

# Modelling Occasionally Binding Constraints Using Regime-Switching

Andrew Binning<sup>1</sup>   Junior Maih<sup>2,3</sup>

<sup>1</sup>Modelling & Research Team, The New Zealand Treasury

<sup>2</sup>Monetary Policy Department, Norges Bank

<sup>3</sup>BI, Norwegian Business School

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# Motivation

- ▶ Many important problems in macroeconomics can be framed or interpreted as occasionally binding constraints (OBCs)
- ▶ Common examples include:
  - ▶ Zero lower bound
  - ▶ Downward nominal wage rigidities
  - ▶ Borrowing/collateral constraints
  - ▶ Irreversible investment
  - ▶ Cash in advance constraint

# Ways to Address the Problem I

- ▶ Eternally binding constraints (EBC)
  - ▶ Iacoviello (2005), Faia (2011)
- ▶ Smooth approximation of OBCs (penalty & linex functions)
  - ▶ Kim & Ruge-Murcia (2011), Den Haan & De Wind (2012), Brzoza-Brzezina et al. (2015)
- ▶ (Stochastic) Extended path
  - ▶ Adjemian & Juillard (2013), Coenen & Warne (2014)
- ▶ Anticipated shocks
  - ▶ Holden & Paetz (2012), Lindé et al. (2016)

# Ways to Address the Problem II

- ▶ Occbin/piecewise linear
  - ▶ Guerrieri & Iacoviello (2015), Kulish et al. (2014)
- ▶ Global/projections/grid methods
  - ▶ Christiano & Fisher (2000), Fernández-Villaverde et al. (2015), Judd et al. (2012)
- ▶ Regime-Switching
  - ▶ Benigno et al. (2015), Binning & Maih (2016)

# Why Regime Switching

- ▶ Lucas critique: ignoring (potential) changes in (policy) parameters can have severe consequences for forecasting and policy analysis.
- ▶ OBCs may depend on policy, eg ZLB, LTV ratio.
- ▶ Natural to think of past and expected future implementation of policy in terms of regimes governed by separate policy parameters.
- ▶ e.g. policy can determine when a constraint does and does not bind.
- ▶ Shock transmission mechanism and behavioral parameters can also change when the constraint is binding. e.g. agents take on more or less risk when a constraint binds, implies a change in relative risk aversion.

# Why Regime Switching

- ▶ The framework naturally lends itself to scenarios such as staying longer than required at the ZLB (e.g. forward guidance).
- ▶ Stable solution, simulations (generally) stable, e.g. no forward guidance puzzles or numerical “pathologies”.
- ▶ Solution methods and filters (estimation) easily extend to large models and higher order approximations.

# Overview/Plan

- ▶ Types of constraints and their recasting into a regime-switching framework
- ▶ The generic regime-switching model
- ▶ Applications
- ▶ Comparison with other methods

# Types of Constraints & Their Recasting into Regime-Switching



# The Game Plan

The ideal procedure:

- ▶ analytical
- ▶ handles large state spaces/models
- ▶ accurate
- ▶ fast

# The Game Plan

The ideal procedure:

- ▶ analytical
- ▶ handles large state spaces/models
- ▶ accurate
- ▶ fast

... does not exist

# The Game Plan

## Our poor-man strategy:

- ▶ Make the distinction between occasionally binding constraints that

- ▶ contain choice variables

$$A_t \leq B_t,$$

- ▶ do not contain choice variables

$$C_t = \max(D_t, F_t).$$

- ▶ Add transition probabilities
- ▶ Solve the resulting model with regime switches using the perturbation approach of Maih (2015)

# Complementary Slackness Type Constraints

- Constraints that involve choice variables

$$A_t \leq B_t,$$

- $A_t$  and/or  $B_t$  are (contain) choice variables.  
Kuhn-Tucker/complementary slackness conditions:

$$\lambda_t (A_t - B_t) = 0,$$

with

$$A_t < B_t, \quad \lambda_t = 0,$$

or

$$A_t = B_t, \quad \lambda_t > 0.$$

# Complementary Slackness Type Constraints

- ▶ Can approximate complementary slackness type conditions using regime-switching with a two state Markov chain
- ▶ Binding ( $B$ ) and non-binding ( $N$ ), and a regime-switching parameter  $\phi(s_t)$  with values

$$\phi(N) = 0, \quad \phi(B) = 1.$$

- ▶ The complementary slackness condition is replaced by

$$\phi(s_t)(A_t - B_t) + (1 - \phi(s_t))\lambda_t = 0.$$

- ▶ Benigno et al. (2015) use a similar approach

# Transition Probabilities

- ▶ Transition matrix

$$\mathbb{Q}_{t,t+1} = \begin{bmatrix} 1 - p_{NB,t} & p_{NB,t} \\ p_{BN,t} & 1 - p_{BN,t} \end{bmatrix},$$

- ▶ Transition probabilities with choice variables (complementary slackness)

$$p_{NB,t} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp(-\psi_{N,B}(A_t - B_t))},$$

$$p_{BN,t} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp(\psi_{B,N}\lambda_t)}.$$

# Min/Max Constraints

- ▶ Can approximate min/max type constraints using regime-switching with a two state Markov chain
- ▶ Binding ( $B$ ) and non-binding ( $N$ ), and a regime-switching parameter  $\phi(s_t)$  with values

$$\phi(N) = 0, \quad \phi(B) = 1.$$

- ▶ The constraint can be written as a simple min or max condition if it doesn't contain choice variables,

$$C_t = \max(D_t, F_t).$$

- ▶ While the min/max constraint is replaced by

$$C_t = \phi(s_t) D_t + (1 - \phi(s_t)) F_t.$$

- ▶ This implies  $C_t = D_t$  in the binding regime.

# Transition Probabilities

- ▶ Transition matrix

$$\mathbb{Q}_{t,t+1} = \begin{bmatrix} 1 - p_{NB,t} & p_{NB,t} \\ p_{BN,t} & 1 - p_{BN,t} \end{bmatrix},$$

- ▶ Transition probabilities with min/max condition

$$p_{NB,t} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp(\psi_{N,B}(F_t - D_t))},$$

$$p_{BN,t} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp(-\psi_{B,N}(F_t - D_t))}.$$



# The Generic Regime-Switching Model

# Generic Model

- History is made of regimes with distinctive properties.

$$E_t \sum_{r_{t+1}=1}^h p_{r_t, r_{t+1}}(\mathcal{I}_t) f_{r_t}(x_{t+1}(r_{t+1}), x_t(r_t), x_{t-1}, \theta_{r_t}, \theta_{r_{t+1}}, \eta_t) = 0,$$

- $E_t$  is the expectations operator and  $f_{r_t}$  is an  $n_f \times 1$  vector of possibly nonlinear functions of their arguments
- $r_t = 1, 2, \dots, h$  is the regime of a time  $t$ ,  $x_t$  is an  $n_x \times 1$  vector of all the endogenous variables and  $\eta_t$  is an  $n_\eta \times 1$  vector of shocks with  $\eta_t \sim N(0, I_{n_\eta})$
- $\theta_{r_t}$  is an  $n_\theta \times 1$  vector of parameter values in regime  $r_t$
- $p_{r_t, r_{t+1}}(\mathcal{I}_t)$  is transition probability of going from regime  $r_t$  to regime  $r_{t+1} = 1, 2, \dots, h$  and  $\sum_{r_{t+1}=1}^h p_{r_t, r_{t+1}}(\mathcal{I}_t) = 1$

# General Solution

- ▶ General solution takes the form

$$x_t(r_t) = \mathcal{T}^{r_t}(z_t),$$

- ▶ where the state vector is defined as follows

$$z_t \equiv \begin{bmatrix} x'_{t-1} & \sigma & \eta'_t \end{bmatrix}',$$

# Perturbation Solution

- ▶  $p$ th order perturbation solution

$$\begin{aligned}\mathcal{T}^{r_t}(z) \simeq \mathcal{T}^{r_t}(\bar{z}_{r_t}) + \mathcal{T}_z^{r_t}(z - \bar{z}_{r_t}) + \frac{1}{2!} \mathcal{T}_{zz}^{r_t}(z - \bar{z}_{r_t})^{\otimes 2} + \dots \\ \dots + \frac{1}{p!} \mathcal{T}_{z^{(p)}}^{r_t}(z - \bar{z}_{r_t})^{\otimes p}.\end{aligned}$$

# Applications

# Applications

- ▶ Zero Lower Bound
- ▶ Collateral/Borrowing constraints
- ▶ Downward nominal wage rigidities
- ▶ Irreversible investment(\*)
- ▶ Higher-order perturbations
- ▶ All in one

# Application 1: ZLB

# Zero Lower Bound

- ▶ Binning & Maih (2016)
- ▶ Canonical three equation NK DSGE model, habit, indexation, smoothing
- ▶ Representative household, firms and monetary authority



# Zero Lower Bound

- ▶ OBC on interest rate (ZLB)

$$R_t = \max(R_{ZLB}, R_t^*),$$

- ▶ Taylor rule

$$R_t^* = R_{t-1}^{*\rho_r} \left( R^* \left( \frac{\pi_t}{\pi} \right)^{\kappa_\pi} \left( \frac{\tilde{Y}_t}{\tilde{Y}_{t-1}} \right)^{\kappa_y} \right)^{1-\rho_r} \exp(\varepsilon_{R,t}),$$

- ▶ States

$$s_t = Z, N.$$

- ▶ Regime specific parameter

$$\mathbf{z}(N) = 0, \quad \mathbf{z}(Z) = 1,$$

- ▶ Regime-Switching approximation of the OBC

$$R_t = \mathbf{z}(s_t) R_{ZLB} + (1 - \mathbf{z}(s_t)) R_t^*.$$

# Zero Lower Bound

- ▶ Transition matrix

$$\mathbb{Q}_{t,t+1} = \begin{bmatrix} 1 - p_{NZ,t} & p_{NZ,t} \\ p_{ZN,t} & 1 - p_{ZN,t} \end{bmatrix},$$

- ▶ Endogenous transition probabilities

$$p_{NZ,t} = \frac{\theta_{N,Z}}{\theta_{N,Z} + \exp(\psi_{N,Z}(R_t^* - R_{ZLB}))},$$

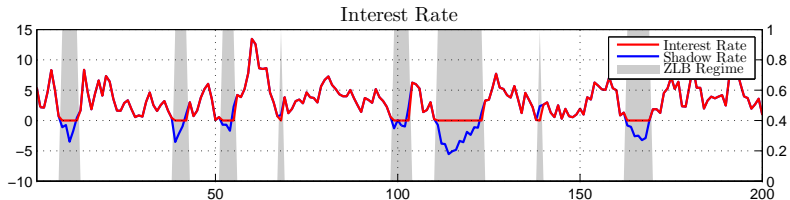
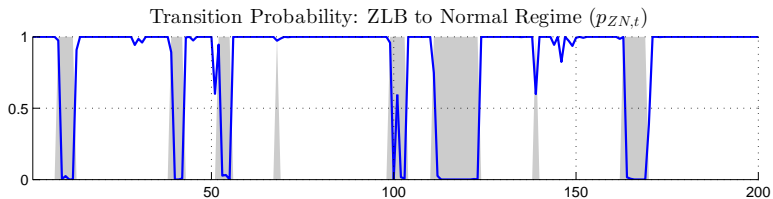
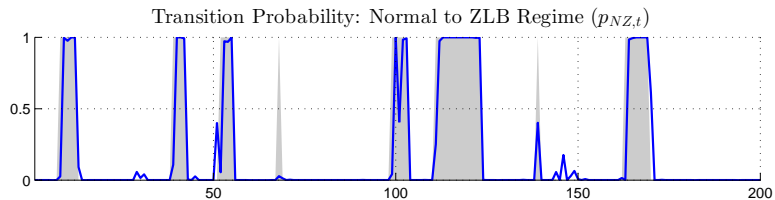
$$p_{ZN,t} = \frac{\theta_{Z,N}}{\theta_{Z,N} + \exp(-\psi_{Z,N}(R_t^* - R_{ZLB}))}.$$

- ▶ Regime specific steady states

$$R(Z) = R_{ZLB}, \quad R(N) = R^* = \frac{\pi_t \mu_t}{\beta}, \text{ where } R_{ZLB} < R^*.$$

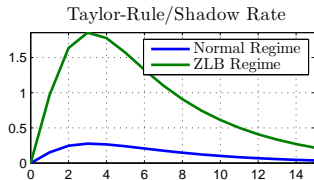
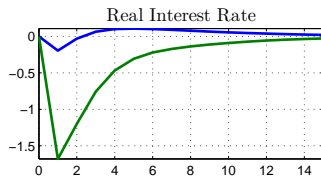
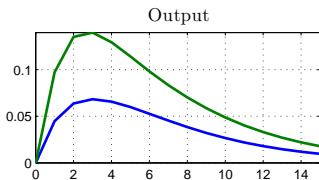
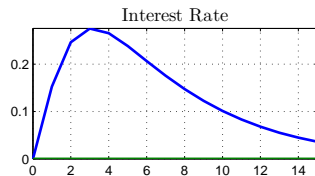
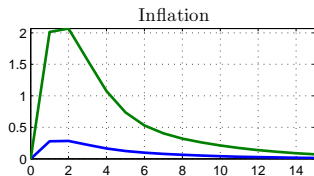
$$d(Z) = \frac{R^*}{R_{ZLB}} > 1, \quad d(N) = 1.$$

# Zero Lower Bound: Transition Probabilities



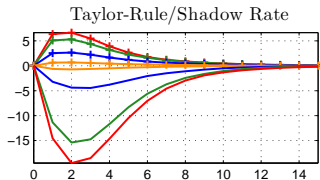
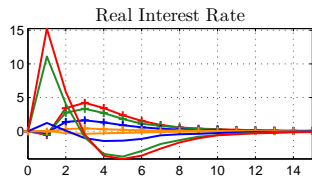
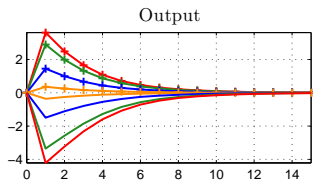
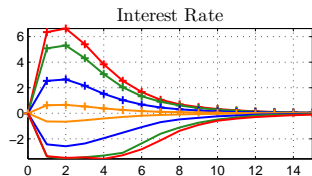
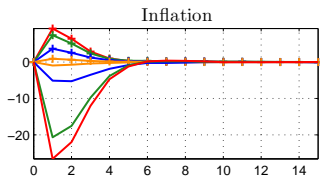
# Zero Lower Bound

## Impulse Responses (Regime Specific): Consumption Preference Shock



# Zero Lower Bound

## Impulse Responses (GIRF): Technology Shock



# Application 2:

## Collateral/Borrowing Constraints

# Collateral/Borrowing Constraints

- ▶ Iacoviello (2005)
- ▶ Households, Entrepreneurs, Firms, Monetary Authority
- ▶ Entrepreneurs impatient relative to households
- ▶ Entrepreneurs borrow from households subject to an occasionally binding collateral constraint

# Collateral/Borrowing Constraints

- ▶ Occasionally binding collateral constraint

$$B_t \leq E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\},$$

- ▶ Complementary slackness condition

$$\Omega_t \left( B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} \right) = 0,$$

- ▶ with

$$B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} = 0, \quad \Omega_t > 0,$$

- ▶ or

$$B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} < 0, \quad \Omega_t = 0.$$



# Collateral/Borrowing Constraints

- ▶ introduce regime specific parameter

$$\mathbf{o}(N) = 0, \quad \mathbf{o}(B) = 1$$

- ▶ Replace complementary slackness condition with

$$\mathbf{o}(s_t) \left( B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\} \right) + (1 - \mathbf{o}(s_t)) \Omega_t = 0,$$

- ▶ Transition matrix

$$\mathbb{Q}_{t,t+1} = \begin{bmatrix} 1 - p_{NB,t} & p_{NB,t} \\ p_{BN,t} & 1 - p_{BN,t} \end{bmatrix},$$

- ▶ Transition probabilities

$$p_{NB,t} = \frac{\theta_{N,B}}{\theta_{N,B} + \exp(-\psi_{N,B} B_t^*)}, \quad p_{BN,t} = \frac{\theta_{B,N}}{\theta_{B,N} + \exp(\psi_{B,N} \hat{\Omega}_t)},$$

# Collateral/Borrowing Constraints

- ▶ Leverage

$$B_t^* = B_t - E_t \left\{ m \frac{Q_{t+1} H_t \pi_{t+1}}{R_t} \right\},$$

- ▶ Multiplier

$$\hat{\Omega}_t,$$

# Collateral/Borrowing Constraints

- Housing steady state

$$\frac{H_t}{Y_t} = \left( \frac{\varepsilon - 1}{\varepsilon} \right) \left( \frac{\gamma \nu}{Q_t (1 - \gamma - (\beta - \gamma) m)} \right),$$

$$\frac{H_t}{Y_t} = \left( \frac{\varepsilon - 1}{\varepsilon} \right) \left( \frac{\gamma \nu}{(1 - \gamma) Q_t} \right),$$

$$(1 + \tau(s_t)) \frac{Q_t}{C_t} = E_t \left\{ \frac{\gamma}{C_{t+1}} \left( \nu M C_{t+1} \frac{Y_{t+1}}{H_t} + (1 + \tau(s_{t+1})) Q_{t+1} \right) + \dots \right\}.$$

$$\tau(s_t) = \mathbf{o}(s_t) m \left( \frac{\beta - \gamma}{1 - \gamma} \right),$$

- Impatient households

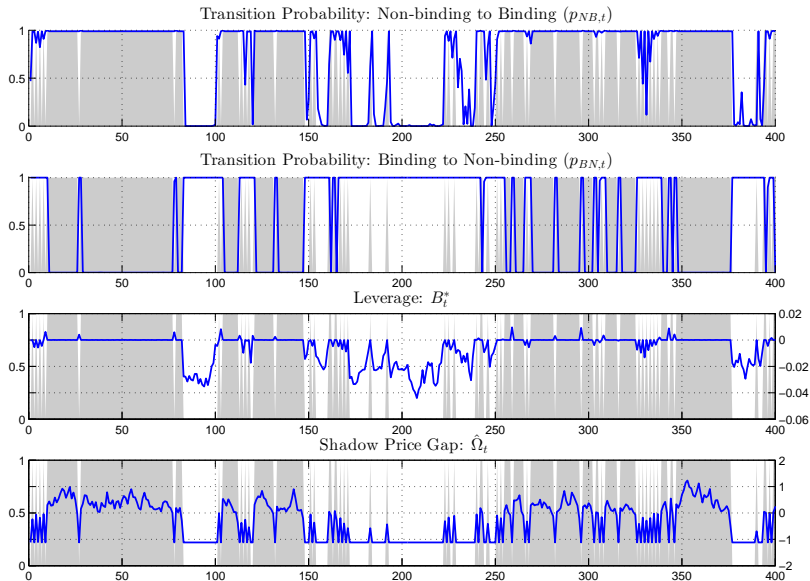
$$\frac{1}{C_t} = E_t \left\{ \gamma \frac{R_t^*}{\pi_{t+1} C_{t+1}} \right\} + \Omega_t R_t,$$

$$R_t^* = R_t \psi_1,$$

$$\psi_1 = \mathbf{o}(s_t) + (1 - \mathbf{o}(s_t)) \beta / \gamma,$$

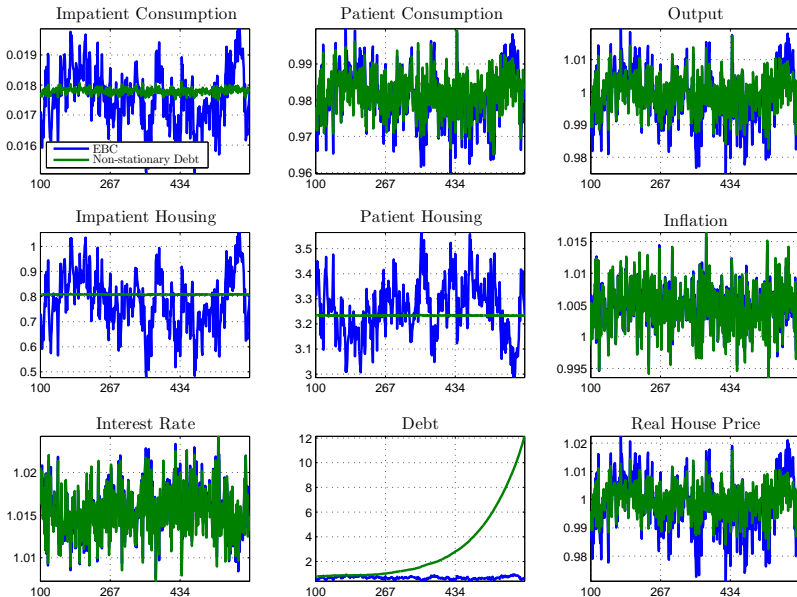
# Collateral/Borrowing Constraints

## Transition Probabilities & Their Determinants



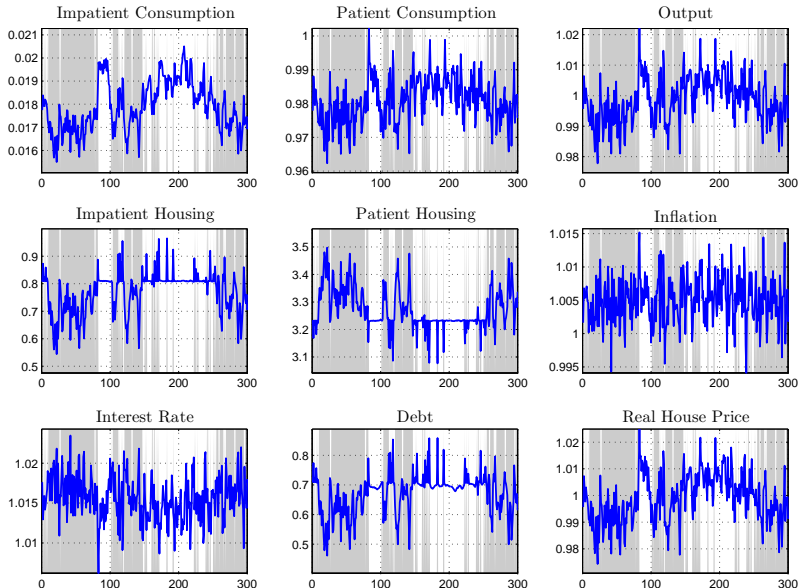
# Collateral/Borrowing Constraints

## Eternally Binding Constraint vs Non-stationary Debt



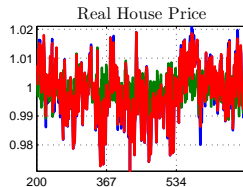
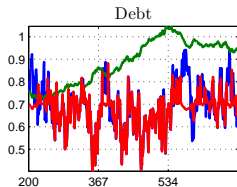
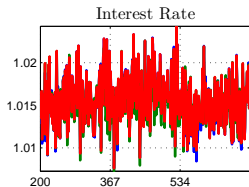
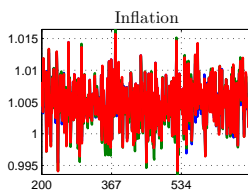
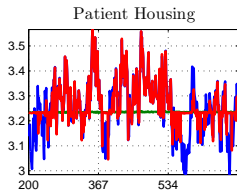
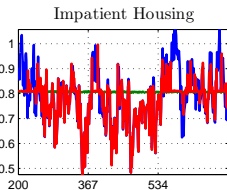
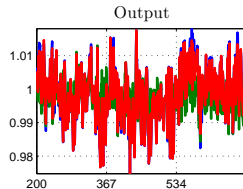
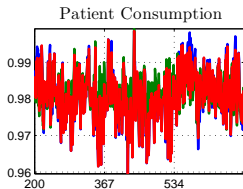
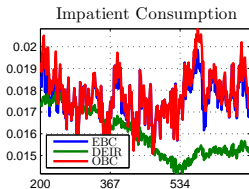
# Collateral/Borrowing Constraints

## Key Model Variables Against Regimes



# Collateral/Borrowing Constraints

## A Comparison: EBC, DEIR & OBC



# Collateral/Borrowing Constraints

## Optimal LTV rule

- ▶ State dependent LTV

$$m_t = m \left( \frac{Y_t}{Y} \right)^{\phi_Y} \left( \frac{Q_t}{Q} \right)^{\phi_Q} \left( \frac{B_t}{B} \right)^{\phi_B}.$$

- ▶ which implies

$$B_t \leq m_t \frac{Q_{t+1} H_t \pi_{t+1}}{R_t},$$

$$(1 + \tau(s_t)) \frac{Q_t}{C_t} = E_t \left\{ \frac{\gamma}{C_{t+1}} \left( \nu M C_{t+1} \frac{Y_{t+1}}{H_t} + (1 + \tau(s_{t+1})) Q_{t+1} \right) + \dots \right\}.$$
$$\dots + \Omega_t m_t Q_{t+1} \pi_{t+1}$$



# Collateral/Borrowing Constraints

## Optimal LTV rule

- ▶ Implementable rules imply loss function

$$\tilde{\mathbb{L}} = \sum_{t=0}^T [\mathbb{L}_t + \mathbf{I}(m_t) \mathbb{P}],$$

$$\mathbb{L}_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \hat{\pi}_t^2 + \omega_Y \hat{Y}_t^2 + \omega_B \hat{B}_t^2 \right] \right\},$$

- ▶ where

$$\mathbf{I}(m_t) = \begin{cases} 1 & \text{if } m_t < 0, \\ 1 & \text{if } m_t > 1, \\ 0 & \text{if } 0 \leq m_t \leq 1. \end{cases}$$

- ▶  $T = 10,000$ ,  $\mathbb{P} = 1000$ ,  $\omega_Y = 1$   $\omega_B = 0.25$

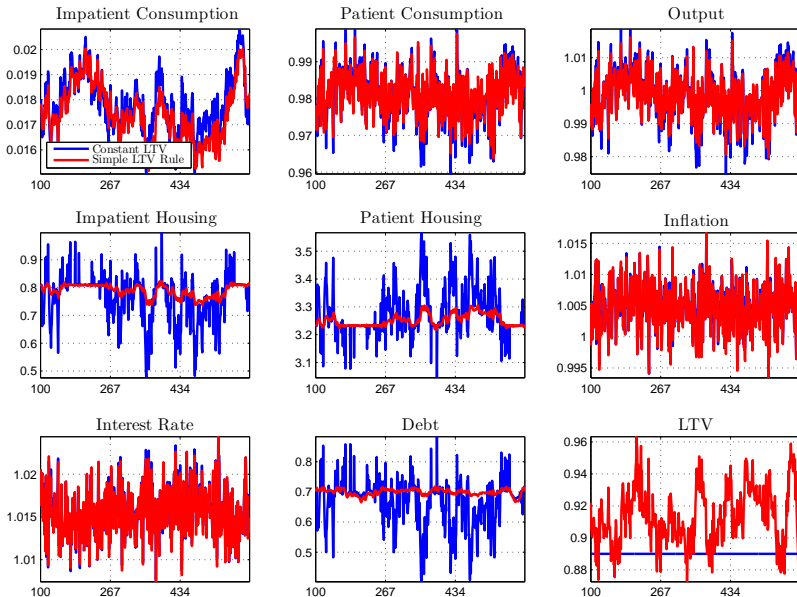
# Optimal LTV Rule

$$m_t = m \left( \frac{Y_t}{Y} \right)^{\phi_Y} \left( \frac{Q_t}{Q} \right)^{\phi_Q} \left( \frac{B_t}{B} \right)^{\phi_B}.$$

- ▶ The parameters that minimize the loss function are  $\phi_Y = 1.4279$ ,  $\phi_Q = -2.9293$  and  $\phi_B = -1.2839$ .

# Collateral/Borrowing Constraints

A Constant LTV Ratio vs. A Simple Implementable LTV Rule



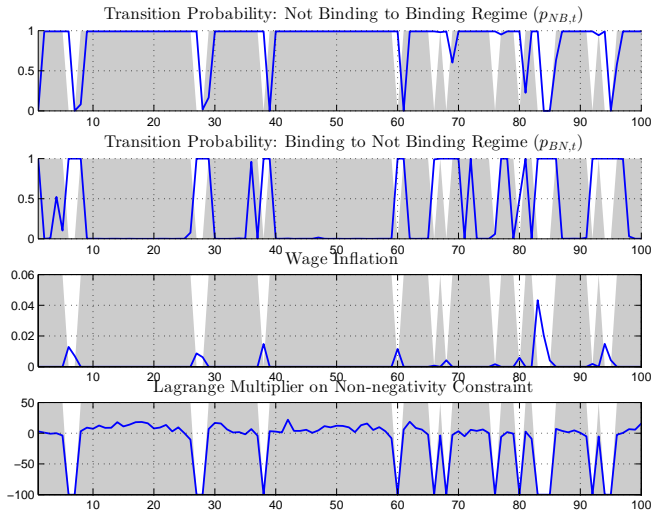
# Application 3: Downward Nominal Wage Rigidities

# Downward Nominal Wage Rigidities

- ▶ Basic NK DSGE model with sticky wages
- ▶ Introduce a constraint that prevents wages from falling
- ▶ Amano & Gnocchi (2017)

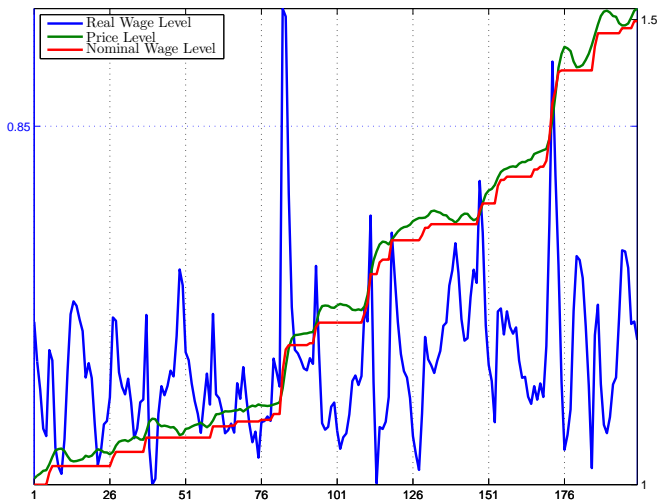
# Downward Nominal Wage Rigidities

## Transition Probabilities



# Downward Nominal Wage Rigidities

Evolution of the Wage and Price Level



# Downward Nominal Wage Rigidities

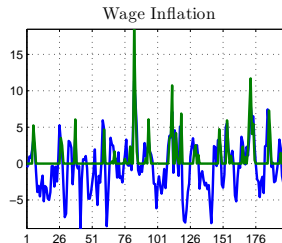
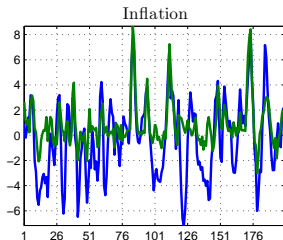
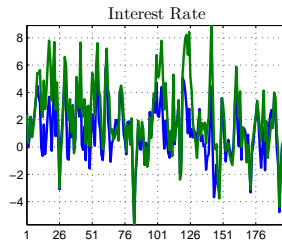
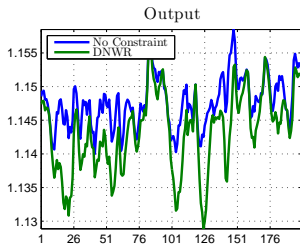
## Evolution of the Wage and Price Level

- ▶ In a model with an inflation target of zero, downward nominal wage rigidity introduces positive wage and price inflation.



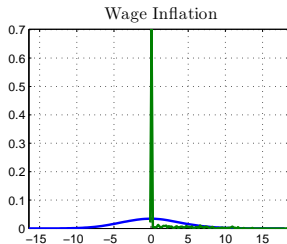
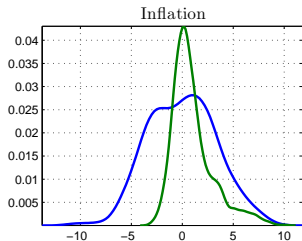
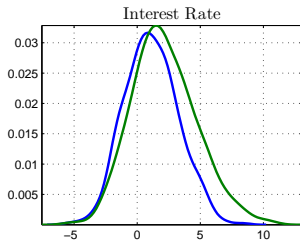
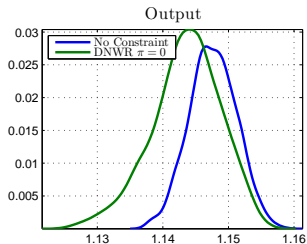
# Downward Nominal Wage Rigidities

Model Simulations: No Constraint vs. Downward Nominal Wage Rigidity



# Downward Nominal Wage Rigidities

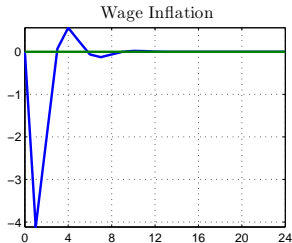
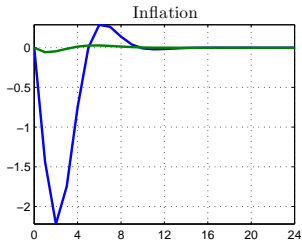
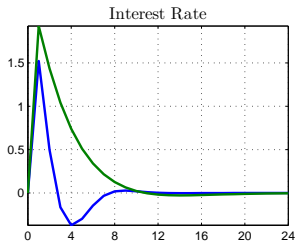
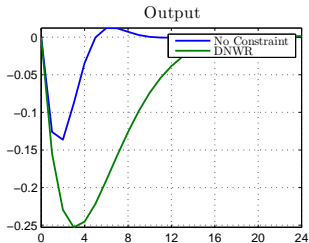
Distributions: No Constraint vs. Downward Nominal Wage Rigidity



# Downward Nominal Wage Rigidities

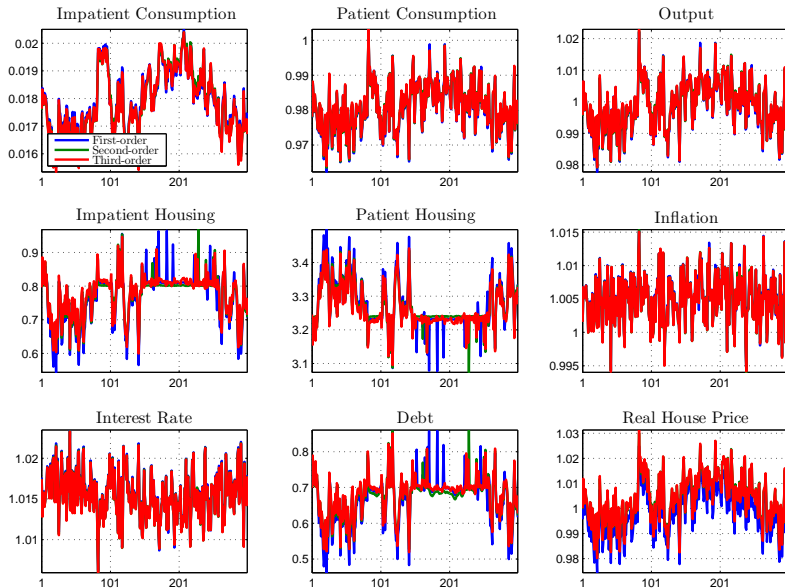
Regime Specific IRFs: (Contractionary) Monetary Policy Shock

irreversInvest

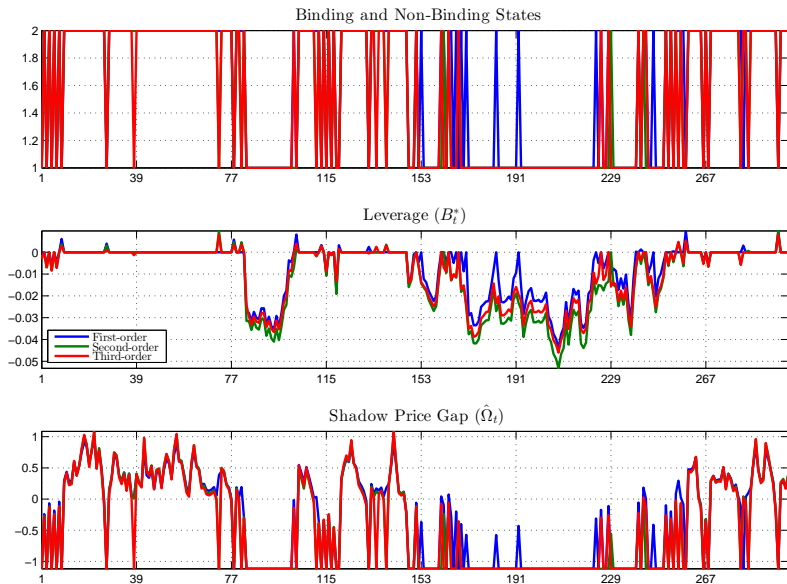


# Higher-order expansions

# Higher-order Approximations (Iacoviello 2005)

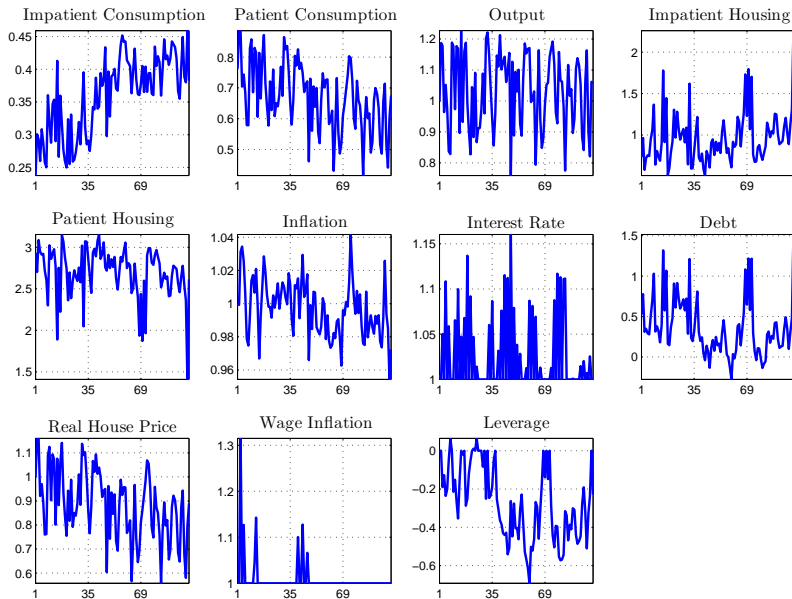


# Higher-order Approximations (Iacoviello 2005)



All in

# All In (Iacoviello 2005 with ZLB, DNWR & Collateral Constraints)

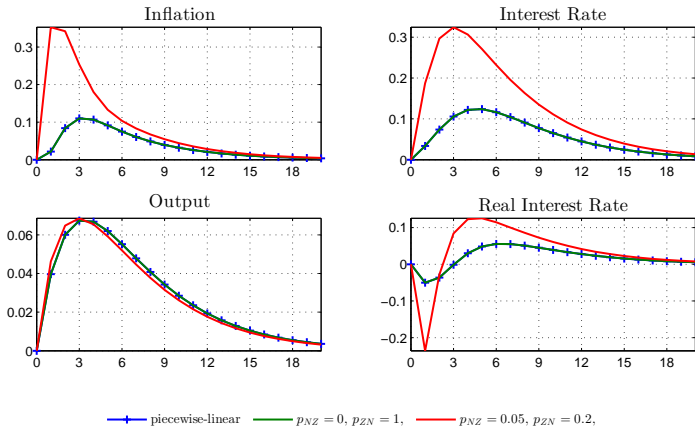




# Comparison with alternative approaches

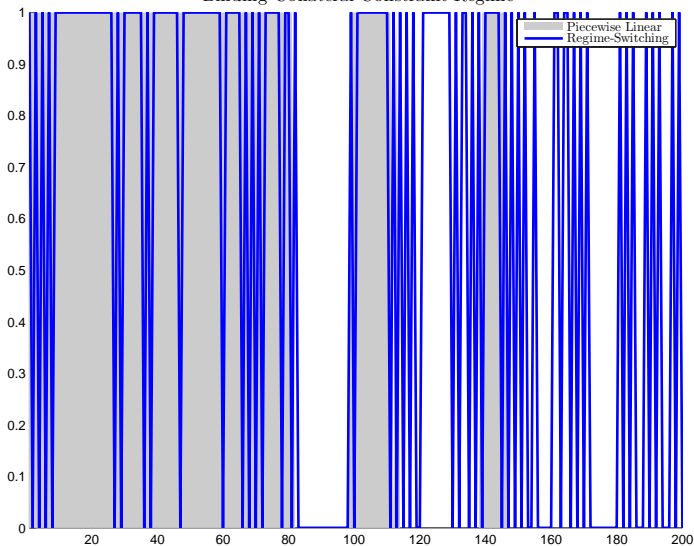
# Regime switching vs Piecewise linear

ZLB model: Consumption Preference Shock in Normal Times



# Regime switching vs Piecewise linear

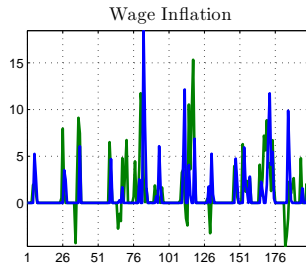
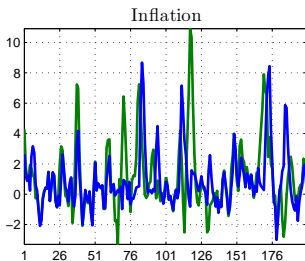
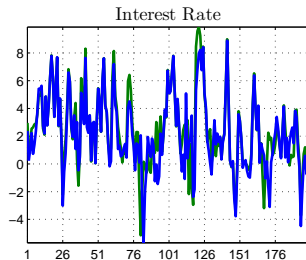
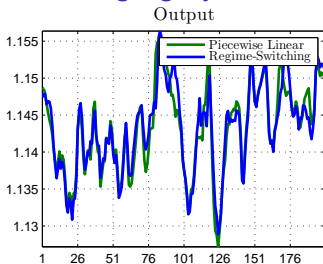
Binding Collateral Constraint Regime



More volatility in the switches between regimes with the regime-switching model

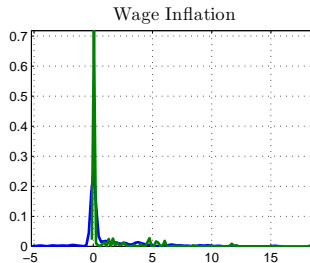
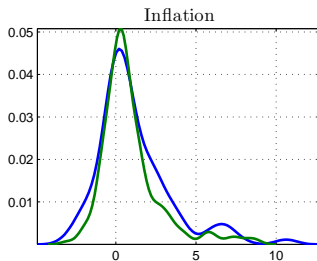
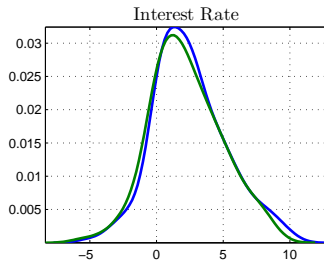
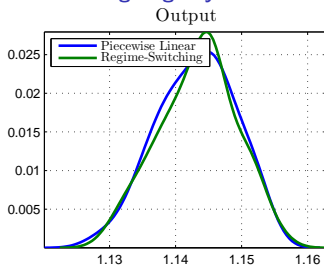
# Regime switching vs Piecewise linear

Downward Nominal wage rigidity: Simulations



# Regime switching vs Piecewise linear

Downward Nominal wage rigidity: distribution

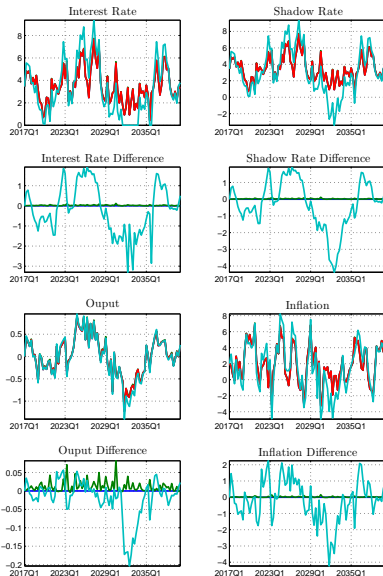


# Ext. path, piecewise Linear and regime switching

ZLB model: small shocks  $\sigma = 0.00375$

Appendix

Further Comparison



Key:

**Blue** = linearized model  
solved using the extended path  
algorithm,

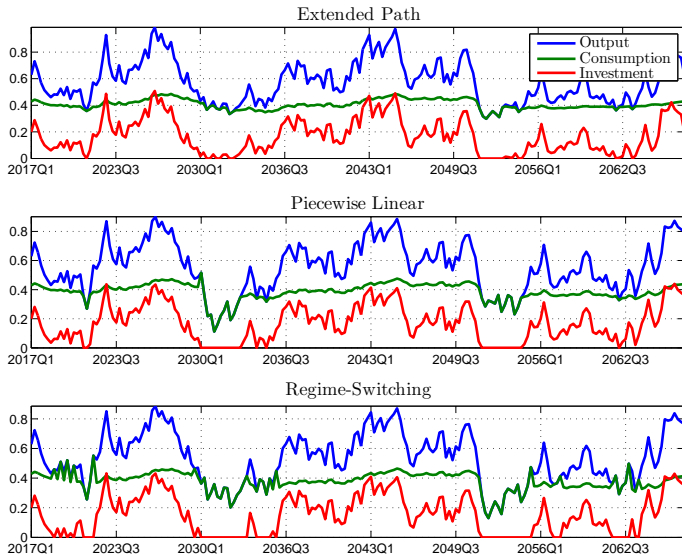
**Green** = non-linear model  
solved using the extended path  
algorithm,

**Red** = piecewise linear solution,

**Turquoise** = Regime-switching  
with endogenous transition  
probabilities.

# Ext. Path, piecewise Linear and regime switching

## Irreversible Investment model



# Conclusion



# Conclusion

- ▶ Show how to model occasionally binding constraints in DSGE models using regime-switching.
- ▶ Large models, accommodates complementary slackness problems, higher-order of perturbation solutions and multiple constraints simultaneously.
- ▶ solve four well known problems: the ZLB on interest rates, Collateral constraints, DNWR and irreversible investment.
- ▶ All codes have been implemented in Matlab using the RISE toolbox.

# References

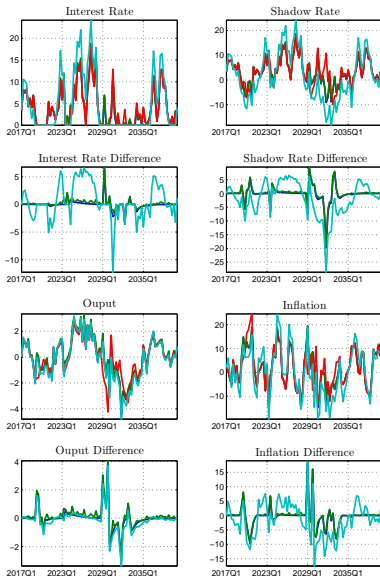
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# Appendix

# Ext. path, piecewise Linear and regime switching

ZLB model: Large shocks:  $\sigma = 0.0125$

◀ Back



Key:

**Blue** = linearized model  
solved using the extended path  
algorithm,

**Green** = non-linear model  
solved using the extended path  
algorithm,

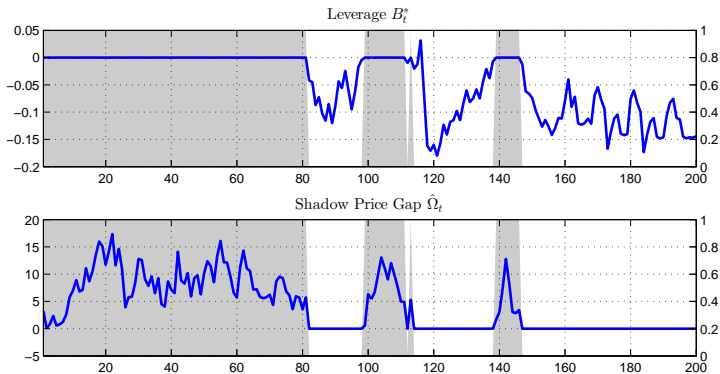
**Red** = piecewise linear solution,

**Turquoise** = Regime-switching  
with endogenous transition  
probabilities.

# Comparison With Other Methods

## Piecewise Linear Regimes

◀ Back

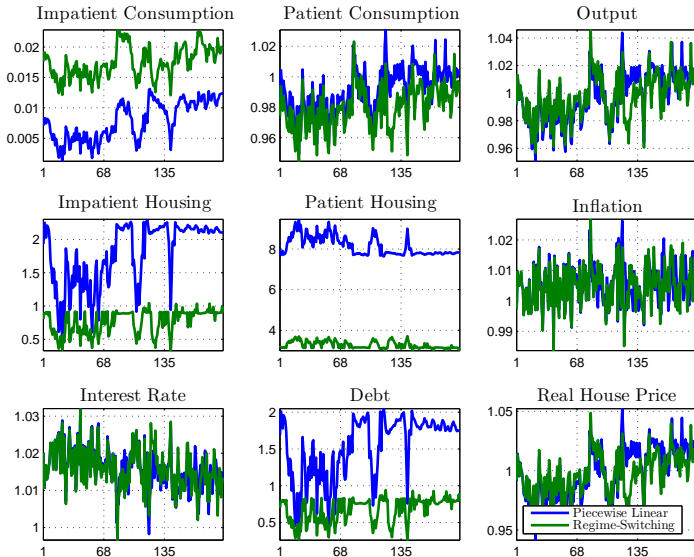


- “Regimes” are more precise compared with regime-switching

# Comparison With Other Methods

## Comparing Solution Methods

[◀ Back](#)



# Comparison with Other Methods

- ▶ Steady state housing different because we add subsidy to ensure the steady state is the same as the non-binding in the regime-switching model.
- ▶ In piece wise linear model is linearized around the binding steady-state.
- ▶ There is more volatility in the piece-wise linear model

# Calibrating Endogenous Probabilities

Assume the endogenous transition probability

$$p(\alpha, \gamma, x) = \frac{\alpha}{\alpha + \exp(\gamma(x - x_1))}$$

when  $x \rightarrow x_1$ ,

$$\Rightarrow p = \frac{\alpha}{\alpha + 1}$$

if

$$p \rightarrow 0, \Rightarrow \alpha = \text{small} \Rightarrow p = \frac{\text{small}}{\text{small} + 1}$$

else

$$p \rightarrow 1, \Rightarrow \alpha = \text{large} \Rightarrow p = \frac{\text{large}}{\text{large} + 1}$$



# Calibrating Endogenous Probabilities

when  $x - x_1 \rightarrow \text{large}$  ,

$$p = \frac{\alpha}{\alpha + \exp(\gamma \text{large})}$$

if

$$p \rightarrow 0, \Rightarrow \gamma = \text{large} \Rightarrow p = \frac{\alpha}{\alpha + \text{large}}$$

else

$$p \rightarrow 1, \Rightarrow \gamma = -\text{large} \Rightarrow p = \frac{\alpha}{\alpha + \text{really small}}$$

◀ perturbation solution

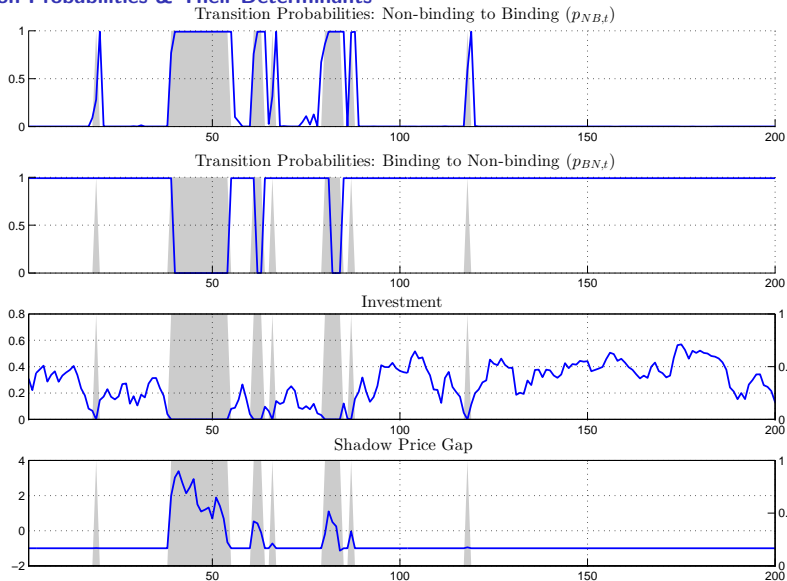
# Application 4: Irreversible Investment

# Irreversible Investment

- ▶ Basic RBC model as used by Adjemian & Juillard (2013) and Christiano & Fisher (2000)
- ▶ Households and firms

# Irreversible Investment

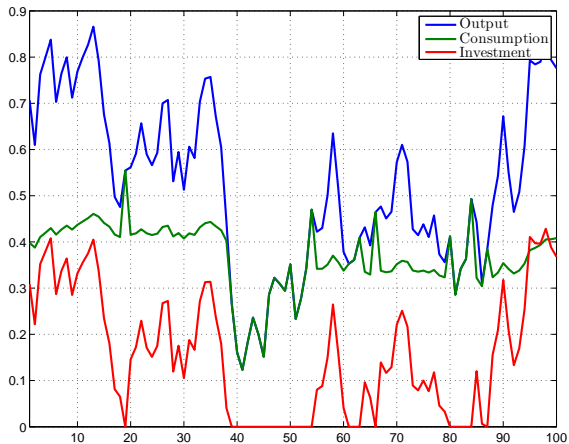
## Transition Probabilities & Their Determinants



# Irreversible Investment

## A Simulation With Irreversible Investment

downWageNomRigid



Extra slides

# Alternative treatment of transition Probabilities

- ▶ The probabilities could be estimated
- ▶ We have procedures with non-parametric transition probabilities
- ▶ The procedure is very slow at the moment and does not always work well in the sense of solving the model at each iteration