1 Finn de partiellderiverte til f

a) $f(x,y) = x^3 y + 3xy^4$

Deriverer for x med y som en konstant så motsatt

$\frac{df}{dx} \; x^3y + 3xy^4$

$\underline{\underline{3yx^2 + 3y^4}}$

$\frac{df}{dy} \; x^3y + 3xy^4$

$\underline{\underline{x^3 + 12xy^3}}$

c) $f(x,y) = \cos(x + y^2)$

$\frac{df}{dx}(x;y) = -\sin(x + y^2) \cdot y = \underline{\underline{-\sin(x + y^2)}}$

$\frac{df}{dy}(y;x) = -\sin(x + y^2) \cdot 2y \; \underline{\underline{= -2y\sin(x + y^2)}}$

e) $f(x,y,z) = (x+y)\, e^{-z}$

$$\frac{df}{dx}(x; y, z) = (x e^{-z} + e^{-z} y)'$$
$$= \underline{\underline{e^{-z}}}$$

$$\frac{df}{dy}(y; x, z) = \underline{\underline{e^{-z}}}$$

$$\frac{df}{dz}(z; x, y) = (x+y)\, e^{-z} \cdot (-z)'$$
$$= (x+y)\, e^{-z} \cdot -1$$
$$= \underline{\underline{-(x+y)\, e^{-z}}}$$

g) $f(x, y, z) = z \arctan(x+y)$

$$\frac{df}{dx}(x; y, z) = z \cdot \frac{1}{1+(x+y)^2} \cdot 1$$
$$= \underline{\underline{\frac{z}{1+(x+y)^2}}}$$

$$\frac{df}{dy}(y; x, z) = \underline{\underline{\frac{z}{1+(x+y)^2}}}$$

$$\frac{df}{dz}(z; x, z) = \underline{\underline{\arctan(x+y)}}$$

2 Finn gradienten til funksjonen

a) $f(x,y) = x^2 y$

$$\frac{df}{dx}(x;y) = 2yx$$

$$\frac{df}{dy}(y;x) = x^2$$

$$\nabla f'(x,y) = (2yx,\ x^2)$$

b) $f(x,y,z) = x\cos(xy^2z)$

$$\frac{df}{dx}(x;y,z) = x' \cdot \cos(xy^2z) + x \cdot \cos(xy^2z)'$$

$$= \cos(xy^2z) + x \cdot -\sin(xy^2z) \cdot y^2z$$

$$= \cos(xy^2z) - xy^2z\sin(xy^2z)$$

$$\frac{df}{dy}(y;xz) = -x\sin(xy^2z) \cdot 2xzy$$

$$-2x^2zy\sin(xy^2z)$$

$$\frac{df}{dz}(z;xy) = -x\sin(xy^2z)\ xy^2$$

$$= -x^2y^2\sin(xy^2z)$$

$$\nabla f(x,y,z) = (\cos xy^2z - xy^2z\sin(xy^2z),$$
$$-2x^2zy\sin(xy^2z),$$
$$-x^2y^2\sin xy^2z\ \sin(xy^2z))$$

c) $f(u, v, w) = w e^{u \cos v}$

$$\frac{df}{du}(u; v, w) = w e^{u \cos v} \cdot \cos v = \underline{w \cos(v) e^{u \cos v}}$$

$$\frac{df}{dv}(v; u, w) = w e^{u \cos v} \cdot (u \cos v)'$$
$$= w e^{u \cos v} \cdot u(-\sin v) = -uw \sin v \, e^{u \cos v}$$

$$\frac{df}{dw}(w; u, v) = \underline{e^{u \cos v}}$$

$$\nabla f(u, v, w) = \underline{\underline{\left(w \cos(v) e^{u \cos v}, \; -uw \sin v \, e^{\cos v}, \; e^{u \cos v} \right)}}$$

3 Finn den retningsderiverte

a) $f(x, y) = 3xy + y^2$, $a = (1, 2)$, $r = (3, -1)$

$$\left. \begin{aligned} \frac{df}{dx}(x; y) &= 3y \\ \frac{df}{dy}(y; x) &= 3x + 2y \end{aligned} \right\} \nabla f(x, y) = (3y, 3x + 2y)$$

$$\nabla f(a) = \nabla f(1, 2) = (3 \cdot 2, 3 \cdot 1 + 2 \cdot 2)$$
$$= (6, 7)$$

$$r \cdot \nabla f(a) = (6, 7) \cdot (3, -1) = 18 - 7 = \underline{\underline{11}}$$

d) $f(x,y,z) = z\sin(xy)$, $a = \left(\frac{\pi}{2}, 1, 0\right)$, $R = (2, 0, -1)$

$$\left.\begin{aligned} \frac{df}{dx}(x; y, z) &= yz\cos(xy) \\ \frac{df}{dy}(y; x\, z) &= xz\cos(xy) \\ \frac{df}{dz}(z; x, y) &= \sin(xy) \end{aligned}\right\} \nabla f(x, y\, z) = (yz\cos(xy),\ xz\cos(xy),\ \sin(xy))$$

$$\nabla f(a) = f\left(\frac{\pi}{2}, 1, 0\right) = (0, 0, 1)$$

$$\nabla f(a; r) = 0 + 0 - 1 = \underline{\underline{-1}}$$

4 I hvilken retning vokser $f'(a; r)$ raskest

c) $f(x, y, z, u) = xuz^2 - y^2 zu$

$$\frac{df}{dx} = \underline{uz^2}$$

$$\frac{df}{dy}(y; x, z, u) = \underline{2yzu}$$

$$\frac{df}{dz}(z; x, y, u) = \underline{2xuz - y^2 u}$$

$$\frac{df}{du}(u; x, y, z) = \underline{xz^2 - y^2 z}$$

$$\nabla f(x, y, z, u) = (uz^2,\ 2yzu,\ 2xuz - y^2u,\ xz^2 - y^2 z)$$

$$a = \overset{x}{1}, \overset{y}{0}, \overset{z}{-2}, \overset{u}{3}$$

$\nabla f(a) = \nabla f(1,0,-2,1) = (3 \cdot 4,\ 0,\ 2\cdot 1\cdot 3\cdot -2,\ 4)$

$= (12,\ 0,\ -12,\ 4)$

$= 4(3,\ 0,\ -3,\ 1)$

den raskeste retningen er gitt ved $(3,\ 0\ -3,\ 1)$

5 radius : r Volum : $V = \pi r^2 h$

høyde : h $f(r,h)$ $f(\Delta r, \Delta h)$

$r \in (2,\ 2,05),\ h \in (5,\ 5,05)$

$V(r,h) = \pi r^2 h$ $\bar{u} = (5,05,\ 2,05)$

$\frac{dV}{dr}(r;h) = 2\pi h r$

$\frac{dV}{dh}(h;r) = \pi r^2$

$\underbrace{2\pi h r}_{\frac{dV}{dr}} \cdot \Delta r + \underbrace{\pi r^2}_{\frac{dV}{dh}} \cdot \Delta h$

$2\pi h r \Delta r + \pi r^2 \Delta h$

maks r og maks h

$= 2\pi \cdot 5 \cdot 2 \cdot 0,05 + 4\pi \cdot 0,05$

$= 3,8$

6 $f(v, h) = \frac{v}{h^2}$

$\frac{df}{dv}(v; h) = \frac{1}{h^2}$

$\frac{df}{dh}(h;) = -2h^{-3} V = \frac{-2V}{h^3}$

$\frac{df}{dv} \cdot \Delta v + \frac{df}{dh} \Delta h$

$\frac{1}{h^2} \Delta V \cdot -2 \frac{V}{h^3} \Delta h$

b)