Human Capital and Economic Growth

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Macroeconomics II

Human Capital and Economic Growth

- Human capital: all the attributes of workers that potentially increase their productivity in all or some productive tasks.
- Can play a major role in economic growth and cross-country income differences.
- Which factors affect human capital investments and how these influence the process of economic growth and economic development.
- Human capital theory is the basis of much of labor economics and plays an equally important role in macroeconomics.
- Important connections between human capital and economic growth, especially related to its effect on technological progress, will be discussed later.

A Simple Separation Theorem I

- Partial equilibrium schooling decisions.
- Continuous time.
- Schooling decision of a single individual facing exogenously given prices for human capital.
- Perfect capital markets.
- Separation theorem: with perfect capital markets, schooling decisions will maximize the net present discounted value of wages of the individual.
- Instantaneous utility function $u(c)$ that satisfies standard Assumptions on utility.
- Planning horizon of T (where $T = \infty$ is allowed), discount $\rho > 0$ and constant flow rate of death equal to $\nu \geq 0$.

A Simple Separation Theorem II

- Standard arguments imply the objective function of this individual at time $t = 0$ is

$$\max \int_0^T e^{-(\rho+\nu)t} u(c(t))\, dt. \tag{1}$$

- Individual is born with some human capital $h(0) \geq 0$.
- Human capital evolves over time according to

$$\dot{h}(t) = G(t, h(t), s(t)), \tag{2}$$

- $s(t) \in [0, 1]$ is the fraction of time spends for investments in schooling.
- $G : \mathbb{R}_+^2 \times [0, 1] \to \mathbb{R}_+$ determines how human capital evolves.

A Simple Separation Theorem III

- Further restriction on schooling decisions,

$$s\left(t\right) \in \mathcal{S}\left(t\right), \tag{3}$$

- $\mathcal{S}\left(t\right) \subset \left[0, 1\right]$: captures the fact that all schooling may have to be full-time, i.e., $s\left(t\right) \in \left\{0, 1\right\}$, or other restrictions on schooling decisions.
- Exogenous sequence of wage per unit of human capital given by $\left[w\left(t\right)\right]_{t=0}^{T}$.
- Labor earnings at time t are

$$W\left(t\right) = w\left(t\right)\left[1 - s\left(t\right)\right]\left[h\left(t\right) + \omega\left(t\right)\right],$$

- $1 - s\left(t\right)$ is the fraction of time spent supplying labor to the market
- $\omega\left(t\right)$ is non-human capital labor that the individual may be supplying.

A Simple Separation Theorem IV

- Sequence of $[\omega(t)]_{t=0}^{T}$, is exogenous: only margin of choice is between market work and schooling (i.e., there is no leisure).
- Individual faces a constant (flow) interest rate equal to r on his savings (potentially including annuity payments)).
- Using the equation for labor earnings, the lifetime budget constraint of the individual is

$$\int_0^T e^{-rt} c(t)\, dt \leq \int_0^T e^{-rt} w(t) [1 - s(t)] [h(t) + \omega(t)]\, dt \tag{4}$$

A Simple Separation Theorem V

Theorem **(Separation Theorem)** Suppose that the instantaneous utility function $u(\cdot)$ is strictly increasing. Then the sequence $[c^*(t), s^*(t), h^*(t)]_{t=0}^{T}$ is a solution to the maximization of (1) subject to (2), (3) and (4) if and only if $[s^*(t), h^*(t)]_{t=0}^{T}$ maximizes

$$\int_0^T e^{-rt} w(t) [1 - s(t)] [h(t) + \omega(t)] \, dt \tag{5}$$

subject to (2) and (3), and $[c^*(t)]_{t=0}^{T}$ maximizes (1) subject to (4) given $[s^*(t), h^*(t)]_{t=0}^{T}$. That is, human capital accumulation and supply decisions can be *separated* from consumption decisions.

A Simple Separation Theorem VI

- Remember that under perfect capital markets the optimal consumption path depends only on total discounted life-time wealth (not on how that wealth evolves across time).
- So the optimal $\left[s^*(t), h^*(t)\right]_{t=0}^{T}$ should maximize the life-time discounted wealth.
- This does not hold if markets are imperfect or agents also make leisure decisions.

Schooling Investments and Returns to Education I

- Adaptation of Mincer (1974).
- Assume that $T = \infty$
- Flow rate of death, ν, is positive, so that individuals have finite expected lives.
- (2) is such that the individual has to spend an interval S with $s(t) = 1$—i.e., in full-time schooling, and $s(t) = 0$ thereafter.
- At the end of the schooling interval, the individual will have a schooling level of
$$h(S) = \eta(S),$$
- $\eta(\cdot)$ is an increasing, continuously differentiable and concave function.
- For $t \in [S, \infty)$, human capital accumulates over time (as the individual works) according to
$$\dot{h}(t) = g_h h(t), \tag{6}$$
for some $g_h \geq 0$.

Schooling Investments and Returns to Education II

- Wages grow exponentially,
$$\dot{w}(t) = g_w w(t), \tag{7}$$
with boundary condition $w(0) > 0$.
- Suppose that
$$g_w + g_h < r + \nu,$$
so that the net present discounted value of the individual is finite.
- Her optimal schooling policy must solve
$$\begin{aligned} \max_{\{\tau,S\}} & \int_0^\infty \mathrm{e}^{-(r+v)t} w(t)(1-s(t))(h(t)+\omega(t))dt \\ \text{s.t. } & s(t) = 1 \text{ for } t \in [\tau, \tau+S] \\ & h(\tau+S) = \eta(S) \\ & \dot{h}(t) = g_h h(t) \text{ for } t \geq \tau + S \\ & \dot{w}(t) = g_w w(t) \text{ for } t \geq 0 \\ & h(0) = 0 \end{aligned}$$

Schooling Investments and Returns to Education III

- The last problem can be written as following

$$\max_{\{\tau,S\}} \underbrace{\int_0^{\tau} e^{-(r+v)t} w(t)\omega(t)dt+}_{S=0}$$

$$\underbrace{\int_{\tau}^{\tau+S} e^{-(r+v)t}(0)dt+}_{S=1}$$

$$\underbrace{\int_{\tau+S}^{\infty} e^{-(r+v)t} w(t)(h(t)+\omega(t))dt}_{S=0}$$

$$\text{s.t. } \dot{h}(t) = g_h h(t)$$
$$\dot{w}(t) = g_w w(t)$$
$$h(\tau+S) = \eta(S)$$

Schooling Investments and Returns to Education IV

- which is the same as

$$\max_{\{\tau,S\}} \int_0^{\tau} e^{-(r+v)t} w(t)\omega(t)dt + \int_{\tau+S}^{\infty} e^{-(r+v)t} w(t)(h(t)+\omega(t))dt$$

$$\text{s.t. } \dot{h}(t) = g_h h(t)$$

$$\dot{w}(t) = g_w w(t)$$

$$h(\tau+S) = \eta(S).$$

Replacing the constraints in the objective function, the problem is reduced to

$$\max_{\{\tau,S\}} \int_0^{\tau} e^{-(r+v)t} e^{g_w t} w(0)\omega(t)dt + \int_{\tau+S}^{\infty} e^{-(r+v)t} e^{g_w t} w(0)(h(t)+\omega(t))dt$$

Schooling Investments and Returns to Education IV

$$
\begin{aligned}
\iff & \max_{\{\tau,S\}} \int_0^{\tau} e^{-(r+v-g_w)t}\, w(0)\omega(t)dt \\
& \quad + \int_{\tau+S}^{\infty} e^{-(r+v-g_w)t}\, w(0)(h(S)\, e^{g_h(t-S)} + \omega(t))dt \\
\iff & \max_{\{\tau,S\}} \int_0^{\tau} e^{-(r+v-g_w)t}\, w(0)\omega(t)dt \\
& \quad + \int_{\tau+S}^{\infty} e^{-(r+v-g_w-g_h)t}\, e^{-g_h S}\, w(0)h(S)dt \\
& \quad + \int_{\tau+S}^{\infty} e^{-(r+v-g_w)t}\, w(0)\omega(t)dt
\end{aligned}
$$

Schooling Investments and Returns to Education V

$$
\begin{aligned}
\iff \max_{\{\tau,S\}} & \int_0^\infty e^{-(r+v-g_w)t} w(0)\omega(t)dt \\
& - \underbrace{\int_\tau^{\tau+S} e^{-(r+v-g_w)t} w(0)\omega(t)dt}_{<0} \\
& + e^{-g_h S} h(s) w(0) \int_{\tau+S}^\infty e^{-(r+v-g_w-g_h)t} dt,
\end{aligned}
$$

where if $\omega(t)$ grows at a rate larger that $r+v-g_w$, then $\tau^* = 0$. This implies that

$$
\begin{aligned}
& \max_{\{S\}} e^{-g_h S} w(0)\eta(S) \int_S^\infty e^{-(r+v-g_w-g_h)t} dt \\
= & \max_{\{S\}} e^{-g_h S} w(0)\eta(S) \frac{1}{r+v-g_w-g_h} e^{-(r+v-g_w-g_h)S} \\
= & \max_{\{S\}} \frac{w(0)\eta(S) e^{-(r+v-g_w)S}}{r+v-g_w-g_h}
\end{aligned}
$$

Schooling Investments and Returns to Education VI

- Thus solution to the previous problem of optimal schooling is equivalent to solving

$$\max_S \int_S^\infty e^{-(r+\nu)t} w(t) h(t) dt. \tag{8}$$

using the Separation Theorem.

- Now using (6) and (7), this is equivalent to:

$$\max_S \frac{w(0)\eta(S)e^{-(r+\nu-g_w)S}}{r+\nu-g_h-g_w}. \tag{9}$$

- Since $\eta(S)$ is concave, the objective function in (9) is strictly concave.

Schooling Investments and Returns to Education VII

- Therefore, the unique solution to this problem is characterized by the first-order condition
$$\frac{\eta'(S^*)}{\eta(S^*)} = r + \nu - g_w, \tag{10}$$
where
$$S^* = S(r, v, g_w)$$
and S^* is not function of g_h. Using implicit function theorem, it can be shown that $S_r < 0$, $S_v < 0$ and $S_{g_w} > 0$.
- Higher interest rates and higher values of ν (shorter planning horizons) reduce human capital investments.
- Higher values of g_w increase the value of human capital and thus encourage further investments.
- Integrating both sides of this equation with respect to S,
$$\ln \eta(S^*) = \text{constant} + (r + \nu - g_w) S^*. \tag{11}$$

Schooling Investments and Returns to Education VIII

- Now note that the wage earnings of the worker of age $\tau \geq S^*$ in the labor market at time t will be given by

$$W(S,t) = e^{g_w t} e^{g_h(t-S)} \eta(S).$$

- Taking logs and using equation (11) implies that the earnings of the worker will be given by

$$\ln W(S^*,t) = \text{ constant} + (r + \nu - g_w) S^* + g_w t + g_h (t - S^*),$$

- $t - S^*$ can be thought of as worker experience (time after schooling).
- If we make a cross-sectional comparison across workers, the time trend term $g_w t$, will also go into the constant.
- Hence obtain the canonical Mincer equation where, in the cross section, log wage earnings are proportional to schooling and experience.

Schooling Investments and Returns to Education IX

- Written differently, we have the following cross-sectional equation

$$\ln W_j = \text{ constant} + \gamma_s S_j + \gamma_e \text{experience}, \tag{12}$$

  where j refers to individual j.
- But have not introduced any source of heterogeneity that can generate different levels of schooling across individuals.
- Economic insight: functional form of the Mincerian wage equation is not just a mere coincidence, but has economic content.
  - Opportunity cost of one more year of schooling is foregone earnings.
  - Thus benefit has to be commensurate with these foregone earnings, should lead to a proportional increase in earnings in the future.
  - This proportional increase should be at the rate $(r + \nu - g_w)$.

Schooling Investments and Returns to Education VI

- Empirical work using equations of the form (12) leads to estimates for γ in the range of 0.06 to 0.10.
- Equation (12) suggests that these returns to schooling are not unreasonable.
  - r as approximately 0.10, ν as corresponding to 0.02 that gives an expected life of 50 years, and g_w approximately about 2%.
  - Implies γ around 0.10.

The Ben-Porath Model I

- Ben-Porath: enriches the model by allowing human capital investments and non-trivial labor supply decisions.
- Now let $s(t) \in [0,1]$ for all $t \geq 0$.
- Human capital accumulation equation, (2), takes the form

$$\dot{h}(t) = \phi(s(t)\,h(t)) - \delta_h h(t), \tag{13}$$

- $\delta_h > 0$ captures "depreciation of human capital."
- The individual starts with an initial value of human capital $h(0) > 0$.
- The function $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, continuously differentiable and strictly concave.
- Furthermore, we simplify by assuming Inada-type conditions,

$$\lim_{x \to 0} \phi'(x) = \infty \text{ and } \lim_{x \to \infty} \phi'(x) = 0.$$

The Ben-Porath Model II

- Latter condition makes sure that we do not have to impose additional constraints to ensure $s(t) \in [0,1]$.
- No non-human capital component of labor, so that $\omega(t) = 0$ for all t.
- $T = \infty$, and that there is a flow rate of death $\nu > 0$.
- Wage per unit of human capital is constant at w and the interest rate is constant and equal to r.
- Normalize $w = 1$.
- Again using the Separation Theorem, human capital investments can be determined as a solution to

$$\max \int_0^\infty e^{-(r+\nu)t} (1 - s(t)) h(t) \, dt$$

subject to (13) and $0 \leq s(t) \leq 1$.

The Ben-Porath Model III

- Current-value Hamiltonian,

$$\mathcal{H}(h, s, \mu) = (1 - s(t)) h(t) + \mu(t) (\phi(s(t) h(t)) - \delta_h h(t)) + \lambda_1(t) (1 - s(t)) + \lambda_2(t) s(t).$$

- Necessary conditions for this problem are

$$\begin{aligned} \mathcal{H}_s(h, s, \mu) &= -h(t) + \mu(t) h(t) \phi'(s(t) h(t)) - \lambda_1(t) + \lambda_2(t) = 0 \\ \mathcal{H}_h(h, s, \mu) &= (1 - s(t)) + \mu(t) \left(s(t) \phi'(s(t) h(t)) - \delta_h\right) \\ &= (r + \nu) \mu(t) - \dot{\mu}(t) \\ 0 &= \lim_{t \to \infty} e^{-(r+\nu)t} \mu(t) h(t), \end{aligned}$$

  and $\lambda_1(t)(1 - s(t)) = 0$, $\lambda_2(t)s(t) = 0$, with $\lambda_1(t) \geq 0$ and $\lambda_2(t) \geq 0$, where $\lambda_1(t) \geq 0$ and $\lambda_2(t) = 0$ if $s(t) = 1$, $\lambda_1(t) = 0$ and $\lambda_2(t) \geq 0$ if $s(t) = 0$, and $\lambda_1(t) = \lambda_2(t) = 0$ if $s(t) \in (0, 1)$.
- Assuming an interior solution for $s(t)$, that is for $s(t) \in (0, 1)$, from the first FOC, $\mu(t)\phi'(s(t)h(t)) = 1$ with $\lambda_1(t) = \lambda_2(t) = 0$. More details in Exercise 10.6.

The Ben-Porath Model IV

- Adopt the following transformation of variables:

$$x(t) \equiv s(t) h(t).$$

- Study the dynamics of the optimal path in $x(t)$ and $h(t)$.
- The first necessary condition then implies that

$$1 = \mu(t) \phi'(x(t)), \tag{14}$$

- Second necessary condition can be expressed as

$$\frac{\dot{\mu}(t)}{\mu(t)} = r + \nu + \delta_h - s(t) \phi'(x(t)) - \frac{1 - s(t)}{\mu(t)}.$$

- Substituting for $\mu(t)$ from (14), and simplifying,

$$\frac{\dot{\mu}(t)}{\mu(t)} = r + \nu + \delta_h - \phi'(x(t)). \tag{15}$$

The Ben-Porath Model V

- Steady-state (stationary) solution involves $\dot{\mu}(t) = 0$ and $\dot{h}(t) = 0$, and thus

$$x^* = \phi'^{-1}(r + \nu + \delta_h), \tag{16}$$

- $\phi'^{-1}(\cdot)$ exists and is strictly decreasing since $\phi(\cdot)$ is strictly concave.
- Using implicit funtion theorem, it is possible to show that $x_r^* < 0$, $x_v^* < 0$ and $x_{\delta_h}^* < 0$.
- Implies $x^* \equiv s^* h^*$ will be higher when r is low, when $1/v$ is high, and when δ_h is low. Set $\dot{h}(t) = 0$ in the human capital accumulation equation (13), which gives

$$\begin{aligned} h^* &= \frac{\phi(x^*)}{\delta_h} \\ &= \frac{\phi\left(\phi'^{-1}(r + \nu + \delta_h)\right)}{\delta_h} \qquad (17) \\ h^* &= h(r, v, \delta_h) \qquad (18) \end{aligned}$$

The Ben-Porath Model V

- Since $\phi'^{-1}(\cdot)$ is strictly decreasing and $\phi(\cdot)$ is strictly increasing, steady-state h^* is uniquely determined and is decreasing in r, ν and δ_h.
- Using implicit function theorem, it is possible to show that $h_r^* < 0$, $h_\nu^* < 0$ and $h_{\delta_h}^* < 0$.
- It is also true that $s^* = \frac{x^*}{h^*} = \frac{x^* \delta_h}{\phi(x^*)} = \frac{\delta_h \phi'^{-1}(r+\nu+\delta_h)}{\phi(\phi'^{-1}(r+\nu+\delta_h))}$.

The Ben-Porath Model VI

- Path of human capital investment: differentiate (14) with respect to time to obtain

$$\begin{aligned}\frac{\dot{\mu}(t)}{\mu(t)} &= -\frac{\phi''(x(t))}{\phi'(x(t))}\dot{x}(t) \\ \frac{\dot{\mu}\left(t\right)}{\mu\left(t\right)} &= \varepsilon_{\phi'}\left(x\right)\frac{\dot{x}\left(t\right)}{x\left(t\right)},\end{aligned}$$

  where

$$\varepsilon_{\phi'}\left(x\right) = -\frac{x\phi''\left(x\right)}{\phi'\left(x\right)} > 0$$

  is the elasticity of the function $\phi'\left(\cdot\right)$ and is positive since $\phi'\left(\cdot\right)$ is strictly decreasing (thus $\phi''\left(\cdot\right) < 0$).

- Combining this equation with (15),

$$\frac{\dot{x}\left(t\right)}{x\left(t\right)} = \frac{1}{\varepsilon_{\phi'}\left(x\left(t\right)\right)}\left(r+\nu+\delta_h-\phi'\left(x\left(t\right)\right)\right). \tag{19}$$

- Figure plots (13) and (19) in the h-x space.

The Ben-Porath Model VII

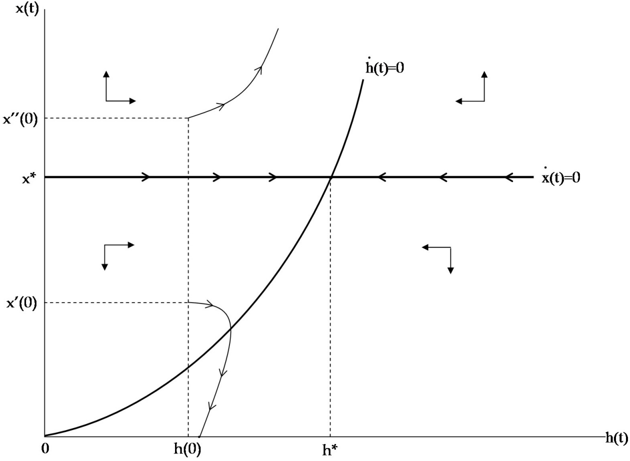

Figure: Steady state and equilibrium dynamics in the simplified Ben Porath model.

The Ben-Porath Model VIII

- Recall that $x^* = \phi'^{-1}(r + v + \delta_h)$ and $h^* = \frac{\phi'(x^*)}{\delta_h} = \frac{\phi'(\phi'^{-1}(r+v+\delta_h))}{\delta_h}$.
- The system exhibits a globally saddle path stable, so for any $h^* > h(0) > 0$ given, $s(0) = \frac{x^*}{h(0)}$ and then as $h(t)$ increases, $s(t)$ decreases.
- On the other hand, if $1 < \frac{x^*}{h(t)}$, then $h(t) < x^*$ and $s(t) = 1$, so that $\mu(t)\phi'(h(t)) > 1$, then the first order conditions imply

$$
\begin{aligned}
\frac{\dot{\mu}(t)}{\mu(t)} &= r + v + \delta_h - \phi'(h(t)) \\
\dot{h}(t) &= \phi(h(t)) - \delta_h h(t) \\
\lambda_1(t) &= \underbrace{\mu(t)\phi'(h(t))h(t) - h(t)}_{>0},
\end{aligned}
$$

which implies that $\lambda_1(t) > 0$. Then $\frac{x^*}{h(0)} > 1$ if and only if $x^* > h(0)$.

The Ben-Porath Model IX

- Here all happens smoothly.
- Original Ben-Porath model involves the use of other inputs in the production of human capital and finite horizons.
  - Constraint for $s(t) \leq 1$ typically binds early on in the life, and the interval during which $s(t) = 1$ can be interpreted as full-time schooling.
  - After full-time schooling, the individual starts working (i.e., $s(t) < 1$), but continues to accumulate human capital.
  - Because the horizon is finite, if the Inada conditions were relaxed, the individual could prefer to stop investing in human capital at some point.
  - Time path of human capital generated by the standard Ben-Porath model may be hump-shaped
  - Path of human capital (and the earning potential of the individual) in the current model is always increasing.

The Ben-Porath Model X

- Importance of Ben-Porath model
  1. Schooling is not the only way to invest in human capital; continuity between schooling investments and other investments.
  2. In societies where schooling investments are high we may also expect higher levels of on-the-job investments in human capital.
     - Thus there may be systematic mismeasurement of the amount or the quality human capital across societies.

The Ben-Porath Model XI

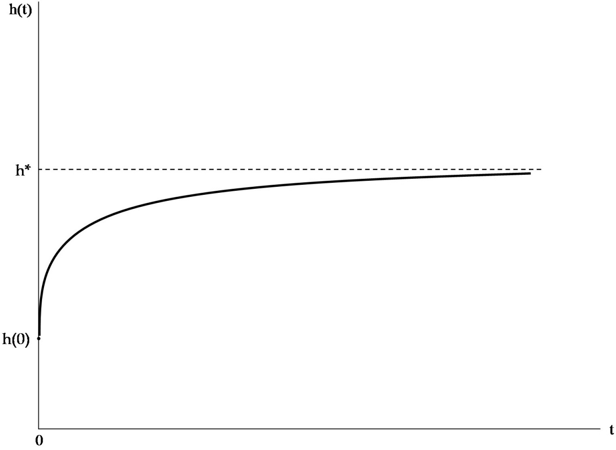

Figure: Time path of human capital investments in the simplified Ben Porath model.

Neoclassical Growth with Physical and Human Capital I

- Physical-human capital interactions could potentially be important.
- Evidence suggests are complementary: greater capital increases productivity of high human capital workers more than of low skill workers.
- May induce a "virtuous cycle" of investments in physical and human capital.
- Potential for complementarities also raises the issue of "imbalances".
  - Highest productivity when there is a balance between the two types of capital.
  - Will decentralized equilibrium ensure such a balance?
- Continuous time economy admitting a representative household with preferences

$$\int_0^\infty e^{-\rho t} u(c(t))\, dt, \tag{20}$$

- $u(\cdot)$ satisfies standard Assumptions on utility and $\rho > 0$.

Neoclassical Growth with Physical and Human Capital II

- Ignore technological progress and population growth.
- Aggregate production function:

$$Y(t) = F(K(t), H(t), L(t)),$$

- $K(t)$ is the stock of physical capital, $L(t)$ is total employment, and $H(t)$ represents human capital.
- No population growth and labor is supplied inelastically, $L(t) = L$ for all t.
- Production function satisfies Assumptions 1 and 2 generalized to production function with three inputs.
- "Raw" labor and human capital as separate factors of production may be less natural than human capital increasing effective units of labor. But allows a simple analysis.

Neoclassical Growth with Physical and Human Capital III

- Express all objects in per capita units, thus we write

$$
\begin{aligned}
y(t) &\equiv \frac{Y(t)}{L} \\
&= f(k(t), h(t)),
\end{aligned}
$$

  where

$$
k(t) \equiv \frac{K(t)}{L} \text{ and } h(t) \equiv \frac{H(t)}{L}
$$

- In view of standard assumptions $f(k, h)$ is strictly increasing, continuously differentiable and jointly strictly concave in both of its arguments.
- Physical and human capital are complementary, that is, $f_{kh}(k, h) > 0$ for all $k, h > 0$.

Neoclassical Growth with Physical and Human Capital IV

- Physical and human capital per capita evolve according to

$$\dot{k}(t) = i_k(t) - \delta_k k(t), \tag{21}$$

and

$$\dot{h}(t) = i_h(t) - \delta_h h(t) \tag{22}$$

- $i_k(t)$ and $i_h(t)$ are the investment levels in physical and human capital, while δ_k and δ_h are the depreciation rates.
- Resource constraint for the economy, in per capita terms,

$$c(t) + i_k(t) + i_h(t) \leq f(k(t), h(t)) \text{ for all } t. \tag{23}$$

- Equilibrium and optimal growth will coincide.
- Focus on the optimal growth problem: maximization of (20) subject to (21), (22), and (23).

Neoclassical Growth with Physical and Human Capital V

- First observe that since $u(c)$ is strictly increasing, (23) will always hold as equality.
- Substitute for $c(t)$ using this constraint and write the current-value Hamiltonian,

$$\begin{aligned} & \mathcal{H}(k(t), h(t), i_k(t), i_h(t), \mu_k(t), \mu_h(t)) \\ = \; & u(f(k(t), h(t)) - i_h(t) - i_k(t)) \\ & + \mu_h(t)(i_h(t) - \delta_h h(t)) + \mu_k(t)(i_k(t) - \delta_k k(t)), \end{aligned} \tag{24}$$

- Two control variables, $i_k(t)$ and $i_h(t)$ and two state variables, $k(t)$ and $h(t)$, two costate variables, $\mu_k(t)$ and $\mu_h(t)$, corresponding to (21) and (22).

Neoclassical Growth with Physical and Human Capital VI

- The conditions for a candidate optimal solution are

$$
\begin{aligned}
\mathcal{H}_{i_k}(\cdot) &= -u'(c(t)) + \mu_k(t) = 0 \\
\mathcal{H}_{i_h}(\cdot) &= -u'(c(t)) + \mu_h(t) = 0 \\
\mathcal{H}_k(\cdot) &= f_k(k(t), h(t))\, u'(c(t)) - \mu_k(t)\,\delta_k \\
&= \rho\mu_k(t) - \dot{\mu}_k(t) \\
\mathcal{H}_h(\cdot) &= f_h(k(t), h(t))\, u'(c(t)) - \mu_h(t)\,\delta_h \\
&= \rho\mu_h(t) - \dot{\mu}_h(t) \\
0 &= \lim_{t\to\infty} \mathrm{e}^{-\rho t}\mu_k(t)\, k(t) \\
0 &= \lim_{t\to\infty} \mathrm{e}^{-\rho t}\mu_h(t)\, h(t).
\end{aligned}
$$

- Two necessary transversality conditions, two state variables (and two costate variables).

Neoclassical Growth with Physical and Human Capital VII

- Needed to verify that $\mathcal{H}(\cdot)$ is concave given the costate variables $\mu_k(t)$ and $\mu_h(t)$, so the above conditions give the unique optimal path.
- The first two conditions immediately imply that

$$\mu_k(t) = \mu_h(t) = \mu(t).$$

- Combining this with the next two conditions gives

$$f_k(k(t), h(t)) - f_h(k(t), h(t)) = \delta_k - \delta_h, \tag{25}$$

- Together with $f_{kh} > 0$ implies that there is a one-to-one relationship between physical and human capital, of the form

$$h = \xi(k),$$

where $\xi(\cdot)$ is uniquely defined, strictly increasing and differentiable.

Neoclassical Growth with Physical and Human Capital VIII

Proposition In the neoclassical growth model described above, the optimal path of physical capital and consumption are given as in the one-sector neoclassical growth model, and satisfy the following two differential equations

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\varepsilon_u(c(t))}\left[f_k(k(t), \xi(k(t))) - \delta_k - \rho\right],$$

$$\dot{k}(t) = \frac{1}{1+\xi'(k)}\left[\begin{array}{c} f(k(t), \xi(k(t))) - \delta_h \xi(k(t)) \\ -\delta_k k(t) - c(t) \end{array}\right],$$

where $\varepsilon_u(c(t)) = -u''(c(t))\, c(t) / u'(c(t))$, together with $\lim_{t\to\infty}\left[k(t)\exp\left(-\int_0^t f_k(k(s), \xi(k(s)))\, ds\right)\right] = 0$, while $h(t) = \xi(k(t))$.

Neoclassical Growth with Physical and Human Capital IX

- Surprising: (25) implies that human and physical capital are always in "balance".
    - May have conjectured that economy that starts with high stock of physical relative to human capital will have a relatively high physical to human capital ratio for an extended period of time.
    - But we have not imposed any non-negativity constraints on the investment levels.
    - Such economy at the first instant it will experience a very high level of $i_h(0)$, compensated with a very negative $i_k(0)$.
    - After this, the dynamics of the economy will be identical to those of the baseline neoclassical growth model.

Neoclassical Growth with Physical and Human Capital X

- Different when there are non-negativity or "irreversibility" constraints.
  - If we assume that $i_k(t) \geq 0$ and $i_h(t) \geq 0$ for all t, initial imbalances will persist for a while.
  - Starting with a ratio $k(0)/h(0)$ that does not satisfy (25), optimal path will involve investment only in one of the two stocks until balance is reached.
  - Some amount of imbalance can arise, but the economy quickly moves towards correcting this imbalance.

Neoclassical Growth with Physical and Human Capital XI

- Impact of policy distortions: suppose resource constraint of the economy modified to

$$c(t) + (1+\tau)(i_k(t) + i_h(t)) \leq f(k(t), h(t)),$$

- $\tau \geq 0$ is a tax affecting both types of investments.
- Suppose that the aggregate production function takes the Cobb-Douglas form

$$\begin{aligned} Y &= F(K, H, L) \\ &= K^{\alpha_k} H^{\alpha_h} L^{1-\alpha_k-\alpha_h}. \end{aligned}$$

- Ratio steady-state income of income in the two economies with taxes/distortions of τ and τ' is given by:

$$\frac{Y(\tau)}{Y(\tau')} = \left(\frac{1+\tau'}{1+\tau}\right)^{\frac{\alpha_k+\alpha_h}{1-\alpha_k-\alpha_h}}. \tag{26}$$

Neoclassical Growth with Physical and Human Capital XII

- Responsiveness of human capital accumulation to these distortions increases impact of distortions. E.g., with $\alpha_k = \alpha_h = 1/3$ and eightfold distortion differences,
$$\frac{Y(\tau)}{Y(\tau')} \approx 8^2 \approx 64,$$
- But has to be interpreted with caution:
  1. Driven by a very elastic response of human capital accumulation:
     - e.g. if distortions correspond to differences in corporate taxes or corruption, may affect corporations rather than individual human capital decisions.
  2. Obvious similarity to Mankiw-Romer-Weil's approach:
     - existing evidence does not support the notion that human capital differences across countries can have such a large impact.

Capital-Skill Complementarity in an Overlapping Generations Model I

- Capital-skill imbalances in a simple overlapping generations model with impure altruism.
- Also generates only limited capital-skill imbalances.
- But capital-skill imbalances become much more important.
- Economy is in discrete time and consists of a continuum 1 of dynasties.
- Each individual lives for two periods, childhood and adulthood.
- Individual i of generation t works during their adulthood at time t, earns labor income equal to $w(t)\, h_i(t)$.
- Individual also earns capital income equal to $R(t)\, b_i(t-1)$.
- Human capital of the individual is determined at the beginning of his adulthood by an effort decision.
- Labor is supplied to the market after this effort decision.

Capital-Skill Complementarity in an Overlapping Generations Model II

- At the end of adulthood, after labor and capital incomes are received, individual decides his consumption and the level of bequest.
- Preferences of individual i (or of dynasty i) of generation t are given by

$$\eta^{-\eta}(1-\eta)^{-(1-\eta)} c_i(t)^{\eta} b_i(t)^{1-\eta} - \gamma(e_i(t)),$$

- $\eta \in (0,1)$, $c_i(t)$ is own consumption, $b_i(t)$ is the bequest to the offspring, $e_i(t)$ is effort expended for human capital acquisition.
- $\gamma(\cdot)$ is a strictly increasing, continuously differentiable and strictly convex cost of effort function.
- $\eta^{-\eta}(1-\eta)^{-(1-\eta)}$ is included as a normalizing factor to simplify the algebra.
- Human capital of individual i is given by

$$h_i(t) = ae_i(t), \tag{27}$$

Capital-Skill Complementarity in an Overlapping Generations Model III

- a corresponds to "ability".
- Substituting for $e_i(t)$ in the above expression, the preferences of individual i of generation t can be written as

$$\eta^{-\eta}(1-\eta)^{-(1-\eta)}c_i(t)^{\eta}b_i(t)^{1-\eta}-\gamma\left(\frac{h_i(t)}{a}\right). \tag{28}$$

- The budget constraint of the individual is

$$c_i(t)+b_i(t)\leq m_i(t)=w(t)h_i(t)+R(t)b_i(t-1), \tag{29}$$

- Defines $m_i(t)$ as the current income of individual i at time t consisting of labor earnings, $w(t)h_i(t)$, and asset income, $R(t)b_i(t-1)$.

Capital-Skill Complementarity in an Overlapping Generations Model IV

- Aggregate production function

$$Y(t) = F(K(t), H(t)),$$

that satisfies Assumptions 1 and 2.

- $H(t)$ is "effective units of labor" or alternatively the total stock of human capital given by,

$$H(t) = \int_0^1 h_i(t)\, di,$$

- $K(t)$, the stock of physical capital, is given by

$$K(t) = \int_0^1 b_i(t-1)\, di.$$

Capital-Skill Complementarity in an Overlapping Generations Model V

- Production function with two factors and constant returns to scale necessarily implies that the two factors are complements,

$$\frac{\partial^2 F(K,H)}{\partial K \partial H} \geq 0. \tag{30}$$

- Simplify the notation by assuming capital depreciates fully after use, that is, $\delta = 1$.
- More useful to define a normalized production function expressing output per unit of human capital.

Capital-Skill Complementarity in an Overlapping Generations Model VI

- Let $\kappa \equiv K/H$ be the capital to human capital ratio (or the "effective capital-labor ratio"), and

$$\begin{aligned} y(t) &\equiv \frac{Y(t)}{H(t)} \\ &= F\left(\frac{K(t)}{H(t)}, 1\right) \\ &= f(\kappa(t)), \end{aligned}$$

- Second line uses the linear homogeneity of $F(\cdot, \cdot)$, last line uses the definition of κ.

Capital-Skill Complementarity in an Overlapping Generations Model VII

- From the definition of κ, the law of motion of effective capital-labor ratios can be written as

$$\kappa(t) \equiv \frac{K(t)}{H(t)} = \frac{\int_0^1 b_i(t-1)\,di}{\int_0^1 h_i(t)\,di}. \tag{31}$$

- Factor prices are then given by the usual competitive pricing formulae:

$$R(t) = f'(\kappa(t)) \text{ and } w(t) = f(\kappa(t)) - \kappa(t) f'(\kappa(t)), \tag{32}$$

- $w(t)$ is now wage per unit of human capital, in a way consistent with (29).

Capital-Skill Complementarity in an Overlapping Generations Model VIII

- An equilibrium in this overlapping generations economy is a sequence $\left\{[h_i(t)]_{i\in[0,1]}, [c_i(t)]_{i\in[0,1]}, [b_i(t)]_{i\in[0,1]}\right\}_{t=0}^{\infty}$, that solve (28) subject to (29) a sequence $\{\kappa(t)\}_{t=0}^{\infty}$ given by (31) with some initial distribution of bequests $[b_i(0)]_{i\in[0,1]}$, and sequences $\{w(t), R(t)\}_{t=0}^{\infty}$ that satisfy (32).
- Solution to the maximization problem of (28) subject to (29) involves

$$c_i(t) = \eta m_i(t) \text{ and } b_i(t) = (1-\eta) m_i(t), \tag{33}$$

- Substituting these into (28), we obtain the indirect utility function:

$$m_i(t) - \gamma\left(\frac{h_i(t)}{a}\right), \tag{34}$$

Capital-Skill Complementarity in an Overlapping Generations Model IX

- Individual maximizes it by choosing $h_i(t)$ and recognizing that $m_i(t) = w(t)\,h_i(t) + R(t)\,b_i(t-1)$.
- First-order condition of this maximization gives the human capital investment of individual i at time t as:

$$aw(t) = \gamma'\left(\frac{h_i(t)}{a}\right), \tag{35}$$

- Or inverting this relationship and using (32),

$$h_i(t) = h(t) \equiv a\gamma'^{-1}\left[a\left(f(\kappa(t)) - \kappa(t)\,f'(\kappa(t))\right)\right]. \tag{36}$$

- Important implication: human capital investment of each individual is identical, and only depends on the effective of capital-labor ratio in the economy.

Capital-Skill Complementarity in an Overlapping Generations Model X

- Consequence of the specific utility function in (28):
  - no income effects so all agents choose the same "income-maximizing" level of human capital (as in Separation Theorem).
- Since bequest decisions are linear as shown (33),

$$
\begin{aligned}
K(t+1) &= \int_0^1 b_i(t)\,di \\
&= (1-\eta)\int_0^1 m_i(t)\,di \\
&= (1-\eta)\,f(\kappa(t))\,h(t),
\end{aligned}
$$

Capital-Skill Complementarity in an Overlapping Generations Model XI

- Last line uses the fact that, since all individuals choose the same human capital level given by (36), $H(t) = h(t)$, and thus $Y(t) = f(\kappa(t))\,h(t)$.
- Combining this with (31),

$$\kappa(t+1) = \frac{(1-\eta)\,f(\kappa(t))\,h(t)}{h(t+1)}.$$

- Using (36), this becomes

$$\begin{aligned} & \kappa(t+1)\,\gamma'^{-1}\left[a\left(f(\kappa(t+1)) - \kappa(t+1)\,f'(\kappa(t+1))\right)\right] \qquad (37) \\ = \; & (1-\eta)\,f(\kappa(t))\,\gamma'^{-1}\left[af(\kappa(t)) - \kappa(t)\,f'(\kappa(t))\right]. \end{aligned}$$

Capital-Skill Complementarity in an Overlapping Generations Model XII

- A steady state involves $\kappa(t) = \kappa^*$ for all t.
- Substituting this into (37) yields

$$\kappa^* = (1 - \eta) f(\kappa^*), \tag{38}$$

- Defines the unique positive steady-state effective capital-labor ratio, κ^* (since $f(\cdot)$ is strictly concave).

Proposition There exists a unique steady state with positive activity, and the physical to human capital ratio is κ^* as given by (38).

Capital-Skill Complementarity in an Overlapping Generations Model XIII

- This steady-state equilibrium is also typically stable, but some additional conditions need to be imposed on the $f(\cdot)$ and $\gamma(\cdot)$.
- Capital-skill (k-h) complementarity in the production function $F(\cdot,\cdot)$ implies that a certain target level of physical to human capital ratio, κ^*, has to be reached in equilibrium.
- I.e., does not allow equilibrium "imbalances" between physical and human capital either.
- Introducing such imbalances: depart from perfectly competitive labor markets.

Physical and Human Capital with Imperfect Labor Markets I

- Deviate from the competitive pricing formulea (32).
- Economy is identical to that described in the previous section, except that there is a measure 1 of firms as well as a measure 1 of individuals.
- Each firm can only hire one worker.
- Production function of each firm is still given by

$$y_j(t) = F(k_j(t), h_i(t)),$$

- $y_j(t)$ refers to the output of firm j, $k_j(t)$ is its capital stock (also per worker, since the firm is hiring only one worker).
- $h_i(t)$ is the human capital of worker i that the firm has matched with.
- Again satisfies Assumptions 1 and 2.

Physical and Human Capital with Imperfect Labor Markets II

- Now assume the following structure for the labor market:
  1. Firms choose physical capital level irreversibly (incurring cost $R(t)\,k_j(t)$), and simultaneously workers choose their human capital level irreversibly.
  2. After workers complete human capital investments, they are randomly matched with firms. High human capital workers are *not* more likely to be matched with high physical capital firms.
  3. After matching, each worker-firm pair bargains over the division of output. Divide output according to some pre-specified rule, worker receives total earnings of

$$W_j\left(k_j(t), h_i(t)\right) = \lambda F\left(k_j(t), h_i(t)\right),$$

for some $\lambda \in (0, 1)$.

Physical and Human Capital with Imperfect Labor Markets III

- Introduce heterogeneity in the cost of human capital acquisition by modifying (27) to

$$h_i(t) = a_i e_i(t),$$

- a_i differs across dynasties (individuals).
- Equilibrium is defined similarly but factor prices are no longer determined by (32).
- Firm chooses physical unsure about the human capital of the worker he will be facing.

Physical and Human Capital with Imperfect Labor Markets IV

- Therefore, the expected return of firm j can be written as

$$(1-\lambda)\int_0^1 F\left(k_j\left(t\right), h_i\left(t\right)\right) di - R\left(t\right) k_j\left(t\right). \tag{39}$$

- Notice (39) is strict concave in $k_j\left(t\right)$ given the strict concavity of $F\left(\cdot,\cdot\right)$ from Assumption 1.
- Therefore, each firm will choose the same level of physical capital, $\hat{k}\left(t\right)$, such that

$$(1-\lambda)\int_0^1 \frac{\partial F\left(\hat{k}\left(t\right), h_i\left(t\right)\right)}{\partial k\left(t\right)} di = R\left(t\right).$$

- Given this and following (34) from the previous section, each worker's objective function can be written as:

$$\lambda F\left(\hat{k}\left(t\right), h_i\left(t\right)\right) + R\left(t\right) b_i\left(t-1\right) - \gamma\left(\frac{h_i\left(t\right)}{a_i}\right),$$

- Have substituted for the income $m_i\left(t\right)$.

Physical and Human Capital with Imperfect Labor Markets V

- Implies the following choice of human capital investment by a worker i:

$$\lambda a_i \frac{\partial F\left(\hat{k}\left(t\right), h_i\left(t\right)\right)}{\partial h_i(t)} = \gamma'\left(\frac{h_i\left(t\right)}{a_i}\right).$$

- Yields unique equilibrium human capital investment $\hat{h}_i\left(\hat{k}\left(t\right)\right)$ for each i.
- Directly depends on the capital choices of all the firms, $\hat{k}\left(t\right)$ and also depends implicitly on a_i.
- Moreover, given (30), $\hat{h}_i\left(\hat{k}\left(t\right)\right)$ is strictly increasing in $\hat{k}\left(t\right)$.
- Also, since $\gamma\left(\cdot\right)$ is strictly convex, $\hat{h}_i\left(\hat{k}\left(t\right)\right)$ is a strictly concave function of $\hat{k}\left(t\right)$.

Physical and Human Capital with Imperfect Labor Markets VI

- Substituting this into the first-order condition of firms,

$$(1-\lambda)\int_0^1 \frac{\partial F\left(\hat{k}(t), \hat{h}_i\left(\hat{k}(t)\right)\right)}{\partial k(t)} di = R(t).$$

- Finally, to satisfy market clearing in the capital market, the rate of return to capital, $R(t)$, has to adjust, such that

$$\hat{k}(t) = \int_0^1 b_i(t-1)\, di,$$

- Follows from the facts that all firms choose the same level of capital investment and that the measure of firms is normalized to 1.
- Implies that in the closed economy version of the current model, capital per firm is fixed by bequest decisions from the previous period.

Physical and Human Capital with Imperfect Labor Markets VII

- Main economic forces are seen more clearly when physical capital is not predetermined.
- Thus imagine economy in question is small and open, so that $R(t) = R^*$.
- Under this assumption, the equilibrium level of capital per firm is determined by

$$(1-\lambda)\int_0^1 \frac{\partial F\left(\hat{k}, \hat{h}_i\left(\hat{k}\right)\right)}{\partial k} di = R^*. \tag{40}$$

Physical and Human Capital with Imperfect Labor Markets VIII

Proposition In the open economy version of the model described here, there exists a unique positive level of capital per worker $\hat{k}$ given by (40) such that the equilibrium capital per worker is always equal to $\hat{k}$. Given $\hat{k}$, the human capital investment of worker i is uniquely determined by $\hat{h}_i\left(\hat{k}\right)$ such that

$$\lambda a_i \frac{\partial F\left(\hat{k}, \hat{h}_i\left(\hat{k}\right)\right)}{\partial h} = \gamma' \left(\frac{\hat{h}_i\left(\hat{k}\right)}{a_i}\right). \tag{41}$$

We have that $\hat{h}_i\left(\hat{k}\right)$ is increasing in $\hat{k}$, and a decline in R^* increases $\hat{k}$ and $\hat{h}_i$ for all $i \in [0,1]$.

In addition to this equilibrium, there also exists a no-activity equilibrium in which $\hat{k} = 0$ and $\hat{h}_i = 0$ for all $i \in [0,1]$.

Proof of Proposition

- Since $F(k,h)$ exhibits constant returns to scale and $\hat{h}_i\left(\hat{k}\right)$ is a concave function of $\hat{k}$ for each i, $\int_0^1 \left(\partial F\left(\hat{k},\hat{h}_i\left(\hat{k}\right)\right)/\partial k\right) di$ is decreasing in $\hat{k}$ for a distribution of $[a_i]_{i\in[0,1]}$.
- Thus $\hat{k}$ is uniquely determined.
- Given $\hat{k}$, (41) determines $\hat{h}_i\left(\hat{k}\right)$ uniquely.
- Applying the Implicit Function Theorem to (41) implies that $\hat{h}_i\left(\hat{k}\right)$ is increasing in $\hat{k}$.
- Finally, (40) implies that a lower R^* increases $\hat{k}$, and from the previous observation $\hat{h}_i$ for all $i \in [0,1]$ increase as well.
- The no-activity equilibrium follows, since when all firms choose $\hat{k} = 0$, output is equal to zero and it is best response for workers to choose $\hat{h}_i = 0$, and when $\hat{h}_i = 0$ for all $i \in [0,1]$, $\hat{k} = 0$ is the best response for all firms.

Physical and Human Capital with Imperfect Labor Markets VIII

- Underinvestment both in human capital and physical capital (even in positive activity equilibrium).
- Consider a social planner wishing to maximize output.
- Restricted by the same random matching technology.
- Similar analysis to above implies social planner would also like each firm to choose an identical level of capital per firm, say $\bar{k}$.
- But it will be different than in the competitive equilibrium and also choose a different relationship between human capital and physical capital investments.
- In particular, given $\bar{k}$, human capital decisions satisfy

$$a_i \frac{\partial F\left(\bar{k}, \bar{h}_i\left(\bar{k}\right)\right)}{\partial h} = \gamma'\left(\frac{\bar{h}_i\left(\bar{k}\right)}{a_i}\right),$$

Physical and Human Capital with Imperfect Labor Markets IX

- Similar to (41), except that λ is absent from the left-hand side.
- Social planner considers the entire output.
- Consequently, as long as $\lambda < 1$,

$$\bar{h}_i(k) > \hat{h}_i(k) \text{ for all } k > 0.$$

- Similarly, the social planner would also choose a higher level of capital investment for each firm, in particular,

$$\int_0^1 \frac{\partial F(\bar{k}, \bar{h}_i(\bar{k}))}{\partial k} di = R^*,$$

- Differs from (40) both because now the term $1 - \lambda$ is not present and because the planner takes into account the differential human capital investment behavior of workers given by $\bar{h}_i(\bar{k})$.

Physical and Human Capital with Imperfect Labor Markets X

Proposition In the equilibrium described, there is underinvestment both in physical and human capital.

Proposition Consider the positive activity equilibrium . Output is equal to 0 if either $\lambda = 0$ or $\lambda = 1$. Moreover, there exists $\lambda^* \in (0, 1)$ that maximizes output.

- Different levels of λ create different types of "imbalances:"
  - High λ implies workers have a strong bargaining position, encourages their human capital investments. But it discourages physical capital investments of firms
  - As $\lambda \to 1$, workers' investment is converging to social planner (i.e., $\hat{h}_i(k) \to \bar{h}_i(k)$ for all $k > 0$), but $\hat{k}$ is converging to zero, implies $\hat{h}_i(k) \to 0$, and production collapses.
  - Same happens, in reverse, when λ is too low.
  - Intermediate value of λ^* achieves a balance, though the equilibrium continues to be inefficient.

Physical and Human Capital with Imperfect Labor Markets XI

- Physical-human capital imbalances can also increase the role of human capital in cross-country income differences.
- Proportional impact of a change in human capital on aggregate output is greater than the return to human capital, latter is determined not by the marginal product but by λ.
- At the root are *pecuniary externalities*: external effects that work through prices.
- By investing more, workers (and symmetrically firms) increase the return to capital (symmetrically wages).
- Underinvestment because they do not take these external effects into consideration.

Physical and Human Capital with Imperfect Labor Markets XII

- Pecuniary external effects are also present in competitive markets, but typically "second order:"
  - prices are equal to both the marginal benefit of buyers and marginal cost of suppliers.
- In this model take the form of *human capital externalities*: human capital investments by a group of workers increase other workers' wages.
- Opposite in economy analyzed in the last section.
- To illustrate, suppose there are two types of workers: fraction of workers χ with ability a_1 and $1-\chi$ with ability $a_2 < a_1$.
- First-order condition of firms, (40),

$$(1-\lambda)\left[\chi\frac{\partial F\left(\hat{k},\hat{h}_1\left(\hat{k}\right)\right)}{\partial k}+(1-\chi)\frac{\partial F\left(\hat{k},\hat{h}_2\left(\hat{k}\right)\right)}{\partial k}\right]=R^*, \qquad (42)$$

Physical and Human Capital with Imperfect Labor Markets XIII

- First-order conditions for human capital investments for the two types of workers take the form

$$\lambda a_j \frac{\partial F\left(\hat{k}, \hat{h}_j\left(\hat{k}\right)\right)}{\partial h} = \gamma'\left(\frac{\hat{h}_j\left(\hat{k}\right)}{a_j}\right) \text{ for } j = 1, 2. \tag{43}$$

- Clearly, $\hat{h}_1(k) > \hat{h}_2(k)$ since $a_1 > a_2$.
- Now imagine an increase in χ.
- Holding $\hat{h}_1\left(\hat{k}\right)$ and $\hat{h}_2\left(\hat{k}\right)$ constant, (42) implies that $\hat{k}$ should increase, since the left-hand side has increased (in view of the fact that $\hat{h}_1\left(\hat{k}\right) > \hat{h}_2\left(\hat{k}\right)$ and $\partial^2 F(k, h) / \partial k \partial h > 0$).
- Each firm expects average worker to have higher human capital.
- Since physical and human capital are complements, more profitable for each firm to increase their physical capital investment.

Physical and Human Capital with Imperfect Labor Markets XIV

- Greater investments by firms, in turn, raise $F\left(\hat{k}, h\right)$ for each h, in particular for $\hat{h}_2\left(\hat{k}\right)$.
- Earnings of type 2 workers is equal to $\lambda F\left(\hat{k}, \hat{h}_2\left(\hat{k}\right)\right)$, their earnings will also increase.
- Human capital externalities are even stronger, because the increase in $\hat{k}$ also raises $\partial F\left(\hat{k}, \hat{h}_2\left(\hat{k}\right)\right)/\partial h$ and thus encourages further investments by type 2 workers.
- But these feedback effects do not lead to divergence or multiple equilibria.

Proposition The positive activity equilibrium exhibits human capital externalities in the sense that an increase in the human capital investments of a group of workers raises the earnings of the remaining workers.

Human Capital Externalities I

- Human capital externalities may arise as a direct non-pecuniary (technological) spillover on the productivity of each worker.
- Empirical evidence on the extent of human capital externalities.
- Rauch (1993): quasi-Mincerian wage regressions, with the major difference that average human capital of workers in the local labor market is also included on the right-hand side:

$$\ln W_{j,m} = \mathrm{X}_{j,m}^{\prime}\mathrm{fi} + \gamma_p S_{j,m} + \gamma_e S_m,$$

- $\mathrm{X}_{j,m}$ is a vector of controls, $S_{j,m}$ is the years of schooling of individual j living/working in labor market m.
- S_m is the average years of schooling of workers in labor market m.
- *private return* to schooling γ_p
- γ_e measures the *external return* .

Human Capital Externalities II

- Rauch estimated external returns often exceeding the private returns.
- But exploited differences in average schooling levels across cities, which could reflect many factors that also directly affect wages.
- Acemoglu and Angrist (2000) exploited differences in average schooling levels across states and cohorts resulting from changes in compulsory schooling and child labor laws.
- Estimate external returns to schooling that are typically around 1 or 2 percent and statistically insignificant (as compared to private returns of about 10%).
- Confirmed by a study by Duflo (2004) using Indonesian data and by Ciccone and Perri (2006).

Human Capital Externalities III

- Moretti (2002) also estimates human capital externalities, and he finds larger effects:
  - focuses on college graduation,
  - also partly reflects the fact that the source of variation that he exploits, changes in age composition and the presence of land-grant colleges, may have other effects on average earnings in area.
- Overall, evidence appears to suggest that local human capital externalities are not very large.
- "Local" is key:
  - if a few generate ideas that are then used in other parts of the country or even in the world, there may exist significant global human capital externalities.

Nelson-Phelps Model of Human Capital I

- Alternative perspective: major role of human capital is not to increase productivity in existing tasks, but to enable workers to cope with change, disruptions and especially new technologies.
- Continuous time model.
- Output is given by
$$Y(t) = A(t) L, \tag{44}$$
- L is the constant labor force, supplying its labor inelastically, and $A(t)$ is the technology level of the economy.
- No capital and also no labor supply margin.
- The only variable that changes over time is technology $A(t)$.
- World technological frontier is given by $A_F(t)$.

Nelson-Phelps Model of Human Capital II

- $A_F(t)$ evolves exogenously according to the differential equation
$$\frac{\dot{A}_F(t)}{A_F(t)} = g_F,$$
with initial condition $A_F(0) > 0$.
- Human capital of the workforce denoted by h.
- This human capital does not feature in the production function, (44).
- Evolution of the technology in use, $A(t)$, is governed by the differential equation
$$\dot{A}(t) = gA(t) + \phi(h) A_F(t),$$
with initial condition $A(0) \in (0, A_F(0))$.
- Parameter g is strictly less than g_F and measures the growth rate of technology $A(t)$.

Nelson-Phelps Model of Human Capital III

- Assume that $\phi(\cdot)$ is increasing, with
$$\phi(0) = 0 \text{ and } \phi(h) = g_F - g > 0 \text{ for all } h \geq \bar{h},$$
where $\bar{h} > 0$.
- Since $A_F(t) = \exp(g_F t) A_F(0)$, the differential equation for $A(t)$ can be written as
$$\dot{A}(t) = gA(t) + \phi(h) A_F(0) \exp(g_F t).$$
- Solving this differential equation,
$$A(t) = \left[\left(\frac{A(0)}{g} - \frac{\phi(h) A_F(0)}{g_F - g}\right) \exp(gt) + \frac{\phi(h) A_F(0)}{g_F - g} \exp(g_F t)\right],$$
- Thus growth rate of $A(t)$ is faster when $\phi(h)$ is higher.

Nelson-Phelps Model of Human Capital IV

- Moreover, it can be verified that

$$A(t) \to \frac{\phi(h)}{g_F - g} A_F(t),$$

- Thus ratio of the technology in use to the frontier technology is also determined by human capital.
- This role of human capital is undoubtedly important in a number of situations:
  - educated farmers are more likely to adopt new technologies and seeds (e.g., Foster and Rosenzweig, 1995).
  - stronger correlation between economic growth and levels of human capital than between economic growth and changes in human capital.
- Human capital could be playing a more major role in economic growth and development than the discussion so far has suggested.

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- But:
  - If taking place within the firm's boundaries, this will be reflected in the marginal product of more skilled workers and taken into account in estimations.
  - If at the level of the labor market, this would be a form of local human capital externalities and it should have shown up in the estimates on local external effects of human capital.
  - So unless is also external and these external effects work at a global level, should not be seriously underestimating the contribution of human capital.

Conclusions

- Human capital differences are a major proximate cause of cross-country differences in economic performance.
- May also play an important role in the process of economic growth and economic development.
- Issues:
  1. If some part of the earnings of labor we observe are rewards to accumulated human capital, then the effect of policies (and perhaps technology) on income per capita could be larger.
  2. Measurement of the contribution of education and skills to productivity:
     - mismeasurement from human capital externalities, differences in human capital quality, differences in formal schooling.
  3. Possibility of an imbalance between physical and human capital and impact of human capital on aggregate productivity.
  4. Role of human capital, skills facilitating the adoption and implementation of new technologies.