Mat 1100 h 1016

Del 2

11) La $P(z) = z^3 - 13z^2 + 52z - 70$

a) Vis at $1+i$ er en rot til P_z og finn de øvrige røttene

Siden $3+i$ skal være en kompleks rot må $P(3+i) = 0$

$$P(3+i) = (3+i)^3 - 13(3+i)^2 + 52(3+i) - 70$$

$$= 3^3 + 3 \cdot 3^2 i + 3 \cdot 3 \cdot i^2 + i^3$$

$$= 27 + 27i - 9 - i$$

$$= 18 + 26i - 13(9 + 6i - 1)$$

$$= 18 + 26i - 117 - 78i + 13$$

$$= -86 - 52i + 52 \cdot 3 + 52i - 70$$

$$= \underbrace{-86 + 156 - 70}_{0} \underbrace{- 52i + 52i}_{0}$$

$$= 0$$

Siden $(3+i)$ er en kompleks rot er også $(3-i)$ og så er en rot derved kan vi utføre polynomdivisjonen av $(z-3-i)(z-3+i) = z^2 - 6z + 10$

$$(z^3-13z^2+52z-70):z^2-6z+10=(z-7)$$
$$-(z^3-6z^2+10z)$$
$$-7z^2+42z-70$$
$$-7z^2+42z-70$$
$$0$$

da er røttene $(z-7)\underbrace{(z-3+i)(z-3-i)}_{\text{komplekx}}$

$$\underbrace{(z^2-6z+10)(z-7)}_{\text{reell}}$$

Oppg 12.

La funksjonen $f:\mathbb{R}\to\mathbb{R}$ være definert ved

$$f(x)=\begin{vmatrix} x^3 & 3x & 1 \\ e^{x^3} & 0 & 0 \\ x^2 & x & -1 \end{vmatrix}$$

a) $a>0$ Volum V gitt ved $V=16\pi\int_0^a x^2e^{2x^3}\,dx$

Ekspander nå den første determinanten til $f(x)$

$$x^3\begin{vmatrix}0&0\\x&-1\end{vmatrix}-3x\begin{vmatrix}e^{x^3}&0\\x^2&-1\end{vmatrix}+1\begin{vmatrix}e^{x^2}&0\\x^2&x\end{vmatrix}$$

$$x^3 \begin{vmatrix} 0 & 0 \\ x & -1 \end{vmatrix} = x^7(0 \cdot -1 - 0 \cdot -1) = 0$$

$$3x \begin{vmatrix} e^{x^3} & 0 \\ x^2 & -1 \end{vmatrix} = 3x\left(e^{x^3} \cdot (-1) - (0 \cdot x^2)\right) = 3x e^{x^3}$$

$$1 \begin{vmatrix} e^{x^2} & 0 \\ x^2 & x \end{vmatrix} = x e^{x^2} - 0$$

$$\Rightarrow 0 + 3x e^{x^2} + x e^{x^2}$$

$$\det(f(x)) = 4x e^{x^2}$$

Setter $\det(f(x))$ inn i funksjonen for rotasjon om x aksen:

$$V = \pi \int_b^a (f(x))^2 \, dx$$

det gir

$$V = \pi \int_0^a \left(4x e^{x^2}\right)^2 dx$$

$$= \pi \int_0^a 16 x^2 e^{2x^3} dx$$

$$= 16\pi \int_0^a x^2 e^{2x^3} dx$$

b) finn V ved å beregne integralet

$$16\pi \int_0^a x^2 e^{2x^3}\,dx$$

$$\int_0^a$$

13 a) $\lim_{\alpha \to 0^+} \alpha \left(\ln(\alpha)\right)^2 = \lim_{x \to 0^+} \dfrac{(\ln(x))^2}{\frac{1}{x}}$

L'H

$$= \lim_{x \to 0^+} \frac{2\ln(x)\,\cancel{\frac{1}{x}}}{-\frac{1}{x^{\cancel{2}}}} = \frac{2\ln x}{\frac{1}{x}}$$

L'H

$$= \frac{\frac{2}{x}}{-\frac{1}{x^2}} = \frac{2}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{1} = \lim_{x \to 0^+} 2x = 0$$

b) $f : (-\infty, 1) \to \mathbb{R}$ være definert ved

$$f(x) = \begin{cases} \frac{1}{(\ln x)^2} & \text{for } x \in (0,1) \\ 0 & \text{for } x \leq 0 \end{cases}$$

Sekker for gitte $\alpha \in (0,1)$

$$\left.\begin{array}{l}\lim\limits_{x\to 0^+} \dfrac{1}{\underbrace{\ln(x)^2}_{\infty}} = 0 \\[2em] \lim\limits_{x\to 0^+} 0 = 0\end{array}\right\}$$ denne er funksjonen kontinuerlig

Del 1

1 La $f(x,y) = x^3 y$. Den retningsderiverte $f'(a;r)$ der $a = (3,1)$ og $r = (1,-1)$ er lik

$$\left.\begin{array}{l}\dfrac{df}{dx}(x;y) = 3yx^2 \\[1.5em] \dfrac{df}{dy}(y;x) = x^3\end{array}\right\} \nabla f(x,y) = (3yx^2, x^3)$$

$$\nabla f(\overset{x}{3}, \overset{y}{1}) = 3 \cdot 1 \cdot 9, 27$$
$$= (27, 27)$$

$$\nabla f(a,r) = (27 \cdot 1, 27 \cdot (-1))$$
$$= (27, -27)$$
$$\Rightarrow 0 \qquad \boxed{A}$$

3 Matrizen

$$F' = \begin{pmatrix} y \cos x & \sin x \\ 3x^2 y & x^3 \end{pmatrix}$$

$$\int y \cos x \, dx \qquad \int \sin x \, dy$$

$$\int 3x^2 y \, dx \qquad \int x^3 \, dy$$

$$y \sin(x) = \sin(x) y$$

$$x^3 y = x^3 y \qquad \boxed{A}$$

4 Volumet til parallelepipedet utspent av

$\bar{a} = (1, 1, 1)$ $\quad \bar{b} = (2, -2, 4)$

$\bar{c} = (3, 0, 3)$

F and $\frac{1}{6}(\bar{a} \times \bar{b}) \cdot \bar{c}$

F and 2 $\frac{1}{6} \det(a, b, c)$

$\det(a, b, c)$:

$$\det \begin{vmatrix} 1 & 1 & 1 \\ 2 & -2 & 4 \\ 3 & 0 & 3 \end{vmatrix} = 1 \begin{vmatrix} -2 & 4 \\ 0 & 3 \end{vmatrix} - 1 \begin{vmatrix} 2 & 4 \\ 3 & 3 \end{vmatrix} + 1 \begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix}$$

$1\cdot(-2\cdot 3-0) - 1\,(2\cdot 3-4\cdot 3)+1\,(2\cdot 0-(-2)\cdot 3)$

$-6 \qquad -\cdot 1(6-12) \qquad -6$

$= 6$

Setter vi inn i funksjonen

$\frac{1}{6}\cdot 6 = 1$ $\qquad \boxed{B}$

5 Den inverse matrisen til $M = \begin{pmatrix}3 & 2\\ 8 & 5\end{pmatrix}$

a) $3\cdot\frac{5}{\frac{1}{11}} + 2\cdot\frac{-8}{\frac{1}{11}} =$

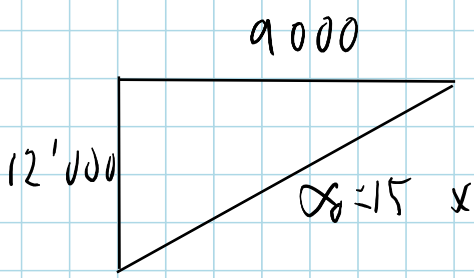
~~b~~ $3\cdot -3 + 2\cdot 8 =$

c $3\cdot 5 + 2\cdot 8 =$

~~D)~~ $3\cdot -5 + 2\cdot 8 = \underline{1}$ $\qquad \boxed{D}$

E) $3\cdot\frac{1}{3} + 2\cdot\frac{1}{8} =$

6.



$x_0 = 15 \quad x' = 600$

$x_0 = \sqrt{12^2 + 9^2}$

$x_0 = 15$

7 Spalte brøken $\frac{12x^2 - 5x + 7}{x^2(x-1)}$

$$\frac{12x^2 - 5x + 7}{x^2(x-1)}$$

$12x^2 - 5x + 7$

faktorisere $12x^2 - 5x + 7$
gir ikke noen reelle faktorer

splitte opp: brøker

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \qquad \boxed{D}$$

8 integralet $\int_0^{\pi/2} \cos x \arctan(\sin(x))\,dx$ er likt?

$$\int_0^{\pi/2} \cos(x) \arctan(\sin(x))$$

$u = \sin(x)$

$$\int_0^{\pi/2} 1 \cdot \arctan(u)$$

$\frac{du}{dx} = \cos(x)$

$\frac{dx}{du} = \frac{1}{\cos x}$

$$u \arctan u - \int \frac{u}{1+u^2}$$

$f = 1 \qquad G = \arctan(u)$

$F = u \qquad g = \frac{1}{1+x^2}$

$$u \arctan u - \int \frac{u}{1 + u^2}$$

$$u \arctan(u) - \int \frac{1}{2u}$$

$$u \arctan u - \frac{\ln u}{2}$$

$$\sin(x) \arctan(\sin(x)) - \frac{1}{2}$$

$$w = (1 + u^2)$$

$$\frac{dw}{du} = 2u \; du$$

$$\frac{du}{dw} = \frac{1}{2u}$$