1.

$$f_X(x) = \begin{cases} \theta K^{\theta} x^{-\theta-1} & \text{for } x > k \\ 0 & \text{ellers} \end{cases}$$

a) Vis at den kumulative sannsynlighetsfordelingen er til X er gitt ved

$$F_X(x) = \begin{cases} 1 - K^{\theta} x^{-\theta} & \text{for } x > k \\ 0 & \text{ellers} \end{cases}$$

For å komme fra sannsynlighetstettheten til den kumulative — " — fordelingen må vi integrere f.

$$\int_k^x f(x) = \int_k^x \theta K^{\theta} x^{-\theta-1} dx$$

Mellomregning

$$\theta K^{\theta} \int_{-\infty}^{x} x^{-\theta-1} dx = \theta K^{\theta} \left[-\frac{x^{-\theta}}{\theta} \right]_{-\infty}^{x}$$

$$\theta K^{\theta}\left(\left(\frac{-x^{-\theta}}{\theta}\right) - \left(\frac{-1}{K^{\theta}\theta}\right)\right) = -x^{-\theta}K + 1$$

derved er

$$F_X(x) = \begin{cases} 1 - K^{\theta} x^{-\theta} & \text{for } x > K \\ 0 & \text{ellers.} \end{cases}$$

#

For å finne medianen Utreder jeg $F_X(x) = 0,5$
siden det viser til midten av sannsynligheten

$$\Rightarrow 1 - K^{\theta} x^{-\theta} = 0,5 \;/ -1$$

$$K^{\theta} x^{-\theta} = 0,5 \Rightarrow \underline{\underline{x^{-\theta} = \frac{0,5}{K^{\theta}}}}$$

b) For å finne $E(x)$ tar jeg

$$\int_K^{\infty} x \, f(x) \, dx$$

$$\Rightarrow \int_K^{\infty} x \cdot \theta K^{\theta} x^{-\theta-1} = \theta K^{\theta} \int_K^{\infty} x^{-\theta} dx = \theta K^{\theta} \left[\frac{1}{-\theta+1} x^{-\theta+1} \right]_K^{\infty}$$

$$= \underbrace{\lim_{x \to \infty} \frac{\theta K^{\theta}}{-\theta+1} \frac{x}{x^{\theta}}}_{0} - \left(\frac{1}{-\theta+1} K^{-\theta+1} \right) = \underline{\underline{\frac{\theta K}{\theta - 1}}}$$

c) $K = 400\,000$, $\theta = 3$

Median:

$$k^{\theta} x^{-\theta} = 0{,}5$$

$$400'000^{3} x^{-3} = 0{,}5$$

$$\underline{x = 503968}$$

E(x):

$$\frac{\theta K}{\theta - 1} \Rightarrow \frac{3 \cdot 400'000}{3-1}$$

$$\underline{= 600'000}$$

d) $V(X) = E(X^2) - E(X)^2$

$$E(X^2) = \int_{400'000}^{\infty} x^2 \cdot \theta K^{\theta} x^{-\theta-1} \, dx$$

$$= \theta K^{\theta} \int x^{-\theta+1} \, dx$$

$$= \theta K^{\theta} \left[\frac{x^{-\theta+2}}{-\theta+2} \right]_K^{\infty}$$

$$= \theta K^{\theta} \left(\lim_{x \to \infty} \frac{x^2}{-\theta x^{\theta} - 2} + \frac{k^2 \theta}{\theta - 2} \right)$$

siden $\theta > x$ vil
blir det lik 0

$$= \frac{K^2 \theta}{\theta - 2}$$

setter inn verdier

$$\frac{400'000^2 \cdot 3}{3 - 2} = 4{,}8e^{11}$$

$E(X)^2 = 600'000^2$

$= 3.6e^{11}$

$\underline{\underline{V(X) = 1,2 \cdot 10^{11}}}$

$\underline{\underline{\sigma = 346'410,1615}}$

2. Simultan sansynlighet

$$f(x,y) = \begin{cases} k(x-y) & \text{for } 0 \le y \le x \le 1 \\ 0 & \text{ellers} \end{cases}$$

a) Vis at $k = 6$

Siden hele området av en tetthet skal være lik 1 derved tar vi integralet over hele arealet og setter det lik 1.

$$\int_0^1 \int_0^x k(x-y)\,dy\,dx =$$

$$= k \int_0^1 \int_0^x x - y \, dy \, dx = k \int_0^1 \left[xy - \frac{y^2}{2} \right]_0^x dx = 1$$

$$= k \int_0^1 x^2 - \frac{x^2}{2} dx = k \int_0^1 \frac{x^2}{2} dx = 1$$

$$= k \left[\frac{x^3}{6} \right]_0^1 = \frac{k}{6} = 1$$

$$\underline{\underline{k = 6}}$$

c) bestem $P(2Y \leq X)$

$2Y \leq X \;/ :2$

$Y \leq \frac{X}{2}$

$$P(2Y \leq X) = \int_0^1 \int_0^{\frac{x}{2}} 6(x-y) \, dy \, dx$$

$$= 6 \int_0^1 \left[xy - \frac{y^2}{2} \right]_0^{\frac{x}{2}} dx$$

$$= 6 \int_0^1 \frac{3}{8} x^2 dx$$

$$= 6 \frac{3}{8} \left[\frac{x^3}{3} \right]_0^1 = 6 \cdot \frac{3}{8} \cdot \frac{1}{3} = \underline{\underline{\frac{3}{4}}}$$

c) Vis den marginale sannsynlighetstettheten for x

$$\int_0^x f(x,y)\,dy = \int_0^x 6(x-y)\,dy$$

$$= 6\left[xy - \frac{y^2}{2}\right]_0^x dy$$

$$= 6\left(x^2 - \frac{x^2}{2}\right) = 6\frac{x^2}{2} = \underline{\underline{3x^2}}$$

$$f_X(x) = \begin{cases} 3x^2 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{ellers} \end{cases}$$

d) Vis den marginale sannsynlighetstettheten for y

$$f_Y(y) = \int_y^1 f(x,y)\,dx$$

$$= 6\int_y^1 (x-y)\,dx$$

$$= 6\left[\frac{x^2}{2} - yx\right]_y^1$$

$$= 6\left(\frac{1}{2} - y - \left(\frac{y^2}{2} - y^2\right)\right)$$

$$= \underline{\underline{3y^2 - 6y + 3}}$$

e) X og Y er uavhengige hvis

$$f(x,y) = f(x)f(y)$$

Setter inn

$$6(x-y) \neq 3x^2 + 3y^2 - 6y + 3$$

derved er de ikke uavhengige

Oppg 3

a) $F(x) = P(X \leq x) = P(F^{-1}(u) \leq x) = F[F^{-1}(u)] = u$

#

b) $u = 1 - \frac{k^\theta}{x^\theta}$

$x^\theta u = x^\theta - k^\theta$

$x^\theta(u-1) = -k^\theta$

$x^\theta = \frac{k^\theta}{u-1}$

$x = \frac{k}{\sqrt[\theta]{u-1}}$

```matlab
%inndata
K = 400000;
theta = 3;
n = 10000;
%getting n uniformed u between 0 and 1
u = unifrnd(0,1,[1 n]);

x = K./(nthroot(1-u,theta));
%finding the mean
mean(x)
%finding the median
median(x)
hold on
%d) plotting the histogram
histogram(x,"normalization","pdf","binlimits", [400000 2000000])
X = (400000:0.1:2000000);
%e) printer tetheten til Paretofordelingen
line(X,theta*(K.^theta)*(X.^(-theta-1)), 'color', 'red')
hold off

```

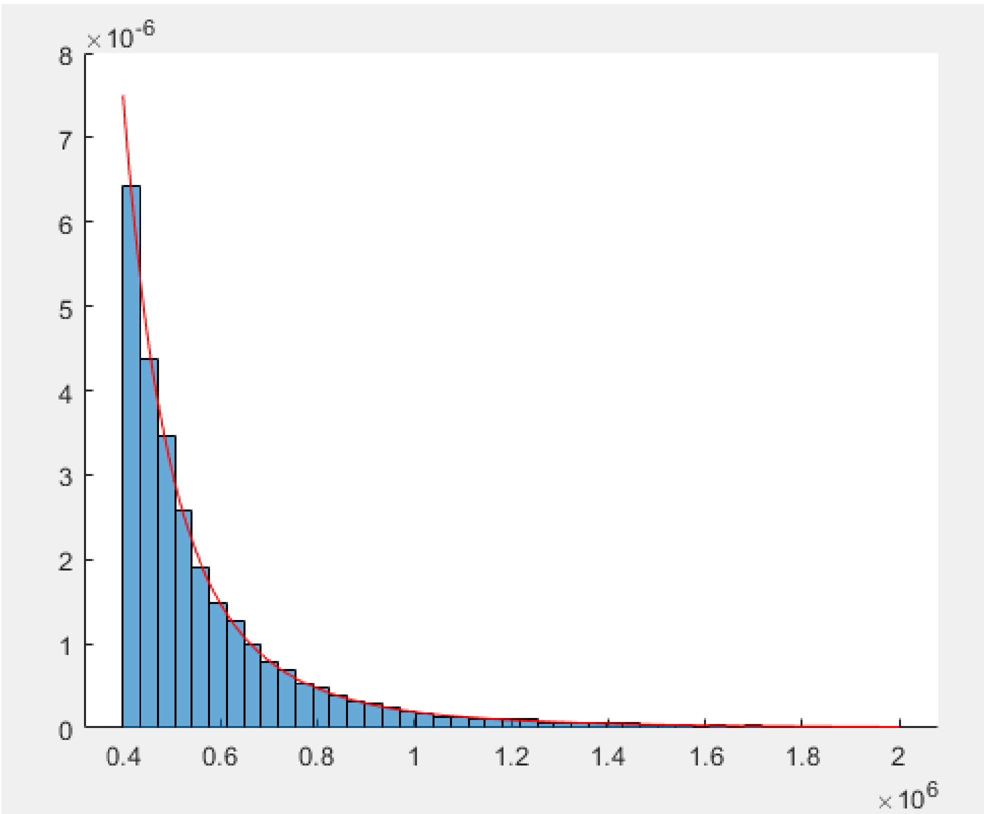
```
>> oppg3

ans = mean

   5.9822e+05


ans = median

   5.0478e+05
```



1e)

$$y = \theta \ln(x/h)$$

$$y' = \theta \frac{1}{x} \left(\frac{x}{h}\right)'$$

$$= \frac{\theta}{xh}$$