Problem Set 4

Statistics 509 – Winter 2018

Due by Wednesday, February 7 in class

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. Suppose Z, X are independent random variables where $X \sim \mathcal{N}(0,1)$, and Z is discrete random variable satisfying that

$$P(Z=-1) = P(Z=1) = \frac{1}{2}$$

and $Y = ZX$.

(a) Show that X and Y are uncorrelated.

(b) Show that X and Y are not independent.

(c) Derive the Spearman correlation between X and Y.

Hint. For part **(b)**, note that if X and Y were independent, then

$$E(g(X)h(Y)) = E(g(X))E(h(Y))$$

for any functions g, h. Pick the right g, h and show that above is not true.

2. Suppose $\mathbf{Y} \sim t_\nu(\boldsymbol{\mu}, \boldsymbol{\Lambda})$, i.e., is p-dimensional multivariate t-distribution with ν degrees of freedom and $\mathbf{X} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Lambda})$, i.e., is multivariate normal with mean of zero and covariance of $\boldsymbol{\Lambda}$. Assuming that $\nu > 2$, show that $\mathbf{X}$ and $\mathbf{Y}$ have the same correlation matrices.

3. In the Data directory are Nasdaq weekly return data and SP400 weekly return data from 1992 to 2012.

(a) Carry out a fitting of a multivariate normal distribution to the log-returns (computed from the Adusted Closing prices) and carry out diagnostic plots – univariate QQ and a plot comparing the empirical vs. theoretical bivariate cumulative distribution function. Provide a discussion of the fit based on these plots.

(b) Same as **(a)**, but now use a multivariate t distribution – also derive a confidence interval for the degrees of freedom via the method of profile likelihood. Provide a discussion of the fit based on these plots.

(c) Based on results in **(a)** and **(b)**, which model do you prefer and why.

(d) For portfolio of 10 million dollars evenly split between these two indices, derive the VaR at $q = .001$ for the model derived in **(a)** and the model derived in **(b)**.

(e) For a portfolio of 10 million, find the optimal portfolio (i.e., proportion $0 \leq w \leq 1$ invested in Nasdaq and $(1-w)$ invested in SP400) for the following criteria:

- Maximum expected return.
- Minimum volatility.
- Minimum VaR at $q = .002$.