Del 1

1 $f(x,y) = x\sin(xy^2)$

$$\frac{df}{dy}(y;x) = x\cos(xy^2)\,2xy$$
$$= 2x^2 y\cos(xy^2)$$

$\boxed{C}$

2 Dersom $f(x,y) = xe^{xy}$ så er retningsderivert $f'(a:r)$ der $a=(1,1)$ og $r=(-2,1)$

$$\frac{df}{dx}(x;y) = e^{xy} + xe^{xy}y = e^{xy}(1+xy)$$

$$\frac{df}{dy}(y;x) = x^2 e^{xy}$$

$$\nabla f(x,y) = (e^{xy}(1+xy),\ x^2e^{xy})$$

$$\nabla f(1,1) = e+e,\ e$$
$$= 2e,\ e$$

$$f'(a,r) = -4e + e$$
$$= -3e$$

$\boxed{D}$

3 $f(x, y) = x^3 y + 2y^2$, punkt $(2, -1)$

razlest vrednostnih

$$\frac{df}{dx}(x, y) = 3x^2 y$$

$$\frac{df}{dy}(y; x) = x^3 + 4y$$

$$\nabla f(x, y) = (3x^2 y, x^3 + 4y)$$

$$\nabla f(2, -1) = -3 \cdot 4, 8 - 4$$
$$= (-12, 4)$$
$$= 4(-3, 1)$$

$\boxed{B}$

6 $$\frac{P(x)}{Q(x)} = \frac{x^2 + 2x - 1}{(x+1)(x^2+1)^2}$$

$$\frac{A}{x+1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2+1)^2}$$

4 $a = (1, 0, -1)$

$b = (1, 2, 1)$

$c = (2, -1, 3)$

$$\begin{vmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix} = 1 \begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} \quad 0 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + (-1) \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$$

$$= (2 \cdot 3 - (-1 \cdot 1)) - (1 \cdot (-1) - 2 \cdot 2)$$

$$7 + 5$$

$$\boxed{\cancel{20}}$$

7. $F(x) = \int_0^{x^2} \frac{\sin t}{1+t^2}\, dt$

$$\int_0^x \frac{\sin t}{1+t^2} = \frac{\sin x}{1+x^2} \quad \Rightarrow \int_0^{x^2} \frac{\sin(t)}{1+t^2}\, dt = \frac{\sin(x)}{1+(x^2)^2} \, (x^2)'$$

$$= \underline{\underline{\frac{2x \sin(x^2)}{1+x^4}}}$$

8 använd detta för att ... $f(x) = \sin(x)$

$[0, \pi]$

$$2\pi \int_0^{\pi} x \sin(x)$$

$u = x \qquad v = -\cos(x)$

$u' = 1 \qquad v' = \sin(x)$

$$2\pi\left[-x\cos(x) - \int 1 \cos(x)\right]_0^{\pi}$$

$$2\pi\left(-x\cos x - \sin(x)\right)_0^{\pi}$$

$$2\pi\,(+\pi + 0)$$

$$2\pi^2$$

9. integralet $\displaystyle\int \frac{1}{\sqrt{x}\,(1+x)}$

$$\int \frac{1}{x\,(1+u^2)} \cdot 2x$$

$u = \sqrt{x}$

$\dfrac{du}{dx} = 2x$

$\dfrac{dx}{du} = \dfrac{1}{2x}$

$$\int \frac{1}{1+u^2} \cdot 2$$

$$2 \cdot \arctan u$$

$$= \underline{\underline{2\arctan(\sqrt{x}) + C}}$$

10 $\displaystyle\int_0^1 \frac{1}{\sqrt{x}-1}$

$u = \sqrt{x} - 1$

$x = (u-1)^2$

$dx = 2(u+1)\,du$

$$\int \frac{1}{u} \cdot 2(u+1))$$

$$\int \frac{1}{u}(2u+2)$$

$$\int \frac{2\cancel{u}}{\cancel{u}} + \frac{2}{u}$$

$$\int_0^1 2 + \frac{2}{u}$$

$$2\left(1 + \frac{1}{u}\right)$$

$$2\int_0^1 1 + \frac{1}{u}$$

$$2\left[u + \ln u\right]_0^1$$

$$2\left[(\sqrt{x}-1) + \ln \sqrt{x}-1\right]_0^1$$

$$2\left[(\sqrt{1}-1) + \ln(1-1) - ((\sqrt{0}-1) + \ln\sqrt{0}\right.$$

divergent $\boxed{D}$

12. b) 3 tilstander x, y, z

$$r = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad Ar = \begin{pmatrix} 4000 \\ 2000 \\ 1000 \end{pmatrix}$$

$$r_0 = A^{-1} r_1 = D\, r_1$$

$$\begin{pmatrix} 1 & -0{,}2 & -0{,}1 \\ -0{,}1 & 1 & 0 \\ 0{,}2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4000 \\ 2000 \\ 1000 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 \cdot 4000 - 0{,}2 \cdot 2000 - 0{,}1 \cdot 1000 \\ -0{,}1 \cdot 4000 + 1 \cdot 2000 + 0 \cdot 1000 \\ 0{,}2 \cdot 4000 + 0 \cdot 2000 + 1 \cdot 1000 \end{pmatrix}$$

$$= \begin{pmatrix} 1500 \\ ? \\ ? \end{pmatrix}$$

13 $\int 1 \cdot \ln(x^2+1)\,dx$

$x \ln x^2+1 - \int x \frac{(2x)}{x^2+1}$