

$$1. \quad f_x(x) = \begin{cases} \theta K^\theta x^{-\theta-1} & \text{for } x > k \\ 0 & \text{ellers} \end{cases}$$

a) Vi visar att den kumulativa sannsynlighetsfördelning är
 till X är gitt ved
 $F_x(x) = \begin{cases} 1 - K^\theta x^{-\theta} & \text{for } x > k \\ 0 & \text{ellers} \end{cases}$

För att komma fram sannsynlighetsfördelningen till
 den kumulativa — \rightarrow fördelningen
 måste man integrera f .

$$\int_k^x f(x) dx = \int_k^x \theta K^\theta x^{-\theta-1} dx$$

Mellanregning

$$\theta K^\theta \int_{-\infty}^x x^{-\theta-1} dx = \theta K^\theta \left[-\frac{x^{-\theta}}{\theta} \right]_{-\infty}^x$$

$$\theta K^\theta \left(\left(-\frac{x^{-\theta}}{\theta} \right) - \left(-\frac{1}{K^\theta \theta} \right) \right) = -x^{-\theta} K + 1$$

därmed är

$$F_x(x) = \begin{cases} 1 - K^\theta x^{-\theta} & \text{for } x > k \\ 0 & \text{ellers.} \end{cases}$$

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Før vi finne medianen bruker jeg $F_X(x) = 0,5$
 siden det viser til midten av sannsynsligheten

$$\Rightarrow 1 - K^\theta x^{-\theta} = 0,5 / -1$$

$$K^\theta x^{-\theta} = 0,5 \Rightarrow x^{-\theta} = \frac{0,5}{K^\theta} \quad \rightarrow$$

v) For å finne $E(x)$ tar jeg

$$\int_K^{\infty} x f(x) dx$$

$$\Rightarrow \int_K^{\infty} x \cdot \theta K^\theta x^{-\theta-1} = \theta K^\theta \int_K^{\infty} x^{-\theta} dx = \theta K^\theta \left[\frac{1}{-\theta+1} x^{-\theta+1} \right]_K^{\infty}$$

$$= \lim_{x \rightarrow \infty} \frac{\theta K^\theta}{-\theta+1} \frac{x}{x^\theta} - \left(\frac{1}{-\theta+1} K^{-\theta+1} \right) = \underline{\underline{\frac{\theta K}{\theta-1}}}$$

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$$c) K = 400\,000, \theta = 3$$

Median:

$$k^{\theta} \alpha^{-\theta} = 0,5$$

$$400'000 \left\{ \alpha^{-3} \right\} = 0,5$$

$$\underline{x = 583966}$$

E(x):

$$\frac{\theta K}{\theta - 1} \Rightarrow \frac{3 \cdot 400'000}{3 - 1}$$

$$\underline{= 600'000}$$

$$J) V(x) = E(x^2) - E(x)^2$$

$$E(x^2) = \int_{1000}^{\infty} x^2 \cdot \theta K^{\theta} x^{-\theta-1} dx$$

$$= \theta K^{\theta} \int x^{-\theta+1} dx$$

$$= \theta K^{\theta} \left[\frac{x^{-\theta+2}}{-\theta+2} \right]_{K}^{\infty}$$

$$= \theta K^{\theta} \left(\lim_{x \rightarrow \infty} \frac{x^2}{-\theta x^{\theta-2}} + \frac{K^{\theta} \theta}{\theta-2} \right)$$

*Siehe $\theta > 2$ x gro
Wir setzt hier $\theta = 0$*

$$= \frac{K^2 \theta}{\theta-2}$$

Setzen wir ein und rücke

$$\frac{400'000^2 \cdot 3}{3-2} \approx 4,8 c^{**}$$

$$\tilde{E(x)}^2 = 607'000^2 \\ = 3.6 \text{ e}^{11}$$

$$\underline{\underline{V_{xx} = 1,2 \cdot 10^{11}}} \\ \underline{\underline{\sigma = 346'470,1615}}$$

2. Simultan sansyndighet

$$f(x, y) = \begin{cases} k(x-y) & \text{for } 0 \leq y \leq x \leq 1 \\ 0 & \text{ellers} \end{cases}$$

a) V_{xy} at $k = 6$

Siden hele avsikt av en tetthet skal være lik 1
dovet for vi integrerer over hele avsikt og setter det
litt til.

$$\iint_0^x k(x-y) dy dx =$$

$$= k \int_0^1 \int_0^x (x-y) dy dx = k \int_0^1 \left[xy - \frac{y^2}{2} \right]_0^x dx = 1$$

$$= k \int_0^1 x^2 - \frac{x^2}{2} dx = k \int_0^1 \frac{x^2}{2} dx = 1$$

$$= k \left[\frac{x^3}{6} \right]_0^1 = \frac{k}{6} = 1$$

$$\underline{k = 6}$$

g) bestimmen $P(2Y \leq x)$

$$2Y \leq x \quad | :2$$

$$Y \leq \frac{x}{2}$$

$$P(2Y \leq x) = \int_0^1 \int_0^{\frac{x}{2}} 6(x-y) dy dx$$

$$= 6 \int_0^1 \left[xy - \frac{y^2}{2} \right]_0^{\frac{x}{2}} dx$$

$$= 6 \int_0^1 \frac{3}{8} x^2 dx$$

$$= 6 \cdot \frac{3}{8} \cdot \frac{1}{3} = \underline{\underline{\frac{3}{4}}}$$

c) V is den marginalen sozialen Nutzenfaktor für x

$$\begin{aligned} \int_0^x f(x, y) dy &= \int_0^x 6(x-y) dy \\ &= 6 \left[xy - \frac{y^2}{2} \right]_0^x \\ &= 6 \left(x^2 - \frac{x^2}{2} \right) = 6 \frac{x^2}{2} = \underline{\underline{3x^2}} \end{aligned}$$

$$f_x(x) = \begin{cases} 3x^2 & 0 \leq x \leq 1 \\ 0 & \text{else} \end{cases}$$

d) V ist der marginale soziale Nutzenfaktor für y

$$\begin{aligned} f_y(y) &= \int_y^1 f(x, y) dx \\ &= 6 \int_y^1 (x-y) dx \\ &\leq 6 \left[\frac{x^2}{2} - yx \right]_y^1 \\ &= 6 \left(\frac{1}{2} - y - \left(\frac{y^2}{2} - y^2 \right) \right) \\ &= \underline{\underline{3y^2 - 6y + 3}} \end{aligned}$$

e) X en Y er wachttige huij

$$f(x, y) = f(x)f(y)$$

zetter in

$$6(x - y) \neq 3x^2 + 3y^2 - 6y + 3$$

deed er de idde wachttige

Opmg 3

3a)

$$X = F^{-1}(u)$$

$$\Rightarrow F(x) = P(X \leq x) = P(F^{-1}(u) \leq x) = F[F^{-1}(u)]$$

$$= u \quad \#$$

$$V = \frac{1 - K^\theta}{X^{\frac{1}{\theta}}} / \cdot X^\theta$$

$$X^\theta u = X^\theta - K^\theta / -X^\theta$$

$$X^\theta(u - 1) = -K^\theta / : (u - 1)$$

$$X^\theta = \frac{-K^\theta}{u - 1}$$

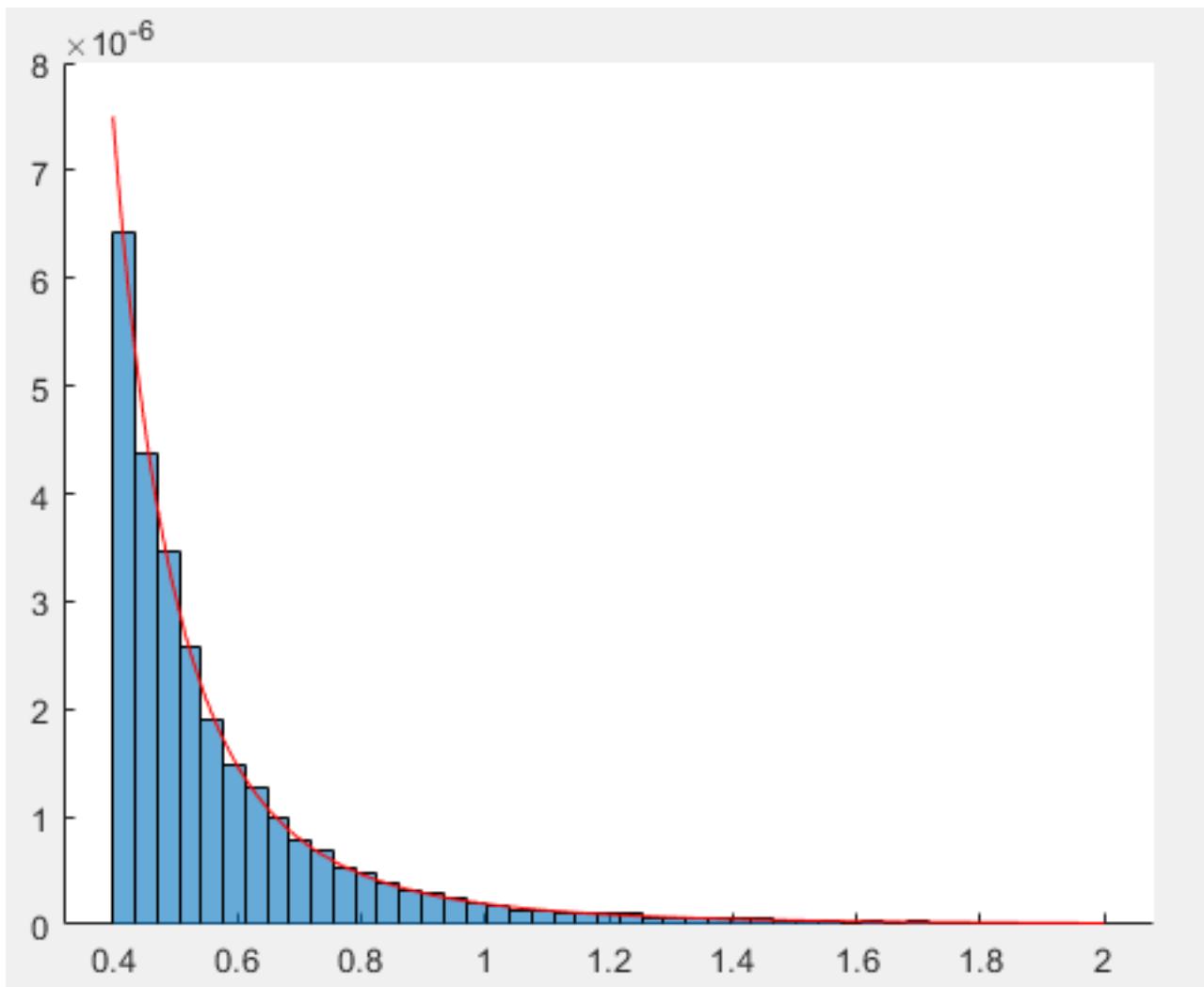
$$X = \sqrt[\theta]{\frac{K^\theta}{1 - u}}$$

$$\underline{X = \frac{K}{\sqrt[\theta]{1 - u}}}$$

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1 %inndata
2 K = 400000;
3 theta = 3;
4 n = 10000;
5 %getting n uniformed u between 0 and 1
6 u = unifrnd(0,1,[1 n]);
7
8 x = K ./ (nthroot(1-u,theta));
9 %finding the mean
10 mean(x)
11 %finding the median
12 median(x)
13 hold on
14 %d) plotting the histogram
15 histogram(x,"normalization","pdf","binlimits", [400000 2000000])
16 X = (400000:0.1:2000000);
17 %e) printer tetheten til Paretofordelingen
18 line(X,theta*(K.^theta)*(X.^(-theta-1)), 'color', 'red')
19 hold off

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>> oppg3
ans = mean(x)
5.9822e+05
ans = median(x)
5.0478e+05
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