⋮

Begrenser oss til tyngdekraft og trykkraft

- Vi ser bort ifra viskositet / ikke friksjon = idealt fluid

$$\underline{F} = -\int_S p\,\underline{n}\, d\sigma + m\underline{g}$$

gauss satsen

$$= -\int \nabla p \, d\sigma + m\underline{g}$$

$$\approx -V\nabla p + m\underline{g}$$

$$m\underline{a} = -V\nabla p + m\underline{g}$$

$$\underline{a} = -\frac{V}{m}\nabla p + \underline{g}$$

$$\underline{a} = -\frac{1}{\rho}\nabla p + \underline{g}$$

$$\frac{V}{m} = m\underline{g}$$

$$\frac{dv}{dt} + \underline{v}\nabla v = \frac{1}{\rho}\nabla p + g$$

Kontinuitetslikning

$$\frac{d\rho}{dt} + \underline{v}\cdot\nabla\rho + \rho\nabla\cdot\underline{v} = 0$$

Eulers likning

$$\frac{dv}{dt} + \underline{v}\cdot\nabla\cdot\underline{v} = -\frac{1}{\rho}\nabla p + \underline{g}$$

E. 1 Enkelt eksempel

Stille-stående fluid $\underline{v} = 0$

Kontinuitetsligningen

$$\frac{d\rho}{dt} = 0 \Rightarrow \text{tettheten er stasjonær}$$

Euler

$$\underline{0} = -\frac{1}{\rho} \nabla p + \underline{g}$$

$$\nabla p = \rho \underline{g} = -\rho g \underline{k}$$

$$\left.\begin{aligned} \frac{dp}{dx} &= 0 \\ \frac{dp}{dy} &= 0 \\ \frac{dp}{dz} &= -\rho g \end{aligned}\right\} \Rightarrow p = p(z)$$

$$p = p_0 - \int_{z_0}^{z} \rho(z)\, g \, dz$$

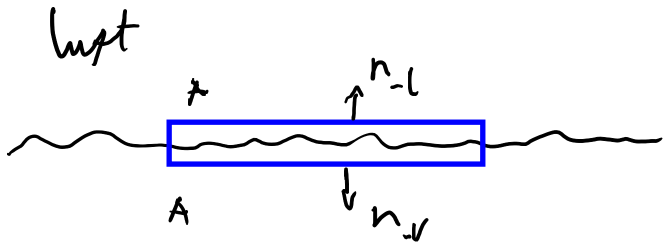
For konstant tetthet

$$p = p_0 - \rho g z$$

hvor $p = p_0$ for $z = 0$

Hydrostatisk trykk

Hva skjer der vann møter luft



Summen av krefter på kontroll volumet

(Trykkraft ovenfra) (Trykk fra vannet og oppover)

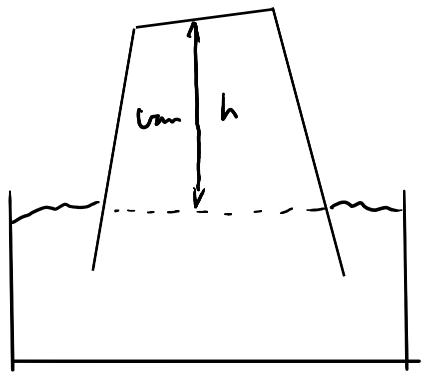
$$(-p_l \underline{n}_l A) \ (-p_v \underline{n}_v A) + m\underline{g} = m\underline{a}$$

I grensen $m \to 0$, $V \to 0$, $d \to 0$

$$-p_l \underline{n}_l - p_v \underline{n}_v = 0 \qquad \underline{n}_l = \underline{n}_v$$

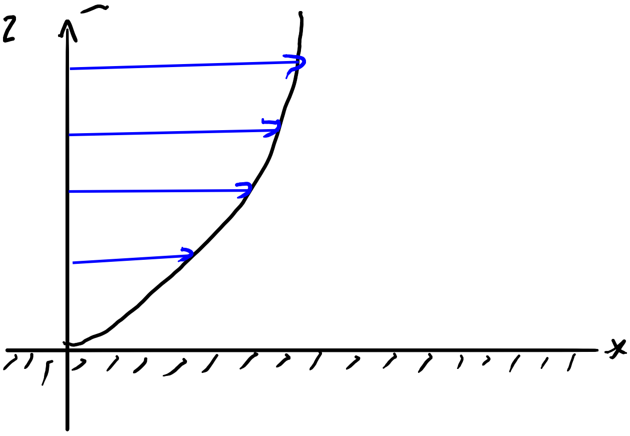
$\underline{p_l = p_v}$ trykket er kontinuerlig

$$p = p_0 - \rho g h$$



Eksempel: Vind blåser over bakken

$\underline{v} = V(z)\,\underline{i}$



Finn trykket

$$\frac{d\underline{v}}{dt} + \underline{v}\cdot\nabla\underline{v} = \frac{1}{\rho}\nabla p - g\underline{k}$$

$$\underline{v}\cdot\nabla = V(z)\,\underline{i}\cdot\left(\underline{i}\frac{d}{dx} + \underline{j}\frac{d}{dy} + \underline{k}\frac{d}{dz}\right) = V(z)\frac{d}{dx}$$

$$(\underline{v}\cdot\nabla)\,\underline{v} = V(z)\frac{d}{dx}V(z)\,\underline{i} = 0$$

$$\nabla p = -\rho g\underline{k}$$

Samme som for det hydrostatiske problemet

Eksempel:

Karusell-bevegelse i vann

$$\underline{v} = \underline{w}\times\underline{r} = w\underline{k}\times\underline{r}$$

$$= -wy\,\underline{i} + wx\,\underline{j}$$

$\uparrow^{z} = \underline{w} = w\underline{k}$

Regn ut trykket p

$$\frac{d\underline{v}}{dt} = 0 \quad \text{stasjonær hastighet}$$

$$\underline{v}\cdot\nabla = (-wy\,\underline{i} + wx\,\underline{j})\cdot\left(\underline{i}\frac{d}{dx} + \underline{j}\frac{d}{dy} + \underline{k}\frac{d}{dz}\right)$$

$$(\underline{v}\cdot\nabla)\underline{v} = \left(-wy\frac{d}{dx} + wx\frac{d}{dy}\right)(-wy\underline{i} + wx\underline{j})$$

$$= -wy\,w\underline{j} + wx(-w\underline{i}) = -w^2(x\underline{i} + y\underline{j})$$

Eulers likning

$$\underbrace{\cancel{\frac{d\underline{v}}{dt}}}_{\to 0} + \underbrace{\underline{v}\cdot\nabla\underline{v}}_{=-w^2(x\underline{i}+y\underline{j})} = -\frac{1}{\rho}\nabla p - g\underline{k}$$

$$\nabla p = \rho w^2(x\underline{i} + y\underline{j}) - \rho g\underline{k}$$

$$\left.\begin{aligned} \frac{dp}{dx} &= \rho w^2 x \\ \frac{dp}{dy} &= \rho w^2 y \\ \frac{dp}{dz} &= -\rho g \end{aligned}\right\} \quad p = \overset{,c}{p_0} + \frac{1}{2}\rho w^2(x^2 + y^2) - \rho g z$$

$P_{vann} = P_{luft} = P$ = konstant der ver vann møter luft

$$z = \eta(x,y)$$

$$P_{luft} = P_0 + \frac{1}{2}\rho w^2(x^2 + y^2) - \rho g\,\eta(x,y)$$

$$\eta(x,y) = \frac{P_0 - P_{luft}}{\rho g} + \frac{w^2}{2g}(x^2 + y^2)$$ – Paraboloide