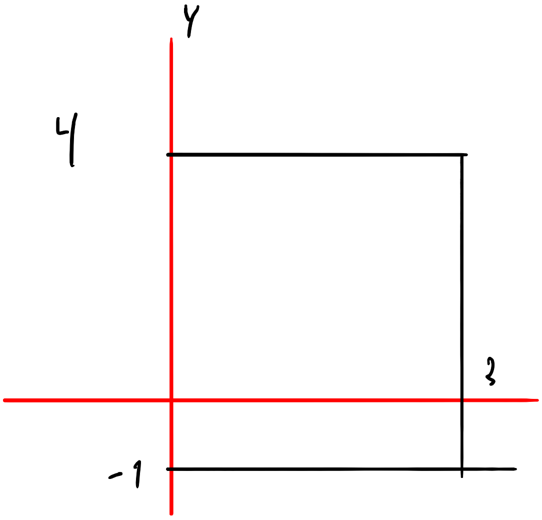
Eks. $\iint_R (4x^2y + 2xy)\,dx\,dy$

der R er gitt ved $x \in [0,3]$ $y \in [-1,4]$



$$\iint_R (4x^2y + 2xy\,dx\,dy$$

$$= \int_0^3 \left[\int_{-1}^4 (4x^2y + 2xy)\,dy \right] dx$$

$$= \int_0^3 \left[2x^2y^2 + xy^2 \right]_{y=-1}^{y=4} dx = \int_0^3 \left[32x^2 + 16x - (2x^2 + x) \right] dx$$

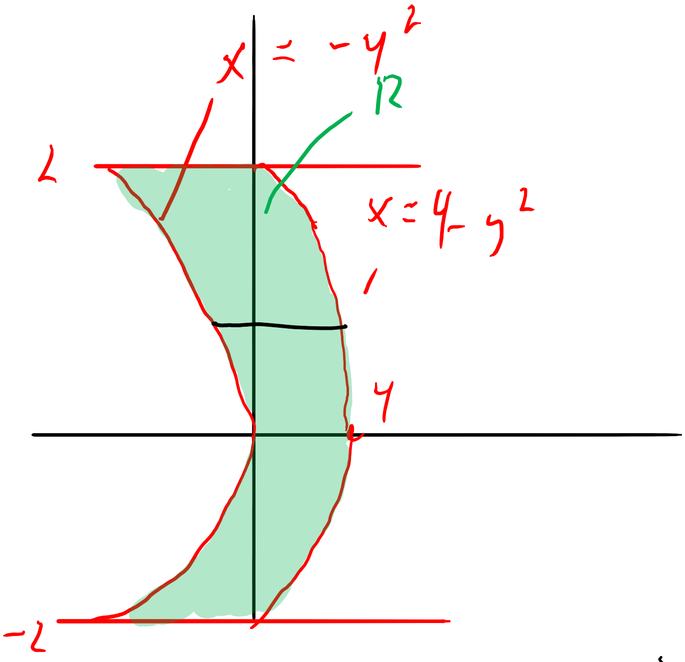
$$= \int_0^3 30x^2 + 15x\,dx = \left[10x^3 + \frac{15}{2}x^2 \right]_0^3 = \text{etc.}$$

?

eks 2. Finne $\iint_R 2xy^2\,dx\,dy$

der R er område i $\mathbb{R}^2$ begrenset av de fire kurvene $x = -y^2$, $x = 4-y^2$,
$y=2$, $y=-2$

Løsning



Beskrivelse av R

$y \in [-2, 2]$

$x \in [-y^2, 4-y^2]$

$$\iint_R 2xy^2\,dx\,dy = \int_{-2}^{2}\left[\int_{-y^2}^{4-y^2} 2xy^2\,dx\right]dy$$

$$= \int_{-2}^{2}\left[x^2y^2\right]_{x=-y^2}^{x=4-y^2} dy$$

$$= \int_{-2}^{2}\left[(4-y^2)^2y^2 - (-y^2)^2y^2\right]dy = \underline{\underline{etc.}}$$

oppg. 3

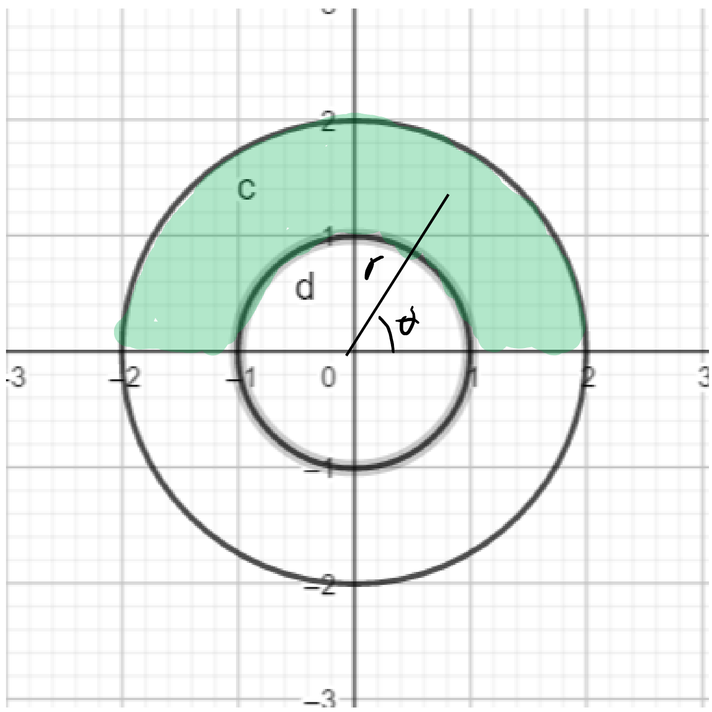
Finne $\iint_R 15x^2y \, dx\,dy$

der R er det lukkede området i $\mathbb{R}^2$ begrenset av sirkelen $x^2+y^2=4$, sirkelen $x^2+y^2=1$ og x-aksen, for $y \geq 0$

Løsning

Polarkoordinater

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$



Beskrivelse R:

$$\begin{cases} \theta \in [0, \pi] \\ r \in [1, 2] \end{cases}$$

Jakobideterminanten

$$J = \begin{vmatrix} \frac{dx}{dr} & \frac{dx}{d\theta} \\ \frac{dy}{dr} & \frac{dy}{d\theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$$= r\cos^2\theta \, r\sin^2\theta = r(\overbrace{\sin^2\theta + \cos^2\theta}^{1}) = 1$$

$$\iint_R 15x^2y\,dx\,dy = \int_0^{\pi}\left[\int_1^2 15(r\cos\theta)^2(r\sin\theta)\cdot|J|\,dr\right]d\theta$$

$$= \int_0^{\pi}\left[\int_1^2 15r^3\sin\theta\cos^2\theta\cdot r\,dr\right]d\theta$$

$$= \int_0^{\pi}\left[15\tfrac{1}{5}r^5\cdot\cos^2\theta\sin\theta\right]_{r=1}^{r=2}d\theta$$

$$= \int_0^{\pi}\left[96\cos^2\theta\sin\theta - 3\cos^2\theta\sin\theta\right]d\theta$$

$$u = \cos\theta$$

$$\frac{du}{d\theta} = -\sin\theta$$

$$du = -\sin\theta\,d\theta$$

$$= \int_0^{\pi} 93\cos^2\theta\sin\theta\,d\theta$$

$$= \left[-31\cos^3\theta\right]_0^{\pi} = 31-(-31) = \underline{\underline{62}}$$