Problem Set 1

Statistics 509 – Winter 2018

Due Wednesday, January 17 in class

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. Suppose $X \sim \mathcal{N}(1, 2^2)$.

(a) Find the .90-quantile for $Y = X^3 + 2$.

(b) Find the .90-quantile of $Z = e^{-X}$

For all of the problems above, your solutions must be in terms of the quantile functions for normal distribution.

2. Suppose have portfolio of 100 million dollars. Compute the value-at-risk and relative value-at-risk, VaR and $\tilde{\text{VaR}}$, at $\alpha = .003$ for the following cases in **(a)** and **(b)**.

(a) Distribution of log-returns is normally distributed with a mean of 0 and standard deviation of .025.

(b) Distribution of log-returns has a t distribution with a mean of 0 and standard deviation of .025 for the cases of $\nu = 12, 6, 3$.

Clarification/Hints: The relative value-at-risk will be the negative of the .003 quantile of the returns, and the value-at-risk will 100 million times the relative value-at-risk; for this homework, you will simply compute these values for the distributions as specified in **(a)** and **(b)**, and we will discuss these concepts in more detail in the lectures.

For **(b)**, you will need to set the scale parameter, λ, appropriately to get standard deviations being .025, and you will need to relate quantiles from the R-function "qt" which is for standard t-distribution with quantile of $t_\nu(\mu, \lambda)$ for the appropriate μ and λ.

3. In Canvas in the Data subdirectory under Files is the data set of the daily price data of the NASDAQ Composite from Jan/2012 to Dec/2017.

(a) Generate plots of adjusted closing price and log returns (based on adjusted closing price) as a function of time and give a brief summary what the plots show.

(b) Generate summary statistics, skewness, kurtosis, histogram, and boxplot of the log-returns – give a brief summary of interesting data features discovered based on this analysis. *Hint:* Can get skewness and kurtosis from R-package "fBasics".

(c) Based on the kurtosis, what would you say about the tails of the distribution of log-returns relative to normal distribution and double-exponential distribution, and be sure to explain your answer.

4. Problems 12, 13, 16, and 17 on pages 15-16 in Ruppert/Matteson. *Hint:* For problems 16 and 17, look at code on page 13 of textbook on a loop in R for carrying out a Monte Carlo simulation.