Lecture 6
Perturbation Methods

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Local Approximations

- DSGE models are characterized by a set of nonlinear equations
- Use local approximation to transform the nonlinear system into a system of linear equations
- Two requirements: small shocks, no occasionally binding constraints

Perturbing Equations

- Want: find a solution $y = g(x)$ for equation system $f(x, y) = 0$
- Steady state $f(\bar{x}, \bar{y}) = 0$

- (First-order) Taylor expansion of f

$$f(x, g(x)) \approx f(\bar{x}, \bar{y}) + \frac{\partial f}{\partial x}(\bar{x})(x - \bar{x}) + \frac{\partial f}{\partial y}(\bar{y})\, g'(\bar{x})(x - \bar{x})$$

- Back out $g'(\bar{x})$ and approximate the policy function $g(x) \approx g(\bar{x}) + (x - \bar{x})\, g'(\bar{x})$

Perturbing DSGE

- Equation system: $\mathbb{E}_t \mathcal{H}\left(y', y, x', x; \sigma\right) = 0$
- Want: policy function $y = g\left(x; \sigma\right)$ and law of the motions of the states $x' = h\left(x; \sigma\right)$

- Perturbation parameter is the variance scalar of the shocks, σ
- There can be multiple uncertainties, but only one perturbation variable

- Find approximate solution of g and h around $x = \bar{x}, \sigma = 0$

Perturbing DSGE

- Plug in the proposed solution

$$F\left(x;\sigma\right)\equiv\mathbb{E}_t\mathcal{H}\left(g\left(h\left(x;\sigma\right)\right),g\left(x;\sigma\right),h\left(x;\sigma\right),x;\sigma\right)=0$$

- $F=0$ for any values of x and σ, so the directives at any direction should be 0
- In particular, we take the derivatives around $x=\bar{x},\sigma=0$

$$F_{x^i\sigma^j}\left(\bar{x};0\right)=0,\forall i,j$$

Perturbing DSGE

- We propose solution of g, h

$$g(x;\sigma) = g(\bar{x};0) + g_x(x-\bar{x}) + g_\sigma \sigma$$
$$h(x;\sigma) = h(\bar{x};0) + h_x(x-\bar{x}) + h_\sigma \sigma$$

- $g_x, h_x, g_\sigma, h_\sigma$ should solve $F_x = 0, F_\sigma = 0$

$$F_x = \mathcal{H}_{y'} g_x h_x + \mathcal{H}_y g_x + \mathcal{H}_{x'} h_x + \mathcal{H}_x = 0$$
$$F_\sigma = \mathcal{H}_{y'} g_x h_\sigma + \mathcal{H}_y g_\sigma + \mathcal{H}_y g_\sigma + \mathcal{H}_{x'} h_\sigma = 0$$

- Observations:
  1. The first equation is a linear-quadratic system. Typical solution method applies (Blanchard-Kahn, Klein, Sims *gensys*)
  2. The second equation is homogeneous in g_σ and h_σ – certainty equivalence