1) $\bar{v} = xy^2\,\bar{i} - x^2 y\,\bar{j}$

a) Finn div og virvling a

$$\boxed{\nabla \cdot \bar{v} = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right)}$$

$$\nabla \cdot \bar{v} = \frac{\partial}{\partial x}(xy^2) - \frac{\partial}{\partial y}x^2 y = \underline{\underline{y^2 - x^2}}$$

$$\boxed{\nabla \times \bar{v} = \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\bar{k}}$$

$$\nabla \times \bar{v} = \left(\frac{\partial}{\partial x}(-x^2 y) - \frac{\partial}{\partial y}xy^2\right)\bar{k} = -2xy - 2xy = \underline{\underline{-4xy\,\bar{k}}}$$

b) ingen strømfunksjon pga ikke divergensfritt

ingen potensial siden vektorfeltet ikke er virvelfritt

$v = -wy\,\bar{i} + wx\,\bar{j}$

$\psi = \int \frac{\partial \psi}{\partial y} = wy \qquad \psi = \int \frac{\partial \psi}{\partial x}\ wx$

$\psi = \frac{1}{2}wy^2 + f_1(x)\ ,\ \psi = \frac{1}{2}wx^2 + f_2(x)$

Velger $f_i(x)$: $\frac{1}{2}wx^2\ ,\ \frac{1}{2}wy^2$

$\psi = \frac{1}{2}w(x^2 + y^2)$

$\psi_0 = \frac{1}{2}w(x^2 + y^2) = C$

$\frac{2\psi_0}{w} = x^2 + y^2 = \frac{2c}{\cancel{w}}$

Potensial

$\bar{F} = -\lambda(x\,\bar{i} + y\,\bar{j})$

$\frac{\partial V}{\partial x} = \lambda x\ ,\ \frac{\partial V}{\partial y} = \lambda y$

$V_1 = \frac{\lambda}{2}x^2 + f_1(y)\ ,\ V_1 = \frac{\lambda}{2}y^2 + f_2(x)$

$V_1 = V_2$ ved å bestemme f_1 og f_2

$V = \frac{\lambda}{2}(x^2 + y^2) + c$

c) $\int x y^2 \, dy = - \int x^2 y \, dx$

$$\frac{x y^3}{3} = - \frac{x^3 y}{3} + C$$

$$x y^3 + x^3 y = 3C$$

$$x y (y^2 + x^2) = 3c$$

Stagnasjonspunkt

$x y (y^2 + x^2) = 0$ når $x = 0$, $y = 0$, $y^2 + x^2 = 0$

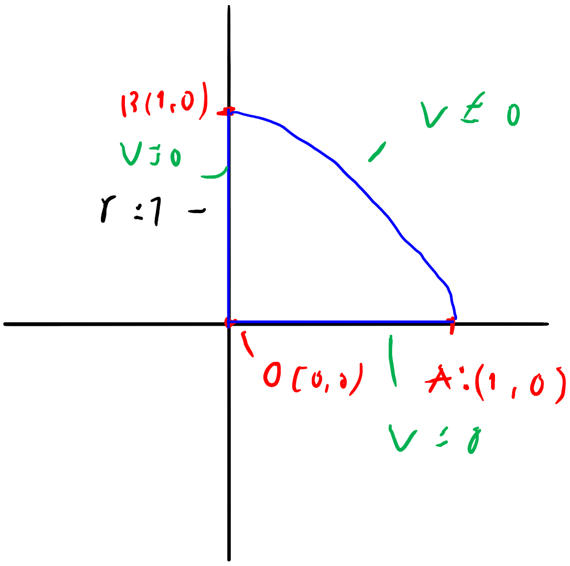
Strømlinje

$$\bar{v} \times d\bar{r} = 0$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ v_x & v_y & 0 \\ dx & dy & 0 \end{vmatrix} = (v_x \, dy - v_y \, dx) \bar{k}$$

$\int v_x \, dy = \int v_y \, dx \Rightarrow$ strømfunksjonen

1 d)



En passende parametrisering av den

$$r(\theta) = \cos(\theta) \bar{i} + \sin(\theta) \bar{j}$$

for $\theta [0, \pi/2]$

Vi har

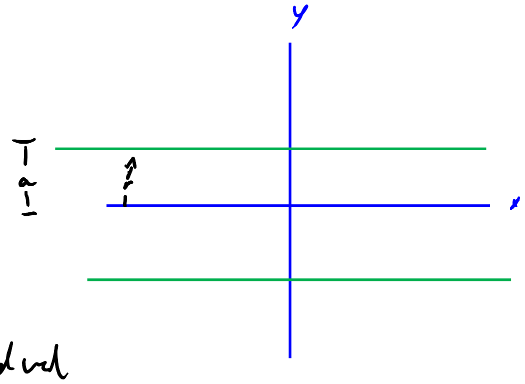
$$r'(\theta) = - \sin(\theta) \bar{i} + \cos\theta \, \bar{j}$$

$$r \cdot dr = (\cos(\theta), \sin\theta) \cdot (-\sin\theta, \cos\theta)$$

$$= (-\sin\theta\cos\theta + \sin\theta\cos\theta)$$

$$= 0$$

2) $\bar{v} = v_0 \left[1 - \left(\frac{r}{a}\right)^2 \right] \bar{i}$



v_0 er maksimal fart midt i røret

Ved $t = t_0$ temperaturfordelingen gitt ved

$$T = T_0 + \alpha x$$

T_0 og α er konstanter

Varmefluksdensitet gitt på formen

$$\bar{H} = \rho c \bar{v} T - k \nabla^2 T$$

Varmetransportligninga

$$\frac{\partial T}{\partial t} + v \cdot \nabla T = \kappa \nabla^2 T$$

a) ρ : tetthet kg/m^3

T : absolutt temp K

k : Varmeledningstall (termisk konduktivitet) $J/(msK)$

κ : Varmediffusitet m^2/s

$\bar{H}$: Varmefluksstetthet $J/m^2 s$

2 b) $\int_\sigma H \cdot n \, d\sigma$

$$H = H_s + H_c \, , \quad H_s = \rho c (T - T_0) \bar{v}$$

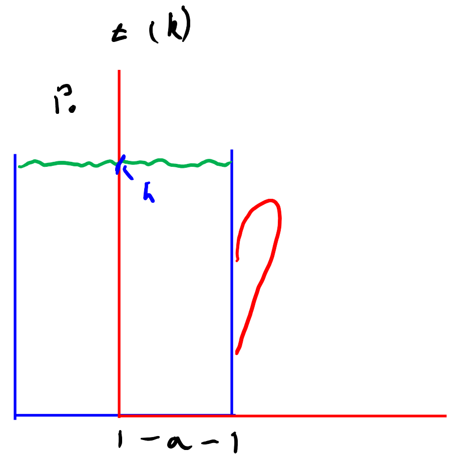
$$H_c = -k \nabla T$$

$$n = \bar{i}$$

$$\int_\sigma \left(\rho c (T - T_0) \bar{v} - k \nabla T \right) \cdot i \, d\sigma$$

$$\int_\sigma \left(\rho c (T_0 + \alpha x - T_0) (v_0 \left[1 - \left(\frac{r}{a}\right)^2 \right] \bar{i} - k \alpha \, d\sigma \right.$$

3



tetthet : ρ

tyngdens akselerasjon : $\bar{g} = -g \bar{k}$

Luftens tetthet : P_0

Eulers bevegelsesligning, for $\bar{v} = 0$

$$\frac{D \cancel{\bar{v}}}{dt} = -\frac{1}{\rho} \nabla p + \bar{g} \qquad \bar{g} = -g \bar{k}$$

da får vi

$$0 = -\frac{1}{\rho} \frac{dp}{dx} + 0$$

$$0 = -\frac{1}{\rho} \frac{dp}{dy} + 0$$

$$0 = -\frac{1}{\rho} \frac{dp}{dy} - g \Rightarrow \int_z^h \frac{dp}{dz} = \int_z^h -g\rho = -g\rho z + c$$

$-g\rho(z-h)+p_0=p$

b) regn ut akselerasjonen til vannpartiklene

$\bar{v} = w \times \bar{R}$: Rotasjonen

$\bar{v} = \bar{w} \times \bar{R}$ $\qquad \bar{w} = w\bar{k}$, $\bar{R} = r\bar{i}_r + z\bar{k}$

$\bar{v} = w\bar{k} \times (r\bar{i}_r + z\bar{k})$

$$= \begin{vmatrix} \bar{i}_r & \bar{\theta} & \bar{k} \\ 0 & 0 & w \\ r & 0 & z \end{vmatrix} = i \cdot 0 + (wr)\bar{\theta} + k \cdot 0$$

$wr\bar{\theta}$

Regner ut den partikkelderiverte

$$\bar{a} = \frac{D\bar{v}}{dt} = \cancel{\frac{\partial v}{\partial t}} + v \cdot \nabla v$$

Regner derd ut $v \cdot \nabla$

$$v \cdot \nabla = w r_\theta \cdot \left(i_r \frac{\partial}{\partial r} + \frac{i_\theta}{r} \frac{\partial}{\partial \theta} + \bar{k} \frac{\partial}{\partial z} \right)$$

$$= w \frac{d}{d\theta}$$

så resten

$$(v \cdot \nabla)\bar{v} = w \frac{d}{d\theta}(w r i_\theta) = w^2 r \frac{d i_\theta}{d\theta} = -w^2 r i_r$$