Fluctuation Relaxation in the AD/AS Model

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October 22, 2018

In this memo we examine the relationship between two statements of equilibrium that are encountered in discussions of the AD/AS model: (i) the notion of $\omega \to \infty$ (or $\gamma \to \infty$) and (ii) the notion of $\pi_t = \pi_{t-1}$.

In our discussion of the AD/AS model we have encountered two seemingly different expressions of what happens to the Phillips curve[1]

$$\pi_t = \pi_{t-1} - \omega\left(U_t - U_{\mathrm{N}}\right) \tag{1}$$

in "the long run." Mishkin (2012, 2014, 2015) casts this as a result of wage and price flexibility[2]

> "If wages and prices are completely flexible, then ω becomes so large that the short-run Phillips curve is vertical, and it would be identical to the long-run Phillips curve."

and I have cast it as a consequence of the notion that in the long run the economy is in equilibrium and when the economy is in equilibrium inflation is not changing: $\pi_t = \pi_{t-1}$.

The relationship between these perspectives regarding equilibrium can be seen if we write the Phillips curve as the SRAS curve

$$\pi_t = \pi_{t-1} + \gamma\left(Y_t - Y^{\mathsf{P}}\right) \tag{2}$$

where, via Okun's law $\gamma = |C_{\mathrm{Okun}}\omega|$ and then convert the output gap $\left(Y_t - Y^{\mathsf{P}}\right)$ into an inflation gap using the AD curve

$$Y_t = \left(\overline{Y} - \zeta_Y \overline{r}\right) - \zeta_Y \lambda \pi_t \tag{3}$$

which when $Y_t = Y^{\mathsf{P}}$ becomes[3]

$$Y^{\mathsf{P}} = \left(\overline{Y} - \zeta_Y \overline{r}\right) - \zeta_Y \lambda \pi^{\mathsf{T}} \tag{4}$$

where π^{T} is the inflation target of a central bank that wants $Y_t = Y^{\mathsf{P}}$. Subtracting Eq. (4) from Eq. (3) we see that the AD curve can be written in a "gapped form" that relates the output gap to the inflation gap, $\pi_t - \pi^{\mathsf{T}}$, as

$$Y_t - Y^{\mathsf{P}} = -\zeta_Y \lambda\left(\pi_t - \pi^{\mathsf{T}}\right) \ , \tag{5}$$

and with this we can write the SRAS curve (Eq. (2)) as

$$\pi_t = \pi_{t-1} - \zeta_Y \lambda \gamma\left(\pi_t - \pi^{\mathsf{T}}\right) \tag{6}$$

[1] Since we are discussing "the long run" we can assume that $\rho_t = 0$.

[2] cf. page 272 of Mishkin (2012), page 286 of Mishkin (2014), or page 330 of Mishkin (2015).

[3] The form of the AD curve given in Eq. (4) follows from (i) defining the equilibrium real interest rate, r^* as the rate that causes the IS curve to yield potential output

$$Y^{\mathsf{P}} = \overline{Y} - \zeta_Y r^*$$

and (ii) defining target inflation π^{T} to be the level of inflation that causes the MP curve to yield the equilibrium real interest rate

$$r^* = \overline{r} + \lambda \pi^{\mathsf{T}} \ .$$

Combining these two equations results in the potential-output version of the AD curve shown in Eq. (4). Thus we see that an inflation target of π^{T} is equivalent to an output target of Y^{P}.

which shows how inflation changes as a function of the inflation gap . We can simplify this to an expression of how the inflation gap evolves though time by subtracting π^{T} from each side of the equation

$$\left(\pi_t - \pi^{\mathsf{T}}\right) = \left(\pi_{t-1} - \pi^{\mathsf{T}}\right) - \zeta_Y \lambda \gamma \left(\pi_t - \pi^{\mathsf{T}}\right) \ , \tag{7}$$

moving the lagged output gap to the left-hand side of the equation

$$\left(\pi_t - \pi^{\mathsf{T}}\right) - \left(\pi_{t-1} - \pi^{\mathsf{T}}\right) = -\zeta_Y \lambda \gamma \left(\pi_t - \pi^{\mathsf{T}}\right) \ , \tag{8}$$

and noting that $\left(\pi_t - \pi^{\mathsf{T}}\right) - \left(\pi_{t-1} - \pi^{\mathsf{T}}\right)$ is the finite-difference form[4] of the time derivative of the inflation gap, or

[4] We used this finite-difference form of the time derivative earlier in the course to calculate inflation.

$$\left(\pi_t - \pi^{\mathsf{T}}\right) - \left(\pi_{t-1} - \pi^{\mathsf{T}}\right) = \frac{d\left(\pi_t - \pi^{\mathsf{T}}\right)}{dt} \tag{9}$$

we can write

$$\frac{d\left(\pi_t - \pi^{\mathsf{T}}\right)}{dt} = -\zeta_Y \lambda \gamma \left(\pi_t - \pi^{\mathsf{T}}\right) \tag{10}$$

which has the solution for a fluctuation π_0 away from equilibrium π^{T} at $t = 0$ of

$$\left(\pi_t - \pi^{\mathsf{T}}\right) = \left(\pi_0 - \pi^{\mathsf{T}}\right) e^{-\zeta_Y \lambda \gamma t}. \tag{11}$$

THE RELATIONSHIP OF EQUILIBRIUM as a statement of large ω to that of "the long run" (i.e., large t) can be seen directly in Eq. (11). If ω is very large so too is γ and $\exp\left(-\zeta_Y \lambda \gamma t\right)$ becomes very small for $t > 0$: any inflation-gap fluctuation rapidly (instantaneously if $\omega \to \infty$) returns to the equilibrium inflation-gap level of zero. Similarly, for any $\omega > 0$ we will have $\gamma > 0$ and when t gets very large $\exp\left(-\zeta_Y \lambda \gamma t\right)$ becomes very small and we have the same result: relaxation of an inflation-gap fluctuation back to zero where inflation is not changing and $\pi_t = \pi^{\mathsf{T}}$.

Acknowledgements

My thanks to the Econ 100B students who requested further elaboration of this point.

References

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