Konte MAT 1110 2017

La A være matrisen

$$A = \frac{1}{3}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

a) Finn egenverdiene og egenvektorene

$$\begin{vmatrix} \frac{1}{3} - \lambda & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} - \lambda \end{vmatrix} = \left(\tfrac{1}{3} - \lambda\right)\left(\tfrac{1}{3} - \lambda\right) - \tfrac{2}{3} \cdot \tfrac{2}{3}$$

$$\frac{1}{9} - \frac{1}{3}\lambda - \frac{1}{3}\lambda + \lambda^2 - \frac{4}{9}$$

$$\lambda^2 - \frac{2}{3}\lambda - \frac{1}{3} = 0$$

$$\frac{1}{3}(1 - \lambda)\left(-\tfrac{1}{3} - \lambda\right)$$

egenverdiene er 1 og $-1/3$

Egenvektoren finner man ved

$$A\bar{x} = \lambda\bar{x}$$

$$\frac{1}{3}\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = 1\begin{pmatrix}x\\ y\end{pmatrix}$$

I $\frac{1}{3}x + \frac{2}{3}y = x$

II $\frac{2}{3}x + \frac{1}{3}y = y$

$x + y = x + y \Rightarrow x = y$

1 b) Definer en følge $\{x_n, y_n\}_{n=1}^{\infty}$ ved

$$\begin{pmatrix}x_0\\ y_0\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix}, \begin{pmatrix}x_{n+1}\\ y_{n+1}\end{pmatrix} = A\begin{pmatrix}x_n\\ y_n\end{pmatrix}, n \geq 0$$

Finn grensen

$$\lim_{n\to\infty}\begin{pmatrix}x_n\\ y_n\end{pmatrix}$$

Ved $n = 1$

$$\frac{1}{3}\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}\frac{1}{3}\cdot 1 + \frac{2}{3}\cdot 0\\ \frac{2}{3}\cdot 1 + \frac{1}{3}\cdot 0\end{pmatrix} = \begin{pmatrix}\frac{1}{3}\\ \frac{2}{3}\end{pmatrix}$$

$n = 2$

$$\begin{pmatrix}\frac{1}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{2}{3}\\ \frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3}\end{pmatrix} = \begin{pmatrix}\frac{1}{9} + \frac{4}{9}\\ \frac{2}{9} + \frac{2}{9}\end{pmatrix} = \begin{matrix}\frac{5}{9}\\ \frac{4}{9}\end{matrix}$$

$$\frac{1}{3}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} n_x \\ n_y \end{pmatrix} = \begin{pmatrix} \frac{1}{3}\cdot n_x + \frac{2}{3}\cdot n_y \\ \frac{2}{3}\cdot n_x + \frac{1}{3} n_y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} n_x + 2 n_y \\ 2 n_x + 1 n_y \end{pmatrix}$$

2 a) Fin konvergensradien till rekka

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}$$

Konvergensradie

$$\lim_{x \to \infty} a_{n+1}/a_n < 1$$

Finner

$$\lim_{n\to\infty} \frac{(-1)^{n+1} x^{2(n+1)}}{(n+1)!} \cdot \frac{n!}{(-1)^n x^{2n}}$$

Bryter opp stykket

$$\frac{(-1)^{(n+1)}}{(-1^n)} = (-1)^1 \quad , \quad \frac{x^{2(n+1)}}{x^{2n}} = x^{2(n+1)-2n} = x^2$$

$$\cancel{2n} + 2 - \cancel{2n}$$

$$\frac{n!}{(n+1)!} = \frac{1}{n}$$

Setter stykket sammen igjen

$$\lim_{n\to\infty} \frac{(-1)\cdot x^2}{n} = 0$$, dvs blir samme 0 uavhengig av x

så konvergensradien er ∞

2 c) La

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{2n+1} x^{2n}$$

Finn $f(1/2)$

$$\sum_{n=0}^{\infty} \frac{1}{2n+1} \frac{1}{2}^{2n}$$

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$f(x,y,z) = xyz^2$ Lagranges multiplikatormetode

$g(x,y) = x^2 + y^2 + z^2 = 1$ I

$$\nabla f = \lambda \nabla g(x,y,z)$$

Si vi om

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= \lambda \frac{\partial g}{\partial x} \\ \frac{\partial f}{\partial y} &= \lambda \frac{\partial g}{\partial y} \\ \frac{\partial f}{\partial z} &= \lambda \frac{\partial g}{\partial z} \end{aligned}\right\} \quad \begin{aligned} yz^2 &= 2x\lambda \quad \text{II} \\ xz^2 &= 2y\lambda \quad \text{III} \\ 2xy &= 2z\lambda \quad \text{IV} \end{aligned}$$

$$\frac{\text{II}}{\text{III}} \quad \frac{y\cancel{z^2}}{x\cancel{z^2}} = \frac{\cancel{2}x\cancel{\lambda}}{\cancel{2}y\cancel{\lambda}} \Rightarrow \frac{y}{x} = \frac{x}{y} \Rightarrow y^2 = x^2$$

$$\frac{\text{III}}{\text{IV}} \quad \frac{\cancel{x}z^{\cancel{2}}}{2\cancel{x}y} = \frac{\cancel{2}y\cancel{\lambda}}{\cancel{2}\cancel{z}\cancel{\lambda}} \Rightarrow \frac{z}{2y} = y \Rightarrow z^2 = 2y^2$$

innsatt i I gir (I: $x^2 + y^2 + z^2 = 1$)

$$y^2 + y^2 + 2y^2 = 1$$

$$4y^2 = 1 \Rightarrow y = \pm\sqrt{\tfrac{1}{4}}$$

det gir $x = \pm\frac{1}{2}$, $y = \pm\frac{1}{2}$, $z = \pm\frac{1}{\sqrt{2}}$

ser dermed att

$f(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}) = \frac{1}{8}$ er den største verdien

4) La F være vektorfeltet

$$F(x,y) = \frac{y}{x^2+y^2+1}\,\bar{i} - \frac{x}{x^2+y^2+1}\,\bar{j}$$