Problem Set 2

Statistics 509 – Winter 2018

Due by Wednesday, January 24 in class

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. Continuation of **3.** in Homework 1. In Canvas in the Data subdirectory under Files is the data set of the daily price data of the NASDAQ Composite from Jan/2012 to Dec/2017.

(a) Estimate the mean and standard deviation of the log-returns, and then using a double exponential distribution with these parameters, estimate the relative VaR corresponding to $\alpha = .005$. Also, compare this estimate of the relative VaR with the estimate generated by simply using the .005-quantile of the log-returns.

(b) Use Monte-Carlo simulation to derive an estimate of the expected shortfall in **(a)**.

(c) Repeat **(a)** and **(b)** utilizing a one-sided exponential distribution for positive losses.

2. Exercise 1 on page 81 in Ruppert/Matteson. The data set ford.csv is in Data directory on CTools.
Note: The normal plot and t-plots correspond to QQ plots relative to those distributions.

3. Suppose $X_1, X_2, \ldots, X_n$ are iid $\mathcal{N}(0,1)$ and you use a kernel density estimate with a rectangular kernel.

(a) Analytically derive an expression for the expected value of the kernel density estimate $\hat{f}_b(x)$ in terms of the cdf of $\mathcal{N}(0,1)$.
Hint: You can use that the width, w, of rectangular kernel with bandwidth b is approximately $w = b \cdot 3.464$, and also note that rectangular kernel with bandwidth parameter b and width w satisfies that

$$K_b(x) = \begin{cases} \frac{1}{w} & -\frac{w}{2} \leq x \leq \frac{w}{2} \\ 0 & \text{otherwise} \end{cases}$$

(b) Based on **(a)**, derive an expression for the bias in terms of cdf and pdf of the standard normal distribution, and then plot the bias of the kernel density estimate as a function of x for $b = .1, .2, .4$. Provide an explanation of what these plots show in terms of the bias, i.e., where is the bias positive or negative and how is bias related to the bandwidth.
Recall: $\text{Bias}(x) = E(\hat{f}_b(x)) - f(x)$.