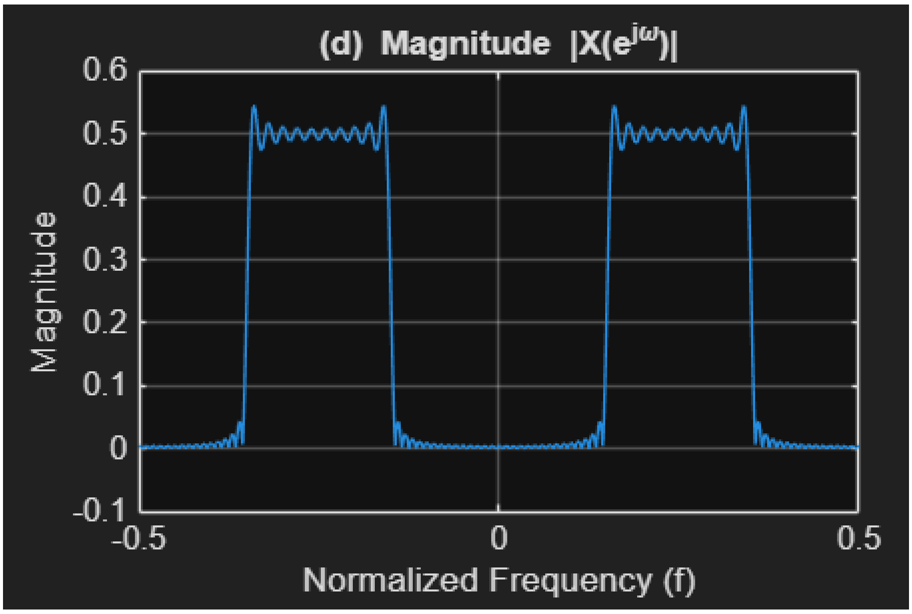
Assignment - 2

Lab Report

Question -1:

Magnitude Spectrum:

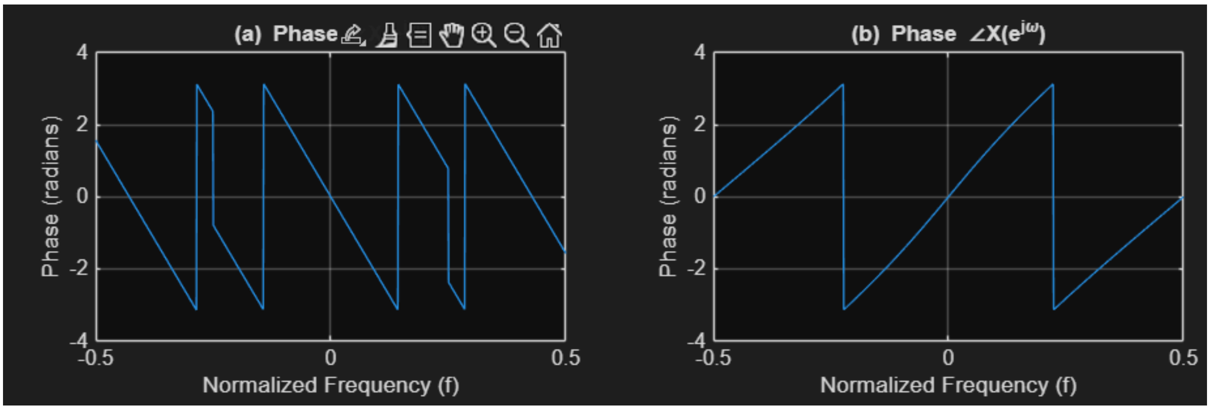
Here we got consistent plots as we analytically derived. Except for the (d) part:



We did not get a perfect rect waves because of the ideal nature of the rect signal. Instead we got a close approximation.

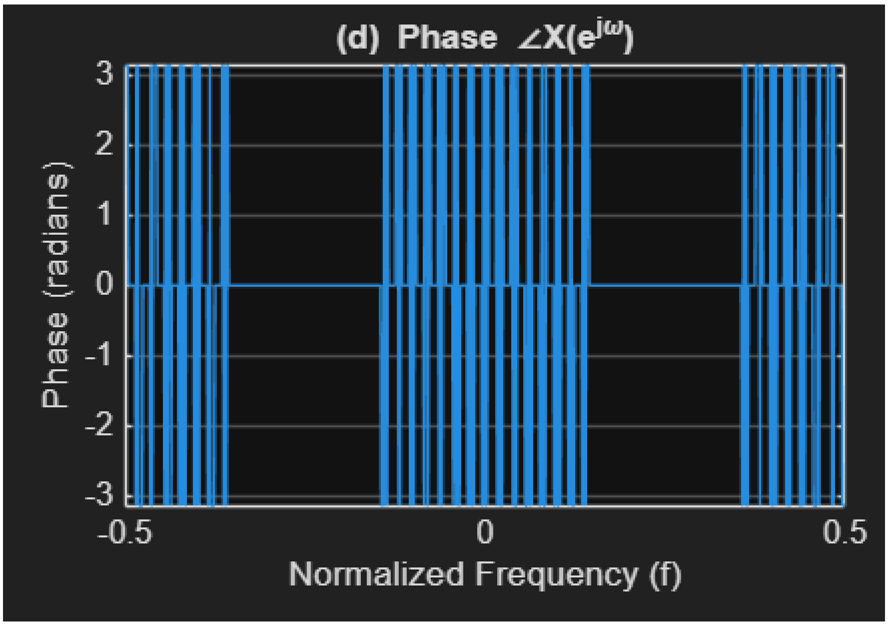
Phase Spectrum:

For phase, we got a sawtooth kind of plot in part (a) and (b) instead of a consistent straight line. This happened because we used MATLAB's angle() function, which always returns a **principal value**, which by convention is restricted to the range **(-π, π]** (approximately -3.14 to +3.14 radians).



For part (d), we got a chaotic and noisy plot what should have been 0 phase theoretically. The noise-like phase is caused by two main factors: **signal truncation** and **numerical precision**.

Our signal is based on a sinc function, which is **infinite in length**. To compute its DTFT in MATLAB, we have to **truncate** it, meaning we only use a finite section (e.g.,n = -50:50). Multiplication in the time domain is **convolution** in the frequency domain. This means our perfect rectangular spectrum gets convolved with a sharp sinc function. This convolution "smears" the sharp edges of the rectangles and introduces ripples across the entire spectrum. These ripples are not perfectly real; they introduce tiny, non-zero imaginary components into the DTFT.

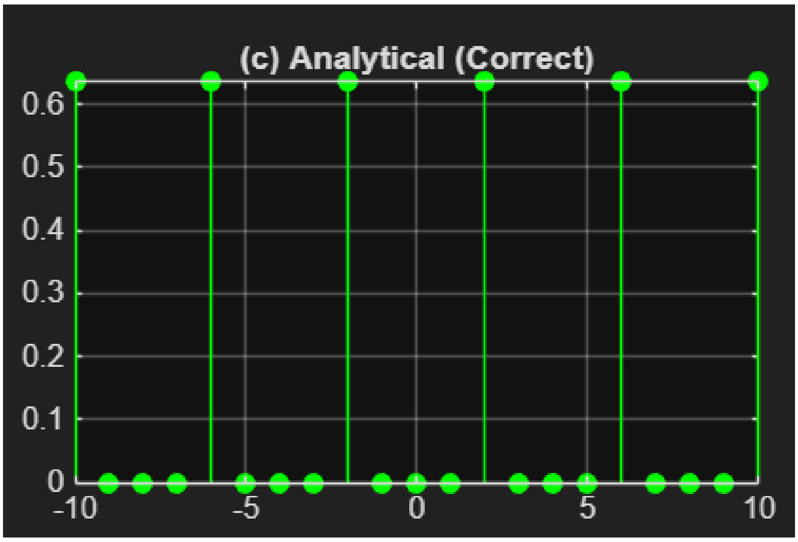


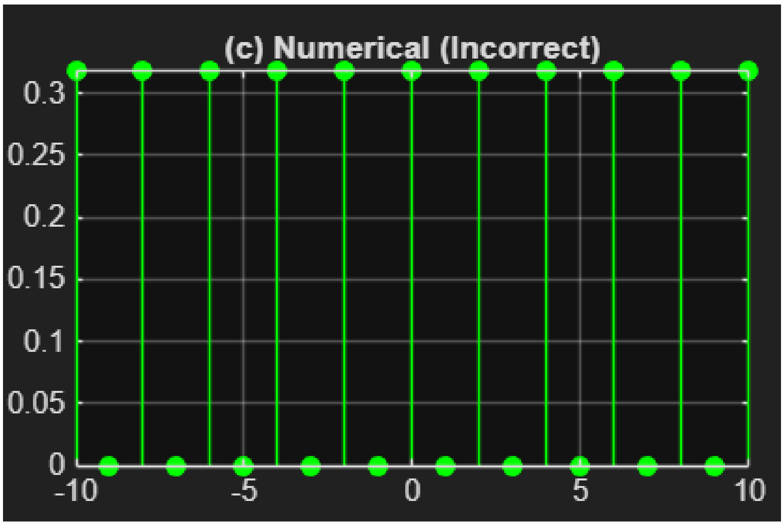
These ripples are not perfectly real; they introduce tiny, non-zero imaginary components into the DTFT.When you ask MATLAB to find the phase of this numerical noise, the result is practically **random**. A tiny change in the noise can swing the angle from -π to +π.

<u>Question -2:</u>

We got the exact same signals analytically and numerically in matlab in part (a),(b) and (d).

In part c, we do not get the same plot because the given delta dirac function is an ideal function for modelling impulse response which is not real and can not be represented in matlab precisely.





Question -3.2(b):

For this problem we got the exact same plot of Power Spectral Density (PSD) from calculating the DTFT of Autocorrelation and from squared magnitude. Hence, the theorem is verified.

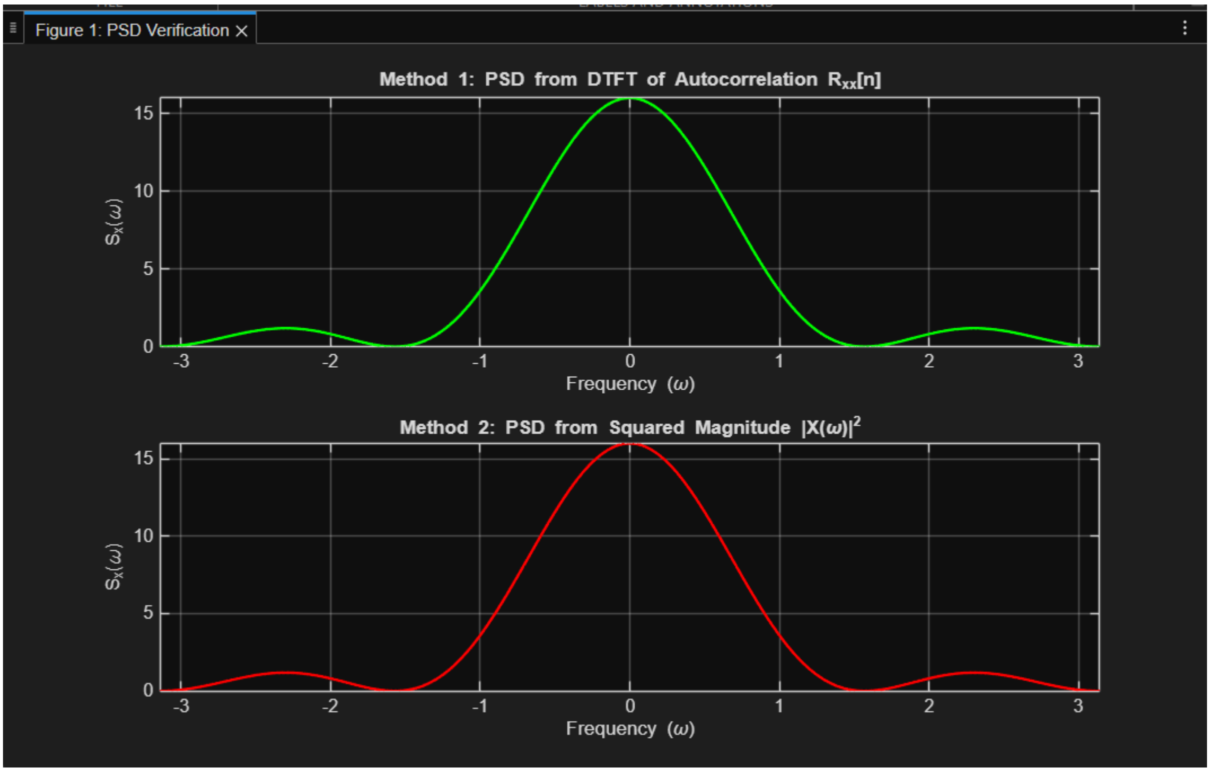
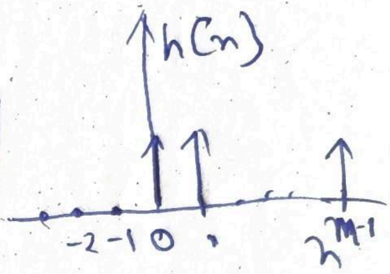


Figure 1: PSD Verification
Method 1: PSD from DTFT of Autocorrelation $R_{xx}[n]$
$S_x(\omega)$
Frequency (ω)
Method 2: PSD from Squared Magnitude $|X(\omega)|^2$
$S_x(\omega)$
Frequency (ω)

Question -5: (a)

$$y[n] = \frac{1}{M}\sum_{m=0}^{M-1} x[n-m]$$



$$\Rightarrow \boxed{h[n] = \frac{1}{M}\sum_{m=0}^{M-1} \delta[n-m]}$$

$$\Rightarrow H(\omega) = \sum_{n=-\infty}^{\infty} h[n]\, e^{-j\omega n}$$

$$= \frac{1}{M}\left(\sum_{m=0}^{M-1} e^{-j\omega m}\right)$$

$$H(\omega) = \frac{1}{M}\,\frac{1-e^{-j\omega M}}{1-e^{-j\omega}} = \frac{1}{M}\,\frac{e^{-j\omega\left(\frac{M}{2}\right)}\left[e^{\frac{j\omega M}{2}} - e^{-\frac{j\omega M}{2}}\right]}{e^{-j\omega/2}\left[e^{j\omega/2} - e^{-j\omega/2}\right]}$$

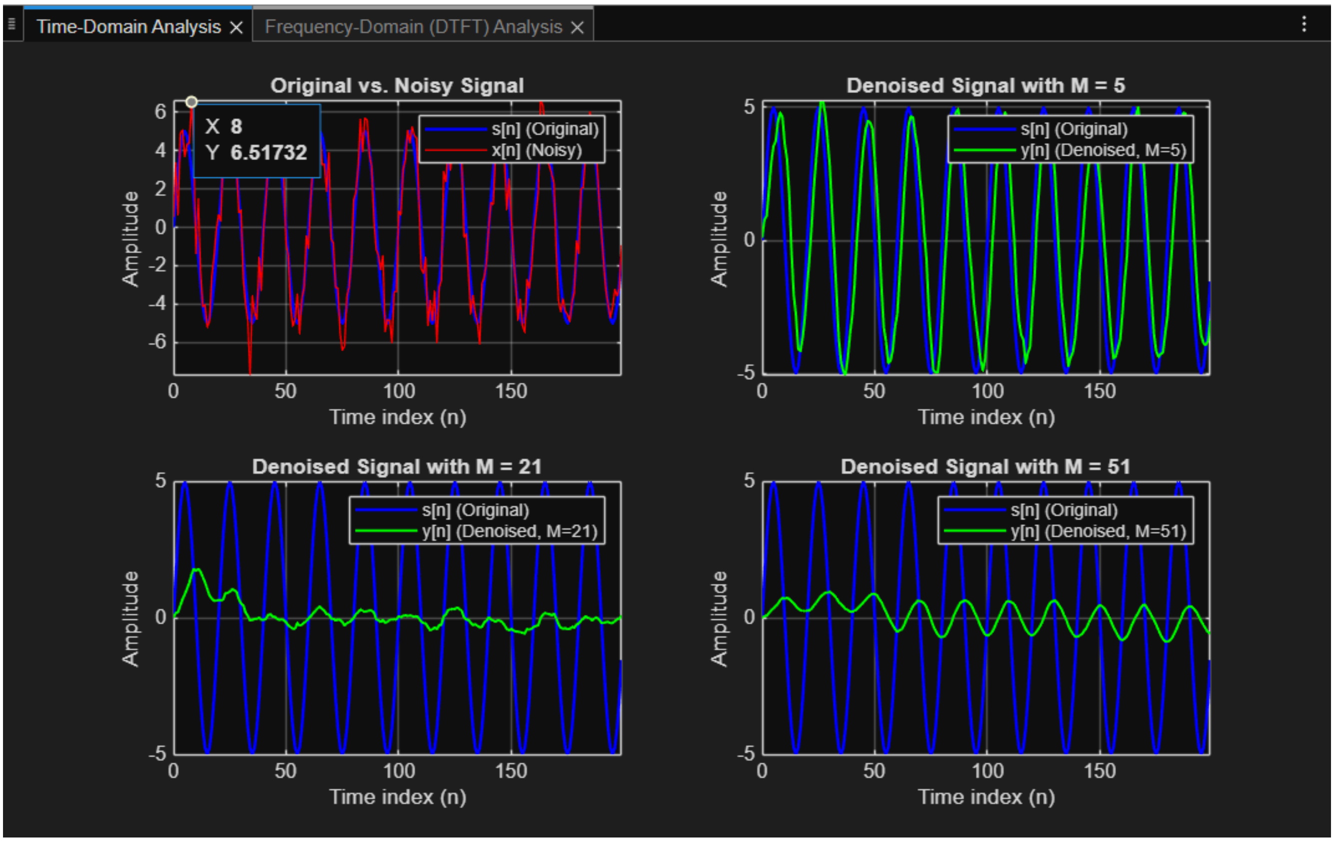
$$\Rightarrow \boxed{H(\omega) = \frac{1}{M}\, e^{-j\omega\frac{(M-1)}{2}}\, \frac{\sin\left(\frac{\omega M}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}}$$

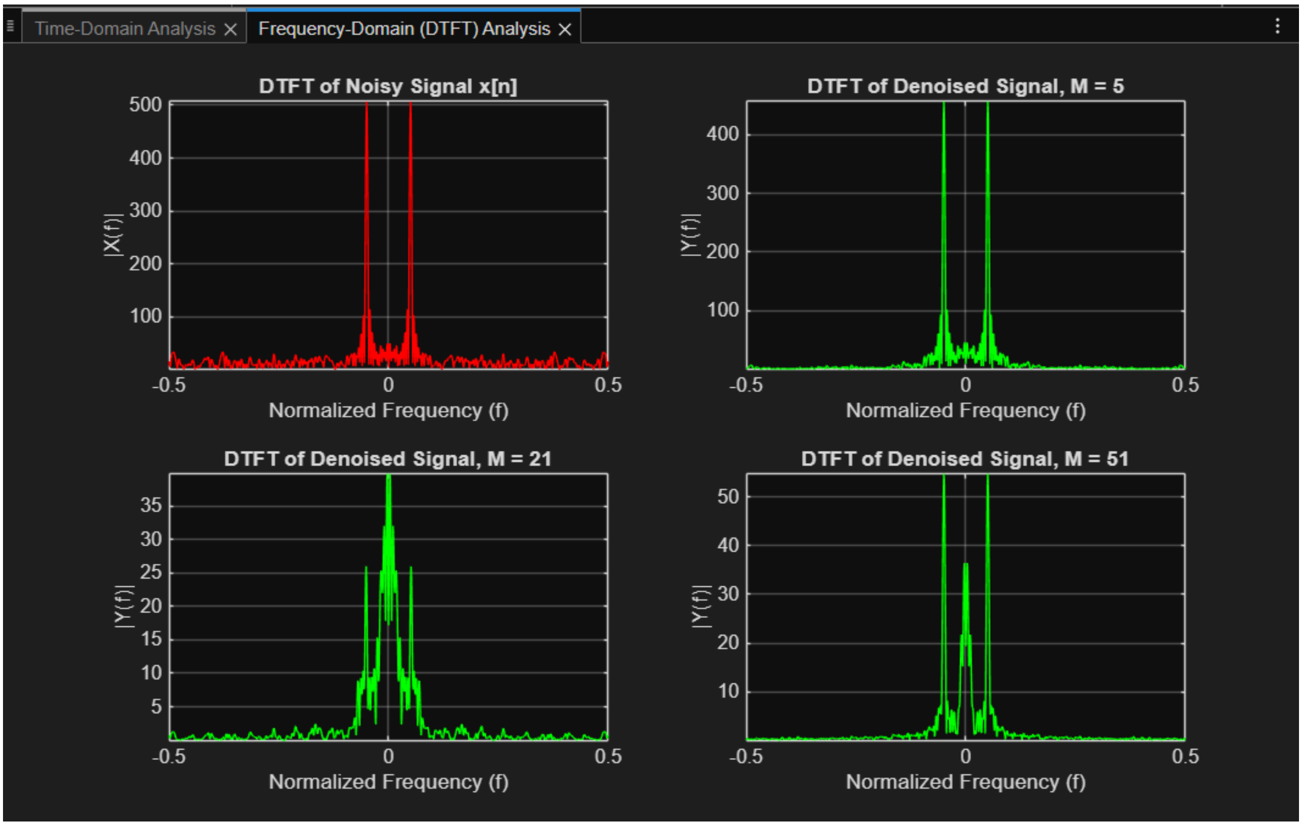
(d) From your time-domain plots, we can observe a clear pattern as the filter order changes.

- **For M = 5:** The denoised signal follows the original signal very closely in terms of shape and amplitude. However, it is still visibly noisy. This small averaging window smooths the signal only slightly.
- **For M = 21:** The noise is significantly reduced. But the amplitude of the sine wave attenuated. The peaks of the green wave no longer reach the peaks of the original blue wave.

- **For M = 51:** The denoised signal is extremely smooth, indicating excellent noise suppression. However, the amplitude attenuation is now severe. The resulting sine wave is much smaller than the original.

The fundamental trade-off is between Noise Reduction and Signal Fidelity.**Increasing M** leads to better **noise suppression**. By averaging over a larger number of samples, the random fluctuations of the noise cancel each other out more effectively. **Increasing M** also leads to more **signal distortion**. which in this case includes the peaks of the sine wave, causing amplitude reduction.





(e) This plot clearly shows two components. The **two tall spikes** at f = +-0.5 represent the frequency of the pure sine wave.This visually confirms that the filter is successfully removing the high-frequency noise. The filter also affects the amplitude signal's frequency components as we can notice that the **height of the sine wave's spikes decreases as M increases.**