Homework 2

DDL: March 2

孙济宸　520030910016

1

1.1 a

$$H(X) - H(X|Y) = \mathbb{E} - \log p(X) + \mathbb{E}\log p(X|Y)$$
$$= \mathbb{E} - \log p(X) + \mathbb{E}\log \frac{p(X,Y)}{p(Y)} = \mathbb{E}\log \frac{p(X,Y)}{p(X)p(Y)} = I(X;Y)$$

1.2 b

$$H(X) + H(Y) - H(X,Y) = \mathbb{E} - \log p(X) + +\mathbb{E} - \log p(Y)\mathbb{E}\log p(X,Y)$$
$$= \mathbb{E}\log \frac{p(X,Y)}{p(X)p(Y)} = I(X;Y)$$

1.3 c

$$H(X|Z) + H(Y|Z) - H(X,Y|Z) = \mathbb{E} - \log p(X|Z) + +\mathbb{E} - \log p(Y|Z)\mathbb{E}\log p(X,Y|Z)$$
$$= \mathbb{E}\log \frac{p(X,Y|Z)}{p(X|Z)p(Y|Z)} = I(X;Y|Z)$$

2

2.1 a

if:

$$p(x) = q(x)$$
$$D(p||q) = \sum_{x\in X} p(x)\log \frac{p(x)}{q(x)} = 0$$

only if:

$$D(p||q) = -\sum_{x\in X} p(x)\log \frac{q(x)}{p(x)} \geq \sum_{x\in X} p(x)(1 - \frac{q(x)}{p(x)}) = \sum_{x\in X} p(x) - \sum_{x\in X} q(x) = 0$$

with equality only if $p = q$ for all $x \in X$.

2.2 b

if:

X and Y are independent,

$$I(X;Y) = H(X) - H(X|Y) = H(X) - H(X) = 0$$

only if:

$$I(X;Y) = D(p(x,y)||p(x)p(y)) = 0$$

From (a), we know

$$p(x,y) = p(x)p(y)$$

. i.e. X and Y are independent

2.3 c

if:

$$p(y|x) = q(y|x), \forall x, y s.t. p(x) > 0$$

$$D(p(y|x)||q(y|x)) = \sum_{x \in X} p(x) \sum_{y \in Y} p(y|x) \log \frac{p(y|x)}{q(y|x)} = 0$$

only if:

$$D(p(y|x)||q(y|x)) = -\sum_{x \in X} p(x) \sum_{y \in Y} p(y|x) \log \frac{q(y|x)}{p(y|x)}$$

$$\geq \sum_{x \in X} p(x) \sum_{y \in Y} p(y|x)(1 - \frac{q(y|x)}{p(y|x)}) = \sum_{x \in X} p(x)[\sum_{y \in Y} p(y|x) - \sum_{y \in Y} q(y|x)] = 0$$

with equality only if $p(y|x) = q(y|x)$ for all $p(x) > 0$.

2.4 d

if:

X and Y are conditionally independent given Z,

$$I(X;Y|Z) = H(X|Z) - H(X|Y,Z) = H(X|Z) - H(X|Z) = 0$$

only if:

$$I(X;Y|Z) = D(p(x,y|z)||p(x|z)p(y|z)) = 0$$

$$p(x,y|z) = p(x|z)p(y|z)$$

. i.e. X and Y are conditionally independent given Z