1

La C være kurven

$$r(t) = t\cos(t)\,\bar{i} + t\sin(t)\,\bar{j}, \quad t \in [0, 2\pi]$$

og la $\bar{F}$ være vektorfeltet

$$\bar{F}(x,y) = -y\,\bar{i} + x\,\bar{j}$$

a) regn ut

$$\int_C \bar{F}\cdot d\bar{r} = \bar{F}(\bar{r}(t))\cdot r'(t)\,dt$$

$$\bar{F}(\bar{r}(t))\cdot d'(t) = (-(t\sin(t)), t\cos(t))\cdot(t'\cos t + t(\cos t)', t'\sin(t) + t(\sin t)')\,dt$$

$$= (-t\sin t, t\cos t)\cdot(\cos t - t\sin t, \sin t + \cos(t))\,dt$$

$$= (t\sin t)\cdot(\cos t - t\sin t) + (t\cos t)\cdot(\sin t + \cos t)\,dt$$

$$= -\cancel{t\sin(t)\cos(t)} + \underline{t^2\sin^2(t)} + \cancel{t\cos(t)\sin(t)} + \underline{t^2\cos^2(t)}\,dt$$

$$= t^2(\sin^2 t + \cos^2(t))$$

$$\underline{= t^2}$$

$$\int_C F\cdot d\bar{r} = \int_0^{2\pi} t^2 = \left[\frac{t^3}{3}\right]_0^{2\pi}$$

$$= \frac{(2\pi)^3}{3} - 0 = \underline{\underline{\frac{8}{3}\pi^3}}$$

b) Regne ut arealet av området avgrenset av c og den rette linja fra $(2\pi, 0)$ til $(0, 0)$

ser at vi må skrive om til polarkoordinater

ser at $c = r = \theta$

$r = [0, \theta]$, $\theta = [0, 2\pi]$

$$A = \iint_R 1 \, dx\, dy = \int_0^{2\pi}\int_0^{\theta} 1 \cdot r \, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{\theta^2}{2} \, d\theta = \frac{1}{2}\left[\frac{\theta^3}{3}\right]_0^{2\pi} = \frac{1}{6} \, 8\pi^3$$

$$= \frac{4}{3}\pi^3$$

La A være matrisen

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{pmatrix}$$

For hvilke $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in \mathbb{R}^3$ har ligningen $Ax = \bar{b}$

entydig løsning $\tilde{x} = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$?

$$A \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\Rightarrow \left.\begin{aligned} x + 2y &= b_1 \\ 2x + y &= b_2 \\ x &= b_3 \end{aligned}\right\} y = b_2 - 2b_3 \left.\vphantom{\begin{aligned} x \\ x \\ x \end{aligned}}\right\} b_3 + 2(b_2 - 2b_3) = b_1$$

$b_1 = 2b_2 - 3b_3$ er dette oppfylt har vi entydig løsning

c) $b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$f(x) = |A\tilde{x} - \vec{b}|^2$$

$$= \left| \begin{matrix} x + 2y - 0 \\ 2x + y - 0 \\ x - 1 \end{matrix} \right|^2$$

$$(x - 2y)^2 + (2x + y)^2 + (x - 1)^2$$

$$6x^2 + 8xy + 5y^2 - 2x + 1$$

$$\frac{dt}{dx} = 12x + 6y - 2 = 0 \quad \mathrm{I}$$

$$\frac{dt}{dy} = 8x + 10y = 0 \quad \mathrm{II}$$

II $\quad \frac{8x}{8} = -\frac{10}{8}y$

$$x = -\frac{5}{4}y$$

I in demed

$$12\left(-\frac{5}{4}y\right) + 6y = 2$$

$$-15y + 6y = 2$$

$$\frac{-7y}{-7} = \frac{2}{-7}$$

$$y = \frac{-2}{7}$$

sai $x = -\frac{5}{4} \cdot -\frac{2}{7} = \frac{10}{28} = \frac{5}{14}$