1 b) $A = \begin{pmatrix} 0 & 9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

finn egenverdiene $\lambda_1, \lambda_2, \lambda_3$

$$A = \begin{pmatrix} 0 & 9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 0 & 1 & 0 \\ 9 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\det(\lambda I_3 - A^T) = \begin{vmatrix} \lambda & -1 & 0 \\ -9 & \lambda & 1 \\ 0 & 0 & \lambda \end{vmatrix} \begin{matrix} I \leftrightarrow III \\ \\ \sim \end{matrix}$$

$$= - \begin{vmatrix} 0 & 0 & \lambda \\ -9 & \lambda & 1 \\ \lambda & -1 & 0 \end{vmatrix} = -\lambda((-9 \cdot -1) - (\lambda \cdot \lambda))$$

$$= -\lambda(9 - \lambda^2)$$

setter uttrykket lik 0

$0 = -\lambda(9-\lambda^2)$ når $\lambda = 0$ eller når $9 - \lambda^2 = 0$

$$0 = 9 - \lambda^2$$

$$\pm\sqrt{\lambda^2} = \pm\sqrt{9}$$

$$\lambda = \pm 3$$

$\lambda_1, \lambda_2, \lambda_3 = 0, 3, -3$

$$\lambda_1 \Rightarrow \begin{aligned} 0x + 9y + 0z &= 0x \\ 1x + 0y + 0z &= 0y \\ 0x + 1y + 0z &= 0z \end{aligned}$$

$$\left.\begin{aligned} 9y &= 0 \\ 1x &= 0 \\ y &= 0 \end{aligned}\right\} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = v_1$$

$$\lambda_2 \Rightarrow \begin{aligned} 0x + 9y + 0z &= 3x \\ 1x + 0y + 0z &= 3y \\ 0x + 1y + 0z &= 3z \end{aligned}$$

$$\left.\begin{aligned} 9y = 3x &\Rightarrow 3y = x \\ x = 3y &\Rightarrow 3y = x \\ 1y = 3z &\Rightarrow y = 3z \end{aligned}\right\}$$

Velger $z = 1$

$$v_2 = \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix}$$

$$\lambda_3 \Rightarrow \begin{aligned} 0x + 9y + 0z &= -3x \\ 1x + 0y + 0z &= -3y \\ 0x + 1y + 0z &= -3z \end{aligned}$$

$$\left.\begin{aligned} 9y &= -3x \\ x &= -3y \\ y &= -3z \end{aligned}\right\} \quad \left.\begin{aligned} 3y &= -x \\ 3y &= -x \\ y &= -3z \end{aligned}\right\} \quad \text{velger } z = 1 \quad \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix} = v_3$$

dermed har vi egenvektorene

$$v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

Ganger A med tilstanden for å finne neste tilstand

$$\begin{pmatrix} 0 & 9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix} = \begin{pmatrix} 0 \cdot 0 + 9 \cdot 0 + k \cdot 0 \\ 1 \cdot 0 + 0 \cdot 0 + k \cdot 0 \\ 0 \cdot 0 + 1 \cdot 0 + k \cdot 0 \end{pmatrix} = 0$$

det er ingen kaniner til neste sesong hvis tilstanden er $\begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}$

1a)

$$\begin{pmatrix} x_{n+1}\\ y_{n+1}\\ z_{n+1}\end{pmatrix} = A\cdot\begin{pmatrix}x_n\\ y_n\\ z_n\end{pmatrix} \quad \text{der } A=\begin{pmatrix}0&9&0\\1&0&0\\0&1&0\end{pmatrix}$$

I En voksen hunn lager 9 barn per hunn

II Alle unge blir voksne

III Alle voksne blir gamle

x gruppe gir til y gruppe

y gruppe får fra x gruppe

	u	v	g
u	0	I 9	0
v	II 1	0	0
g	0	III 1	0

dermed fungerer A som en faktor til å regne seg til $n+1$

eks startverdi $= \begin{pmatrix}1\\1\\1\end{pmatrix}$

$$\begin{pmatrix}0&9&0\\1&0&0\\0&1&0\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0\cdot1+9\cdot1+0\cdot1\\ 1\cdot1+1\cdot0+1\cdot0\\ 0\cdot1+1\cdot1+0\cdot1\end{pmatrix} = \begin{pmatrix}9\\1\\1\end{pmatrix}$$

c) $A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 45 \\ 18 \\ 5 \end{pmatrix}$

$$\begin{pmatrix} 0 & 9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 45 \\ 18 \\ 5 \end{pmatrix}$$

$$\left.\begin{aligned} 0x + 9y + 0z &= 45 \\ x + 0y + 0z &= 18 \\ 0x + y + 0z &= 5 \end{aligned}\right\}$$

$9y = 45 \Rightarrow y = 45$

$\underline{x = 18}$

$\underline{y = 5}$

Z er en fri variabel som ikke har noen innvirkning

d) $M^{-1} A M = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$

$\mathrm{I} \leftrightarrow \mathrm{III}$
$\mathrm{II} : 3$
$\mathrm{III} : 9$
$\mathrm{III} - \mathrm{II}$
$\mathrm{III} : 2$
$\mathrm{II} + \mathrm{III}$
$\mathrm{I} - \mathrm{II} - \mathrm{III}$

$M = \begin{pmatrix} 0 & 9 & 9 \\ 0 & 3 & -3 \\ 1 & 1 & 1 \end{pmatrix}$

$M^{-1} \Rightarrow \left(\begin{array}{ccc|ccc} 0 & 9 & 9 & 1 & 0 & 0 \\ 0 & 3 & -3 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$ $\mathrm{I} \Leftrightarrow \mathrm{III}$ $\sim$

$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 3 & -3 & 0 & 1 & 0 \\ 0 & 9 & 9 & 1 & 0 & 0 \end{array}\right)$ $\mathrm{III} : 9$, $\mathrm{II} : 3$ $\sim$

$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 1 & \frac{1}{9} & 0 & 0 \end{array}\right)$ $\mathrm{III} - \mathrm{II}$ $\sim$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 2 & \frac{1}{9} & -\frac{1}{3} & 0 \end{pmatrix} \quad \mathrm{III} : 2$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{1}{18} & -\frac{1}{6} & 0 \end{pmatrix} \quad \begin{matrix} \mathrm{I} - \mathrm{II} \\ \\ \sim \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & 0 & -\frac{1}{3} & 1 \\ 0 & 1 & -1 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{1}{18} & -\frac{1}{6} & 0 \end{pmatrix} \quad \begin{matrix} \mathrm{I} - 2\,\mathrm{III} \\ \mathrm{II} + \mathrm{III} \\ \sim \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{1}{9} & 0 & 1 \\ 0 & 1 & 0 & \frac{1}{18} & \frac{1}{6} & 0 \\ 0 & 0 & 1 & \frac{1}{18} & -\frac{1}{6} & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -\frac{1}{9} & 0 & 1 \\ \frac{1}{18} & \frac{1}{6} & 0 \\ \frac{1}{18} & -\frac{1}{6} & 0 \end{pmatrix} = M^T$$

$$M^T A M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

$$\left(\begin{pmatrix} -1/9 & 0 & 1 \\ 1/18 & 1/6 & 0 \\ 1/18 & -1/6 & 0 \end{pmatrix} \begin{pmatrix} 0 & 9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \right) \begin{pmatrix} 0 & 9 & 9 \\ 0 & 3 & -3 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

$$\begin{array}{ccc|ccc} & & & 0 & 9 & 0 \\ & & & 1 & 0 & 0 \\ & & & 0 & 1 & 0 \\ \hline -1/9 & 0 & 1 & 0 & 0 & 0 \\ 1/18 & 1/6 & 0 & 1/6 & 1/2 & 0 \\ 1/18 & -1/6 & 0 & -1/6 & 1/2 & 0 \end{array} \quad \cdot \begin{pmatrix} 0 & 9 & 9 \\ 0 & 3 & -3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{array}{ccc|ccc} & & & 0 & 9 & 9 \\ & & & 0 & 3 & -3 \\ & & & 1 & 1 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 1/6 & 1/2 & 0 & 0 & 3 & 0 \\ -1/6 & 1/2 & 0 & 0 & 0 & -3 \end{array}$$

$\frac{1}{6} \cdot 9 = \frac{9}{6} = \frac{3}{2}$

$\frac{3}{2} + \frac{3}{2} = 2$

Det siær oss at

$$M^T A M = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

1e) $t = 0$, 18 kunstnviner $\Rightarrow \begin{pmatrix} 0 \\ 18 \\ 0 \end{pmatrix}$

Vi vet at egenverdiene er

$0, 3, -3$

og at egenvektorene er

$$\begin{pmatrix} 0 & 9 & 9 \\ 0 & 3 & -3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$r_n = c_1 \lambda_1^n v_1 + c_2 \lambda_2^n v_2 + c_3 \cdot \lambda_3^n v_3$$

$$= c_1 0^n v_1 + c_2 3^n v_2 + c_3 (-3)^n v_3$$

$$= c_2 3^n \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} + c_3 -3^n \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

Vi vet at når $n = 0$ så er $r = 18$

$$\begin{pmatrix} 0 \\ 18 \\ 0 \end{pmatrix} = c_2 3^0 \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} + c_3 (-3)^0 \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 18 \\ 0 \end{pmatrix} = c_2 \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

for å skaffe en ligning til så kan vi regne opp et skritt med en opprinnelige relasjon

$$\begin{pmatrix} 0 & 9 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 18 \\ 0 \end{pmatrix} = \begin{pmatrix} 9 \cdot 18 \\ 0 \\ 18 \end{pmatrix} = \begin{pmatrix} 162 \\ 0 \\ 18 \end{pmatrix}$$

$$162 + 0 + 18 = 188$$

dermed har vi at $r_1 = 188$, så da har vi ligningen

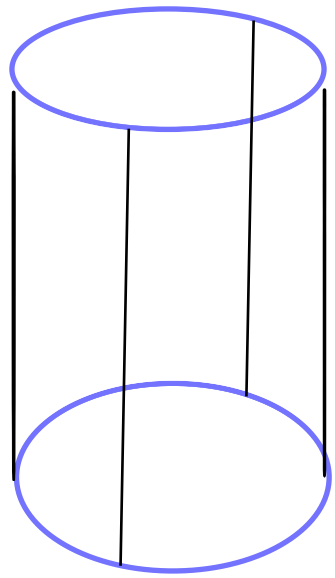
$$\begin{pmatrix} 0 \\ 18 \\ 0 \end{pmatrix} = c_2 \begin{pmatrix} 9 \\ 3 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$$

$$\left.\begin{aligned} 0 &= c_2 9 + c_3 9 \\ 18 &= c_2 3 + -3c_3 \\ 0 &= c_2 + c_3 \end{aligned}\right\} \quad \left.\begin{aligned} c_2 &= -c_3 \\ c_2 + 6 &= c_3 \\ c_2 &= -c_1 \end{aligned}\right\} \quad \begin{aligned} c_1 &= 3 \\ c_2 &= -3 \end{aligned}$$

dermed er $r_n = (3)^n \begin{pmatrix} 27 \\ 9 \\ 3 \end{pmatrix} - (-3)^n \begin{pmatrix} 27 \\ -9 \\ 3 \end{pmatrix}$

3. $C: x^2 + y^2 = 4$

planet $z = y + 3$ i $\mathbb{R}^3$



a)

$$r(t) = (2\cos(t),\ 2\sin(t),\ 3 + 2\sin(t))$$

c) Vektorfeltet $F(x, y, z) = x\underline{i} + y\underline{j} + x\underline{k}$

$$F(r(t)) = 2\cos(t)\underline{i} + 2\sin t\,\underline{j} + 2\cos(t)$$

$$r'(t) = -2\sin(t)\,\underline{i} + 2\cos(t)\,\underline{j} + 2\cos(t)$$

$$\int_C F(r(t)) \cdot dr = \int_0^{2\pi} (2\cos(t),\ 2\sin t,\ 2\cos(t)\underline{k}) \cdot (2\sin(t),\ -2\cos(t)\underline{j},\ 2\cos(t)\underline{k})$$

$$= \int_0^{2\pi} \cancel{4\cos(t)\sin(t)} - \cancel{4\sin(t)\cos(t)} + 4\cos^2(t)$$

$$= 4\int_0^{2\pi} \cos^2(t)$$

Bruker at:

$$\cos^2(t) = \frac{1}{2}(\cos(2t) + 1)$$

$$= 4\int_0^{2\pi} \frac{1}{2}(\cos(2t) + 1)$$

$$= 2\int_0^{2\pi} \cos(2t) + 1$$

$$\left[2\left(\frac{\sin(2t)}{2}+t\right)\right]\Big|_0^{2\pi}$$

$$=\left(-\sin(2t)+2t\right)\Big|_0^{2\pi} = -\left(\sin(2\cdot 0)+2\cdot 0\right)+\left(\sin(4\pi)+2\cdot 2\pi\right)$$

$$= \underline{4\pi}$$

c) avgjøre om $F(r(t))$ er konservativ

$$F(r(t)) = 2\cos(t)\,\underline{i} + 2\sin t\,\underline{j} + 2\cos(t)$$

$$\frac{dF_x}{dy} = 0 = \frac{dF_y}{dx}$$

$$\frac{dF_x}{dy} = 0\ \frac{dF_z}{dx}$$

$$\frac{dF_z}{dx} \neq 0 = \frac{dF_x}{dz}$$

$$\parallel$$

$$1$$

4)

$$x_{n+1} = 40x_n + y_n \qquad x_0 = 2$$

$$y_{n+1} = 40y_n - x_n \qquad y_0 = 80$$

a)

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = M \cdot \begin{pmatrix} x_n \\ y_n \end{pmatrix}, \text{ der } M = \begin{pmatrix} 40 & 1 \\ -1 & 40 \end{pmatrix}$$

M er en constant vektor som beskriver endringene i x og y per periode n

smille
-slemme

$$\begin{array}{c|l} \begin{pmatrix} 40 & 1 \\ -1 & 40 \end{pmatrix} & \begin{array}{l} \text{snille} \\ \text{slemme} \end{array} \end{array}$$

a11 = (40) antall nye snille biller per antall nye biller

a12 = (1) ekstra bille per slem bille snillen de snille biler å bo med andre

a21 = (-1) de slemme blir 1 mindre for hver snill siden de ikke liker å bo med andre

a22 = de slemme blir 40 flere per slem

b) egenvärden till 14

$$(\lambda - 40)(\lambda - 40) - (-1 \cdot 1)$$

$$\lambda^2 - 40\lambda - 40\lambda + 1600 - (-1)$$

$$\lambda^2 - 80\lambda + 16001$$

$$\frac{-(-80) \pm \sqrt{80^2 - 4 \cdot 1601}}{2 \cdot 1}$$

$$\frac{80 \pm \sqrt{6,400 - 6,404}}{2} = \frac{80 \pm 2i}{2}$$

$$\frac{80 + 2i}{2} \vee \frac{80 - 2i}{2}$$

$$\lambda_1 = 40 + i \vee 40 - i = \lambda_2$$

$$\lambda_1 = \left.\begin{matrix} 40x + y = (40+i)x \\ -x + 40y = (40+i)y \end{matrix}\right\} \quad \begin{matrix} y - ix = 0 \\ x + iy = 0 \end{matrix} \quad \underline{V_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}}$$

$$\lambda_2 = \left.\begin{matrix} 40x + y = (40-i)x \\ -x + 40y = (40-i)y \end{matrix}\right\} \quad \begin{matrix} y + ix = 0 \\ -x - iy = 0 \end{matrix} \quad \underline{V_2 = \begin{pmatrix} i \\ 1 \end{pmatrix}}$$

c) $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 2 \\ 80 \end{pmatrix}$

$$r_n = c_1 \lambda_1^n v_1 + c_2 \lambda_2^n v_2$$

$$\begin{pmatrix} 1 & i & 2 \\ i & 1 & 80 \end{pmatrix} \overset{\mathrm{II} \cdot i}{\sim} \left\| \begin{pmatrix} 1 & i & 2 \\ -1 & i & 80i \end{pmatrix} \right. \overset{\mathrm{II} + \mathrm{I}}{\sim}$$

$$\begin{pmatrix} 1 & i & 2 \\ 0 & 2i & 80i+2 \end{pmatrix} \overset{\mathrm{II} : 2}{\sim} \left\| \begin{pmatrix} 1 & i & 2 \\ 0 & i & 40i+1 \end{pmatrix} \right. \overset{\underline{\mathrm{I} - \mathrm{II}}}{\sim}$$

$$\begin{pmatrix} 1 & 0 & 1-40i \\ 0 & i & 40i+1 \end{pmatrix} \overset{\mathrm{II} \cdot i}{\sim} \left\| \begin{pmatrix} 1 & 0 & 1-40i \\ 0 & -1 & -40+i \end{pmatrix} \right. \overset{\mathrm{II} \cdot (-1)}{\sim}$$

$$\underline{\underline{\begin{pmatrix} 1 & 0 & 1-40i \\ 0 & 1 & 40+i \end{pmatrix}}} \qquad r_0 = c_1 \begin{pmatrix} 1 \\ i \end{pmatrix} + c_2 \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = (1-40i) \cdot (40+i)^n \begin{pmatrix} 1 \\ i \end{pmatrix} + (40-i)(40-i) \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$y_n = (40+i)^{n+1} + (40-i)^{n+1}$$