1)

La C være kurven

$$\bar{r} = t\cos(t)\,\bar{i} + t\sin(t)\,\bar{j}\ ,\quad t \in [0, 2\pi$$

og la F være vektorfeltet

$$F(x,y) = -y\bar{i} + x\bar{j}$$

a) Regn ut

$$\int_C F \cdot dr$$

$$\int_0^{2\pi} \bar{F}(\bar{r}(t)) \cdot \bar{r}'(t)\, dt$$

Finner $\bar{F}(\bar{r}(t))$

$$\bar{F}(\bar{r}(t)) = (-t\sin(t),\ t\cos(t))$$

Finner $\bar{r}'(t)$

$$(t\cos(t),\ t\sin(t))' = (\cos(t) - t\sin(t),\ \sin(t) + t\cos(t))$$

Finner dermed $\bar{F}(\bar{r}(t)) \cdot \bar{r}'(t)$

$$(-t\sin(t), t\cos t) \cdot (\cos(t) - t\sin(t),\ \sin(t) + t\cos(t))$$

$$= -t\sin t\cos t + t^2\sin(t)\sin t + t\cos t\sin t + t\cos(t)\,t\cos t$$

$$= -t\,\cancel{\sin(t)\cos(t)} + t^2\sin^2(t) + \cancel{t\cos t\sin t} + t^2\cos^2(t)$$

$$= t^2(\sin^2(t) + \cos^2(t))$$

$$= t^2$$

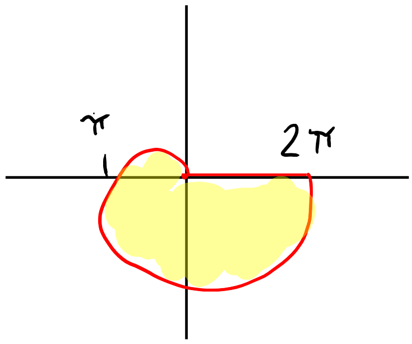
$$= \int_0^{2\pi} t^2\, dt = \left[\frac{t^3}{3}\right]_0^{2\pi} = \frac{2^3\pi^3}{3} = \frac{8\pi^3}{3}$$

2b Regn ut arealet av området avgrenset av C og den rette linja $(2\pi, 0)$ til $(0,0)$

Skriver da om fra kulekoordinater

$r = \theta = t$

$t \in [0, 2\pi]$



Beskrivelse av C: polarkoordinater

$\theta = [0, 2\pi]$

$r = [0, \theta]$

$$\int_0^{2\pi}\int_0^{\theta} 1 \cdot r \, dr \, d\theta = \int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^{\theta} d\theta = \frac{1}{2}\left[\frac{\theta^3}{3}\right]_0^{2\pi} = \frac{1}{6} 2^3 \pi^3 = \frac{4\pi^3}{3}$$

2) La A være matrisen

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & d \end{pmatrix}$$

a) For hvilke $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in \mathbb{R}^3$ har ligningen $Ax = b$ entydig løsning

$x = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$?

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Skriver ut

$$x + 2y = b_1 \quad \mathrm{I}$$
$$2x + y = b_2 \quad \mathrm{II}$$
$$x = b_3 \quad \mathrm{III}$$

III i I $\quad b_3 + 2y = b_1 \quad | - b_3$

$$2y = b_1 - b_3$$
$$y = \frac{b_1 - b_3}{2}$$

(III i I) i II $\quad 2b_3 + \frac{b_1 - b_3}{2} = 2b_2$

$$4b_3 + b_1 - b_3 = 2b_2 \quad | - b_2$$
$$4b_3 + b_1 - b_3 - b_2 = 0$$
$$3b_3 + b_1 - 2b_2 = 0$$

~ entydig så länge denna ligningen är uppfyllt

2b) sett $b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \in \mathbb{R}^3$

Vi ønsker å finne $x \in \mathbb{R}^2$ slik at vi er "nærmest mulig en løsning. sett

$$f(x) = |Ax - b|^2$$

$$f(x) = \left\| \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\|^2$$

$$= \left\| \begin{pmatrix} x + 2y \\ 2x + y \\ x \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\|^2$$

$$= \left\| \begin{pmatrix} x + 2y \\ 2x + y \\ x - 1 \end{pmatrix} \right\|^2$$

$$= (x + 2y)^2 + (2x + y)^2 + (x - 1)^2$$

$$= \cancel{x^2} + 4xy + \cancel{(2y)^2} + \cancel{(2x)^2} + 4xy + \cancel{y^2} + \cancel{x^2} - 2x + 1$$

$$= x^2 + 4x^2 + x^2 + 4y^2 + y^2 + 8xy - 2x + 1$$

$$f(x) = 6x^2 + 5y^2 + 8xy - 2x + 1$$

$$\begin{array}{r|l} 10 & 2 \\ 5 & 5 \end{array} \qquad \begin{array}{r|l} 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array}$$

$$\frac{\partial f}{\partial x} = 12x + 8y - 2 = 0, \quad \frac{\partial f}{\partial y} = 10y + 6x = 0$$

$$\frac{8x}{8} = \frac{-10y}{8}$$

$$x = -\frac{5}{4}y$$

$$-\frac{12 \cdot 5}{4} + 8y = 2, \quad y = -\frac{2}{7}$$

$$x = -\frac{5}{4} \cdot \left(-\frac{2}{7}\right) = \frac{10}{28} = \frac{5}{14}$$

Minimalpunkt i $\left(\frac{5}{14}, -\frac{2}{7}\right)$

3a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n)}{n}$ Konvergent

Alternerende rekke

Sjekker om $|a_n| = 0$

$$\lim_{n \to \infty} \left| \frac{\ln(n)}{n} \right| \overset{\left[\frac{\infty}{\infty}\right]}{=} \lim_{n \to \infty} \frac{\frac{1}{n}}{1} = 0$$

Dermed konvergerer rekken

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