oppg 2.

a: recessivt gen, for sykdomen

A: Dominant gen, ingen

a) situasjon 1.

Aa: ingen men bærer av sykdomen

		Mor	
		a	A
Far	a	1/4	1/4
	A	1/4	1/4

Det er 1/4 sannsynlighet for at barnet får aa og 2 · 1/4 sannsynlighet at barnet får Aa. Vist i diagramet over.

b) situasjon 2

8%: a

92%: A

$$= \underbrace{0.92 \cdot 0}_{\text{Mor AA}} + \underbrace{0.08 \cdot 0.25}_{\text{Mor Aa}}$$

$$= P(aa) = \frac{1}{\cancel{4}} \cdot \frac{\cancel{8}}{100} = 2/100 = \underline{\underline{2\%}}$$

c) Far Aa

Mor Aa eller AA

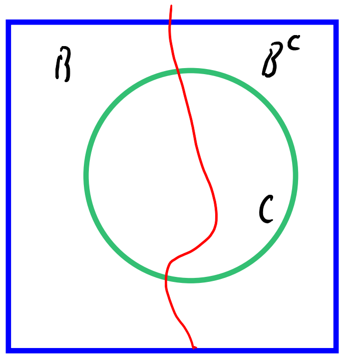
8% 92%

Bays setning

Har at

$$P(B|C) = \frac{P(B\cap C)}{P(C)}$$

$$= \frac{P(B)P(C|B)}{P(B)P(C|B) + P(B^c)P(C|B)}$$



$= P(\text{mor } aA \mid 3 \text{ friske barn})$

$$= \frac{P(\text{mor } Aa)(P \text{ tre friske barn} \mid \text{Mor } Aa)}{P(\text{mor } Aa)\, P(3 \text{ friske barn} \mid \text{mor } aA) + P(\text{mor } AA)\, P(3 \text{ friske barn} \mid \text{mor } AA)}$$

$$= \frac{0.08 \cdot \left(\frac{3}{4}\right)^3}{0.08 \left(\frac{3}{4}\right)^3 + 0.92 \cdot 1} = 3.5\%$$

3 stokastiske variabler X, Y med simultantetthet

$$f_{X,Y}(x,y) = \begin{cases} K & \text{hvis } x \geq 0,\ y \geq 0,\ x+y \leq \theta \\ 0 & \text{ellers} \end{cases}$$

Prøver

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{x,y}(x,y) = 1$$

$$= \int_0^{\infty}\int_0^{\theta - x} K \, dy\, dx = \int_0^{\infty} \Big[Ky\Big]_0^{\theta-x} dx = \int_0^{\infty} K(\theta - y)\, dx = K(\theta - y)\Big[x\Big]_0^{\infty} =$$

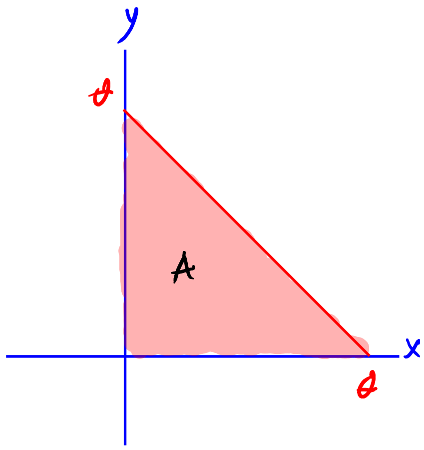
$$= K(\theta - y)\infty = 1$$

Feil !.

Forts. 2. (Med Hjelp)

$$1 = \iint_A f_{X,Y}(x,y)$$

$$\Rightarrow \iint_A K\, dx\,dy = 1$$



$$= K \underbrace{\iint_A dx\,dy}_{\text{Arealet } A}$$

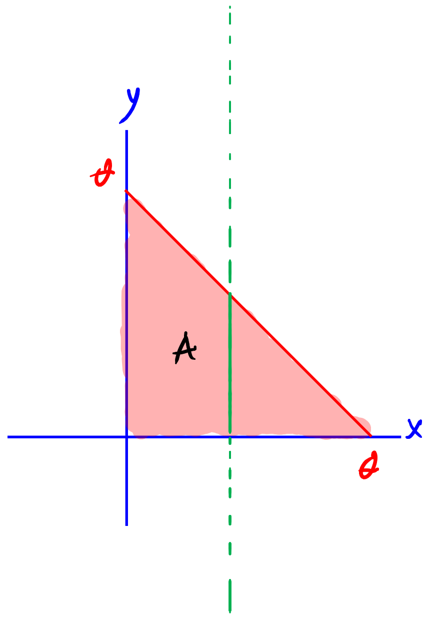
$$= K \cdot \theta \cdot \theta \cdot \frac{1}{2} = K\frac{\theta^2}{2}$$

$$K = 2/\theta^2$$

c) Vi vil finne marginaltettheten $f_X(x)$

Generelt

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$



Finner for $0 < x < \theta$

$$f_X(x) = \int_0^{\theta - x = y} \underbrace{\frac{2}{\theta^2}}_{K}\,dy = \frac{2}{\theta^2}\left[y\right]_0^{\theta - x}$$

$$= \frac{2}{\theta^2}(\theta - x)$$

Finner også for y

$$f_Y(y) = \int_{-\infty}^{\infty} K\,dy = \int_0^{\theta - y} \frac{2}{\theta^2}\,dx = \frac{2}{\theta^2}(\theta - y)$$

Er X, Y uavhengige

Generelt er X og Y uavhengig hvis

$f_{xy}(y,x) = f_x(x) f_y(y)$

for alle x og y

Finner her at

$f_{xy}(x,y) \neq f_x(x) f_y(y)$

Altså er ikke x og y uavhengige

c) Vis at

$$E(x^r) = \frac{2\theta^r}{(r+1)(r+2)}$$

Generelt

$$E(x^r) = \int_{-\infty}^{\infty} x^r f_x(x)\,dx$$

$$E(x^r) = \int_0^{\theta} x^r \frac{2}{\theta^2}(\theta - x)\,dx = \frac{2}{\theta^2}\int_0^{\theta} x^r\theta - x^{r+1}\,dx$$

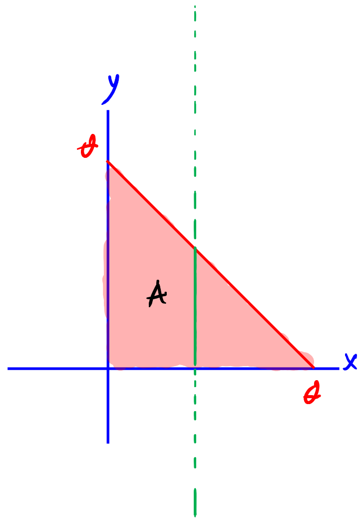
$$= \frac{2}{\theta^2}\left(\left[\frac{x^{r+1}\theta}{r+1}\right]_0^{\theta} - \left[\frac{x^{r+2}}{r+2}\right]\right) = \frac{2}{\cancel{\theta^2}}\left(\frac{\theta^{r+\cancel{2}}}{r+1} - \frac{\theta^{r+\cancel{2}}}{r+2}\right)$$

$$= 2\theta^r \frac{1}{(r+1)(r+2)}$$

$E(x) = \frac{\theta}{3}$, $E(x^2) = \frac{\theta^2}{6}$, $V(x) = E(x^2) - E(x)^2 = \frac{\theta^2}{6} - \left(\frac{\theta}{3}\right)^2 = \frac{\theta^2}{18}$

d) Betinget fordeling Y for $X = x$

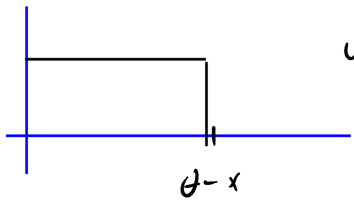
uttrykt ved $f_{Y|X=x}(y)$



$$f_{Y|X=x} = \frac{f_{XY}(x,y)}{f_Y}$$

$$\Rightarrow \begin{cases} \dfrac{2/\theta^2}{(2/\theta^2)(\theta - x)} & \text{for } 0 \le y < \theta - x \\ 0 & \text{ellers} \end{cases}$$

$$\frac{1}{\theta - x}$$



uniformt fordelt

$$E(Y|X=x) = \int_0^{\theta - x} y \frac{1}{\theta - x}\, dy = \frac{1}{\theta - x}\left[\frac{y^2}{2}\right]_0^{\theta - x} = \frac{\theta - x}{2}$$

e) Finn $P(X^2 + Y^2 \le \theta^2/2)$

$x^2 + y^2 = \frac{\theta^2}{2}$ er en sirkel

Generelt for $A \subset \mathbb{R}^2$:

$$P((X,Y) \in A) = \iint_A f_{XY}(x,y)\, dx\, dy$$

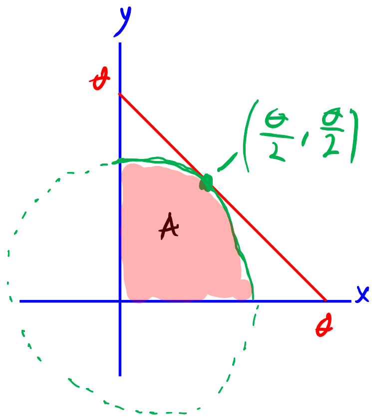
Finner her følgende område

A rådet av kvartsirkelen m radius $\frac{\theta}{\sqrt{2}}$

$$\frac{1}{4}\pi \frac{\theta^2}{\sqrt{2}^2} = \frac{\pi\theta^2}{8}$$

Dermed

$$P(X^2 + Y^2 \le \theta^2/2) = \iint_A \frac{2}{\theta^2}\, dx\, dy$$



(ligger på sirkelen) — $\left(\frac{\theta}{2}\right)^2 + \left(\frac{\theta}{2}\right)^2 = \frac{\theta^2}{2}$

(ligger på linjen) — $\frac{\theta}{2} + \frac{\theta}{2} = \theta$

$$= \frac{2}{a^2} \underbrace{\iint_A dx\,dy}_{\text{area of } A}$$

$$= \frac{2}{a^2} \frac{\pi a^2}{8} = \underline{\pi/4}$$