s of the n clones. Put differently, conditioning on the distress of a large systemic institution is the same as conditioning on one of the n clones. This property also holds for the Gaussian case, as can be seen from equation (6). Both the covariance and the ΔVaR are divided by n, leaving $\Delta CoVaR_{q,t}^{j|i}$ unchanged.

Systemic As a Herd.—Consider a large number of small financial institutions that are exposed to the same factors (because they hold similar positions and are funded in a similar way). Only one of these institutions falling into distress will not necessarily *cause* a systemic crisis. However, if the distress is due to a common factor, then the other institutions will also be in distress. Overall, the set of institutions is *systemic as a herd*. Each individual institution's corisk measure should capture this notion of being systemic as a herd, even in the absence of a direct causal link. The Δ*CoVaR* measure achieves exactly that. Moreover, when we estimate Δ*CoVaR*, we control for lagged state variables that capture variation in tail risk not directly related to the financial system risk exposure. This discussion connects naturally with the clone property: if we split a systemically important institution into n clones, then each clone is systemic as part of the herd. The Δ*CoVaR* of each clone is the same as that of the original institution, capturing the intuition of systemic risk in a herd.

Endogeneity of Systemic Risk.—Note that each institution's Δ*CoVaR* is endogenous and depends on other institutions' risk-taking. Hence, imposing a regulatory framework that forces institutions to lower their leverage and liquidity mismatch, L^i, lowers reduced-form $\sigma^{i\cdot}(\cdot)$ in equations (1) and (2) and spillover effects captured in primitive $\bar{\sigma}^{i\cdot}(\cdot)$ in equations (3) and (4).

A regulatory framework that tries to internalize externalities also alters the Δ*CoVaR* measures. Δ*CoVaR* is an equilibrium concept which adapts to changing environments and provides incentives for institutions to reduce their exposure to risk if other institutions load excessively on it. Overall, we believe that Δ*CoVaR* can be a useful reduced-form analytical tool, but should neither serve as an explicit target for regulators, nor guide the setting of systemic taxes.[7]

III. Δ*CoVaR* Estimation

In this section we outline the estimation of Δ*CoVaR*. In Section IIIA we start with a discussion of alternative estimation approaches and then in Section IIIB present the quantile regression estimation method that we use in this paper. We go on to describe the estimation of the time-varying Δ*CoVaR* in Section IIIC. Details on the econometrics are given in Appendix I; robustness checks, including the *GARCH* estimation of Δ*CoVaR*, are provided in Appendix II. Section IIID provides estimates of Δ*CoVaR* and discusses properties of the estimates.

A. *Alternative Empirical Approaches*

Our main estimation approach relies on quantile regressions, as we explain in Sections IIIB and IIIC. Quantile regressions are a numerically efficient way to estimate *CoVaR*. Bassett and Koenker (1978) and Koenker and Bassett (1978) are the first to derive the statistical properties of quantile regressions. Chernozhukov (2005) provides statistical properties for extremal quantile regressions, and Chernozhukov and Umantsev (2001) and Chernozhukov and Du (2008) discuss *VaR* applications.

However, quantile regressions are not the only way to estimate *CoVaR*. There is an emerging literature that proposes alternative ways to estimate *CoVaR*. It can be computed from models with time-varying second moments, from measures of extreme events, by using Bayesian methods, or by using maximum likelihood estimation. We will now briefly discuss the most common alternative estimation procedures.

[7] The virtues and limitations of the Δ*CoVaR* thus are not in conflict with Goodhart's law (see Goodhart 1975).

A particularly popular approach to estimating *CoVaR* is from multivariate *GARCH* models. We provide such alternative estimates using bivariate *GARCH* models in Appendix II. Girardi and Ergün (2013) also provide estimates of *CoVaR* from multivariate *GARCH* models. An advantage of the *GARCH* estimation is that it captures the dynamic evolution of systemic risk contributions explicitly.

CoVaR can also be calculated from copulas. Mainik and Schaanning (2012) present analytical results for *CoVaR* using copulas, and compare the properties to alternative systemic risk measures. Oh and Patton (2013) present estimates of *CoVaR* and related systemic risk measures from CDS spreads using copulas. An advantage of the copula methodology is that it allows estimation of the whole joint distribution, including fat tails and heteroskedasticity.

Bayesian inference can also be used for *CoVaR* estimation. Bernardi, Gayraud, and Petrella (2013) present a Bayesian quantile regression framework based on a Markov chain Monte Carlo algorithm, exploiting the asymmetric Laplace distribution and its representation as a location-scale mixture of normals.

A number of recent papers make distributional assumptions and use maximum likelihood techniques to estimate *CoVaR*. Bernardi, Maruotti, and Petrella (2013) estimate *CoVaR* using a multivariate Markov switching model with a student-*t* distribution accounting for heavy tails and nonlinear dependence. Cao's (2013) *Multi-CoVaR* estimates a multivariate student-*t* distribution to calculate the joint distribution of *CoVaR* across firms. The maximum likelihood methodology has efficiency advantages relative to the quantile regressions if the distributional assumptions are correct.

In addition, there is a growing literature that develops the econometrics of quantile regressions for *CoVaR* estimation. Castro and Ferrari (2014) derive test statistics for *CoVaR* which can be used to rank firms according to systemic importance. White, Kim, and Manganelli (2015) propose a dynamic *CoVaR* estimation using a combination of quantile regressions and *GARCH*.

B. *Estimation Method: Quantile Regression*

We use quantile regressions to estimate *CoVaR*. In this section, the model underlying our discussion of the estimation procedure is a stylized version of the reduced-form model discussed in Section IIA, a more general version will be used in Section IIIC, and a full discussion is relegated to Appendix I.

To see the attractiveness of quantile regressions, consider the predicted value of a quantile regression of financial sector losses X_q^{system} on the losses of a particular institution i for the $q\%$-quantile,

$$\hat{X}_q^{system|X^i} = \hat{\alpha}_q^i + \hat{\beta}_q^i X^i, \tag{7}$$

where $\hat{X}_q^{system|X^i}$ denotes the predicted value for a $q\%$-quantile of the system conditional on a return realization X^i of institution i.[8] From the definition of value at risk, it follows directly that

[8] Note that a median regression is the special case of a quantile regression where $q = 50$. We provide a short synopsis of quantile regressions in the context of linear factor models in Appendix I. Koenker (2005) provides a

$$(8) \qquad CoVaR_q^{system|X^i} = \hat{X}_q^{system|X^i}.$$

That is, the predicted value from the quantile regression of system return losses on the losses of institution i gives the value at risk of the financial system conditional on X^i. The $CoVaR\,_q^{system|i}$ given X^i is just the conditional quantile. Using the predicted value of $X^i = VaR_q^i$ yields our $CoVaR_q^i$ measure $\left(CoVaR\,_q^{system|X^i=VaR_q^i}\right)$. More formally, within the quantile regression framework, our $CoVaR_q^i$ measure is given by

$$(9) \qquad CoVaR_q^i = VaR_q^{system|X^i=VaR_q^i} = \hat{\alpha}_q^i + \hat{\beta}_q^i VaR_q^i.$$

VaR^i can be obtained simply as the $q\%$-quantile of institution i's losses. So $\Delta CoVaR_q^i$ is

$$(10) \qquad \Delta CoVaR_q^i = CoVaR_q^i - CoVaR_q^{system|VaR_{50}^i} = \hat{\beta}_q^i\left(VaR_q^i - VaR_{50}^i\right).$$

As explained in Section II, we refer here to the conditional VaR expressed in percentage loss rates. The unconditional VaR_q^i and $\Delta CoVaR_q^i$ estimates for Figure 1 are based on equation (10).

Measuring Losses.—Our analysis relies on publicly available data and focuses on return losses on market equity, $X_{t+1}^i = -\Delta N_{t+1}^i / N_t^i$. Alternatively, one could also conduct the analysis with book equity data, defined as the residual between total assets and liabilities. Supervisors have a larger set of data at their disposal; hence they could also compute the VaR^i and $\Delta CoVaR^i$ from a broader definition of book equity that would include equity in off-balance-sheet items, exposures from derivative contracts, and other claims that are not properly captured by publicly traded equity values. A more thorough approach would potentially improve measurement. The analysis could also be extended to compute the risk measures for assets or liabilities separately. For example, the $\Delta CoVaR^i$ for liabilities captures the extent to which financial institutions rely on debt funding—such as repos or commercial paper—which can collapse during systemic crises. Total assets are most closely related to the supply of credit to the real economy, and risk measures for regulatory purposes are typically computed for total assets. (Earlier versions of this paper used the market value of total assets as a basis for the calculations.)

Financial Institutions Data.—We focus on publicly traded financial institutions, consisting of four financial sectors: commercial banks, security broker-dealers (including investment banks), insurance companies, and real estate companies. Our sample starts in 1971:I and ends in 2013:II. The data thus cover six recessions (1974–1975, 1980, 1981, 1990–1991, 2001, and 2007–2009) and several financial crises (1987, 1994, 1997, 1998, 2000, 2008, and 2011). We also perform a robustness check using data going back to 1926:III. We obtain daily market equity data from the Center for Research in Security Prices (CRSP) and quarterly balance sheet

more detailed overview of the econometric issues. While quantile regressions are often used in many applied fields of economics, their application in financial economics has been limited.

data from COMPUSTAT. We have a total of 1,823 institutions in our sample. For bank holding companies, we use additional asset and liability variables from the FR Y9-C reports. The main part of our empirical analysis is carried out with weekly observations, allowing reasonable inference despite the relatively short samples available. Appendix III provides a detailed description of the data.

C. *Time Variation Associated with Systematic State Variables*

The previous section presented a methodology for estimating $\Delta CoVaR$ that is constant over time. To capture time-variation in the joint distribution of X^{system} and X^i, we estimate *VaRs* and Δ*CoVaRs* as a function of state variables, allowing us to model the evolution of the joint distributions over time. We indicate time-varying $CoVaR^{\,i}_{q,t}$ and $VaR^{\,i}_{q,t}$ with a subscript t, and estimate the time variation conditional on a vector of lagged state variables $\mathbf{M}_{t-1}$. We estimate the following quantile regressions on weekly data:

$$X^i_t = \alpha^i_q + \gamma^i_q \mathbf{M}_{t-1} + \varepsilon^i_{q,t}, \tag{11a}$$

$$X^{system|i}_t = \alpha^{system|i}_q + \gamma^{system|i}_q \mathbf{M}_{t-1} + \beta^{system|i}_q X^i_t + \varepsilon^{system|i}_{q,t}. \tag{11b}$$

We then use the predicted values from these regressions to obtain

$$VaR^i_{q,t} = \hat{\alpha}^i_q + \hat{\gamma}^i_q \mathbf{M}_{t-1}, \tag{12a}$$

$$CoVaR^i_{q,t} = \hat{\alpha}^{system|i}_q + \hat{\gamma}^{system|i}_q \mathbf{M}_{t-1} + \hat{\beta}^{system|i}_q VaR^i_{q,t}. \tag{12b}$$

Finally, we compute $\Delta CoVaR^i_{q,t}$ for each institution:

$$\Delta CoVaR^i_{q,t} = CoVaR^i_{q,t} - CoVaR^i_{50,t} \tag{13}$$

$$= \hat{\beta}^{system|i}_q \left(VaR^i_{q,t} - VaR^i_{50,t} \right). \tag{14}$$

From these regressions, we obtain a panel of weekly $\Delta CoVaR^i_{q,t}$. For the forecasting regressions in Section IV, we generate a weekly panel of $\Delta^{\$} CoVaR^i_{q,t}$ by multiplying $\Delta CoVaR^i_{q,t}$ by the respective market equity ME^i_t. We then obtain a quarterly panel of $\Delta^{\$} CoVaR^i_{q,t}$ by averaging the weekly observations within each quarter. In order to obtain stationary variables, we divide each $\Delta^{\$} CoVaR^i_{q,t}$ by the cross-sectional average of market equity N^i_t.

State Variables.—To estimate the time-varying $\Delta CoVaR_t$ and VaR_t, we include a set of state variables $\mathbf{M}_t$ that are (i) known to capture time variation in the conditional moments of asset returns, (ii) liquid, and (iii) tractable. The state variables $\mathbf{M}_{t-1}$ are lagged. They should not be interpreted as systematic risk factors, but rather as variables that condition the mean and volatility of the risk measures. Note that different firms can load on these risk factors in different directions, so that particular correlations of the risk measures across firms—or correlations of the different risk measures for the same firm—are not imposed by construction. We restrict ourselves to a small set of state variables to avoid overfitting the data. Our variables are:

(i) The *change in the three-month yield* from the Federal Reserve Board's H.15 release. We use the change in the three-month Treasury bill rate because we find that the change, not the level, is most significant in explaining the tails of financial sector market-valued asset returns;

(ii) The *change in the slope of the yield curve*, measured by the spread between the composite long-term bond yield and the three-month bill rate obtained from the Federal Reserve Board's H.15 release;

(iii) A short-term TED spread, defined as the difference between the three-month LIBOR rate and the three-month secondary market Treasury bill rate. This spread measures short-term funding liquidity risk. We use the three-month LIBOR rate that is available from the British Bankers' Association, and obtain the three-month Treasury rate from the Federal Reserve Bank of New York;

(iv) The *change in the credit spread* between Moody's *Baa*-rated bonds and the ten-year Treasury rate from the Federal Reserve Board's H.15 release;

(v) The weekly *market return* computed from the S&P500;

(vi) The weekly *real estate sector return* in excess of the market financial sector return (from the real estate companies with SIC code 65–66);

(vii) *Equity volatility*, which is computed as the 22-day rolling standard deviation of the daily CRSP equity market return.

Table 1 provides summary statistics of the state variables. The 1 percent stress level is the level of each respective variable during the 1 percent worst weeks for financial system asset returns. For example, the average of the equity volatility during the stress periods is 2.27, as the worst times for the financial system occur when the equity volatility was highest. Similarly, the stress level corresponds to a high level of the liquidity spread, a sharp decline in the Treasury bill rate, sharp increases of the term and credit spreads, and large negative market return realizations.

D. $\Delta CoVaR$ Summary Statistics

Table 2 provides the estimates of our weekly conditional $\Delta CoVaR^{i}_{99,t}$ measures obtained from quantile regressions. The summary statistics are calculated on the universe of financial institutions.

Line (1) of Table 2 give the summary statistics for the market equity loss rates; line (2) gives the summary statistics for the $VaR^{i}_{99,t}$ for each institution; line (3) gives the summary statistics for $\Delta CoVaR^{i}_{99,t}$; line (4) gives the summary statistics for the *stress*-$\Delta CoVaR^{i}_{99,t}$; and line (5) gives the summary statistics for the

Table 1—State Variable Summary Statistics

	Mean	Standard deviation	Skewness	Min.	Max.	1 percent stress
Three-month yield change	−0.22	21.76	−0.68	−182	192	−8.89
Term spread change	0.09	19.11	0.16	−168	146	5.83
TED spread	103.98	91.09	1.86	6.34	591	138.59
Credit spread change	−0.04	8.41	0.80	−48	60	7.61
Market return	0.15	2.29	−0.23	−15.35	13.83	−7.41
Real estate excess return	−0.03	2.58	0.27	−14.49	21.25	−3.01
Equity volatility	0.89	0.53	3.40	0.28	5.12	2.27

Notes: The spreads and spread changes are expressed in weekly basis points, and returns are in weekly percent. The 1 percent stress in the last column corresponds to the state variable realizations in the worst 1 percent of financial system returns.

Table 2—Summary Statistics for Estimated Risk Measures

		Mean	Standard deviation	Observations
1.	X_t^i	−0.286	6.111	1,342,547
2.	$VaR_{99,t}^i$	11.136	6.868	1,342,449
3.	$\Delta CoVaR_{99,t}^i$	1.172	1.021	1,342,449
4.	Stress–$\Delta CoVaR_{99}^i$	3.357	4.405	1,823
5.	$VaR_{99,t}^{system}$	4.768	2.490	2,209

Notes: The table reports summary statistics for the market equity losses and 99 percent risk measures of the 1,823 financial firms for weekly data from 1971:I–2013:II. X^i denotes the weekly market equity losses. The individual firm risk measure $VaR_{99,t}^i$ and the system risk measure $VaR_{99,t}^{system}$ are obtained by running 99 percent quantile regressions of returns on the one-week lag of the state variables and by computing the predicted value of the regression. $\Delta CoVaR_{99,t}^i$ is the difference between $CoVaR_{99,t}^i$ and $CoVaR_{99,t}^{i|median}$, where $CoVaR_{q,t}^i$ is the predicted value from a $q\%$ quantile regression of the financial system equity losses on the institution equity losses and on the lagged state variables. The *stress*-$\Delta CoVaR_{99}^i$ is the $\Delta CoVaR_{99,t}^i$ computed conditional on state variable realizations in the worst 1 percent tail of the financial system returns as reported in the last column of Table 1. All quantities are expressed in units of weekly percent returns.

financial system value at risk, $VaR_{99,t}^{system}$. The *stress*-$\Delta CoVaR_{99,t}^i$ is estimated by substituting the worst 1 percent of state variable realizations into the fitted model for $\Delta CoVaR_{99,t}^i$ (see equations (12a) and (12b)).

Recall that $\Delta CoVaR_t^i$ measures the change in the value at risk of the financial system associated with stress at institution i (relative to its median state) and conditional on state variables $\mathbf{M}_t$. We report the mean, standard deviation, and number of observations for each of the items in Table 2. We have a total of 1,823 institutions in the sample, with observations over an average time span of 736 weeks. The institution with the longest history spans all 2,209 weeks of the 1971:I–2013:II sample period. We require institutions to have at least 260 weeks of equity return data in order to be included in the panel. In the following analysis, we focus primarily on the 99 percent and the 95 percent quantiles, corresponding to the worst 22 weeks and the worst 110 weeks over the sample horizon, respectively. It is straightforward to estimate more extreme tails following Chernozhukov and Du (2008) by extrapolating the quantile estimates using extreme value theory, an analysis that we leave for future research. In the following analysis, we largely find results to be qualitatively similar for the 99 percent and the 95 percent quantiles. We also report the stress-$\Delta CoVaR_{99}^i$, which is the $\Delta CoVaR_{99,t}^i$ conditional on state variable realizations

Table 3—Average t-Statistics of State Variable Exposures

	VaR^{system}	VaR^i	$\Delta CoVaR^i$
Three-month yield change (lag)	(1.95)	(−0.26)	(2.10)
Term spread change (lag)	(1.73)	(−0.04)	(1.72)
TED spread (lag)	(6.87)	(1.97)	(8.86)
Credit spread change (lag)	(5.08)	(−0.28)	(4.08)
Market return (lag)	(−16.98)	(−3.87)	(−18.78)
Real estate excess return (lag)	(−3.78)	(−1.86)	(−4.41)
Equity volatility (lag)	(12.81)	(7.47)	(15.81)
Market equity loss X^i			(7.38)
Pseudo-R^2	39.94%	21.23%	43.42%

Notes: The table reports average t-statistics from 99 percent quantile regressions. For the risk measure $VaR^i_{99,t}$ and the system risk measure $VaR^{system}_{99,t}$, 99 percent quantile regressions of losses are estimated on the state variables. For $CoVaR^i_{99,t}$, 99 percent quantile regressions of the financial system equity losses are estimated on the state variables and firm i's market equity losses.

in the worst 1 percent tail of financial system returns (as reported in the last column of Table 1).

We obtain time variation of the risk measures by running quantile regressions of equity losses on the lagged state variables. We report average t-statistics of these regressions in Table 3. A higher equity volatility, higher TED spread, and lower market return tend to be associated with high risk. In addition, increases in the three-month yield, increases in the term spread, and increases in the credit spread tend to be associated with higher risk. Overall, the average significance of the conditioning variables reported in Table 3 show that the state variables do indeed proxy for the time variation in the quantiles and particularly in *CoVaR*.

E. $\Delta CoVaR$ *versus VaR*

Figure 1 shows that, *across institutions*, there is only a very loose link between an institution's VaR^i and its $\Delta CoVaR^i$. Hence, applying financial regulation solely based on the risk of an institution in isolation might not be sufficient to insulate the financial sector against systemic risk. Figure 2 shows the scatter-plot of the time series average of $\Delta CoVaR^i_t$ against the time series average of VaR^i_t for all institutions in our sample, for each of the four financial industries. While there is only a weak correlation between $\Delta CoVaR^i_t$ and VaR^i_t in the cross section, there is a strong time series relationship. This can be seen in Figure 3, which plots the time series of the $\Delta CoVaR^i_t$ and VaR^i_t for a sample of the largest firms over time.

F. *Comparison of Out-of-Sample and In-Sample* $\Delta CoVaR$

Figure 4 shows the weekly $\Delta CoVaR$ for Lehman Brothers, Bank of America, JP Morgan, and Goldman Sachs for the crisis period 2007–2008. The three vertical bars indicate when BNP Paribas reported funding problems (August 7, 2007), the bail-out of Bear Stearns (March 14, 2008), and the Lehman bankruptcy (September 15, 2008). Each of the plots shows both the in- and out-of-sample estimate of $\Delta CoVaR$ using expanding windows.

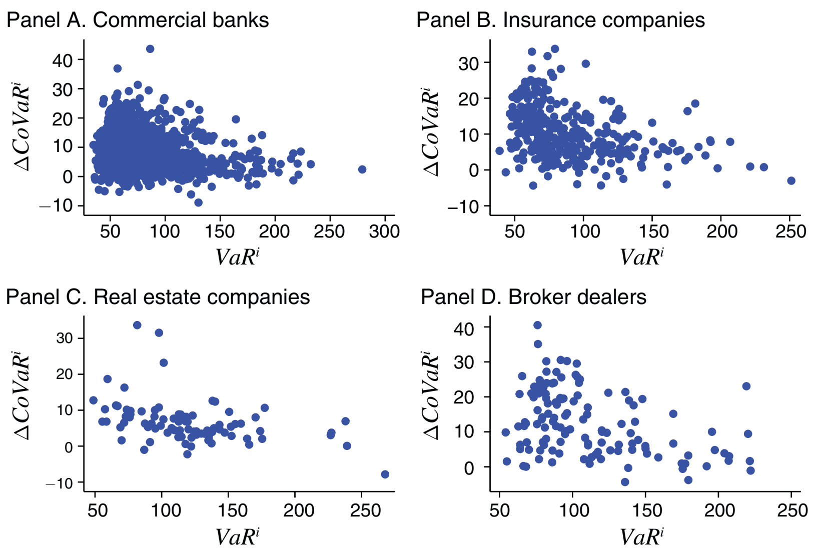

FIGURE 2. CROSS SECTION OF $\Delta CoVaR$ AND VaR

Notes: The scatter-plot shows the weak cross-sectional link between the time-series average of a portfolio's risk in isolation, measured by $VaR^i_{95,t}$ (x-axis), and the time-series average of a portfolio's contribution to system risk, measured by $\Delta CoVaR^i_{95,t}$ (y-axis). The $VaR^i_{95,t}$ and $\Delta CoVaR^i_{95,t}$ are in units of quarterly percent of total market equity loss rates.

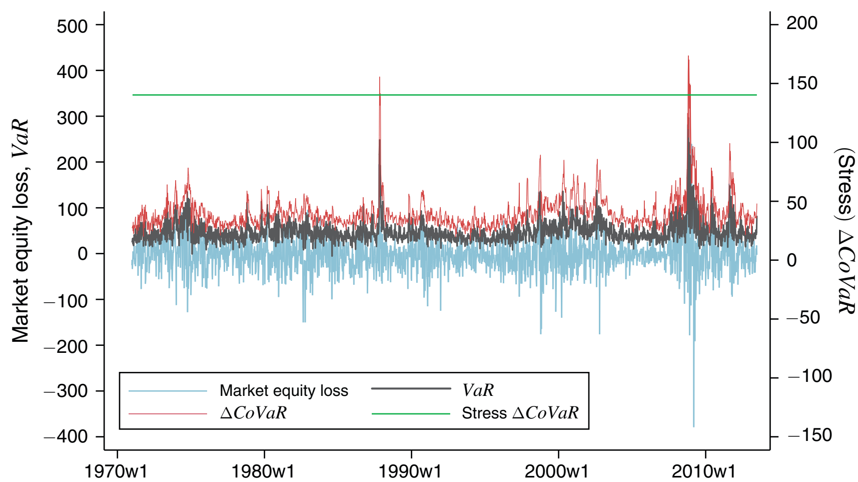

FIGURE 3. TIME-SERIES OF $\Delta CoVaR$ AND VaR FOR LARGE FINANCIAL INSTITUTIONS

Notes: This figure shows the market equity losses, the $VaR^i_{95,t}$, and the $\Delta CoVaR^i_{95,t}$ for a sample of the 50 largest financial institutions as of the beginning of 2007. The *stress*-$\Delta CoVaR^i_{95,t}$ is also plotted. All variables are quarterly percent of market equity loss rates.

Among these four figures, Lehman Brothers clearly stands out: its $\Delta CoVaR$ rises sharply with the onset of the financial crisis in the summer of 2007, and remains elevated throughout the middle of 2008. While the $\Delta CoVaR$ for Lehman declined following the bailout and distressed sale of Bear Stearns, it steadily increased from

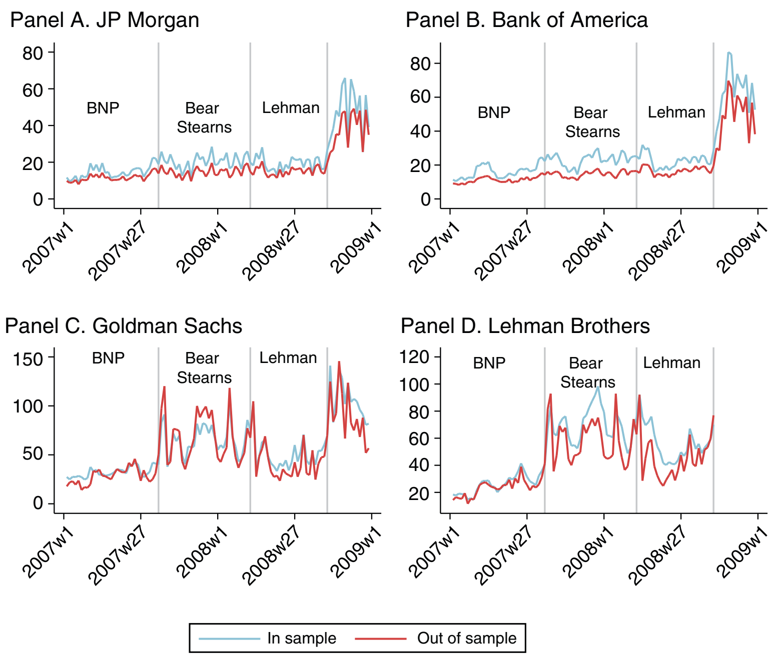

Figure 4. Time-Series of $\Delta CoVaR$ of Four Large Financial Institutions

Notes: This figure shows the time series of weekly $\Delta CoVaR^{i}_{95,t}$ estimated in sample and out of sample. All variables are quarterly percent of market equity loss rates. The first vertical line refers to the week of August 7, 2007, when BNP experienced funding shortages. The second vertical line corresponds to the week of March 15, 2008, when Bear Stearns was distressed. The third vertical line corresponds to the week of September 15, 2008, when Lehman Brothers filed for bankruptcy.

mid-2008. It is also noteworthy that the level of $\Delta CoVaR$ for Goldman Sachs and Lehman is materially larger than those for Bank of America and JP Morgan, reflecting the fact that until October 2008 Goldman Sachs was not a bank holding company and did not have access to public backstops.

G. *Historical* $\Delta CoVaR$

Major financial crises occur rarely, making the estimation of tail dependence between individual institutions and the financial system statistically challenging. In order to understand the extent to which $\Delta CoVaR$ estimates are sensitive to the length of the sample period, we select a subset of financial firms with equity market returns that extend back to 1926:III.[9] Figure 5 compares the newly estimated $\Delta CoVaR^{i}_{t}$ to the one estimated using data from 1971:I.

The comparison of the $\Delta CoVaR$s reveals two things. Firstly, systemic risk measures were not as high in the Great Depression as they were during the recent financial crisis. This could be an artifact of the composition of the firms, as the four firms with a very long time series are not necessarily a representative sample of firms from

[9] The two financial firms that we use in the basket are Adams Express Company (ADX) and Alleghany Corporation (Y).

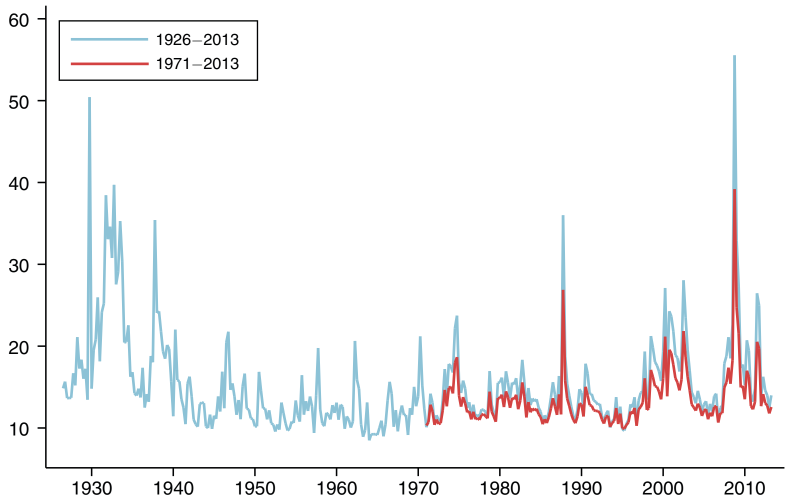

Figure 5. Historical $\Delta CoVaR$

Notes: This figure shows the $\Delta CoVaR^{i}_{95,t}$ for a portfolio of two firms estimated in two ways from weekly data, shown as average within quarters. The darker line shows the estimated $\Delta CoVaR^{i}_{95,t}$ since 1971, while the lighter line shows the estimated $\Delta CoVaR^{i}_{95,t}$ since 1926:III. The $\Delta CoVaR$s are estimated with respect to the value-weighted CRSP market return. The risk measures are in percent quarterly equity losses.

the Great Depression era.[10] Secondly, the longer time series exhibits fatter tails, and generates a slightly higher measure of systemic risk over the whole time horizon. Tail risk thus appears to be biased downward in the shorter sample. Nevertheless, the correlation between the shorter and longer time series is 96 percent. We conclude that the shorter time span for the estimation since 1971 provides adequate *CoVaR* estimates compared with the longer estimation since 1926.

IV. *Forward-$\Delta CoVaR$*

In this section we link $\Delta CoVaR$ to financial institutions characteristics to address two key issues: procyclicality and measurement accuracy. Procyclicality refers to the time series component of systemic risk. Systemic risk builds in the background during seemingly tranquil times when volatility is low (the "volatility paradox"). Any regulation that relies on contemporaneous risk measures would be too loose in periods when imbalances are building up and too tight during crises. In other words, such regulation would exacerbate the adverse impacts of bad shocks, while amplifying balance sheet growth and risk-taking in expansions.[11] We propose to focus on variables that predict *future*, rather than *contemporaneous*, $\Delta CoVaR$. In this section, we calculate a forward-looking systemic risk measure that can serve as a useful analytical tool for financial stability monitoring, and may provide guidance

[10] Bank equity was generally not traded in public equity markets until the 1960s. Moreover, there may be a survivorship bias, as the firms which survived the Great Depression may have had lower $\Delta CoVaR$s.

[11] See Estrella (2004), Kashyap and Stein (2004), and Gordy and Howells (2006) for studies of the procyclical nature of capital regulation.

for (countercyclical) macroprudential policy. We first present the dependence of $\Delta CoVaR$ on lagged characteristics. We then use these characteristics to construct the *forward-*$\Delta CoVaR$.

Any tail risk measure estimated at a high frequency is by its very nature imprecise. Quantifying the relationship between $\Delta CoVaR$ and more easily observable institution-specific variables, such as size, leverage, and maturity mismatch, deals with measurement inaccuracy in the direct estimation of $\Delta CoVaR$, at least to some extent. For this purpose, we project $\Delta CoVaR$ onto explanatory variables. Since the analysis involves the comparison of $\Delta CoVaR$ across firms, we use $\Delta^{\$}CoVaR$, which takes the size of firms into account.

For each firm we regress $\Delta^{\$}CoVaR$ on the institution i's characteristics, as well as conditioning macro-variables. More specifically, for a forecast horizon $h = 1, 4, 8$ quarters, we estimate regressions

$$\Delta^{\$}CoVaR_{q,t}^{i} = a + \mathbf{c}\mathbf{M}_{t-h} + \mathbf{b}\mathbf{X}_{t-h}^{i} + \eta_t^i, \tag{15}$$

where $\mathbf{X}_{t-h}^{i}$ is the vector of characteristics for institution i, $\mathbf{M}_{t-h}$ is the vector of macrostate variables lagged h quarters, and η_t^i is an error term.

We label the h quarters predicted value *forward-*$\Delta^{\$}CoVaR$,

$$\Delta_h^{\mathrm{Fwd}}CoVaR_{q,t}^{i} = \hat{a} + \hat{\mathbf{c}}\mathbf{M}_{t-h} + \hat{\mathbf{b}}\mathbf{X}_{t-h}^{i}. \tag{16}$$

A. $\Delta CoVaR$ Predictors

As previously, the macrostate variables are the change in the three-month yield, the change in the slope of the yield curve, the TED spread, the change in the credit spread, the market return, the real estate sector return, and equity volatility.

Institutions Characteristics.—The main characteristics that we consider are the following:

(i) *Leverage*. For this, we use the ratio of the market value of assets to market equity.

(ii) The *maturity mismatch*. This is defined as the ratio of book assets to short-term debt less short-term investments less cash.

(iii) *Size*. As a proxy for size, we use the log of total market equity for each firm divided by the log of the cross-sectional average of market equity.

(iv) A *boom* indicator. Specifically, this indicator gives (for each firm) the number of consecutive quarters of being in the top decile of the market-to-book ratio across firms.

Table 4 provides the summary statistics for $\Delta^{\$}CoVaR_{q,t}^{i}$ at the quarterly frequency, and the quarterly firm characteristics. In Table 5, we ask whether our systemic risk measure can be forecast cross-sectionally by lagged characteristics

Table 4—Quarterly Summary Statistics

	Mean	Standard deviation	Observations
$\Delta^{\$}CoVaR^{i}_{95,t}$	792.93	3,514.15	106,531
$\Delta^{\$}CoVaR^{i}_{99,t}$	1,023.58	4,030.08	106,531
$VaR^{i}_{95,t}$	84.84	44.79	106,889
$VaR^{i}_{99,t}$	145.46	80.01	106,889
Leverage	9.12	10.43	94,772
Size	−2.70	1.97	96,738
Maturity mismatch	3.51	11.17	96,738
Boom	0.30	1.34	116,366

Notes: The table reports summary statistics for the quarterly variables in the *forward*-$\Delta CoVaR$ regressions. The data span 1971:I–2013:II, covering 1,823 financial institutions. $VaR^{i}_{q,t}$ is expressed in units of quarterly percent. $\Delta\$CoVaR^{i}_{q,t}$ is normalized by the cross-sectional average of market equity for each quarter and is expressed in quarterly basis points. The institution characteristics are described in Section IVA.

at different time horizons. Table 5 shows that firms with higher leverage, more maturity mismatch, larger size, and higher equity valuation according to the boom variable tend to be associated with higher $\Delta^{\$}CoVaRs$ one quarter, one year, and two years later. These results hold for the 99 percent $\Delta^{\$}CoVaR$ and the 95 percent $\Delta^{\$}CoVaR$. The coefficients in Table 5 are the sensitivities of $\Delta^{\$}CoVaR^{i}_{q,t}$ with respect to the characteristics expressed in units of basis points. For example, the coefficient of 14.5 on the leverage forecast at the two-year horizon implies that an increase in an institution's leverage (say, from 15 to 16) is associated with an increase in $\Delta^{\$}CoVaR$ of 14.5 basis points of quarterly market equity losses at the 95 percent quantile. Columns 1–3 and 4–6 of Table 5 can be interpreted as a "term structure" of our systemic risk measure when read from right to left. The comparison of panels A and B provide a gauge of the "tailness" of our systemic risk measure.

Importantly, these results allow us to connect $\Delta^{\$}CoVaR$ with frequently and reliably measured institution-level characteristics. $\Delta^{\$}CoVaR$—like any tail risk measure—relies on relatively few extreme data points. Hence, adverse movements, especially followed by periods of stability, can lead to sizable increases in tail risk measures. In contrast, measurement of characteristics such as size are very robust, and they can be measured more reliably at higher frequencies. The "too big to fail" suggests that size is considered by some to be the dominant variable, and, consequently, that large institutions should face more stringent regulations than smaller institutions. However, focusing only on size fails to acknowledge that many small institutions can be systemic as a herd. Our solution to this problem is to combine the virtues of both types of measures by projecting $\Delta^{\$}CoVaR$ on multiple, more frequently observed variables, providing a tool that might prove useful in identifying systemically important financial institutions. The regression coefficients of Table 5 can be used to weigh the relative importance of various firm characteristics. For example, the trade-off between size and leverage is given by the ratio of the two respective coefficients of our forecasting regressions. In order to lower its systemic risk per unit of total asset, a bank could reduce its maturity mismatch or improve its systemic risk profile along other dimensions. In fact, in determining systemic importance of global banks for regulatory purposes, the Basel Committee on Banking Supervision (BCBS 2013) relies on frequently observed firm characteristics.

TABLE 5—$\Delta CoVaR^i$ FORECASTS FOR ALL PUBLICLY TRADED FINANCIAL INSTITUTIONS

	Panel A. $\Delta^{\$}CoVaR^i_{95,t}$			*Panel B.* $\Delta^{\$}CoVaR^i_{99,t}$		
	2-year	1-year	1-quarter	2-year	1-year	1-quarter
VaR	7.760	8.559	9.070	2.728	3.448	4.078
	(9.626)	(10.566)	(11.220)	(7.484)	(9.443)	(10.225)
Leverage	14.573	13.398	13.272	17.504	15.958	15.890
	(5.946)	(6.164)	(6.317)	(5.854)	(6.299)	(6.627)
Size	1,054.993	1,014.396	990.862	1,238.674	1,195.072	1,170.075
	(22.994)	(23.420)	(23.630)	(27.549)	(28.243)	(28.603)
Maturity mismatch	7.306	5.779	4.559	9.349	7.918	6.358
	(2.187)	(1.968)	(1.760)	(2.725)	(2.537)	(2.225)
Boom	154.863	160.391	151.389	155.184	169.315	165.592
	(4.161)	(4.431)	(4.414)	(3.653)	(3.962)	(3.908)
Equity volatility	74.284	67.707	135.484	212.860	203.569	286.300
	(1.317)	(1.346)	(2.889)	(3.211)	(3.478)	(4.960)
Three-month yield change	−111.225	−144.750	−127.052	−75.877	−123.081	−111.545
	(−5.549)	(−6.686)	(−7.059)	(−3.390)	(−5.518)	(−5.748)
TED spread	−431.094	−187.730	−232.091	−436.214	−175.884	−196.513
	(−6.989)	(−2.403)	(−3.207)	(−6.282)	(−2.163)	(−2.488)
Credit spread change	−145.121	−165.345	−78.447	−147.680	−172.642	−102.399
	(−3.319)	(−3.959)	(−2.036)	(−2.659)	(−3.336)	(−2.023)
Term spread change	−275.593	−243.509	−187.183	−251.197	−236.494	−179.935
	(−7.570)	(−9.017)	(−8.500)	(−6.678)	(−8.094)	(−7.216)
Market return	88.971	30.799	−97.783	97.187	33.065	−111.336
	(3.881)	(1.401)	(−4.327)	(3.735)	(1.381)	(−4.469)
Housing	27.373	32.940	17.800	4.517	15.249	6.644
	(1.845)	(2.479)	(1.156)	(0.269)	(0.992)	(0.390)
Foreign FE	−439.424	−424.325	−405.148	−828.557	−811.836	−788.096
	(−2.376)	(−2.378)	(−2.295)	(−4.492)	(−4.579)	(−4.532)
Insurance FE	−724.971	−681.143	−649.836	−435.109	−408.868	−391.193
	(−7.610)	(−7.629)	(−7.639)	(−4.086)	(−4.114)	(−4.138)
Real estate FE	−50.644	−42.466	−24.328	66.136	80.733	98.908
	(−0.701)	(−0.647)	(−0.395)	(0.794)	(1.067)	(1.387)
Broker dealer FE	128.640	99.435	84.613	396.346	343.386	310.082
	(0.850)	(0.695)	(0.612)	(2.500)	(2.311)	(2.187)
Others FE	−373.424	−388.902	−381.562	−209.309	−235.844	−240.875
	(−5.304)	(−5.934)	(−6.121)	(−2.727)	(−3.338)	(−3.586)
Constant	4,608.697	4,348.786	3,843.332	5,175.855	4,970.410	4,443.515
	(16.292)	(17.542)	(19.802)	(18.229)	(19.768)	(21.566)
Observations	79,317	86,474	91,750	79,317	86,474	91,750
Adjusted R^2	24.53%	24.36%	24.35%	26.89%	26.75%	26.76%

Notes: This table reports the coefficients from panel forecasting regressions of $\Delta\$CoVaR^i_{95,t}$ on the quarterly, one-year, and two-year lags of firm characteristics in panel A and for the $\$CoVaR^i_{99,t}$ in panel B. Each regression has a panel of firms. FE denotes fixed effect dummies. Newey-West standard errors allowing for up to five periods of autocorrelation are displayed in parentheses.

Additional Characteristics of Bank Holding Companies.—Ideally, one would like to link $\Delta^{\$}CoVaR$ to more institutional characteristics than size, leverage, and maturity mismatch. More granular balance sheet items are available for the subsample of bank holding companies (BHC). On the asset side of banks' balance sheets, we use loans, loan loss allowances, intangible loss allowances, intangible assets, and trading assets. Each of these variables is expressed as a percentage of total book assets. The cross-sectional regressions with these asset-side variables are reported in panel B of Table 6. In order to capture the liability side of banks' balance sheets, we use interest-bearing core deposits, non-interest-bearing deposits, large time deposits, and demand deposits. Again, each of these variables is expressed as a percentage of total book assets. The variables can be interpreted as refinements of the maturity mismatch variable used earlier. The cross-sectional regressions with the liability-side variables are reported in panel A of Table 6.

Panel A of Table 6 shows which types of liability variables are significantly increasing or decreasing in systemic risk. Bank holding companies with a higher fraction of non-interest-bearing deposits are associated with a significantly higher *forward*-$\Delta^{\$}CoVaR$, while interest bearing core deposits and large time deposits are decreasing the forward estimate of $\Delta^{\$}CoVaR$. Non-interest-bearing deposits are typically held by nonfinancial corporations and households, and can be quickly reallocated across banks conditional on stress in a particular institution. Interest-bearing core deposits and large time deposits, on the other hand, are more stable sources of funding and are thus associated with lower *forward*-$\Delta^{\$}CoVaR$. The maturity mismatch variable that we constructed for the universe of financial institutions is no longer significant once we include the more refined liability measures for the bank holding companies.

Panel B of Table 6 shows that the fraction of trading assets is a particularly good predictor for *forward*-$\Delta^{\$}CoVaR$, with the positive sign indicating that increased trading activity is associated with a greater systemicity of bank holding companies. Larger shares of loans also tend to increase the association between a bank's distress and aggregate systemic risk, while intangible assets do not have much predictive power. Finally, loan loss reserves do not appear significant, likely because they are backward-looking.

In summary, the results of Table 6, in comparison to Table 5, show that more information about the balance sheet characteristics of financial institutions can improve the estimated *forward*-$\Delta^{\$}CoVaR$. We expect that additional data capturing particular activities of financial institutions, such as supervisory data, would lead to further improvements in the estimation precision of *forward*-$\Delta^{\$}CoVaR$.

B. *Time-Series Predictive Power of Forward-*$\Delta CoVaR$

The predicted values of regression (15) yields a time-series of *forward*-$\Delta CoVaR$ for each institution i. In Figure 6 we plot the $\Delta CoVaR$ together with the two-year *forward*-$\Delta CoVaR$ for the average of the largest 50 financial institutions, where size is observed at 2007:I. The *forward*-$\Delta CoVaR$ is estimated in-sample until the end of 2001, and out-of-sample from 2002:I. The figure clearly shows the strong negative correlation of the contemporaneous $\Delta CoVaR$ and the *forward*-$\Delta CoVaR$. In particular, during the credit boom of 2003–2006, the contemporaneous $\Delta CoVaR$ is estimated to be small, while the forward $\Delta CoVaR$ is relatively large. Macroprudential regulation based on the *forward*-$\Delta CoVaR$ can thus be countercyclical.

From an economic perspective, the countercyclicality of the forward measure reflects the fact that institutions' risk-taking is endogenously high during expansions, which makes them vulnerable to adverse economic shocks. For example, in the equilibrium model of Adrian and Boyarchenko (2012), contemporaneous

TABLE 6—$\Delta CoVaR^i$ FORECASTS FOR BANK HOLDING COMPANIES

	Panel A. BHC liability variables			*Panel B. BHC asset variables*		
	2-year	1-year	1-quarter	2-year	1-year	1-quarter
VaR	9.995 (4.774)	11.068 (5.099)	11.699 (5.310)	4.251 (2.211)	6.402 (3.305)	7.715 (3.977)
Leverage	56.614 (9.654)	44.044 (9.683)	38.639 (9.340)	40.779 (6.476)	29.967 (6.103)	23.919 (5.532)
Size	1,457.875 (13.539)	1,394.324 (13.781)	1,360.318 (13.850)	1,203.344 (13.069)	1,141.656 (13.352)	1,107.145 (13.591)
Boom	88.923 (1.238)	108.619 (1.617)	90.283 (1.560)	124.161 (1.758)	143.028 (2.179)	123.243 (2.209)
Equity volatility	92.400 (0.821)	−55.501 (−0.527)	48.969 (0.436)	292.221 (2.672)	90.937 (0.887)	165.272 (1.521)
Three-month yield change	−541.105 (−6.815)	−512.923 (−6.442)	−442.339 (−7.344)	−357.807 (−5.567)	−343.193 (−5.145)	−278.257 (−5.585)
TED Spread	−709.255 (−3.784)	−230.846 (−0.830)	−448.387 (−1.926)	−622.720 (−3.688)	−142.805 (−0.548)	−369.578 (−1.716)
Credit spread change	−338.997 (−2.860)	−268.226 (−2.082)	−136.785 (−1.132)	−208.571 (−1.879)	−113.161 (−0.935)	35.318 (0.297)
Term spread change	−838.872 (−6.863)	−681.318 (−7.575)	−556.294 (−8.060)	−640.835 (−6.521)	−505.287 (−6.886)	−391.500 (−6.718)
Market return	−16.289 (−0.329)	−61.080 (−1.156)	−205.718 (−3.698)	25.380 (0.541)	−34.474 (−0.706)	−188.620 (−3.601)
Housing	88.442 (2.853)	116.009 (4.091)	69.066 (2.297)	64.023 (2.236)	95.015 (3.687)	51.096 (1.795)
Core deposits	−50.915 (−8.005)	−53.888 (−8.217)	−53.439 (−8.244)			
Non-interest deposits	51.610 (3.840)	46.680 (4.278)	43.844 (4.519)			
Time deposits	−68.087 (−8.347)	−65.152 (−8.123)	−62.553 (−8.064)			
Demand deposits	−13.782 (−0.929)	−14.811 (−1.254)	−16.195 (−1.511)			
Total loans				9.419 (2.285)	6.568 (1.792)	3.811 (1.106)
Loan loss reserves				−133.352 (−1.086)	−131.401 (−1.263)	−72.177 (−0.776)
Intanglible assets				48.721 (0.884)	43.285 (0.874)	41.082 (0.915)
Trading assets				576.060 (5.332)	565.476 (5.636)	549.158 (6.088)
Constant	10,820.323 (10.244)	10,077.831 (10.388)	9,222.897 (11.572)	5,825.307 (7.899)	5,127.386 (7.849)	4,404.582 (8.573)
Observations	25,578	28,156	30,128	25,481	28,060	30,030
Adjusted R^2	28.94%	28.17%	28.13%	36.29%	35.47%	35.41%

Notes: This table reports the coefficients from panel forecasting regressions of $\Delta\$CoVaR^i_{95,t}$ on the quarterly, one-year, and two-year lags of liability-side and other firm characteristics in panel A, and for asset-side and other firm characteristics in panel B. The methodologies for computing the risk measures $VaR^i_{95,t}$ and $\Delta\$CoVaR^i_{95,t}$ are given in the captions of Tables 2 and 3. The risk measures are calculated for the 95 percent quantile. Newey-West standard errors allowing for up to five periods of autocorrelation are displayed in parentheses.

volatility is low in booms, which relaxes risk management constraints on intermediaries, allowing them to increase risk-taking, and making them more vulnerable to shocks.

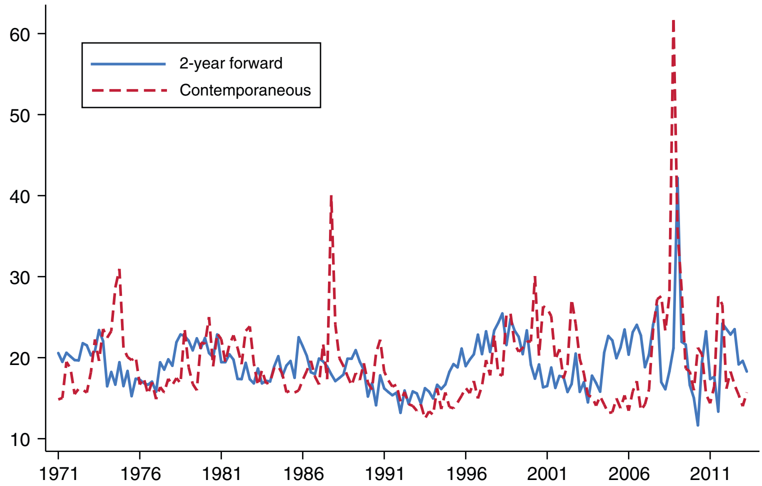

Figure 6. Countercyclicality of *forward*-$\Delta CoVaR$

Notes: This figure shows average forward and contemporaneous $\Delta CoVaR^i_{95,t}$ estimated out-of-sample since 2002:I for the largest 50 financial institutions, and in-sample prior to 2002. *Forward*-$\Delta CoVaR^i_{95,t}$ is estimated as described in the main body of the text. The *forward*-$\Delta CoVaR^i_{95,t}$ at a given date uses the data available at that time to predict $\Delta CoVaR^i_{95,t}$ two years in the future. All units are in percent of quarterly market equity losses.

C. *Cross-Sectional Predictive Power of Forward-*$\Delta CoVaR$

Next, we test the extent to which the *forward*-$\Delta CoVaR^i$ predicts realized $\Delta CoVaR^i$ across institutions during the financial crisis. To do so, we calculate *forward*-$\Delta CoVaR^i$ for each firm up to 2006:IV. We also calculate the crisis $\Delta CoVaR^i$ for each firm for the 2007:II–2009:II period. In order to show the out-of-sample forecasting performance of *forward*-$\Delta CoVaR^i$ in the cross section, we regress the *crisis*-$\Delta CoVaR^i_{95}$ (computed for 2007:I–2008:IV) on the *forward*-$\Delta CoVaR^i_{95}$ (as of 2006:IV). We report the 95 percent level, though we found that the 99 percent level gives very similar results.

Table 7 shows that the two year ahead *forward*-$\Delta CoVaR$ as of the end of 2006:IV was able to explain over one-third of the cross sectional variation of realized Δ $CoVaR$ during the crisis. The one-year ahead forecast of 2008:IV using data as of 2007:IV only predicts one-fifth of the cross-sectional dispersion, while the one-quarter ahead forecast for 2008:IV as of 2008:III predicts over three-quarters of the cross section. The last two columns of Table 7 also show the one-year and one-quarter ahead forecasts of realized $\Delta CoVaR$ as of 2006:IV. We view these findings as very strong, indicating that the systemic risk measures have significant forecasting power for the cross section of realized $\Delta CoVaR$. Panel B of Table 7 provides similar forecasts using *forward-VaR*, showing that *VaR* has no forecasting power for realized $\Delta CoVaR$.

V. Conclusion

During financial crises, tail events tend to spill across financial institutions. Such spillovers are preceded by a phase in which risk builds up. Both elements are

TABLE 7—$\Delta CoVaR_i$ FORECASTS FOR BANK HOLDING COMPANIES

	Crisis $\Delta CoVaR$				
	2008:IV	2008:IV	2008:IV	2007:IV	2007:I
Panel A					
2Y *Forward-$\Delta CoVaR$* (2006:IV)	1.206				
1Y *Forward-$\Delta CoVaR$* (2007:IV)		0.664			
1Q *Forward-$\Delta CoVaR$* (2008:III)			1.708		
1Y *Forward-$\Delta CoVaR$* (2006:IV)				0.848	
1Q *Forward-$\Delta CoVaR$* (2006:IV)					0.541
Constant	13.08	18.51	2.409	4.505	2.528
Observations	378	418	430	428	461
R^2	36.6%	17.8%	78.9%	49.6%	55.5%
Panel B					
2Y *Forward-VaR* (2006:IV)	−0.029				
1Y *Forward-VaR* (2007:IV)		−0.001			
1Q *Forward-VaR* (2008:III)			0.001		
Constant	26.71	23.58	23.40		
Observations	378	418	430		
R^2	1.0%	0.0%	0.0%		

Notes: This table reports a regression of the $\Delta CoVaR$ during the financial crisis of 2007–2009 on *forward-$\Delta CoVaR$* for the universe of bank holding companies in panel A. Panel B reports the regression of the $\Delta CoVaR$ during the financial crisis of 2007–2009 on *forward-VaR*. The columns correspond to different forecasting horizons at different dates. The first column of panel A uses a two-year forecast as of 2006:IV, the second column uses a one-year forecast as of 2007:IV, the third column uses a one-month forecast as of 2008:III, the fourth column uses a one-year forecast as of 2006:IV, and the last column uses a one-quarter forecast as of 2006:IV. In panel B, the first column uses a two-year *VaR* forecast as of 2006:IV, the second column uses a one-year *VaR* forecast as of 2007:IV, the third column uses a one-month *VaR* forecast as of 2008:III.

important components of financial system risk. $\Delta CoVaR$ is a parsimonious measure of systemic risk that captures the (directed) tail-dependency between an institution and the financial system as a whole. $\Delta CoVaR$ broadens risk measurement to afford a macroprudential perspective in the cross section and complements measures designed to assess microprudential risk of individual financial institutions. The *forward-$\Delta CoVaR$* is a forward-looking measure of systemic risk. It is constructed by projecting $\Delta CoVaR$ on lagged firm characteristics such as size, leverage, maturity mismatch, and industry dummies. This forward-looking measure can be used in a time-series application of macroprudential policy.

APPENDIX

I. *CoVaR* Estimation via Quantile Regressions

This Appendix explains how to use quantile regressions to estimate *VaR* and *CoVaR*. As discussed in footnote 5, the model considered here is a special case of the stylized financial system analyzed in Section II, with particularly simple expressions for $\mu^j(\cdot)$, $\sigma^{ji}(\cdot)$, and $\sigma^{jj}(\cdot)$. Specifically, we assume that losses X_t^i have the following linear factor structure

$$X_{t+1}^j = \phi_0 + \mathbf{M}_t \phi_1 + X_{t+1}^i \phi_2 + (\phi_3 + \mathbf{M}_t \phi_4) \Delta Z_{t+1}^j, \tag{I.1}$$

where $\mathbf{M}_t$ is a vector of state variables. The error term ΔZ^j_{t+1} is assumed to be i.i.d. with zero mean and unit variance, and $E\left[\Delta Z^j_{t+1} \mid \mathbf{M}_t, X^i_{t+1}\right] = 0$. The conditional expected return $\mu^j\left[X^j_{t+1} \mid \mathbf{M}_t, X^i_{t+1}\right] = \phi_0 + \mathbf{M}_t \boldsymbol{\phi}_1 + X^i_{t+1}\phi_2$ depends on the set of state variables $\mathbf{M}_t$ and on X^i_{t+1}, and the conditional volatility $\sigma^{jj}_t\left[X^j_{t+1} \mid \mathbf{M}_t, X^i_{t+1}\right] = (\phi_3 + \mathbf{M}_t\phi_4)$ is a direct function of the state variables $\mathbf{M}_t$.[12] The coefficients ϕ_0, ϕ_1, and ϕ_2 could be estimated consistently via OLS of X^i_{t+1} on $\mathbf{M}_t$ and X^i_{t+1}. The predicted value of such an OLS regression would be the mean of X^j_{t+1} conditional on $\mathbf{M}_t$ and X^i_{t+1}. In order to compute the *VaR* and *CoVaR* from OLS regressions, one would have to also estimate ϕ_3, and ϕ_4, and then make distributional assumptions about ΔZ^j_{t+1}.[13] The quantile regressions incorporate estimates of the conditional mean and the conditional volatility to produce conditional quantiles, without the distributional assumptions that would be needed for estimation via OLS.

Instead of using OLS regressions, we use quantile regressions to estimate model (I.1) for different percentiles. We denote the cumulative distribution function (CDF) of ΔZ^j by $F_{\Delta Z^j}(\cdot)$, and its inverse CDF by $F^{-1}_{\Delta Z^j}(q)$ for the q%-quantile. It follows immediately that the inverse CDF of X^j_{t+1} is

$$\text{(I.2)} \qquad F^{-1}_{X^j_{t+1}}\left(q \mid \mathbf{M}_t, X^i_{t+1}\right) = \alpha_q + \mathbf{M}_t \boldsymbol{\gamma}_q + X^i_{t+1}\beta_q,$$

where $\alpha_q = \phi_0 + \phi_3 F^{-1}_{\Delta Z^j}(q)$, $\boldsymbol{\gamma}_q = \boldsymbol{\phi}_1 + \phi_4 F^{-1}_{\Delta Z^j}(q)$, and $\beta_q = \phi_2$ for quantiles $q \in (0, 100)$. We call $F^{-1}_{X^j_{t+1}}\left(q \mid \mathbf{M}_t, X^i_{t+1}\right)$ the conditional quantile function. From the definition of *VaR*, we obtain

$$VaR^j_{q,t+1} = \inf_{VaR^j_{q,t+1}} \left\{ \Pr\left(X^j_{t+1} \middle| \left\{ \mathbf{M}_t, X^i_{t+1} \right\} \leq VaR^j_{q,t+1} \right) \geq q\% \right\} = F^{-1}_{X^j_{t+1}}\left(q \mid \mathbf{M}_t, X^i_{t+1}\right).$$

The conditional quantile function $F^{-1}_{X^j_{t+1}}\left(q \mid \mathbf{M}_t, X^i_{t+1}\right)$ is the $VaR^j_{q,t+1}$ conditional on $\mathbf{M}_t$ and X^i_{t+1}. By conditioning on $X^i_{t+1} = VaR^i_{q,t+1}$, we obtain the $CoVaR^{j|i}_{q,t+1}$ from the quantile function:

$$\text{(I.3)} \quad CoVaR^{j|i}_{q,t+1} = \inf_{VaR^j_{q,t+1}} \left\{ \Pr\left(X_{t+1} \middle| \left\{ \mathbf{M}_t, X^i_{t+1} = VaR^i_{q,t+1} \right\} \leq VaR^j_{q,t+1} \right) \geq q\% \right\}$$
$$= F^{-1}_{X^j_{t+1}}\left(q \mid \mathbf{M}_t, X^i_{t+1} = VaR^i_{q,t+1}\right).$$

We estimate the quantile function as the predicted value of the q%-quantile regression of X^i_{t+1} on $\mathbf{M}_t$ and X^j_{t+1} by solving

$$\min_{\alpha_q, \beta_q, \gamma_q} \sum_t \begin{cases} q\% \left| X^j_{t+1} - \alpha_q - \mathbf{M}_t\gamma_q - X^i_{t+1}\beta_q \right| & \text{if } \left(X^j_{t+1} - \alpha_q - \mathbf{M}_t\gamma_q - X^i_{t+1}\beta_q \right) \geq 0 \\ (1 - q\%) \left| X^j_{t+1} - \alpha_q - \mathbf{M}_t\gamma_q - X^i_{t+1}\beta_q \right| & \text{if } \left(X^j_{t+1} - \alpha_q - \mathbf{M}_t\gamma_q - X^i_{t+1}\beta_q \right) < 0 \end{cases}.$$

[12] Alternatively, X^i_{t+1} could have also been introduced as a direct determinant of the volatility. The model would then just be $X^j_{t+1} = \phi_0 + \mathbf{M}_t\boldsymbol{\phi}_1 + X^i_{t+1}\phi_2 + (\phi_3 + \mathbf{M}_t\boldsymbol{\phi}_4 + X^i_{t+1}\phi_5)\,\Delta Z^j_{t+1}$.

[13] The model (I.1) could alternatively be estimated via maximum likelihood if distributional assumptions about ΔZ are made.

Bassett and Koenker (1978) and Koenker and Bassett (1976) provide statistical properties of quantile regressions. Chernozhukov and Umantsev (2001) and Chernozhukov and Du (2008) discuss *VaR* applications of quantile regressions.

II. Robustness Checks

A. *GARCH* Δ*CoVaR*

One potential shortcoming of the quantile estimation procedure described in Section III is that it models time-varying moments only as a function of aggregate state variables. An alternative approach is to estimate bivariate *GARCH* models to obtain the time-varying covariance between institutions and the financial system. As a robustness check, we estimate $\Delta CoVaR$ using a bivariate diagonal *GARCH* model (*DVECH*) and find that this method produces estimates quite similar to the quantile regression method, leading us to the conclusion that the quantile regression framework is sufficiently flexible to estimate $\Delta CoVaR$. We begin by outlining a simple Gaussian framework under which $\Delta CoVaR$ has a closed-form expression, and then present the estimation results. The Gaussian framework is a special case of the stylized financial system we develop in Section II, with deterministic mean and covariance terms, and jointly normally distributed latent shock processes.

Gaussian Model.—Assume firm and system losses follow a bivariate normal distribution:

$$\text{(II.1)} \qquad \left(X_t^i, X_t^{system}\right) \sim N\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\left(\sigma_t^i\right)^2 & \rho_t^i\sigma_t^i\sigma_t^{system}\\ \rho_t^i\sigma_t^i\sigma_t^{system} & \left(\sigma_t^{system}\right)^2\end{pmatrix}\right).$$

By properties of the multivariate normal distribution, the distribution of system losses conditional on firm losses is also normally distributed:

$$\text{(II.2)} \qquad X_t^{system}\,|\,X_t^i \sim N\left(\frac{X_i^t\sigma_t^{system}\rho_t^i}{\sigma_t^i}, \left(1-\left(\rho_t^i\right)^2\right)\left(\sigma_t^{system}\right)^2\right).$$

Using the definition of $CoVaR_{q,t}^i$

$$\text{(II.3)} \qquad \Pr\left(X_t^{system}\,|\,X_t^i = VaR_{q,t}^i \le CoVaR_{q,t}^i\right) = q\%,$$

we find

$$\text{(II.4)} \quad \Pr\left(\left[\frac{X_t^{system} - X_t^i\rho_t^i\sigma_t^{system}/\sigma_t^i}{\sigma_t^{system}\sqrt{1-\left(\rho_t^i\right)^2}}\,\middle|\, X_t^i = VaR_{q,t}^i\right] \le \frac{CoVaR_{q,t}^i - X_t^i\rho_t\sigma_t^{system}/\sigma_t^i}{\sigma_t^{system}\sqrt{1-\left(\rho_t^i\right)^2}}\right) = q\%.$$

Note that $\left[\frac{X_t^{system} - X_t^i\rho_t^i\sigma_t^{system}/\sigma_t^i}{\sigma_t^{system}\sqrt{1-\left(\rho_t^i\right)^2}}\right] \sim N(0,1)$. Also, the firm value-at-risk is given by $VaR_{q,t}^i = \Phi^{-1}(q\%)\,\sigma_t^i$. Combining the two, and using the simple expression for *VaR* if losses are distributed as in equation (II.2), we can write:

$$\text{(II.5)} \qquad CoVaR_{q,t}^i = \Phi^{-1}(q\%)\,\sigma_t^{system}\sqrt{1-\left(\rho_t^i\right)^2} + \Phi^{-1}(q\%)\,\rho_t^i\sigma_t^{system}.$$

Because $\Phi^{-1}(50\%) = 0$, solving for $\Delta CoVaR$ gives

$$\text{(II.6)} \qquad \Delta CoVaR_{q,t}^{i} = \Phi^{-1}(q\%)\, \rho_t^i \sigma_t^{system}.$$

In the Gaussian framework, $\Delta CoVaR$ is thus pinned down by three determinants: the correlation, the volatility of the financial system, and the Gaussian quantile. Cross-sectionally, the only ingredient that varies is the correlation of firms with the system, while over time, both the correlation and the system volatility are changing. While the time variation of $\Delta CoVaR$ is a function of the state variables M_t from Section IIIC in the quantile regression approach, it is only a function of the time varying variances and covariances in the *GARCH* approach. Despite these very different computations, we will see that the resulting $\Delta CoVaR$ estimates are—perhaps surprisingly—similar.

Estimation.—We estimate a bivariate diagonal vech *GARCH*(1,1) for each institution in our sample.[14] As a robustness check, we estimated the panel regressions of Section IV on a matched sample of 1,035 institutions for which our *GARCH* estimates converged.

The results in Table 8 show the coefficients of size, leverage, maturity mismatch, and boom are qualitatively similar between the *GARCH* and quantile estimation methods. Hence, the economic determinants of $\Delta CoVaR$ across firms does not appear to be dependent on the particular estimation method that is used to compute *CoVaR*. Figure 7 shows the GARCH and quantile estimates of $\Delta CoVaR$ for Citibank, Goldman Sachs, Metlife, and Wells Fargo, showing similarity across firms and over time.

Specification Tests.—In order to compare the performance of quantile estimates and the *GARCH* model more formally, we perform the conditional specification tests for value at risk measures proposed by Christoffersen (1998). Table 9 shows the fraction of firms whose *VaR* estimates have correct conditional coverage, i.e., the fraction of firms for which the probability of incurring losses that exceed the *VaR* is below the 95 percent or 99 percent values. The conditional coverage is computed via a likelihood ratio test whose alternative hypothesis is that the probability of the *VaR* forecast is correct conditional on past observations and is equal to the specified probability. The table then presents the fraction of firms for which the null hypothesis is rejected.

Table 9 shows that *GARCH* estimates perform better for the ninety-fifth percentile, while the quantile estimates perform considerably better for the ninety-ninth percentile. For example, at the 5 percent level, the conditional coverage of the quantile model is 0.46 for the ninety-ninth percentile and 0.52 for the ninety-fifth percentile; for the *GARCH* model, the respective fractions are 0.20 and 0.70.

[14] We were able to get convergence of the *GARCH* model for 56 percent of firms. We found that convergence of the models in our data is very sensitive to both missing values and extreme returns. Truncation of returns generally, but not consistently, resulted in an increase in the fraction of the models that converged.

TABLE 8—$\Delta CoVaR_i$ FORECASTS USING *GARCH* ESTIMATION

	2-year		1-year		1-quarter	
	Quantile	*GARCH*	Quantile	*GARCH*	Quantile	*GARCH*
VaR	10.655	22.126	10.462	22.290	10.760	22.953
	(7.146)	(6.446)	(8.292)	(6.762)	(9.025)	(7.725)
Size	2,171.414	3,650.023	2,088.345	3,495.286	2,044.432	3,432.149
	(21.404)	(16.656)	(21.865)	(16.792)	(22.064)	(17.000)
Maturity mismatch	22.292	40.001	18.708	30.449	15.859	28.336
	(2.709)	(2.207)	(2.540)	(1.975)	(2.405)	(1.898)
Boom	224.188	371.613	250.597	459.495	243.163	440.275
	(3.060)	(2.934)	(3.417)	(3.683)	(3.448)	(3.599)
Equity volatility	355.404	681.617	421.286	375.124	638.354	1,676.640
	(2.772)	(2.710)	(3.493)	(1.477)	(5.375)	(7.103)
Leverage	17.133	19.563	17.383	18.788	18.257	22.212
	(2.655)	(1.693)	(2.975)	(1.654)	(3.249)	(2.170)
Housing	16.228	82.042	3.684	−63.677	−29.998	−246.555
	(0.460)	(1.192)	(0.116)	(−1.018)	(−0.784)	(−3.338)
Three-month yield change	−204.962	−441.897	−282.786	−649.486	−248.337	−444.962
	(−4.476)	(−5.124)	(−5.584)	(−6.105)	(−5.854)	(−6.116)
TED spread	−683.690	−1,119.422	−171.649	250.885	−249.432	−567.982
	(−4.966)	(−4.386)	(−0.937)	(0.659)	(−1.482)	(−1.881)
Credit spread change	−341.279	−715.071	−392.110	−616.055	−215.385	−45.730
	(−3.295)	(−3.572)	(−3.959)	(−3.461)	(−2.319)	(−0.239)
Term spread change	−554.797	−1,174.171	−493.187	−847.419	−369.603	−557.334
	(−6.508)	(−6.821)	(−7.809)	(−7.419)	(−7.159)	(−6.241)
Market return	97.262	154.923	−36.331	−162.681	−329.304	−490.855
	(1.875)	(1.580)	(−0.680)	(−1.295)	(−5.717)	(−4.905)
Foreign FE	−600.936	−110.671	−608.110	−275.112	−580.752	−445.307
	(−1.253)	(−0.144)	(−1.311)	(−0.377)	(−1.265)	(−0.652)
Insurance FE	−1,550.564	−3,500.083	−1,421.556	−3,254.618	−1,339.724	−3,054.498
	(−7.233)	(−8.064)	(−7.129)	(−7.974)	(−7.093)	(−8.030)
Real estate FE	63.919	−34.037	156.418	68.235	223.824	234.309
	(0.369)	(−0.116)	(0.982)	(0.250)	(1.483)	(0.928)
Broker dealer FE	202.435	−775.272	253.854	−708.925	281.495	−664.763
	(0.626)	(−1.527)	(0.820)	(−1.442)	(0.930)	(−1.424)
Others FE	−682.642	−1,535.849	−656.751	−1,546.880	−626.059	−1,418.808
	(−4.026)	(−5.076)	(−4.198)	(−5.226)	(−4.212)	(−5.196)
Constant	7,920.183	14,203.481	7,553.186	13,032.912	6,519.919	9,495.516
	(13.759)	(11.613)	(14.828)	(12.660)	(16.862)	(13.664)
Observations	51,294	51,286	55,347	55,343	58,355	58,352
Adjusted R^2	27.10%	21.23%	26.94%	21.20%	27.00%	21.93%

Notes: This table reports the coefficients from panel forecasting regressions of the two estimation methods of $\Delta^{\$}CoVaR^{i}_{95,t}$ on the quarterly, one-year, and two-year lag of firm characteristics. FE denotes fixed effect dummies. The *GARCH*-$\Delta^{\$}CoVaR^{i}_{95,t}$ is computed by estimating the covariance structure of a bivariate diagonal *VECH GARCH* model. Newey-West standard errors allowing for up to five periods of autocorrelation are displayed in parentheses.

B. *Alternative Financial System Losses*

The financial system loss variable X_t^{system} used in the paper is the weekly loss on the market equity of the financial system, as proxied by the universe of publicly traded US financial institutions. This measure is generated by taking average market equity losses, weighted by lagged market equity. One concern with this

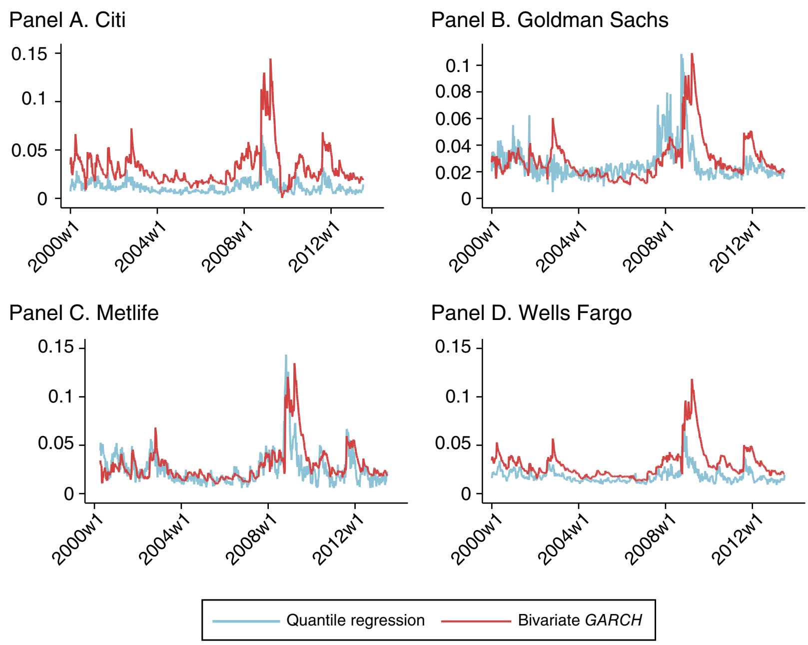

Figure 7. $\Delta CoVaR$ via *GARCH* and Quantile Regressions

Note: The plots show a comparison of $\Delta CoVaR$ estimates using quantile regressions and using *GARCH* for four large financial firms.

Table 9—Fraction of Firms Whose *VaR* Estimates Have Correct Conditional Coverage

	Quantile		*GARCH*	
	5% level	10% level	5% level	10% level
95% − *VaR*	0.52	0.61	0.70	0.78
99% − *VaR*	0.46	0.59	0.20	0.26

Notes: By definition of the VaR_t, losses at t should be less than the VaR_t with probability 95 percent or 99 percent. The table reports the fraction of firms that have correct conditional coverage at the 5 percent and 10 percent confidence levels based on Christoffersen's (1998) likelihood ratio test.

methodology is that it might introduce a mechanical correlation between each institution and the financial system proportional to the relative size of the financial institution. We check to see if such a mechanical correlation is driving our results by reestimating institutions' $\Delta CoVaR$ using system return variables formed from the value-weighted returns of all other institutions in the sample, leaving out the institution for which $\Delta CoVaR$ is being estimated.

We find a very strong correlation across institutions, and across time, for the two different measures of $\Delta CoVaR$. In fact, even for the largest institutions we find a very strong correlation between the baseline system return variable and the modified system return, with correlation coefficients over 99 percent. Table 10 reports the *forward-*$\Delta CoVaR$ regressions for the 95 percent level using both specifications. The coefficients under the two specifications are statistically indistinguishable, indicating that this mechanical correlation is not driving our results.

TABLE 10—$\Delta CoVar_i$ FORECASTS USING ALTERNATIVE SYSTEM RETURNS VARIABLE

	2-year		1-year		1-quarter	
	X^{system}	$X^{system-i}$	X^{system}	$X^{system-i}$	X^{system}	$X^{system-i}$
VaR	7.590	7.079	8.416	7.901	8.952	8.432
	(9.375)	(9.534)	(10.326)	(10.573)	(10.974)	(11.292)
Leverage	14.586	14.150	13.380	12.910	13.241	12.743
	(5.988)	(6.093)	(6.178)	(6.271)	(6.316)	(6.388)
Size	1,045.383	1,003.729	1,005.236	965.690	982.123	943.777
	(22.846)	(23.639)	(23.263)	(24.102)	(23.461)	(24.318)
Maturity mismatch	7.203	6.349	5.636	4.970	4.417	3.900
	(2.149)	(2.138)	(1.920)	(1.902)	(1.710)	(1.684)
Boom	155.288	150.837	160.661	156.523	151.023	147.602
	(4.203)	(4.266)	(4.470)	(4.543)	(4.445)	(4.533)
Equity volatility	77.476	77.382	67.143	67.784	135.018	135.387
	(1.389)	(1.474)	(1.337)	(1.428)	(2.897)	(3.062)
Three-month yield change	−110.309	−104.495	−143.529	−137.320	−125.988	−120.507
	(−5.543)	(−5.535)	(−6.673)	(−6.717)	(−7.050)	(−7.102)
TED spread	−425.174	−406.246	−185.526	−172.485	−228.606	−214.126
	(−6.953)	(−7.103)	(−2.387)	(−2.361)	(−3.175)	(−3.181)
Credit spread change	−144.973	−137.867	−160.705	−154.783	−75.179	−72.486
	(−3.391)	(−3.416)	(−3.929)	(−3.987)	(−1.984)	(−2.006)
Term spread change	−274.462	−262.126	−242.640	−231.757	−185.570	−176.715
	(−7.606)	(−7.673)	(−9.052)	(−9.161)	(−8.492)	(−8.605)
Market return	88.292	84.502	29.547	27.823	−98.583	−95.902
	(3.848)	(3.948)	(1.349)	(1.347)	(−4.367)	(−4.496)
Housing	27.667	25.551	33.085	30.527	18.043	15.607
	(1.860)	(1.827)	(2.491)	(2.447)	(1.168)	(1.077)
Foreign FE	−433.032	−420.804	−418.956	−407.989	−400.882	−391.199
	(−2.336)	(−2.366)	(−2.343)	(−2.378)	(−2.266)	(−2.304)
Insurance FE	−712.369	−656.614	−669.666	−617.652	−639.305	−589.511
	(−7.496)	(−7.395)	(−7.516)	(−7.428)	(−7.527)	(−7.439)
Real estate FE	−46.567	−37.370	−39.152	−31.187	−21.467	−14.662
	(−0.649)	(−0.546)	(−0.601)	(−0.503)	(−0.351)	(−0.252)
Broker dealer FE	113.499	139.780	84.969	110.399	69.907	94.869
	(0.757)	(0.973)	(0.599)	(0.813)	(0.510)	(0.724)
Others FE	−362.552	−341.190	−379.304	−358.435	−373.640	−353.344
	(−5.182)	(−5.130)	(−5.815)	(−5.798)	(−6.015)	(−6.012)
Constant	4,573.181	4,393.954	4,310.730	4,146.631	3,805.569	3,658.264
	(16.248)	(16.599)	(17.495)	(17.884)	(19.746)	(20.250)
Observations	79,317	79,317	86,474	86,474	91,750	91,750
Adjusted R^2	24.41%	25.13%	24.24%	24.96%	24.24%	24.96%

Notes: This table reports the coefficients from forecasting regressions of the two estimation methods of $\Delta^{\$}CoVaR^i_{95,t}$ on the quarterly, one-year, and two-year lag of firm characteristics. In the columns labeled X^{system}, $\Delta^{\$}CoVaR^i_{95}$ is estimated using the regular system returns variable described in Section III, while in columns labeled $X^{system-i}$, $\Delta^{\$}CoVaR^i_{95}$ is estimated using a system return variable that does not include the firm for which $\Delta^{\$}CoVaR^i_{95}$ is being estimated. FE denotes fixed effect dummies. Newey-West standard errors allowing for up to five periods of autocorrelation are displayed in parentheses.

III. Data Description

A. *CRSP and COMPUSTAT Data*

As discussed in the paper, we estimate $\Delta CoVaR$ for market equity losses of financial institutions. We start with daily equity data from CRSP for all financial institutions with two-digit COMPUSTAT SIC codes between 60 and 67 inclusive, indexed by PERMNO. Banks correspond to SIC codes 60, 61, and 6712; insurance companies correspond to SIC codes 63–64, real estate companies correspond to SIC codes 65–6, and broker-dealers are SIC code 67 (except for the bank holding companies, 6712). All other financial firms in our initial sample are placed in an "other" category. We manually adjust the COMPUSTAT SIC codes to account for the conversions of several large institutions into bank holding companies in late 2008, but otherwise do not observe time-varying industry classifications. Following the asset pricing literature, we keep only ordinary common shares (which exclude certificates, e.g., American Depositary Receipts (ADRs), Shares of Beneficial Interest (SBIs), Real Estate Investment Trusts (REITs), etc.) and drop daily equity observations with missing or negative prices or missing returns. Keeping only ordinary common shares excludes several large international institutions, such as Credit Suisse and Barclays, which are listed in the United States as American Depository Receipts.

The daily data are collapsed to weekly frequency and merged with quarterly balance sheet data from the CRSP/COMPUSTAT quarterly dataset. The quarterly data are filtered to remove leverage and book-to-market ratios less than zero and greater than 100. We also apply 1 percent and 99 percent truncation to the maturity mismatch variable.

Market equity and balance sheet data are adjusted for mergers and acquisitions using the CRSP daily dataset. We use a recursive algorithm to traverse the CRSP DELIST file to find the full acquisition history of all institutions in our sample. The history of acquired firms is collapsed into the history of their acquirers. For example, we account for the possibility that firm A was acquired by firm B, which was then acquired by firm C, etc. Our final panel therefore does not include any firms that we are able to identify as having been ultimately acquired by another firm in our universe. The final estimation sample is restricted to include firms with at least 260 weeks of nonmissing market equity returns. To construct the overall financial system portfolio (for $j = system$), we compute the average market equity-valued returns of all financial institutions, weighted by the (lagged) market value of their equity.

B. *Bank Holding Company Y9-C Data*

Balance sheet data from the FR Y-9C reports are incorporated into our panel dataset using a mapping maintained by the Federal Reserve Bank of New York.[15] We are able to match data for 732 US bank holding companies for a total of 40,241 bank-quarter observations. The link is constructed by matching PERMCOs in the linking table to RSSD9001 in the Y9-C data. We then match to the last available PERMCO of each institution in our CRSP/COMPUSTAT sample. It is important to note that our main panel of CRSP and COMPUSTAT data are historically merger-adjusted, but the Y9-C data are not.

In the forecasting regressions of Table 6, these variables are expressed as a percentage of total book assets. All ratios are truncated at the 1 percent and 99 percent

[15] The mapping is available at http://www.ny.frb.org/research/banking_research/datasets.html.

level across the panel. Detailed descriptions of the Y9-C variables listed above can be found in the Federal Reserve Board of Governors Micro Data Reference Manual.[16]

	Date range	FR Y-9C series name
Trading assets	1986:I—1994:IV	bhck2146
	1995:I—2013:II	bhck3545
Loans net loan-loss reserves	1986:I—2013:II	bhck2122−bhck3123
Loan-loss reserve	1986:I—2013:II	bhck3123
Intangible assets	1986:I—1991:IV	bhck3163+bhck3165
	1992:I—2000:IV	bhck3163+bhck3164 +bhck5506+bhck5507
	2001:I—2013:II	bhck3163+bhck0426
Interest-bearing core deposits	1986:I—2013:II	bhcb2210+bhcb3187+bhch6648 +bhdma164+bhcb2389
Non-interest-bearing deposits	1986:I—2013:II	bhdm6631+bhfn6631
Large time deposits	1986:I—2013:II	bhcb2604
Demand deposits	1986:I—2013:II	bhcb2210

IV. List of Financial Institutions for Figure 1[17]

Banks and Thrifts: Bank of America (BAC), Citigroup (C), JPMorgan Chase (JPM), Wachovia (WB), Wells Fargo (WFC)

Investment Banks: Bear Stearns (BSC), Goldman Sachs (GS), Lehman Brothers (LEH), Merrill Lynch (MER), Morgan Stanley (MS)

GSEs: Fannie Mae (FNM), Freddie Mac (FRE)

Insurance Companies: American International Group (AIG), Metlife (MET), Prudential (PRU)

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[16] http://www.federalreserve.gov/reportforms/mdrm.
[17] Industry classifications are as of 2006:IV.

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