8.2

$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

enhetsvektorene

$$\bar{i} = \cos\theta\,\bar{i}_r - \sin\theta\,\bar{i}_\theta$$
$$\bar{j} = \sin\theta\,\bar{i}_r + \cos\theta\,\bar{i}_\theta$$

Opg 1. a) $\beta(x,y) = xy \qquad \beta(r,\theta) = r^2\cos\theta\sin\theta$

b) Finn gradientvektoren $\nabla\beta$ i kartesiske og polare koordinater

$\nabla\beta(x,y)$:

$$\frac{d\beta(x,y)}{dx} = y$$

$$\frac{d\beta(x,y)}{dy} = x$$

$$\nabla\beta(x,y) = (y, x)$$

$\nabla\beta(r,\theta)$

$$\frac{d\beta(r,\theta)}{dr} = 2r\cos\theta\sin\theta$$

$$\frac{d\beta(r,\theta)}{d\theta} = r^2(\cos\theta'\sin\theta) + (\cos\theta\sin\theta')$$

$$= r^2(-\sin\theta\sin\theta) + (\cos\theta\cos\theta)$$

$$= r^2(-\sin^2\theta + \cos^2\theta$$

Gjør om $y\bar{i} + x\bar{j}$ til polare koordinater

$$r\sin\theta(\cos\theta\,\bar{i}_r - \sin\theta\,\bar{i}_\theta) + r\cos\theta(\sin\theta\,\bar{i}_r + \cos\theta\,\bar{i}_\theta$$

$$r\sin(\theta)\cos\theta\,\bar{i}_r - r\sin^2\theta\,\bar{i}_\theta + r\cos\theta\sin\theta\,\bar{i}_r + r\cos\theta\cos\theta\,\bar{i}_\theta$$

$$2r\sin\theta\cos\theta\,\bar{i}_r + (-r\sin^2\theta + r\cos^2)\bar{i}_\theta$$

c) Beregne divergensen til $\nabla\beta$ i begge koordinatsystem

Vis at de blir like

Kartesiske:

$$y\bar{i} + x\bar{j}$$

$$\nabla\cdot(\nabla\beta(x,y)) = \frac{d}{dx}y + \frac{d}{dy}x = 0$$

Polare koordinater

$$\nabla\beta(r,\theta) = 2r\cos\theta\sin\theta\,\bar{i}_r + r(-\sin^2\theta + \cos^2\theta)\,\bar{i}_\theta$$

$$\nabla\cdot\nabla\beta(r,\theta) = \begin{cases} \frac{d}{dr}2r\cos\theta\sin\theta = 2\cos\theta\sin\theta \\ + \\ \frac{d}{d\theta}-r\sin^2\theta + r\cos^2\theta = -4r\sin(\theta)\cos(\theta) \end{cases}$$

$$\neq 0$$