STATS 509 Homework 1　Xiaofeng Nie

1. Solution:

a) Because $Y=X^3+2$ is increasing, so we have $y_{0.90} = x_{0.90}^3 + 2$.
As $X \sim N(1,2^2)$, $x_{0.9} \approx 3.56$
Thus, $y_{0.90} = 3.56^3 + 2 \approx 47.24$

b) Because $Z=\exp(-X)$ is decreasing, so we have $z_{0.90} = \exp(-x_{0.10})$.
As $X \sim N(1,2^2)$, $x_{0.1} \approx -1.56$
Thus, $z_{0.90} = e^{1.56} \approx 4.77$

2. Solution:

a) $r_{0.003} \approx -0.069 \Rightarrow R_{0.003} = \exp(-r_{0.003}) - 1 \approx -0.0664$
Thus, $\widetilde{VaR} = -R_{0.003} \approx 0.0664$; $VaR = 100m * \widetilde{VaR} \approx 6640000$

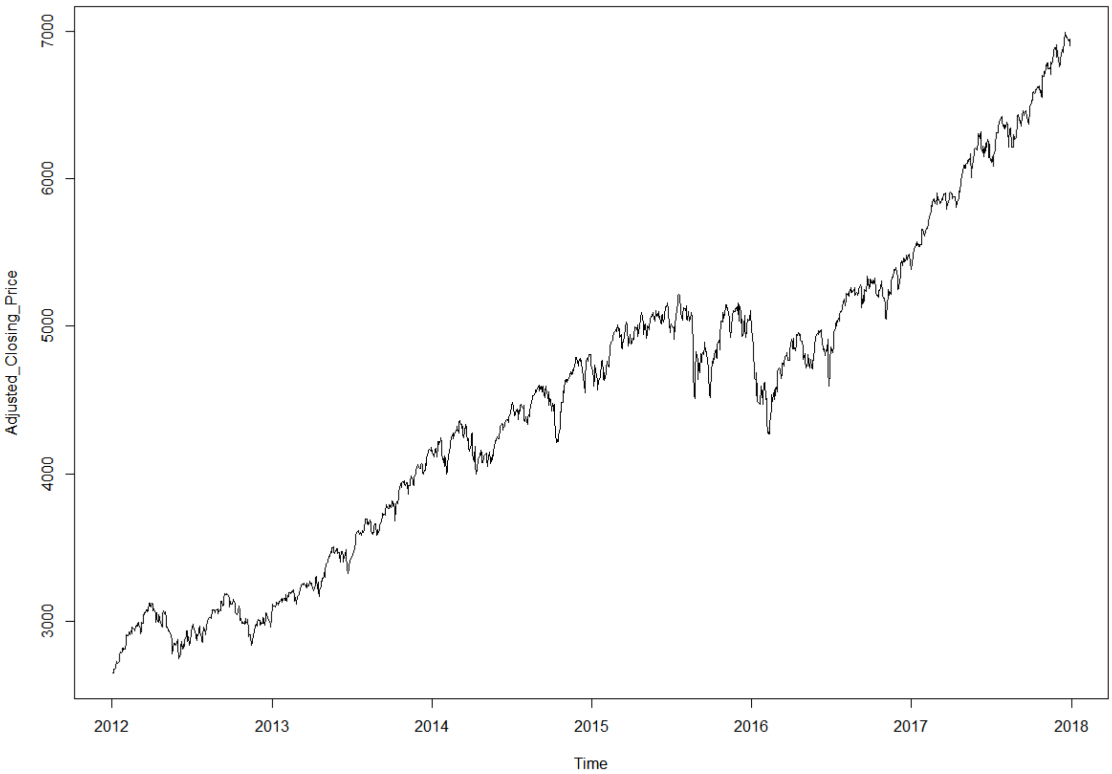
b) **ν=12:** Let $SD = \lambda \cdot \sqrt{\frac{\nu}{\nu-2}} = 0.025$, we set $\lambda \approx 0.023$, i.e. $Y_{0.003} = 0.023X_{0.003}$
$r_{0.003} \approx -0.076 \Rightarrow R_{0.003} = \exp(r_{0.003}) - 1 \approx -0.0732$
Thus, $\widetilde{VaR} = -R_{0.003} \approx 0.0732$; $VaR = 100m * \widetilde{VaR} \approx 7320000$

ν=6: Let $SD = \lambda \cdot \sqrt{\frac{\nu}{\nu-2}} = 0.025$, we set $\lambda \approx 0.020$, i.e. $Y_{0.003} = 0.020X_{0.003}$
$r_{0.003} \approx -0.085 \Rightarrow R_{0.03} = \exp(r_{0.003}) - 1 \approx -0.0813$
Thus, $\widetilde{VaR} = -R_{0.003} \approx 0.0813$; $VaR = 100m * \widetilde{VaR} \approx 8130000$

ν=3: Let $SD = \lambda \cdot \sqrt{\frac{\nu}{\nu-2}} = 0.025$, we set $\lambda \approx 0.014$, i.e. $Y_{0.003} = 0.014X_{0.003}$
$r_{0.003} \approx -0.101 \Rightarrow R_{0.03} = \exp(r_{0.003}) - 1 \approx -0.0960$
Thus, $\widetilde{VaR} = -R_{0.003} \approx 0.0960$; $VaR = 100m * \widetilde{VaR} \approx 9600000$

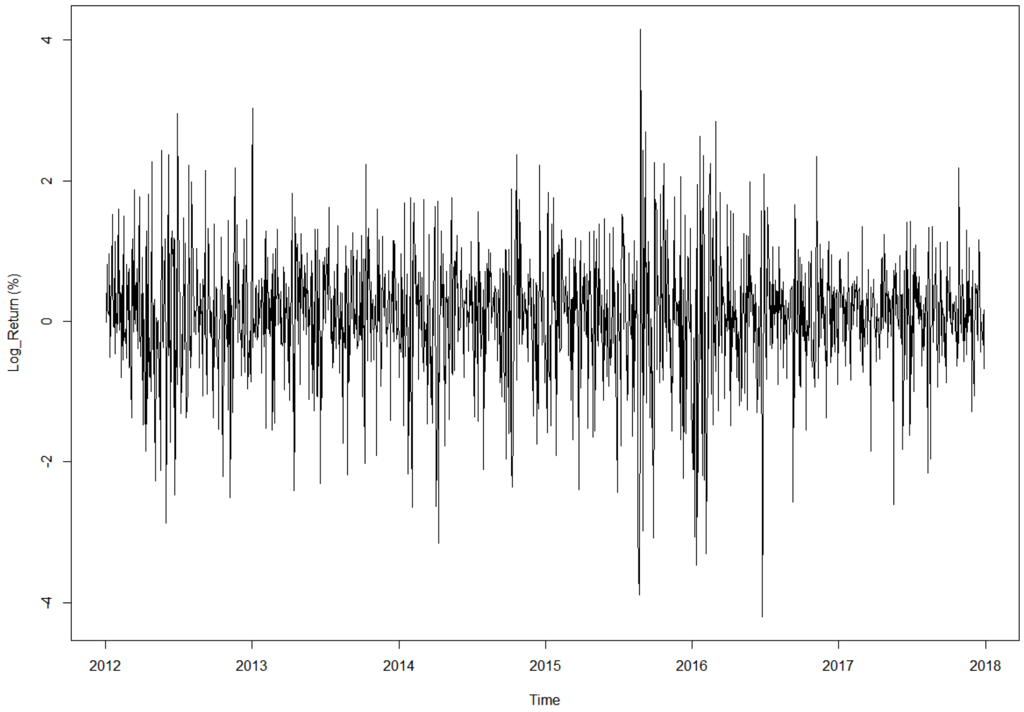
3. Solution:

a)



The plot of shows that the trend of adjusted closing price from 2012 to 2018 is increasing, though the index plummets several times in 2015 and 2016. And the index price seems increase faster after

2016.

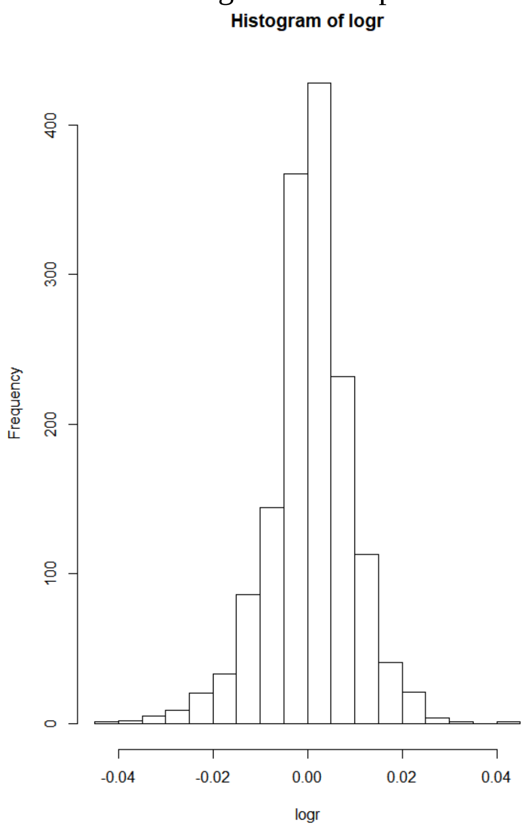


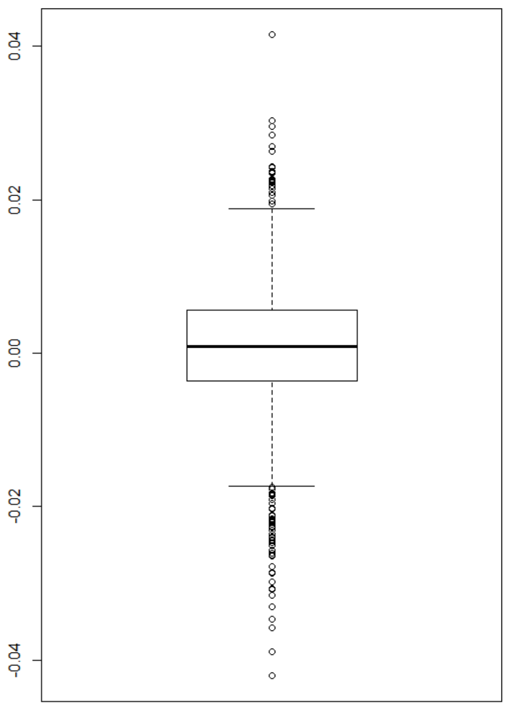
The log return plot shows that the expectation of log return is 0, and the variation of log return was larger in 2012, in the second half year of 2015, and in the first half year of 2016.

b)

```
> summary(logr)
      Min.    1st Qu.     Median       Mean    3rd Qu.       Max.
-0.0420232 -0.0035846  0.0008784  0.0006352  0.0055978  0.0415203
> skewness(logr)
[1] -0.4036703
> kurtosis(logr)
[1] 2.002712
```

From the outcome above, we can find that mean of log return is 0.0006352, which is almost 0. Skewness is -0.4036703, which presents a little left skewed. Kurtosis is 2.002712, which means, the distribution of log return is leptokurtic and heavy-tailed.

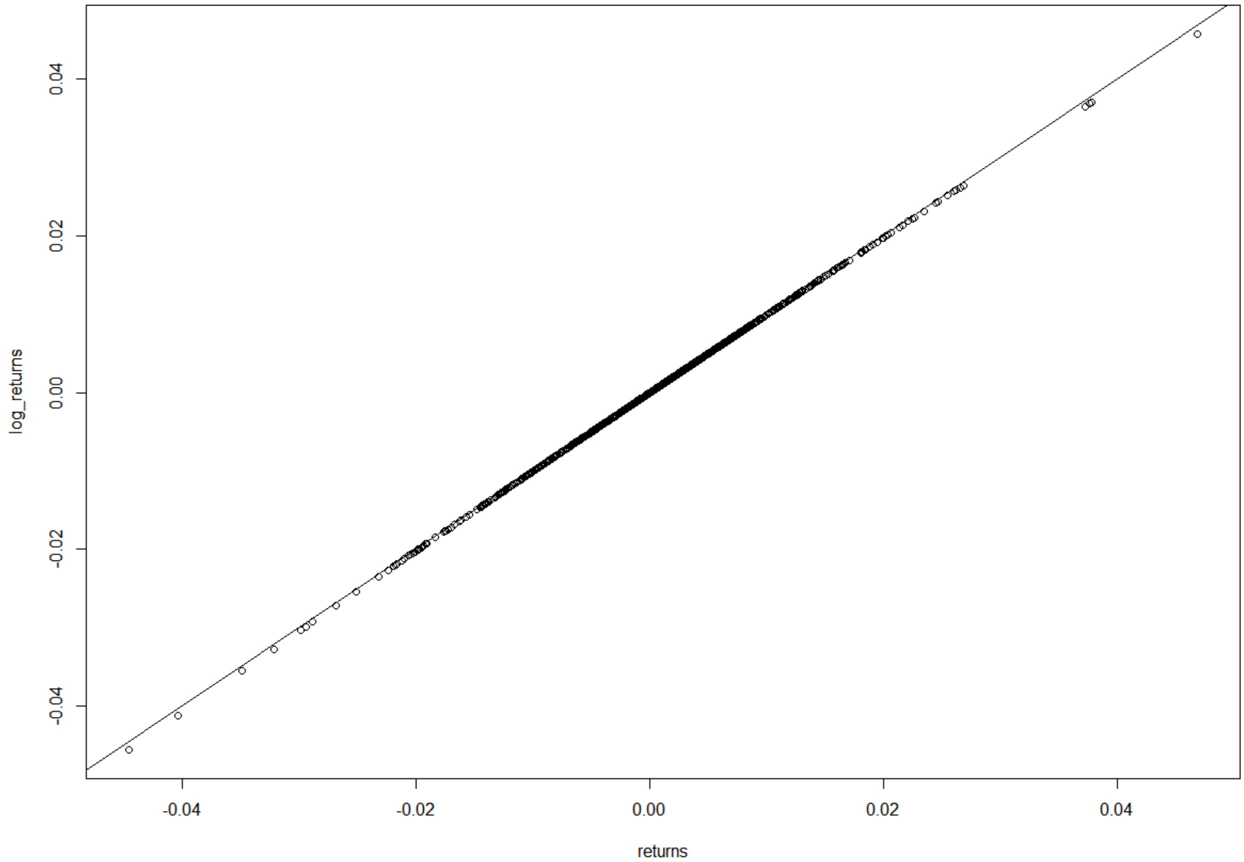




Histogram and boxplot present the distribution of log return has heavy tail and a higher peak and it is almost symmetric, while mean is almost 0.

c) Because kurtosis of the distribution of log returns is 2.002712, which is larger than kurtosis of normal distribution and less than double-exponential distribution, we can say that the distribution of log returns has heavier tail than normal distribution but lighter tail than double-exponential distribution.

4. Solution:
Problem 12:



The plot above presents that the value of returns and log returns are approximately equal. Because when x is small, log(1+x)≈x, where x is defined as returns and log(1+x) is defined as log returns.

Problem 13:

```
> mean(r)
[1] 0.0005027479
> sd(r)
[1] 0.008900319
```

```
> mean(logr)
[1] 0.0004630553
> sd(logr)
[1] 0.008901467
```

It is clear that mean of returns is slightly larger than log returns, while standard deviation of returns is slightly smaller than log returns (the difference of standard deviations is negligible).
As returns are approximately equal to log return, descriptive statistics could be approximately equal, due to the same reason.

Problem 16:
Using Monte Carlo simulation, we predict the proportions that simulations where the minimum price is less than $85 is 0.0105.
The expectation of our wealth is 100*0.0105-1*(1-0.0105)=0.0605, which is bigger than 0, so we should make the bet.

Problem 17:

Using Monte Carlo simulation, we predict the proportions that simulations where the minimum price is less than $84.5 is 0.0073.
The expectation of our wealth is 100*(0.0105-0.0073)+125*0.0073-1*(1-0.0105)=0.243, which is bigger than 0, so we should make the bet.

Appendix:

```
dat = read.csv("O:\\18WIN\\STATS 509\\HW1\\Nasdaq_daily_Jan1_2012_Dec31_2017.csv",header = T)
d = as.Date(dat$Date,format="%m/%d/%y")
logr = diff(log(dat$Adj.Close))
plot(d,dat$Adj.Close,type = "l",xlab="Time",ylab="Adjusted_Closing_Price")
plot(d[-1],100*logr,type="l",xlab="Time",ylab="Log_Return (%)")

library(fBasics)
summary(logr)
skewness(logr)
kurtosis(logr)
par(mfrow=c(1,2))
hist(logr)
boxplot(logr)

dat = read.csv("O:\\18WIN\\STATS 509\\datasets\\MCD_PriceDaily.csv",header=T)
nd = dim(dat)
n = nd[1]
r = dat$Adj.Close[-1]/dat$Adj.Close[-n] - 1
logr = diff(log(dat$Adj.Close))
plot(r,logr,xlab="returns",ylab="log_returns")
abline(0,1)

niter = 1e4
below = rep(0,niter)
set.seed(2015)
for (i in 1:niter)
{
  r = rnorm(20,mean = mean(logr),sd = sd(logr))
  logPrice = log(93.07) + cumsum(r)
  minlogP = min(logPrice)
  below[i] = as.numeric((minlogP) < log(85)) %or 84.5 in problem 17
}
mean(below)
```