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Joint Research Centre

Screening methods in sensitivity analysis

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DYNARE Summer School

Paris, June 14th 2018

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Screening methods

Screening designs can be considered as the development of Design of Experiments (DOE)

DOE determines how much the variables involved in a physical experiment affect one or more measurements

The setup for sensitivity analysis of simulation results is similar to that of physical experimentation but…

Screening methods

.... simulations allows to explore more complex system with many more variables

screening designs

able to "screen" a subset of few important input variables among the many (hundreds, thousands) often contained in models

Goal: Model simplification / Model lumping / Pre-calibration

Screening methods

How much information sensitivity analysis reveals depends on:

- the number of sample points simulated
- where they are located

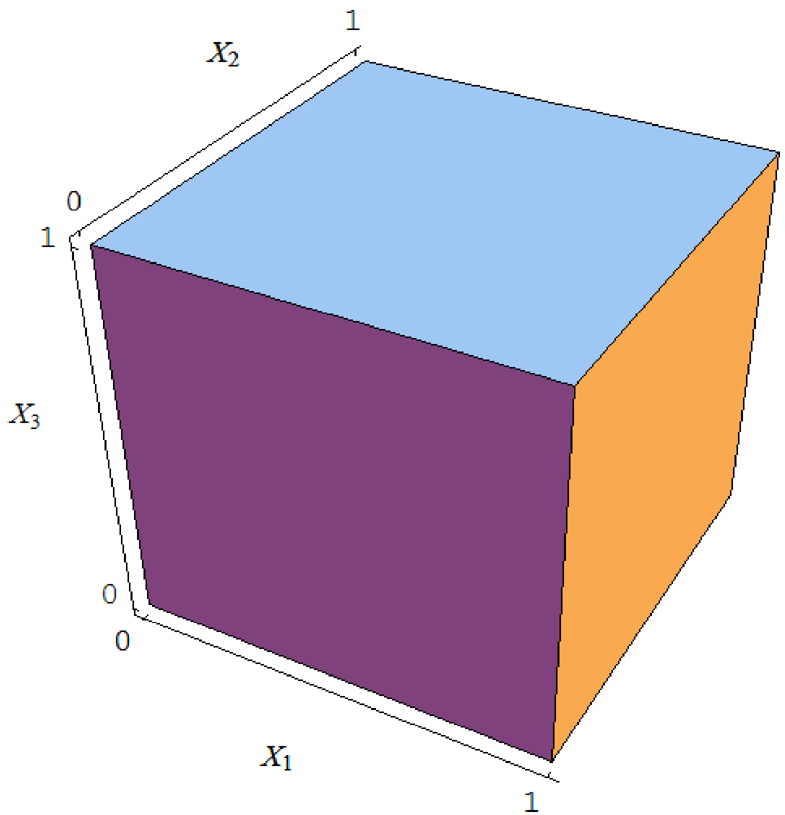
Screening methods aim to extract sensitivity information with a small number of sample points (low computational cost)

Fractional factorial sampling

The Full Factorial Design

A 2-Level Full Factorial Design for 3 parameters

X_1	X_2	X_3
1	1	1
–1	1	1
1	–1	1
–1	–1	1
1	1	–1
–1	1	–1
1	–1	–1
–1	–1	–1



Fractional factorial sampling

A disadvantage of a factorial design is the enormous number of simulations required

Using 2 levels: 10 parameters → $2^{10} = 1024$ simulations

20 parameters → more than a million!!

Solution

To select only a fraction of these simulations to generate a smaller design that can still produce valuable results

Fractional factorial sampling

A Fractional Factorial Design for 7 Parameters

X_1	X_2	$X_3 = X_1X_2$	X_4	$X_5 = X_1X_4$	$X_6 = X_2X_4$	$X_7 = X_1X_2X_4$
1	1	1	1	1	1	1
–1	1	–1	1	–1	1	–1
1	–1	–1	1	1	–1	–1
–1	–1	1	1	–1	–1	1
1	1	1	–1	–1	–1	–1
–1	1	–1	–1	1	–1	1
1	–1	–1	–1	–1	1	1
–1	–1	1	–1	1	1	–1

The method of Elementary Effects

(Morris, 1991)

The EE method can be seen as an extension of a derivative-based analysis

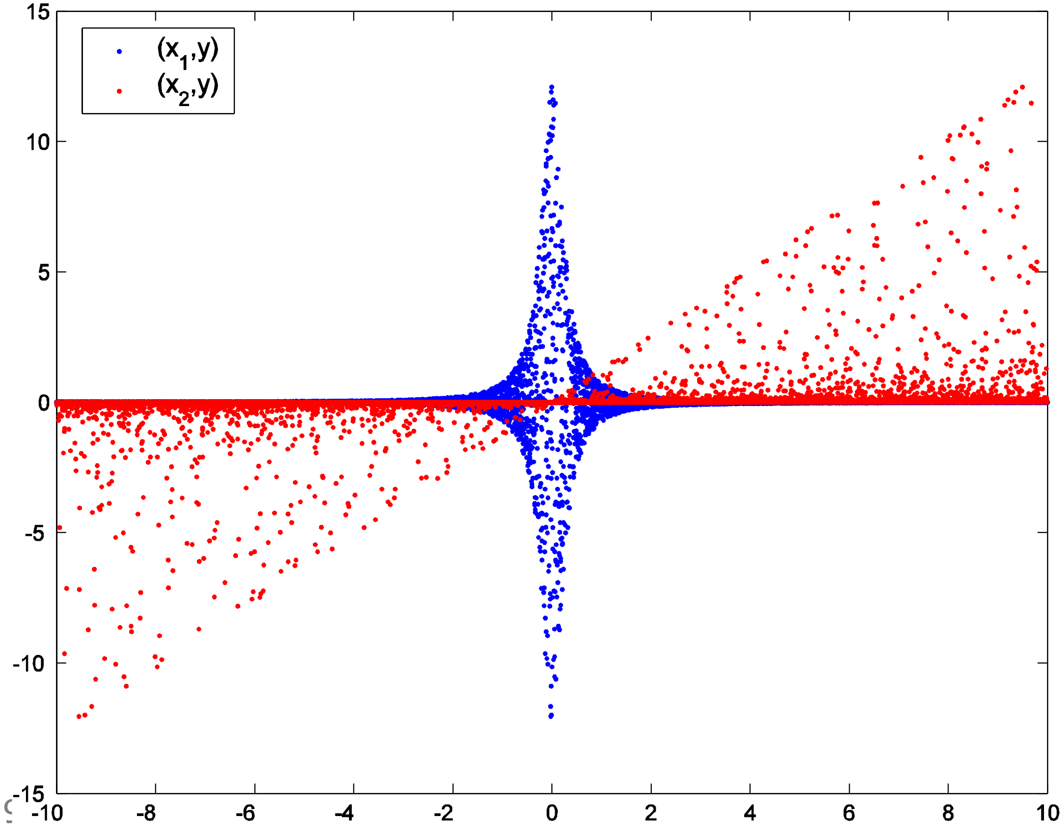
Problems related to a derivative-based approach:

- small perturbation around base values
- local measure

Derivative = 0 only implies that a factor is locally non influent

Example

$$y = x_2 * \left\{ \frac{\pi}{4} * \left[1 + (4 * x_1)^2 \right] \right\}^{-1}$$



Derivatives

$$\left. \frac{\partial y}{\partial x_1} \right|_{x_1 = x_2 = 0} = 0$$

$$\left. \frac{\partial y}{\partial x_2} \right|_{x_1 = x_2 = 0} = \frac{4}{\pi}$$

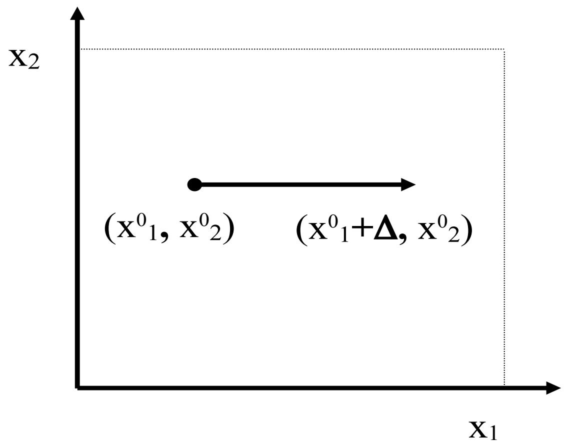
The method of Elementary Effects

(Morris, 1991)

Model $y = y(x_1,..,x_k)$

Elementary Effect for the i^th input factor in a point X^o

$$EE_i(x_1^0,\dots,x_k^0) = \frac{y(x_1^0,x_2^0,..,x_{i-1}^0,x_i^0+\Delta,x_{i+1}^0,..,x_k^0) - y(x_1^0,\dots,x_k^0)}{\Delta}$$



Δ is larger than in local methods

The method of Elementary Effects

Each input varies across l possible values (levels) within its range of variation

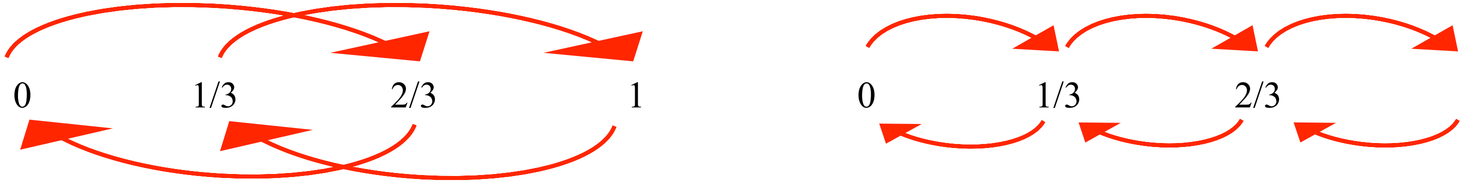
x_i U(0,1) $\quad l = 4 \rightarrow \quad l_1 = 0 \quad l_2 = 1/3 \quad l_3 = 2/3 \quad l_4 = 1$

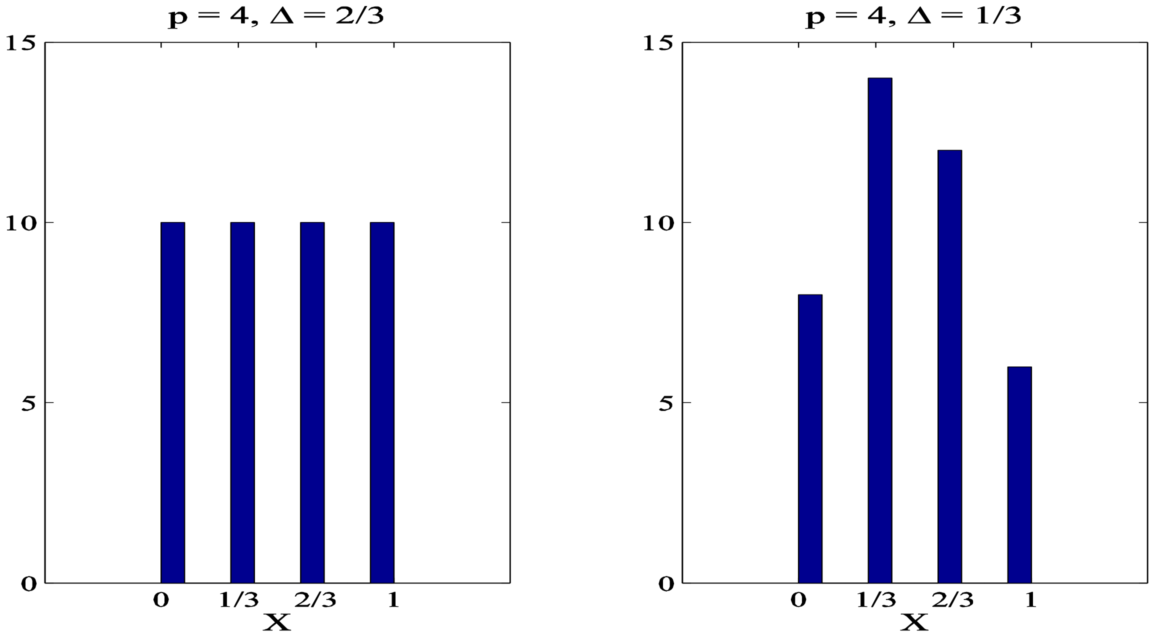
Distribution not uniform → levels correspond to distribution quantiles

The value of Δ (sampling step) is a function of l

Optimal choice for Δ is $\Delta = l \,/\, 2\,(l - 1)$

The method of Elementary Effects





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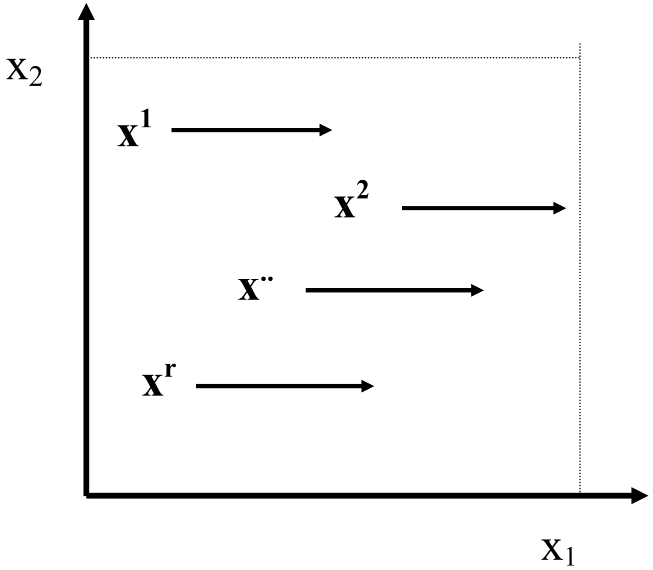
The method of Elementary Effects

The EE_i is still a local measure

Solution: take the average of several EE



r elem. effects EE^1_i EE^2_i … EE^r_i

are computed at $\mathbf{X}^1$, … , $\mathbf{X}^r$

and then averaged

Average of $|EE_i|$'s $\rightarrow$ $\mu^*(x_i)$

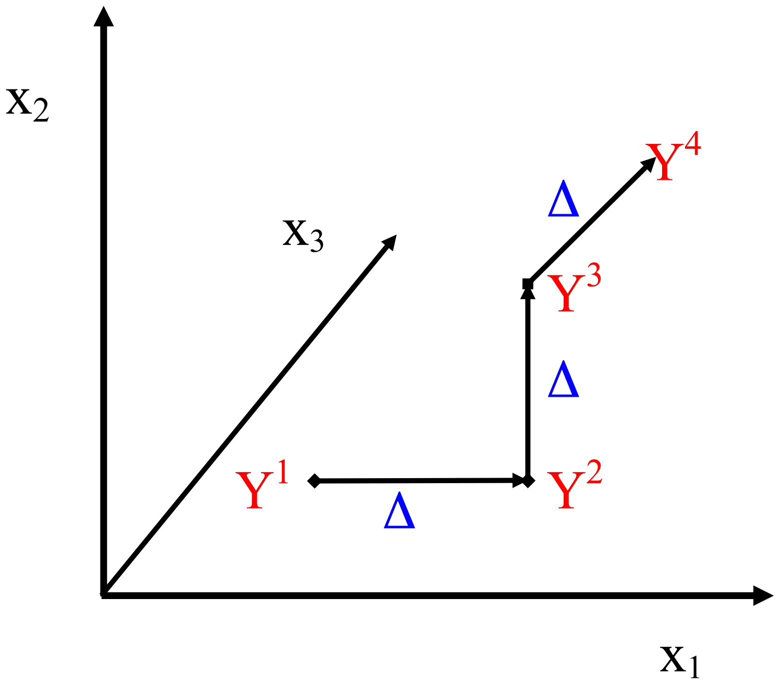
$\mu^*(x_i)$ is effective in identifying irrelevant inputs

Implementing the EE method

(Morris, 1991)

Goal: estimate r EE's per input

Morris builds *r* trajectories of (k+1) sample points each providing one EE per input



A trajectory of the EE design

$EE^1_1 = (Y^2 - Y^1) / \Delta$

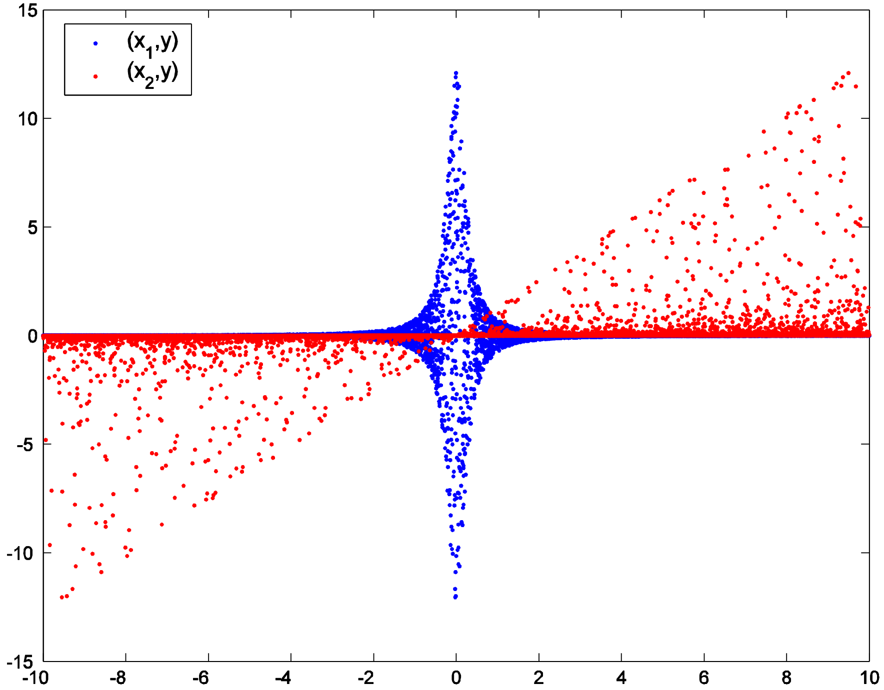
$EE^1_2 = (Y^3 - Y^2) / \Delta$

$EE^1_3 = (Y^4 - Y^3) / \Delta$

Total cost = r (k + 1)
r is in the range 4 -20

The example

$$y = x_2 * \left\{ \frac{\pi}{4} * \left[1 + (4 * x_1)^2 \right] \right\}^{-1}$$



Derivatives

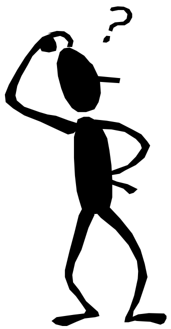
$$\frac{\partial y}{\partial x_1}(0,0) = 0 \quad \frac{\partial y}{\partial x_2}(0,0) = \frac{4}{\pi}$$

EE (l=4, r=10)

$$\mu^*(x_1) = 0.070$$

$$\mu^*(x_2) = 0.024$$

The method of Elementary Effects



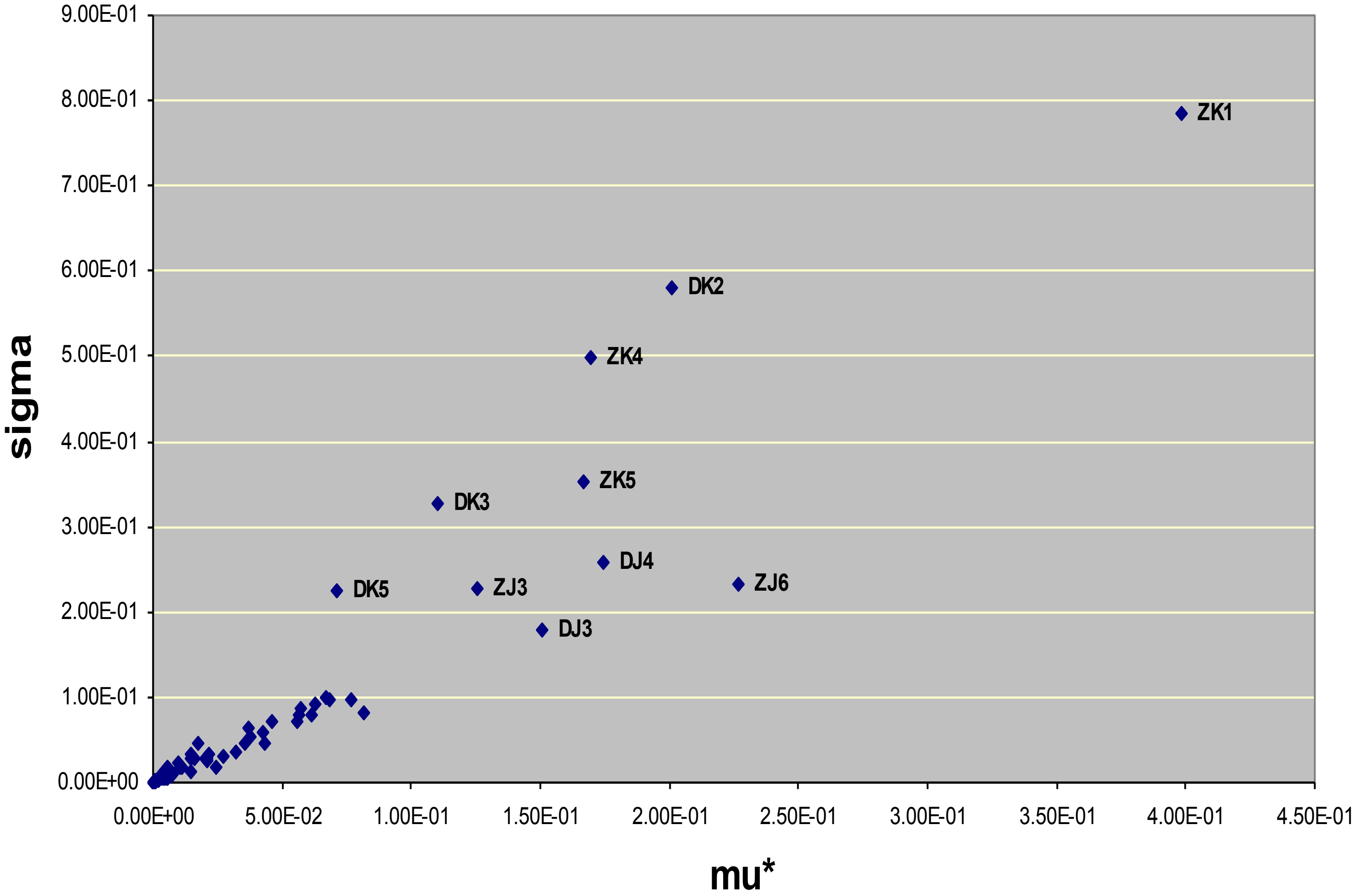
May I gain additional sensitivity information from the EE_i's?

What type of information I gain from the st. dev. σ of the EE_i's ?

σ is a measure of the sum of all interactions of x_i with other factors and of all its nonlinear effects

The method of Elementary Effects

A graphical representation of results



An analytical example (Morris, 1991)

$$y = \beta_0 + \sum_{i=1}^{20} \beta_i w_i + \sum_{i<j}^{10} \beta_{i,j} w_i w_j + \sum_{i<j<l}^{10} \beta_{i,j,l} w_i w_j w_l$$

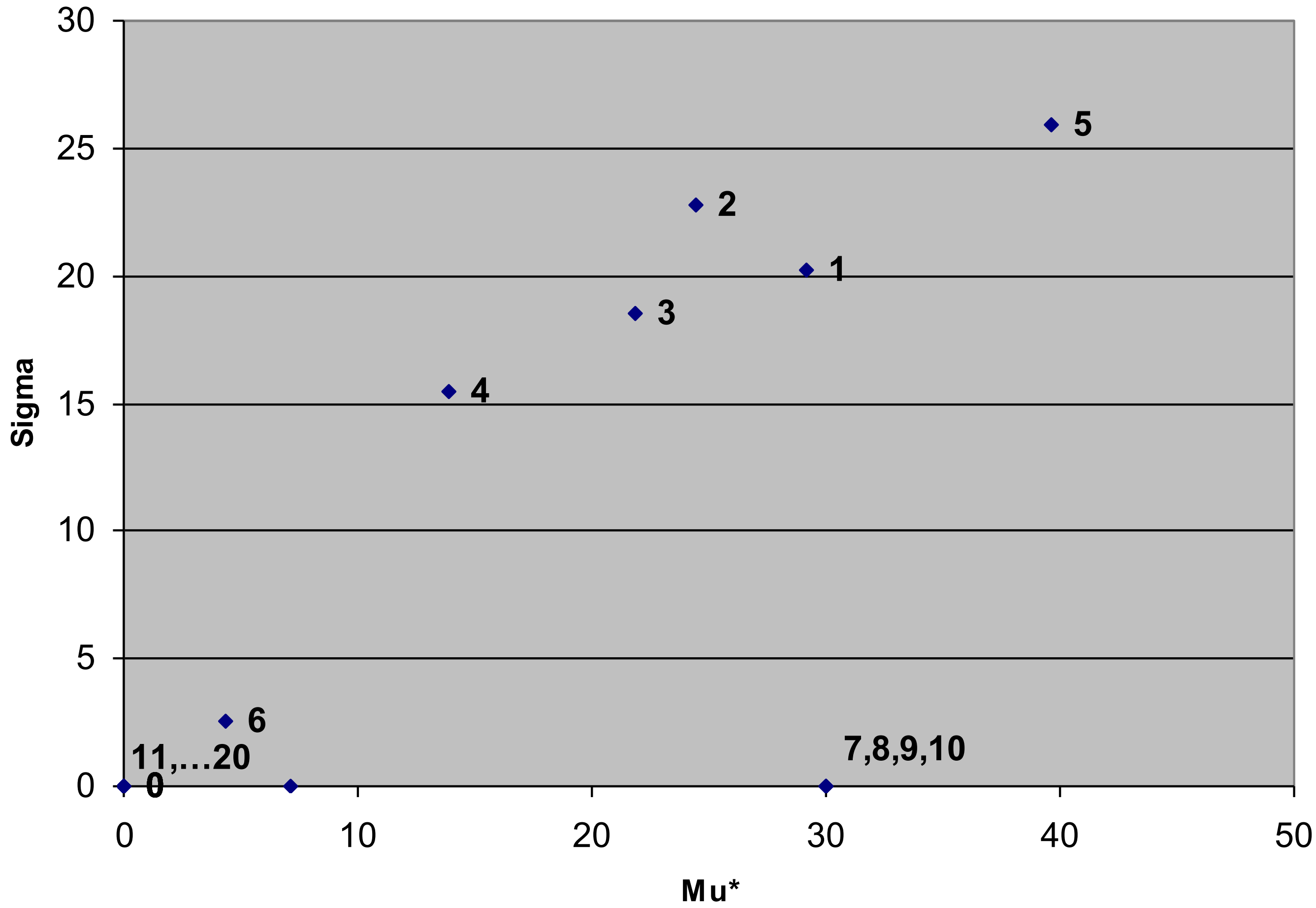
$w_1, w_2, \ldots, w_{10}$ in [-1,1]

$w_i = 2\,(x_i - 0.5)$ for $i \neq 3$
$w_3 = 2\,(1.1x_3/(x_3 + 0.\,1) - 0.5)$
$x_i \sim U[0;\ 1]$

$\beta_i = -15;$ $\quad i = 1, 2$
$\beta_{i,j} = 30;$ $\quad i, j = 3, 4$
$\beta_{i,j,l} = 10;$ $\quad i, j, l = 1, \ldots, 4$

Other coefficients ~ N(0,1)

An analytical example

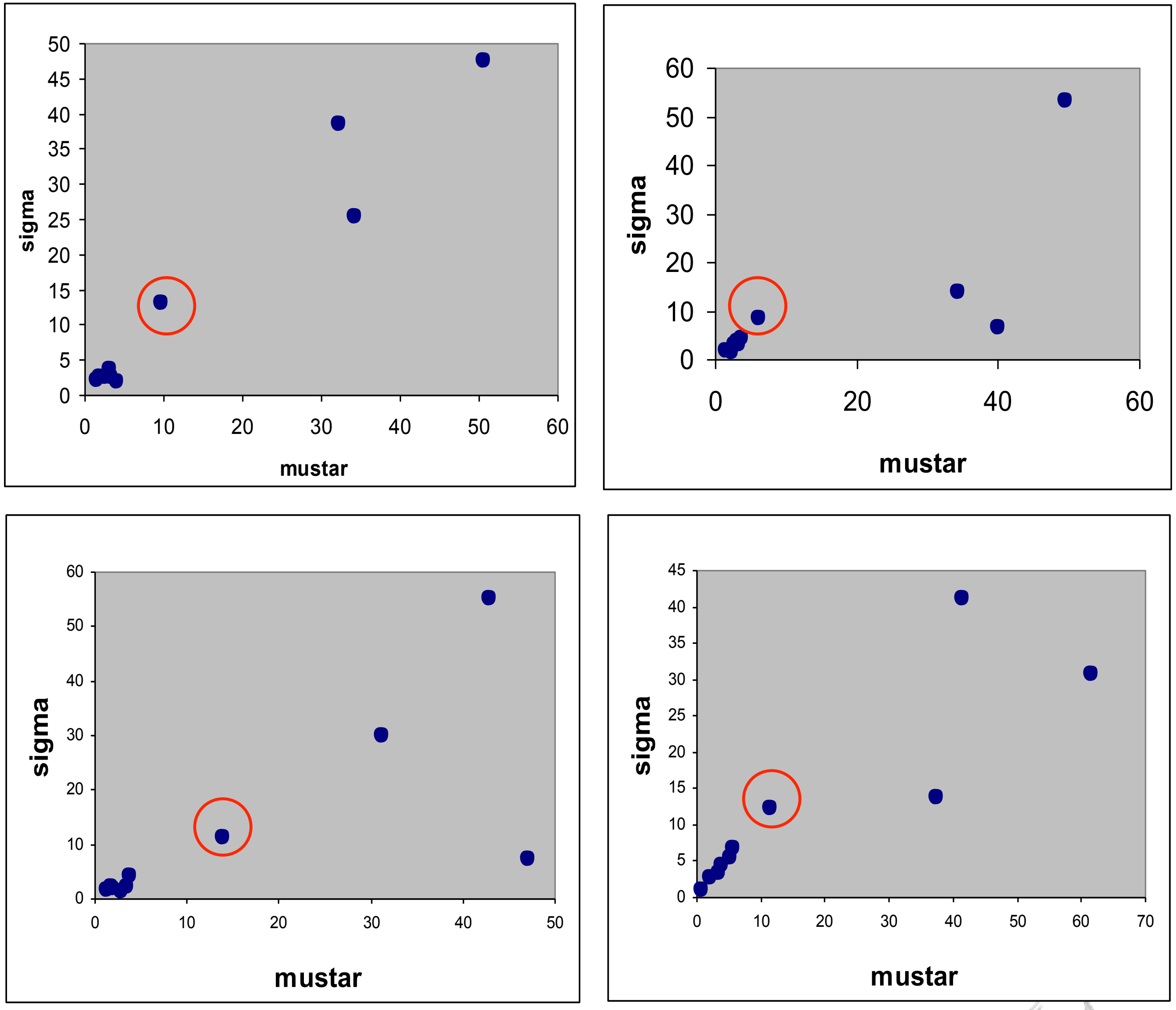


$r = 4$

$l = 4$

$\Delta = 2/3$

“missed” factors, r=4



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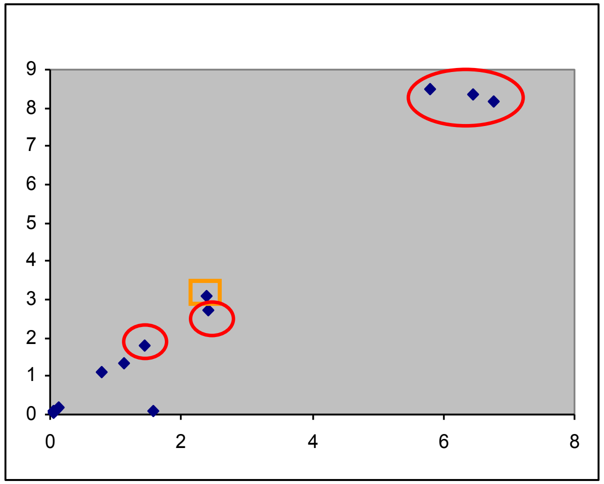
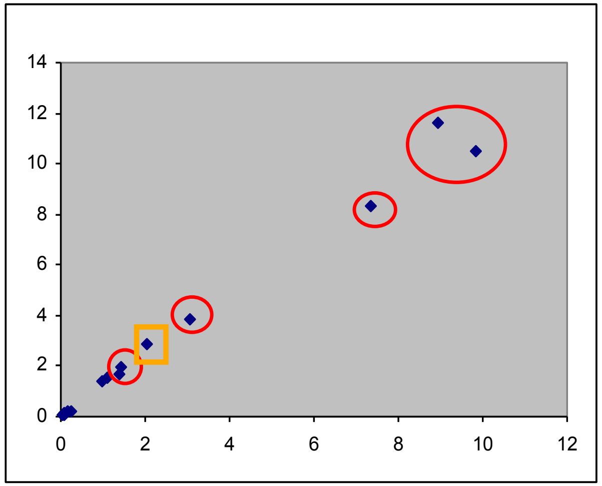
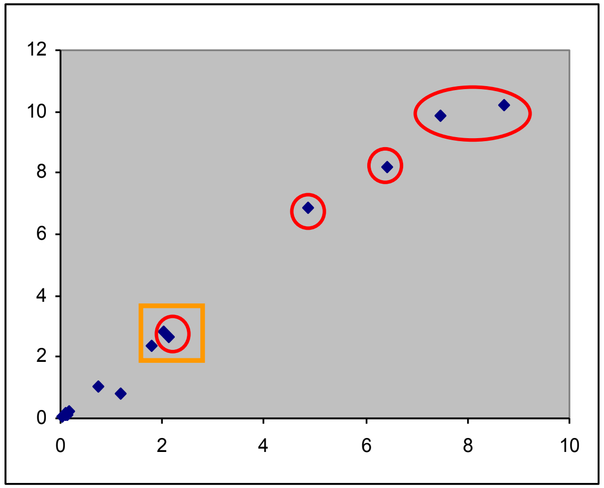
Test case 3



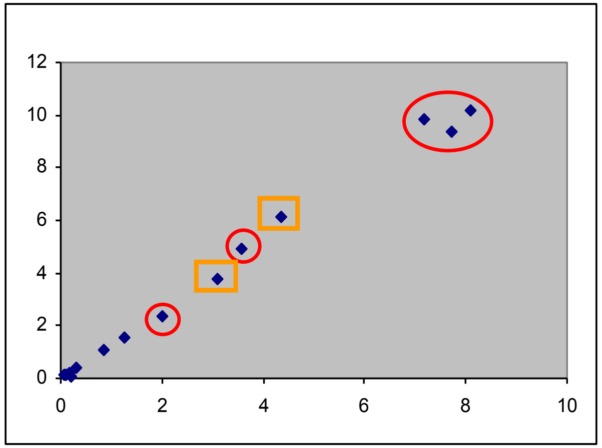
0 1 2 3 4 5 6 7 8 9
0 2 4 6 8



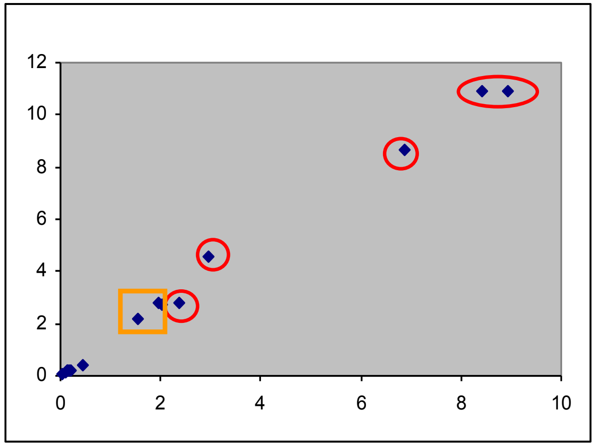
0 2 4 6 8 10 12 14
0 2 4 6 8 10 12



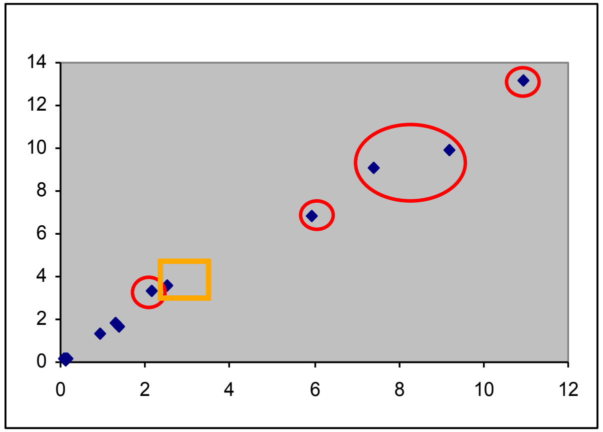
0 2 4 6 8 10 12
0 2 4 6 8 10



0 2 4 6 8 10 12
0 2 4 6 8 10



0 2 4 6 8 10 12
0 2 4 6 8 10



0 2 4 6 8 10 12 14
0 2 4 6 8 10 12

Identification ??

Ex-ante identification screening …

Lacks the collinearity component (only sensitivity effect).

Potential for very expensive ABM models ?

Identification

Covariance/autocovariance matrices among observed variables y_t:

$corr(y_t)$; $corr(y_t, y_{t-k})$;

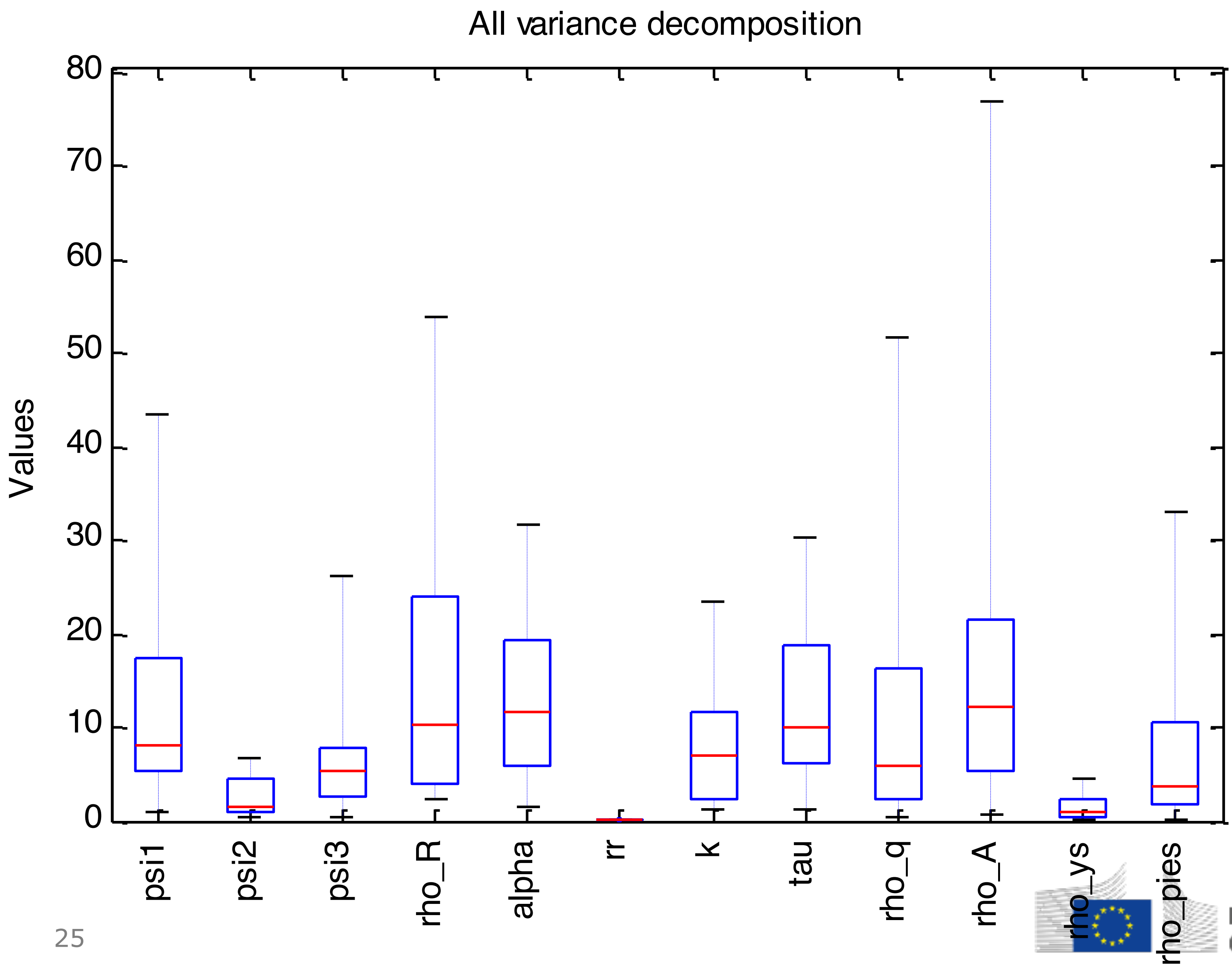
[nobs x (nobs-1)/2];

[nobs x nobs]

Identification

Under Gaussian hypotheses, first and second moments contain all information that can be used for the estimation of model parameters

Identification



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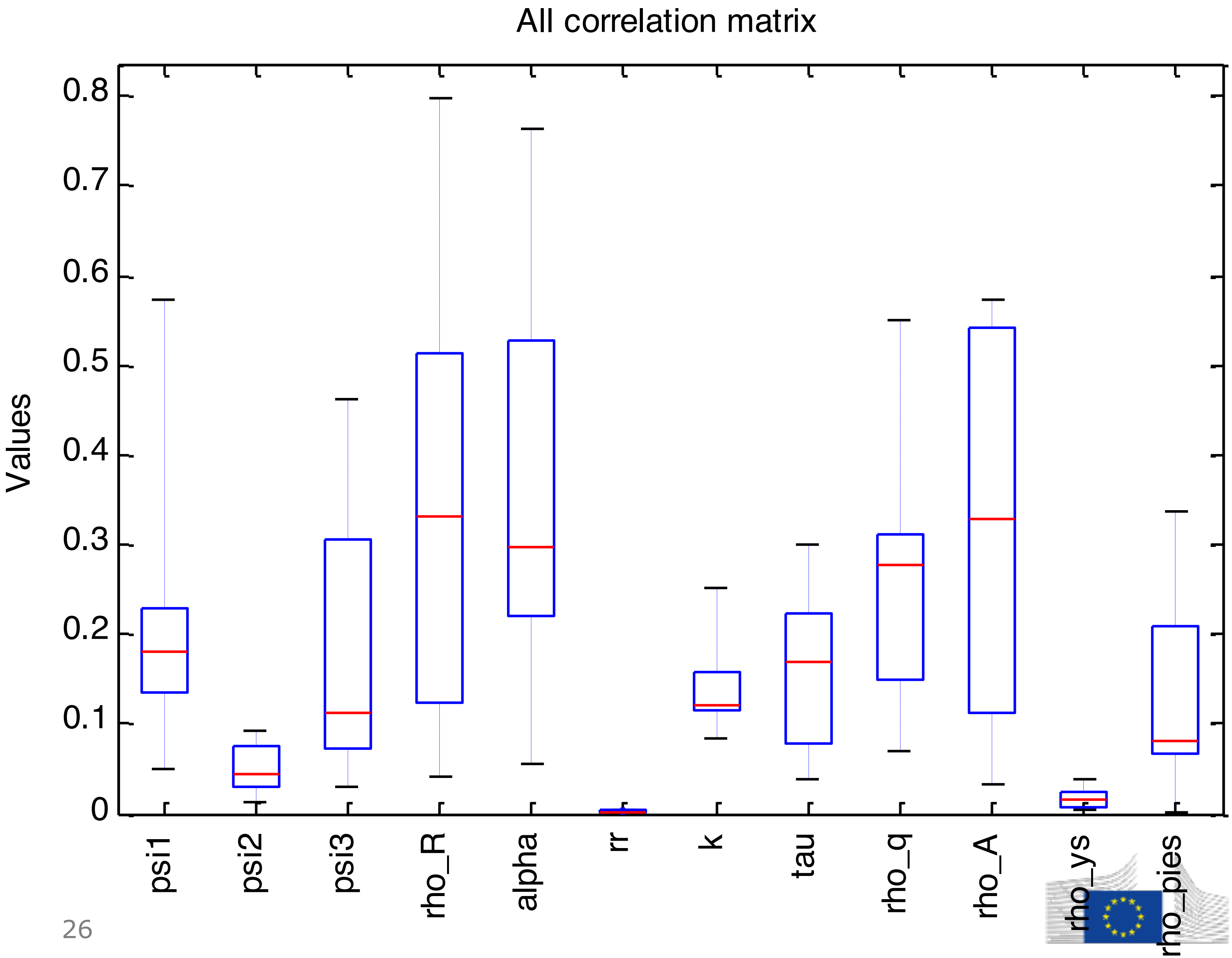
Identification



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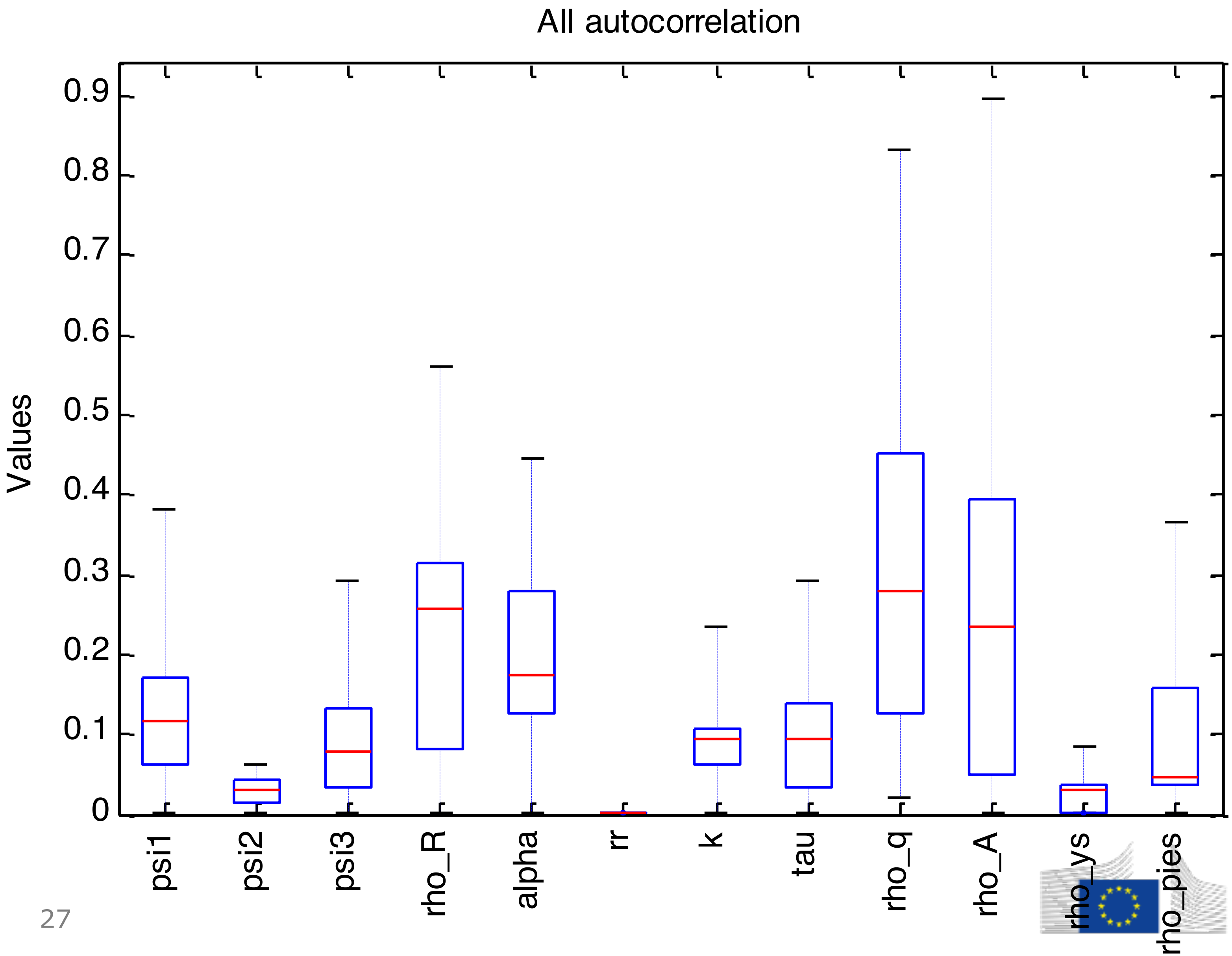
Identification



Conclusions

Screening designs are useful to “screen” a subset of few important input variables among the many contained in models

Often they rely on the assumption that the number of important parameters is small

Their feature is the low computational cost (low number of model evaluations)

Conclusions

Quick screening tests for ex-ante identification (sensitivity component) of DSGE models

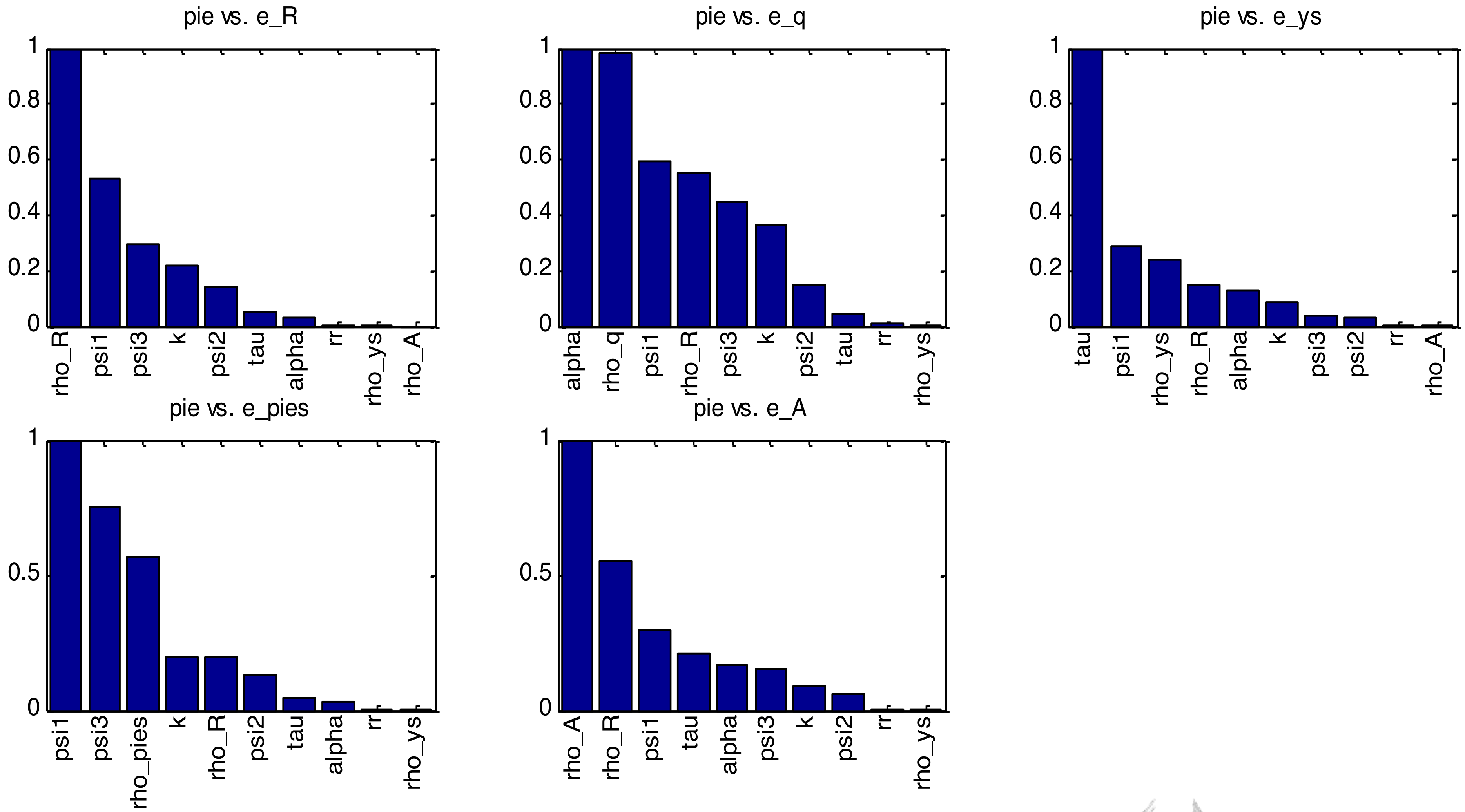
Mapping the reduced form of RE models

Relationship between the reduced form of a rational expectation model and the structural coefficients.

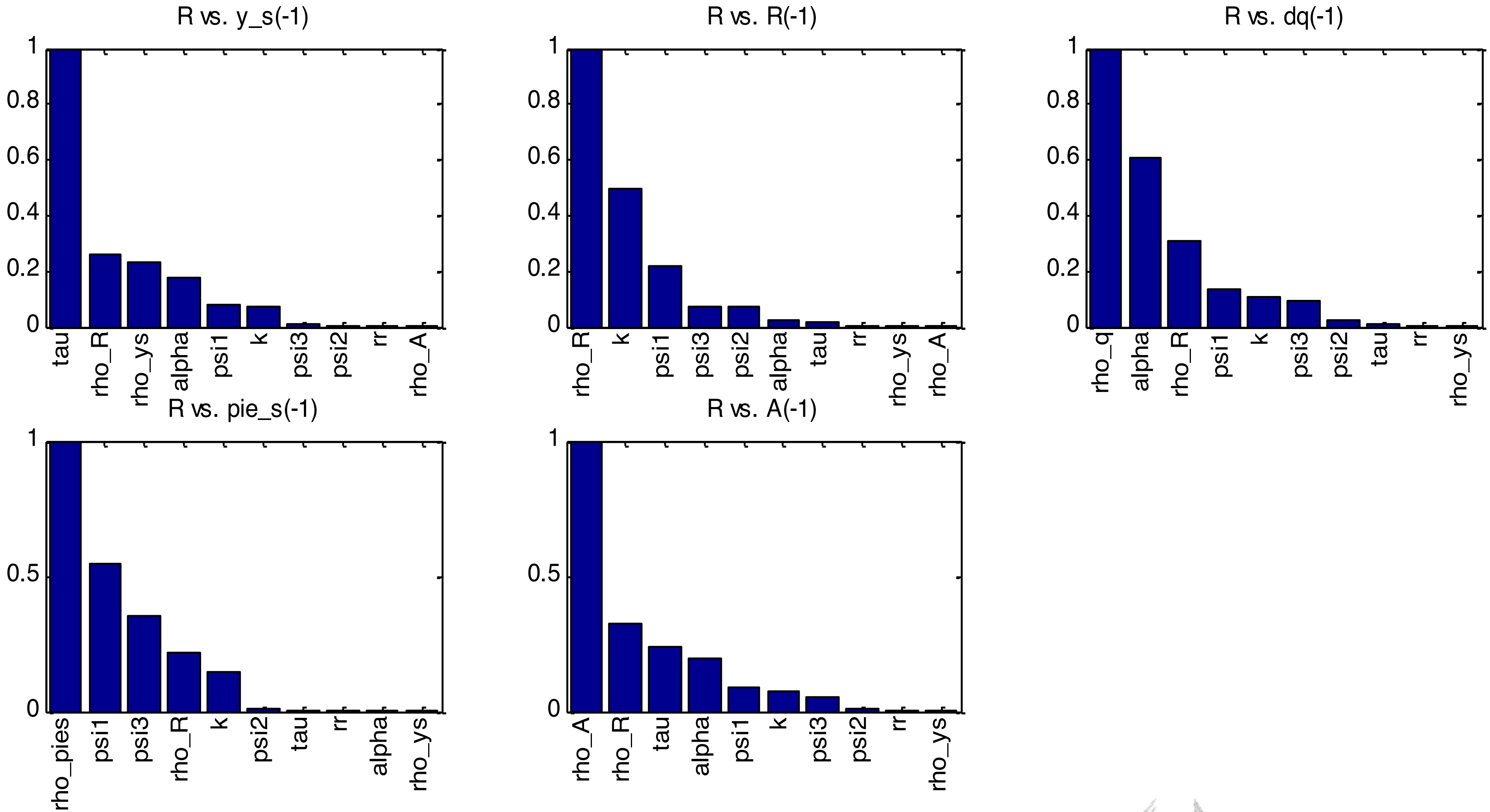
let the reduced form be
$y_t=Ty_{t-1}+Bu_t$,

'outputs' Y of our analysis will be the entries in the transition matrix $T(X_1,\ldots,X_k)$ or in the matrix $B(X_1,\ldots,X_k)$.

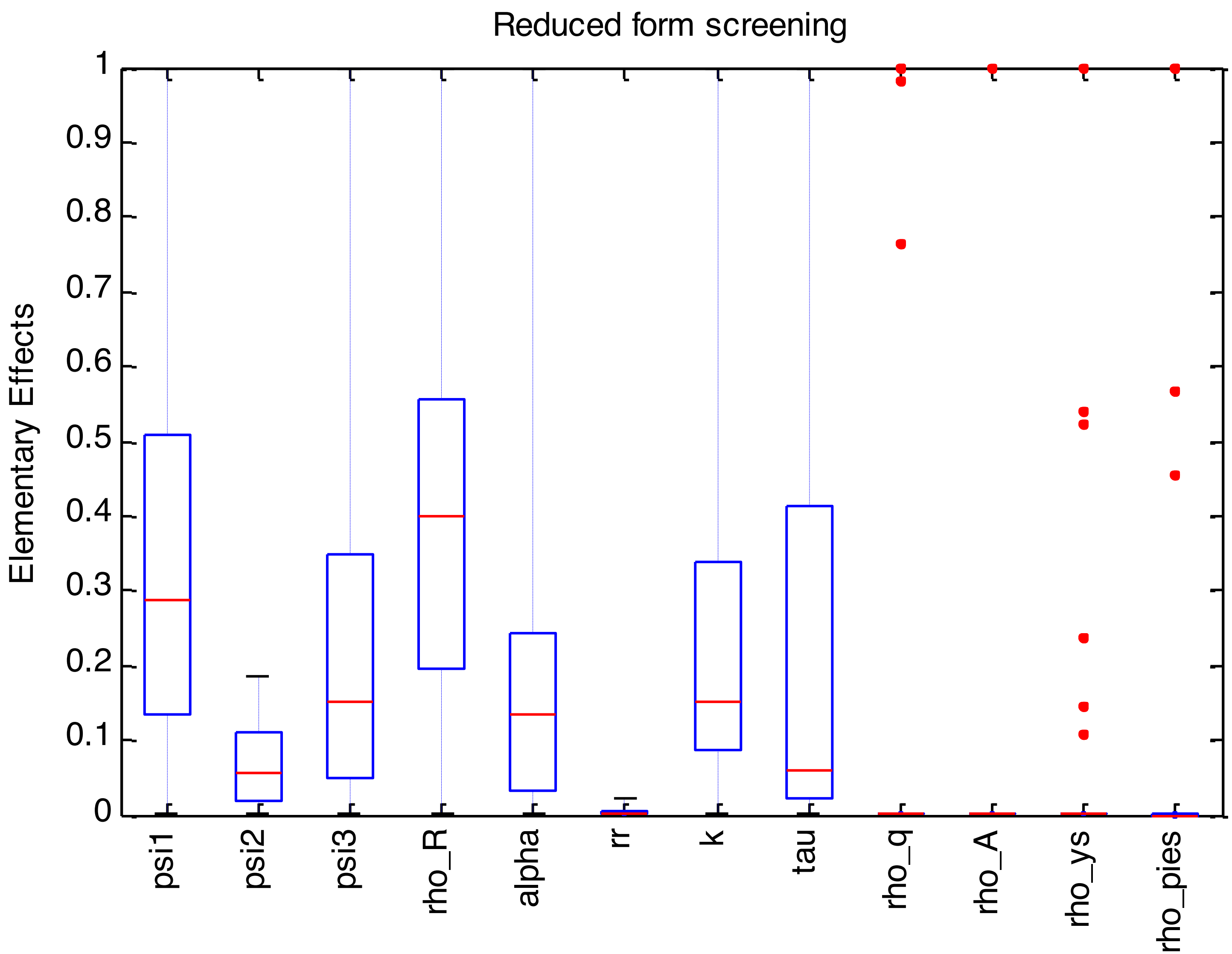
LS 2005: pie vs. shocks



LS 2005: pie vs. lags



LS 2005: overall picture



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