Eksempel

$$\underline{v} = \underline{w} \times \underline{r}$$

$$\underline{w} = w\,\underline{k}$$

$$\underline{r} = R\,\underline{i}_R(\theta) + z\,\underline{k}$$

$$\underline{v} = \begin{vmatrix} \underline{i}_R & \underline{i}_\theta & \underline{i}_z \\ 0 & 0 & w \\ R & 0 & z \end{vmatrix} = wR\,\underline{i}_\theta$$

$$\nabla \cdot \underline{v} = \frac{1}{R}\frac{d v_\theta}{d\theta} = \frac{1}{R}\frac{d}{d\theta}(wR) = 0$$

$$\nabla \times \underline{v} = \frac{1}{R}\begin{vmatrix} \underline{i}_R & R\,\underline{i}_\theta & \underline{i}_z \\ \frac{d}{dR} & \frac{d}{d\theta} & \frac{d}{dz} \\ 0 & RwR & 0 \end{vmatrix} = \frac{\underline{i}_z}{R}\frac{d}{dR}(wR^2)$$

$$= \frac{\underline{i}_z}{R}\,2wR$$

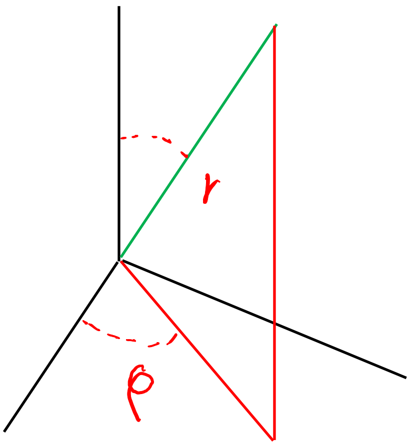
$$= 2w\,\underline{i}_z = 2\underline{w}$$

Partikkel akselerasjon

$$\frac{D\underline{v}}{dt} = \underbrace{\cancel{\frac{d\underline{v}}{dt}}}_{0} + (\underline{v}\cdot\nabla)\,\underline{v}$$

$$(\underline{v}\cdot\nabla) = wR\,\underline{i}_\theta \cdot \left(\underline{i}\frac{d}{dR} + \underline{i}_\theta\frac{d}{d\theta} + \underline{i}\frac{d}{dz}\right) = w\frac{d}{d\theta}$$

$$\underline{v}\cdot\nabla\underline{v} = w\frac{d}{d\theta}\left(wR\, i_\theta\right) = -w^2 R\, i_R$$



$$\{r, \theta, \varphi\} \qquad \begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \\ z &= r \end{aligned}$$

$$\begin{aligned} \underline{r} = {} & r\sin\theta\cos\varphi\,\underline{i} \\ & + r\sin\theta\sin\varphi\,\underline{j} \\ & + r\cos\theta\,k \end{aligned}$$

$$\frac{d\underline{r}}{d\theta} = r\cos\theta\cos\varphi\,\underline{i} + r\cos\theta\sin\varphi - r\sin\theta\,\underline{k}$$

$$h_\theta = \frac{d\underline{r}}{d\theta} = r$$

$$\underline{i}_\theta = \cos\theta\cos\varphi\,\underline{i} + \cos\theta\sin\underline{\varphi} - \sin\theta\,\underline{k}$$

$$\frac{d\underline{r}}{d\varphi} = -r\sin\theta\sin\varphi\,\underline{i} + r\sin\theta\cos\varphi\,\underline{j}$$

$$h_\varphi = r\sin\theta$$

$$\underline{i}_\varphi = -\sin\varphi\,\underline{i} + \cos\varphi\,\underline{j}$$

Ortogonalt (Ja)

$$i_r \cdot i_\theta = \sin\theta\cos\varphi\cos\theta\cos\varphi + \sin\theta\sin\varphi\cos\theta\sin\varphi - \cos\theta\sin\theta = 0$$

$$i_r \cdot i_\varphi = \sin\theta\cos\varphi + \sin\theta\sin\varphi\cos\varphi = 0$$

$$i_\theta \cdot i_\varphi = 0$$

Er det høyrehånd

$$i_R \times i_\theta = \begin{vmatrix} i & j & k \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \end{vmatrix} = \dots = i_\varphi$$

Rettlinjet eller krumlinjet

	i_r	i_θ	i_φ
$\frac{d}{dr}$	0	0	0
$\frac{d}{d\theta}$	i_θ	$-i_r$	0
$\frac{d}{d\varphi}$	$\sin\theta\, i_\varphi$	$\cos\theta\, i_\varphi$	$-\sin\theta\, i_r - \cos\theta\, i_\theta$

Mye arbeid