Nurse scheduling and patient assignment

Defining the problem

I wanted to implement a new strategy for the original problem with 2 different layers - 2 substep:

1. scheduling nurses and
2. patient assignment to nurses in order to optimize the initial solution.

First layer:

Some basic constraints and assumptions:

- Each day is divided into 3 separate shifts of 8 hours (s1,s2,s3).
- There are a number of required nurses for each of them(5,4,3).
- A nurse is off on a specific day if no shift is assigned or if the nurse has requested a day off
- A nurse will have just a shift in a day
- A nurse who works on the 3rd shift will take the next day off.
- Max numbers of night shift for each nurse is at most ONE per week

Second layer:

After we schedule the nurses per shift, we will try to assign the patients to them.

The same constraints from initial solution are kept:

- Each patient is treated by exactly one nurse
- A nurse can not work more than 8 hours per day
- A patient from a shift is treated by the nurses assigned to that shift

Model

The first objective was to **minimize** the numbers of scheduled nurses by satisfying all constraints and the second to **maximize** the number of assigned patients.

The implementation is based on linear programming with pulp python package to find the solution of this constrained optimization problem.

In the both cases, we will use a binary variable for example:

var_n_s = 1, if the nurse will work on shift s, 0 otherwise

var_p_n = 1, if the patient p is treated by the nurse n, 0 otherwise

	M_s1	M_s2	M_s3	Tu_s1	Tu_s2	Tu_s3	W_s1	W_s2	W_s3	Th_s1	Th_s2	Th_s3	F_s1	F_s2	F_s3	Sa_s1	Sa_s2	Sa_s3	Su_s1	Su_s2	Su_s3
nurse0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0
nurse1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0
nurse2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
nurse3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
nurse4	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
nurse5	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	1
nurse6	1	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0
nurse7	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
nurse8	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
nurse9	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
nurse10	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
nurse11	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
nurse12	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
nurse13	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
nurse14	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0
nurse15	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
nurse16	0	1	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0
nurse17	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
nurse18	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
nurse19	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
nurse20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
nurse21	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0
nurse22	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
nurse23	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
nurse24	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0
nurse25	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	1	0	0	0
nurse26	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0
nurse27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
nurse28	0	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0
nurse29	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
nurse30	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0
nurse31	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1
nurse32	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0

The number of nurses required for each shift is:
[5 4 3 5 4 3 5 4 3 5 4 3 5 4 3 5 4 3 5 4 3]

In matrix form:

	M	Tu	W	Th	F	Sa	Su
0	5	5	5	5	5	5	5
1	4	4	4	4	4	4	4
2	3	3	3	3	3	3	3

	0	1	2	3	4
0	0	1	0	0	0
1	0	1	0	0	0
2	1	0	0	0	0
3	1	0	0	0	0
4	1	0	0	0	0
5	0	0	0	0	1
6	0	0	1	0	0
7	0	0	0	0	1
8	0	0	1	0	0
9	0	0	0	1	0
10	0	1	0	0	0
11	0	0	0	0	1
12	0	1	0	0	0
13	0	0	0	1	0
14	1	0	0	0	0
15	0	0	0	0	1
16	0	0	0	1	0
17	0	0	1	0	0
18	0	0	0	1	0
19	0	0	0	0	1
20	0	0	0	1	0

	0	1	2	3	4
0	0	1	0	0	0
1	1	0	0	0	0
2	1	0	0	0	0
3	0	0	1	0	0
4	0	0	0	1	0
5	0	1	0	0	0
6	0	0	1	0	0
7	0	0	0	1	0
8	0	0	0	1	0
9	0	1	0	0	0
10	0	1	0	0	0
11	0	0	0	0	1
12	0	1	0	0	0
13	0	0	0	1	0
14	1	0	0	0	0
15	0	0	0	0	1
16	0	0	0	1	0
17	0	0	1	0	0
18	0	1	0	0	0
19	0	0	0	0	1
20	0	0	0	1	0
21	1	0	0	0	0
22	1	0	0	0	0
23	1	0	0	0	0

24	0	0	0	0	1
25	0	0	1	0	0
26	0	0	0	0	1
27	0	0	1	0	0
28	0	0	0	1	0
29	0	0	0	1	0
30	0	1	0	0	0
31	0	0	0	1	0
32	1	0	0	0	0
33	0	1	0	0	0
34	0	0	0	1	0
35	0	0	1	0	0
36	1	0	0	0	0
37	0	1	0	0	0
38	0	0	1	0	0
39	0	1	0	0	0
40	0	0	0	1	0
41	0	0	0	0	1
42	0	1	0	0	0
43	0	0	0	0	1
44	1	0	0	0	0
45	0	0	0	1	0
46	1	0	0	0	0
47	0	0	0	0	1
48	1	0	0	0	0
49	0	0	0	0	1

Comparison with the initial solutions

For the same input data, in both cases, all the patients can be treated.

```
On day 0, number of treated patients: 50 / 50
On day 1, number of treated patients: 50 / 50
On day 2, number of treated patients: 50 / 50
On day 3, number of treated patients: 50 / 50
On day 4, number of treated patients: 50 / 50
On day 5, number of treated patients: 50 / 50
On day 6, number of treated patients: 50 / 50
```

The required number of nurses estimated for the 50 patients was also 5, but the initial restrictions are weak.

```
On day 6, nurse 0 will be assigned to patients: [2, 5, 6, 15, 21, 25, 28, 30, 31, 37].
On day 6, nurse 25 will be assigned to patients: [0, 1, 7, 9, 10, 11, 13, 16, 20, 23, 24, 27].
On day 6, nurse 27 will be assigned to patients: [4].
On day 6, nurse 29 will be assigned to patients: [3, 8, 12, 14, 17, 18, 19, 22, 26, 34, 35, 38, 46, 47].
On day 6, nurse 30 will be assigned to patients: [29, 32, 33, 36, 39, 40, 41, 42, 43, 44, 45, 48, 49].
```

Also, we observe, the initial assignment is **not fair**: nurse 27 will treat just an patient but with the second solution the number of patients is more balanced

Efficiency of resource use (Eficienta utilizarii resurselor)

I analyzed how the available resources are used, such as the number of support needed, the level of **nurses overload or the inefficient allocation of patients**. Greater efficiency in the use of resources indicates better performance.

To measure if the workload is equitable among the nurses based on the number of patients they treat, I used statistical measures such as the mean, standard deviation and variance. These measures can indicate the fairness of the workload.

For the **improved solution**, the limits of the **confidence interva**l are closer than those of the first solution, meaning **a more accurate estimate or greater confidence in the accuracy of the estimate**. On the other hand, when the limits are further apart, it means that there is more uncertainty or more variation in the estimate.

NrAsistente/Shft	NrTotal Pacienti /Shift	NrPacienti Tratati	NrPacienti Per Asistenta	Media	Var	Std	Interval confidenta 95%	Durata program asistente	Durata asignare pacienti	Total
5-4-3	100	79	17 15 14 17 16	15.8	1.36	1.16	[15.47-18.12]	0.2142	0.16807	0.38229
Solutia initiala (5)	100	74	15 9 9 18 23	14.8	28.96	5.38	[9.951-19.649]		0.10	
40-30-25	100	100	[1, 5,2,2, 2,1,3…….…..]	2.5	3.15	1.77	[1.954- 3.046]	15.75	8.47	24.23
Solutia initiala (40)	100	100	1 3 1 17….	10	47	6.85	[8,318 - 11,682]		2.3	
20-15-10	200	200	13,13,12, 11,14,16, 14,14,12, 14,11,13, 16,14,13	13.33	2.08	1.44	[12.483-13.65]	2.094291	3.4341	5.528
Solutia initiala (15)	200	193	11 24 9 3 27 8	13.78	47.31	6.87	[10.94 - 16,618]		3.5	
20-15-10	200	200	10, 13,11,11, 12,10,12, 6,7,10, 8,13,12, 12,12,8,10, 7,10, 6	10	4.9	2.21	[9.009- 10.991]	2.094291	8.35645	10.45074
Solutia initiala (20)	200	200	1 4 11 18	10	31.1	5.5767	[7.849-12.151]		3.05	
20-15-10	300	282	…….	14.1	8.59	2.93	[13.58-14.62]	3.64	~30	33.64
Solutia initiala (20)	300	274	…….	13.7	30.41	5.51	[11.93-15.47]		10	