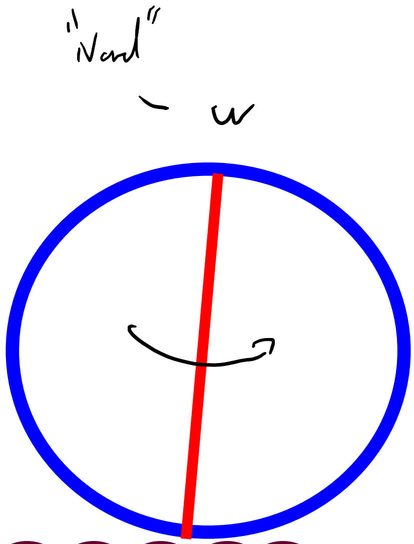
Kryssprodukt triks

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

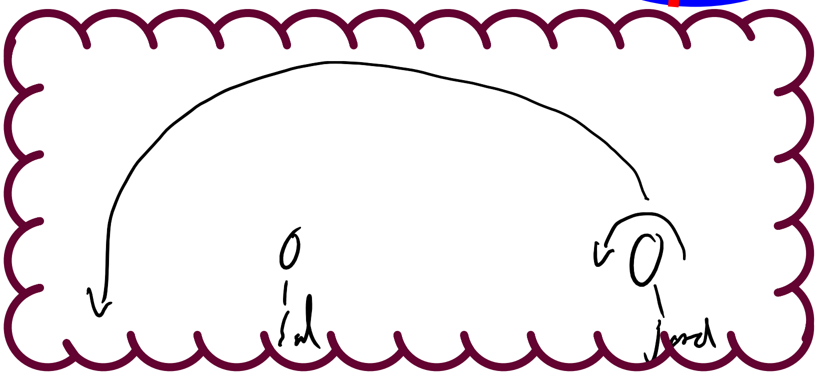


Nord

Legg 4 fingre på høyre hånd i rotasjonsretning slik at høyre tommel peker i $\underline{w}$ sin retning. Tuppen av tommelen er "Nord"

Prograd system

jorda roterer samme vei rundt seg selv som den gjør rundt sola



Eksempel: Vi står på jorda

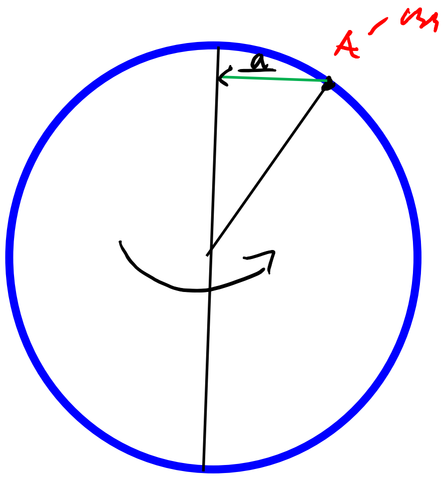
Hvilken vei beveger vi oss og hvilken vei er vi akselerert

La origo være i jordens sentrum

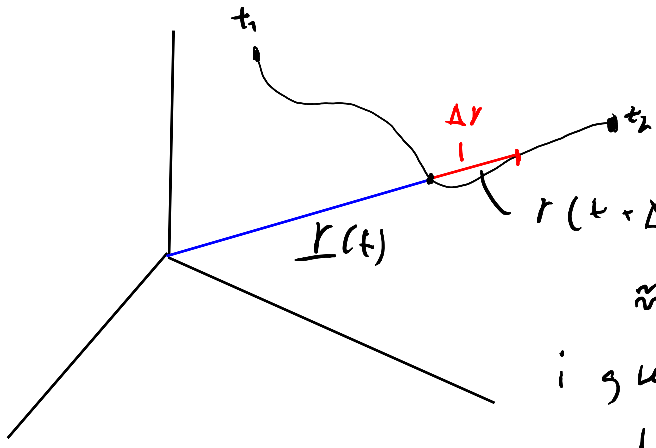
$\underline{w}$ = peker mot polarstjernen

$\underline{v} = \underline{w} \times \underline{r}$

$\underline{a} = \underline{w} \times \underline{v}$



Kurveintegral (Parametriserte kurver)



$$r(t+\Delta t) - r(t) = \Delta r$$

$$\approx \frac{dr}{dt} \cdot \Delta t$$

i grensen $\Delta t \to 0$

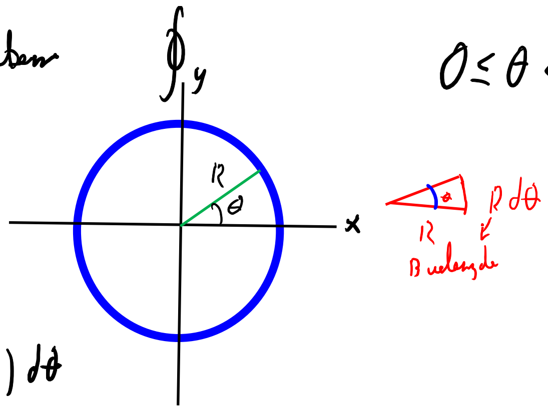
$$d\underline{r} = \frac{dr}{dt} dt$$

$$\int_\gamma f(\underline{r})\, d\underline{r} \quad , \quad \int_\gamma F \cdot dr \quad , \quad \int_\gamma F \times d\underline{r}$$

Noen ganger starter vi i samme punkt som vi sluttet i

da har vi et nytt integraltegn $\oint_\gamma$

$0 \leq \theta < 2\pi$



Eks. Omkretsen til en sirkel

$$r(\theta) = R\cos\theta\, i + R\sin\theta\, j$$

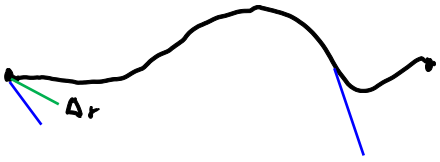
$$d\underline{r} = \frac{dr}{d\theta} d\theta = R(-\sin\theta\, i + \cos\theta\, j)\, d\theta$$

$$|d\underline{r}| = R\sqrt{\sin^2\theta + \cos^2\theta}\, d\theta = R\,d\theta$$

$$\oint_\gamma |d\underline{r}| = \int_0^{2\pi} R\,d\theta = \underline{\underline{2\pi R}}$$

Eksempel: Arbeid (kraft · vei)

J = N · m



$$\text{Arbeid} = \int_\gamma \underline{F} \cdot d\underline{r}$$

J N m

Kurveintegraler

Eksempel: $\underline{v} = x\underline{i} + y\underline{j}$

γ: rett linje fra origo til $(x=2, y=0)$

Regn ut $\int_\gamma \underline{v} \cdot d\underline{r}$

$\underline{r}(t) = t\underline{i}$ for $0 \le t \le 2$

$d\underline{r} = \frac{d\underline{r}}{dt} \dots = \underline{i}\,dt$

$\underline{v} = t\underline{i}$, $\underline{v} \cdot d\underline{r} = t\underline{i} \cdot \underline{i}\,dt$

$$\int \underline{v}\,d\underline{r} = \int_0^2 t\,dt = \frac{1}{2}t^2 \Big|_0^2 = 2$$

Alt 2.

$\underline{r}(t) = 2t^2\underline{i}$ for $0 \le t \le 1$

$= x\underline{i} + y\underline{j}$

$d\underline{r} = \frac{d\underline{r}}{dt}dt = 4t\underline{i}\,dt$

$\underline{v}\,d\underline{r} = 2t^2\underline{i} \cdot 4t\underline{i}\,dt = 8t^3\,dt$

$$\int \underline{v} \cdot d\underline{r} = \int_0^1 8t^2\,dt = 2t^4\Big|_0^1 = 2$$

Exempel: $\underline{V} = x\underline{i} + y\underline{j}$

γ : halvcirkel rundt $(x=1, y=0)$ med radius 1

vela $\underline{r}(\theta) = (1-\cos\theta)\,i + (\sin\theta)\,j$ för $0 \le \theta \le \pi$

$$d\underline{r} = \frac{d\underline{r}}{d\theta}\,d\theta = (\sin\theta\, i + \cos\theta\, j)\,d\theta$$

$$\underline{V} = (1-\cos\theta)\,\underline{i} + \sin\theta\,\underline{j}$$

$$\underline{V}\,d\underline{r} = ((1-\cos\theta)\sin\theta + \sin\theta\cos\theta)\,d\theta = \sin\theta\,d\theta$$

$$= \int \underline{V}\cdot d\underline{r} = \int_0^{\pi} \sin\theta\,d\theta = -\cos\theta\Big|_0^{\pi}$$

$$= -(-1-1) = \underline{\underline{2}}$$