$\underline{v} = w\underline{k} \times r$

Regne ut sirkelrotasjonen rundt en sirkel med radius a

$$r(\theta) = a\cos\theta\,\underline{i} + a\sin\theta\,\underline{j}$$

$$\underline{v}(\theta) = -wa\sin\theta\,\underline{i} + wa\cos(\theta)\,\underline{j}$$

$$d\underline{r} = (-a\sin\theta\, i + a\cos\theta\,\underline{j})\,d\theta$$

$$v \cdot dr = (wa^2\sin^2\theta + wa^2\cos\theta)\,d\theta$$

Sirkelrotasjon $C = \oint_{\partial} v \cdot d\underline{r} = \int_0^{2\pi} wa^2\,d\theta$

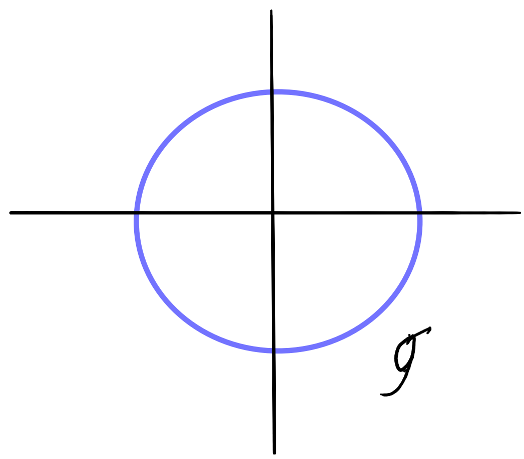
Areal til sirkelskiven $A = \pi a^2$

$$\frac{C}{A} = \frac{2\pi wa^2}{\pi a^2} = 2w$$

$$w = \frac{C}{2A}$$

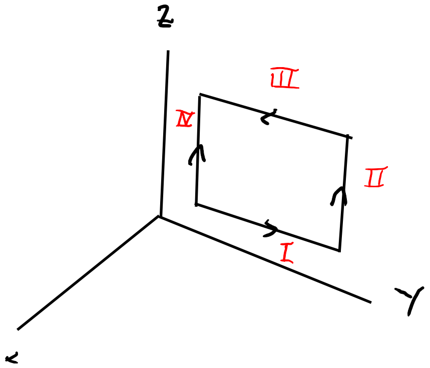
Definisjon av virvling

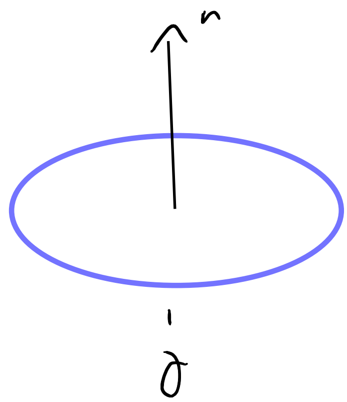
$$\mathbf{n} \cdot \operatorname{curl} \mathbf{v} = \lim_{A \to 0} \frac{1}{A} \oint \mathbf{v} \cdot d\mathbf{r}$$



(x_0, y_0, z_0)

rektangel med sider $\Delta y, \Delta z$





$\mathbf{n} = \mathbf{i}$

side	$r(t)$	dr	$\mathbf{v} \cdot d\mathbf{r}$	$\int \mathbf{v} \cdot d\mathbf{r}$
I	$x_0 \mathbf{i} + (y_0 + t)\mathbf{j} + (z_0 - \frac{\Delta z}{2})\mathbf{k}$ for $\frac{\Delta y}{2} < t < \frac{\Delta y}{2}$	$\mathbf{j}\, dt$	$v_y\, dt$	$\int_{-\frac{\Delta y}{2}}^{\frac{\Delta y}{2}} v_y \Big\vert_{r(t)} dt \approx v_y(x_0, y_0, z_0 - \frac{\Delta z}{2}) \Delta y$
II	$x_0 \mathbf{i} + (y_0 - t)\mathbf{j} + (z_0 + \frac{\Delta z}{2})\mathbf{k}$ for $-\frac{\Delta y}{2} < t < \frac{\Delta y}{2}$	$-\mathbf{j}\, dt$	$-v_y\, dt$	$\int_{-\frac{\Delta y}{2}}^{\frac{\Delta y}{2}} -v_y \Big\vert_{r(t)} dt \approx -v_y(x_0, y_0, z_0 + \frac{\Delta z}{2}) \Delta y$
⋮				

$$\left(\int_{I} + \int_{III}\right) \underline{v}\, d\underline{r} = \left\{ v_y\left(x_0, y_0, z_0 - \frac{\Delta z}{2}\right) - v_y\left(x_0, y_0, z_0 + \frac{\Delta z}{2}\right) \right\}$$

$$\approx -\frac{d v_y}{d z}\Big|_{(x_0, y_0, z_0)} \Delta y \Delta z$$

$$\left(\int_{II} \int_{IV}\right) v\, dr \approx + \frac{d v_z}{d y} \Delta y \Delta z$$

Vi har funnet

$$C: \oint_{\partial} \underline{v} \cdot dr \approx \left(\frac{d v_z}{d y} - \frac{d v_y}{d z}\right) \Delta y \Delta z$$

$$A = \Delta y \Delta z$$

$$\frac{C}{A} \approx \frac{d v_z}{d y} - \frac{d v_z}{d z}$$

$$\underset{\smile}{i}\ \mathrm{curl}\, \underline{v} = \lim_{A \to 0} \frac{1}{A} \oint \underline{v}\, dr = \frac{d v_z}{d y} - \frac{d v_z}{d z}$$

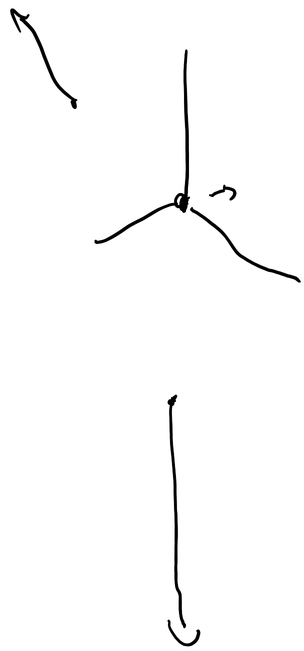
$$\text{Curl } \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ v_x & v_y & v_z \end{vmatrix} = \nabla \times \underline{v}$$

Konstant strøm

$\underline{v}$ = konstant

$\nabla \cdot \underline{v} = 0$, $\nabla \times \underline{v} = \underline{0}$

Eksempel: posisjonsvektoren



$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$, $\underline{v} = \underline{r}$

$$\nabla \cdot r = \frac{dx}{dx} + \frac{dy}{dy} + \frac{dz}{dz} = 1+1+1 = 3 > 0$$

ekspansjon

$$\nabla \times \underline{r} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x & y & z \end{vmatrix} = \underline{0}$$ virvelfritt

Eksempel kartesisk

$$\underline{w} = \omega \underline{k}$$

$$\underline{v} = \omega \underline{k} \times r = -\omega y \underline{i} + \omega \underline{j}$$

$\nabla \cdot v = 0$ ikke ekspansjon

$$\nabla \times v = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -\omega y & \omega x & 0 \end{vmatrix}$$

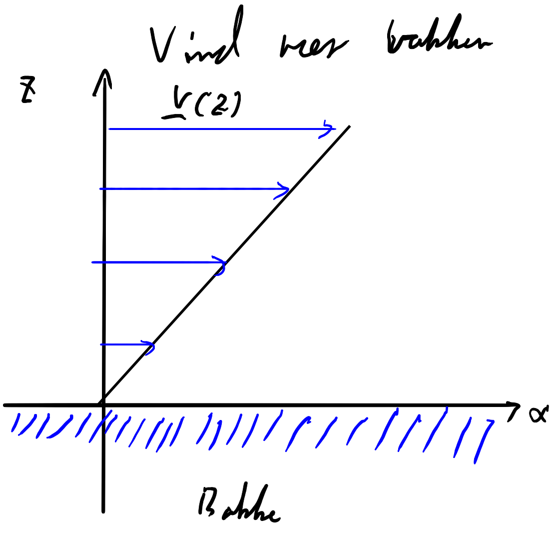
$$= \underline{k}\left(\frac{d(\omega x)}{dx} - \left(-\frac{d(\omega y)}{dy}\right)\right)$$

$$= 2\omega \underline{k} = 2\underline{w}$$

$$\boxed{\underline{w} = \frac{1}{2} \nabla \times \underline{v}}$$

Gjelder for rotasjon som et fast legeme

Eksempel:



$$\underline{v}(z) = \alpha z \underline{i}$$

$$\nabla \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \alpha z & 0 & 0 \end{vmatrix} = \underline{j}\,\alpha$$

Dersom den virvlingen hyggen roterer med vinden som et fast legeme, så kan vi forvente

$$\underline{w} = \frac{1}{2} \nabla \times \underline{v} = \frac{1}{2} \alpha \underline{j}$$

Ren spekulasjon

Eksempel: Karusell $\underline{w} = w_x \underline{i} + w_y \underline{j} + w_z \underline{k}$

$$\underline{v} = \underline{w} \times \underline{r}$$

$$\nabla \times (\underline{w} \times \underline{r})$$

$$= \underline{w}\, \underbrace{\nabla \cdot \underline{r}}_{3} - \underline{r}(\nabla \cdot \underline{w})$$

$$= 3\underline{w} - \underline{w} \cdot \nabla\, \underline{r}$$

$$\nabla \cdot v = \nabla \cdot (\underline{w} \times \underline{r})$$

$$= \underline{r} \cdot (\nabla \times \underline{w})$$

$$= \underline{w} \cdot (\underline{r} \times \nabla)$$

$$= \underline{w} \cdot (-\nabla \times r)$$

$$= 0$$

Regn ut $(\underline{w} \cdot \nabla)\underline{r}$

$$w \cdot \nabla = (w_x \underline{i} + w_y \underline{j} + w_z \underline{k}) \cdot \left(\underline{i} \frac{d}{dx} + \underline{j} \frac{d}{dy} + \underline{k} \frac{d}{dz}\right)$$

$$= w_x \frac{d}{dx} + w_y \frac{d}{dy} + w_z \frac{d}{dz}$$

$$(w \cdot \nabla)\underline{r} = w_x \frac{d\underline{r}}{dx} + w_y \frac{d\underline{r}}{dy} + w_z \frac{d\underline{r}}{dz}$$

$$= w_x \underline{i} + w_y \underline{j} + w_z \underline{k} = \underline{w}$$

$$\nabla \times (\underline{w} \times \underline{r})$$

$$= 3\underline{w} - w = 2w$$

$$\underline{w} = \frac{1}{2} \nabla \times \underline{v}$$

Kun for rotasjon som
faste legemer