1.3-25 $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}$, $\bar{b} = \begin{pmatrix} 4 \\ 1 \\ -4 \end{pmatrix}$

a) Basic set Theory

b in span $(\bar{v})$

Determining whethe a vector $\bar{b}$ is in Span $(\bar{v}_1, \dots, \bar{v}_p)$ amounts to determining whether the vector equation

$$x\bar{v}_1 + x_2\bar{v}_2 + \dots + x_p\bar{v}_p = \bar{b}$$

has a solution, or, equivalently, asking whether the linear system with augmented matrix $\begin{bmatrix} \bar{v}_1, \dots, v_p, b \end{bmatrix}$ has a solution

a) i can not see any $\bar{b}$ in A

b) $\begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{bmatrix} \begin{matrix} III+2I \\ II:3 \\ \end{matrix} \sim \begin{pmatrix} 1 & 0 & -4 & 4 \\ 0 & 1 & -\frac{2}{3} & 1 \\ 0 & 6 & -5 & 4 \end{pmatrix} \begin{matrix} III-6II \\ \\ \end{matrix} \sim$

$\begin{pmatrix} 1 & 0 & -4 & 4 \\ 0 & 1 & -\frac{2}{3} & 1 \\ 0 & 0 & -1 & -2 \end{pmatrix} \begin{matrix} III\cdot(-1) \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 0 & -4 & 4 \\ 0 & 1 & -\frac{2}{3} & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

Det er en løsning. $\bar{b}$ er i W

c) setter $x_1 = 1$, $x_2 = 0$, $x_3 = 0$

$$1a_1 + 0a_2 + 0a_3 = a_1$$

$$a_1 = a_1$$

1.4.

La $A = \begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & 2 & -7 \end{bmatrix}$ og $\bar{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$$\text{ref}(A\,\bar{b}) = \begin{pmatrix} 1 & 3 & 4 & b_1 \\ 0 & 14 & 10 & b_2 + 4b_2 \\ 0 & 0 & 0 & b_3 + 3b_1 - \frac{1}{2}(b_2 + 4b_1) \end{pmatrix}$$

Da er ikke $A\bar{x} = \bar{b}$ konsistent for alle $\bar{b}$ siden $b_3 + 3b_1 - \frac{1}{2}(b_2 + 4b_1)$ kan være ikke null

2.2.3 1 Finn A^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} II + 3I \\ III - 2 \cdot I \\ \sim \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} III + 3II \\ \\ \sim \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \begin{array}{l} \checkmark \\ I + III \\ II + III \\ \sim \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \begin{array}{l} III : 2 \\ \\ \sim \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 3{,}5 & 1{,}5 & 0{,}5 \end{array}\right]$$

2.2.3 5

$$A = \begin{bmatrix} -1 & -7 & -3 \\ 2 & 15 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

Finn 3. kolonne sin A^{-1}

$$\begin{bmatrix} -3 & 0 \\ 6 & 0 \\ 2 & 1 \end{bmatrix} \begin{array}{l} I \cdot -\frac{1}{3} \\ II - 6I \\ III - 2I \\ \sim \end{array} \left| \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \right.$$

2.3.33 $\quad T(\bar{x}) = (-5x_1 + 9x_2,\ 4x_1 - 7x_2)$

$$= \begin{bmatrix} -5 & 9 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A^{-1} : \left[\begin{array}{cc|cc} -5 & 9 & 1 & 0 \\ 4 & -7 & 0 & 1 \end{array}\right] \begin{matrix} \mathrm{I} : -5 \\ \sim \end{matrix} \left[\begin{array}{cc|cc} 1 & 9/5 & 1/5 & 0 \\ 4 & -7 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 9 & 1/5 & 0 \\ 0 & -7.8 & -4/5 & 1 \end{array}\right] \begin{matrix} \mathrm{II} : -7.8 \\ \sim \end{matrix} \left[\begin{array}{cc|c} 1 & 9 & 1/5 \\ 0 & 1 & 4/39 \end{array}\right.$$

2.3.35

Teorem 8 och 12

2.4, Ex. 5

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$

Övre triangulär, antag att A_{11} är $p \times p$, A_{22} $q \times q$

och A är inverterbar, Finn en formel för A^{-1}

$$\begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} I_p & 0 \\ 0 & I_q \end{bmatrix}$$

A_{22} : $q \times q$, A_{11} $p \times p$

Matrisen gir oss fire ligninger

$$A_{11} B_{11} + A_{12} B_{21} = I_p \qquad \mathrm{I}$$

$$A_{11} B_{12} + A_{12} B_{22} = 0 \qquad \mathrm{II}$$

$$0 B_{11} + A_{22} B_{21} = 0 \qquad \mathrm{III}$$

$$0 B_{12} + A_{22} B_{22} = 0 \qquad \mathrm{IV}$$

Siden A_{22} er kvadratisk, sier invertible matrise teoremet og IV at A_{22} er invertibel, og $B_{22} = A_{22}^{-1}$

$$\mathrm{IV} \quad A_{22} B_{22} = I_q \quad / \; A_{22}^{-1}$$

$$\Rightarrow A_{22}^{-1} (A_{22} B_{22}) = A_{22}^{-1} (I_q)$$

$$\Rightarrow \underbrace{A_{22}^{-1} A_{22}}_{I} (B_{22}) = A_{22}^{-1}$$

$$\Rightarrow \underline{B_{22} = A_{22}^{-1}}$$

For III multipliserer vi begge sider med A_{22}^{-1}, da får vi

$$\underline{B_{21} = A_{22}^{-1} \cdot 0 = 0}$$

Det forenkler I til

$$A_{11} B_{11} + 0 = I_p$$

siden A_{11} er kvadratisk så er

$$B_{11} = A_{11}^{-1}$$

Bruker resultatet på II

$$A_{11} B_{12} + A_{12} B_{22} = 0$$

$$A_{11} B_{12} = - A_{12} B_{22} = - A_{12} A_{22}^{-1} \quad / \cdot A_{11}^{-1}$$

$$\underline{B_{12} = - A_{11}^{-1} A_{12} A_{22}^{-1}}$$

Dermed har vi

$$A^{-1} = \begin{bmatrix} A_{11}^{-1} & -A_{11}^{-1} A_{12} A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{bmatrix}$$

2, 4, 3

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 \cdot A + 1 \cdot C & 0 \cdot B + 1 \cdot D \\ 1 \cdot A + 0 \cdot C & B \cdot 1 + 0 \cdot D \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} C & D \\ A & B \end{bmatrix}}}$$

R

2.4.5

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ x & y \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix}$$

$$\begin{bmatrix} A \cdot I + Bx & A \cdot 0 + By \\ C \cdot I + 0x & C \cdot 0 + 0 \cdot y \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix}$$

$$\begin{bmatrix} AI + Bx & By \\ CI & 0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ Z & 0 \end{bmatrix}$$

$AI + Bx = 0$ I

$By = I$ II

$CI = Z$ III

$0 = 0$ ~~IV~~

Finn formeler for X, Y, Z avhengig av A, B, C

II $By = I \Rightarrow \underbrace{B^{-1}B}_{I} y = B^{-1}$, III $CI = C = Z$

I $A + Bx = 0$ $(X = -BA^{-1})$

$B^{-1}A + x = 0$

$x = A^{-1}B$, $x = -BA^{-1}$ (Vet ikke om det er lov)

2.4.21 $A^2 = I$

$$A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\boxtimes$