Eks 4.9.10

Vi skal regne ut det(A)

$$A = \begin{pmatrix} 2 & 1 & 3 & -1 \\ 0 & -2 & 2 & 3 \\ 3 & -1 & 2 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix} \sim$$

Vi velger å utvikle langs $\underline{\text{VI}}$ siden den inneholder 2 nuller

$$\det(A) = \cancel{-0 \begin{vmatrix} 1 & 3 & -1 \\ -2 & 2 & 3 \\ -1 & 2 & 1 \end{vmatrix}} + 1 \cdot \begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 & -1 \\ 0 & -2 & 3 \\ 3 & -1 & 1 \end{vmatrix} \cancel{+ 0 \cdot \begin{vmatrix} 2 & 1 & 3 \\ 0 & -2 & 2 \\ 3 & -1 & 2 \end{vmatrix}}$$

$$= \underbrace{\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix}}_{\alpha} - 2 \cdot \underbrace{\begin{vmatrix} 2 & 1 & -1 \\ 0 & -2 & 3 \\ 3 & -1 & 1 \end{vmatrix}}_{\gamma}$$

Vi utvikler langs $\underline{\text{I}}$

$$\alpha = \cancel{-0 \cdot \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix}} + 2 \cdot \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} - 3 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = 2 \cdot (2 \cdot 1 - 3 \cdot (-1)) - 3 \cdot (2 \cdot (-1) - 3 \cdot 1)$$

$$= 25$$

$$\gamma = -0 \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} + (-2) \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} - 3 \cdot \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -2 \cdot (2 \cdot 1 - 3 \cdot (-1)) - 3 \cdot (2 \cdot (-1) - 2 \cdot 1$$

$$= 5$$

$$\det(A) = 25 - 2 \cdot 5 = \underline{\underline{25}}$$

Eks 4.9,21

Vi skal regne ut det A

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 4 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix}$$

vi utvikler langs siste søyle

$$= \cancel{0 \cdot \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}} - 3 \cdot \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} + \cancel{0 \cdot \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}}$$

$$= -3 \cdot (2 \cdot 1 - 3 \cdot 1) = \underline{\underline{3}}$$

4.9.

1 a) $\begin{pmatrix} 1 & -2 & -1 \\ 3 & 1 & -2 \\ 1 & 0 & 1 \end{pmatrix}$ Vi regner med $\boxed{III}$

$$\begin{matrix} + & - & + & - & + & - \\ - & + & - & + & - & + \\ + & - & + & - & + & - \\ - & + & - & + & - & + \end{matrix}$$

$$+1 \cdot \underbrace{\begin{vmatrix} -2 & -1 \\ 1 & -2 \end{vmatrix}}_{x} - \cancel{0 \cdot \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix}} + 1 \underbrace{\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix}}_{y}$$

$x \Rightarrow -2 \cdot -2 - 1 \cdot (-1) = \underline{5}$

$y \Rightarrow 1 \cdot 1 - 3 \cdot -2 = 7$

$$5 + 7 = \underline{\underline{12}}$$

2.

a) $\begin{pmatrix} 1 & -3 & 0 \\ 2 & -1 & -2 \\ 1 & -1 & 1 \end{pmatrix}$ Bruk radoperasjoner

$II - 2 \cdot I$

$\sim$

$-1 - 2\cdot(-3) = 1 - (-6) = 7$

$-1 - (-3) = 2$

$-2 - 3 \cdot 1 = -5$

$1 - 2\cdot(-5) = 9$

$$\begin{pmatrix} 1 & -3 & 0 \\ 0 & 7 & -2 \\ 1 & -1 & 1 \end{pmatrix} \overset{III - I}{\sim} \begin{pmatrix} 1 & -3 & 0 \\ 0 & 7 & -2 \\ 0 & 2 & 1 \end{pmatrix} \overset{II - 3 \cdot III}{\sim}$$

$$\begin{pmatrix} 1 & -3 & 0 \\ 0 & 1 & -5 \\ 0 & 2 & 1 \end{pmatrix} \overset{III - 2II}{\sim} \begin{pmatrix} 1 & -3 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 9 \end{pmatrix} \overset{I + 3II}{\sim}$$

$$\begin{pmatrix} 1 & 0 & -15 \\ 0 & 1 & -5 \\ 0 & 0 & 9 \end{pmatrix} =: \alpha \qquad \det(\alpha) = 1 \cdot \begin{vmatrix} 1 & -5 \\ 0 & 9 \end{vmatrix}$$

$$= 1 \cdot (1 \cdot 9 - 0 \cdot (-5))$$

$$= \underline{\underline{9}}$$

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