Lecture 5
Approximating Distribution

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Approaches

- Continuum of households makes the wealth distribution an infinite dimensional object in the state space
- Need approximation
- Several approaches beyond Monte-Carlo simulation
  1. Discretization of the density function
  2. Eigenvector method
  3. Parametrizing in exponential family

Discretization of the Invariant Density Function

- The grid should be finer than the one we used to compute the optimal savings rule
- Choose some initial density function $\lambda^0\left(a_k, y_i\right)$
- For every $\left(a_k, y_j\right)$ on the grid:

$$\lambda^1\left(a_k, y_j\right) = \sum_{y_i \in Y} \pi_{ij} \sum_{m \in M_i} \frac{a_{k+1} - g\left(a_m, y_i\right)}{a_{k+1} - a_k} \lambda^0\left(a_m, y_i\right)$$

$$\lambda^1\left(a_{k+1}, y_j\right) = \sum_{y_i \in Y} \pi_{ij} \sum_{m \in M_i} \frac{g\left(a_m . y_i\right) - a_k}{a_{k+1} - a_k} \lambda^0\left(a_m, y_i\right)$$

where $M_i = \left\{m = 1, \ldots, N | a_k \leqslant g\left(a_m, y_i\right) \leqslant a_{k+1}\right\}$

Lottery Rule

- We can think of this way of handling the discrete approximation to the density function as forcing the agents in the economy to play a lottery.

- If the optimal policy is to save $a' \in [a_k, a_{k+1}]$, then with probability $\frac{a_{k+1}-a'}{a_{k+1}-a_k}$ you go to a_k, and with probability $\frac{a'-a_k}{a_{k+1}-a_k}$ you go to a_{k+1}.

Eigenvector Method

- The invariant pdf for a Markov transition matrix Q is λ^* that satisfies $\lambda^* Q = \lambda^*$
- Perron-Frobenius Theorem: Q has a unique dominant eigenvalue $\epsilon = 1$ such that
  - its associated eigenvector has all positive entries
  - all other eigenvalues are smaller than ϵ in absolute value
  - Q has no other eigenvector with all non-negative entries
- This eigenvector (renormalized so that it sums to one) is the unique invariant distribution