

# Human Capital and Economic Growth

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Macroeconomics II

# Human Capital and Economic Growth

- Human capital: all the attributes of workers that potentially increase their productivity in all or some productive tasks.
- Can play a major role in economic growth and cross-country income differences.
- Which factors affect human capital investments and how these influence the process of economic growth and economic development.
- Human capital theory is the basis of much of labor economics and plays an equally important role in macroeconomics.
- Important connections between human capital and economic growth, especially related to its effect on technological progress, will be discussed later.

# A Simple Separation Theorem I

- Partial equilibrium schooling decisions.
- Continuous time.
- Schooling decision of a single individual facing exogenously given prices for human capital.
- Perfect capital markets.
- Separation theorem: with perfect capital markets, schooling decisions will maximize the net present discounted value of the individual.
- Instantaneous utility function  $u(c)$  that satisfies standard Assumptions on utility.
- Planning horizon of  $T$  (where  $T = \infty$  is allowed), discount  $\rho > 0$  and constant flow rate of death equal to  $\nu \geq 0$ .

# A Simple Separation Theorem II

- Standard arguments imply the objective function of this individual at time  $t = 0$  is

$$\max \int_0^T \exp(-(\rho + \nu) t) u(c(t)) dt. \quad (1)$$

- Individual is born with some human capital  $h(0) \geq 0$ .
- Human capital evolves over time according to

$$\dot{h}(t) = G(t, h(t), s(t)), \quad (2)$$

- $s(t) \in [0, 1]$  is the fraction of time spends for investments in schooling.
- $G : \mathbb{R}_+^2 \times [0, 1] \rightarrow \mathbb{R}_+$  determines how human capital evolves.

# A Simple Separation Theorem III

- Further restriction on schooling decisions,

$$s(t) \in \mathcal{S}(t), \quad (3)$$

- $\mathcal{S}(t) \subset [0, 1]$ : captures the fact that all schooling may have to be full-time, i.e.,  $s(t) \in \{0, 1\}$ , or other restrictions on schooling decisions.
- Exogenous sequence of wage per unit of human capital given by  $[w(t)]_{t=0}^T$ .
- Labor earnings at time  $t$  are

$$W(t) = w(t) [1 - s(t)] [h(t) + \omega(t)],$$

- $1 - s(t)$  is the fraction of time spent supplying labor to the market
- $\omega(t)$  is non-human capital labor that the individual may be supplying.

# A Simple Separation Theorem IV

- Sequence of  $[\omega(t)]_{t=0}^T$ , is exogenous: only margin of choice is between market work and schooling (i.e., there is no leisure).
- Individual faces a constant (flow) interest rate equal to  $r$  on his savings (potentially including annuity payments)).
- Using the equation for labor earnings, the lifetime budget constraint of the individual is

$$\int_0^T \exp(-rt) c(t) dt \leq \int_0^T \exp(-rt) w(t) [1 - s(t)] [h(t) + \omega(t)] dt \quad (4)$$

# A Simple Separation Theorem V

**Theorem (Separation Theorem)** Suppose that the instantaneous utility function  $u(\cdot)$  is strictly increasing. Then the sequence  $[c^*(t), s^*(t), h^*(t)]_{t=0}^T$  is a solution to the maximization of (1) subject to (2), (3) and (4) if and only if  $[s^*(t), h^*(t)]_{t=0}^T$  maximizes

$$\int_0^T \exp(-rt) w(t) [1 - s(t)] [h(t) + \omega(t)] dt \quad (5)$$

subject to (2) and (3), and  $[c^*(t)]_{t=0}^T$  maximizes (1) subject to (4) given  $[s^*(t), h^*(t)]_{t=0}^T$ . That is, human capital accumulation and supply decisions can be *separated* from consumption decisions.

# A Simple Separation Theorem VI

- Remember that under perfect capital markets the optimal consumption path depends only on total discounted life-time wealth (not on how that wealth evolves across time).
- So the optimal  $[s^*(t), h^*(t)]_{t=0}^T$  should maximize the life-time discounted wealth.
- This does not hold if markets are imperfect or agents also make leisure decisions.



# Schooling Investments and Returns to Education I

- Adaptation of Mincer (1974).
- Assume that  $T = \infty$
- Flow rate of death,  $\nu$ , is positive, so that individuals have finite expected lives.
- (2) is such that the individual has to spend an interval  $S$  with  $s(t) = 1$ —i.e., in full-time schooling, and  $s(t) = 0$  thereafter.
- At the end of the schooling interval, the individual will have a schooling level of

$$h(S) = \eta(S),$$

- $\eta(\cdot)$  is an increasing, continuously differentiable and concave function.
- For  $t \in [S, \infty)$ , human capital accumulates over time (as the individual works) according to

$$\dot{h}(t) = g_h h(t), \quad (6)$$

for some  $g_h \geq 0$ .

# Schooling Investments and Returns to Education II

- Wages grow exponentially,

$$\dot{w}(t) = g_w w(t), \quad (7)$$

with boundary condition  $w(0) > 0$ .

- Suppose that

$$g_w + g_h < r + \nu,$$

so that the net present discounted value of the individual is finite.

- Her optimal schooling policy must solve

$$\max_{\{\tau, S\}} \int_0^{\infty} \exp(-(r + \nu)t) w(t) (1 - S(t)) (h(t) + \omega(t)) dt$$

$$\text{s.t. } S(t) = 1 \text{ for } t \in [\tau, \tau + S]$$

$$h(t + S) = \eta(S)$$

$$\dot{h}(t) = g_h h(t) \text{ for } t \geq \tau + S$$

$$\dot{w}(t) = g_w w(t) \text{ for } t \geq 0$$

$$h(0) = 0$$

# Schooling Investments and Returns to Education III

- The last problem can be written as following

$$\begin{aligned}
 \max_{\{t, S\}} & \underbrace{\int_0^{\tau} \exp(-(r+v)t) w(t) \omega(t) dt}_{S=0} + \\
 & \underbrace{\int_{\tau}^{\tau+S} \exp(-(r+v)t) (0) dt}_{S=1} + \\
 & \underbrace{\int_{\tau+S}^{\infty} \exp(-(r+v)t) w(t) (h(t) + \omega(t)) dt}_{S=0}
 \end{aligned}$$

$$\text{s.t. } \dot{h}(t) = g_h h(t)$$

$$\dot{w}(t) = g_w w(t)$$

$$h(t+S) = \eta(S)$$

# Schooling Investments and Returns to Education IV

- which is the same as

$$\begin{aligned}
 & \max_{\{t, S\}} \int_0^{\tau} \exp(-(r + v)t) w(t) \omega(t) dt \\
 & \quad + \int_{\tau+S}^{\infty} \exp(-(r + v)t) w(t) (h(t) + \omega(t)) dt \\
 & \text{s.t. } \dot{h}(t) = g_h h(t) \\
 & \quad \dot{w}(t) = g_w w(t) \\
 & \quad h(t + S) = \eta(S).
 \end{aligned}$$

Replacing the constraints in the objective function, the problem is reduced to

$$\begin{aligned}
 & \max_{\{t, S\}} \int_0^{\tau} \exp(-(r + v)t) \exp(g_w t) w(0) \omega(t) dt \\
 & \quad + \int_{\tau+S}^{\infty} \exp(-(r + v)t) \exp(g_w t) w(0) (h(t) + \omega(t)) dt
 \end{aligned}$$

# Schooling Investments and Returns to Education IV

$$\begin{aligned}
 &\Leftrightarrow \max_{\{t,S\}} \int_0^{\tau} \exp(-(r + v - g_w)t) w(0) \omega(t) dt \\
 &\quad + \int_{\tau+S}^{\infty} \exp(-(r + v - g_w)t) w(0) (h(S) \exp(g_h(t - S)) + \omega(t)) dt \\
 &\Leftrightarrow \max_{\{t,S\}} \int_0^{\tau} \exp(-(r + v - g_w)t) w(0) \omega(t) dt \\
 &\quad + \int_{\tau+S}^{\infty} \exp(-(r + v - g_w - g_h)t) \exp(-g_h S) w(0) h(S) dt \\
 &\quad + \int_{\tau+S}^{\infty} \exp(-(r + v - g_w)t) w(0) \omega(t) dt
 \end{aligned}$$

# Schooling Investments and Returns to Education V

$$\begin{aligned}
 \Leftrightarrow \max_{\{t, S\}} & \int_0^{\infty} \exp(-(r + v - g_w)t) w(0) \omega(t) dt \\
 & - \underbrace{\int_{\tau}^{\tau+S} \exp(-(r + v - g_w)t) w(0) \omega(t) dt}_{<0} \\
 & + \exp(-g_h S) h(s) w(0) \int_{\tau+S}^{\infty} \exp(-(r + v - g_w - g_h)t) dt,
 \end{aligned}$$

where if  $\omega(t)$  grows at a rate larger than  $r + v - g_w$ , then  $\tau^* = 0$ .

This implies that

$$\begin{aligned}
 & \max_{\{S\}} \exp(-g_h S) w(0) \eta(S) \int_S^{\infty} \exp(-(r + v - g_w - g_h)t) dt \\
 & = \max_{\{s\}} \exp(-g_h S) w(0) \eta(S) \frac{1}{r + v - g_w - g_h} \exp(-(r + v - g_w - g_h)S) \\
 & = \max_{\{s\}} \frac{w(0) \eta(S) \exp(-(r + v - g_w)S)}{r + v - g_w - g_h}
 \end{aligned}$$

# Schooling Investments and Returns to Education VI

- Thus solution to the previous problem of optimal schooling is equivalent to solving

$$\max_S \int_S^{\infty} \exp(-(r + \nu)t) w(t) h(t) dt. \quad (8)$$

using the Separation Theorem.

- Now using (6) and (7), this is equivalent to:

$$\max_S \frac{w(0) \eta(S) \exp(-(r + \nu - g_w)S)}{r + \nu - g_h - g_w}. \quad (9)$$

- Since  $\eta(S)$  is concave, the objective function in (9) is strictly concave.

# Schooling Investments and Returns to Education VII

- Therefore, the unique solution to this problem is characterized by the first-order condition

$$\frac{\eta'(S^*)}{\eta(S^*)} = r + \nu - g_w, \quad (10)$$

where

$$S^* = S(r, \nu, g_w)$$

and  $S^*$  is not function of  $g_h$ . Using implicit function theorem, it can be shown that  $S_r < 0$ ,  $S_\nu < 0$  and  $S_{g_w} > 0$ .

- Higher interest rates and higher values of  $\nu$  (shorter planning horizons) reduce human capital investments.
- Higher values of  $g_w$  increase the value of human capital and thus encourage further investments.
- Integrating both sides of this equation with respect to  $S$ ,

$$\ln \eta(S^*) = \text{constant} + (r + \nu - g_w) S^*. \quad (11)$$



# Schooling Investments and Returns to Education VIII

- Now note that the wage earnings of the worker of age  $\tau \geq S^*$  in the labor market at time  $t$  will be given by

$$W(S, t) = \exp(g_w t) \exp(g_h(t - S)) \eta(S).$$

- Taking logs and using equation (11) implies that the earnings of the worker will be given by

$$\ln W(S^*, t) = \text{constant} + (r + \nu - g_w) S^* + g_w t + g_h(t - S^*),$$

- $t - S$  can be thought of as worker experience (time after schooling).
- If we make a cross-sectional comparison across workers, the time trend term  $g_w t$ , will also go into the constant.
- Hence obtain the canonical Mincer equation where, in the cross section, log wage earnings are proportional to schooling and experience.

# Schooling Investments and Returns to Education IX

- Written differently, we have the following cross-sectional equation

$$\ln W_j = \text{constant} + \gamma_s S_j + \gamma_e \text{experience}, \quad (12)$$

where  $j$  refers to individual  $j$ .

- But have not introduced any source of heterogeneity that can generate different levels of schooling across individuals.
- Economic insight: functional form of the Mincerian wage equation is not just a mere coincidence, but has economic content.
  - Opportunity cost of one more year of schooling is foregone earnings.
  - Thus benefit has to be commensurate with these foregone earnings, should lead to a proportional increase in earnings in the future.
  - This proportional increase should be at the rate  $(r + v - g_w)$ .

# Schooling Investments and Returns to Education VI

- Empirical work using equations of the form (12) leads to estimates for  $\gamma$  in the range of 0.06 to 0.10.
- Equation (12) suggests that these returns to schooling are not unreasonable.
  - $r$  as approximately 0.10,  $\nu$  as corresponding to 0.02 that gives an expected life of 50 years, and  $g_w$  approximately about 2%.
  - Implies  $\gamma$  around 0.10.

# The Ben-Porath Model I

- Ben-Porath: enriches the model by allowing human capital investments and non-trivial labor supply decisions.
- Now let  $s(t) \in [0, 1]$  for all  $t \geq 0$ .
- Human capital accumulation equation, (2), takes the form

$$\dot{h}(t) = \phi(s(t)h(t)) - \delta_h h(t), \quad (13)$$

- $\delta_h > 0$  captures “depreciation of human capital.”
- The individual starts with an initial value of human capital  $h(0) > 0$ .
- The function  $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is strictly increasing, continuously differentiable and strictly concave.
- Furthermore, we simplify by assuming Inada-type conditions,

$$\lim_{x \rightarrow 0} \phi'(x) = \infty \text{ and } \lim_{x \rightarrow h(0)} \phi'(x) = 0.$$

# The Ben-Porath Model II

- Latter condition makes sure that we do not have to impose additional constraints to ensure  $s(t) \in [0, 1]$ .
- No non-human capital component of labor, so that  $\omega(t) = 0$  for all  $t$ .
- $T = \infty$ , and that there is a flow rate of death  $\nu > 0$ .
- Wage per unit of human capital is constant at  $w$  and the interest rate is constant and equal to  $r$ .
- Normalize  $w = 1$ .
- Again using the Separation Theorem, human capital investments can be determined as a solution to

$$\max \int_0^{\infty} \exp(-(r + \nu)) (1 - s(t)) h(t) dt$$

subject to (13) and  $0 \leq s(t) \leq 1$ .

# The Ben-Porath Model III

- Current-value Hamiltonian,

$$\mathcal{H}(h, s, \mu) = (1 - s(t)) h(t) + \mu(t) (\phi(s(t) h(t)) - \delta_h h(t)) \\ + \lambda_1(t) (1 - s(t)) + \lambda_2(t) s(t).$$

- Necessary conditions for this problem are

$$\begin{aligned} \mathcal{H}_s(h, s, \mu) &= -h(t) + \mu(t) h(t) \phi'(s(t) h(t)) - \lambda_1(t) + \lambda_2(t) = 0 \\ \mathcal{H}_h(h, s, \mu) &= (1 - s(t)) + \mu(t) (s(t) \phi'(s(t) h(t)) - \delta_h) \\ &= (r + \nu) \mu(t) - \dot{\mu}(t) \\ 0 &= \lim_{t \rightarrow \infty} \exp(-(r + \nu)t) \mu(t) h(t), \end{aligned}$$

and  $\lambda(t)(1 - s(t)) = 0$ , with  $\lambda_1(t) \geq 0$  and  $\lambda_2(t) = 0$  if  $s(t) = 1$ ,  $\lambda_1(t) = 0$  and  $\lambda_2(t) \geq 0$  if  $s(t) = 0$  and  $\lambda_1(t) = \lambda_2(t) = 0$  if  $s(t) \in (0, 1)$ .

- Assuming an interior solution for  $s(t)$ , that is for  $s(t) \in (0, 1)$ , from the first FOC,  $\mu(t) \phi'(s(t) h(t)) = 1$  with  $\lambda_1(t) = \lambda_2(t) = 0$ . More details in Exercise 10.6.

# The Ben-Porath Model IV

- Adopt the following transformation of variables:

$$x(t) \equiv s(t) h(t).$$

- Study the dynamics of the optimal path in  $x(t)$  and  $h(t)$ .
- The first necessary condition then implies that

$$1 = \mu(t) \phi'(x(t)), \quad (14)$$

- Second necessary condition can be expressed as

$$\frac{\dot{\mu}(t)}{\mu(t)} = r + v + \delta_h - s(t) \phi'(x(t)) - \frac{1 - s(t)}{\mu(t)}.$$

- Substituting for  $\mu(t)$  from (14), and simplifying,

$$\frac{\dot{\mu}(t)}{\mu(t)} = r + v + \delta_h - \phi'(x(t)). \quad (15)$$

# The Ben-Porath Model V

- Steady-state (stationary) solution involves  $\dot{\mu}(t) = 0$  and  $\dot{h}(t) = 0$ , and thus

$$x^* = \phi'^{-1}(r + v + \delta_h), \quad (16)$$

- $\phi'^{-1}(\cdot)$  exists and is strictly decreasing since  $\phi(\cdot)$  is strictly concave.
- Using implicit function theorem, it is possible to show that  $x_r^* < 0$ ,  $x_v^* < 0$  and  $x_{\delta_h}^* < 0$ .
- Implies  $x^* \equiv s^* h^*$  will be higher when  $r$  is low, when  $1/v$  is high, and when  $\delta_h$  is low. Set  $\dot{h}(t) = 0$  in the human capital accumulation equation (13), which gives

$$\begin{aligned} h^* &= \frac{\phi(x^*)}{\delta_h} \\ &= \frac{\phi(\phi'^{-1}(r + v + \delta_h))}{\delta_h} \end{aligned} \quad (17)$$

$$h^* = h(r, v, \delta_h) \quad (18)$$



# The Ben-Porath Model V

- Since  $\phi'^{-1}(\cdot)$  is strictly decreasing and  $\phi(\cdot)$  is strictly increasing, steady-state  $h^*$  is uniquely determined and is decreasing in  $r$ ,  $\nu$  and  $\delta_h$ .
- Using implicit function theorem, it is possible to show that  $h_r^* < 0$ ,  $h_\nu^* < 0$  and  $h_{\delta_h}^* < 0$ .
- It is also true that  $s^* = \frac{x^*}{h^*} = \frac{x^* \delta_h}{\phi(x^*)} = \frac{\delta_h \phi'^{-1}(r+\nu+\delta_h)}{\phi(\phi'^{-1}(r+\nu+\delta_h))}$ .

# The Ben-Porath Model VI

- Path of human capital investment: differentiate (14) with respect to time to obtain

$$\begin{aligned}\frac{\dot{\mu}(t)}{\mu(t)} &= -\frac{1}{\phi'(x(t))}\phi''(x(t)) \\ \frac{\dot{\mu}(t)}{\mu(t)} &= \varepsilon_{\phi'}(x) \frac{\dot{x}(t)}{x(t)},\end{aligned}$$

where

$$\varepsilon_{\phi'}(x) = -\frac{x\phi''(x)}{\phi'(x)} > 0$$

is the elasticity of the function  $\phi'(\cdot)$  and is positive since  $\phi'(\cdot)$  is strictly decreasing (thus  $\phi''(\cdot) < 0$ ).

- Combining this equation with (15),

$$\frac{\dot{x}(t)}{x(t)} = \frac{1}{\varepsilon_{\phi'}(x(t))} (r + v + \delta_h - \phi'(x(t))). \quad (19)$$

- Figure plots (13) and (19) in the  $h$ - $x$  space.

# The Ben-Porath Model VII

- Recall that  $x^* = \phi'^{-1}(r + v + \delta_h)$  and  $h^* = \frac{\phi'(x^*)}{\delta_h} = \frac{\phi'(\phi'^{-1}(r+v+\delta_h))}{\delta_h}$ .
- The system exhibits a globally saddle path stable, so for any  $h^* > h(0) > 0$  given,  $s(0) = \frac{x^*}{h(0)}$  and then as  $h(t)$  increases,  $s(t)$  decreases.
- On the other hand, if  $1 < \frac{x^*}{h(t)}$ , then  $s(t) = 1$ , then the first order conditions imply

$$\begin{aligned}\frac{\dot{\mu}(t)}{\mu(t)} &= r + v + \delta_h - \phi'(h(t)) \\ \dot{h}(t) &= \phi(h(t)) - \delta_h h(t) \\ \lambda_1(t) - \lambda_2(t) &= \underbrace{\mu(t)\phi(h(t))}_{<1} h(t) - h(t),\end{aligned}$$

which implies that  $\lambda_1(t) \leq 1$ . Then  $\frac{x^*}{h(0)} > 1$  if and only if  $x^* > h(0)$ .

**CHECK**

# The Ben-Porath Model VIII

FIGURE

# The Ben-Porath Model IX

- Here all happens smoothly.
- Original Ben-Porath model involves the use of other inputs in the production of human capital and finite horizons.
  - Constraint for  $s(t) \leq 1$  typically binds early on in the life, and the interval during which  $s(t) = 1$  can be interpreted as full-time schooling.
  - After full-time schooling, the individual starts working (i.e.,  $s(t) < 1$ ), but continues to accumulate human capital.
  - Because the horizon is finite, if the Inada conditions were relaxed, the individual could prefer to stop investing in human capital at some point.
  - Time path of human capital generated by the standard Ben-Porath model may be hump-shaped
  - Path of human capital (and the earning potential of the individual) in the current model is always increasing.

# The Ben-Porath Model X

- Importance of Ben-Porath model
  - ① Schooling is not the only way to invest in human capital; continuity between schooling investments and other investments.
  - ② In societies where schooling investments are high we may also expect higher levels of on-the-job investments in human capital.
    - Thus there may be systematic mismeasurement of the amount or the quality human capital across societies.

FIGURE

# Neoclassical Growth with Physical and Human Capital I

- Physical-human capital interactions could potentially be important.
- Evidence suggests are complementary: greater capital increases productivity of high human capital workers more than of low skill workers.
- May induce a “virtuous cycle” of investments in physical and human capital.
- Potential for complementarities also raises the issue of “imbalances”.
  - Highest productivity when there is a balance between the two types of capital.
  - Will decentralized equilibrium ensure such a balance?
- Continuous time economy admitting a representative household with preferences

$$\int_0^{\infty} \exp(-\rho t) u(c(t)) dt, \quad (20)$$

- $u(\cdot)$  satisfies standard Assumptions on utility and  $\rho > 0$ .



# Neoclassical Growth with Physical and Human Capital II

- Ignore technological progress and population growth.
- Aggregate production function:

$$Y(t) = F(K(t), H(t), L(t)),$$

- $K(t)$  is the stock of physical capital,  $L(t)$  is total employment, and  $H(t)$  represents human capital.
- No population growth and labor is supplied inelastically,  $L(t) = L$  for all  $t$ .
- Production function satisfies Assumptions 1 and 2 generalized to production function with three inputs.
- “Raw” labor and human capital as separate factors of production may be less natural than human capital increasing effective units of labor. But allows a simple analysis.

# Neoclassical Growth with Physical and Human Capital III

- Express all objects in per capita units, thus we write

$$\begin{aligned} y(t) &\equiv \frac{Y(t)}{L} \\ &= f(k(t), h(t)), \end{aligned}$$

where

$$k(t) \equiv \frac{K(t)}{L} \text{ and } h(t) \equiv \frac{H(t)}{L}$$

- In view of standard assumptions  $f(k, h)$  is strictly increasing, continuously differentiable and jointly strictly concave in both of its arguments.
- Physical and human capital are complementary, that is,  $f_{kh}(k, h) > 0$  for all  $k, h > 0$ .

# Neoclassical Growth with Physical and Human Capital IV

- Physical and human capital per capita evolve according to

$$\dot{k}(t) = i_k(t) - \delta_k k(t), \quad (21)$$

and

$$\dot{h}(t) = i_h(t) - \delta_h h(t) \quad (22)$$

- $i_k(t)$  and  $i_h(t)$  are the investment levels in physical and human capital, while  $\delta_k$  and  $\delta_h$  are the depreciation rates.
- Resource constraint for the economy, in per capita terms,

$$c(t) + i_k(t) + i_h(t) \leq f(k(t), h(t)) \text{ for all } t. \quad (23)$$

- Equilibrium and optimal growth will coincide.
- Focus on the optimal growth problem: maximization of (20) subject to (21), (22), and (23).

# Neoclassical Growth with Physical and Human Capital V

- First observe that since  $u(c)$  is strictly increasing, (23) will always hold as equality.
- Substitute for  $c(t)$  using this constraint and write the current-value Hamiltonian,

$$\begin{aligned} & \mathcal{H}(k(t), h(t), i_k(t), i_h(t), \mu_k(t), \mu_h(t)) \\ = & u(f(k(t), h(t)) - i_h(t) - i_k(t)) \\ & + \mu_h(t)(i_h(t) - \delta_h h(t)) + \mu_k(t)(i_k(t) - \delta_k k(t)), \end{aligned} \quad (24)$$

- Two control variables,  $i_k(t)$  and  $i_h(t)$  and two state variables,  $k(t)$  and  $h(t)$ , two costate variables,  $\mu_k(t)$  and  $\mu_h(t)$ , corresponding to (21) and (22).

# Neoclassical Growth with Physical and Human Capital VI

- The conditions for a candidate optimal solution are

$$\mathcal{H}_{i_k}(\cdot) = -u'(c(t)) + \mu_k(t) = 0$$

$$\mathcal{H}_{i_h}(\cdot) = -u'(c(t)) + \mu_h(t) = 0$$

$$\begin{aligned}\mathcal{H}_k(\cdot) &= f_k(k(t), h(t)) u'(c(t)) - \mu_k(t) \delta_k \\ &= \rho \mu_k(t) - \dot{\mu}_k(t)\end{aligned}$$

$$\begin{aligned}\mathcal{H}_h(\cdot) &= f_h(k(t), h(t)) u'(c(t)) - \mu_h(t) \delta_h \\ &= \rho \mu_h(t) - \dot{\mu}_h(t)\end{aligned}$$

$$0 = \lim_{t \rightarrow \infty} \exp(-\rho t) \mu_k(t) k(t)$$

$$0 = \lim_{t \rightarrow \infty} \exp(-\rho t) \mu_h(t) h(t).$$

- Two necessary transversality conditions, two state variables (and two costate variables).

# Neoclassical Growth with Physical and Human Capital VII

- Needed to verify that  $\mathcal{H}(\cdot)$  is concave given the costate variables  $\mu_k(t)$  and  $\mu_h(t)$ , so the above conditions give the unique optimal path.
- The first two conditions immediately imply that

$$\mu_k(t) = \mu_h(t) = \mu(t).$$

- Combining this with the next two conditions gives

$$f_k(k(t), h(t)) - f_h(k(t), h(t)) = \delta_k - \delta_h, \quad (25)$$

- Together with  $f_{kh} > 0$  implies that there is a one-to-one relationship between physical and human capital, of the form

$$h = \xi(k),$$

where  $\xi(\cdot)$  is uniquely defined, strictly increasing and differentiable.

# Neoclassical Growth with Physical and Human Capital VIII

**Proposition** In the neoclassical growth model described above, the optimal path of physical capital and consumption are given as in the one-sector neoclassical growth model, and satisfy the following two differential equations

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\varepsilon_u(c(t))} [f_k(k(t), \xi(k(t))) - \delta_k - \rho],$$

$$\dot{k}(t) = \frac{1}{1 + \xi'(k)} \begin{bmatrix} f(k(t), \xi(k(t))) - \delta_h \xi(k(t)) \\ -\delta_k k(t) - c(t) \end{bmatrix},$$

where  $\varepsilon_u(c(t)) = -u''(c(t))c(t)/u'(c(t))$ , together with  $\lim_{t \rightarrow \infty} \left[ k(t) \exp \left( - \int_0^t f_k(k(s), \xi(k(s))) ds \right) \right] = 0$ , while  $h(t) = \xi(k(t))$ .

# Neoclassical Growth with Physical and Human Capital IX

- Surprising: (25) implies that human and physical capital are always in “balance”.
  - May have conjectured that economy that starts with high stock of physical relative to human capital will have a relatively high physical to human capital ratio for an extended period of time.
  - But we have not imposed any non-negativity constraints on the investment levels.
  - Such economy at the first instant it will experience a very high level of  $i_h(0)$ , compensated with a very negative  $i_k(0)$ .
  - After this, the dynamics of the economy will be identical to those of the baseline neoclassical growth model.



# Neoclassical Growth with Physical and Human Capital X

- Different when there are non-negativity or “irreversibility” constraints.
  - If we assume that  $i_k(t) \geq 0$  and  $i_h(t) \geq 0$  for all  $t$ , initial imbalances will persist for a while.
  - Starting with a ratio  $k(0)/h(0)$  that does not satisfy (25), optimal path will involve investment only in one of the two stocks until balance is reached.
  - Some amount of imbalance can arise, but the economy quickly moves towards correcting this imbalance.

# Neoclassical Growth with Physical and Human Capital XI

- Impact of policy distortions: suppose resource constraint of the economy modified to

$$c(t) + (1 + \tau)(i_k(t) + i_h(t)) \leq f(k(t), h(t)),$$

- $\tau \geq 0$  is a tax affecting both types of investments.
- Suppose that the aggregate production function takes the Cobb-Douglas form

$$\begin{aligned} Y &= F(K, H, L) \\ &= K^{\alpha_k} H^{\alpha_h} L^{1-\alpha_k-\alpha_h}. \end{aligned}$$

- Ratio steady-state income of income in the two economies with taxes/distortions of  $\tau$  and  $\tau'$  is given by:

$$\frac{Y(\tau)}{Y(\tau')} = \left( \frac{1 + \tau'}{1 + \tau} \right)^{\frac{\alpha_k + \alpha_h}{1 - \alpha_k - \alpha_h}}. \quad (26)$$

# Neoclassical Growth with Physical and Human Capital XII

- Responsiveness of human capital accumulation to these distortions increases impact of distortions. E.g., with  $\alpha_k = \alpha_h = 1/3$  and eightfold distortion differences,

$$\frac{Y(\tau)}{Y(\tau')} \approx 8^2 \approx 64,$$

- But has to be interpreted with caution:
  - ① Driven by a very elastic response of human capital accumulation:
    - e.g. if distortions correspond to differences in corporate taxes or corruption, may affect corporations rather than individual human capital decisions.
  - ② Obvious similarity to Mankiw-Romer-Weil's approach:
    - existing evidence does not support the notion that human capital differences across countries can have such a large impact.

# Capital-Skill Complementarity in an Overlapping Generations Model I

- Capital-skill imbalances in a simple overlapping generations model with impure altruism.
- Also generates only limited capital-skill imbalances.
- But capital-skill imbalances become much more important.
- Economy is in discrete time and consists of a continuum 1 of dynasties.
- Each individual lives for two periods, childhood and adulthood.
- Individual  $i$  of generation  $t$  works during their adulthood at time  $t$ , earns labor income equal to  $w(t) h_i(t)$ .
- Individual also earns capital income equal to  $R(t) b_i(t-1)$ .
- Human capital of the individual is determined at the beginning of his adulthood by an effort decision.
- Labor is supplied to the market after this effort decision.

# Capital-Skill Complementarity in an Overlapping Generations Model II

- At the end of adulthood, after labor and capital incomes are received, individual decides his consumption and the level of bequest.
- Preferences of individual  $i$  (or of dynasty  $i$ ) of generation  $t$  are given by

$$\eta^{-\eta} (1 - \eta)^{-(1-\eta)} c_i(t)^\eta b_i(t)^{1-\eta} - \gamma(e_i(t)),$$

- $\eta \in (0, 1)$ ,  $c_i(t)$  is own consumption,  $b_i(t)$  is the bequest to the offspring,  $e_i(t)$  is effort expended for human capital acquisition.
- $\gamma(\cdot)$  is a strictly increasing, continuously differentiable and strictly convex cost of effort function.
- $\eta^{-\eta} (1 - \eta)^{-(1-\eta)}$  is included as a normalizing factor to simplify the algebra.
- Human capital of individual  $i$  is given by

$$h_i(t) = a e_i(t), \quad (27)$$

# Capital-Skill Complementarity in an Overlapping Generations Model III

- $a$  corresponds to “ability”.
- Substituting for  $e_i(t)$  in the above expression, the preferences of individual  $i$  of generation  $t$  can be written as

$$\eta^{-\eta} (1 - \eta)^{-(1-\eta)} c_i(t)^\eta b_i(t)^{1-\eta} - \gamma \left( \frac{h_i(t)}{a} \right). \quad (28)$$

- The budget constraint of the individual is

$$c_i(t) + b_i(t) \leq m_i(t) = w(t) h_i(t) + R(t) b_i(t-1), \quad (29)$$

- Defines  $m_i(t)$  as the current income of individual  $i$  at time  $t$  consisting of labor earnings,  $w(t) h_i(t)$ , and asset income,  $R(t) b_i(t-1)$ .

# Capital-Skill Complementarity in an Overlapping Generations Model IV

- Aggregate production function

$$Y(t) = F(K(t), H(t)),$$

that satisfies Assumptions 1 and 2.

- $H(t)$  is “effective units of labor” or alternatively the total stock of human capital given by,

$$H(t) = \int_0^1 h_i(t) di,$$

- $K(t)$ , the stock of physical capital, is given by

$$K(t) = \int_0^1 b_i(t-1) di.$$

# Capital-Skill Complementarity in an Overlapping Generations Model V

- Production function with two factors and constant returns to scale necessarily implies that the two factors are complements,

$$\frac{\partial^2 F(K, H)}{\partial K \partial H} \geq 0. \quad (30)$$

- Simplify the notation by assuming capital depreciates fully after use, that is,  $\delta = 1$ .
- More useful to define a normalized production function expressing output per unit of human capital.



# Capital-Skill Complementarity in an Overlapping Generations Model VI

- Let  $\kappa \equiv K/H$  be the capital to human capital ratio (or the “effective capital-labor ratio”), and

$$\begin{aligned} y(t) &\equiv \frac{Y(t)}{H(t)} \\ &= F\left(\frac{K(t)}{H(t)}, 1\right) \\ &= f(\kappa(t)), \end{aligned}$$

- Second line uses the linear homogeneity of  $F(\cdot, \cdot)$ , last line uses the definition of  $\kappa$ .

# Capital-Skill Complementarity in an Overlapping Generations Model VII

- From the definition of  $\kappa$ , the law of motion of effective capital-labor ratios can be written as

$$\kappa(t) \equiv \frac{K(t)}{H(t)} = \frac{\int_0^1 b_i(t-1) di}{\int_0^1 h_i(t) di}. \quad (31)$$

- Factor prices are then given by the usual competitive pricing formulae:

$$R(t) = f'(\kappa(t)) \text{ and } w(t) = f(\kappa(t)) - \kappa(t) f'(\kappa(t)), \quad (32)$$

- $w(t)$  is now wage per unit of human capital, in a way consistent with (29).

# Capital-Skill Complementarity in an Overlapping Generations Model VIII

- An equilibrium in this overlapping generations economy is a sequence  $\left\{ [h_i(t)]_{i \in [0,1]}, [c_i(t)]_{i \in [0,1]}, [b_i(t)]_{i \in [0,1]} \right\}_{t=0}^{\infty}$ , that solve (28) subject to (29) a sequence  $\{\kappa(t)\}_{t=0}^{\infty}$  given by (31) with some initial distribution of bequests  $[b_i(0)]_{i \in [0,1]}$ , and sequences  $\{w(t), R(t)\}_{t=0}^{\infty}$  that satisfy (32).
- Solution to the maximization problem of (28) subject to (29) involves

$$c_i(t) = \eta m_i(t) \text{ and } b_i(t) = (1 - \eta) m_i(t), \quad (33)$$

- Substituting these into (28), we obtain the indirect utility function:

$$m_i(t) - \gamma \left( \frac{h_i(t)}{a} \right), \quad (34)$$

# Capital-Skill Complementarity in an Overlapping Generations Model IX

- Individual maximizes it by choosing  $h_i(t)$  and recognizing that  $m_i(t) = w(t) h_i(t) + R(t) b_i(t-1)$ .
- First-order condition of this maximization gives the human capital investment of individual  $i$  at time  $t$  as:

$$aw(t) = \gamma' \left( \frac{h_i(t)}{a} \right), \quad (35)$$

- Or inverting this relationship and using (32),

$$h_i(t) = h(t) \equiv a\gamma'^{-1} [a(f(\kappa(t)) - \kappa(t)f'(\kappa(t)))] . \quad (36)$$

- Important implication: human capital investment of each individual is identical, and only depends on the effective of capital-labor ratio in the economy.

# Capital-Skill Complementarity in an Overlapping Generations Model X

- Consequence of the specific utility function in (28):
  - no income effects so all agents choose the same “income-maximizing” level of human capital (as in Separation Theorem).
- Since bequest decisions are linear as shown (33),

$$\begin{aligned}
 K(t+1) &= \int_0^1 b_i(t) di \\
 &= (1-\eta) \int_0^1 m_i(t) di \\
 &= (1-\eta) f(\kappa(t)) h(t),
 \end{aligned}$$

# Capital-Skill Complementarity in an Overlapping Generations Model XI

- Last line uses the fact that, since all individuals choose the same human capital level given by (36),  $H(t) = h(t)$ , and thus  $Y(t) = f(\kappa(t)) h(t)$ .
- Combining this with (31),

$$\kappa(t+1) = \frac{(1-\eta) f(\kappa(t)) h(t)}{h(t+1)}.$$

- Using (36), this becomes

$$\begin{aligned} & \kappa(t+1) \gamma'^{-1} [a(f(\kappa(t+1)) - \kappa(t+1) f'(\kappa(t+1)))] \quad (37) \\ = & (1-\eta) f(\kappa(t)) \gamma'^{-1} [a f(\kappa(t)) - \kappa(t) f'(\kappa(t))]. \end{aligned}$$

# Capital-Skill Complementarity in an Overlapping Generations Model XII

- A steady state involves  $\kappa(t) = \kappa^*$  for all  $t$ .
- Substituting this into (37) yields

$$\kappa^* = (1 - \eta) f(\kappa^*), \quad (38)$$

- Defines the unique positive steady-state effective capital-labor ratio,  $\kappa^*$  (since  $f(\cdot)$  is strictly concave).

**Proposition** There exists a unique steady state with positive activity, and the physical to human capital ratio is  $\kappa^*$  as given by (38).

# Capital-Skill Complementarity in an Overlapping Generations Model XIII

- This steady-state equilibrium is also typically stable, but some additional conditions need to be imposed on the  $f(\cdot)$  and  $\gamma(\cdot)$ .
- Capital-skill ( $k$ - $h$ ) complementarity in the production function  $F(\cdot, \cdot)$  implies that a certain target level of physical to human capital ratio,  $\kappa^*$ , has to be reached in equilibrium.
- I.e., does not allow equilibrium “imbalances” between physical and human capital either.
- Introducing such imbalances: depart from perfectly competitive labor markets.



# Physical and Human Capital with Imperfect Labor Markets

I

- Deviate from the competitive pricing formula (32).
- Economy is identical to that described in the previous section, except that there is a measure 1 of firms as well as a measure 1 of individuals.
- Each firm can only hire one worker.
- Production function of each firm is still given by

$$y_j(t) = F(k_j(t), h_i(t)),$$

- $y_j(t)$  refers to the output of firm  $j$ ,  $k_j(t)$  is its capital stock (also per worker, since the firm is hiring only one worker).
- $h_i(t)$  is the human capital of worker  $i$  that the firm has matched with.
- Again satisfies Assumptions 1 and 2.

# Physical and Human Capital with Imperfect Labor Markets II

- Now assume the following structure for the labor market:
  - Firms choose physical capital level irreversibly (incurring cost  $R(t) k_j(t)$ ), and simultaneously workers choose their human capital level irreversibly.
  - After workers complete human capital investments, they are randomly matched with firms. High human capital workers are *not* more likely to be matched with high physical capital firms.
  - After matching, each worker-firm pair bargains over the division of output. Divide output according to some pre-specified rule, worker receives total earnings of

$$W_j(k_j(t), h_i(t)) = \lambda F(k_j(t), h_i(t)),$$

for some  $\lambda \in (0, 1)$ .

# Physical and Human Capital with Imperfect Labor Markets III

- Introduce heterogeneity in the cost of human capital acquisition by modifying (27) to

$$h_i(t) = a_i e_i(t),$$

- $a_i$  differs across dynasties (individuals).
- Equilibrium is defined similarly but factor prices are no longer determined by (32).
- Firm chooses physical unsure about the human capital of the worker he will be facing.

# Physical and Human Capital with Imperfect Labor Markets IV

- Therefore, the expected return of firm  $j$  can be written as

$$(1 - \lambda) \int_0^1 F(k_j(t), h_i(t)) di - R(t) k_j(t). \quad (39)$$

- Notice (39) is strict concave in  $k_j(t)$  given the strict concavity of  $F(\cdot, \cdot)$  from Assumption 1.
- Therefore, each firm will choose the same level of physical capital,  $\hat{k}(t)$ , such that

$$(1 - \lambda) \int_0^1 \frac{\partial F(\hat{k}(t), h_i(t))}{\partial k(t)} di = R(t).$$

- Given this and following (34) from the previous section, each worker's objective function can be written as:

$$\lambda F(\hat{k}(t), h_i(t)) + R(t) b_i(t-1) - \gamma \left( \frac{h_i(t)}{a_i} \right),$$

- Have substituted for the income  $m_i(t)$ .

# Physical and Human Capital with Imperfect Labor Markets

## V

- Implies the following choice of human capital investment by a worker  $i$ :

$$\lambda a_i \frac{\partial F(\hat{k}(t), h_i(t))}{\partial h_i(t)} = \gamma' \left( \frac{h_i(t)}{a_i} \right).$$

- Yields unique equilibrium human capital investment  $\hat{h}_i(\hat{k}(t))$  for each  $i$ .
- Directly depends on the capital choices of all the firms,  $\hat{k}(t)$  and also depends implicitly on  $a_i$ .
- Moreover, given (30),  $\hat{h}_i(\hat{k}(t))$  is strictly increasing in  $\hat{k}(t)$ .
- Also, since  $\gamma(\cdot)$  is strictly convex,  $\hat{h}_i(\hat{k}(t))$  is a strictly concave function of  $\hat{k}(t)$ .

# Physical and Human Capital with Imperfect Labor Markets VI

- Substituting this into the first-order condition of firms,

$$(1 - \lambda) \int_0^1 \frac{\partial F(\hat{k}(t), \hat{h}_i(\hat{k}(t)))}{\partial k(t)} di = R(t).$$

- Finally, to satisfy market clearing in the capital market, the rate of return to capital,  $R(t)$ , has to adjust, such that

$$\hat{k}(t) = \int_0^1 b_i(t-1) di,$$

- Follows from the facts that all firms choose the same level of capital investment and that the measure of firms is normalized to 1.
- Implies that in the closed economy version of the current model, capital per firm is fixed by bequest decisions from the previous period.

# Physical and Human Capital with Imperfect Labor Markets VII

- Main economic forces are seen more clearly when physical capital is not predetermined.
- Thus imagine economy in question is small and open, so that  $R(t) = R^*$ .
- Under this assumption, the equilibrium level of capital per firm is determined by

$$(1 - \lambda) \int_0^1 \frac{\partial F(\hat{k}, \hat{h}_i(\hat{k}))}{\partial k} di = R^*. \quad (40)$$

# Physical and Human Capital with Imperfect Labor Markets

## VIII

**Proposition** In the open economy version of the model described here, there exists a unique positive level of capital per worker  $\hat{k}$  given by (40) such that the equilibrium capital per worker is always equal to  $\hat{k}$ . Given  $\hat{k}$ , the human capital investment of worker  $i$  is uniquely determined by  $\hat{h}_i(\hat{k})$  such that

$$\lambda a_i \frac{\partial F(\hat{k}, \hat{h}_i(\hat{k}))}{\partial h} = \gamma' \left( \frac{\hat{h}_i(\hat{k})}{a_i} \right). \quad (41)$$

We have that  $\hat{h}_i(\hat{k})$  is increasing in  $\hat{k}$ , and a decline in  $R^*$  increases  $\hat{k}$  and  $\hat{h}_i$  for all  $i \in [0, 1]$ .

In addition to this equilibrium, there also exists a no-activity equilibrium in which  $\hat{k} = 0$  and  $\hat{h}_i = 0$  for all  $i \in [0, 1]$ .



# Proof of Proposition

- Since  $F(k, h)$  exhibits constant returns to scale and  $\hat{h}_i(\hat{k})$  is a concave function of  $\hat{k}$  for each  $i$ ,  $\int_0^1 (\partial F(\hat{k}, \hat{h}_i(\hat{k})) / \partial k) di$  is decreasing in  $\hat{k}$  for a distribution of  $[a_i]_{i \in [0,1]}$ .
- Thus  $\hat{k}$  is uniquely determined.
- Given  $\hat{k}$ , (41) determines  $\hat{h}_i(\hat{k})$  uniquely.
- Applying the Implicit Function Theorem to (41) implies that  $\hat{h}_i(\hat{k})$  is increasing in  $\hat{k}$ .
- Finally, (40) implies that a lower  $R^*$  increases  $\hat{k}$ , and from the previous observation  $\hat{h}_i$  for all  $i \in [0, 1]$  increase as well.
- The no-activity equilibrium follows, since when all firms choose  $\hat{k} = 0$ , output is equal to zero and it is best response for workers to choose  $\hat{h}_i = 0$ , and when  $\hat{h}_i = 0$  for all  $i \in [0, 1]$ ,  $\hat{k} = 0$  is the best response for all firms.

# Physical and Human Capital with Imperfect Labor Markets

## VIII

- Underinvestment both in human capital and physical capital (even in positive activity equilibrium).
- Consider a social planner wishing to maximize output.
- Restricted by the same random matching technology.
- Similar analysis to above implies social planner would also like each firm to choose an identical level of capital per firm, say  $\bar{k}$ .
- But it will be different than in the competitive equilibrium and also choose a different relationship between human capital and physical capital investments.
- In particular, given  $\bar{k}$ , human capital decisions satisfy

$$a_i \frac{\partial F(\bar{k}, \bar{h}_i(\bar{k}))}{\partial h} = \gamma' \left( \frac{\bar{h}_i(\bar{k})}{a_i} \right),$$

# Physical and Human Capital with Imperfect Labor Markets IX

- Similar to (41), except that  $\lambda$  is absent from the left-hand side.
- Social planner considers the entire output.
- Consequently, as long as  $\lambda < 1$ ,

$$\bar{h}_i(k) > \hat{h}_i(k) \text{ for all } k > 0.$$

- Similarly, the social planner would also choose a higher level of capital investment for each firm, in particular,

$$\int_0^1 \frac{\partial F(\bar{k}, \bar{h}_i(\bar{k}))}{\partial k} di = R^*,$$

- Differs from (40) both because now the term  $1 - \lambda$  is not present and because the planner takes into account the differential human capital investment behavior of workers given by  $\bar{h}_i(\bar{k})$ .

# Physical and Human Capital with Imperfect Labor Markets

## X

**Proposition** In the equilibrium described, there is underinvestment both in physical and human capital.

**Proposition** Consider the positive activity equilibrium. Output is equal to 0 if either  $\lambda = 0$  or  $\lambda = 1$ . Moreover, there exists  $\lambda^* \in (0, 1)$  that maximizes output.

- Different levels of  $\lambda$  create different types of “imbalances:”
  - High  $\lambda$  implies workers have a strong bargaining position, encourages their human capital investments. But it discourages physical capital investments of firms
  - As  $\lambda \rightarrow 1$ , workers' investment is converging to social planner (i.e.,  $\hat{h}_i(k) \rightarrow \bar{h}_i(k)$  for all  $k > 0$ ), but  $\hat{k}$  is converging to zero, implies  $\hat{h}_i(k) \rightarrow 0$ , and production collapses.
  - Same happens, in reverse, when  $\lambda$  is too low.
  - Intermediate value of  $\lambda^*$  achieves a balance, though the equilibrium continues to be inefficient.

# Physical and Human Capital with Imperfect Labor Markets

## XI

- Physical-human capital imbalances can also increase the role of human capital in cross-country income differences.
- Proportional impact of a change in human capital on aggregate output is greater than the return to human capital, latter is determined not by the marginal product but by  $\lambda$ .
- At the root are *pecuniary externalities*: external effects that work through prices.
- By investing more, workers (and symmetrically firms) increase the return to capital (symmetrically wages).
- Underinvestment because they do not take these external effects into consideration.

# Physical and Human Capital with Imperfect Labor Markets XII

- Pecuniary external effects are also present in competitive markets, but typically “second order:”
  - prices are equal to both the marginal benefit of buyers and marginal cost of suppliers.
- In this model take the form of *human capital externalities*: human capital investments by a group of workers increase other workers' wages.
- Opposite in economy analyzed in the last section.
- To illustrate, suppose there are two types of workers: fraction of workers  $\chi$  with ability  $a_1$  and  $1 - \chi$  with ability  $a_2 < a_1$ .
- First-order condition of firms, (40),

$$(1 - \lambda) \left[ \chi \frac{\partial F(\hat{k}, \hat{h}_1(\hat{k}))}{\partial k} + (1 - \chi) \frac{\partial F(\hat{k}, \hat{h}_2(\hat{k}))}{\partial k} \right] = R^*, \quad (42)$$

# Physical and Human Capital with Imperfect Labor Markets

## XIII

- First-order conditions for human capital investments for the two types of workers take the form

$$\lambda a_j \frac{\partial F(\hat{k}, \hat{h}_j(\hat{k}))}{\partial h} = \gamma' \left( \frac{\hat{h}_j(\hat{k})}{a_j} \right) \text{ for } j = 1, 2. \quad (43)$$

- Clearly,  $\hat{h}_1(k) > \hat{h}_2(k)$  since  $a_1 > a_2$ .
- Now imagine an increase in  $\chi$ .
- Holding  $\hat{h}_1(\hat{k})$  and  $\hat{h}_2(\hat{k})$  constant, (42) implies that  $\hat{k}$  should increase, since the left-hand side has increased (in view of the fact that  $\hat{h}_1(\hat{k}) > \hat{h}_2(\hat{k})$  and  $\partial^2 F(k, h) / \partial k \partial h > 0$ ).
- Each firm expects average worker to have higher human capital.
- Since physical and human capital are complements, more profitable for each firm to increase their physical capital investment.

# Physical and Human Capital with Imperfect Labor Markets

## XIV

- Greater investments by firms, in turn, raise  $F(\hat{k}, h)$  for each  $h$ , in particular for  $\hat{h}_2(\hat{k})$ .
- Earnings of type 2 workers is equal to  $\lambda F(\hat{k}, \hat{h}_2(\hat{k}))$ , their earnings will also increase.
- Human capital externalities are even stronger, because the increase in  $\hat{k}$  also raises  $\partial F(\hat{k}, \hat{h}_2(\hat{k})) / \partial h$  and thus encourages further investments by type 2 workers.
- But these feedback effects do not lead to divergence or multiple equilibria.

**Proposition** The positive activity equilibrium exhibits human capital externalities in the sense that an increase in the human capital investments of a group of workers raises the earnings of the remaining workers.



# Human Capital Externalities I

- Human capital externalities may arise as a direct non-pecuniary (technological) spillover on the productivity of each worker.
- Empirical evidence on the extent of human capital externalities.
- Rauch (1993): quasi-Mincerian wage regressions, with the major difference that average human capital of workers in the local labor market is also included on the right-hand side:

$$\ln W_{j,m} = \mathbf{X}_{j,m}' \mathbf{f} + \gamma_p S_{j,m} + \gamma_e S_m,$$

- $\mathbf{X}_{j,m}$  is a vector of controls,  $S_{j,m}$  is the years of schooling of individual  $j$  living/working in labor market  $m$ .
- $S_m$  is the average years of schooling of workers in labor market  $m$ .
- *private return* to schooling  $\gamma_p$
- $\gamma_e$  measures the *external return*.

# Human Capital Externalities II

- Rauch estimated external returns often exceeding the private returns.
- But exploited differences in average schooling levels across cities, which could reflect many factors that also directly affect wages.
- Acemoglu and Angrist (2000) exploited differences in average schooling levels across states and cohorts resulting from changes in compulsory schooling and child labor laws.
- Estimate external returns to schooling that are typically around 1 or 2 percent and statistically insignificant (as compared to private returns of about 10%).
- Confirmed by a study by Duflo (2004) using Indonesian data and by Ciccone and Perri (2006).

# Human Capital Externalities III

- Moretti (2002) also estimates human capital externalities, and he finds larger effects:
  - focuses on college graduation,
  - also partly reflects the fact that the source of variation that he exploits, changes in age composition and the presence of land-grant colleges, may have other effects on average earnings in area.
- Overall, evidence appears to suggest that local human capital externalities are not very large.
- “Local” is key:
  - if a few generate ideas that are then used in other parts of the country or even in the world, there may exist significant global human capital externalities.

# Nelson-Phelps Model of Human Capital I

- Alternative perspective: major role of human capital is not to increase productivity in existing tasks, but to enable workers to cope with change, disruptions and especially new technologies.
- Continuous time model.
- Output is given by

$$Y(t) = A(t) L, \quad (44)$$

- $L$  is the constant labor force, supplying its labor inelastically, and  $A(t)$  is the technology level of the economy.
- No capital and also no labor supply margin.
- The only variable that changes over time is technology  $A(t)$ .
- World technological frontier is given by  $A_F(t)$ .

# Nelson-Phelps Model of Human Capital II

- $A_F(t)$  evolves exogenously according to the differential equation

$$\frac{\dot{A}_F(t)}{A_F(t)} = g_F,$$

with initial condition  $A_F(0) > 0$ .

- Human capital of the workforce denoted by  $h$ .
- This human capital does not feature in the production function, (44).
- Evolution of the technology in use,  $A(t)$ , is governed by the differential equation

$$\dot{A}(t) = gA(t) + \phi(h) A_F(t),$$

with initial condition  $A(0) \in (0, A_F(0))$ .

- Parameter  $g$  is strictly less than  $g_F$  and measures the growth rate of technology  $A(t)$ .

# Nelson-Phelps Model of Human Capital III

- Assume that  $\phi(\cdot)$  is increasing, with

$$\phi(0) = 0 \text{ and } \phi(h) = g_F - g > 0 \text{ for all } h \geq \bar{h},$$

where  $\bar{h} > 0$ .

- Since  $A_F(t) = \exp(g_F t) A_F(0)$ , the differential equation for  $A(t)$  can be written as

$$\dot{A}(t) = gA(t) + \phi(h) A_F(0) \exp(g_F t).$$

- Solving this differential equation,

$$A(t) = \left[ \left( \frac{A(0)}{g} - \frac{\phi(h) A_F(0)}{g_F - g} \right) \exp(gt) + \frac{\phi(h) A_F(0)}{g_F - g} \exp(g_F t) \right],$$

- Thus growth rate of  $A(t)$  is faster when  $\phi(h)$  is higher.

# Nelson-Phelps Model of Human Capital IV

- Moreover, it can be verified that

$$A(t) \rightarrow \frac{\phi(h)}{g_F - g} A_F(t),$$

- Thus ratio of the technology in use to the frontier technology is also determined by human capital.
- This role of human capital is undoubtedly important in a number of situations:
  - educated farmers are more likely to adopt new technologies and seeds (e.g., Foster and Rosenzweig, 1995).
  - stronger correlation between economic growth and levels of human capital than between economic growth and changes in human capital.
- Human capital could be playing a more major role in economic growth and development than the discussion so far has suggested.

# Nelson-Phelps Model of Human Capital V

- But:
  - If taking place within the firm's boundaries, this will be reflected in the marginal product of more skilled workers and taken into account in estimations.
  - If at the level of the labor market, this would be a form of local human capital externalities and it should have shown up in the estimates on local external effects of human capital.
  - So unless is also external and these external effects work at a global level, should not be seriously underestimating the contribution of human capital.



# Conclusions

- Human capital differences are a major proximate cause of cross-country differences in economic performance.
- May also play an important role in the process of economic growth and economic development.
- Issues:
  - ① If some part of the earnings of labor we observe are rewards to accumulated human capital, then the effect of policies (and perhaps technology) on income per capita could be larger.
  - ② Measurement of the contribution of education and skills to productivity:
    - mismeasurement from human capital externalities, differences in human capital quality, differences in formal schooling.
  - ③ Possibility of an imbalance between physical and human capital and impact of human capital on aggregate productivity.
  - ④ Role of human capital, skills facilitating the adoption and implementation of new technologies.