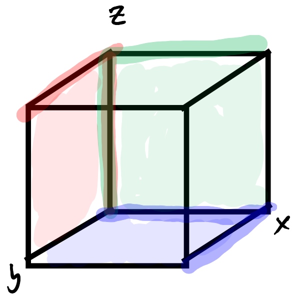
1) Vektorfelt

$$\bar v = 4x^2y\,\bar i + xyz\,\bar j + yz^2\,\bar k$$

a) integrert fluks



$$0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1.$$

$$= \int_0^1\int_0^1 4\cdot 1^2\, y \; dy\,dz$$

$$= 4\int_0^1 y\int_0^1 1 \; dz\,dy = 4\int_0^1 y\,[z]_0^1\,dy = 4\int_0^1 y = 4\left[\frac{y^2}{2}\right]_0^1 = \underline{2}$$

$$\int_0^1\int_0^1 x\,\overset{=1}{y}\,z \; dx\,dz = \int_0^1 x\int_0^1 z\,dz\,dx = \int_0^1 x\left[\frac{z^2}{2}\right]_0^1 = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 = \underline{\frac{1}{4}}$$

$$\int_0^1\int_0^1 y\,1^2\,dx\,dy = \int_0^1 y\int_0^1 1\;dx\,dy = \left[\frac{y^2}{2}\right]_0^1 = \underline{\frac{1}{2}}$$

$$\int_0^1\int_0^1 -4\,0^2 y\,\bar i\cdot -\bar i\,dy\,dz = 0$$

$$\int_0^1\int_0^1 -x\,\overset{0}{y}\,z\;dx\,dz = 0$$

$$\int_0^1\int_0^1 -y\,\overset{0}{z}\;dx\,dy = 0$$

$$2 + \frac{1}{4} + \frac{1}{2} = \frac{8}{4} + \frac{1}{4} + \frac{2}{4} = \underline{\underline{\frac{11}{4}}}$$

R

2 proper Gaus

$$\nabla \cdot \bar{v} = \frac{d\bar{v}_x}{dx}\,\bar{i} + \frac{d\bar{v}_y}{dy}\,\bar{j} + \frac{d\bar{v}_z}{dz}\,\bar{k}$$

$$\frac{d}{dx}\,4x^2y = 8xy$$

$$\frac{d}{dy}\,xyz = xz$$

$$\frac{d}{dz}\,yz^2 = 2yz$$

$$\underline{\nabla \cdot \bar{v}} = 8xy + xz + 2yz$$

$$\int_0^1\int_0^1\int_0^1 8xy + xz + 2yz \; dx\,dy\,dz$$

$$\int_0^1\int_0^1\int_0^1 8xy\;dx\,dy\,dz = \int_0^1\int_0^1 8xy\,[z]_0^1 = \int_0^1 8x \int_0^1 y\,dy\,dx = \int_0^1 8x\cdot\frac{1}{2}$$

$$= 4\int_0^1 x\,dx = 4\,\frac{1}{2} = \underline{2}$$

$$\int_0^1\int_0^1\cancel{\int_0^1} yz\;\cancel{dx}\,dy\,dz = \int_0^1 y\int_0^1 z\,dz\,dy = \cancel{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \underline{\frac{1}{4}}$$

$$\int_0^1\int_0^1\cancel{\int_0^1} 2yz\;dx\,\cancel{dy}\,dz = 2\int_0^1 y\int_0^1 z = 2\cdot\frac{1}{2}\cdot\frac{1}{2} = \underline{\frac{1}{2}}$$

$$2 + \frac{1}{4} + \frac{1}{2} = \underline{\underline{\frac{11}{4}}}$$ ~ Gaus' sats

1 $\quad -a \le x \le a, \; -h \le z \le 0$

$$\bar{v} = \frac{U(z+h)}{h}\bar{j}$$

U er vannets hastighet i vannoverflaten

i $z = 0$ er $p = p_0$

h er dybden

ρ vannets tetthet

2a er bredden

$g = -g\bar{k}$ tyngdeakselerasjon

1a) Finn ut om det er et potensial og isåfall finn det

$\boxed{\nabla \times \bar{F} = 0, \text{ isåfall er det potensial}}$

$$\left(\frac{\cancel{dA_z}}{\cancel{dy}} - \frac{dA_y}{dz}\right)\bar{i} + \cancel{\left(\frac{dA_x}{dz} - \frac{A_z}{dx}\right)}\bar{j} + \left(\frac{dA_y}{dx} - \cancel{\frac{dA_x}{dy}}\right)\bar{k}$$

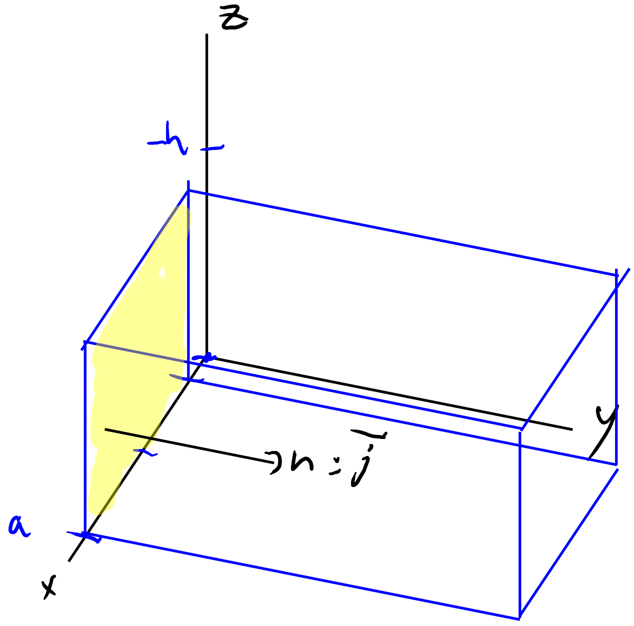
$$-\frac{d}{dz}\frac{U(z+h)}{h}\bar{i} + \frac{d}{dx}\frac{U(z+h)}{h}\bar{k}$$

$$\begin{cases} -\left(\frac{d}{dz}\, Uh^{-1}z + \cancel{h}z\cancel{h^{-1}}\right) = -\left(\frac{U}{h} + 1\right)\bar{i} \\ \frac{d}{dx}\frac{U(z+h)}{h}\bar{k} = 0 \end{cases}$$

$$\nabla \times \bar{F} = \left(\frac{U}{h} + 1\right)\bar{i}$$

b) Tverrsnitt $y=0$

$$\bar v = \frac{V(z+h)}{h}\,\bar j$$



$$\int_0^h \int_0^{2a} \frac{V(z+h)}{h}\,\bar j\cdot\bar j\,dx\,dz$$

$$\int_0^h \left(\frac{Vz}{h}+V\right)\int_0^{2a} 1\,dx\,dz = \frac{V}{h}\int_0^h (z+h)\cdot 2a\,dz$$

$$= \frac{2Va}{h}\int_0^h z\,dz = \frac{2VA}{h}\left[\frac{z^2}{2}\right]_0^h = \underline{\underline{Vah}}$$

2c) Kontinuitetsligningen

$$\frac{D\rho}{Dt} = -\rho\,\nabla\cdot\bar v$$

$$\underbrace{\frac{D\rho}{Dt}}_{0} + \underbrace{\rho}_{\rho_1}(\nabla\cdot\bar v) = 0$$

$\nabla\cdot\bar v = 1$ (1)

(2)

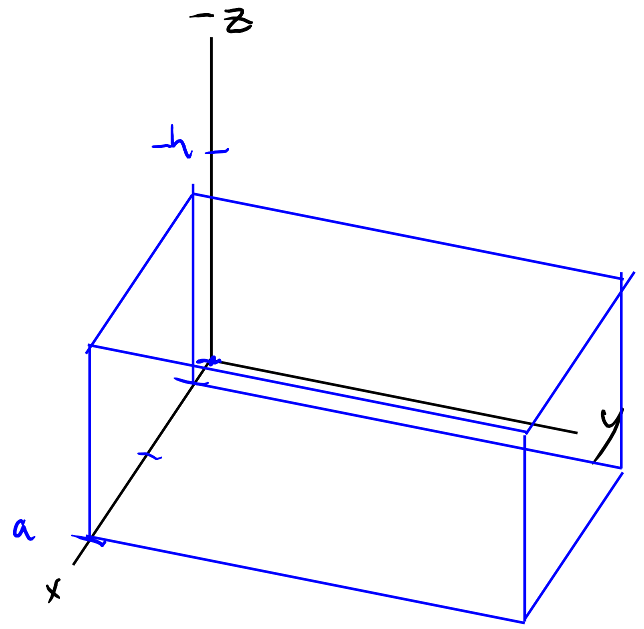
Nå å dermed regne ut $\nabla\cdot\bar v$

$$\frac{\partial v_y}{\partial y} = 0$$

Dermed er kontinuitetslikninga oppfylt

2d) Bruk Eulers likning

e)



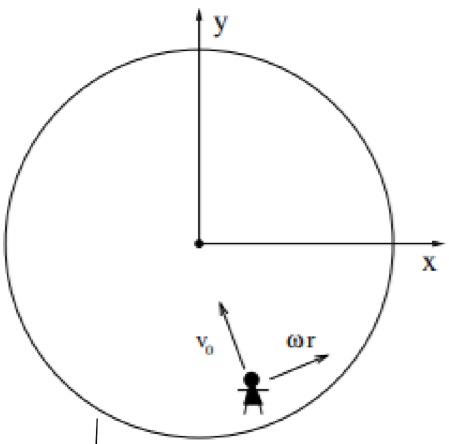
Oppgave 3

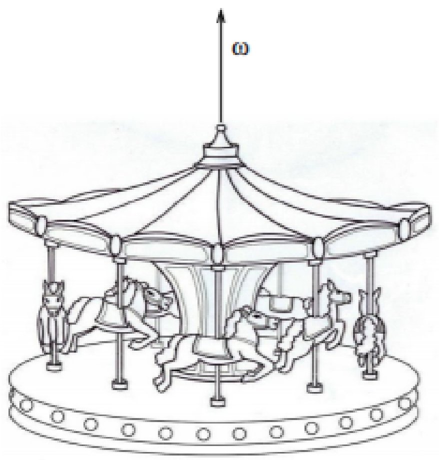
En karusell i en fornøyelsespark roterer med konstant vinkelhastighet $\boldsymbol{\omega} = \omega \boldsymbol{k}$. Omdreiningsaksen sammenfaller med z-aksen som peker oppover, $\boldsymbol{k}$ er enhets-koordinatvektor i z-retning, xy-planet er horisontalt, r er radiell horisontal avstand fra omdreiningsaksen, θ er vinkelen i xy-planet slik at $x = r\cos\theta$ og $y = r\sin\theta$.

En person på karusellen flytter seg innover mot sentrum med konstant fart v_0 i radiell retning, samtidig som hun roterer med karusellens bevegelse i vinkelretning, hun beveger seg derfor i henhold til hastighetsfeltet

$$\boldsymbol{v} = \omega r \boldsymbol{i}_\theta - v_0 \boldsymbol{i}_r$$

hvor $\boldsymbol{i}_r$ er enhets-koordinatvektor i radiell retning og $\boldsymbol{i}_\theta$ er enhets-koordinatvektor i vinkelretning.





Illustrasjon fra `http://www.imagui.com/a/dibujo-de-feria-iRRjA8xLx`

Merk: Dersom du prøver dette på en karusell som roterer fritt vil du oppleve at vinkelhastigheten øker. I denne oppgaven antar vi derimot at karusellen ikke roterer fritt, men at vinkelhastigheten holdes konstant!

I følgende deloppgaver ønsker vi svarene presentert i sylinderkoordinater:

3a

Regn ut divergensen til $\boldsymbol{v}$.

3b

Regn ut virvlinga til $\boldsymbol{v}$.

3c

Vi skal finne akselerasjonen til personen ved å regne ut den partikkelderiverte av hastighetsfeltet $\boldsymbol{v}$: Regn ut lokalakselerasjonen og den konvektive akselerasjonen.

SLUTT

3 Rotasjon $\bar{w} = w k$

Radiell avstand $= r$

Vinkel i xy-planet $= \theta$

$x = r\cos(\theta)$, $y = r\sin\theta$

en person beveger seg mot sentrum (radielt) med en hastighet: v_0

a) Regn ut divergensen til $\bar{v}$

$$\boxed{\nabla \cdot \bar{v} = \frac{1}{r}\left(\frac{d}{dr}(rA_r) + \frac{dA_\theta}{d\theta}\right) + \cancel{\frac{dA_z}{dz}}}$$

$$\bar{v} = wr\,i_\theta - v_0\,i_r$$

$$\nabla \cdot \bar{v} = \frac{1}{r}\left[\frac{d}{dr}\,r(-v_0) + \frac{d}{d\theta}\,wr\right]$$

$$= \frac{1}{r}(-v_0 + 0) = \underline{\underline{\frac{-v_0}{r}}}$$

b) Regn ut virvlinga

$$\nabla \times \bar{v} = \left(\cancel{\frac{1}{r}\frac{dA_z}{d\theta}} - \cancel{\frac{dA_\theta}{dz}}\right)\bar{i}_r + \left(\cancel{\frac{dA_r}{dz}} - \cancel{\frac{dA_z}{dr}}\right)\bar{i}_\theta + \frac{1}{r}\left(\frac{d}{dr}(rA_\theta) - \cancel{\frac{dA_r}{d\theta}}\right)\bar{k}$$

$$\nabla \times \bar{v} = \frac{1}{r}\left(\frac{d}{dr}(rA_\theta)\right)\bar{k} = \frac{1}{r}\frac{d}{dr}r\cdot(wr)$$

$$= \frac{1}{\cancel{r}}2\cancel{r}\,w = 2w$$

3c

$(\bar{v}\cdot\nabla)\bar{v}$

$$(wr\bar{i}_\theta - V_0\bar{i}_r)\cdot\left(\bar{i}_r\frac{\partial}{\partial r} + \frac{\bar{i}_\theta}{r}\frac{\partial}{\partial\theta}\right)$$

$$\bar{i}_r\cdot\frac{\partial}{\partial r}wr\bar{i}_\theta = 0$$

$$wr\bar{i}_\theta\frac{\bar{i}_\theta}{r}\frac{\partial}{\partial\theta} = \frac{w\cancel{r}\cdot 1}{\cancel{r}}\cdot\frac{\partial}{\partial\theta} = w\frac{\partial}{\partial\theta}$$

$$-V_0\bar{i}_r\cdot\bar{i}_r\frac{\partial}{\partial r} = -V_0\frac{\partial}{\partial r}$$

$$-V_0\bar{i}_r\cdot\frac{\bar{i}_\theta}{r}\frac{\partial}{\partial\theta} = 0$$

$$= \left(w\frac{\partial}{\partial\theta} - V_0\frac{\partial}{\partial r}\right)(wr\bar{i}_\theta - V_0\bar{i}_r)$$

$$= w\frac{\partial}{\partial\theta}(wr\bar{i}_\theta) - w\frac{\partial}{\partial\theta}(V_0\bar{i}_r) - V_0\frac{\partial}{\partial r}(wr\bar{i}_\theta) - \cancel{V_0\frac{\partial}{\partial r}(-V_0\bar{i}_r)}$$

$$\frac{\partial}{\partial\theta}\bar{i}_\theta = -\bar{i}_r$$

$$-w^2 r\bar{i}_r - wV_0\bar{i}_\theta - V_0 w\bar{i}_\theta - V_0\cdot 0$$

$$-w^2 r\bar{i}_r - 2wV_0\bar{i}_\theta$$

$w: \frac{1}{t}$ $\quad \left(\frac{1}{t}\right)^2\cdot m \quad \frac{m}{t^2} \quad \frac{1}{t}\ m/t$

r : Length (m)

t : t

$$\bar{V} = \left(\frac{Vz}{h} + V \right) \bar{j}$$

$$\nabla \times \bar{V} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 0 & V_{\bar{j}} & 0 \end{vmatrix} = \bar{i} \cdot \left(0 - \frac{d}{dz} \frac{Vz}{h} \cancel{+ V} \right) = -\frac{V}{h} \bar{i}$$

$$x = 0, \bar{i} : \int_0^b \int_0^{-h} -\frac{V}{h} \cdot -1 = \frac{V}{h} \cdot b \cdot -h = -Vb$$