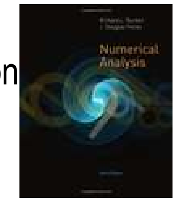
Analysis Numerical Analysis Pg. 16 Ex. 18 solutions

College

Numerical Analysis, 9th Edition



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Shashank Jaiswal 5.0 0

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Given $f(x) = (1-x)^{-1}$ and $x_0 = 0$

$$\begin{aligned} f(x) &= (1-x)^{-1} \\ f'(x) &= 1!(1-x)^{-2} \\ f''(x) &= 2!(1-x)^{-3} \\ f'''(x) &= 3!(1-x)^{-4} \\ &.. \\ &.. \\ f^n(x) &= n!(1-x)^{-(n+1)} \\ f^{n+1}(x) &= (n+1)!(1-x)^{-(n+2)} \end{aligned}$$

We have,

$f(0) = 1, \quad f'(0) = 1, \quad f''(0) = 2! \quad f'''(0) = 3!, ..., f^n(0) = n!, \quad f^{n+1}(0) = (n+1)!$

2

$k=0$ $k=0$

and **Remainder Term** associated with $P_n(x)$ is:

$$R_n(x) = \frac{f^{n+1}(\epsilon)}{(n+1)!}x^{n+1} = \frac{x^{n+1}}{(1-\epsilon)^{n+2}} \quad \text{for some } \epsilon \in [0, x]$$

$$|f(x) - P_n(x)| = |R_n(x)| = \left|\frac{x^{n+1}}{(1-\epsilon)^{n+2}}\right| \quad \text{for some } \epsilon \in [0, x]$$

On interval $[0, 0.5]$, we have to approximate $f(x)$ by $P_n(x)$ within error 10^{-6}. So,

$$\begin{aligned}\sup|R_n(x)| = \sup\left|\frac{x^{n+1}}{(1-\epsilon)^{n+2}}\right| &= (0.5)^{n+1} \leq 10^{-6} \\ &\implies (n+1)\log(0.5) \leq -6 \\ &\implies n+1 \geq \frac{-6}{\log(0.5)} \approx 19.93 \\ &\implies n \geq 18.93 \\ &\implies n = 19\end{aligned}$$

RESULT

For approximating $f(x) = (1-x)^{-1}$ by Taylor's Polynomial $P_n(x)$ within tolerance of 10^{-6}, on interval $[0, 0.5]$, minimum value of $n = 19$. Click here to see detailed solution.

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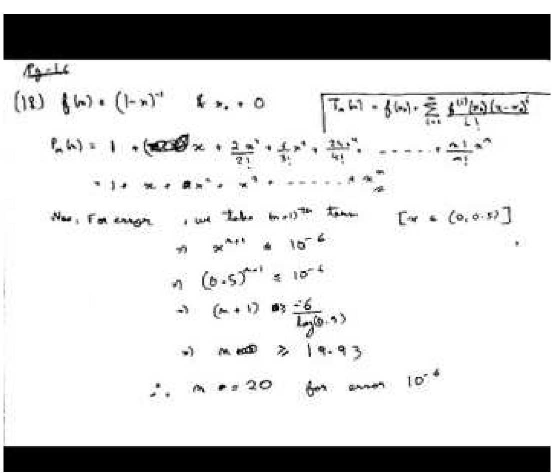

Pg-16

(18) $f(x) = (1-x)^{-1}$ & $x_0 = 0$ $\quad T_n(x) = f(x_0) + \sum_{i=1}^{n} \frac{f^{(i)}(x_0)(x-x_0)^i}{i!}$

$P_n(x) = 1 + x + \frac{2x^2}{2!} + \frac{6}{3!}x^3 + \frac{24x^4}{4!} + \cdots + \frac{n!}{n!}x^n$

$= 1 + x + x^2 + x^3 + \cdots + x^n$

Now, For error, we take $(n+1)^{th}$ term $[x \in (0, 0.5)]$

$\Rightarrow x^{n+1} \leq 10^{-6}$

$\Rightarrow (0.5)^{n+1} \leq 10^{-6}$

$\Rightarrow (n+1) \geq \frac{-6}{\log(0.5)}$

$\Rightarrow n \geq 19.93$

$\therefore n = 20$ for error 10^{-6}

RESULT

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