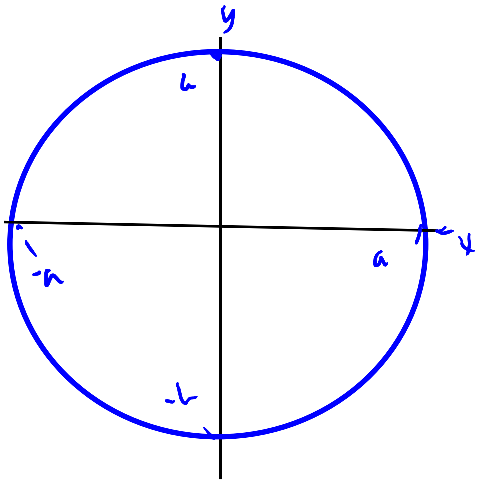
3.6 Kjeglesnitt

Likningen

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

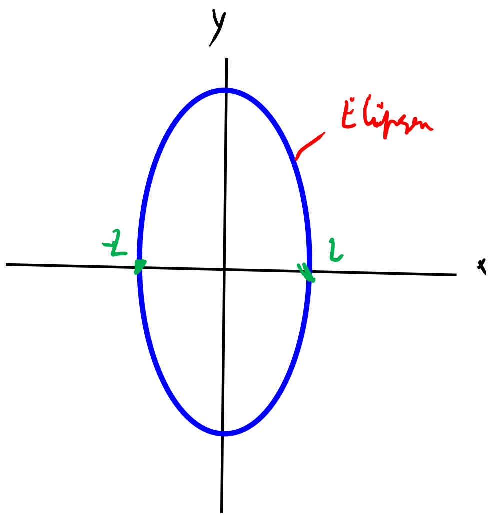
er standardlikningen for en ellipse med halvakser a og b



Tegn ellipsen $\frac{x^2}{4} + \frac{y^2}{9} = 1$ $(a = 2, b = 3)$

Skjæringspunkter med y-aksen $(x = 0)$: $\frac{y^2}{9} = 1$, $y = \pm 3$

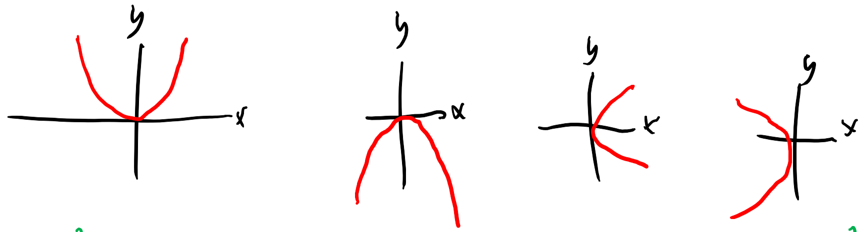
— " — x-aksen $(y = 0)$: $\frac{x^2}{4} = 1$, $x = \pm 2$



Parabler

$$y = ax^2 \quad \text{og} \quad x = ay^2 \quad (x \neq 0)$$

4 standard ligninger for parabler

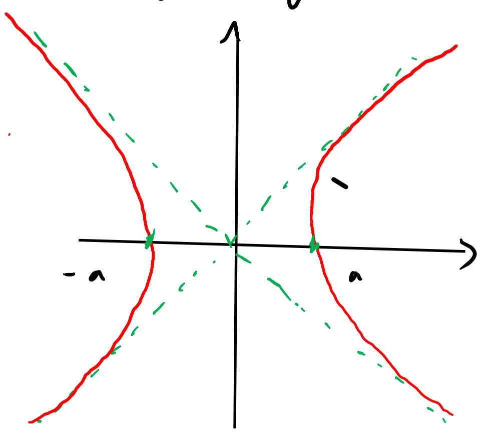


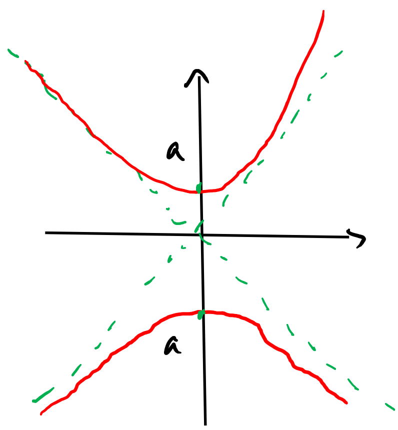
$y = ax^2$, $a > 0$ | $y = ax^2$, $a < 0$ | $x = ay^2$, $a > 0$ | $x = ay^2$, $a < 0$

Hyperbler

Ligninger

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{og} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$





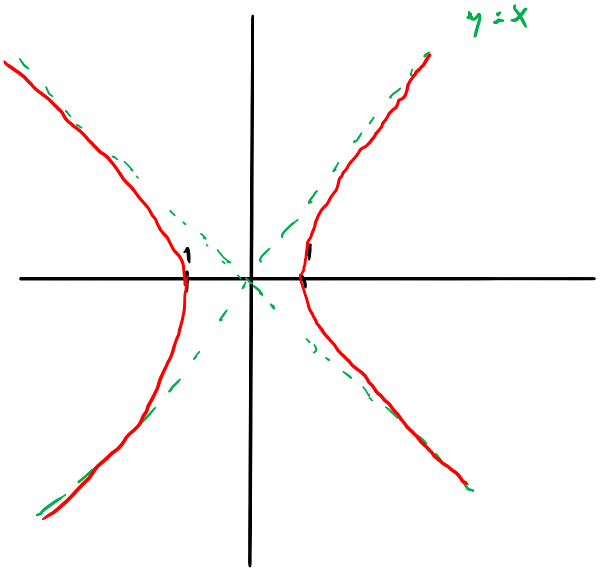
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

eks. Tegn hyperbelen $x^2 - y^2 = 1$

Løsning.

$$y = \pm\sqrt{x^2 - 1}$$

$y(x) = \sqrt{x^2-1}$ ikke def $x \in [-1,1]$ og $y(-1) = y(1) = 0$



Har også $y(-x) = y(x)$ så symmetrisk om y-aksen

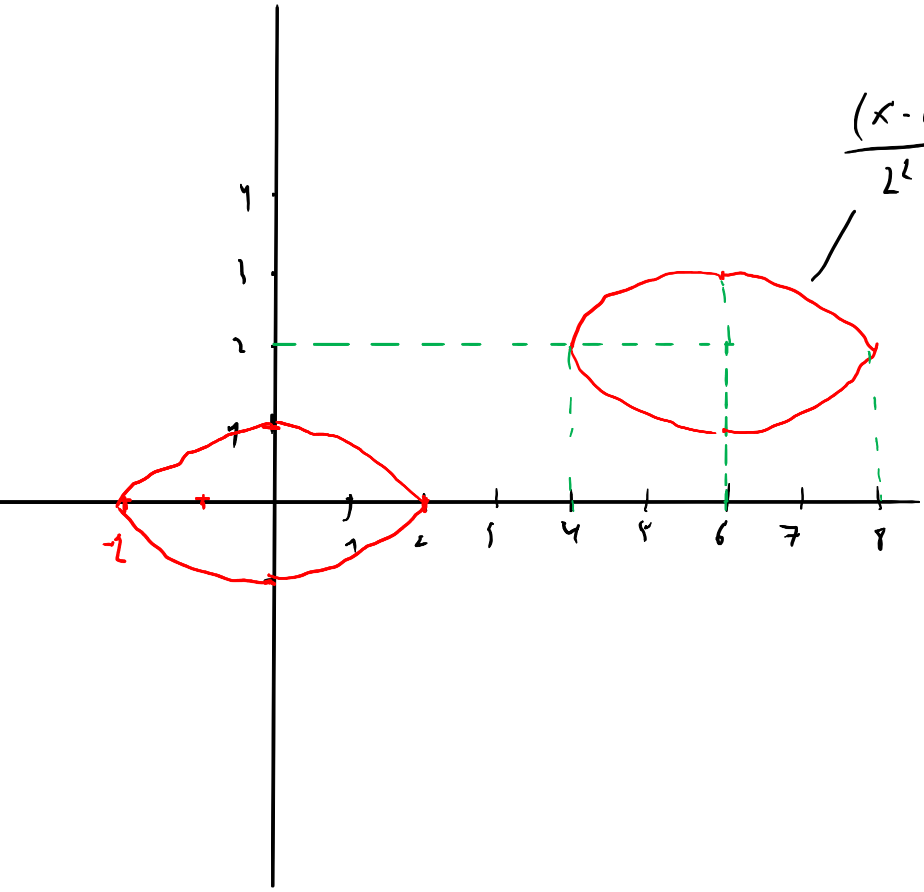
Skrå asymptote $y = x$ når $x \to \infty$

Translaterte kjeglesnitt

$$\left.\begin{array}{l} x \to x - h \\ y \to y - k \end{array}\right\} \text{ gir at } (h,k) \text{ spiller rollen til } (0,0)$$

$$\frac{(x-6)^2}{2^2} + \frac{(y-2)^2}{1^2} = 1$$

$(h,k) = 6,2$



Translaterte utgaver av standard likningene

Ellipse: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b} = 1 \qquad a > b$

Parabel $y - k = a(x-h)^2$ og $x - h = a(y-k)^2$

Hyperbel $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = \dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

Klassifikasjon kjegelsnitt

$$Ax^2 + By^2 + Cx + Dy + E = 0$$

utvid til fullstendig kvadrat i x og y

Eks: $x^2 + 9y^2 + 6x + 18y + 9 = 0$ Hvilket kjegelsnitt

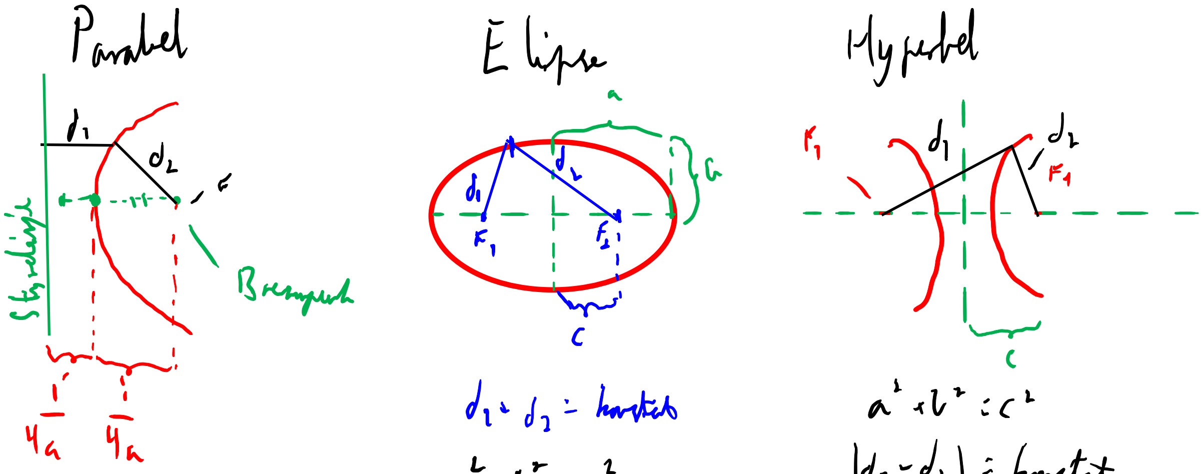
Løsning. $(x^2 - 6x + 9) + 9(y^2 + 2y + 1) + 9 = 2 \cdot 9$

$$(x-3)^2 + 9(x+1)^2 = 9 \quad | :9$$

$$\dfrac{(x-3)^2}{3^2} + \dfrac{(x+1)^2}{1^2} = 1$$

=> ellipse med sentrum $(h,k) = (3,-1)$ og halvakser 3 og 1

Geometriske egenskaper ved kjeglesnitt



$d_1 + d_2 = \text{konstant}$

$a^2 + b^2 = c^2$

$a^2 + b^2 = c^2$

$|d_1 - d_2| = \text{konstant}$

$d_1 = d_2$

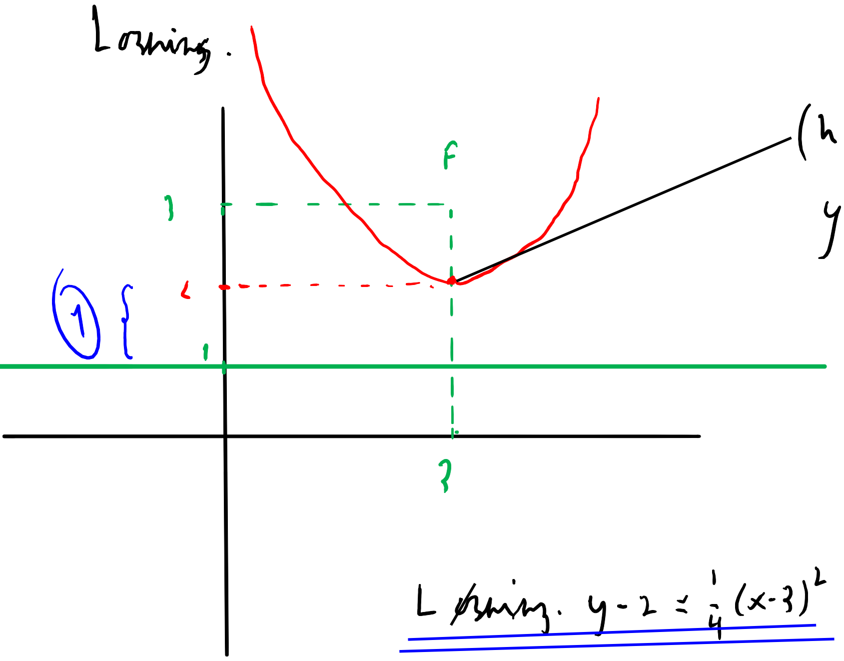
F: brennpunktet

Eks 1.

En parabel har brennpunkt $(3,3)$ og styrelinje $y=1$

Finn ligningen

Løsning.



$(h,k) = 3,2$

$y - 2 = a(x-3)^2$

Ser at $\frac{1}{4a} = 1$ så $a = \frac{1}{4}$

Løsning. $y - 2 = \frac{1}{4}(x-3)^2$

Finn brennpunktene til ellipsen $\frac{(x-3)^2}{1^2}+\frac{(y-1)^2}{\underset{a}{2^2}}=1$

Løsning.

$(h,k)=(3,1)$, $a=2$, $b=1$ $\leftarrow$ $\boxed{a>b}$