Wednesday, 25 September 2024 4:12 PM

4d. $H_0 : \beta = 0$

$Y = \beta x + \epsilon$

$t = \dfrac{\hat{\beta}}{SE(\hat{\beta})}$ Given $\hat{\beta} = \dfrac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$

$SE(\hat{\beta}) = \sqrt{\dfrac{s^2}{\sum_{i=1}^{n} x_i^2}}$ where $s^2 = \dfrac{1}{n-1} \sum (y_i - x_i\hat{\beta})^2$

$\therefore RSS = (\sum (y_i - x_i\hat{\beta})^2)$

expanding

$= \sum y_i^2 - 2\hat{\beta}\sum_{i=1}^{n} x_i y_i + \hat{\beta}^2 \sum_{i=1}^{n} x_i^2$

substituting $\hat{\beta}$

$\sum_{i=1}^{n} (y_i - x\hat{\beta})^2 = \sum_{i=1}^{n} y_i^2 - 2\left(\dfrac{\sum x_i y_i}{\sum x_i^2}\right)\sum x_i y_i + \left(\dfrac{\sum x_i y_i}{\sum x_i^2}\right)^2 \sum x_i^2$

$= \sum_{i=1}^{n} y_i^2 - \dfrac{(\sum x_i y_i)^2}{\sum x_i^2}$

$\therefore$ substituting $SE(\hat{\beta}) = \sqrt{\dfrac{1}{n-1} \cdot \dfrac{\sum y_i^2 \sum x_i^2 - (\sum x_i y_i)^2}{(\sum x_i^2)^2}}$

$= \sqrt{\dfrac{\sum y_i^2 \sum x_i^2 - (\sum x_i y_i)^2}{(n-1)(\sum x_i^2)^2}}$

$$\boxed{\therefore t = \hat{\beta}/SE(\hat{\beta}) = \dfrac{\sqrt{n-1}\ \sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2 - (\sum x_i y_i)^2}}}$$

4e.

$t_{y|x} = \dfrac{\sqrt{n-1}\ \sum x_i y_i}{\sqrt{\sum x_i^2 \sum y_i^2 - (\sum x_i y_i)^2}}$

$t_{x|y} = \dfrac{\sqrt{n-1}\ \sum y_i x_i}{\sqrt{\sum y_i^2 \sum x_i^2 - (\sum y_i x_i)^2}}$

Since $\sum y_i x_i = \sum x_i y_i$ – numerator identical

The d...

denominator are also same.

$\therefore t x|y = t y|x$