GATE CSE NOTES

by

UseMyNotes

EXTRAS

* DFA for $L = \{w \in \{a,b\}^* \mid n_a \% 3 > n_b \% 3\}$

POSSIBILITIES

$n_a \% 3$	$n_b \% 3$	Cond^n satisfied?
0	0	×
0	1	×
0	2	×
1	0	✓
1	1	×
1	2	×
2	0	✓
2	1	✓
2	2	×

Make product DFA

$$\begin{bmatrix} n_a \% 3 = 0 \\ n_b \% 3 = 0 \end{bmatrix}$$

Final states are –

q_{10}, q_{20}, q_{21}.

* Formal language is the abstraction of generalized characteristics of programming languages.

* Automaton

- FA
  - DFA
  - NFA
  - ε-NFA
- PDA
  - DPDA
  - NPDA
- LBA
- TM
  - DTM
  - NTM

$\boxed{L(DFA) \equiv L(NFA) \equiv L(\varepsilon\text{-}NFA)}$

expressive power

$\boxed{L(DPDA) < L(NPDA) \quad DCFL \subset CFL}$

$L(DTM) \equiv L(NTM)$

or TM

- Halting TM (Accept Recursive langs)
- Standard TM (Accept REL)

HTM < TM

* Expressive power of automata

(FA < DPDA < NPDA or PDA < LBA < HTM < TM)

(FA ≡ TM with read only tape ≡ TM with unidirectional tape
≡ TM with finite tape ≡ PDA with finite stack)

(PDA ≡ FA with stack)

(TM ≡ PDA with additional stack ≡ FA with 2 stacks)

* ME 136 Dgm

* Grammar - decidable / undecidable

Decision Problem	RL	DCFL	CFL	CSL	RECL	REL
Membership $w \in L$?	D	D	D	D	D	UD
Emptiness $L = \phi$?	D	D	D	UD	UD	UD
Finiteness	D	D	D	UD	UD	UD
Equivalence $L_1 = L_2$?	D	D	UD	UD	UD	UD
Intersecⁿ empty $L_1 \cap L_2 = \phi$? (Disjoint)	D	UD^I	UD	UD	UD	UD
Totality $L = \Sigma^*$?	D	D	UD	UD	UD	UD
Subset $L_1 \subseteq L_2$? (Containment)	D	UD^S	UD	UD	UD	UD
Intersecⁿ finiteness ($L_1 \cap L_2$ finite?)	D	UD^I	UD	UD	UD	UD
Cofiniteness ($\bar{L}$ finite?)	D	D	UD	UD	UD	UD
Regularity (L = reg?)	D	D	UD	UD	UD	UD
Ambiguity	D	UD^A	UD	UD	UD	UD
C	$\bar{L}$ same type?	D	D	UD	D	D	UD
I	$L_1 \cap L_2$ is same type?	D	UD^I	UD	UD	UD	UD
Halting	D	D	D	D	D	UD.

- Arbitrary CFGs G, G_1, G_2 & arbitrary Regex R, undecidable - whether $L(R) \subseteq L(G)$, whether $L(G)$ is DCFL, whether $L(G) = L(R)$.
  decidable - whether $L(G) \subseteq L(R)$ [test $L(G) \cap \overline{L(R)} = \phi$ or not]

- Arbitrary DCFGs G, G_1, G_2 & arbitrary regex R, decidable - whether $L(G) = L(R)$, whether $L(G) \subseteq L(R)$, whether $L(R) \subseteq L(G)$, whether $L(G)$ is CFL. (trivial)

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TOC	(related to) $\longrightarrow$	Compiler
FA	RL	Lexical Analysis
PDA	CFG	Syntax "
LBA	CSL	Semantic "
TM	REL	Logic (Whole compiler)

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TOC	Algo / Program
FA	Algo w/o using any memory
PDA	Algo using 1 stack (Palindrome)
LBA	Bounded memory
TM	Any algo

* (DFA = NFA) < DPDA < NPDA < LBA < (NTM = DTM)

* FA with 1 stack = PDA
FA with 2 stacks = TM.

* Automata with a queue ≈ TM

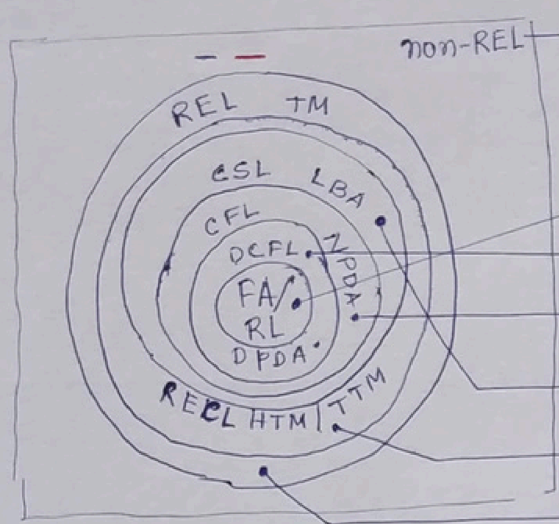
* TM with 3 states ≈ TM ≈ Multitape TM with 'stay' & at most 2 states. ≈ NPDA with 2 independent stacks.

* NDTM with only stack - PDA
TM with finite tape = FA
TM with part of tape only where ip is present = LBA (use to check CSL)

* L accepted by LBA

$\{a^n b^n c^n \mid n \geq 1\}$, $\{a^{n!} \mid n \geq 0\}$, $\{a^n \mid n \text{ prime}\}$, $\{a^n, n = m^2, m \geq 1\}$, $\{a^n \mid n \text{ not prime}\}$, $\{ww \mid w \in (a,b)^+\}$, $\{w^n \mid w \in (a,b)^+, n \geq 1\}$, $\{www^r \mid w \in (a,b)^+\}$

Closure property



→ (undecidable) (no TM exists) eg. set of real numbers, 2^{N} (power set of N), 2^{Σ^*}.

→ $a^n b^m$

→ $a^n b^n$

→ ww^r

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→ $a^n b^n c^n$.

→ Turing decidable (Decidable)

→ Turing recognizable (undecidable)

* Closure properties of Formal lang.

	RL	DCFL	CFL	CSL	RECL (Turing Decidable)	REL (Turing recognizable, Enumerable, Semi-d…)
✓ Union (finite)	✓	✗	✓	✓	✓	✓
✓ Intersection (finite)	✓	✗	✗	✓	✓	✓
✓ Set difference (finite)	✓	✗	✗	✓	✓	✗
✓ Complementation	✓	✓	✗	✓	✓	✗
✓ Intersection with a RL	✓	✓	✓	✓	✓	✓
✓ Concatenation (finite)	✓	✗	✓	✓	✓	✓
Kleene closure	✓	✗	✓	✓	✓	✓
Positive closure	✓	✗	✓	✓	✓	✓
✓ Reversal	✓	✗	✓	✓	✓	✓
Homomorphism	✓	✗	✓	✗	✗	✓
ε-free homomorphism	✓	✗	✓	✓	✓	✓
Inverse homomorphism	✓	✓	✓	✓	✓	✓
Substitution	✓	✗	✓	✗	✗	✓
ε-free substitution	✓	✗	✓	✓	✓	✓
Right quotient with a RL	✓	✓	✓	✗		✓
Left quotient with a RL	✓	✓	✓	✗		✓
✓ Union with RL	✓	✓	✓			
Left difference with RL		✓	✗			
Right difference with RL		✓	✓			
✓ Infinite union	✗	✗	✗	✗	✗	✗
✓ Infinite intersection	✗	✗	✗	✓	✓	✓
Infinite difference	✗					
Infinite concatenation	✗					
✓ Prefix	✓	✓	✓			
Suffix						
Cycle	✓		✓			
Min		✓				
Max		✓				
XOR / Symmetric Difference	✓		✗			
NAND			✗			
NOR	✓		✗			
COR	✓					

	RL	DCFL	CFL	CSL	RECL	REL
Square root of L, $\sqrt{L}$	✓					
Square of L	✗					
Shuffle(L_1, L_2)	✓					
One-third of L	✓					
Half of L	✓					
Subsequence	✓					
Subword	✓					
✓ Subset	✗		✗			
✓ Superset			✗			

• Decd^g

	RL	DCFL	CFL	RECL	REL
Membership	D	D	D	D	UD (Semidec) ~~but~~ und.
Halting	D	D	D	D.	UD (Semi dec.) but deci.
Emptiness	D	D	D	UD (non-re)	UD (non-re)
Finiteness	D	D	D	UD (non-re)	UD (non-re)
Totality	D	D	UD (non-re)	UD (non-re)	UD (non-re)
Equivalence	D	D	UD (non-re)	UD (non-re)	UD (non-re)
Disjoint	D	UD (non-re)	UD (non-re)	UD (non-re)	UD (non-re)
Set containment	D	UD (non-re)	UD (non-re)	UD (non-re)	UD (non-re)
Ambiguity	D	UD	UD	UD	UD

Marvelous humble employee failed to equate dogs & cats. & ants.

	RL	DCFL	CFL	RECL	REL
Regularity	D	D	UD ? ~~non-re~~	UD ? ~~non-re~~	UD non-re

Non-membership

Decidable upto RECL.

For REL, undecidable (non-RE).

Non-emptiness

$\{\langle M \rangle \mid L(M) \neq \phi\}$

For TM, it is semidecidable.

Non-equivalence

non-re for TM.

SD for PDA, HTM.

Decision problem	RL	DCFL	CFL	RECL	REL
Membership	D	D	D	D	UD RE, not REC
Halting	D	D	D	D	UD RE, not REC
Emptiness	D	D	D	UD non-RE	UD non-RE
Finiteness	D	D	D	UD non RE (nr)	UD (nr)
Totality	D	D	UD	UD nr	UD nr
Equivalence	D	D	UD nr	UD nr	UD nr
Disjoint	D	UD nr	UD nr	UD nr	UD nr
Set containment	D	UD nr	UD nr	UD nr	UD nr
Ambiguity	D	UD nr	UD nr	UD nr	UD nr
Regularity	D	D	UD	UD	UD non-re

- Non-membership: Decidable upto RECL for REL, non-RE.
- Non emptiness: $\{ \langle M \rangle \mid L(M) \neq \phi \}$ For RE, semidecidable
- Non - equivalence: For CFL, RECL semidecidable For REL, non-RE.

Marvelous humble employee failed to equate dogs, cats, ants, rats.