Assignment 2

EC5.201 Signal Processing

The Discrete-Time Fourier Transform & LTI systems
Deadline: (Theory) 27th Aug '25, 11.55PM IST

Instructions:

- All the questions are compulsory
- The questions contain both theory (analytical) and practical (MATLAB coding) parts
- The submission format is as follows:
  - **Theory (on Moodle)** – PDF file containing the handwritten theory assignment solutions
  - **Lab (on GitHub)**
    - **Code** – folder containing all the codes
    - **Images** – folder containing all the .png images
    - **Report** – PDF/text file containing observations for the MATLAB section

  The naming convention for the code and image files is q<qn no.> _ <sub-part no.>.
- **Late submission:** For the theory component, a 10% penalty per day will be applicable (accepted up to at most 3 days after the deadline). No late submissions will be accepted for the lab component.

Question 1: DTFT

Compute the discrete-time Fourier transform of the following signals.

(a) $x[n] = u[n-2] - u[n-6]$

(b) $x[n] = (\frac{1}{3})^{|n|}u[-n-2]$

(c) $x[n] = \begin{cases} n & -3 \leq n \leq 3 \\ 0 & \text{else} \end{cases}$

(d) $x[n] = \frac{\sin(\frac{\pi n}{5})}{\pi n}\cos(\frac{7\pi}{2}n)$

Sketch the time-domain, magnitude spectrum and phase spectrum plots in each case.

MATLAB:

Define a function *dtft* as follows,

```
function X = dtft(n, x, f)
    % evaluates the discrete-time Fourier transform of given signal
    %
    % n - vector of time indices where the signal is defined
```

```
    % x - discrete-time signal vector
    % f - vector of frequency values where DTFT is computed
    %
    % X - complex vector of Fourier domain values

    %% code goes here
end
```

The vector f is supposedly a continuous-frequency vector, i.e., it is supposed to contain a continuous set of frequency values. Define it accordingly.

Write a script to compute the DTFT of signals $a-d$ given above. Make a 2×2 plot showing the time-domain signals. Make a 2×2 plot for the magnitude spectra and another 2×2 plot for the phase spectra. Verify that all your plots are consistent with those obtained analytically (note any inconsistencies or the lack thereof in your report).

Question 2: Inverse DTFT

The following are the Fourier transforms of discrete-time signals. Determine the signal corresponding to each transform.

(a) $X(e^{j\omega}) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-j3\omega} + e^{-j10\omega}$ (Do NOT use the inverse DTFT formula for this one!!)

(b) $X(e^{j\omega}) = \cos^2 \omega + \sin^2 3\omega$

(c) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(\omega - \frac{\pi}{2}k)$

(d) $X(e^{j\omega}) = \frac{1-\frac{1}{3}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-j2\omega}}$ (Do NOT use the inverse DTFT formula for this one!!)

MATLAB:
The formula for the inverse DTFT is as follows,

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} \, \mathrm{d}\omega.$$

Write a MATLAB script that uses this formula to compute the inverse DTFT of a given frequency domain expression.

Then use this to find the time-domain signals corresponding to the DTFT expressions given above. Make a 2×2 plot of the resultant time-domain signals. Also make a 2×2 plot of the time-domain expressions you obtained analytically. Compare the two sets of plots and note down any differences in your report.

Question 3: Properties of DTFT

1. Prove the multiplication (in time) property of DTFT.

2. **Autocorrelation and Power Spectral Density:** The cross-correlation of two time-domain signals is given as follows,

$$R_{x_1 x_2}[n] = \sum_{k=-\infty}^{\infty} x_1[k] x_2[k-n]$$

(You might notice this is very similar in form to the convolution operation!!)

When $x_1[n] = x_2[n] = x[n]$, R_{xx} is called the autocorrelation function of $x[n]$.

The power spectral density (PSD) of a signal is defined as,

$$S_x(\omega) = |X(\omega)|^2$$

where $X(\omega)$ is the DTFT of $x[n]$.

This function tells you how much power each frequency component contributes to the signal. This interpretation of the PSD justifies Parseval's theorem, according to which the total energy of the signal in the time domain and frequency domain representations are equal.

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, \mathrm{d}\omega$$

(a) Prove the Parseval's relation.

(b) Prove that the autocorrelation and the power spectral density of a given signal are Fourier transform pairs, i.e.,

$$\mathrm{DTFT}(R_{xx}[n]) = S_x(\omega)$$

MATLAB:
Write a script to verify the above theorem for any one signal from question 1.

Procedure:

- Use the *xcorr* function in MATLAB to compute the autocorrelation of the signal.
- Find the PSD $S_x(\omega)$ of the signal by computing the DTFT of the autocorrelation function $R_{xx}[n]$
- Also find the PSD by finding the DTFT of the signal and then using that to find $S_x(\omega) = |X(\omega)|^2$.
- Plot the PSD computed using each method in a 2×1 plot and verify visually that they are the same (report your observations).

Mention the signal(s) that you are using to demonstrate this theorem in your report.

3. **Symmetry property:** For real signals, the DTFT magnitude plot is symmetric about the y-axis, i.e., $X(\omega)$ is an even function and the phase plot is odd. Prove both these facts. Mathematically, prove that,

$$|X(\omega)| = |X(-\omega)|,$$
$$\angle X(\omega) = -\angle X(-\omega)$$

Question 4: Applications

1. Consider the signal $x[n] = \{-1, 2, \underset{\uparrow}{-3}, 2, -1\}$ (the arrow indicates the $n = 0$ value). Compute the following quantities without explicitly computing $X(\omega)$.

(a) $X(0)$

(b) $\angle X(\omega)$

(c) $\int_{-\pi}^{\pi} X(\omega) \, \mathrm{d}\omega$

(d) $X(\pi)$

(e) $\int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega$

2. The center of gravity of a signal $x[n]$ is defined as

$$c = \frac{\sum_{n=-\infty}^{\infty} n x[n]}{\sum_{n=-\infty}^{\infty} x[n]}$$

and provides a measure of the signal's time delay.

(a) Express c in terms of $X(\omega)$.

(b) Compute c for the signal $x[n]$ whose Fourier transform is shown in Fig. 1.

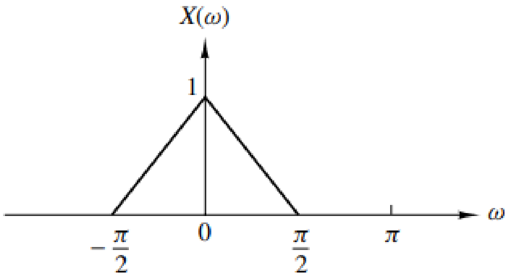

Figure 1: DTFT plot

3. From a discrete-time signal $x[n]$ with Fourier transform $X(\omega)$, shown in Fig. 2, determine and sketch the Fourier transform of the following signals:

(a) $y_1[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

(b) $y_2[n] = x[2n]$

(c) $y_3[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

Note that $y_1[n] = x[n]s[n]$, where $s[n] = \{\ldots, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1, \ldots\}$.

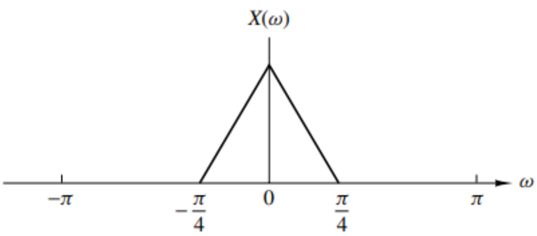

Figure 2: DTFT plot

Question 5: Discrete LTI systems

MATLAB:
In this question, you will implement a discrete-time filter (just a fancy name for a system!!) by using *conv*, the in-built convolution function in MATLAB.

The filter you shall be implementing is the order-M moving average filter, with input-output relation given by,

$$y[n] = \frac{1}{M}\sum_{m=0}^{M-1} x[n-m]$$

(a) What is the impulse response of this system?

(b) Write a MATLAB script to generate the sine wave $s[n] = 5\sin(\omega_0 n)$ and its noisy version $x[n] = s[n] + w[n]$, where $v[n]$ is Gaussian noise generated using *randn*. In the first tile of a 2×2 plot, plot $s[n]$ and $x[n]$ on top of each other.

(c) Filter the signal with a moving average filter, for $M = 5, 21, 51$ and in the remaining 3 tiles of the figure, plot these 3 signals over $s[n]$. (While using *conv*, set the shape parameter as *full*).

(d) Report your observations as M changes. What are the trade-offs?

(e) Make a 2×2 plot to display the DTFTs of the noisy signal, and the 3 denoised signals. Report your observations.