Oblig 1. Mat1110 Sondre S. Skjåvik

1b)

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = A \cdot \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}, \text{ der } A = \begin{pmatrix} 0 & 9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Finne egenverdiene $\lambda_1, \lambda_2, \lambda_3$

$$A^T = \begin{pmatrix} 0 & 1 & 0 \\ 9 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

, transponerer for å gjøre det enklere

$$\det(\lambda I_3 - A^T) = \begin{pmatrix} \lambda & -1 & 0 \\ -9 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \sim \quad I \leftrightarrow II$$

$$= \begin{vmatrix} 0 & -1 & \lambda \\ -9 & \lambda & 1 \\ \lambda & & 0 \end{vmatrix} = -\lambda\left((-9 \cdot -1) - (\lambda \cdot \lambda)\right)$$

$$= -\lambda(9 - \lambda^2)$$

Setter $0 = -\lambda(9-\lambda^2)$ og finner de tre løsningene for λ

$0 = -\lambda(9-\lambda^2)$, når $\underline{\lambda = 0}$ og når $(9-\lambda^2) = 0$

$$0 = 9 - \lambda^2$$

$$\sqrt{\lambda^2} = \sqrt{9}$$

$$\underline{\lambda = \pm 3}$$

Dermed har vi egenverdiene $\lambda_1, \lambda_2, \lambda_3 = 0, 3, -3$

Finner egenvektoren tilhørende λ_1

$$\left.\begin{aligned} 0x + 9y + 0z &= 0x \\ x + 0y + 0z &= 0y \\ 0x + y + 0z &= 0z \end{aligned}\right\} \quad \begin{aligned} 9y &= 0 \\ x &= 0 \\ y &= 0 \end{aligned}$$

Ser at z er en fri variabel setter den lik 1, dermed får vi:

$$\underline{v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$$

Finner egenvektoren tilhørende λ_2

$$\left.\begin{aligned} 0x + 9y + 0z &= 3x \\ x + 0y + 0z &= 3y \\ 0x + y + 0z &= 3z \end{aligned}\right\} \left.\begin{aligned} 9y &= 3x \\ x &= 3y \\ y &= 3z \end{aligned}\right\} \begin{aligned} 3y &= x \\ 3y &= x \\ y &= 3z \end{aligned}$$

Setter $z = 1$ og får dermed:

$$\underline{V_2 = \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix}}$$

Finner egenvektoren tilhørende λ_3

$$\left.\begin{aligned} 0x + 9y + 0z &= -3x \\ x + 0y + 0z &= -3y \\ 0x + y + 0z &= -3z \end{aligned}\right\} \begin{aligned} 3y &= -x \\ 3y &= -x \\ y &= -3z \end{aligned}$$

Setter $z = 1$ og får dermed:

$$V_3 = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

Dermed har vi egenverdiene:

$$\lambda_1, \lambda_2, \lambda_3 = 0, 3, -3$$

Og egenvektorene:

$$V_1, V_2, V_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

1 a)

$$A = \begin{array}{cc} \begin{array}{ccc} u & v & g \end{array} & \\ \begin{pmatrix} 0 & 9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} & \begin{array}{l} \text{unge } (u) \\ \text{voksne } (v) \\ \text{gamle } (g) \end{array} \end{array}$$

a_{11}: antall unge endrer ikke på antall unge til neste periode

a_{12}: antall voksne gir 9 nye unge per voksen

a_{13}: antall gamle endrer ikke på antall unge til neste periode

a_{21}: antall barn gir 1 nye voksne per unge

a_{22}: antall voksne endrer ikke på antall voksne til neste periode

a_{23}: antall gamle endrer ikke antall voksne

a_{31}: antall unge endrer ikke på antall gamle

a_{32}: per voksen blir det en ny gamel

a_{33}: gamle endrer ikke på antall gamle

Ehs.

$$A \cdot \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0\cdot 1 + 9\cdot 1 + 0\cdot 1 \\ 1\cdot 1 + 1\cdot 0 + 0\cdot 0 \\ 0\cdot 1 + 1\cdot 1 + 0\cdot 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$$

1c)

$$A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 45 \\ 18 \\ 5 \end{pmatrix}$$

$$\Rightarrow \left.\begin{aligned} 0x + 9y + 0z &= 45 \\ x + 0y + 0z &= 18 \\ 0x + y + 0z &= 5 \end{aligned}\right\}$$

$9y = 45 \Rightarrow \underline{y = 5}$

$\underline{x = 18}$

$\underline{y = 5}$

$$\underline{\underline{x = 18,\ y = 5,\ z = \text{udefinert}}}$$

1 d)

$$M^{-1} A M = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

$$M = \begin{pmatrix} 0 & 9 & 9 \\ 0 & 3 & -3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$M^{-1} \Rightarrow \left(\begin{array}{ccc|ccc} 0 & 9 & 9 & 1 & 0 & 0 \\ 0 & 3 & -3 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \mathrm{I} \leftrightarrow \mathrm{III} \\ \\ \sim \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 3 & -3 & 0 & 1 & 0 \\ 0 & 9 & 9 & 1 & 0 & 0 \end{array}\right) \begin{array}{l} \mathrm{III} : 9 \\ \mathrm{II} : 3 \\ \sim \end{array} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1/3 & 0 \\ 0 & 1 & 1 & 1/9 & 0 & 0 \end{array}\right) \begin{array}{l} \mathrm{III} - \mathrm{II} \\ \\ \sim \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1/3 & 0 \\ 0 & 0 & 2 & 1/9 & -1/3 & 0 \end{array}\right) \begin{array}{l} \mathrm{I} - 2\,\mathrm{III} \\ \mathrm{II} + \mathrm{III} \\ \sim \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/9 & 0 & 1 \\ 0 & 1 & 0 & 1/18 & 1/6 & 0 \\ 0 & 0 & 1 & 1/18 & -1/6 & 0 \end{array}\right)$$

$$\Rightarrow M^{-1} = \begin{pmatrix} 1/9 & 0 & 1 \\ 1/18 & 1/6 & 0 \\ 1/18 & -1/6 & 0 \end{pmatrix}$$

$$M^{-1}AM = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\left(\begin{pmatrix} 1/9 & 0 & 1 \\ 1/18 & 1/6 & 0 \\ 1/18 & -1/6 & 0 \end{pmatrix} \begin{pmatrix} 0 & 9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \right) \begin{pmatrix} 0 & 9 & 9 \\ 0 & 3 & -3 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\begin{array}{ccc|ccc} & & & 0 & 0 & 0 \\ & & & 1 & 9 & 0 \\ & & & 0 & 1 & 0 \\ \hline -1/9 & 0 & 0 & 0 & 0 & 0 \\ 1/18 & 1/6 & 0 & 1/6 & 1/2 & 0 \\ 1/18 & -1/6 & 0 & 0 & 1/2 & 0 \end{array} \qquad \begin{array}{ccc|ccc} & & & 0 & 9 & 9 \\ & & & 0 & 3 & -3 \\ & & & 1 & 1 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 1/6 & 1/2 & 0 & 0 & 3 & 0 \\ -1/6 & 1/2 & 0 & 0 & 0 & -3 \\ \hline \end{array}$$

Det viser oss at $M^{-1}AM = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$

1e) $t = 0$, 18 unge kaniner

$$\Rightarrow \begin{matrix} 0 \\ 18 \\ 0 \end{matrix}$$

Vi vet at egenverdiene er

$0, 3, -3$

og at egenvektorene er

$$\begin{pmatrix} 0 & 9 & 9 \\ 0 & 3 & -3 \\ 1 & 1 & 1 \end{pmatrix}$$

Vi skal finne

$$r_n = c_1 \lambda_1^n v_1 + c_2 \lambda_2^n v_2 + c_3 \lambda_3^n v_3$$

Fyller inn verdier

$$r_n = c_1 0^n \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + c_2 3^n \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} + c_3 (-3)^n \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

$$= c_2 3^n \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} + c_3 (-3)^n \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

Vi vet at når $n = 0$ så er $r = \begin{pmatrix} 0 \\ 18 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 0 \\ 18 \\ 0 \end{pmatrix} = C_2 3^0 \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} + C_3 (-3)^0 \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

$$= C_2 \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} + C_3 \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

$$\left.\begin{aligned} 0 &= C_2 9 + C_3 9 \\ 18 &= C_2 3 + C_3 (-3) \end{aligned}\right\} \left.\begin{aligned} C_2 &= -C_3 \\ C_2 + 6 &= C_3 \\ C_2 &= -C_1 \end{aligned}\right\} \begin{aligned} c_1 &= 3 \\ c_2 &= -3 \end{aligned}$$

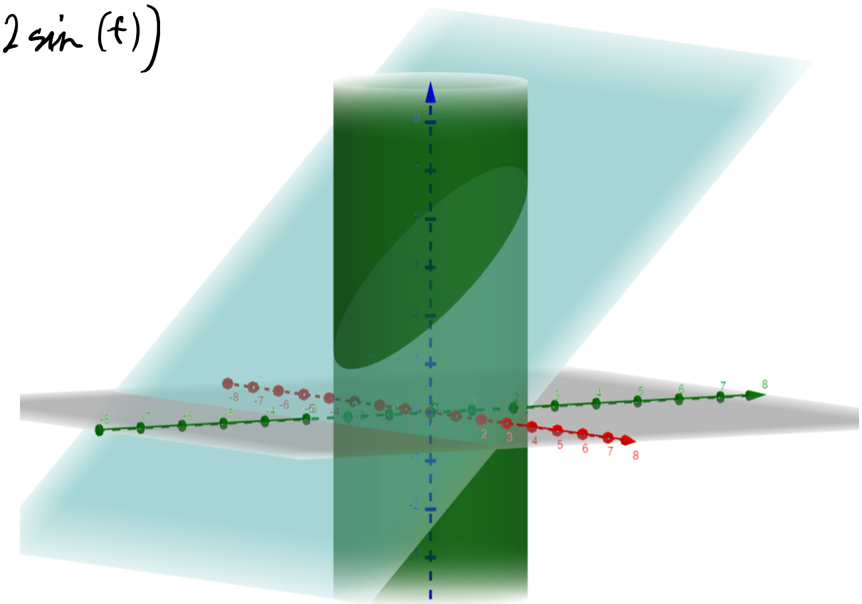
derved kan vi skrive

$$r_n = 3 \cdot (3^n) \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} - 3(-3)^n \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

$$= \underline{\underline{3^{n+1} \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} - (-3)^{n+1} \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}}}$$

3. $C = x^2 + y^2 = 4$, planet: $z = y + 3$ i $\mathbb{R}^3$

a) $r(t) = (2\cos(t), 2\sin(t), 3 + 2\sin(t))$



b) Vektorfeltet $F(x,y,z) = x\underline{i} + y\underline{j} + x\underline{k}$

$r(t) = 2\cos(t), 2\sin(t), 3 + 2\sin(t)$

$F(r(t)) = 2\cos(t)\underline{i} + 2\sin(t)\underline{j} + 2\sin(t)\underline{k}$

$r'(t) = -2\sin(t)\underline{i} + 2\cos(t)\underline{j} + 2\cos(t)$

$$\int_C F(r(t))\,dr = \int_0^{2\pi} (2\cos(t), 2\sin, 2\cos(t)) \cdot (2\sin(t), -2\cos(t), 2\cos(t))\,dt$$

$$= \int_0^{2\pi} \cancel{4\cos(t)\sin(t)} - \cancel{4\sin(t)\cos(t)} + 4\cos^2(t)\,dt$$

$$= 4\int_0^{2\pi} \cos^2(t)\,dt$$

Bruker at : $\cos^2(t) = \frac{1}{2}\cos(2t) + 1$

$$= 4\int_0^{2\pi} \frac{1}{2}(\cos(2t) + 1)\,dt$$

$$= 2\int_0^{2\pi} \cos(2t) + 1\,dt$$

$$= \left[2\left(\frac{\sin(2t)}{2} + t\right)\right]_0^{2\pi}$$

$$= -(\sin(2\cdot 0) + 2\cdot 0) + (\sin(4\pi) + 2\cdot 2\pi)$$

$$= \underline{\underline{4\pi}}$$

c) $\frac{dF_x}{dy} = 0 = \frac{dF_y}{dx}$

$\frac{dF_x}{dy} = 0 = \frac{dF_z}{dx}$

$dF_z \neq 0 = \frac{dF_x}{dz}$

$\parallel$

1 dermed er ikke

$\underline{\underline{F(r(t))\text{ konservativ}}}$

4)

$$x_{n+1} = 40x_n + y_n$$

$$y_{n+1} = 40y_n - x_n$$

a)

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = M \cdot \begin{pmatrix} x_n \\ y_n \end{pmatrix}, \text{ der } M = \begin{bmatrix} 40 & 1 \\ -1 & 40 \end{bmatrix}$$

a_{11}: beskriver antall nye snille biller per snill bille

a_{12}: beskriver antall ekstra biller per ikke snille bille

a_{21}: beskriver endring i slemme biller per ikke slemme bille

a_{22}: beskriver antall nye slemme biller per slemme bille

b) egenverdiene til M

$$= (\lambda - 40)(\lambda - 40) - (-1 \cdot 1)$$

$$= \lambda^2 - 80\lambda + 1601$$

setter $= 0$ og løser

$$\frac{-(-80) \pm \sqrt{(-80)^2 - 4 \cdot 1 \cdot 1601}}{2 \cdot 1}$$

$$\frac{8 \pm \sqrt{-4}}{2} = 40 \pm i$$

$\lambda_1 = 40 + i \lor \lambda_2 = 40 - i$

λ_1 sin:

$$\left.\begin{aligned} 40x + y &= (40 + i)x \\ -x + 40y &= (40 - i)y \end{aligned}\right\} \left.\begin{aligned} y + ix &= 0 \\ -x - iy &= 0 \end{aligned}\right\} \underline{\underline{v_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}}}$$

$$\left.\begin{aligned} 40x + y &= (40 + i)x \\ -x + 40y &= (40 - i)y \end{aligned}\right\} \left.\begin{aligned} y + ix &= 0 \\ -x + iy &= 0 \end{aligned}\right\} \underline{\underline{v_2 = \begin{pmatrix} i \\ 1 \end{pmatrix}}}$$

c) $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 2 \\ 80 \end{pmatrix}$

$$r_n = c_1 \lambda_1^n v_1 + c_2 \lambda_2^n v_2$$

$$\begin{pmatrix} 1 & i & 2 \\ i & 1 & 80 \end{pmatrix} \overset{II \cdot i}{\sim} \begin{pmatrix} 1 & i & 2 \\ -1 & i & 80i \end{pmatrix} \overset{II + I}{\sim} \begin{pmatrix} 1 & i & 2 \\ 0 & 2i & 80i + 2 \end{pmatrix} \overset{II : 2}{\sim}$$

$$\begin{pmatrix} 1 & i & 2 \\ 0 & i & 40i + 1 \end{pmatrix} \overset{I - II}{\sim} \begin{pmatrix} 1 & 0 & 1 - 40i \\ 0 & i & 40 + i \end{pmatrix} \overset{II \cdot i}{\sim}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -40 + i \end{pmatrix} \overset{II(-1)}{\sim} \begin{pmatrix} 1 & 0 & 1 - 40i \\ 0 & 1 & 40 + i \end{pmatrix}$$

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = (1 - 40i) \cdot (40 + i)^n \begin{pmatrix} 1 \\ i \end{pmatrix} + (40 - i)(4 - i)^n \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$\Rightarrow y_n = (40 + i)^{n+1} + (40 - i)^{n+1}$$