06.04.2018

Forventningsverdi

Svarer til snittet

Vil bruke rullett til å motivere.

37 felt 0 – 36

1 – 36 er røde og svarte 0 er grønn

k er antall felt spilleren satset på

36/k · satsen er gevinsten hvis personen vinner

satser på 18 felt, 10 euro

Kvinnen spiller 3 ganger

X er den samlede nettogevinsten

De mulige verdiene: . . .

$P(X=-30) = \left(\frac{19}{37}\right)^3 = 0{,}135$, taper 30 euro

$P(X=-10) = 3 \cdot \frac{18}{37} \cdot \left(\frac{19}{37}\right) = 0{,}365$ vinner 1, taper 2

$P(X=10) = 3 \cdot \left(\frac{18}{37}\right)^2 \cdot \frac{19}{37} = 0{,}365$ vinner 2, taper 1

$P(X=30) = \left(\frac{18}{37}\right)^3 = 0{,}115$

Gjennomsnitt:

$= -30 \cdot \frac{1}{10} - 10 \frac{3}{10} + 10 \cdot \frac{5}{10} + 30 \cdot \frac{1}{10}$

Relativ frekvens

$= -0{,}81 \rightarrow$ taper 0,81 euro hver gang

$= E_X = \mu_X = -0{,}81$

$$E(X) = \mu_X = \sum_{x \in D} x \cdot p(x)$$

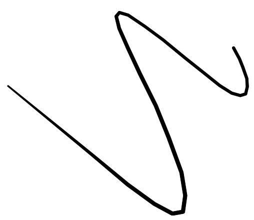
X = antall øyne i en kasting

Y = ...

$$E(X) = \sum_{x=1}^{k} x \cdot P(X=x) = 1\tfrac{1}{6} + 2\tfrac{1}{6} + 3\tfrac{1}{6} + 4\tfrac{1}{6} + 5\tfrac{1}{6} + 6\tfrac{1}{6} = 3{,}6$$

Matlab.

```
>> x=2:12;
>> px=[1 2 3 4 5 6 5 4 3 2 1]/36;
>> x*px
Error using  *
Inner matrix dimensions must agree.

>> x.*px

ans =

  Columns 1 through 10

    0.0556    0.1667    0.3333    0.5556    0.8333    1.1667    1.1111    1.0000    0.8333    0.6111

  Column 11

    0.3333

>> sum(x.*px)

ans =

    7.0000
```



2	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
3	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
4	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
5	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
6	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
4× 7	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)
		8	9	10	11	12

Geometrisk fordeling

$X \qquad p(x) = P(X = x) \qquad x = 1, 2, 3, \ldots$

$$p(x) = (1-p)^{x-1} p$$

Forventning

$$E(X) = \sum_{x=1}^{\infty} x\, p(x) =$$

$$\sum_{x=1}^{\infty} (1-p)^x = \frac{1-p}{p}$$

Deriverer på begge sider

$$\frac{d}{dp} \sum_{x=1}^{\infty} (1-p)^x = -\sum_{x=1}^{\infty} x(1-p)^{x-1} = \frac{1}{p^2}$$

Altså

$$\sum_{x=1}^{\infty} x(1-p)^{x-1} = \frac{1}{p^2} \Rightarrow \text{Dermed } E(X) = p \sum_{x=1}^{\infty} x(1-p)^{x-1} = \frac{1}{p}$$

x	-2	-1	0	1	2
$P(X=x)$	$0{,}1$	$.2$	$.4$	$.2$	$.1$

Punktsannsynligheten Y

Mulige verdier for Y er 0,1,4

Sannsynligheten for disse verdiene er

$$P(Y=0) = P(X=0) = 0{,}40$$

$$P(Y=1) = P(X=-1) + P(X=1) = 0{,}20 + 0{,}20 = 0{,}40$$

$$P(Y=4) = P(X=-2) + P(X=2) = .1 + .1 = .2$$

1) med

$$E(X) = 0 \cdot P(Y=0) + 1 \cdot P(Y=1) + 4P(Y=4)$$

$$= 1.2$$

$$X : p(x) = P(X=x)$$

$$Y = h(X) = aX + b$$

$$E(Y) = \sum_{x=0} (aX + b)\, p(x) = a \overbrace{\sum_x x\, p(x)}^{E(X)} + \overbrace{\sum_x p(x)}^{1}$$

Varians og standardavvik

Forventningsverdi:

snitt verdi $x \to \infty$

Varians:

$$\sigma_x^2 = V(x) = \sum (x-\mu)^2 \cdot p(x) = E\{(x-\mu)^2\}$$

standardavvik $\sigma = \sqrt{V(x)}$

Eks

x = antall øyne, kaster en terning

$E(x) = 7/2$

$$V(x) = \sum_{x=1}^{6} \left(x - \frac{7}{2}\right)^2 P(X=x) = \left(\frac{5}{2}\right)^2 \cdot \frac{1}{6} \cdots$$ (se PP)

$$V(x) = E(x^2) - [E(x)]^2$$

Eks:

Y	1	2	3	4	5	6
$P_{Y=y}$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$$\mu = E(Y) = 1 \cdot \frac{1}{36} + 2 \cdot \frac{3}{36} + 3\frac{5}{36} + \cdots + 6\frac{11}{36} = \frac{161}{36} = 4{,}47$$

$$E(x^2) = 1^2 \cdot \frac{1}{36} + 2^2 \frac{3}{36} + \cdots + 6^2 \frac{11}{36} = \frac{791}{36} = 21{,}97$$

$$V(x) = E(x^2) - \mu^2$$

$$Y = aX + b \qquad \mu_Y = E(Y) = a\overbrace{E(X)}^{\mu_x} + b = a\mu_x + b$$

$$V(aX+b) = V(Y) = E\{(Y-\mu_Y)^2\} = E\{(aX+b)-(a\mu_X+b)\}^2$$

$\vdots$

Momentgenererende funksjoner 3.4

$$M_X(t) = E(\underbrace{e^{tX}}_{h(x)}) = \sum \underbrace{e^{tx}}_{h(x)} \cdot p(x)$$

Den momentgenererende funksjon eksisterer hvis det finnes et tall $t_0 > 0$ slik at

$$\sum_{x \in D} e^{tx} \cdot p(x) < \infty$$

for alle $t \in (-t_0, t_0)$

Eks.

Hvor mange ganger må man kaste en terning, før du får "6"

$$= \sum_{x=1}^{\infty} e^{tx} \cdot (1-p)^{x-1} \cdot p$$

$$= pe^t \sum_{x=1}^{\infty} (e^t(1-p))^{x-1}$$

$$\frac{pe^t}{1-(1-p)e^t} \quad \text{- så sant } |e^t(1-p)| < 1$$