1. La C være kurven

$$\bar{r}(t) = t\cos(t)\,\bar{i} + t\sin(t)\,\bar{j}, \quad t \in [0, 2\pi]$$

og la

$$\bar{F}(x,y) = -y\,\bar{i} + x\,\bar{j}$$

1a. Regn ut $\int_C \bar{F} \cdot d\bar{r}$

$$\int_C \bar{F} \cdot d\bar{r} = \int_0^{2\pi} \bar{F}(\bar{r}(t)) \cdot \bar{r}'(t)\,dt$$

$$\bar{F}(\bar{r}(t)) = -(t\sin(t))\,\bar{i} + t\cos(t)\,\bar{j}$$

$$r'(t) = 1 \cdot \cos t - t\sin(t)\,\bar{i} + 1 \cdot \sin t + t\cos(t)$$

$$= (\cos t - t\sin t)\,\bar{i} + (\sin t + t\cos t)\,\bar{j}$$

$$\bar{F}\,\bar{r}(t) = (-t\sin(t)\,\bar{i} + t\cos(t)\,\bar{j})$$

$$\cdot ((\cos t - t\sin t)\,\bar{i} + \sin t + t\cos t)\,\bar{j})$$

$$= \cancel{-t\sin(t)\cos(t)} + t^2\sin^2(t) + \cancel{t\cos(t)\sin(t)} + t^2\cos^2(t)$$

$$= + t^2\sin^2(t) + t^2\cos^2 t = t^2(1)$$

$$= t^2$$

$$\int_C \bar{F}(t)\cdot d\bar{r} = \int_0^{2\pi} t^2 = \left[\frac{t^3}{3}\right]_0^{2\pi} = \frac{(2\pi)^3}{3} = \underline{\underline{\frac{8}{3}\pi^3}}$$

1b) Regn ut området avgrenset av C og den rette linjen fra $(2\pi, 0)$ til $(0,0)$

$$\int_0^{2\pi}\int_0^{|r(t)|} \rho \, d\rho \, dt = \frac{1}{2}\int_0^{2\pi} t^2 \, dt = \frac{1}{2}\,\frac{8\pi^3}{3} = \underline{\underline{\frac{4}{3}\pi^3}}$$

2

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{pmatrix}$$ 3×2

a) For hvilke $\bar{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in \mathbb{R}^3$ har ligningen $\bar{A}x = \bar{b}$

$\bar{x} : 2\times 1$

entydig løsning $\bar{x} = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$

Se forelesning for bedre forklaring (1. etter 2018.05.14)

Radreduserer:

$$\begin{pmatrix} 1 & 2 & b_1 \\ 2 & 1 & b_2 \\ 1 & 0 & b_3 \end{pmatrix} \begin{matrix} \mathrm{II} - 2\mathrm{III} \\ \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 2 & b_1 \\ 0 & 1 & b_2 - 2b_3 \\ 1 & 0 & b_3 \end{pmatrix} \begin{matrix} \mathrm{III} - \mathrm{I} \\ \\ \\ \end{matrix} \sim$$

$$\begin{pmatrix} 1 & 2 & b_1 \\ 0 & 1 & b_2 - 2b_3 \\ 0 & -2 & b_3 - b_1 \end{pmatrix} \begin{matrix} \mathrm{III} + 2\mathrm{II} \\ \\ \\ \end{matrix} \sim \begin{pmatrix} 1 & 2 & b_1 \\ 0 & 1 & b_2 - 2b_3 \\ 0 & 0 & b_3 + 2b_2 - b_1 \end{pmatrix} \sim$$

$b_3 - b_1 + 2(b_2 - b_3)$
$= b_3 + 2b_2 - b_1$

dermed ser vi at vi har en entydig løsning hvis

Løsningsforslag sier

$$\begin{pmatrix} 1 & 2 & b_1 \\ 2 & 1 & b_2 \\ 1 & 0 & b_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & b_1 \\ 0 & 3 & 2b_1 - b_2 \\ 0 & 0 & b_1 - 2b_2 + 3b_3 \end{pmatrix}$$

$$b_1 - 2b_2 + 3b_3$$

$$b_1 + 2b_1 - b_2 + b_1 - 2b_2 + 3b_3$$

3. Avgjør om denne rekken konvergerer eller divergerer

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n)}{n}$$

$$(-1) \sum_{n=1}^{\infty} \frac{(-1)^{n} \ln(n)}{n}$$

Rekken konvergerer hvis " $\lim_{n\to\infty} \frac{\ln(n)}{n} \to 0$ "

$$\lim_{n\to\infty} \frac{\ln(n)}{n} \overset{L'H}{=} \frac{\left(\frac{1}{n}\right)}{\cancel{n}_{1}} = \frac{1}{n^2} \to 0$$

Dermed konvergerer rekken $\boxtimes$

3b) Finn summen av rekken

$$\sum_{n=0}^{\infty} \frac{\pi^n}{(n+1)4^{n+1}}$$