1. La C være kurven

$$\bar{r}(t) = t\cos(t)\,\bar{i} + t\sin(t)\,\bar{j}, \quad t \in 0, 2\pi$$

La $\bar{F}$ være vektorfeltet

$$\bar{F}(x,y) = -y\bar{i} + x\bar{j}$$

1 a) Regn ut $\int_C \bar{F} \cdot d\bar{r}$

$$\int_C \bar{F} \cdot d\bar{r} = \int_0^{2\pi} \bar{F}_{(\bar{r}(t))}\, \bar{r}'(t)\, dt$$

$$r'(t) = -t\sin(t)\,\bar{i} + t\cos(t)\,\bar{j}$$

$$F(r(t)) = -t\sin(t)\,\bar{i} + t\cos t\,\bar{j}$$

$$F(r(t))\,r'(t) = (-t\sin t,\, t\cos(t)) \cdot (-t\sin t,\, t\cos(t))$$

$$= t^2\sin^2(t) + t^2\cos^2(t)$$

$$= t^2$$

$$\int_0^{2\pi} F(r(t))\, r'(t)\, dt = \int_0^{2\pi} t^2\, dt = \left[\frac{t^3}{3}\right]_0^{2\pi} = \frac{(2\pi)^3}{3} = \frac{8\pi}{3}$$

1b) Regn ut arealet av området avgrenset av C og en rette linja fra $(2\pi, 0)$ til $(0,0)$

Skriver om $C: \bar{r}$ til polarkoordinater

$$\bar{r} = t\cos(t)\,\bar{i} + t\sin(t)\,\bar{j} \quad , \text{ ser at } \quad \theta = r = t$$

dermed har vi at

$$\int_0^{2\pi}\int_0^{\theta} 1\, r\, dr\, d\theta = \int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^{\theta} d\theta = \left[\frac{\theta^3}{6}\right]_0^{2\pi}$$

$$= \frac{(2\pi)^3}{6} = \frac{8\pi^3}{6} = \frac{4\pi^3}{3}$$

2 $$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{pmatrix}$$

a) For hvilke $\bar{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in \mathbb{R}^3$ har ligningen $A\bar{x} = B$

Entydig løsning

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\Rightarrow \begin{cases} x_1 + 2x_2 = b_1 & \mathrm{I} \\ 2x_1 + x_2 = b_2 & \mathrm{II} \\ x_1 = b_3 & \mathrm{III} \end{cases}$$

$\mathrm{III} : \mathrm{II}$ $\quad 2b_3 + x_2 = b_2$

$$x_2 = b_2 - 2b_3$$

$(\mathrm{III} : \mathrm{II})\ \mathrm{I}$ $\quad b_3 + 2(b_2 - 2b_3) = b_1$

$$b_3 + 2b_2 - 4b_3 = b_1$$

$$2b_2 - 3b_3 = b_1$$

er dette oppfylt, har vi entydig løsning

b) sett $\vec{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$f(x) = |A\bar{x} - b|^2$$

$$= \left\| \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\|^2$$

$$\begin{vmatrix} x + 2y - 0 \\ 2x + y - 0 \\ x - 1 \end{vmatrix}^2 = (x + 2y)^2 + (2x + y)^2 + (x - 1)^2$$

$$x^2 + 2yx + (2y)^2 + (2x)^2 + 2 \cdot 2xy + y^2 + x^2 - 2x \cdot 1 + 1$$

$$x^2 + 4x^2 + x^2 - 2x + 4y^2 + y^2 + 2yx + 4yx + 1$$

$$6x^2 - 2x + 5y^2 + 8xy + 1$$

$$\begin{cases} \dfrac{\partial f}{\partial x} = 12x - 2 + 8y = 0 & \text{I} \\[2ex] \dfrac{\partial f}{\partial y} = 10y + 8x = 0 & \text{II} \end{cases}$$

II $\quad \dfrac{8x}{8} = -\dfrac{10y}{8}$

$$x = -\frac{5}{4}y$$

II i I $\quad 12\left(\dfrac{-5y}{4}\right) - 2 + 8y =$

$$-15y + 8y = 2$$

$$-\frac{7y}{7} = \frac{2}{7}$$

$$y = -\frac{2}{7}$$

$$x = \frac{-5}{4} \cdot \frac{-2}{7}$$

$$= \frac{5}{14}$$

altså har f et globalt minimum på punkt $\left(\frac{5}{14}, -\frac{2}{7}\right)$

3 a) avgjør om denne rekka konvergerer eller divergerer

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n)}{n} = -1 \sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$$

Alternerende rekke

1. må gjelde at $\lim_{n\to\infty} |a_n| = 0$
2. $|a_{n+1}| < |a_n|$ for stor nok n

$$a_n = \frac{\ln n}{n}$$

1. $$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{\ln(n)}{n} \overset{\left[\frac{\infty}{\infty}\right]}{=} \frac{1/n}{1} = 0$$

2. $$f(x) = \frac{\ln x}{x}, \quad f'(x) = \frac{\ln(x)' \cdot x - \ln x \cdot x'}{x^2}$$

$$= \frac{1/x \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$$

$f'(x) < 0$ for $x \geq 3$

dermed konvergerer rekken

3b) Finn summen av rekken

$$\sum_{n=0}^{\infty} \frac{\pi^n}{(n+1)4^{n+1}}$$

, ser ut som $\ln(1-x)$

$$= -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{\pi^{n-1}}{n\, 4^n} = \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\pi}{4}\right)^n$$

setter $x = \frac{\pi}{4}$

$$\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{x^n}{n} = -\frac{1}{\pi}\ln(1-x) = -\frac{1}{\pi}\ln\left(1-\frac{\pi}{4}\right)$$

4) Finn det største volumet

Kassen har hjørner $(0,0,0), (x,0,0), (0,y,0), (0,0,z), (x,y,0)$
$(x,0,z), (0,y,z), (x,y,z)$

$f(x,y,z) = xyz$

Bibetingelse

$$g(x,y,z) = x^2 + y^2 + \frac{z^2}{4} - 1 = 0 \quad \text{I}$$

Lagrange

$$\nabla f = \lambda \nabla g$$

$$\nabla f : \begin{cases} \frac{\partial f}{\partial x} = yz \\ \frac{\partial f}{\partial y} = xz \\ \frac{\partial f}{\partial z} = xy \end{cases} \qquad \nabla g : \frac{\partial f}{\partial x} = 2x, \quad \frac{\partial f}{\partial y} = 2y, \quad \frac{\partial f}{\partial z} = \frac{z}{2}$$

$$\Rightarrow \quad yz = 2\lambda x \quad \text{II}$$
$$xz = 2\lambda y \quad \text{III}$$
$$xy = \lambda \frac{z}{2} \quad \text{VI}$$

$$\left.\begin{array}{l} \frac{yz}{x} = 2\lambda \\ \frac{xz}{y} = 2\lambda \end{array}\right\} \quad \left.\begin{array}{l} \frac{yz}{x} = \frac{xz}{y} \\ y^2 z = x^2 z \end{array}\right\} \quad y^2 = x^2$$

$$\frac{\text{II}}{\text{III}} \quad \frac{y\cancel{z}}{x\cancel{z}} = \frac{2\lambda x}{2\lambda y} \Rightarrow \frac{y}{x} = \frac{x}{y} \Rightarrow y^2 = x^2 \Rightarrow \underline{x = \pm y}$$

$$\frac{\text{VI}}{\text{II}} \quad \frac{x\cancel{y}}{\cancel{y}z} = \frac{\cancel{\lambda} z}{2(2\cancel{\lambda} x)} \Rightarrow \frac{x}{z} = \frac{z}{4x} \Rightarrow x^2 = \frac{z^2}{4} \Rightarrow \underline{x = \pm\frac{z}{2}}$$

$$\underline{y = \pm x = \pm\frac{z}{2}}$$

I $\quad y = \quad x^2 + y^2 + \frac{z^2}{4} - 1 = 0$

$$\left.\begin{array}{l} x = \frac{z}{2} \\ y = +\frac{z}{2} \end{array}\right\} : \left(\frac{z}{2}\right)^2 + \left(\frac{z}{2}\right)^2 + \frac{z^2}{4} - 1 = 0$$

$$\frac{z^2}{4} + \frac{z^2}{4} + \frac{z^2}{4} = 1$$

$$\frac{3z^2}{4} = 1 \quad / \cdot \frac{4}{3}$$

$$z^2 = \frac{3}{4}$$

$$z = \sqrt{\frac{3}{4}} \quad : \left\{\begin{array}{l} y = \dfrac{\left(\frac{\sqrt{3}}{2}\right)}{2} = \frac{\sqrt{3}}{4} \\ x = \frac{\sqrt{3}}{4} \end{array}\right.$$