GATE CSE NOTES

by

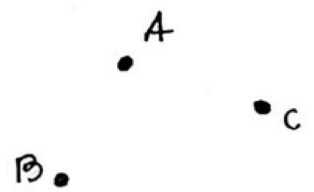
UseMyNotes

* <u>Graph</u> : Ordered pair of set of vertices & a set of edges.

$G = (V, E)$. V Set of vertices
E Set of edges connecting the vertices.

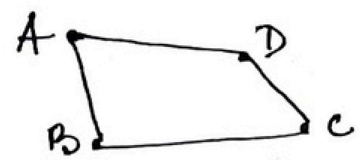
* <u>Types of Graphs</u> :

1. <u>Null Graph</u> : A graph whose edge set is empty.



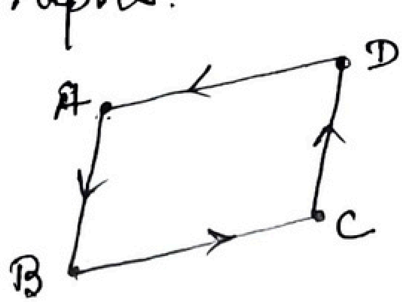
2. <u>Trivial Graph</u> : A graph having only one vertex.

3. <u>Non-directed Graph</u> : All the edges are undirected.

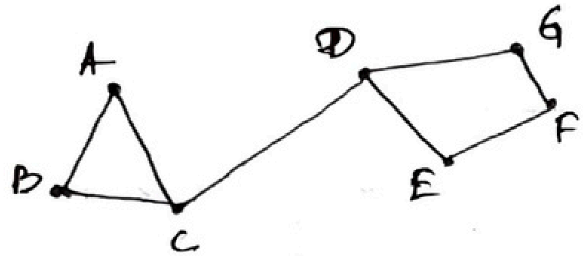


4. <u>Directed Graph</u> : All the edges are directed.

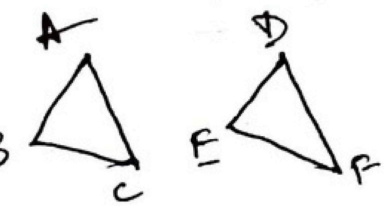
Also called digraphs.



5. **Connected Graph** : We can visit from any one vertex to any other vertex. One path exists between every pair of vertices.

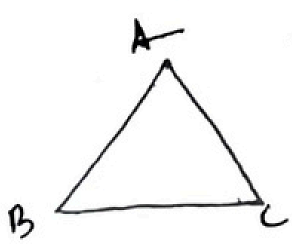


6. **Disconnected Graph** : Does not exist any path between at least one pair of vertices.

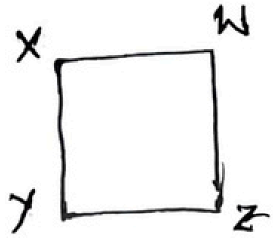


These graph consists of (at least) 2 independent components which are disconnected.

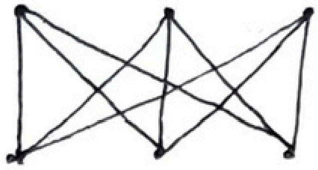
7. **Regular graph** : All the vertices have the same degree. It's a k-regular graph when all vertices are of degree k.



2-regular



2-regular

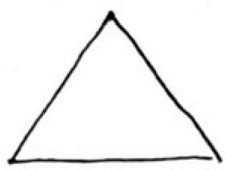


3- regular.

8. <u>Complete Graph</u> : Exactly one edge is present between every pair of vertices.

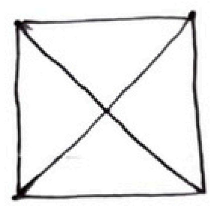
* A complete graph of n vertices contains $^{n}C_2$ edges. $\frac{n(n-1)}{2}$

Represented as K_n [$n \rightarrow$ no. of vertices].



d = 2

K_3

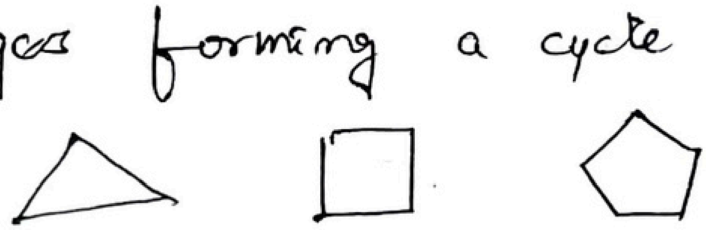


d = 3.

K_4

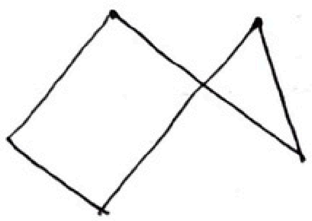
Each vertex is connected with all the remaining vertices through exactly one edge. Each vertex in K_n has degree $(n-1)$.

9. <u>Cycle Graph</u> : Simple graph of n vertices $(n \geq 3)$ & n edges forming a cycle of length n.

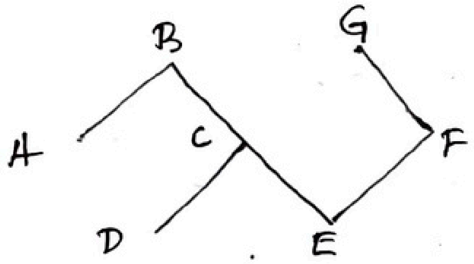


All vertices of degree 2 in cycle graph

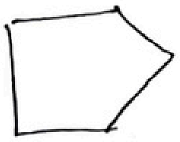
10. <u>Cyclic Graph.</u> : Graph containing at least one cycle in it.



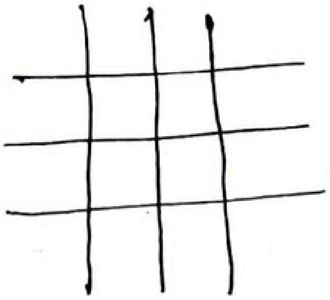
11. <u>Acyclic graph</u> : Graph not containing any cycle in it.



12. <u>Finite Graph</u> : Graph consisting of finite no. of vertices & edges.



13. <u>Infinite Graph.</u> : Graph consisting of infinite vertices & edges.



14. Bipartite Graph :

* A graph where -

- Vertices can be divided into 2 sets X & Y.
- Vertices of set X only join with the vertices of set Y.
- None of vertices belonging to the same set join each other.

15. Planar Graph : A graph that we can draw on a plane such that no 2 edges cross each other.

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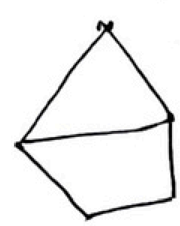
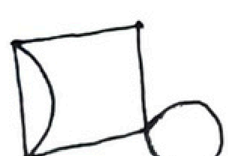


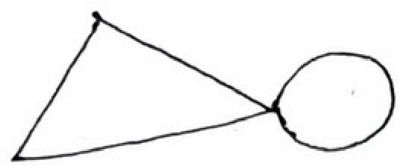
16. Simple Graph : Graph having no self loops & no parallel edges.



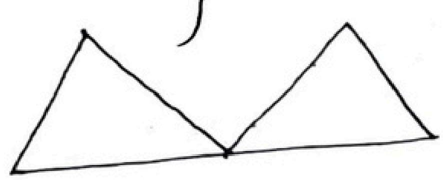
not a simple graph.

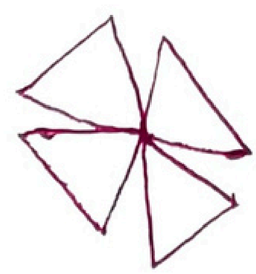
17. <u>Multi Graph</u> : Graph having no self loops but having parallel edges.

18. <u>Pseudo Graph</u> : A graph having no parallel edges but having having self loops



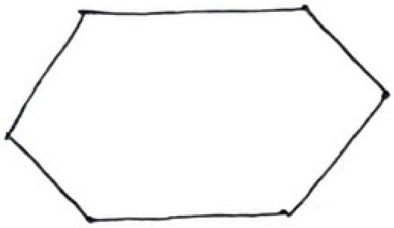
19. <u>Euler Graph</u> : Connected graph in which all vertices are of even degree.



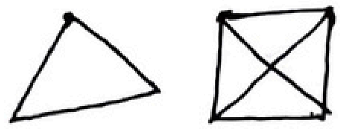


20. <u>Hamiltonian Graph</u> :

There exists a closed walk in the connected graph that visits every vertex of the graph exactly once (except starting vertex) without repeating the edges.



- Every polygon is a 2-regular graph.
- Every complete graph of n vertices is a (n-1) regular graph.



- Every regular graph need not be a complete graph.

-

	Self Loops	Parallel Edges
Graph	✓	✓
Simple	×	×
Multi	×	✓
Pseudo	✓	×

* Planar Graph.

$v + f - e = 2$

$v + f - e = k + 1$ # compound

$3f \leq 2e$

• Regions of Plane (or faces in planar graph)

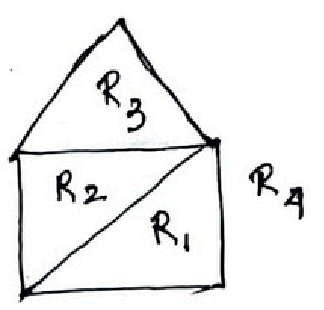
The planar representation of a graph splits the plane into connected area called as regions of the plane.

Each region has some degree associated with it given as -

Degree of interior region = No. of edges enclosing that region

Degree of exterior region = No. of edges exposed to the region

eg.



$D(R_1) = 3$

$D(R_2) = 3$

$D(R_3) = 3$

$D(R_4) = 5$

• Chromatic Number of Planar Graph.

Chromatic no. of any planar graph is always less than or equal to 4. Thus, any planar graph requires maximum

4 colors for coloring its vertices.

- **Planar graph properties.**

(1) In any planar graph, sum of degrees of all the vertices = 2 × total no. of edges in the graph.

$$\sum_{i=1}^{n} \deg(v_i) = 2|E|.$$

(2) Total sum of degrees of all the regions = 2 × no. of edges in graph

$$\sum_{i=1}^{n} \deg(r_i) = 2|E|.$$

2.i. Case i If degree of each region is K, then

*

$$K \times |R| = 2 \times |E|.$$

2.ii. Case ii If degree of each region is at least K ($\geq K$) then

5 + 3 − 6
2 − 1·1

$$K \times |R| \leq 2 \times |E|.$$

↑ sum of degrees must be greater than or equal to this.

2.iii <u>Case iii</u> If degree of each region is at most K ($\leq K$) then

$$K \times |R| \geq 2 \times |E|.$$

③. If G is a connected planar simple graph with e edges, v vertices & r no. of regions in the planar representation of G, then

$$r = e - v + 2. \quad \text{[Euler's Formula.]}$$

$v + r - e = 2$

$v + r - e = K + 1$

It remains same in all the planar representation of the graph.

④ If G is a planar graph with K components, then

$$r = e - v + (K+1)$$

Q. Let G be a connected planar simple graph with 25 vertices & 60 edges. Find the no. of regions in G.

$\rightarrow$ $r = e - v + 2$

$= 60 - 25 + 2$

$= 37.$

Q Let G be a planar graph with 10 vertices, 3 components & 9 edges. Find no. of regions.

$\rightarrow$ $r = e - v + (K+1)$

$= 9 - 10 + (3+1)$

$= 3.$

Q Let G be a connected planar simple graph with 20 vertices & degree of each vertex is 3. Find no. of regions.

| $K \times f = 2e.$

$\rightarrow$ $20 \times 3 = 2 \times e$

$\Rightarrow e = 30.$

$r = e - v + 2. = 30 - 20 + 2 = 12.$

Q G a connected planar simple graph with 35 regions, degree of each region is 6. Find the no. of vertices in G.

$\rightarrow$ $35 \times 6 = 2 \times e \Rightarrow e = 105$

$r = e - v + 2 \Rightarrow 35 = 105 - v + 2$

$\Rightarrow v = 72.$

Q G be a connected planar graph with 12 vertices, 30 edges, degree of each region is k. Find value of k.

→ $r = e - v + 2$

$\Rightarrow r = 30 - 12 + 2 = 20$

$20 \times k = 2 \times 30$

$\Rightarrow k = 3.$

faces in $K_{2,3}$ (compl bipar

$e = 2 \times 3 = 6$

$v = 3 + 2 = 5$

$v + f - e = 2$

$\Rightarrow f = e - v + 2$

$= 3$

Q. What is the maximum no. of regions possible in a simple planar graph with 10 edges?

→ $3 \times |R| \leq 2 \times |E|$

$\Rightarrow 3 \times |R| \leq 2 \times 10$

$\Rightarrow |R| \leq 6.67.$

$|R|_{max} = 6.$

In simple planar graph, deg(region) ≥ 3.

face surrounded by 3 edges at least

Q Min. no. of edges necessary in a simple planar graph with 15 regions.

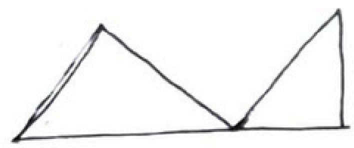
→ $3 \times |R| \leq 2 \times |E|$

$\Rightarrow 3 \times 15 \leq 2 \times |E|$

$\Rightarrow |E| \geq 23$

$|E|_{min} = 23$

* Euler Graphs. (Closed trail connecting all edges)



Euler path || Walk through the graph that uses every edge exactly once.

Euler Path

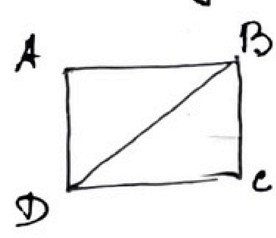
Also, Euler trail, or Euler walk

If there exists a walk in the connected graph that visits every edge of the graph exactly once with or without repeating the vertices, then such a walk is Euler walk.

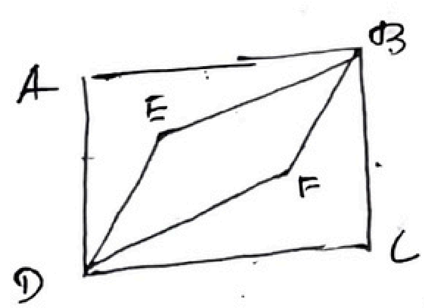
(A graph will contain an Euler path if & only if it contains at most 2 vertices of odd degree.)

8 - 6 + 2

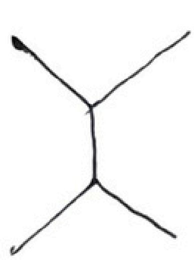
e.g.



B C D B A D



B C D F B E D A B



Euler path does not exist.

Euler Circuit :

Also, Euler cycle or Euler tour.

Euler path that starts & ends at same vertex.

If there exists a circuit in the connected graph that contains all the edges of the graph, then that circuit is called Euler circuit.

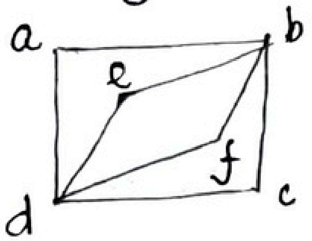
If there exists a walk in the connected graph that starts & ends at the same vertex & visits every edge of the graph exactly once with or without repeating the vertices, then such a walk is an Euler circuit.

An Euler trail that starts & ends at the same vertex is called as an Euler circuit.

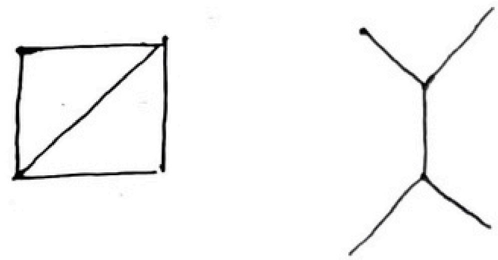
A closed Euler trail is an Euler circuit.

- A graph will contain an Euler circuit iff all its vertices are of even degree.

eg.



Euler Circuit
b c d a b e d f b



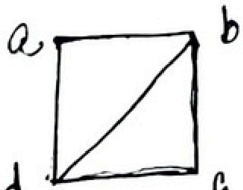
Euler circuit does not exist.

Semi - Euler Graph

If a connected graph contains an Euler trail but does not contain an Euler circuit, then such a graph is a semi - Euler graph.

For a graph to be a semi - Euler graph,

- Graph must be connected.
- Graph must contain an Euler trail.



$\longrightarrow$ euler path - b d a b c d

no euler circuit.

Notes

(1) To check whether any graph is an Euler graph or not, any one of the following two ways may be used —

If the graph is connected & contains an Euler circuit, then it's Euler.

If all the vertices of the graph are of even degree, then it's Euler.

(2) To check whether any graph contains an Euler circuit or not,

Just make sure that all its vertices are of even degree.

(3) To check whether graph is semi-Euler or not,

Just make sure that it is connected & contains an Euler trail.

(4) To check whether any graph contains an Euler trail or not,

Just make sure that the no. of vertices in the graph with odd degree are not more than 2.

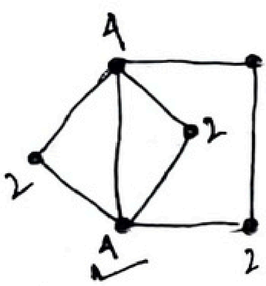
All vertices even degree ⇒ euler circuit

vertices with odd degree ≤ 2 ⇒ euler path

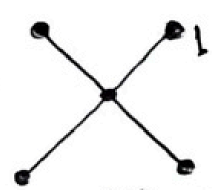
(5) A graph will definitely contain an Euler trail if it contains an Euler circuit. A graph may or may not contain an Euler circuit if it contains an Euler trail.

(6) An Euler graph is definitely a semi-Euler graph. Converse is not true always.

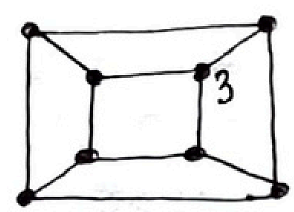
Q Which are Euler graphs?

A


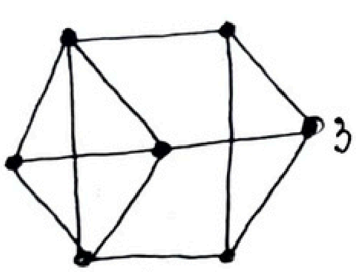
✓

B


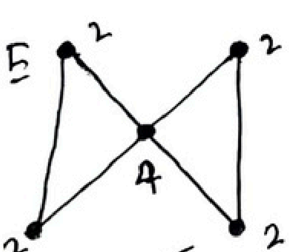
X

C


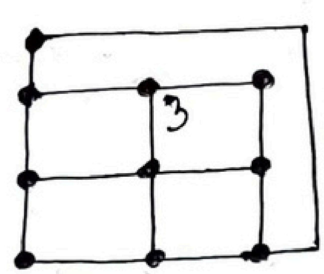
X

D


X

E


✓

F


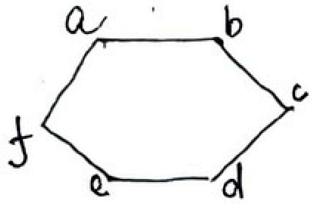
X

If all vertices of the graph are of even degree, then Euler graph.

* Hamiltonian Graph

If there exists a closed walk in the connected graph that visits every vertex of the graph exactly once (except starting vertex) without repeating the edges, then such a group is called as a Hamiltonian graph.

Any connected graph that contains a Hamiltonian circuit is a Hamiltonian graph.

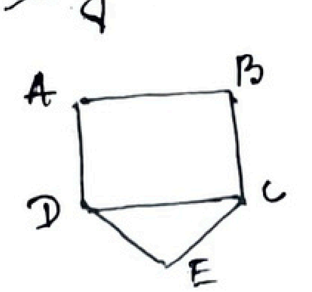


a b c d e f a

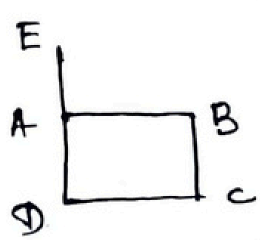
Hamiltonian Path.

If there exists a walk in the connected graph that visits every vertex of the graph exactly once without repeating the edges, then such a walk is called Hamiltonian path.

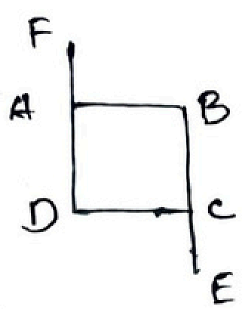
In Hamiltonian path, all the edges may or may not be covered but edges must not repeat.



ABCDE



EABCD



H. path does not exist.

* If $G = (V,E)$ is a simple graph having n vertices & for each $v \in V$ we have $deg(v) \geq n/2$, then G is Hamiltonian.

Hamiltonian Circuit.

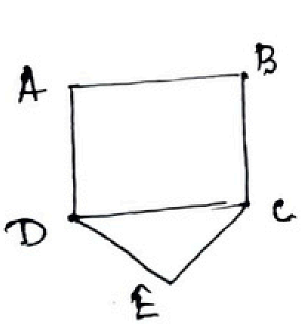
Also, Hamiltonian Cycle.

If there exists a walk in the connected graph that visits every vertex of the graph exactly once (except starting vertex) without repeating the edges & returns to the starting vertex, then such a walk is Hamiltonian circuit.

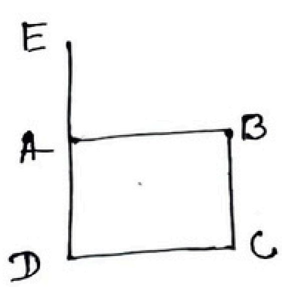
If there exists a cycle in the connected graph that contains all the vertices of the graph, then that cycle is a Hamiltonian circuit.

A Hamiltonian path that starts & ends at the same vertex is a Hamiltonian circuit.

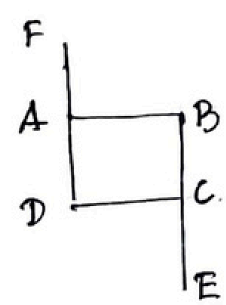
A closed Hamiltonian path is a Hamiltonian circuit.



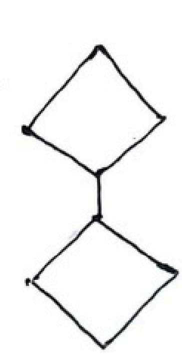
ABCEDA



H. circuit doesn't exist.



H. circuit doesn't exist.

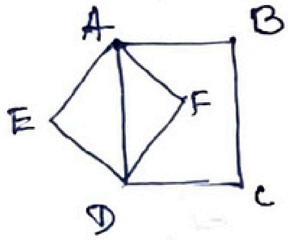


H. circuit doesn't exist.

- Any Hamiltonian circuit can be converted to a Hamiltonian path by removing one of its edges.
- Every graph that contains a Hamiltonian circuit also contains a Hamiltonian path but vice versa is not true.
- There may exist more than one Hamiltonian paths & Hamiltonian circuits in a graph.

Q Which are Hamiltonian graphs?

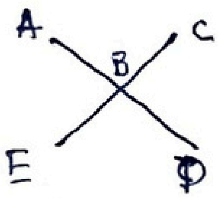
a)



Neither contains a H. path nor a H. circuit.
So, not a H. graph.

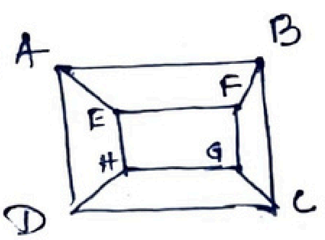
$n/2 = 3$

b)



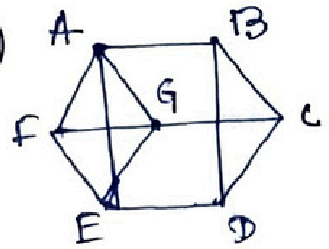
Not a H. graph.

c)

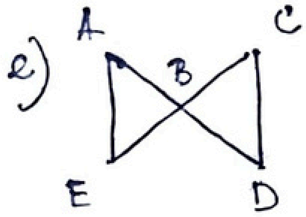


Contains H. ~~circuit~~ path - ABCDHGFE
Contains H. circuit - ABCDHGFEA
A H. graph

d)

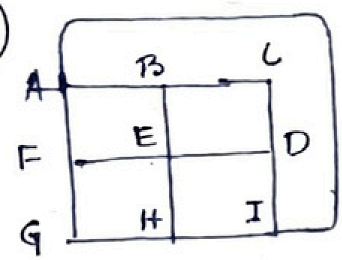


Contains H. path ABCDEFG
Contains H. circuit ABCDEFGA
A H. graph.

e) 

Neither contains a H. path nor a H. circuit.

Not a H. graph.

f) 

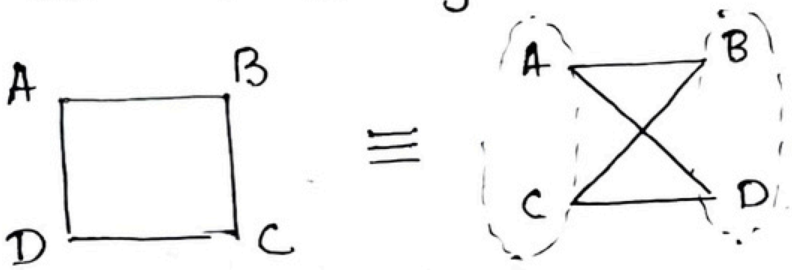
Contains H path ABCDEFGHI

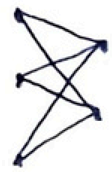
Contains H. circuit ABCDEFGHIA

A H. graph.

* <u>Bipartite Graph</u> ($K_{m,n}$) When complete. bipartite

Consists of 2 sets of vertices X & Y. The vertices of set X join only with the vertices of set Y. The vertices within the same set do not join.



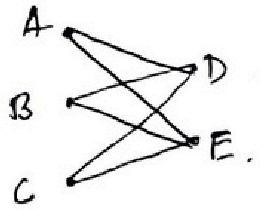


<u>Complete Bipartite Graph</u> ($K_{m,n}$)

A bipartite graph where every vertex of set X is joined to every vertex of set Y is called a complete bipartite graph.

~~It's a graph that is complete as well as bipartite.~~

• <u>A graph is bipartite if & only if it has no odd cycle.</u> → Join of 2 vertices inside one set.

e.g. 

Called $K_{3,2}$

Bipartite Graph Chromatic Number

To properly color any bipartite graph,

- minimum 2 colors are required,
- this ensures that the end vertices of every edge are colored with different colors.
- Thus, bipartite graphs are 2-colorable.

. If graph is bipartite with no edges, then it is 1-colorable.

Bipartite graph properties.

- Bipartite graphs are 2-colorable.
- It contains no odd cycles.
- Every subgraph of a bipartite graph is itself bipartite.

✓ There does not exist a perfect matching for a bipartite graph with bipartition X & Y if $|X| \neq |Y|$.

✓ In any bipartite graph with bipartition X & Y,

Sum of degree of vertices of set X = Sum of degree of vertices of set Y.

Bipartite Graph Perfect Matching.

✓ Number of complete matchings for $K_{n,n} = n!$

Given a bipartite graph G with bipartition X & Y,

✓. There does not exist a perfect matching for G if $|X| \neq |Y|$.

. A perfect matching exists on a bipartite graph G with bipartition X & Y if & only if for all the subsets of X, the no. of elements in the subset is less than or equal to the no. of elements in the neighbourhood of the subset.

Maximum number of edges.

✓ Any bipartite graph consisting of n vertices can have at most $\frac{1}{4}n^2$ edges.

Max. possible no. of edges in a bipartite graph of n vertices $= \frac{1}{4}n^2$

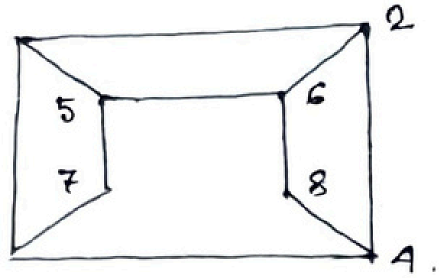
– Explanation → Bipartition be (V_1, V_2) where $|V_1| = K$ & $|V_2| = n-K$.

No. of edges between V_1 & V_2 can be at most $K(n-K)$ which is maximised at $K = \frac{n}{2}$. Thus, maximum $n^2/4$ edges can be present.

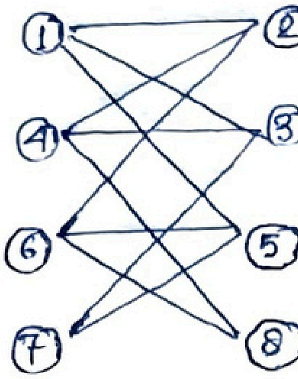
Also, for any graph with n vertices & more than $\frac{1}{4}n^2$ edges, it will contain a ✓ triangle. This is not possible in a bipartite graph since bipartite graphs contain no odd cycles.

Q. Bipartite or not ? ⇒ Redrawn as.

V_1	V_2
1	2
	3
6	8
7	5
A	



Bipartite.

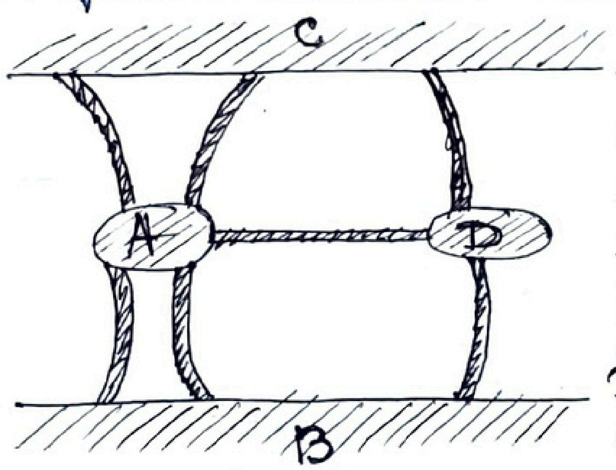


$\{1, 4, 6, 7\}$
$\{2, 3, 5, 8\}$

Q Max no. of edges in a bipartite graph of 12 vertices __36__.

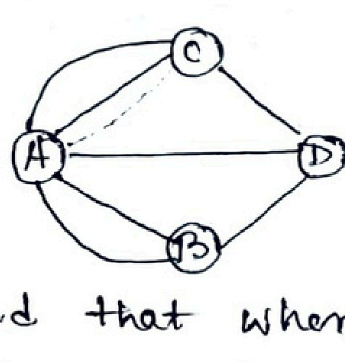
→ $\frac{1}{4}(12)^2 = 36$

* Königsberg Bridge Problem.



Starting from any of the four lands A, B, C, D, is it possible to cross each of the 7 bridges exactly once & come back to the starting point without swimming across the river ?

Graph representation —



Euler observed that when a vertex is visited during the process of tracing a graph,
— There must be one edge that enters into the vertex.

- There must be another edge that leaves the vertex.

- Therefore, order of the vertex must be an even number.

Euler discovered that it depends on the no. of odd vertices present in the network whether any network is traversable or not.

Euler found that only those networks are traversable that have either -

- No odd vertices (then any vertex may be the beginning & the same vertex will also be the ending)

* - Or exactly 2 odd vertices (then one odd vertex will be the starting point & other odd vertex will be the ending point).

- Since, the Königsberg network has 4 odd vertices, therefore the network is not traversable.

* <u>Handshaking Lemma.</u> (Sum of degree Theorem)

In any graph, sum of degree of all the vertices is twice the number of edges contained in it.

$$\sum_{i=1}^{n} d(v_i) = 2 \times |E| \quad \Big| \quad \sum_{v \in V} d(v) = 2|E(G)|$$

<u>Cor.</u>

i) Sum of degree of all vertices is even.

ii) Sum of degree of all vertices with odd degree is even.

iii) No. of vertices with odd degree is even.

Q Graph G (simple) has 24 edges & degree of each vertex is 4. Find no. of vertices.

→ Sum of degree of all vertices = 2 × no. of edges

$\Rightarrow n \times 4 = 2 \times 24$

$\Rightarrow n = 12.$

Q Graph contains 21 edges, 3 vertices of degree 4 & all other vertices of degree 2. Find total no. of vertices.

→ $3 \times 4 + 2(n-3) = 2 \times 21$

$\Rightarrow n = 18.$

Q A simple graph contains 35 edges, 4 vertices of degree 5, 5 vertices of degree 4 & 4 vertices of degree 3. Find no. of vertices with degree 2.

→ $4 \times 5 + 5 \times 4 + 4 \times 3 + n \times 2 = 2 \times 35$

$\Rightarrow n = 9.$

Q Graph has 24 edges & degree of each vertex is k, then which of the following is possible no. of vertices? 20 / 15 / 10 / 8.

→ $k \times n = 2 \times 24 = 48$

$k = \frac{48}{n}$

For compatible k,

$n = 20, \ k = 2.4$ not allowed

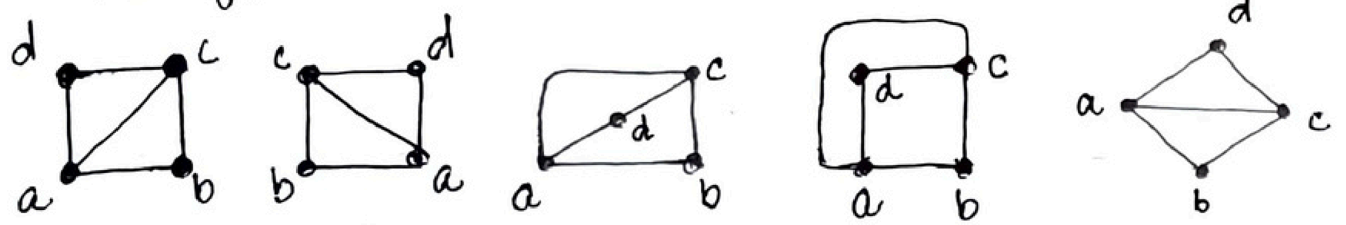
$n = 15, \ k = 3.2$ 〃

$n = 10, \ k = 4.8$ 〃

$n = 8, \ k = 6.$ ✓

* Graph Isomorphism.

Existance of same graph in more than one forms.



✓ **Conditions**

i) No. of vertices in both graphs must be same.

ii) No. of edges same.

iii) Degree sequence of both must be same

iv) If a cycle of length k is formed by the vertices $\{v_1, v_2, \dots, v_k\}$ in one graph, then a cycle of same length k must be formed by the vertices $\{f(v_1), f(v_2) \dots, f(v_k)\}$ in other graph as well.

[Degree sequence – Sequence of the degree of all the vertices in ascending order].

These conditions are just the necessary ones. They're not sufficient.

Sufficient conditions.

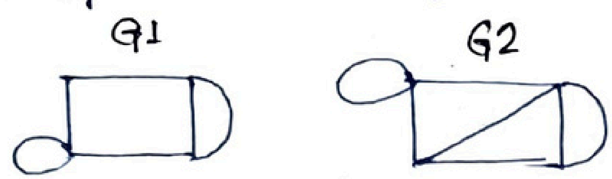
If any of these conditions satisfies, it can be said that graphs are isomorphic –

i) Complement graphs are isomorphic.

ii) If their adjacency matrix are same.

iii) Corresponding subgraphs obtained by deleting some vertices of one graph & their corresponding images in the other graph are isomorphic.

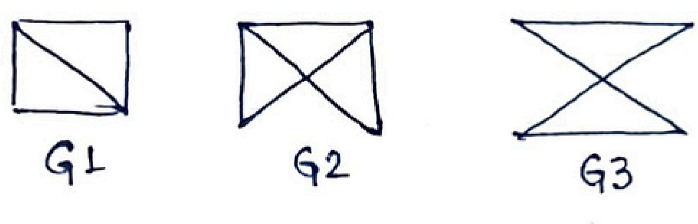
Q Isomorphic or not?



→ No. of vertices same.
No. of edges different.

Not isomorphic.

Q.



→ No. of edges different.

Not isomorphic, altogether. (G_1, G_2) & G_3

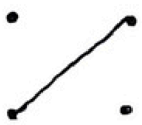
But, G1 & G2 may be isomorphic

G1 & G2 have same degree sequence.
$\{2, 2, 3, 3\}$.

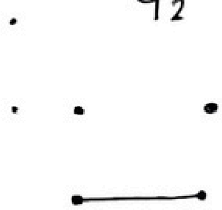
Both contain 2 cycles each of length 3 formed by the vertices having degrees $\{2, 3, 3\}$.

Sufficient conditions -

$\overline{G}_1$



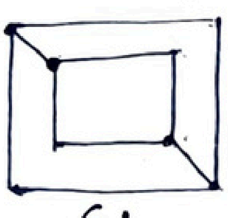
$\overline{G}_2$



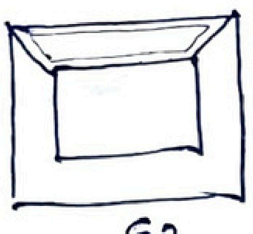
$\therefore$ $\overline{G}_1$ & $\overline{G}_2$ are isomorphic.

G_1 & G_2 are isomorphic.

<u>Q</u> Isomorphic or not ?



G1



G2

→ No. of vertices & edges same.

Degree sequence same $\{2, 2, 2, 2, 3, 3, 3, 3\}$.

In G2, degree 3 vertices form a cycle of length 4 which is not present in G1.

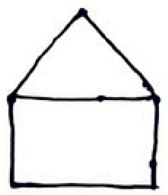
So, G1 & G_2 not isomorphic.

* Complement of Graph.

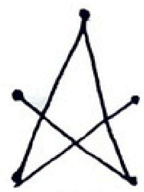
Complement of a simple graph G is a simple graph G' having -

all the vertices of G ,

an edge between 2 vertices v & w iff there exists no edge between v & w in G.



G



$\overline{G}$

Relationship between G & $\overline{G}$

1. No. of vertices same.

2. Sum of total no. of edges in G & G' is equal to the total no. of edges in a complete graph.

$$|E(G)| + |E(G')| = C(n,2) = \frac{n(n-1)}{2}$$

- Order of graph - Total no. of vertices
- Size of graph - Total no. of edges.

Q. Simple G has 10 vertices & 21 edges. Total no. of edges in G'.

$$\rightarrow \quad 21 + n = \frac{10(10-1)}{2}$$

$$\Rightarrow n = 24.$$

Q Simple G has 30 edges & its complement G' has 36 edges. Find no. of vertices in G.

→ $$30 + 36 = \frac{n(n-1)}{2} \Rightarrow n^2 - n - 132 = 0.$$

$$n = 12$$

Q Simple G of order n. If the size of G is 56 & size of G' is 80. What is n?

→ $$56 + 80 = \frac{n(n-1)}{2} \Rightarrow n^2 - n - 272 = 0.$$

$$n = 12$$

* Walks.

A walk is defined as a finite length alternating sequence of vertices & edges.

Total no. of edges covered in a walk is the length of the walk.

Open Walk

If length of walk is greater than zero & the vertices at which the walk starts & ends are different.

Closed walk.

If length of the walk is greater than zero & the vertices at which the walk starts & ends are same.

• If length of walk = 0, then it's trivial walk.

Both vertices & edges can repeat in a walk whether it's open or closed.

✓ Path.

Path is an open walk in which -

neither vertices (except start & end vertices) nor edges are allowed to repeat.

✓ Cycle

A closed walk in which -

neither vertices (except start & end v) nor edges are allowed to repeat.

Cycle is closed path.

✓ Trail

An open walk in which -

vertices may repeat, but edges aren't allowed to repeat.

✓ Circuit

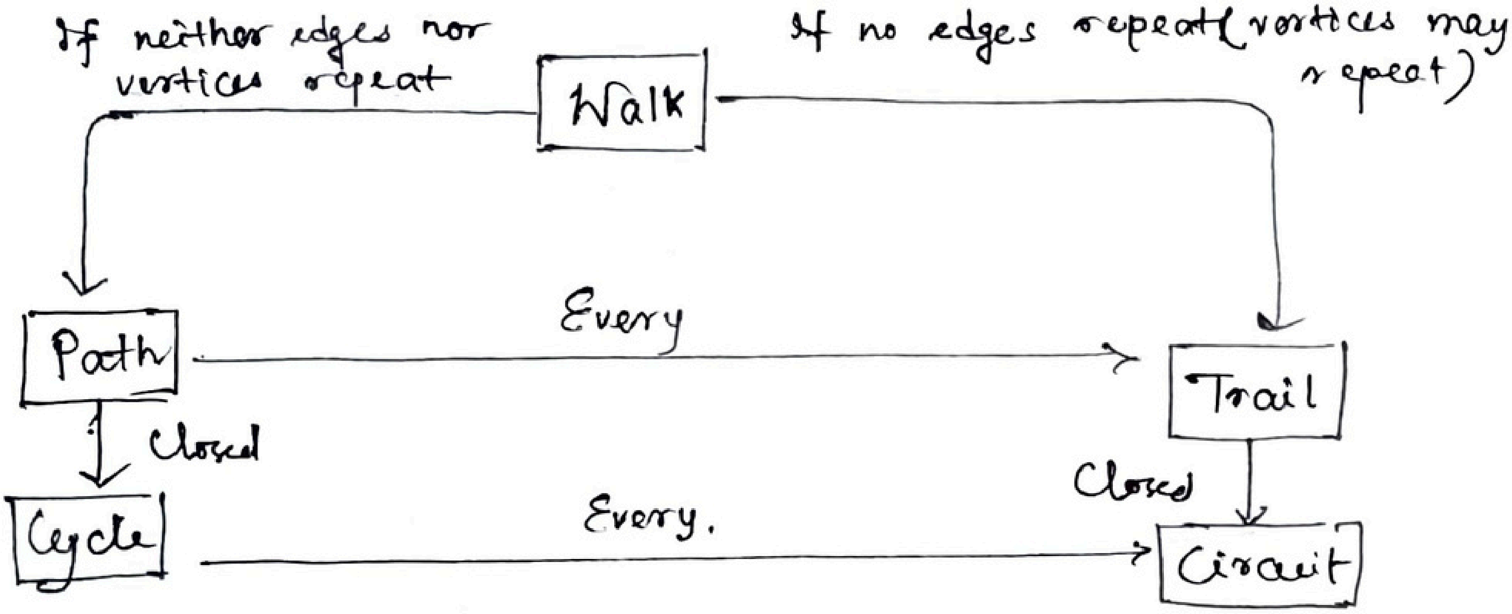
A closed walk in which -

vertices may repeat, but edges aren't allowed to repeat.

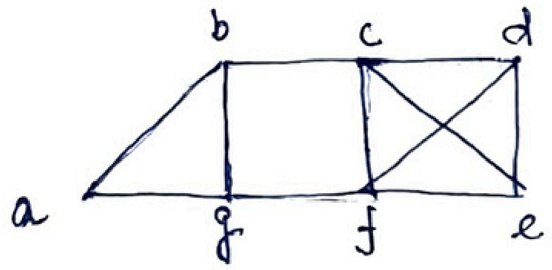
Closed trail is a circuit.

- Every path is a trail but every trail need not be a path.

  Every cycle is a circuit but every circuit need not be a cycle.



Q



Decide walks, path, cycle, trail or circuit.

1. a, b, g, f, c, b Trail.
2. b, g, f, c, b, g, a Walk
3. c, e, f, c Cycle.
4. c, e, f, c, e Walk
5. a, b, f, a Not a walk
6. f, d, e, c, b. Path.

* Graph Coloring. / Vertex Coloring

Process of assigning colors to the vertices of a graph such that no two adjacent vertices are of same color.

It ensures that there exists no edge in the graph whose end vertices are colored with same color. Such a graph is properly colored graph.

Applications.

Map coloring, scheduling tasks, preparing time table, assignment, conflict resolution, sudoku.

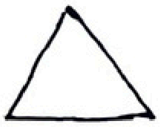
Chromatic Number.

✓ Minimum no. of colors reqd. to properly color any graph.

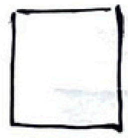
Chr. No. of Graphs.

1. Cycle Graph. : A simple graph of n $(n \geq 3)$ vertices & n edges forming a cycle of length n is called cycle graph.

✓ $\begin{cases} \text{If } n \text{ is even, then C.N.} = 2 \\ \text{If } n \text{ is odd, then C.N.} = 3. \end{cases}$



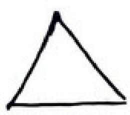
$CN = 3$



$CN = 2$.

2. Planar Graph.

✓ CN of any planar graph is < or = 4.



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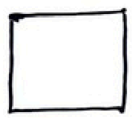
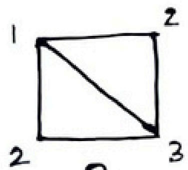


2



3

3. Complete Graph

✓ CN = No. of vertices.

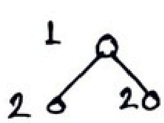
4. Bipartite Graph

✓ CN = 2.

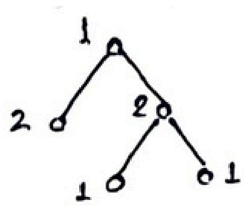
5. Trees

Connected graph in which there are no circuits. Every tree is a bipartite graph.

✓ CN = 2.



CN = 2



CN = 2.

• Graph Coloring Algorithm.

There exists no efficient algorithm for coloring a graph with min. no. of colors. Graph coloring is a NP complete problem.

– Greedy algo

1. Color first vertex with the first color.

2. Consider remaining $(v-1)$ vertices one by one & do –

a) Color the currently picked vertex with the lowest numbered color if it has not been used to color any of its adjacent vertices.

b) If it has been used, then choose the next least numbered color.

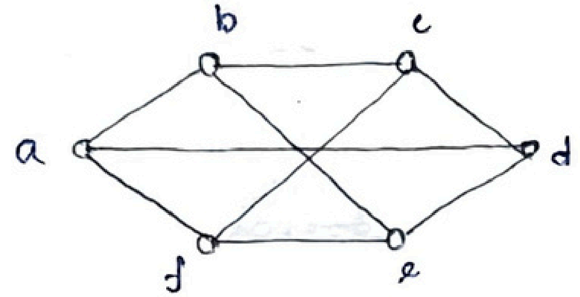
c) If all colors have been used, then assign a new color to the currently picked vertex.

– Drawbacks of Greedy algo

i) Doesn't always use min. no. of colors.

ii) No. of colors used sometimes depend on the order in which the vertices are processed

Q CN for the graph -

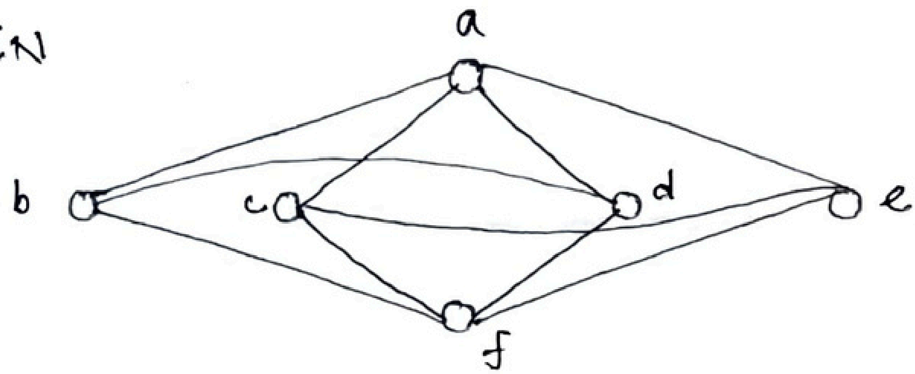


→ Applying greedy algo -

CN = 2.

a	b	c	d	e	F
C1	C2	C1	C2	C1	C2

Q CN



CN = 3.

→

a	b	c	d	e	f
C1	C2	C2	C3	C3	C1.