4.6

b) $f(x) = \frac{1}{2^{2/2}\Gamma(1)} x^{1-1} e^{-x/2} = \frac{1}{2} e^{-x/2}$

$$0{,}05 = \int_c^\infty \frac{1}{2} e^{-x/2}\,dx = \left[-e^{-x/2}\right]_c^\infty = e^{-c/2}$$

$$c = -2\ln(0{,}05) = \underline{\underline{5{,}9914}}\ldots$$

c) qchisq $(\underline{1-0{,}05}, df = 2)$

4.7 Momenter av χ^2_v, for en stor v, hvilket teorem

$Y \sim \chi^2_v$, $A \sim \chi^2_a$, $B \sim \chi^2_b$,

$A + B \sim \chi^2_{a+b}$

$$Y = \sum_{i=1}^{v} X_i \qquad X_i \sim \chi^2_1$$

CTL gir oss at: Sum av iid variable går $\to N$

4.8 Law of large Numbers

Vis: $Y \sim \chi^2_V$, $\frac{Y}{V} \to 1$

giennomsnitt

$X_i \to \chi^2_1$ $Y = \sum_{i=1}^{V} X_i$, $\frac{Y}{V} = \frac{\sum_{i=1}^{V} X_i}{V}$

$LLN: \frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow[n\to\infty]{} E[X_i]$, X_i iid

$$\frac{\sum_{i=1}^{n} X_i}{V} \xrightarrow[V\to\infty]{} E[X_i] = 1$$

4.9 $x^* = V - 2$, $f(x^*) \geq f(x)$

$f(x) = \frac{1}{2^{V/2}\,\Gamma(\frac{V}{2})} x^{V/2 - 1} e^{-x/2}$, $x^* = \arg\max_x f(x)$

$\arg\max f(x) = \arg\max_x \ln f(x)$

$$\ln f(x) = -\frac{V}{2}\ln 2 - \ln(\Gamma(\tfrac{V}{2})) + (\tfrac{V}{2} - 1)\ln x - \frac{x}{2}$$

$$0 = \frac{d \ln f(x)}{dx} = (\frac{V}{2} - 1)\frac{1}{x} - \frac{1}{2} \Rightarrow \underline{V - 2}$$

$x \geq 2 \Rightarrow V - 2 \geq 0 \Rightarrow V \geq 2$

50 $(n-1)S^2/\sigma^2 \sim \chi^2_{n-1}$

a) $E(S^2) = \sigma^2$

$E\left[\frac{n-1}{\sigma^2} S^2\right] = (n-1)$

$\frac{n-1}{\sigma^2} E[S^2] = (n-1) \Rightarrow E[S^2] = \sigma^2$

b) $V\left[\frac{n-1}{\sigma^2} S^2\right] = 2(n-1)$

$\left(\frac{n-1}{\sigma^2}\right)^2 V[S^2] = 2(n-1)$

$V(S^2) = \frac{2\sigma^4}{(n-1)} \xrightarrow[n\to\infty]{} 0$

c) (6.12) $E[X^k] = \frac{2^k \Gamma(k+v/2)}{\Gamma(v/2)}$ $X \sim \chi^2_v$

$E\left[\left(\frac{n-1}{\sigma^2} S^2\right)^{1/2}\right] = \frac{\sqrt{2}\,\Gamma(n/2)}{\Gamma\left(\frac{n-1}{2}\right)} = \frac{\sqrt{n-1}}{\sigma} E[S]$

$\Rightarrow E(S) = \sigma \frac{\sqrt{2}\,\Gamma(n/2)}{\sqrt{n-1}\,\Gamma\left(\frac{n-1}{2}\right)}$

$n=2$: $E(S)\, \sigma \frac{\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)} = \frac{\sqrt{2}}{\pi}$ $\Gamma(k) = (k-1)!$, int

6 5) approx $P(\chi^2_{50} > 70)$

$$\chi^2_{20} \overset{\sim}{\sim} N(50, 2\cdot 50) = N(50, 10^2)$$

$$P(\chi^2_{50} > 70) = P\left(\frac{\chi^2_{50} - 50}{10} > \frac{70-50}{10}\right) =$$

$$P\left(Z > \frac{20}{10}\right) = P(Z > 2) = 1 - 0{,}9772 = \underline{0{,}0228}$$

b) $\chi^2_v \approx v\left(1 - \frac{2}{9v} + Z\frac{\sqrt{2}}{9v}\right)^3$ (, for store verdier av v)

$$P(\chi^2_{50} > 70) = P\left(v\left(1 - \frac{2}{9v} + Z\sqrt{\frac{2}{9v}}\right)^3 > 70\right) \quad v = 50$$

$$= P(Z > 1.847) = P(\ldots) = 0{,}03237$$

6 6) $X \sim \chi^2_a$, $Y \sim \chi^2_v$

$X - Y \overset{?}{\sim} \chi^2_?$, $f(x) \geq 0$

$X - Y \not> 0$, kan være mindre enn 0 , $(X - Y) \in (-\infty, \infty)$

Nei!