Using the Dynare macro-processor

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1 The model

We consider a simple multi-country RBC model. There are n countries. A single homogenous good is produced, traded and consumed accross countries.

Production of country j at date t is $y_{jt} = a_{jt}\,A\,k_{j,t-1}^{\alpha}$, where A is a constant, $k_{j,t-1}$ is capital stock (using end-of-period timing convention for stocks, which is Dynare's natural convention), and a_{it} is productivity level. The law of motion of capital is:

$$k_{jt} = (1-\delta)k_{j,t-1} + i_{jt} \tag{1}$$

where i_{jt} is investment.

The law of motion of productivity is:

$$\log a_{jt} = \rho \log a_{j,t-1} + \sigma(e_{jt} + e_t) \tag{2}$$

where e_{jt} is a country specific shock and e_t is a worldwide shock. Shocks are assumed to be i.i.d. gaussian variables of zero mean and unit variance.

There is an adjustment cost on the capital stock:

$$\Gamma_{jt} = \frac{\phi}{2} k_{j,t-1} \left(\frac{i_{jt}}{k_{j,t-1}} - \delta \right)^2$$

Each country has a representative agent, whose utility is $u(c_{jt}) = \frac{c_{jt}^{1-\frac{1}{\gamma_j}}}{1-\frac{1}{\gamma_j}}$, where c_{jt} is consumption.

The world budget constraint is:

$$\sum_{j=1}^{n} (c_{jt} + i_{jt} - \delta\, k_{j,t-1}) = \sum_{j=1}^{n} \left(a_{jt}\,A\,k_{j,t-1}^{\alpha} - \frac{\phi}{2} k_{j,t-1} \left(\frac{i_{jt}}{k_{j,t-1}} - \delta \right)^2 \right) \tag{3}$$

One can show that the decentralized market equilibrium with complete financial markets is equivalent to a social planner program, where each country has a weight τ_j (Negishi weight) in the planner's objective; the weights depend on initial endowments.

2 FOC and calibration

We call λ_t the Lagrange multiplier of the world budget constraint. The first order conditions are:

$$\tau_j\, c_{jt}^{-\frac{1}{\gamma_j}} = \lambda_t \tag{4}$$

$$\lambda_t \left[1 + \phi\left(\frac{i_{jt}}{k_{j,t-1}} - \delta\right)\right] = \beta\,\mathbb{E}_t \Bigg\{ \lambda_{t+1} \Bigg[1 + a_{j,t+1}\, A\, \alpha\, k_{jt}^{\alpha-1} + \phi\left(1 - \delta + \frac{i_{j,t+1}}{k_{jt}} - \frac{1}{2}\left(\frac{i_{j,t+1}}{k_{jt}} - \delta\right)\right)\left(\frac{i_{j,t+1}}{k_{jt}} - \delta\right)\Bigg]\Bigg\} \tag{5}$$

We calibrate the model with: $\alpha = 0.36$, $\beta = 0.99$, $\delta = 0.025$, $\sigma = 0.01$, $\rho = 0.95$, $\phi = 0.5$, $A = \frac{1-\beta}{\alpha\,\beta}$. Heterogeneity accross countries is introduced in intertemporal elasticity of substitution: $\gamma_j = \frac{j+0.25(n-j)}{n}$, so that $\gamma_j \in (0.25, 1]$.

We choose the Negishi weights such that, at steady state, consumption equals production for each country: $\tau_j = \bar{c_j}^{\frac{1}{\gamma_j}}$ (where $\bar{c_j}$ is steady state consumption).

The deterministic steady state of the model is: $\bar{c_j} = A$, $\bar{k_j} = 1$, $\bar{i_j} = \delta$, $\bar{a_j} = 1$ and $\bar{\lambda} = 1$. This implies $\tau_j = A^{\frac{1}{\gamma_j}}$.

Note that, from Dynare's point of view, we have:

- $4n + 1$ endogenous variables: consumption, investment, capital, productivity for each country, plus the (worldwide) Lagrange multiplier
- $4n + 1$ equations (as many as variables, necessarily!): laws of motion of capital and productivity, first order conditions and global budget constraint
- $n + 1$ exogenous variables: one global shock and one country specific shock

3 Questions

1. Without using the macro-language, write a MOD file for the specific case where $n = 2$. Calibrate the model, check the steady state and the Blanchard-Kahn conditions, and compute the first order stochastic approximation with associated IRFs.

2. Now, using the macro-language, modify your MOD file to make it work for any number of countries. The first line of your file should be:

   ```
   @#define n = 5
   ```

   (where the figure 5 can be replaced by any positive integer)

   When you're done, use the `savemacro` option of the `dynare` command to examine the macro-expanded MOD file.