5) $x^2 y' y^2 = 2x \quad | : x^2$

$y' y^2 = \frac{2}{x}$

$y^2 \frac{dy}{dx} = \frac{2}{x}$

$\int y^2 \, dy = 2 \int \frac{1}{x} dx$

$\frac{1}{3} y^3 = 2 \ln x + C \quad / \cdot 3$

$y^3 = 6 \ln x + D \quad , \quad D = 3C$

$y = (6 \ln(x) + D)^{1/3}$

8) $f'a \approx f(a+h) - 2f$

$f(x) = x^3 \ , \ a = 1$

$f'1 = \frac{(1+h)^3 - 2 \cdot 1^3 + (1+h)^3}{h}$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\frac{1 + \cancel{3h} + 3h^2 + h^3 - 2 - \cancel{3h} + 3h^2 - h^3}{h^2}$

alt "a"

a) trapesmetoden med 4 intervaller

$h = \frac{b-a}{n} \Rightarrow \frac{1}{2}$ $\qquad [\overset{a}{0}, \overset{b}{2}]$

$f(x) = x^2$

$f(0) = 0 \quad , f(2) = 4$

Summen: trapeset

$$\sum_{i=1}^{3} f\left(\frac{i}{2}\right) = f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right)$$

$h = \frac{1}{2}$

$$= \frac{1}{2}^2 + 1^2 + \left(\frac{3}{2}\right)^2$$

$$= \frac{1}{4} + 1 + \frac{9}{4} = \frac{7}{2}, \quad \frac{14}{4}$$

$$I = h \cdot \left(f(a) + 2\sum_{i=1}^{n-1} f(a + ih) + f(b)\right)$$

$$= \frac{1}{2}$$

10 Diff lig.

$$x'' + \sin(x^2 + x') = t$$

$$x(0) = 0, \quad x'(0) = 1$$

$$x_1 = x, \quad x_2 = x'$$

$$x_1(0) = 0, \quad x_2(0) = 1$$

$$x_2 = x_1'$$

$$x_2' = t - \sin(x_1^2 + x_2)$$

Del 2

1 a) $f(x) = x e^x$, $f^{(k)}(x) = (x + k)e^x$ $k \geq 0$

P_0: $f^{(0)}(x) = x e^x$

sidan $f^0(x) = f(x)$

antar at P_k stemmer

P_{k+1}: $f^{(k+1)}(x) = (x + 1 + k)e^x$

$$f^{(k+1)}(x) = (f^{k}(x))'$$

$= ((x + k)e^x)'$ Produkt regelen for derivasjon

$u = x + k$, $u' = 1$

$v = e^x$, $v' = e^x$

$1 \cdot e^x + (x + k)e^x$

$= (1 + (x + k))e^x = (x + k + 1)e^x$

v) $T_n f(x)$ $a = 0$

og finne N slik at restleddet er mindre enn 10^{-3}

$$T_n f(x) = \sum_{k=0}^{n} \frac{f^{(a)}(0)}{k!} x^n$$

fra a er $f^{(n)}(0) = (0 + k) e^0 = k$

$$T_n f(x) = \sum \frac{k}{k!} x^n = \sum_{k=1}^{n} \frac{k}{k!} x^k = \sum \frac{1}{(k-1)!} x^k$$

$$R_n f(x) = \frac{(c + n + 1) e^c}{(n+1)!}, \text{ der } c \in [0, 1]$$

Finne en N slik at $|R_n f(x)| < 10^{-3}$

$f^{(k+1)}(x)$ maksimum på $[0, 1]$

siden $f^{(k+1)}(x)$ er strengt voksende

Vil ;

$$f^{(k+1)}(x) \leq \frac{1 + k + 1}{k+1} < \frac{3(k+2)}{k+1!}$$

$$R_n(x) < \frac{3(N+2)}{N+1} < 10^{-3}$$

$$3000 < \frac{(N+1)!}{N+2}$$

N = 7 er første N som oppfyller kravet