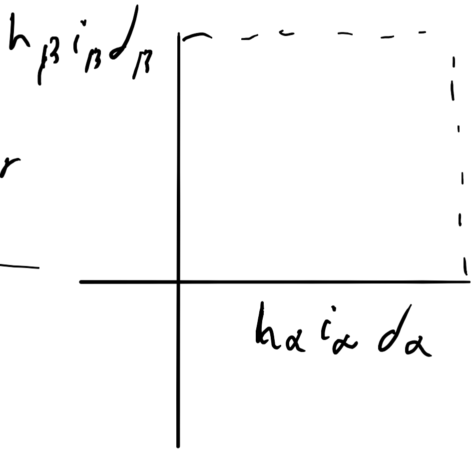
Krumlinjet koordinatsystem

Totalt differensial posisjonsvektor
= kurve element



$$d\underline{r} = h_\alpha i_\alpha d\alpha + h_\beta \underline{i}_\beta d\beta + h_\gamma i_\gamma d\gamma$$

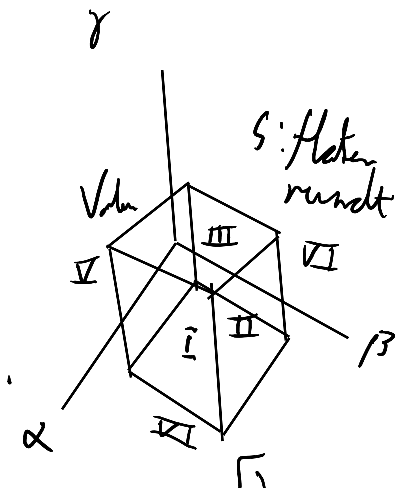
Flateelement

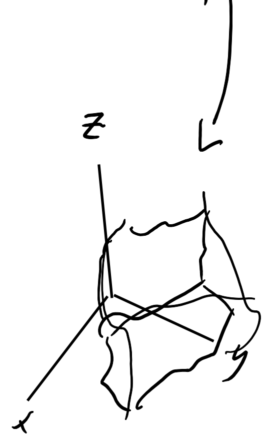
$$\underline{n}\, d\sigma = \begin{cases} h_\alpha h_\beta \underline{i}_\gamma\, d\alpha\, d\beta \\ h_\beta h_\gamma i_\alpha\, d\beta\, d\gamma \\ h_\gamma h_\gamma i_\alpha\, d\gamma\, d\alpha \end{cases}$$

$$d\tau = h_\alpha h_\beta h_\gamma\, d\alpha\, d\beta\, d\gamma$$

Divergens

$$\text{div}\, \underline{v} = \lim_{V \to 0} \frac{1}{V} \int \underline{v} \cdot \underline{n}\sigma$$





$$\text{Volum} = \int_V d\tau = \iiint_{\alpha_1 - \frac{\Delta\alpha}{2}}^{\alpha_0 + \frac{\Delta\alpha}{2}} h_\alpha h_\beta h_\gamma\, d\alpha\, h_\beta\, d\gamma$$

$$V \approx h_\alpha h_\beta h_\gamma \Delta\alpha\, \Delta\beta\, \Delta\gamma$$

$$\int_C \underline{v} \cdot n\sigma = \sum_{i=1}^{6} \int_{S_i} \underline{v} \cdot n\sigma$$

<u>Side 1</u>

$$\underline{n} \cdot d\sigma = h_\beta h_\gamma \underline{i}_\alpha \, d\beta \, d\gamma$$

$$\underline{v} \cdot \underline{n} d$$

$$= h_\beta h_\gamma V_\alpha \, d\beta \, d\gamma$$

Tilsvarande

$$\int \underline{v} \cdot n \, d \doteq \int \int_{\beta_0 - \frac{\Delta\beta}{2}}^{\beta_0 + \frac{\Delta\beta}{2}} h_\beta h_\gamma V_\alpha \Big|_{\alpha_0 + \frac{\Delta\alpha}{2}} d\beta \, d\gamma \approx \left[h_\beta h_\gamma V_\alpha \right]_{\alpha_0 + \frac{\Delta\alpha}{2}}^{\beta} \Delta\beta \Delta\sigma$$

Motstående side (6)

$$S_6 \; \underline{n} d\sigma = -h_\beta h_\alpha \underline{i}_\alpha \, d\beta \, d\gamma$$

$$\int_{S_6} \underline{v} \, d\sigma \approx -\left[h_\beta h_\sigma V_\alpha \right]_{\alpha_0 - \frac{\Delta\alpha}{2}} \Delta\beta \Delta\gamma$$

$$\left(\int_{S_1} + \int_{S_6} \right) \underline{v} \cdot \underline{n} d\sigma \doteq \left[h_\beta h_\sigma V_\alpha \right]_{\alpha_0 - \frac{\Delta\alpha}{2}}^{\alpha_0 + \frac{\Delta\alpha}{2}} \Delta\beta \Delta\gamma$$

$$\approx \frac{d}{d\alpha} \left(h_\beta h_\gamma V_\alpha \right) \Delta\alpha \Delta\beta \Delta\gamma$$

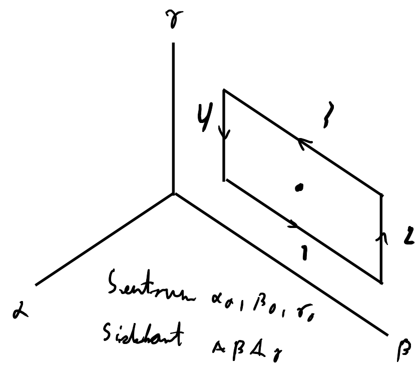
$$\Delta\beta\Delta\gamma \approx \frac{d}{d\alpha} \left(h_\beta h_\gamma V_\alpha \right) \Delta\alpha \Delta\beta \Delta\gamma$$

$$\frac{1}{V}\int \underline{v}\cdot n\, d\sigma \approx \lim_{V\to 0} \frac{1}{h_\alpha h_\beta h_\gamma \cancel{\Delta\alpha\Delta\beta\Delta\gamma}} \left\{ \frac{d}{d_\alpha}(h_\beta h_\gamma v_\alpha) + \frac{d}{d_\beta}(h_\alpha h_\gamma v_\beta) + \frac{d}{d_\gamma}(h_\alpha h_\beta v_\gamma) \right\} \cancel{\Delta\alpha\Delta\beta\Delta\gamma}$$

$$\nabla\cdot v = \frac{1}{h_\alpha h_\beta h_\gamma} \left\{ \frac{d}{d_\alpha}(h_\beta h_\gamma v_\alpha) + \frac{d}{d_\beta}(h_\alpha h_\gamma v_\beta) + \frac{d}{d_\gamma}(h_\alpha h_\beta v_\gamma) \right\}$$

Virvling

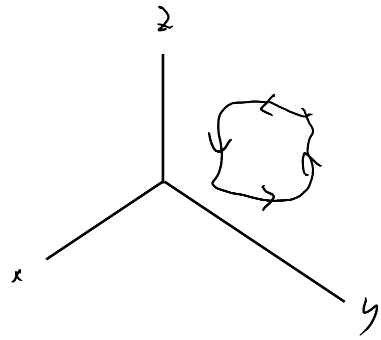
$$\underline{n}\cdot \mathrm{curl}\,\underline{v} = \lim_{A\to 0} \frac{1}{A} \oint_\lambda v\cdot d\underline{r}$$



$$A = \int d\sigma = \iint_{\beta_0-\frac{\Delta\beta}{2}}^{\beta_0+\frac{\Delta\beta}{2}} h_\beta h_\gamma \, d\beta\, d\gamma \approx h_\beta h_\gamma \Delta\beta \Delta\gamma$$

deler i 4 sider

$$\oint_\lambda \underline{v}\cdot d\underline{r} = \sum_{i=1}^{4} \int_{\lambda_i} \underline{v}\cdot d\underline{r}$$



Side 1 i $\alpha = \alpha_0,\ \gamma = \gamma_0 - \frac{\Delta\gamma}{2},\ d\underline{r} = h_\beta \underline{i}_\beta d\beta$

$$\underline{v}\cdot d\underline{r} = \int_{\beta_0-\frac{\Delta\beta}{2}}^{\beta_0+\frac{\Delta\beta}{2}} h_\beta v_\beta \, d\beta \approx \Big[h_\beta v_\beta\Big]_{\gamma_0-\frac{\Delta\gamma}{2}} \Delta\beta$$

Side 3 $\quad \alpha = \alpha_0 ,\ \gamma = \gamma_0 + \frac{\Delta\gamma}{2} \qquad d\underline{r} = -h_\beta \underline{i}_\beta d\beta$

$$\underline{v}\cdot d\underline{r} = \int_{\beta_0-\frac{\Delta\beta}{2}}^{\beta_0+\frac{\Delta\beta}{2}} -h_\beta v_\beta \, d\beta \approx -\left[h_\beta v_\beta\right]_{\gamma_0+\frac{\Delta\gamma}{2}} \Delta\beta$$

$$\left(\int_{\lambda_1}+\int_{\lambda_2}\right)\underline{v}\cdot d\underline{r} = \left[-h_\beta v_\beta\right]_{\gamma_0-\frac{\Delta\gamma}{2}}^{\gamma_0+\frac{\Delta\gamma}{2}} \qquad \Delta\beta \approx \frac{d}{d\gamma}\left(-h_\beta v_\beta\right)\Delta\gamma\Delta\beta$$

Side 2 og 4

$$\left(\int_{\lambda_2}+\int_{\lambda_4}\right)\underline{v}\cdot d\underline{r} = \frac{d}{d\beta}\left(h_\gamma v_\gamma\right)\Delta\beta\Delta\gamma$$

$$i_\alpha\cdot curl(\underline{v}) \approx \frac{\overbrace{\underline{i}_\alpha\cdot\underline{i}_\alpha}^{1}\, h_\alpha}{\cancel{h}\, h_\alpha h_\beta h_\gamma \cancel{\Delta\beta\Delta\gamma}}\left\{\frac{d}{d\beta}\left(h_\gamma v_\gamma\right) - \frac{d}{d\gamma}\left(h_\beta v_\beta\right)\right\}\cancel{\Delta\beta\Delta\gamma}$$

Når vi legger til lim så slutter avhengighet å være tilnærmet likt $\Delta\to 0$

$$\underline{i}_\alpha\cdot(\nabla\times\underline{v}) = \underline{i}_\alpha\cdot\frac{h_\alpha i_\alpha}{h_\alpha h_\beta h_\gamma}\left(\frac{d}{d\beta}\left(h_\gamma v_\gamma\right) - \frac{d}{d\gamma}\left(h_\beta v_\beta\right)\right\}$$

$$i_\alpha\cdot\frac{1}{h_\alpha h_\beta h_\gamma}\begin{vmatrix} h_\alpha i_\alpha & h_\beta i_\beta & h_\gamma \underline{i}_\gamma \\ \frac{d}{d\alpha} & \frac{d}{d\beta} & \frac{d}{d\gamma} \\ h_\alpha v_\alpha & h_\beta v_\beta & d_\gamma v_\gamma \end{vmatrix}$$

Hvzh gradienter

$$\nabla f = \frac{i_\alpha}{h_\alpha}\frac{df}{d\alpha} + \frac{\bar{i}_\beta}{h_\beta}\frac{df}{d\beta} + \frac{i_\gamma}{h_\gamma}\frac{df}{d\gamma}$$