# Problem Set 3

## Statistics 509 – Winter 2018

## Due by Wednesday, January 31 in class

**Instructions.** You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

**1**. The S&P 100 index data set from Oct 1, 2003 to Sept 28, 2013 is in file `SP100_daily_03-13.csv` which is in the Data subdirectory under Files, and you should use the adjusted closing price data for generating the log returns.

*Hint:* To read in the data and get started, use the following commands.

```
XX = read.csv("SP100_daily_03-13.csv",header=TRUE)
SP100_dl <- rev(XX$AdjClose)
SP100_dl_lreturn <- diff(log(SP100_dl)) # generating log returns (daily)
```

**(a)** Compute the median, mean, variance, skewness, and kurtosis and give a brief summary of interesting data features discovered based on these descriptive statistics.

**(b)** Fit a Generalized Pareto Distribution (GPD for short) to the lower tail of the log returns and give detailed plots of the fit (along the lines of what is generated in class) and discuss your results.

**(c)** Compute the relative VaR (expressed in units of the current price) at the level $= .005$ utilizing for the case of fitting a normal distribution to the log returns (not using any POT). Note that VaR is relative to return, not log-return.

**(d)** Compute the relative VaR (expressed in units of the current price) at the level $= .005$ utilizing the GPD distributional fits generated in part **(b)**, and compare with result from part **(c)**.

**(e)** Utilizing 'quant" command in evir, give a discussion on the stability of the VaR as function of threshold.

**(f)** Compute your estimated expected shortfall associated with the VaR computed in part (d), based on your GPD analysis/estimates in parts (b) and (d). *Hint:* Be careful as evir has its own commands for the GPD distribution and commands from other packages will be masked when you load this library.

**2.** Suppose $R_1.R_2$ are returns on an asset, and suppose that $E(R_1) = .02, E(R_2) = .03, \text{Var}(R_1) = (.03)^2, \text{Var}(R_2) = (.04)^2$, and $\text{Corr}(R_1, R_2) = 0.5$.

**(a)** What are $E(0.6R_1 + 0.4R_2)$ and $\text{Var}(0.6R_1 + 0.4R_2)$?

**(b)** For what value of $w$ is $\text{Var}(wR_1 + (1 - w)R_2)$ minimized? Why would it be useful to minimize $\text{Var}(wR_1 + (1w)R_2)$?

**(c)** Assuming a portfolio of a \$1 million, and a multi-variate normal distribution for $R_1$ and $R_2$, find the value $w$ that minimizes the expected shortfall associated with VaR at $q = .005$ and why might that be useful? Also, report the associated VaR with this portfolio. *Hints:* Note that the random variable $[wR_1 + (1 - w)R_2]$ is normally distributed for any $w$, and it will be easiest to use R-package – the solution can be approximate, i.e., accurate to .01.

**(d)** Repeat the above if have multi-variate t-distribution for $R_1$ and $R_2$, with $\nu = 6$.