1. Skalering

$$x(t) = v_0 t \cos(\theta)$$
$$y(t) = v_0 t \sin(\theta) - \frac{1}{2} g t^2$$

a) Finn t_m, $y = 0$, $x(t_m) = x_m$

$$y(t_m) = 0 = v_0 t_m \sin\theta - \frac{1}{2} g t_m^2$$

bruker a, b, c - formelen

$$a = -\frac{1}{2} g, \quad b = v_0 \sin\theta, \quad c = 0$$

$$\frac{-v_0 \sin\theta \pm \sqrt{(-v_0 \sin\theta)^2 - 4 \cdot \frac{1}{2} g \cdot 0}}{2\left(\frac{1}{2} g\right)}$$

$$\frac{-v_0 \sin\theta \pm v_0 \sin\theta}{-g}$$

$$t_{m_1} = 0 \quad \vee \quad t_{m_2} = \frac{-2 v_0 \sin\theta}{-g}$$

Velger t_{m_2} som t_m siden $t_{m_1} = 0$ er starten

$$\underline{\underline{t_m = \frac{-2 v_0 \sin\theta}{-g}}}$$

$$x(t_m) = x_m = v_0 \left(\frac{-2 \sin\theta}{-g}\right) \cos\theta$$

$$= \frac{2 v_0 \sin\theta \cos\theta}{g}$$

c) innføre dimensjonsløse variable (x^*, y^*, t^*) for x, y, t

Skalerer t

$$t^* = \frac{t}{t_m} \iff t = t^* t_m \implies t = \frac{-2t^* v_0 \sin(\theta)}{g}$$

Skalerer y

$$y^* = \frac{y}{x_m}$$

$$\frac{v_0 t \sin\theta - \frac{1}{2} g t^2}{\frac{2 v_0 \sin\theta \cos\theta}{g}}$$

$$\frac{\cancel{v_0} t \cancel{\sin\theta}\, g}{2 \cancel{v_0} \cancel{\sin\theta} \cos\theta} - \frac{g^2 t^2}{4 v_0^2 \sin\theta \cos\theta}$$

$$\frac{tg}{2 \cos\theta\, v_0} - \frac{g^2 t^2}{4 v_0^2 \sin\theta \cos\theta}$$

Setter inn for t

$$\frac{\left(\frac{\cancel{2} t^* \cancel{v_0} \sin(\theta)}{\cancel{g}}\right) \cancel{g}}{\cancel{2} \cancel{v_0} \cos\theta} - \frac{\cancel{g^2} \left(\frac{\cancel{2} t^* \cancel{v_0} \cancel{\sin}(\theta)}{\cancel{g}}\right)^{\cancel{2}}}{\cancel{4} \cancel{v_0^2 \sin}\theta \cos\theta}$$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$ på begge leddene

$$y^* = \tan\theta\, t^* - \tan(\theta)\, t^{*2}$$

Skalerer X

$$X^* = \frac{X}{X_m} = \frac{\cancel{V_0} t \cancel{\cos\theta}}{2 V_0^2 / g \cancel{\cos\theta} \sin\theta}$$

$$= \frac{t g}{2 V_0 \sin\theta}$$

setter inn for t

$$\Rightarrow \frac{\left(\frac{\cancel{2}\cancel{V_0} t^* \cancel{\sin\theta}}{\cancel{g}}\right)\cancel{g}}{\cancel{2}\cancel{V_0}\cancel{\sin\theta}}$$

$$= t^*$$

dermed er $x^*, y^*, z^* = t^*, \tan(\theta)t^* - \tan(\theta)t^{*2}, \frac{2 V_0 \sin\theta}{g}$

Vi trenger ikke skalere θ siden den ikke har en enhet

2 $V = V_x \underline{i} + V_y \underline{j} = xy\,\underline{i} + y\,\underline{j}$

$V_x = xy, \quad V_y = y$

a) finn strømlinjene

Løser kryssproduktet

$$\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} \\ xy & y & 0 \\ dx & dy & 0 \end{pmatrix} = (xy\,dy - y\,dx)\,\underline{k} = 0$$

av dette ser vi at langs en strømlinje må

$$xy\,dy = y\,dx$$

ser at det er en separabel differensiallikning. separerer

$$\frac{xy\,dy}{xy} = \frac{y\,dx}{xy}$$

$$dy = \frac{1}{x}\,dx$$

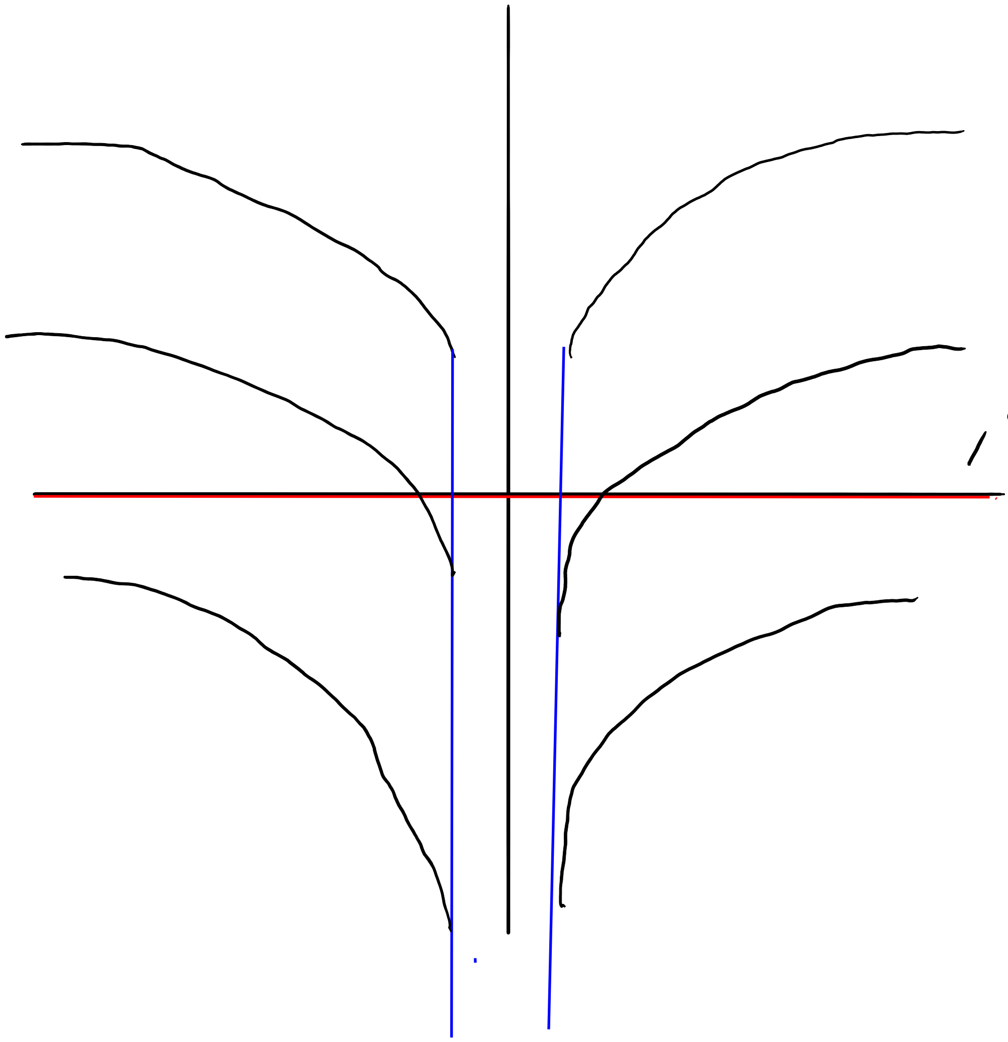
integrerer

$$\int dy = \int \frac{1}{x}\,dx$$

$$y = \ln|x| + C$$

Løser for c

$c = \ln|x| - y$, Velger en konstant "c" så får du en strømlinje



c) Viser at det ikke finnes en strømfunksjon
ved å sjekke $\nabla \cdot v \Leftrightarrow 0$

$$\frac{d v_x}{d x} = y \,,\; \frac{d v_y}{d y} = 0$$

$$y + 1 \not\Leftrightarrow$$

så dermed er det ingen strømfunksjon