AEM 7500 Problem Set #5 Tianli Xia

Dynamic Games

1. The payoff function for city i in neighborhood k at time t:

$$\begin{aligned} \pi\left(a_{kt}, s_{kt}, \varepsilon_{ikt}; \theta\right) &= \pi_0\left(a_{kt}, s_{kt}; \theta\right) + \varepsilon_{ikt} \\ &= \gamma_1 a_{kt} + \gamma_2 s_{kt} + \varepsilon_{ikt} \end{aligned}$$

   is the profit that the city gets from the decision of investment in time t. It consists of two parts,

   (a) a deterministic part which is common to all cities in one neighborhood k: the number of cities that have installed the abatement device a_{kt} and the economy condition s_{kt}.γ_1 and γ_2 show the effects of these two states;

   (b) a stochastic part ε_{ikt} which is city-specific and not observed by the econometrician. The relative magnitude of the shock is captured by the variance term σ_ε.

2. It is dynamic because firm's decision made today will matter for tommorrow's payoff. Since the investment is irreversible, once the firm makes the decision it loeses the option value of waiting. It is a game because in this setting players invest strategically. A player's payoff depends on the other player's move.

3. Each city's value function is given by,

$$V\left(a_{kt}, s_{kt}, \varepsilon_{ikt}; \theta\right) = \max\left\{\underbrace{\pi\left(a_{kt}, s_{kt}, \varepsilon_{ikt}; \theta\right)}_{\text{if } I_{ikt}=1}, \underbrace{\beta V^c\left(a_{kt}, s_{kt}; \theta\right)}_{\text{if } I_{ikt}=0}\right\}$$

   where $\pi(\cdot)$ is given by (1) and

$$V^c\left(a_{kt}, s_{kt}; \theta\right) = E\left[V\left(a_{kt+1}, s_{kt+1}, \varepsilon_{ikt+1}; \theta\right) | a_{kt}, s_{kt}, I_{ikt} = 0\right]$$

   The interpretation is that the value function is the maximum of investing this period and the option value of waiting until the next period.

4. The continuation value V^c is the expected value of the next period conditional on not investing this period. It is the optional value of waiting. With the distribution assumptions on ε_{ikt} we have,

$$\begin{aligned} V^c\left(a_{kt}, s_{kt}; \theta\right) &= E\left[\beta V^c\left(a_{kt+1}, s_{kt+1}, \varepsilon_{ikt+1}; \theta\right) + \sigma_\varepsilon g\left(a_{kt+1}, s_{kt+1}; \theta\right) | a_{kt}, s_{kt}, I_{ikt} = 0\right] \\ V_t^c &= M_{rc}\left(\beta \vec{V}_{t+1}^c + \sigma_\varepsilon \vec{g}_{t+1}\right) \\ M_{rc} &= \Pr\left(\Omega_{k,t+1} = c | \Omega_{k,t} = r, I_{ikt} = 0\right). \end{aligned}$$

5. $g\left(a_{kt}, s_{kt}; \theta\right)$ is the city's probability of investing, i.e. the probability of choosing to install abatement technology at t given the state variables (a_{kt}, s_{kt}). Using distributional assumption we have g in closed form:

$$g\left(a_{kt}, s_{kt}; \theta\right) = \exp\left(-\frac{\beta V^c\left(a_{kt}, s_{kt}; \theta\right) - \pi_0\left(a_{kt}, s_{kt}; \theta\right)}{\sigma_\varepsilon}\right).$$

Panel Data

1. $I_{ikt} \equiv 1\,(\text{installed in t+1}) - 1\,(\text{installed in t})$. Then I manually set $I_{ikt} = -99$ if $t = T$ (last period).

   Should I drop observations after a firm invests (while waiting for the other firm to invest)? I feel yes.

2. $a_{kt} = I_{Akt} + I_{Bkt}$, then I manually drop observations when $a_{kt} = 2$. Since when both city install the abatement device the game is over and they should exit our sample.

3. $\Omega_{kt} = (a_{kt}, s_{kt})$. It takes four possible values: $\{0, 1\} \times \{0, 1\}$.

Structural Econometrics

Table 1: **Firm A's investment decision** $I_A(k,t)$

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18	k=19	k=20	k=21
T=1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-99	0	0
T=2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-99	0	0
T=3	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-99	0	0
T=4	0	0	0	0	0	0	0	0	0	0	0	0	-99	1	1	0	1	0	-99	0	0
T=5	0	0	0	0	0	0	0	0	0	0	0	0	-99	-99	-99	0	-99	0	-99	0	0
T=6	0	0	0	0	0	0	0	0	0	0	0	0	-99	-99	-99	0	-99	0	-99	0	0
T=7	0	0	0	0	0	0	0	0	0	0	0	0	-99	-99	-99	0	-99	0	-99	0	0
T=8	0	0	0	0	0	0	0	0	0	0	0	0	-99	-99	-99	0	-99	0	-99	0	0
T=9	0	0	0	0	0	0	0	0	0	0	0	0	-99	-99	-99	0	-99	0	-99	0	0
T=10	0	0	0	0	0	0	1	1	1	1	0	1	-99	-99	-99	0	-99	0	-99	0	0
T=11	0	0	1	0	0	0	-99	-99	-99	-99	0	-99	-99	-99	-99	0	-99	0	-99	0	0
T=12	0	1	-99	0	0	1	-99	-99	-99	-99	0	-99	-99	-99	-99	0	-99	0	-99	0	0
T=13	1	-99	-99	1	0	-99	-99	-99	-99	-99	1	-99	-99	-99	-99	0	-99	1	-99	0	0
T=14	-99	-99	-99	-99	0	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	0	-99	-99	-99	0	0
T=15	-99	-99	-99	-99	1	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	0	-99	-99	-99	0	0
T=16	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99

Table 2: **Firm B's investment decision** $I_B(k,t)$

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18	k=19	k=20	k=21
T=1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T=2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T=3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T=4	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
T=5	0	0	0	0	0	0	0	0	0	0	0	0	-99	1	1	1	1	0	1	0	0
T=6	0	0	0	0	0	0	0	0	0	0	0	0	-99	-99	-99	-99	-99	0	-99	0	0
T=7	0	0	0	0	0	0	0	0	0	0	0	0	-99	-99	-99	-99	-99	0	-99	0	0
T=8	0	0	1	1	0	0	0	0	0	0	0	0	-99	-99	-99	-99	-99	0	-99	0	0
T=9	0	1	-99	-99	0	0	0	0	0	0	0	0	-99	-99	-99	-99	-99	0	-99	0	0
T=10	1	-99	-99	-99	0	0	0	0	0	0	1	1	-99	-99	-99	-99	-99	1	-99	0	0
T=11	-99	-99	-99	-99	0	0	0	0	0	0	-99	-99	-99	-99	-99	-99	-99	-99	-99	0	0
T=12	-99	-99	-99	-99	1	0	0	0	0	0	-99	-99	-99	-99	-99	-99	-99	-99	-99	0	0
T=13	-99	-99	-99	-99	-99	0	1	1	1	1	-99	-99	-99	-99	-99	-99	-99	-99	-99	0	0
T=14	-99	-99	-99	-99	-99	1	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	0	0
T=15	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	1	1
T=16	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99

Table 3: **State variable a(k,t)**

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18	k=19	k=20	k=21
T=1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
T=2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
T=3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
T=4	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
T=5	0	0	0	0	0	0	0	0	0	0	0	0		1	1	0	1	0	1	0	0
T=6	0	0	0	0	0	0	0	0	0	0	0	0				1		0		0	0
T=7	0	0	0	0	0	0	0	0	0	0	0	0				1		0		0	0
T=8	0	0	0	0	0	0	0	0	0	0	0	0				1		0		0	0
T=9	0	0	1	1	0	0	0	0	0	0	0	0				1		0		0	0
T=10	0	1	1	1	0	0	0	0	0	0	0	0				1		0		0	0
T=11	1	1	1	1	0	0	1	1	1	1	1					1		1		0	0
T=12	1	1		1	0	0	1	1	1	1	1					1		1		0	0
T=13	1			1	1	1	1	1	1	1	1					1		1		0	0
T=14					1	1										1				0	0
T=15					1											1				0	0
T=16																1				1	1

Table 4: **Transition Probability matrix M**

	(a,s)	t+1			
		(0,0)	(0,1)	(1,0)	(1,1)
t	(0,0)	0.857	0.113	0.026	0.004
	(0,1)	0.141	0.704	0.000	0.155
	(1,0)	0.000	0.000	0.750	0.250
	(1,1)	0.000	0.000	0.130	0.870

1. The transition matrix is estimated nonparametrically,

$$\begin{aligned}M_{rc} &= \Pr\left(\Omega_{k,t+1}=c|\Omega_{k,t}=r, I_{ikt}=0\right)\\ &= \frac{\Pr\left(\Omega_{k,t+1}=c,\Omega_{k,t}=r, I_{ikt}=0\right)}{\Pr\left(\Omega_{k,t}=r, I_{ikt}=0\right)}\\ &= \frac{\sum_i \#\left(\Omega_{k,t+1}=c,\Omega_{k,t}=r, I_{ikt}=0\right)}{\sum_i \#\left(\Omega_{k,t}=r, I_{ikt}=0\right)}.\end{aligned}$$

2. Similarly, the empirical investment probability $\bar{g}\left(a_{kt}, s_{kt}\right)$ is estimated nonparametrically,

$$\begin{aligned}\bar{g}\left(a_{kt}, s_{kt}\right) &= \Pr\left(I_{ikt}=1|\Omega_{kt}=i\right) = \frac{\Pr\left(I_{ikt}=1,\Omega_{kt}=i\right)}{\Pr\left(\Omega_{kt}=i\right)}\\ &= \frac{\sum_i \#\left(I_{ikt}=1,\Omega_{kt}=i\right)}{\sum_i \#\left(\Omega_{kt}=i\right)}.\end{aligned}$$

   One question: when calculating the demoninator, should we exclude observations where one player is not making a decision but the other player can still make a decision?

Table 5: **Observed and model predicted choice probabilities on different states:** note since we have only two free poarameters but four choice probabilities to match, we are unable to match all of them perfectly.

	$g_{observed}$	$g_{predicted}$	$V_{c,predicted}$
a=0,s=0	0.036	0.241	1.615
a=0,s=1	0.134	0.151	2.242
a=1,s=0	0.273	0.303	3.128
a=1,s=1	0.324	0.311	3.264

3. Estimation: we search for optimal θ :

   Step 1: find the fixed point of V^c through matrix inversion:

$$\begin{aligned}\vec{V}^c_{4\times 1} &= M_{4\times 4}\left(\beta\vec{V}_c+\sigma_\varepsilon\vec{g}_{4\times 1}\right)\\ \left(I-\beta M\right)\vec{V}_c &= \sigma_\varepsilon M\vec{g}_{4\times 1}\\ \vec{V}_c &= \sigma_\varepsilon\left(I-\beta M\right)^{-1}M\vec{g}_{4\times 1}\end{aligned}$$

   then we can construct the model predicted probability conditional on the unknown parameter θ :

$$\begin{aligned}\hat{g}\left(a_{kt}, s_{kt};\theta\right) &= \exp\left(-\frac{\beta\vec{V}^c\left(a_{kt}, s_{kt};\theta\right)-\pi_0\left(a_{kt}, s_{kt};\theta\right)}{\sigma_\varepsilon}\right)_{4\times 1}\\ &= \exp\left(-\frac{\beta\sigma_\varepsilon\left(I-\beta M\right)^{-1}M\vec{g}_{4\times 1}-\pi_0\left(a_{kt}, s_{kt};\theta\right)}{\sigma_\varepsilon}\right)\\ &= \exp\left(-\beta\left(I-\beta M\right)^{-1}M\vec{g}_{4\times 1}+\frac{\gamma_1}{\sigma_\varepsilon}a_{kt}+\frac{\gamma_2}{\sigma_\varepsilon}s_{kt}\right)\end{aligned}$$

   As we can see, we can only identify $\frac{\gamma_1}{\sigma_\varepsilon}$ and $\frac{\gamma_2}{\sigma_\varepsilon}$. (parameters up to scale), but not able to identify the variance term σ_ε. This can also be seen from the fact that changing the scale profit function won't change the choice probability. So in the following estimation, I only keep two parameters and normalize $\sigma_\varepsilon = 1$.

   Step 2: then we construct the `first set` of moment conditions:

$$\begin{aligned}m\left(\theta\right) &= \begin{pmatrix}\frac{1}{n}\sum_{|\Omega_{kt}|}\left[\hat{g}\left(\Omega_{kt};\theta\right)-\vec{g}\right]n\left(\Omega_{kt}\right)\\ \frac{1}{n}\sum_{|\Omega_{kt}|}\left[\hat{g}\left(\Omega_{kt};\theta\right)-\vec{g}\right]s_{kt}n\left(\Omega_{kt}\right)\\ \frac{1}{n}\sum_{|\Omega_{kt}|}\left[\hat{g}\left(\Omega_{kt};\theta\right)-\vec{g}\right]a_{kt}n\left(\Omega_{kt}\right)\end{pmatrix}\\ &= \frac{1}{n}\sum_{ikt}\left[\hat{g}_{ikt}\left(\Omega_{kt};\theta\right)-\vec{g}_{ikt}\right]\end{aligned}$$

Or we can also match the `second set`:

$$m(\theta) = \begin{pmatrix} [\hat{g}(\Omega_{kt} = (0,0);\theta) - \vec{g}(0,0)] \\ [\hat{g}(\Omega_{kt} = (1,0);\theta) - \vec{g}(1,0)] \\ [\hat{g}(\Omega_{kt} = (0,1);\theta) - \vec{g}(0,1)] \\ [\hat{g}(\Omega_{kt} = (1,1);\theta) - \vec{g}(1,1)] \end{pmatrix}$$

and the objective function becomes,

$$\min_{\theta} m(\theta) W m(\theta)'$$

In the MATLAB code I use the `second sets` of moments, which are similar to POB (Rand)'s recommendation, where they call this a pseudo-chi2 estimator. This set of moment actually gives more reasonable predictions on $\hat{g}$ than the first set.

4. The bootstrap results are shown as follows, the main difference from the last homework is this time we have to bootstrap at the market level so that the time order is not disrupted.

Table 6: **Results: standard error is obtained from bootstrap.**

	Est.	**Std.**	**95% CI Lower**	**95% CI Upper**
gamma1	1.606	0.678	0.436	2.824
gamma2	0.113	0.472	-0.479	0.885
sigma	1 (fixed)			

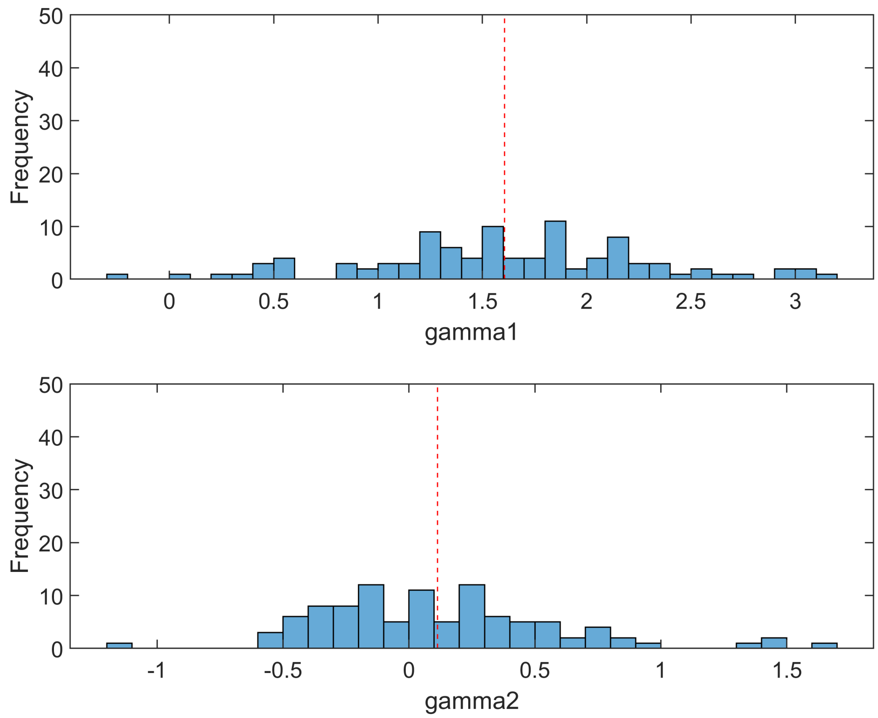

Figure 1: **Bootstrap results**

5. The point estimate of γ_1 is close to 1 and significant, which tells us that the installation of the abatement device is a desirable public good and the other player's installation has a positive spillover effect on my revenue. The estimate of γ_2 is positive but not significant, which means that the exogeneous state does not matter a lot.

6. it helps answer what determines the revenue of firm's decision in installing the abatement device, and in particular, how firm's revenue is dependent on other firm's choices. It also helps answer the policy question why firms delay their investment decisions in pollution abatement device and we can do counterfactual analysis to show what policy instrument can achieve the social optimal outcome.

7. I would like to add the following two sets of variables:

(a) a interaction state between a and s: it is possible that state and installation

(b) more heterogeneous effects: this may require additional observed state variables.