Assignment 1

EC5.201 Signal Processing

Fourier series & Discrete-time systems

Deadline: (Theory) 17th Aug '25, 11.55PM IST

Instructions:

- All the questions are compulsory
- The questions contain both theory (analytical) and practical (MATLAB coding) parts
- The submission format is as follows:
  - **Theory (on Moodle)** – PDF file containing the handwritten theory assignment solutions
  - **Lab (on GitHub)**
    - **Code** – folder containing all the codes
    - **Images** – folder containing all the .png images
    - **Report** – PDF/text file containing observations for the MATLAB section

    The naming convention for the code and image files is q<qn no.> _ <sub-part no.>.
- **Late submission:** For the theory component, 10% penalty per day will be applicable (accepted up to at most 3 days after deadline). No late submissions will be accepted for the lab component.

Question 1: Fourier Series

Compute the complex Fourier series coefficients of the following signals (you can use any of the properties of Fourier series to solve this problem).

1. $x(t) = 1 + \sin(2\pi f_0 t) + 2\cos(2\pi f_0 t) + \cos(4\pi f_0 t + \frac{\pi}{4})$
2. $x(t) = \mathrm{rect}(\frac{t}{\tau}) = \begin{cases} 1 & |t| < \frac{\tau}{2} \\ 0 & \frac{\tau}{2} < |t| < \frac{T}{2} \end{cases}$ where T is the period of the signal and $\tau \in [0, T)$
3. $x(t) = \Delta(\frac{t}{\tau}) = \begin{cases} 1 + \frac{2}{\tau}t & -\frac{\tau}{2} < t < 0 \\ 1 - \frac{2}{\tau}t & 0 < t < \frac{\tau}{2} \\ 0 & \frac{\tau}{2} < |t| < \frac{T}{2} \end{cases}$ with T and τ as in part (b). (Observe that $\Delta(\frac{t}{\tau})$ is the result of the convolution of $\mathrm{rect}\frac{t}{\tau}$ with itself. What can you say about the relationship between their Fourier series coefficients?)
4. $x(t) = \sum_{l=-\infty}^{\infty} \delta(t - lT_s)$

MATLAB:

a) Define a function *fourierCoeff* as follows,

```
function X = fourierCoeff(t, xt, T, t1, t2, N)
    % function to compute Fourier series coefficients
    %
    % t     - symbolic variable for time
    % xt    - continuous-time signal
    % T     - period of the signal
    % t1,t2 - left and right limits where the expression
    %          xt is valid (zero otherwise) for integration
    % N     - coefficients will be computed for k = -N:N
    %
    % X     - 2*N+1 length Fourier series coefficients vector

    %% code goes here
end
```

This function computes the Fourier series coefficients of the given signal. Use the commands ‘sym’ and ‘int’ for this computation. To refer to any documentation, in the command prompt you can type ‘doc command_name’, for example, ‘doc sym’.

Make a 2×2 plot titled *Time-domain* displaying the above four signals. Make another 2×2 plot titled *Fourier-domain* displaying the Fourier series coefficients of the 4 above signals computed by calling *fourierCoeff*.

b) Define a function *partialFourierSum* that performs a finite reconstruction of the signal from the Fourier coefficients.

```
function x_hat = partialFourierSum(A, T, time_grid)
    % performs partial Fourier reconstruction
    %
    % A         - 2N+1 vector of Fourier coefficients
    % T         - period of signal
    % time_grid - time vector (for reconstructed signal; not syms)
    %
    % x_hat     - reconstructed signal (vector)

    %% code goes here
end
```

Make a 2×2 plot where in each tile you plot both the original signal and the reconstructed signal.

c) Write a script that computes the Fourier series coefficients and reconstructs the signal for $N = 1 : 100$. Run this script for $x(t) = \mathrm{rect}(\frac{t}{\tau})$. Now compute the mean absolute error (MAE) of reconstruction, given by,

$$e_{MAE} = \frac{1}{T} \int_{<T>} |x(t) - \hat{x}(t)| dt$$

where $\hat{x}(t)$ is the reconstructed signal.

Plot this error as function of $N = 1 : 100$.

Question 2: Properties of LTI systems

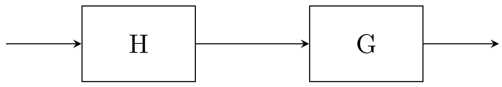
1. Consider the system:

$$S : y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1]$$

   Assume that the system S is causal and is initially relaxed, i.e., $y[-1] = 0$. Sketch the impulse response of S, i.e., the output of S when the input is the Kronecker delta $\delta[n]$.

2. For an LTI system, show that its step response is equal to the integral of the impulse response.

3. Consider a cascade of two discrete-time systems H and G as shown.



   (a) If H and G are both LTI causal systems, prove that the overall system is causal.
   (b) If H and G are both stable systems, show that the overall system is stable.

4. Take the first-order difference equation $y[n] - \xi y[n-1] = x[n]$, $\xi > 0$.

   (a) What is the condition on ξ such that the system is stable.

   Now assume the system is stable (i.e., the condition on ξ found above is satisfied.)
   (b) Find the impulse response of the system.
   (c) Is the system (i) memoryless, and (ii) causal?

Question 3: Discrete-time convolution

The output of a continuous-time LTI system given input $x(t)$ is the convolution integral,

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \tag{1}$$

with $h(t)$ being the impulse response of the system. Similarly, for discrete-time systems the output for input $x[n]$ is given by the convolution sum,

$$y[n] = (h * x)[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]. \tag{2}$$

Compute the following convolution sums, $z[n] = (x * y)[n]$, and also plot the signals $x[n]$, $y[n]$ and $z[n]$.

1. $x[n] = 2\delta[n+10] + 2\delta[n-10], \quad y[n] = 3\delta[n+5] + 2\delta[n-5]$
2. $x[n] = [-1]^n, \quad y[n] = \delta[n] + \delta[n-1]$
3. $x[n] = 4, \quad y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$
4. $x[n] = u[n] - u[n-3], \quad y[n] = \begin{cases} 0 & n < 0 \\ n & n = 0, 1, 2 \\ 1 & n \geq 3 \end{cases}$

MATLAB:
Define a function *disc_conv* that computes the convolution sum of two given signals.

```
function [nz, z] = disc_conv(nx, x, ny, y)
    % computes convolution sum of x[n] and y[n]
    %
    % nx    - n1 length discrete-time vector
    % x     - n1 length input signal
    % ny    - n2 length discrete-time vector
    % y     - n2 length input signal
    %
    % nz   - n1+n2-1 length discrete-time vector
    % z     - n1+n2-1 length output

    %% code goes here
end
```

Write a script to call this function for the 4 above signals. Make 3×1 plots for each part and plot $x[n]$, $y[n]$ and $z[n]$. For infinite duration signals, you can truncate them to some finite length.

Question 4: LTI system

A discrete-time LTI system has an impulse response given by $h[n] = u[n] - u[n-3]$. A signal $x[n] = \sin\left(\frac{\pi}{2}n\right)u[n]$ is given as input to this system.

1. Sketch the input signal for $n = -2$ to 12.
2. Using time domain analysis, find the output of this system and sketch it for $n = -2$ to 12.
3. If the input is instead $\sin\left(\frac{\pi}{2}n\right)$, find the simplified form of the output signal.

Question 5: Eigensignals

We have seen that complex exponential signals act as eigensignals for any LTI system. What are eigensignals?

For each of the following systems, give an example of non-complex exponential signals which act as eigensigals.

1. $h[n] = \delta[n-4]$
2. $h[n] = \delta[n-4] + \delta[n-2]$