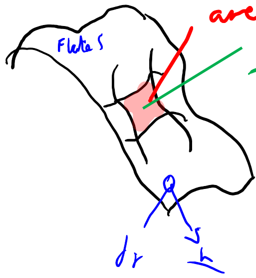
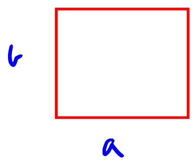
Flateintegral

$$\int_S \underline{F}\cdot n\, d\sigma \;,\; \int F\cdot dr \;,\; \int f\, n\, d\sigma \;,\; \int_S f\, d\sigma \;,\; \int_S \underline{F}\times \underline{n}\, d\sigma$$

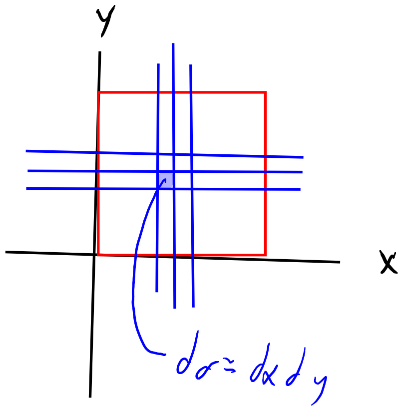


Eks



areal $A = ab$

$$A = \int_S d\sigma = \int_0^b \int_0^a dx\, dy$$

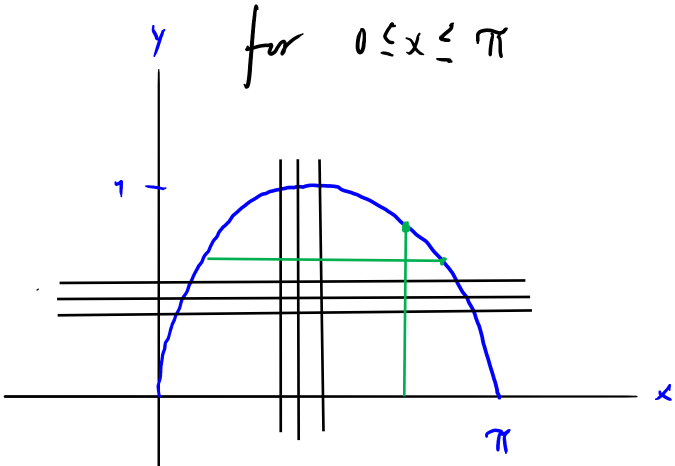


$$\int_0^b x\Big|_0^a \, dy = \int a\, dy$$

$$= a y\Big|_0^b = ab$$

Eks

arealet mellom $y = \sin x$ og x-aksen for $0 \leq x \leq \pi$



$$dσ = dx\,dy = dy\,dx$$

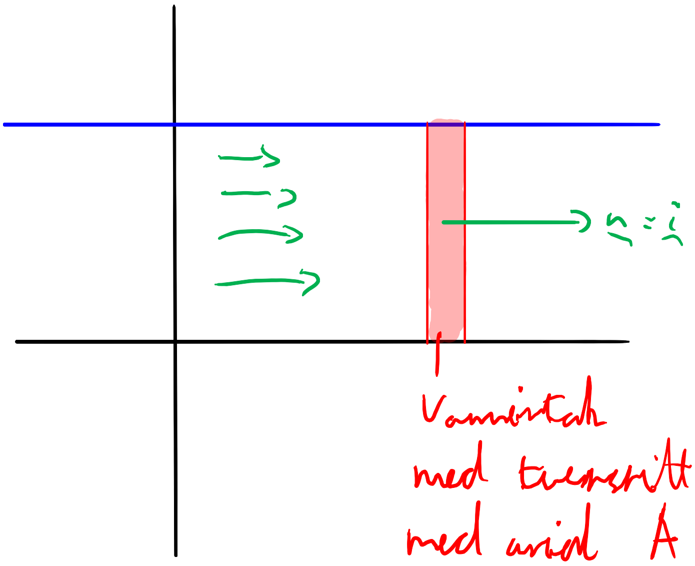
$$A = \int dσ = \int_0^{\pi} \int_0^{\sin x} dy\,dx$$

$$\int_0^{\pi} y\Big|_0^{\sin x} dx = \int_0^{\pi} \sin x \, dx$$

$$= -\cos x \Big|_0^{\pi} = -(-1)-(-1) = 2$$

Eks

Volum fluks av en elv med hastighet $\underline{v} = v\,\underline{i}$ (konstant)



delen av hastigheten som spiller noen rolle

$\underline{v} \cdot \underline{n}$ Volumfluks tetthet

$$Q = \int_S \underline{v} \cdot \underline{n} \, dσ = \int_S v \, dσ$$

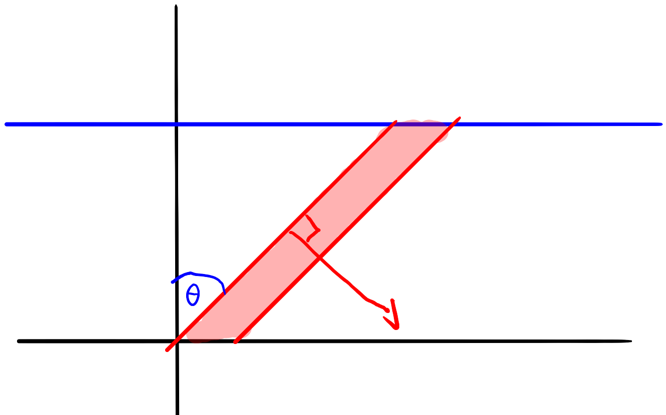
$\frac{m^3}{s}$ $\frac{m}{s}$ dimensjonsløs m^2

v = konstant

$$Q = v \int dσ = vA$$

Integrert Volumfluks

Volumfluks av vann ut med skrått vannistak



Finn $\underline{n}$

$\underline{v} = v \cdot \underline{i}$

delen av hastighetsvektor som speiler seg rette $\underline{v} \cdot \underline{n}$

(I) Ekvivalentflate: $\tan\theta = \frac{x}{y} = \frac{\sin\theta}{\cos\theta} \quad / \, y \cdot \cos\theta$

$$\beta(x,y,z) = x\cos\theta - y\sin\theta = 0$$

$$\underline{n} = \pm \left| \frac{\nabla \beta}{\nabla \beta} \right|$$ (kan peke begge veier)

$$= \pm \frac{\cos\theta \, \underline{i} - \sin\theta \, \underline{j}}{\sqrt{\cos^2\theta + \sin^2\theta}}$$

$$= \pm (\cos\theta \, \underline{i} - \sin\theta \, \underline{j})$$ - på grunn av vannets retning velger vi "+"

$$= \cos\theta \, \underline{i} - \sin\theta \, \underline{j}$$

På en annen måte

Finn $\underline{n}$ (II)   Finn tangentvektor $\underline{t}$, kryss med $\underline{k}$

$$\underline{t} = \sin\theta\, \underline{i} + \cos \underline{j}$$

$$\underline{n} = \underline{t} \times \underline{k}$$

$$\begin{vmatrix} \underline{i} & \underline{j} & k \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos\theta\, \underline{i} - \sin\theta\, \underline{j}$$

Sørge for å få lengde lik 1

Arealet av skrå flate

$$\frac{A}{\cos\theta}$$

$$Q_{skrå} = \int_{S_{skrå}} \underline{v} \cdot n \, d\sigma = \int v\, \underline{i} \cdot (\cos\theta\, \underline{i} - \sin\theta\, \underline{j})\, d\sigma$$

$$= v\cos\theta \int_{skrå} d\sigma = (v\cos\theta)\left(\frac{A}{\cos\theta}\right) = vA$$

Same svar som i sted

Integrert

Masse fluks $= \int_S \rho \underline{v} \cdot n \, d\sigma$ $\qquad \rho =$ massetetthet

$\frac{kg}{s} \qquad \frac{kg}{m^3} \quad \frac{m}{s} \quad 1 \quad m^2 \qquad \frac{kg}{m^3}$

Massefluks tetthet

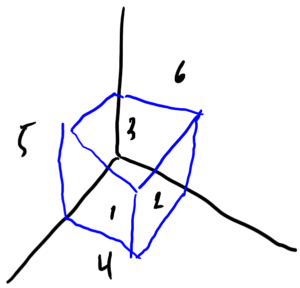
$\frac{kg}{m^2 s}$

Mol fluks tetthet

integrert mol fluks $= \int_S (\text{mol}\ \underline{v}) \cdot n \, d\sigma$

$\frac{mol}{s} \qquad$ mol tetthet $\frac{mol}{m^3} \quad \frac{m}{s} \quad 1 \quad m^2$

Eks finn integralet fluks ut av kuben



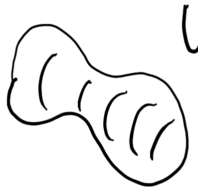
av $\underline{v} = x i$

$$Q = \int_S \underline{v} \cdot n \, d\sigma = \sum_{i=1}^{6} \int_S \underline{v} \cdot n \, d\sigma$$

Finne overflate normalvektorer (Kap 2)

Side	$\beta(x,y,z)=$ konstant	$\underline{n} = \pm \frac{\nabla \beta}{\lvert \nabla \beta \rvert}$	$\underline{v}$ på flate	$d\sigma$	$\int_S \underline{v} \cdot n \, d\sigma$
1	$x = 1$	$+\underline{i}$	$x\underline{i} = \underline{i}$	$dy\,dz$	$\int \underline{i}\cdot\underline{i}\, d\sigma = \int 1 \, d\sigma = 1$
2	$y = 1$	$+\underline{j}$	$x\underline{i}$	$dx\,dz$	$\int x i \cdot \underline{j}\, d\sigma = 0$
3	$z = 1$	$+\underline{k}$	$x\underline{i}$	$dx\,dy$	$\int x \underline{i} \cdot k \, d\sigma = 0$
4	$z = 0$	$-k$	$x\underline{i}$	$dx\,dy$	$\int x \underline{i} \cdot (-k)\, d\sigma = 0$
5	$y = 0$	$-\underline{j}$	$x\underline{i}$	$dx\,dz$	$\int x \underline{i} \cdot (-\underline{j})\, d\sigma = 0$
6	$x = 1$	$-\underline{i}$	0	$dy\,dz$	$\int 0 \, d\sigma = 0$

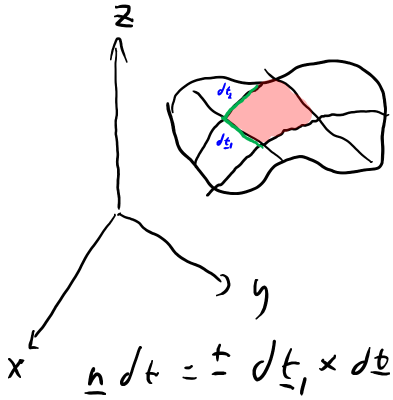
Kavulagon



Lukket flate

$\underline{n}$ peker ut

Mer generell flate



$\underline{n}\, dt = \pm\, d\underline{t}_1 \times d\underline{t}_2$

Husk