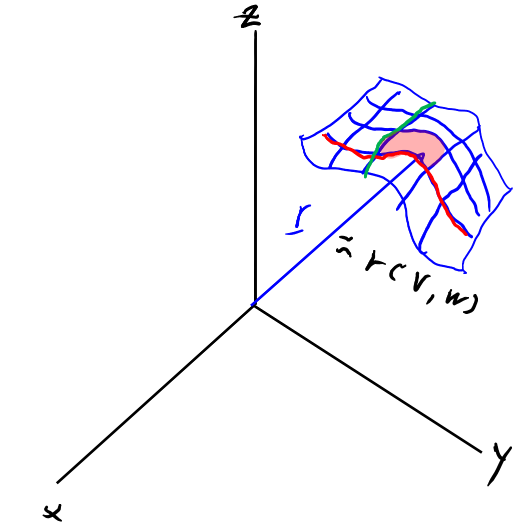
Trykkraft

$$\int_S -p\,\underline{n}\,dt \quad , \quad N = Pa \cdot m^2$$

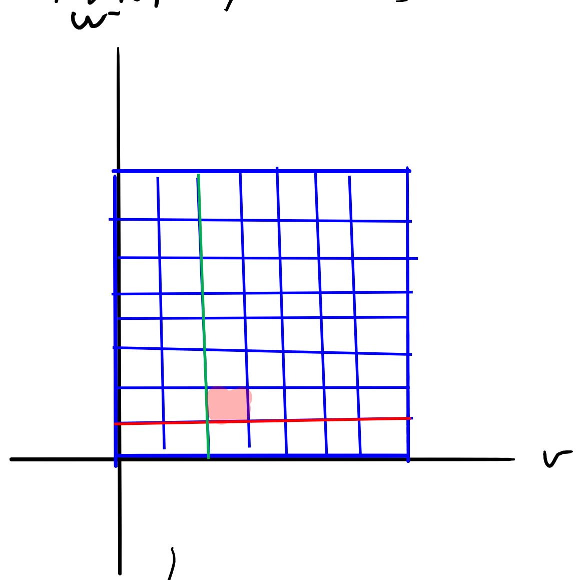
$$Pa = N/m^2$$

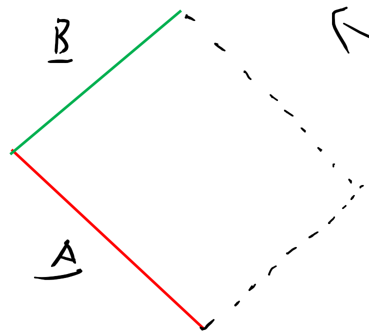
IM s. 35
LH kap. 7.8

Generell flate



Parameterform 2D





areal $|\underline{A} \times B|$

normalvektor $\pm \dfrac{A \times B}{|A \times B|}$

$$\Delta \underline{r}_v = \underline{r}(v+\Delta v, w) - \underline{r}(v,w)$$

$$\approx \frac{d\underline{r}}{dv}\Delta v$$

$$\Delta r_w = \underline{r}(v, w+\Delta w) - r(v,w)$$

$$\approx \frac{d\underline{r}}{dw}$$

La $\Delta v, \Delta w \to 0$
skriv dv og dw

$$d\underline{r}_v = \frac{d\underline{r}}{dv}dv$$

$$d\underline{r}_w = \frac{d\underline{r}}{dw}dw$$

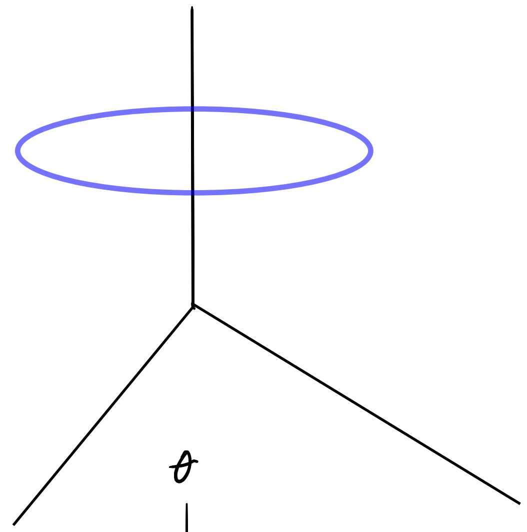
$$\underline{n}\, d\sigma = \pm\, d\underline{r}_v \times d\underline{r}_w = \pm \frac{d\underline{r}}{dv} \times \frac{d\underline{r}}{dw}\, dv\, dw$$

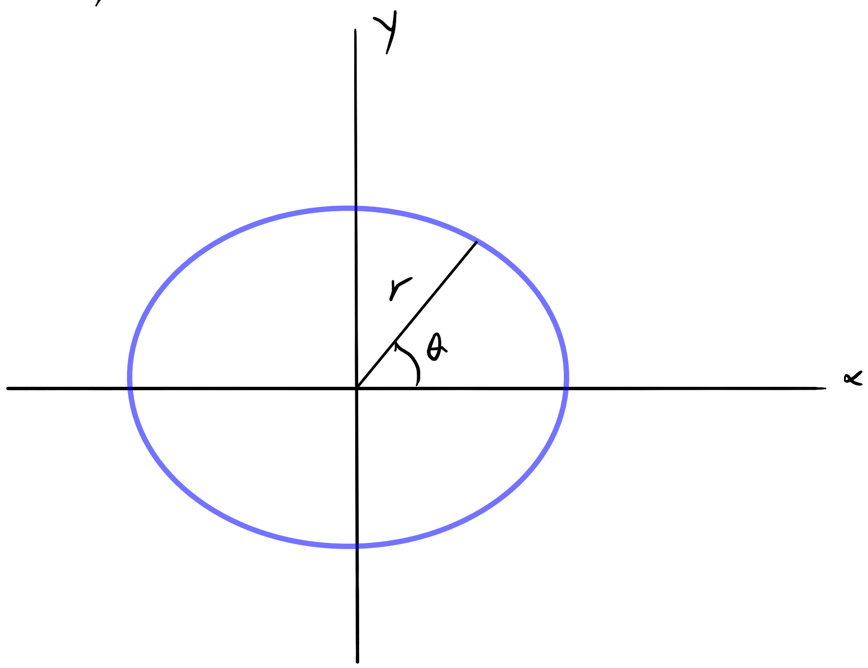
$$d\sigma = \left| \frac{d\underline{r}}{dv} \times \frac{d\underline{r}}{dw} \right| dv\, dw$$

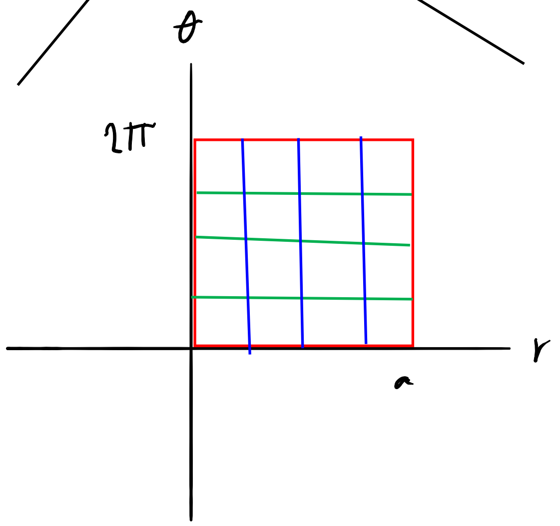
$$\underline{n} = \pm \frac{\frac{d\underline{r}}{dv} \times \frac{d\underline{r}}{dw}}{\left| \frac{d\underline{r}}{dv} \times \frac{d\underline{r}}{dw} \right|}$$

<u>Eks</u> Sirkelskive parallell med x,y-planet og med høyde h

$z = h$, radius a , sentrum i z-aksen







$$\underline{r}(r,\theta) = r\cos\theta\, \underline{i} + r\sin\theta\, \underline{j} + h\underline{k}$$

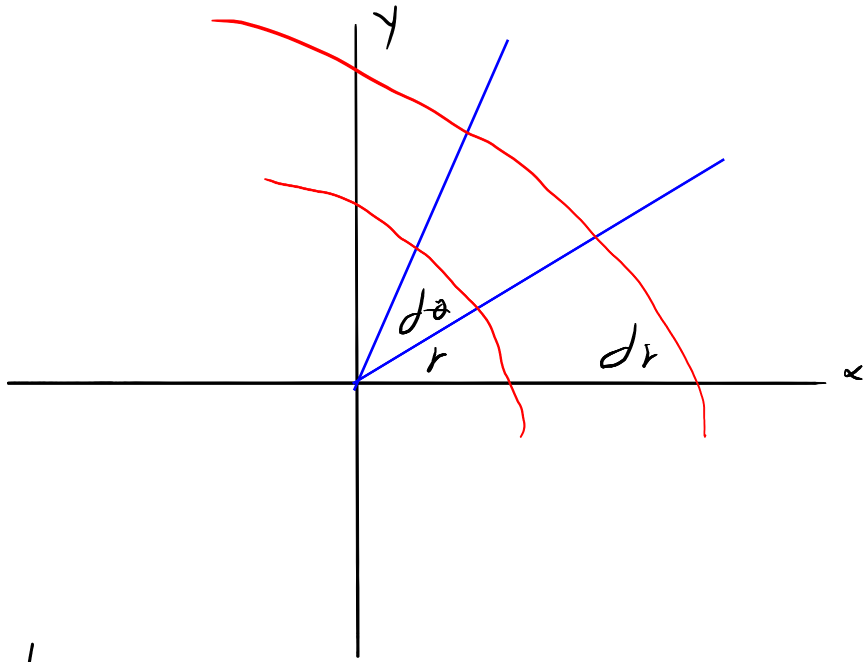
$$\frac{d\underline{r}}{dr} = \cos\theta\,\underline{i} + \sin\theta\,\underline{j}$$

$$\frac{d\underline{r}}{d\theta} = -r\sin\theta\,\underline{i} + r\cos\theta\,\underline{j}$$

$$\underline{n}\,dr = \underline{r}\,\frac{d\underline{r}}{dr} \times \frac{d\underline{r}}{d\theta}\,dr\,d\theta$$

$$\pm \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \cos\theta & \sin\theta & 0 \\ r\sin\theta & r\cos\theta & 0 \end{vmatrix}$$

$$= \pm \underline{k}(r\cos^2\theta + \sin^2\theta) = \pm \underline{k}\,r\,dr\,d\theta$$



buelengde $r\, d\theta$

arealet $(dr)\, r\, d\theta$

$= r\, dr\, d\theta$

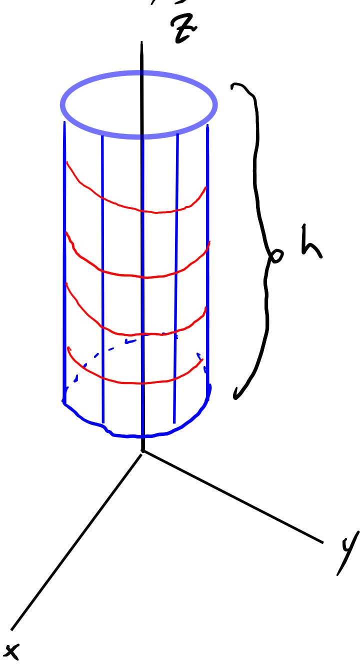
$\underline{n} = \pm k$

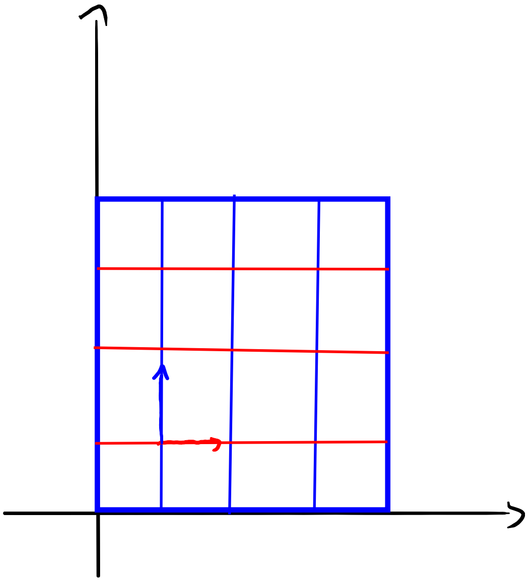
Areal av sirkelskive

$$\int_S d\sigma = \int_0^{2\pi}\int_0^a r\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} r^2 \Big|_0^a d\theta = \int_0^{\pi} \frac{1}{2} a\, d\theta = 2\pi \frac{1}{2} a^2 = \pi a^2$$

E.g. cylinder





$$r(\theta, z) = a\cos\theta\,\underline{i} + a\sin\theta\,\underline{j} + z\,\underline{k}$$

$$\frac{d\underline{r}}{d\theta} = -a\sin\theta\,\underline{i} + a\cos\theta\,\underline{j}$$

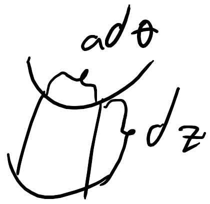
$$\frac{d\underline{r}}{dz} = \underline{k}$$

$$\underline{n}\,d\sigma = \pm\frac{d\underline{r}}{d\theta}\times\frac{d\underline{r}}{dz}\,d\theta\,dz =$$

$$= \begin{vmatrix} \underline{i} & \underline{j} & k \\ -a\sin\theta & a\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} d\theta\,dz = \pm(\underline{i}\,a\cos\theta + a\sin\theta)\,d\theta\,dz$$

$$= \pm a(\underline{i}\cos\theta + \underline{j}\sin\theta)\,d\theta\,dz$$

På en intuitiv måte



$$\Rightarrow d\sigma = a\,d\theta\,dz$$

$$= a\sqrt{\cos^2\theta + \sin^2\theta}\;d\theta\,dz$$

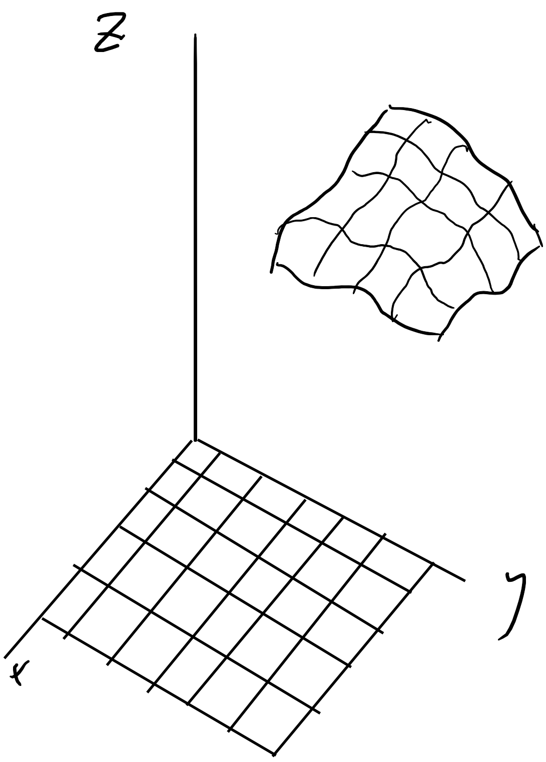
$$= a\,d\theta\,dz$$

$$\underline{n} = \underline{i}\cos\theta + \underline{j}\sin\theta$$

Arealet av en sylinder flate

$$\int_S d\sigma = \int_0^h \int_0^{2\pi} a\,d\theta\,dz = (2\pi a)h$$ - omkretsen · høyden

Eksempel Et terreng $z = h(x,y)$



$$\underline{r}(x,y) = x\underline{i} + y\underline{j} + h(x,y)\underline{k}$$

gender jevne små forandringer

$$\frac{d\underline{r}}{dx} = \underline{i} + \frac{dh}{dx}\underline{k}$$

$$\frac{d\underline{r}}{dy} \qquad \underline{j} + \frac{dh}{dy}\underline{k}$$

$$\underline{n}\,d\sigma = \pm \frac{d\underline{r}}{dx} \times \frac{d\underline{r}}{dy}\,dx\,dy$$

$$= \pm \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 0 & \frac{dh}{dx}\underline{k} \\ 0 & 1 & \frac{dh}{dy}\underline{k} \end{vmatrix} dx\,dy$$

$$= \pm \left(-\underline{i}\frac{dh}{dx} - \underline{j}\frac{dh}{dy} + \underline{k} \right) dx\,dy$$

$$d\sigma = \sqrt{\left(\frac{dh}{dx}\right)^2 + \left(\frac{dh}{dy}\right)^2 + 1^2}\,dx\,dy$$

$$\underline{n} = \pm \frac{-\underline{i}\frac{dh}{dx} - \underline{j}\frac{dh}{dy} + \underline{k}}{\sqrt{\left(\frac{dh}{dx}\right)^2 + \left(\frac{dh}{dy}\right)^2 + 1^2}\,dx\,dy}$$

Beskriv terrenget som en ekviskalarflate

$$\beta(x,y,z) = z - h(x,y) = 0$$

$$\underline{n} = \pm \frac{\nabla \beta}{|\nabla \beta|} = \pm \frac{-\frac{dh}{dx}\underline{i} - \frac{dh}{dy}\underline{j} + \underline{k}}{\sqrt{\frac{dh}{dx}^2 + \frac{dh}{dy}^2 + 1^2}}$$

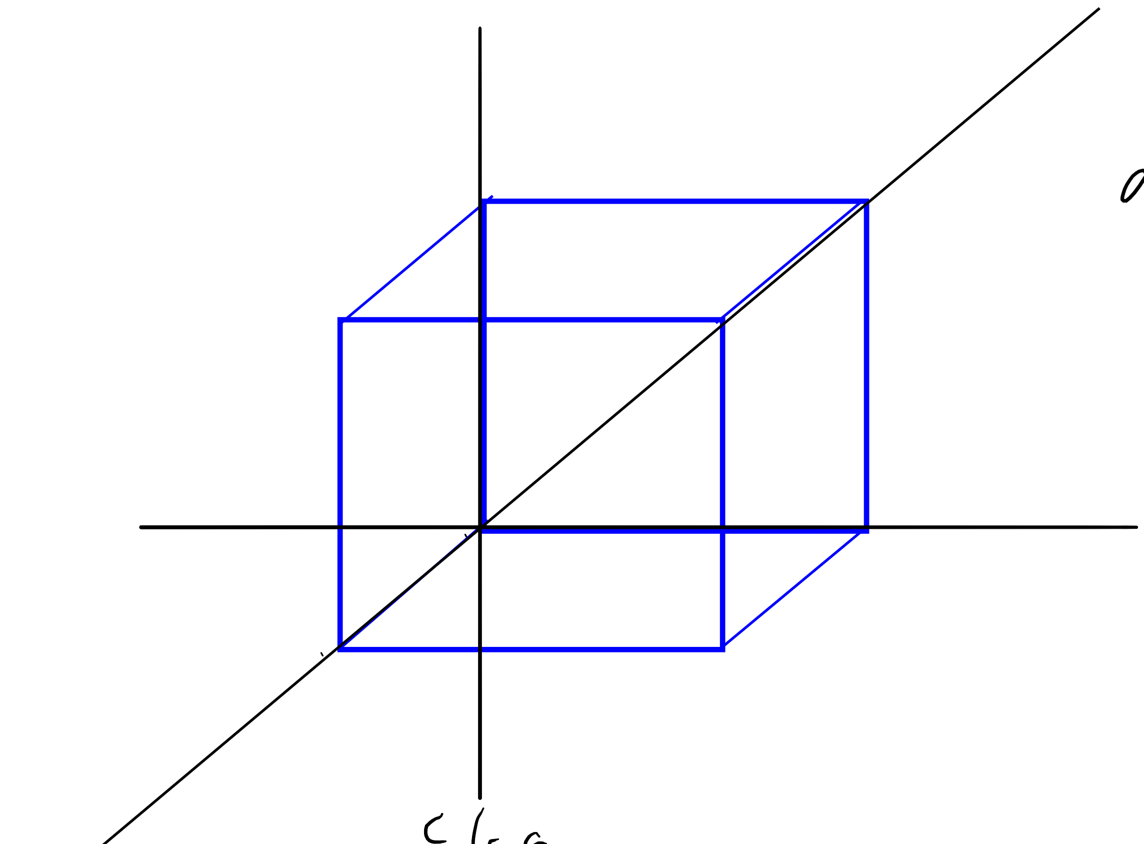
ekviskalar-metoden gir ikke $d\sigma$

Volumintegral

$\int \vec{F}\, d\tau$, $\int f\, d\tau$ $\quad d\tau =$

eks

Eksempel Volumet av en boks med
sidekanter a, b, c



$d\tau = dx\,dy\,dz$

$$\int d\tau = \int_0^c \int_0^b \int_0^a dx\,dy\,dz = cba$$

Eksempel.

Masse til et legeme.

$$M = \int_V \rho \, d\tau \, d\tau \quad \text{hvor } \rho \text{ e massetetthet}$$

$$kg = \frac{kg}{m^3} \cdot m^3$$

Eksempel. Tyngde til et legeme

$$\underline{T} = \int_V \rho \, \underline{g} \, d\tau$$

$$\frac{kg\,m}{s^2} \qquad \frac{kg}{m^3} \cdot \frac{m}{s^2} \cdot m^3$$

$$\frac{kg\,m}{s^2} = N$$