

Problem Set 5

Statistics 509 – Winter 2018

Due by Wednesday, February 14 in class

Instructions. You may work in teams, but you must turn in your own work/code/results. Also for the problems requiring use of the R-package, you need to include a copy of your R-code. This provides us a way to give partial credit in case the answers are not totally correct.

1. (a) Exercise 2 on page 214 in Ruppert/Matteson – ignore the Kendall’s tau part of this problem, and focus only on the Spearman correlation and the Pearson correlation .

(b) Derive the lower and upper coefficients of tail dependence for the Clayton copula model – they will be a function of $\theta > 0$. Also, provide an interpretation of the results in your own words.

2. Utilizing the same data as in Exercise 3. of Homework 4, carry out a copula-based fitting of the bivariate distribution of the log-returns via the following steps:

Step 1: Fit a separate t -distribution, via MLE, to the Nasdaq weekly log-returns and SP400 weekly log-returns.

Step 2: After transforming via the estimated t -cdf’s, fit a t -copula to the data. Compare the fit of this model with the estimated multivariate t -distribution from Problem 3-(b) from Homework 4. For this comparison, compare the fits of the estimated bivariate cumulative distribution function with the bivariate empirical cdf and compare the AIC criteria. For the AIC criteria, recall that multivariate copula pdf model is

$$f(x_1, x_2) = c(F_1(x_1), F_2(x_2))f_1(x_1)f_2(x_2)$$

where c is bivariate pdf of the copula, and f_1, f_2 are pdfs of the marginals.

3. Utilizing the model in Problem 2, carry out the following.

(a) Find the portfolio consisting of Nasdaq and SP400, (i.e., weight w for $wR_{Nasdaq} + (1 - w)R_{SP400}$) that **minimizes VaR** at the $q = .005$ level – allow for short-selling. Also derive the **expected shortfall** for this portfolio, corresponding to this VaR.

(b) Find the portfolio consisting of Nasdaq and SP400, (i.e., weight w for $wR_{Nasdaq} + (1 - w)R_{SP400}$) that has the **minimum variance** – again allow for short-selling.

(c) Determine the probability that the returns of the assets will simultaneously both be below their respective, relative VaRs at $q = .003$.

4. (a) Suppose have 2 risky assets with yearly returns R_1, R_2 having means μ_1, μ_2 , standard deviations σ_1, σ_2 and correlation ρ_{12} , and suppose the risk-free asset has mean μ_f . (Explicitly) Derive the formula for the weight w_T for the tangent portfolio $R_T = w_T R_1 + (1 - w_T) R_2$.

(b) Exercise 4 on page 492 of Ruppert/Matteson.