2 $$\frac{d^2\theta}{dt^2}(t) = -\frac{g}{L}\sin(\theta(t))$$

Vi har at $v(t) = L\frac{d\theta}{dt}(t)$

Vil finne et system av ligningen

$$\frac{d^2\theta}{dt}t = -\frac{g}{L}\sin(\theta(t)) \quad / \cdot L$$

$$L\frac{d^2\theta}{dt^2}(t) = -g\sin(\theta(t))$$

$L\frac{d^2\theta}{dt^2}(t) = v'(t)$, dermed kan vi skrive

$$\underline{v'(t) = -g\sin(\theta(t))}$$

For å finne $\frac{d\theta}{dt}$, bruker jeg utrykket for $v(t)$

$$V(t) = L\frac{d\theta}{dt}(t) \quad /:L$$

$$\underline{\frac{V(t)}{L} = \frac{d\theta}{dt}}$$

#

For å finne løsningen av (1) når vi vet (2) finer jeg sådan:

Vi har

$$\frac{d\theta}{dt} = \frac{v}{L}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = \frac{1}{L}\frac{dv}{dt} \quad | \cdot L$$

$$L\frac{d^2\theta}{dt^2} = \frac{dv}{dt} \quad (*)$$

Setter (*) inn i ligningen $\frac{dv}{dt} = -g\sin(\theta(t))$. dvs.

$$L\frac{d^2\theta}{dt^2} = -g\sin(\theta(t)), \text{ som er ligning (1)}$$

b) Ligningssystemet

$$(3) \quad \frac{dv}{dt} = -g\theta \quad , \quad \frac{d\theta}{dt} = \frac{v}{L} \quad (4)$$

Startverdier

$$\theta(0) = \theta_0 \quad , \quad v(0) = v_0$$

Deriverer (3)

$$\frac{d^2v}{dt^2} = -g\frac{d\theta}{dt}$$, ser at da har vi mulighet til

å utrykke (4) med (3)

$$\frac{d^2 v}{dt^2}\frac{1}{-g} = \frac{d\theta}{dt}$$, Setter inn i (4)

$$\frac{d^2 v}{dt^2}\frac{1}{-g} = \frac{v}{L} \quad / \cdot g$$

$$\frac{d^2 v}{dt^2} = -\frac{gv}{L} \quad / + \frac{gv}{L}$$

$$0 = \frac{d^2 v}{dt^2} + \frac{gv}{L}$$

Setter opp det karakteristiske polynomet.

$$r^2 + 0r + \frac{g}{L} = 0$$

Løser for r

$$\frac{0 \pm \sqrt{0^2 - 4 \cdot g/L}}{2} = r$$

$$\frac{\pm 2\sqrt{g/L}\, i}{2} = r$$

$$r_1 = i\sqrt{g/L} \quad , \quad r_2 = -i\sqrt{g/L}$$

Generelt for 2 komplekse røtter
$r_1 = a + ib$, $r_2 = a - ib$, gir
$$y = e^{ax}(C\cos(bx) + D\sin(bx))$$

Dermed har vi $a = 0$, $b = i\sqrt{g/L}$
dvs.

$$\theta(t) = e^{0}\left(C\cos\left(i\sqrt{g/L}\,x\right) + D\sin\left(i\sqrt{g/L}\,x\right)\right)$$
$$= \left(C\cos\left(i\sqrt{g/L}\,x\right) + D\sin\left(i\sqrt{g/L}\,x\right)\right)$$

Setter da inn startverdien. $\theta(0) = \theta_0$

$$\theta(0) = \theta_0 = C\underbrace{\cos(0)}_{1} + D\underbrace{\sin(0)}_{0}$$

$$\theta_0 = C$$

Vi har at $v'(t) = L \cdot \theta'(t)$

$$L \cdot \theta(0) = v'(0) = L\sqrt{\frac{g}{L}}\left(-\theta_0 \cdot \overbrace{\sin(0)}^{0} + D\overbrace{\cos(0)}^{1}\right)$$

$$v_0 = \sqrt{gL}\,D \quad \Big/ \frac{1}{\sqrt{gL}}$$

$$\underline{D = \frac{v_0}{\sqrt{gL}}}$$

Dermed har vi løsningen

$$\underline{\underline{\theta(t) = \theta_0\cos\left(\sqrt{\frac{g}{L}}\,t\right) + \frac{v_0}{\sqrt{gL}}\cdot\sin\left(\sqrt{\frac{g}{L}}\,t\right)}}$$

$$dt = \frac{f[x_{i+1}] - f[x_i]}{x_{i+1} + x_i}$$

integrasjon:

$$\text{for } i = 1, \ldots, \text{len}(Y)$$

$$d_x = x[i] + x[i-1]$$

$$Y_{x+1} = Y_i \cdot d_x + Y_{i-1}$$