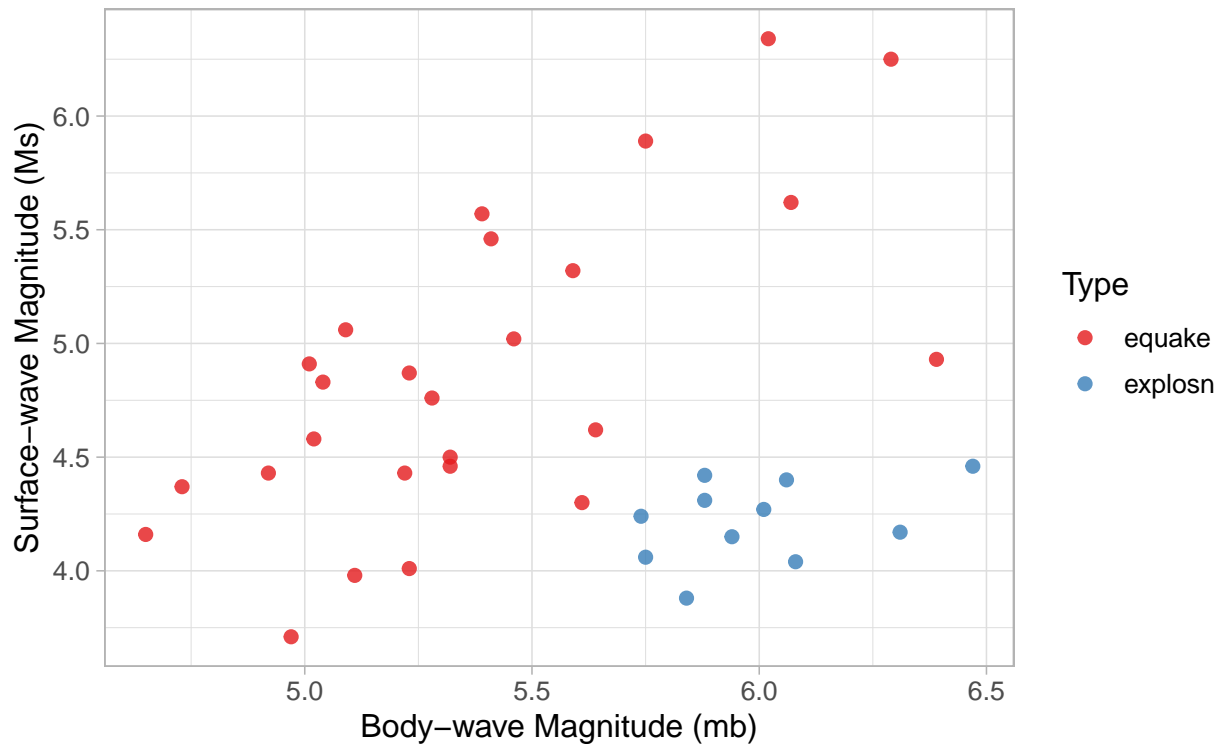# Report_10883408

10883408

2024-04-08

ML part A

Body−wave Magnitude vs. Surface−wave Magnitude
Comparing earthquake types

The result plot by this code would have each earthquake or explosion event as a point in a space defined by its body-wave and surface-wave magnitudes. The colours are deviating the seismic events and this may be very critical for recognition of the patterns or clusters which are earthquake-related rather than explosions.

Clustering

In case there are evident clusters of distinct areas dominated by one type of event it may suggest that these two features are good for discrimination of earthquakes and explosions.

Overlaps

If the colours are overlapping significantly then such two features on their own are insufficient to separate between event types without some additional information or more complex modelling.

Contextual Relevance

In the monitoring of unauthorized nuclear tests, this makes the visualization process a tool used in a rapid assessment. if there exist some clear and distinctive seismic readings patterns that may show nuclear activities. Efficient discrimination between natural seismic events (earthquakes) and man-made seismic events (nuclear explosions) is very important in the area of global security and control of compliance with the international treaties including the Comprehensive Nuclear-Test-Ban Treaty(CTBT).

Numerical Summaries

Though the given code emphasizes on the visual analysis, numerical summaries (such as mean, median, variance, and histograms) are also needed for carrying out the data exploration process. Therefore, the MB and Ms summary (i.e. the summary of mb and Ms for each type) would complete this by defining the central tendencies and dispersion. This could also help in a statistical understanding of the magnitudes of each type significantly.

Justification

The use of a scatter plot is supported since it enables stakeholders to see the relationship between two continuous variables across categories. For such high stakes in nuclear monitoring, instant visual, and venerability were of paramount concern assessment coupled with thorough statistical analysis is mandatory. This is made easy by the plot which gives a brief, instant visual representation of the data in mentioned characteristics.

Box Plot with Jitter of Body–Wave Magnitude by Type
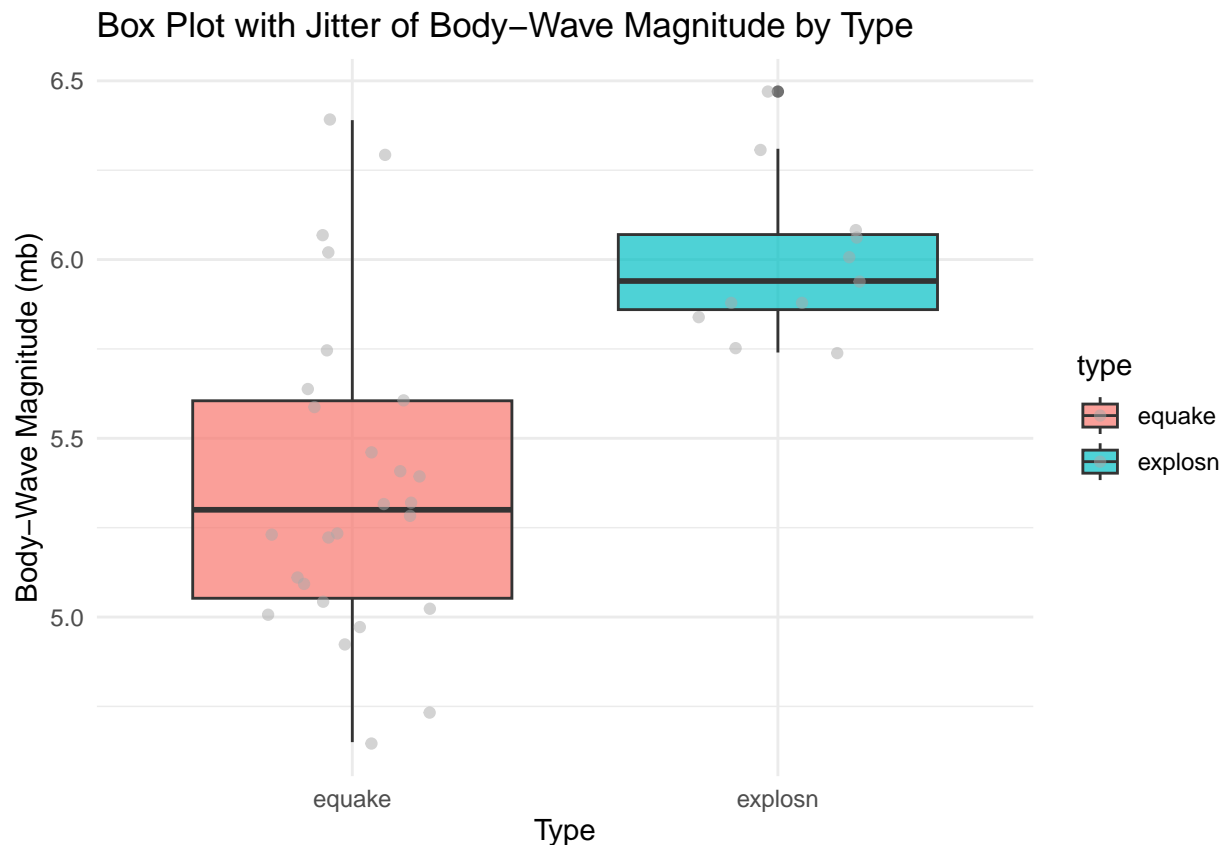
Explanation of the Plot

The jitter with box plot allows to see how body wave magnitudes are distributed within each one type of seismic event. Furthermore, the box plot component represents the median (the middle line in the box),

the 25th and 75th percentile whose positions determine the hinges of the box, and potential outliers (points that fall further than 1.5 times IQR from the hinges). Points jittered represent individual data points and provide a very fine level analyse the distribution of data and the possible anomalies or outliers.

Justification of the Statements

Distribution Insight

The plot helps in rapid detection of any significant differences in the magnitudes of the body-waves among various types of seismic events. For instance, if a certain type of event generally provides higher magnitudes reading, this implies a different energy release feature.

Outlier Detection

Through also showing the summary statistics and the actual data points, this plot assists in identifying the outliers or unusual observations that may need more attention.

Decision Making

This kind of visualization helps in decision support in seismology and geophysics by giving an easy way of comparison of seismic event types. This is likely to be critical in developing monitoring systems or academic research in seismology.

Effective Communication

The plot works as a powerful means of communication in both displaying complex statistical data in a way readable to everyone even those who do not have much of statistical knowledge. This cross between box plot and jitter plot works especially in situations where the variability both within and across categories is to be appreciated. It is an important fact finding tool that offers both an overall look and a more detailed show of how the data is distributed over categories.

Box Plot with Jitter of Surface–Wave Magnitude by Type
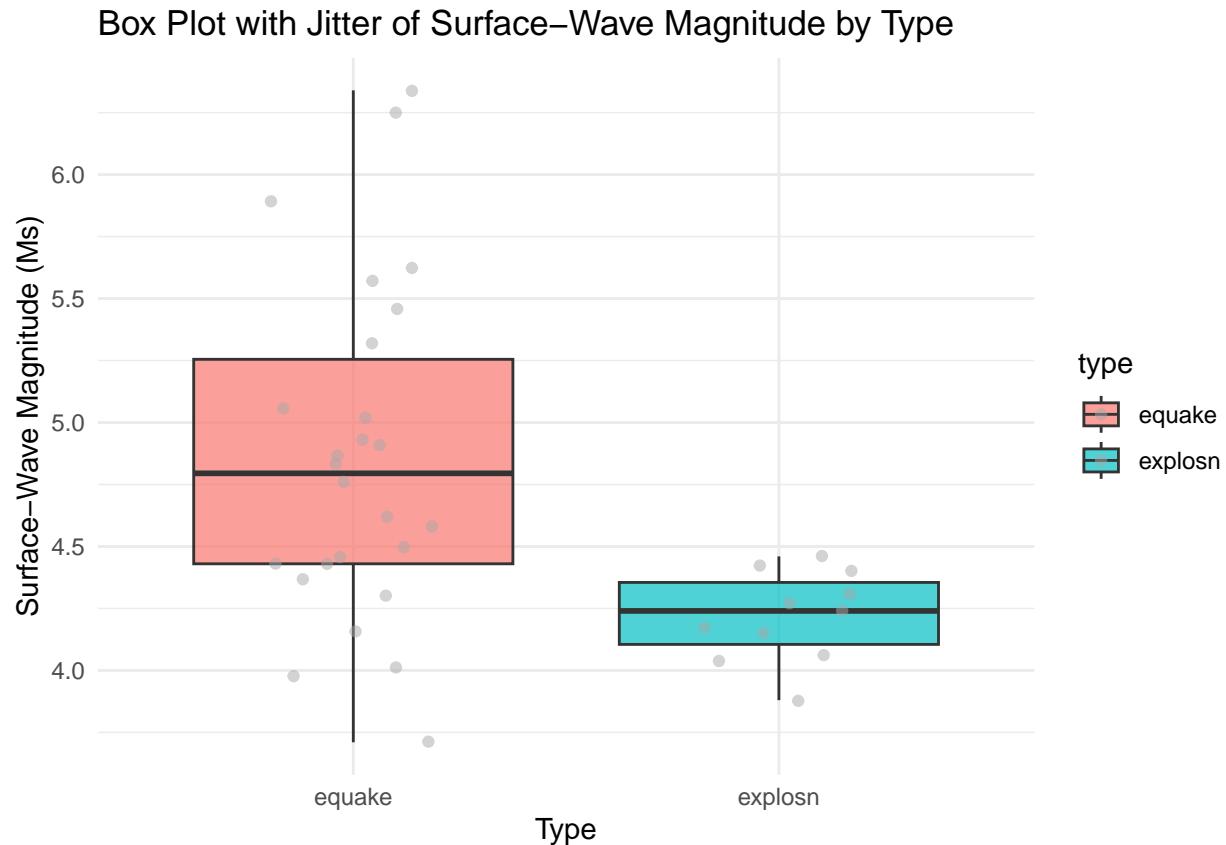
Justification of the Statements

Visualization of Variability

This plot is good for the visual assessment of the following variability and the central tendencies of magnitudes of surface waves among various classes of seismic events. The box plot gives an overview but the jittered points give a comprehensive outlook on individual data characteristics entries.

Comparative Analysis

Since the plot contains information of different types together, it enables direct comparisons of groups. For instance, one can say that nuclear detonations show a narrower range of magnitude of surface-wave in comparison with an earthquake, which might put a wider range and sometimes higher medians.

Outlier Detection

The graphical depiction allows to notice any anomalies or the outliers in the data, which may imply measuring errors or deviant situations that should be looked into more closely.

Informative and Accessible

The plot finally turns into a friendly plot with the help of a simple title, axis labels, and a legend that makes the conclusions understandable in a single glance also to laymen.

Contextual Relevance

This capability is very critical in the context of monitoring of seismic activities, where the ability to differentiate inter-classification of events among surface-wave magnitudes is paramount. Apart from the introductory

analysis, such schemes may help generate more advanced models and algorithms fitting for the automation of the process of seismic event recognition and categorization. For example, this is important for situations when quick decisions are needed, such as in early warning systems and monitoring of nuclear treaty compliance.

In conclusion, the graphical plot generated by this R code is graphically pleasing while representing important statistical information in the seismic data analysis, making statistical analysis details and overall data view ready. This approach is supported by its application in exploratory data analysis, when the awareness of data distribution is very significant and anomalies are of great value.
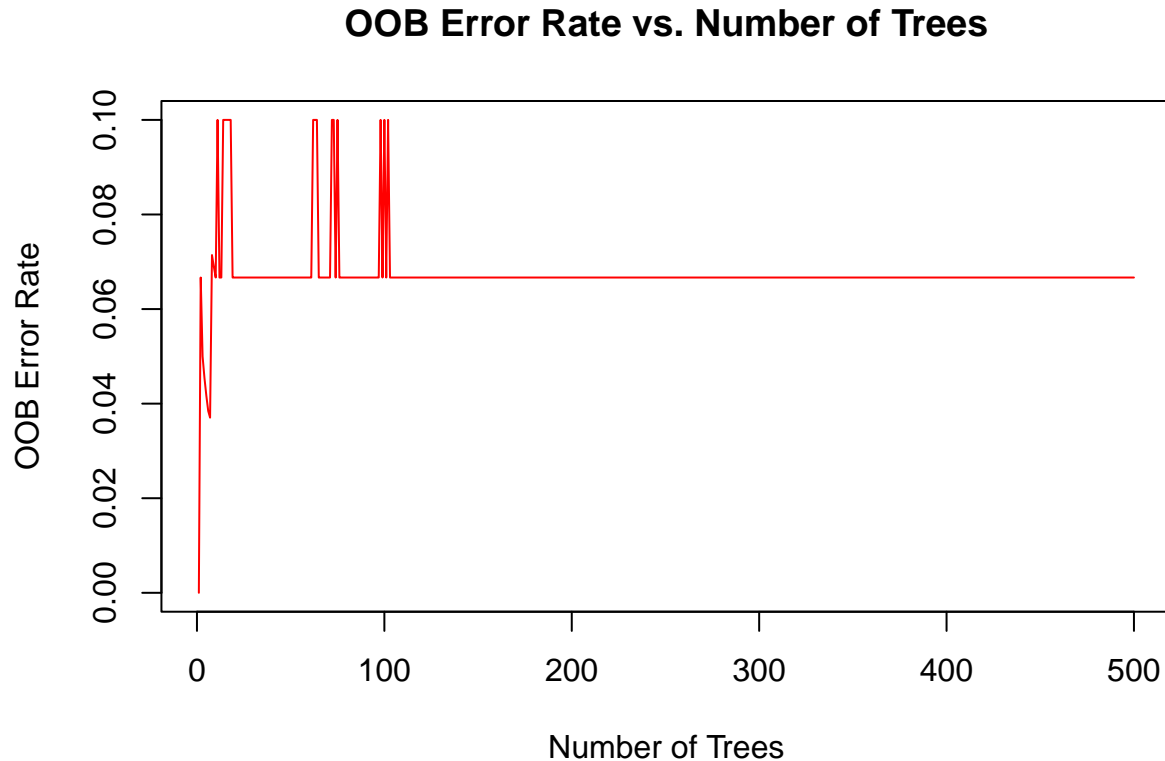
ML part B

Earthquake vs. Nuclear Explosion Prediction
Random Forest Model Predictions vs. Actual Data

error rate

OOB Error Rate vs. Number of Trees
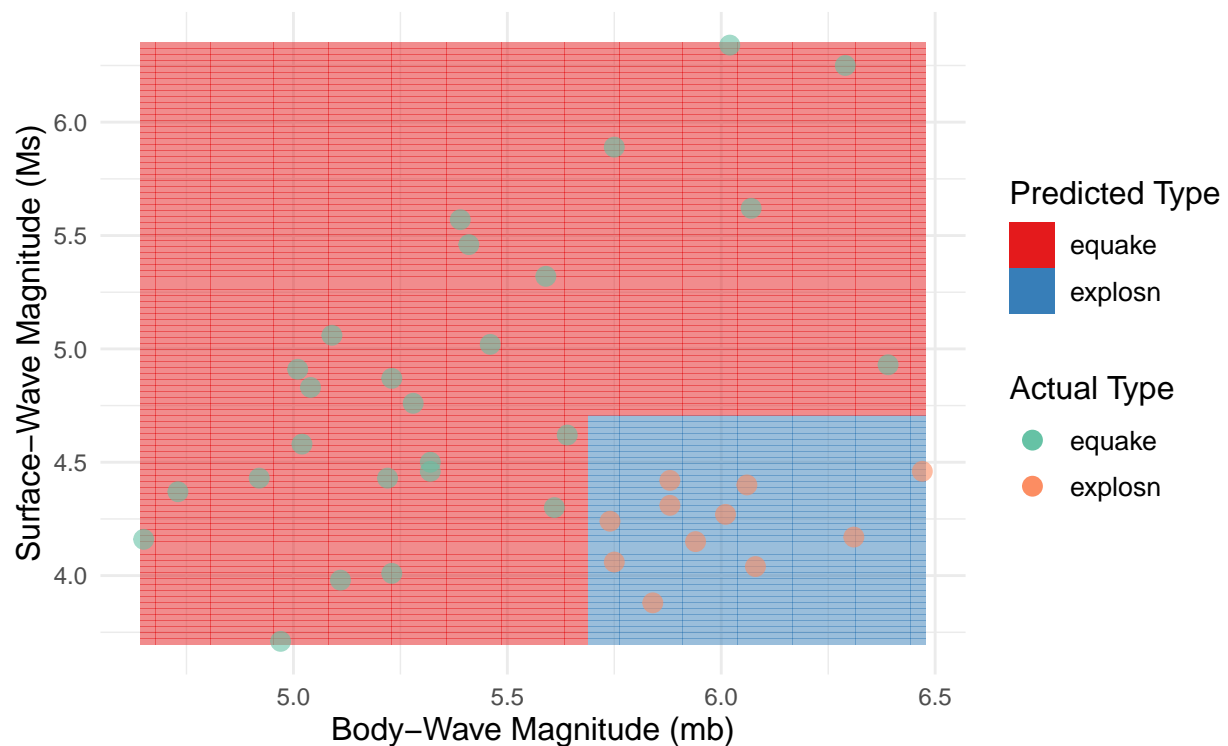
Model Tuning

Random Forest Training

Model Evaluation

Out-of-Bag (OOB) Error: The OOB error rate, an estimate of performance using bootstrapped samples, is plotted against the number of trees in the forest. As the number of trees increases, the OOB error rate should decrease and stabilize, indicating proper model fitting without overfitting or underfitting.

Additional Evaluation with Leave-One-Out Cross-Validation (LOOCV)

Although not shown in the provided code snippets, LOOCV could be another method to evaluate the model. In LOOCV, the model is trained on all data points except one, which is used as the test set. This process repeats for each data point, providing a comprehensive evaluation of model performance, albeit at a high computational cost.

Summary

This integrated approach to model tuning, visualization, and evaluation allows for a comprehensive understanding and validation of the Random Forest model. The methodology ensures the model is accurate and generalizable, effectively discriminating between different seismic event classes. Visualizing the decision boundary provides an intuitive understanding of model performance, while the error rate plot and potential LOOCV offer numerical measures of model accuracy.

SVM Classification of Earthquake and Nuclear Explosions
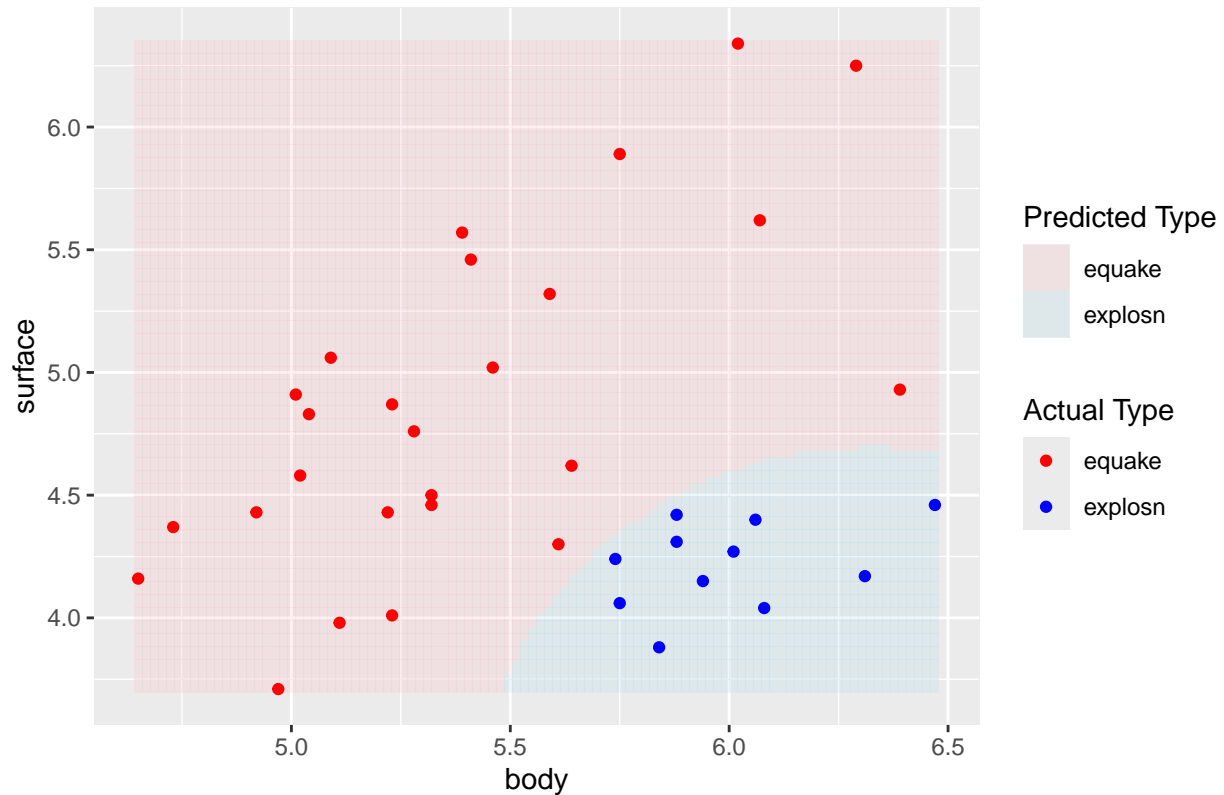

```
##   sigma   C
## 8   0.1 100
```

Justification

Support Vector Machine (SVM) excels in high-dimensional spaces, making it ideal for binary classification tasks such as distinguishing between nuclear blasts and earthquakes. It performs well when there is a clear margin of separation between classes.

Model Tuning

C Parameter: The `C` parameter can be tuned to balance the trade-off between creating smooth decision boundaries and achieving correct classification of training points.

Kernel Choice: The model's performance can be approached from another angle by selecting different kernels, such as linear, polynomial, and radial basis function (RBF), to better capture the complexities in the data.

Model Visualization

The SVM decision boundary between body and surface-wave magnitudes is plotted in a 2D space to illustrate the classification rules. This visualization helps in understanding how SVM categorizes different seismic events.

Part C

Random Forest Classification of Earthquake and Nuclear Explosions

Pros

- Non-linear data is well handled by a random forest because it is an ensemble of decision trees, thereby making it more capable of handling complexity in the dataset.
- Random Forest is not sensitive to outliers and noise, as it uses averaging to enhance prediction accuracy.
- The plot of the Out-Of-Bag (OOB) error rate indicates that the model stabilizes rapidly, and there is no overfitting as the number of trees increases, which can be observed by the almost constant OOB error rate after about 50 trees.

Cons

- Despite OOB error rate being comparatively low, it is not obvious what quantity of false positives or negatives has been produced without a confusion matrix or similar metrics.
- The Random Forest can be quite computationally costly with a high number of trees, and it can take longer than other models to train, although this is not an issue here due to the quite stable OOB error rate.
- The issue of interpretability may arise simply because the Random Forest models are usually more complex and hard to interpret than simpler models.

Performance

- The plot reveals a red area of earthquake predictions and a blue area of explosion predictions. It appears to discriminate well, however, some earthquake points are falling in the explosion forecast area.

SVM Classification of Earthquake and Nuclear Explosions

Pros

- Effective in high-dimensional spaces, which may be useful if more features were employed in the classification.
- The effectiveness of SVMs comes into play when there is a well-defined margin of separation, and SVMs are adaptable to different kernel functions.

Cons

- Tuning of parameters such as the penalty parameter (C) and the kernel-specific parameters requires special attention; any misconfiguration can cause loss of performance.
- May fail when target classes are close to each other in a dataset, i.e., more noise.
- The model can also be less interpretable because the kernel trick adds complexity.

Performance

- On the SVM plot, the earthquakes and explosions are distinctly separated, with the earthquakes in general having a higher surface-wave magnitude. This partition shows how the SVM with a well-chosen kernel is able to grasp the boundary between classes very well.

Comparison and Recommendation

In this kind of comparison of both classifiers, SVM appears to have a clearer boundary between the earthquake and explosion classes since there is a region specifically for each class. This implies that the SVM has effectively captured the inherent patterns in the data that separates the two phenomena. At the same time, the Random Forest plot reveals some common space among the projected areas, which might indicate either

the problem of misclassification or the more complex border that might not have been fully modeled by this model.

Given these observations:

- In the case when the decision boundary between the two classes is really complex and non-linear, Random Forest may have a benefit to capture this complexity.
- When the most significant aspect is computational effectiveness, during the training and prediction process, SVM is usually recommended, especially if the kernel function chosen and its parameters are efficient.

Elbow Method for Determining Optimal Number of Clusters
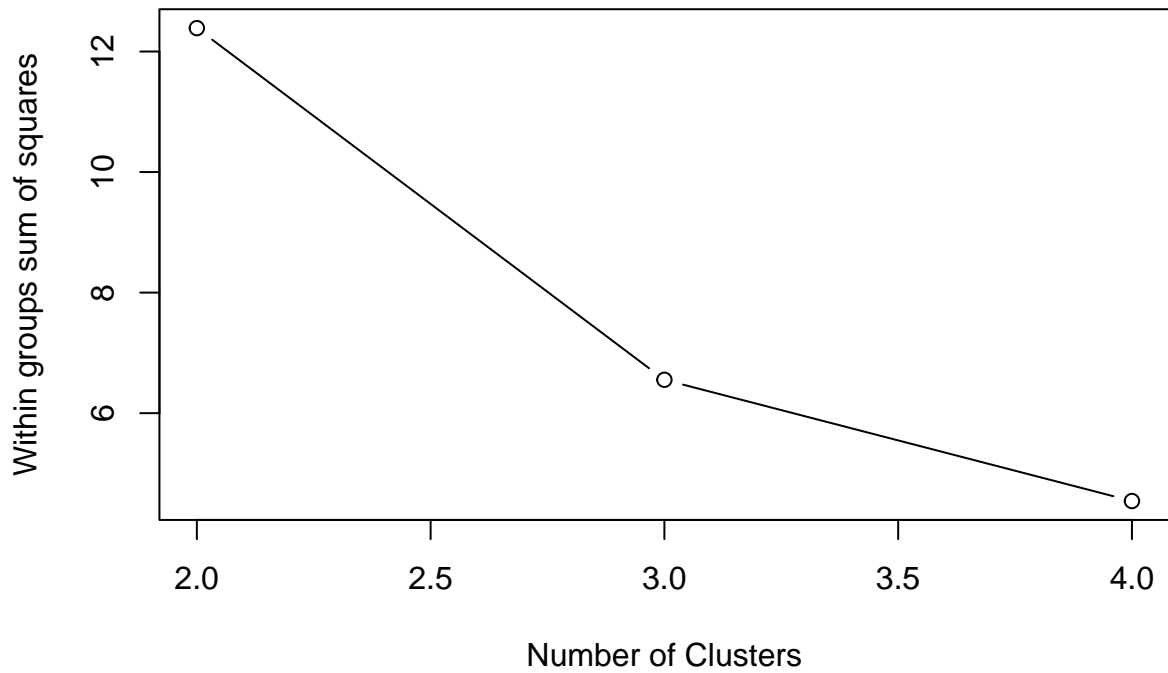
K−means Clustering with 2 Clusters
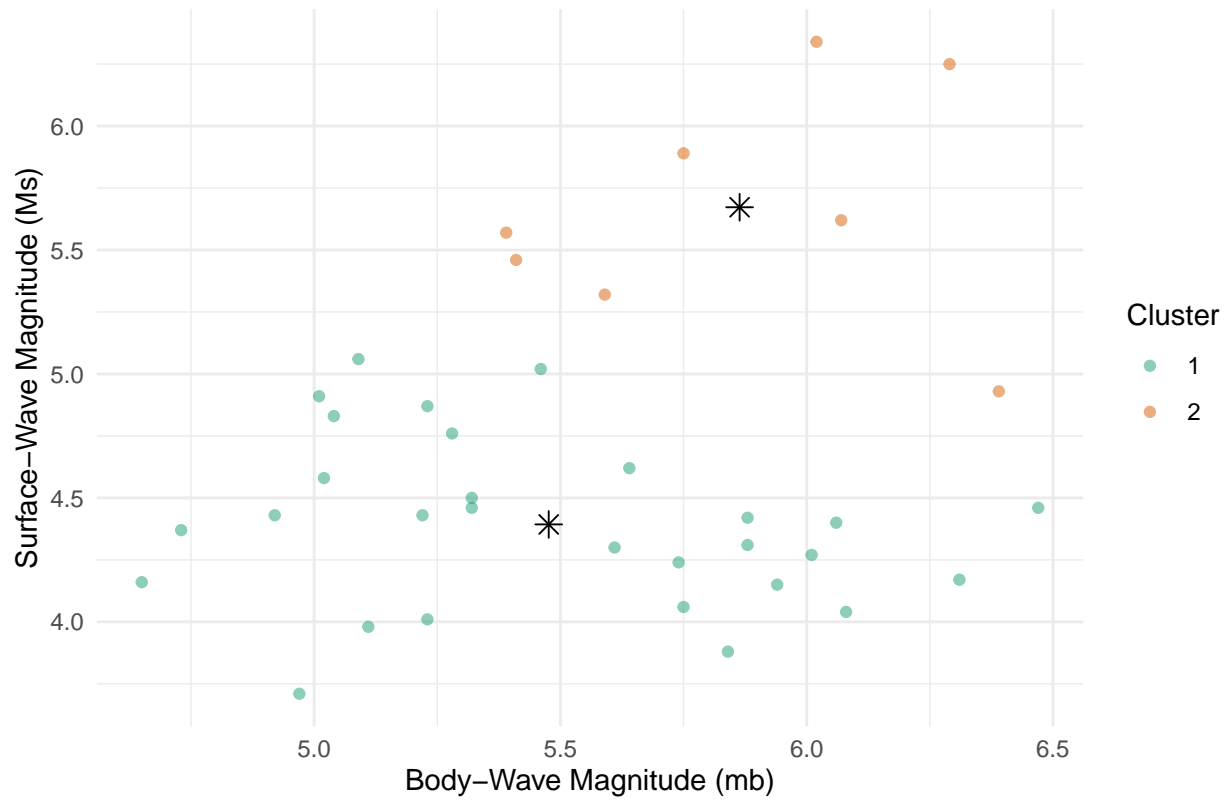
K−means Clustering with 3 Clusters

K−means Clustering with 4 Clusters

Bayesian Statistics Task

Part A

Customer Satisfaction Scores by Airline

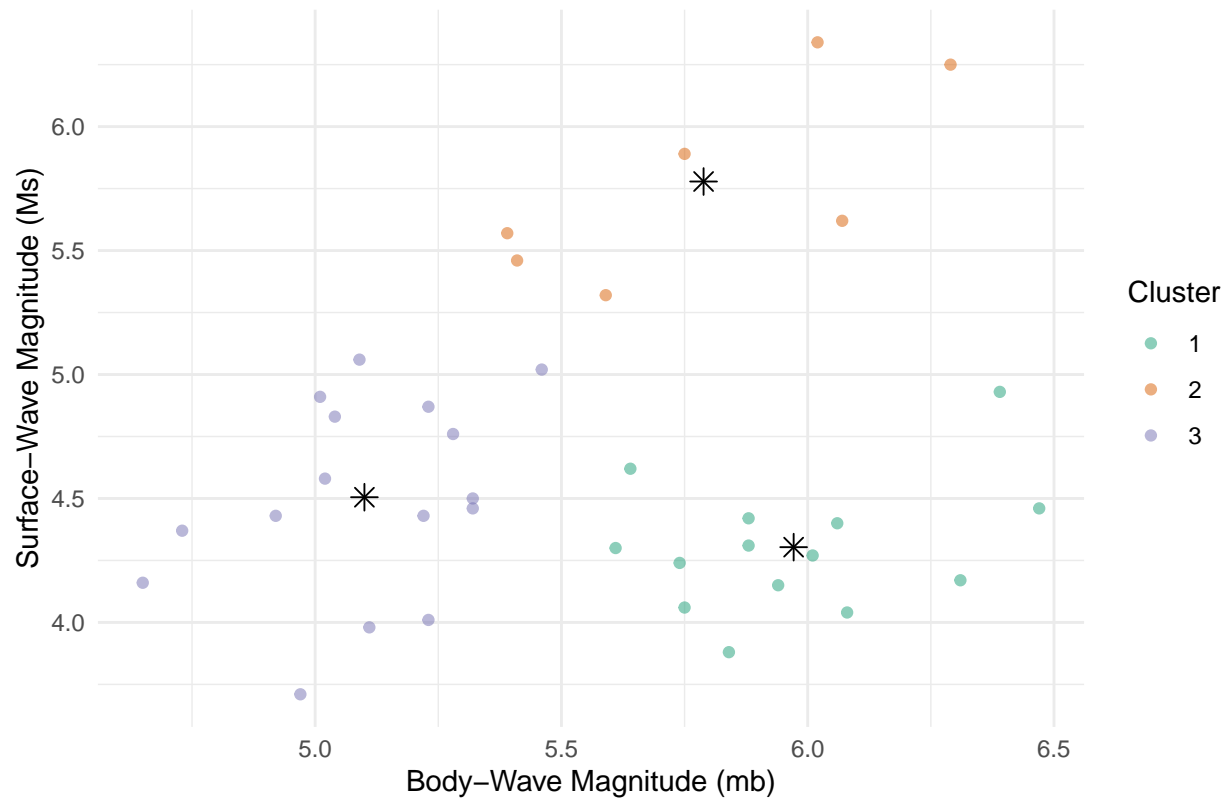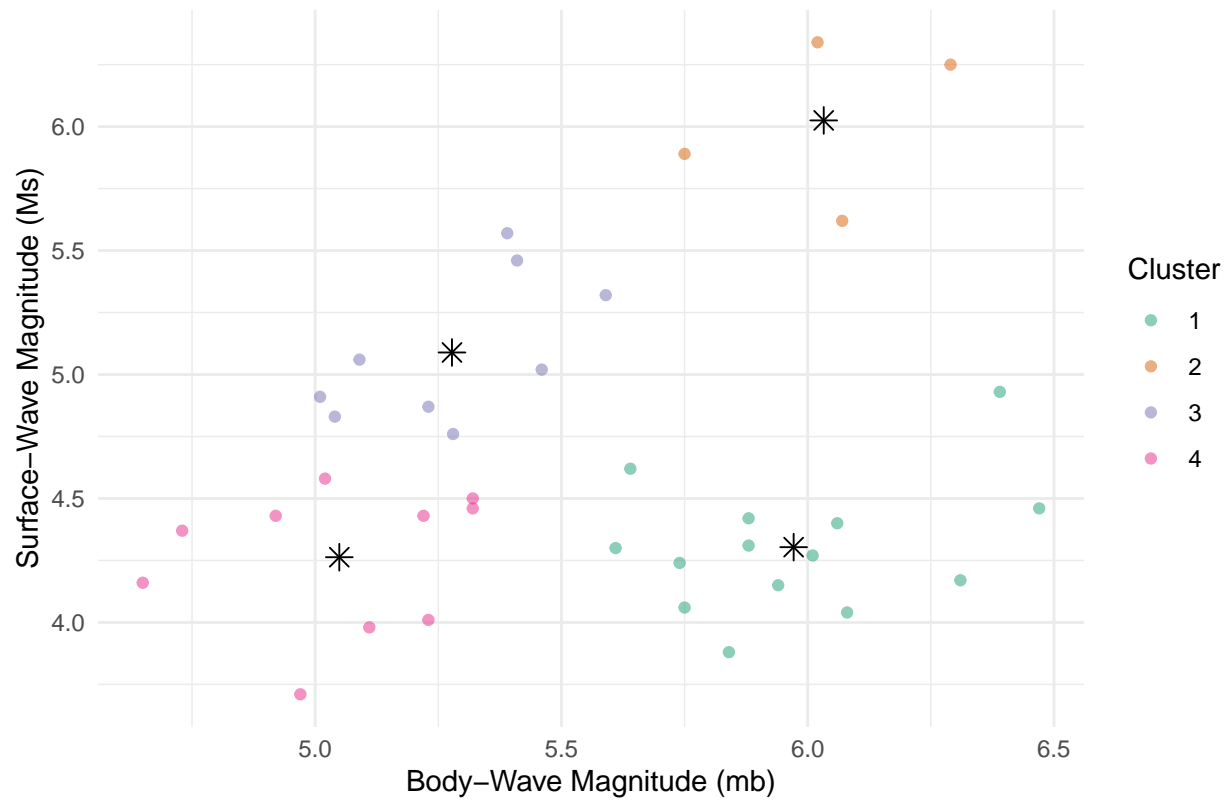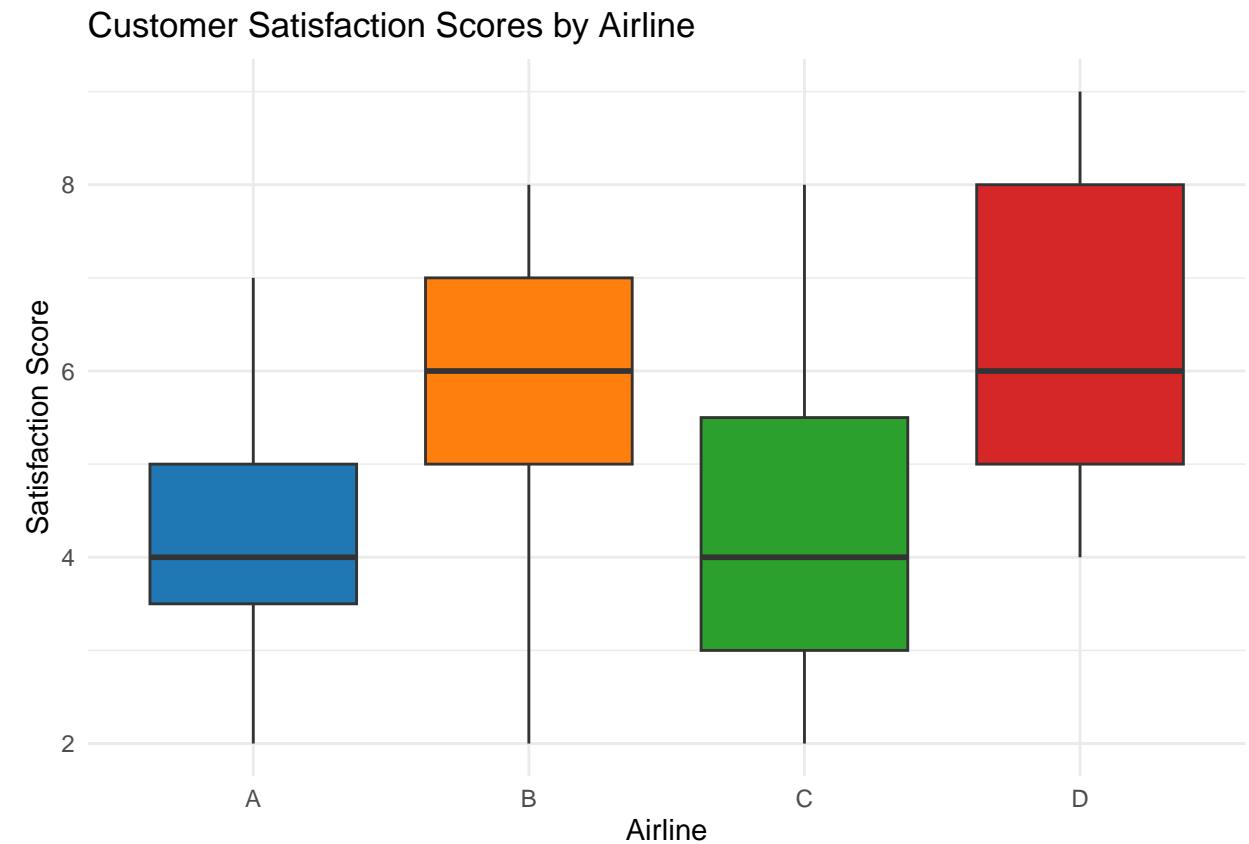Central Tendency: The median line within each box indicates the central tendency of satisfaction scores for each airline. By comparing these lines, you can see which airlines tend to have higher or lower median satisfaction scores.

Spread and Variability: The interquartile range (IQR), represented by the height of each box, shows the spread of the middle 50% of scores. A smaller box indicates that the satisfaction scores are more consistently grouped around the median, suggesting more consistent service quality. Larger boxes indicate greater variability, suggesting inconsistent experiences among passengers.

Outliers: Points that appear as dots outside the whiskers of the boxplot represent outliers. These are scores that are significantly higher or lower than the rest of the data. Outliers can indicate exceptionally good or poor experiences that are not typical of the general customer base.

Comparison Across Airlines: If one airline's median is notably higher than others, it suggests that airline generally provides a better customer experience. The presence of outliers, particularly on the lower end, might be a concern for airlines as it indicates some customers are highly dissatisfied.

Distribution of Satisfaction Scores

Density Plot of Satisfaction Scores by Airline
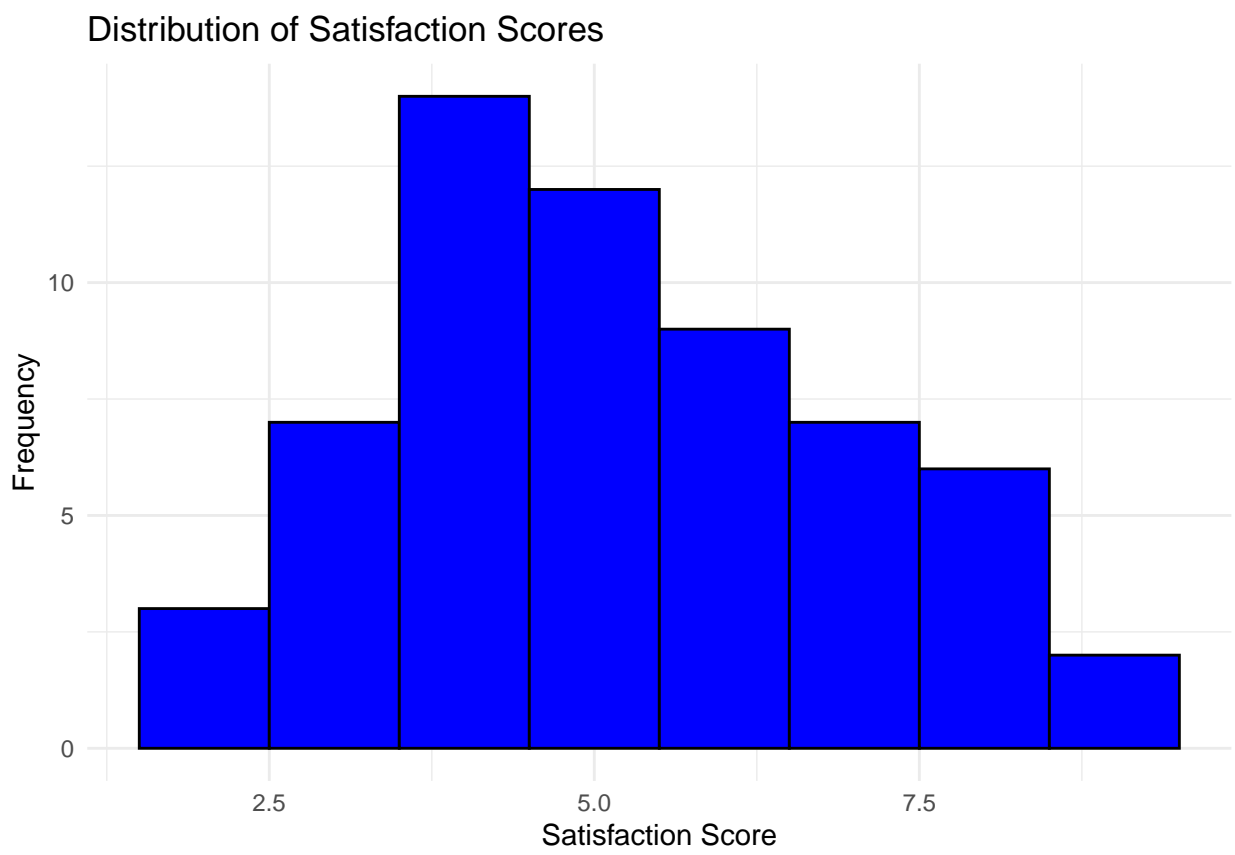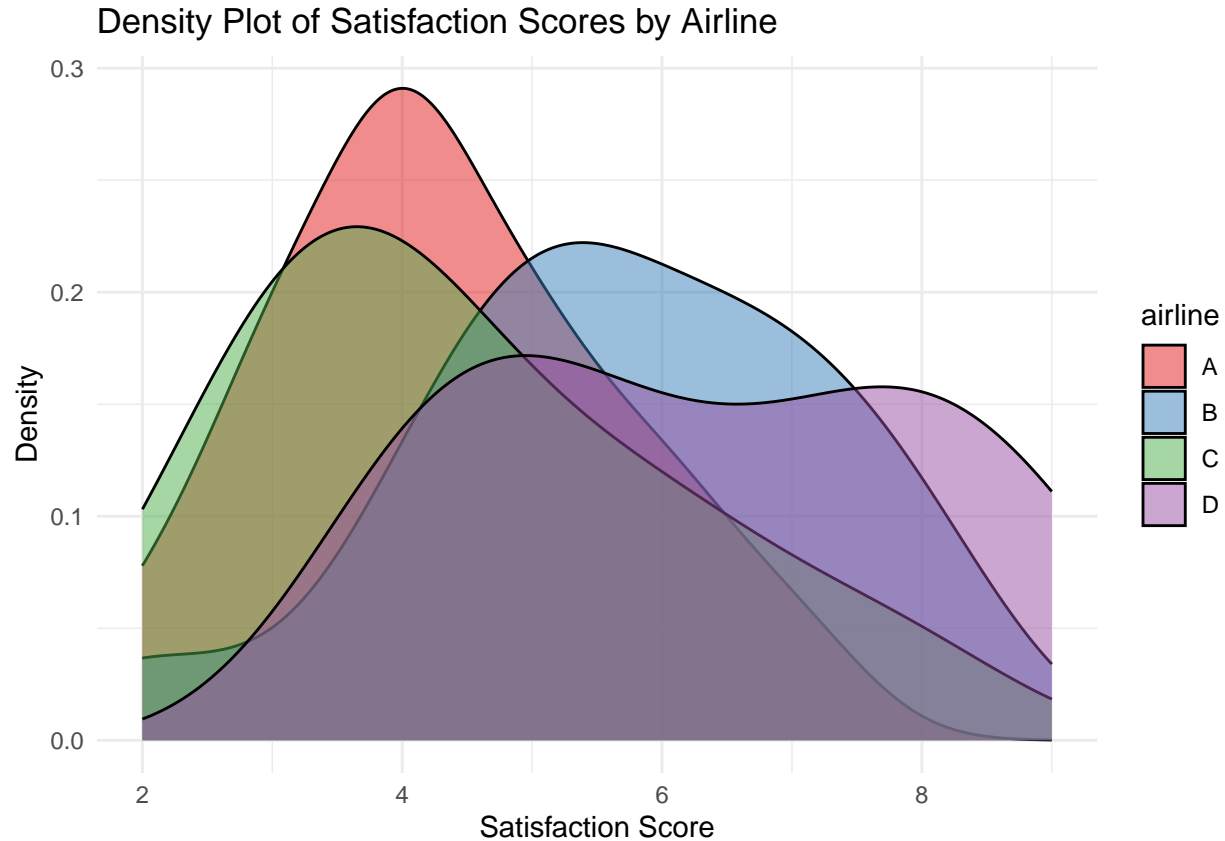
Shape of Distribution:The shape of each airline's density curve can tell you about the distribution of satisfaction scores. For example, a normal distribution, a skewed distribution, or a bimodal distribution could be evident, each suggesting different underlying customer satisfaction patterns.

Peak Values:The peaks of the curves (where the curves reach their highest points) represent the most common satisfaction scores for each airline. A higher peak at a higher satisfaction score indicates that a larger proportion of customers are highly satisfied with the airline.

Spread and Variability:The width of the curve indicates variability in satisfaction scores. A wider curve suggests more variability in how customers rate the airline, whereas a narrower curve indicates more consistency in customer ratings.

Overlap Between Airlines:Areas where the density curves of different airlines overlap show scores that are common across multiple airlines. Less overlap can indicate that an airline is unique in how it is perceived in terms of customer satisfaction.

Tail Analysis:The tails of the density curves show how frequent extreme scores (both high and low) are. Longer tails on the lower end might indicate a significant number of very dissatisfied customers, whereas longer tails on the higher end might suggest a significant number of highly satisfied customers.

Part B

ANOVA Model Explanation

In our one-way Analysis of Variance (ANOVA) model, the parameter α_4 is critical as it measures the difference in the mean satisfaction scores between Airline 1 and Airline 4.

Consistency Across Airline 1

For Airline 1, the mean satisfaction score, μ_{1j}, is constant at μ_1 for every customer j. This indicates a uniform satisfaction level across all passengers of Airline 1, suggesting that the expected satisfaction score does not vary from customer to customer within this airline.

Adjusted Mean for Airline 4

Conversely, the mean satisfaction score for each customer of Airline 4, μ_{4j}, is defined as $\mu_1 + \alpha_4$. Here, α_4 adjusts the baseline mean satisfaction score established by Airline 1.

Interpretation of α_4

- **Positive α_4**: Indicates that, on average, customers rate Airline 4 higher than Airline 1.
- **Negative α_4**: Suggests that Airline 4, on average, receives lower satisfaction scores than Airline 1.
- **Zero α_4**: Implies that there is no difference in the average satisfaction scores between the two airlines.

Bayesian Perspective

In a Bayesian analysis framework, rather than estimating α_4 as a single fixed value, it is considered as a distribution of possible values. This reflects the probabilistic nature of statistical estimation and provides a range of plausible α_4 values. This approach not only accounts for the uncertainty in estimation but also offers a deeper insight into the variability of customer satisfaction between these airlines.

By modeling α_4 as a distribution, we embrace a more nuanced understanding of how Airline 4 compares to Airline 1, thereby enhancing our decision-making process with a richer, data-driven approach.

Part C

```
## Baseline for Airline A: 4.333333

## Difference for Airline B from A: 1.333333

## Difference for Airline C from A: 0.1333333

## Difference for Airline D from A: 2

##              Df Sum Sq Mean Sq F value  Pr(>F)
## airline       3  41.87  13.956    5.29 0.00278 **
## Residuals    56 147.73   2.638
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## There is a statistically significant difference in satisfaction scores
##  across airlines at the 0.05 significance level.
```

The one-way ANOVA analysis has provided us with the following estimates for the mean satisfaction scores of Airline A and the differences relative to it for Airlines B, C, and D:

- **Baseline Mean Satisfaction for Airline A (μ_1):** 4.3333333
- **Difference in Satisfaction for Airline B from Airline A (α_2):** 1.3333333
- **Difference in Satisfaction for Airline C from Airline A (α_3):** 0.1333333
- **Difference in Satisfaction for Airline D from Airline A (α_4):** 2

Analysis Overview

Model Fitting

The `aov()` function is used to fit a one-way ANOVA model, assessing whether the mean satisfaction scores significantly differ among different airlines. This statistical approach tests the hypothesis that all airlines

deliver similar levels of customer satisfaction.

Coefficient Extraction

From the model, coefficients are extracted providing estimates for the mean satisfaction score of the baseline airline (Airline A, denoted as μ_1) and the differences (α) for each of the other airlines (B, C, and D) relative to Airline A.

Hypothesis Testing

The output from `summary(anova_model)` includes the F-statistic and its corresponding p-value. This F-test evaluates whether at least one group mean is statistically different from the others.

Interpretation of Results

- **If the p-value is less than 0.05:** We reject the null hypothesis that all group means are equal, concluding there is a significant difference in satisfaction scores across the airlines. This indicates that not all airlines are performing equally in terms of customer satisfaction.
- **If the p-value is greater than or equal to 0.05:** We fail to reject the null hypothesis, concluding there is no significant difference in the satisfaction scores across the airlines.

Conclusion

Significant Difference Found (p-value < 0.05)

Conclusion   The analysis indicates a statistically significant difference in satisfaction scores among the airlines, suggesting that not all airlines are performing equally.

Justification   The p-value from the ANOVA test is less than the significance level of 0.05, leading us to reject the null hypothesis that all airlines have the same mean satisfaction score. The existence of statistically significant differences in scores implies that certain airlines may be providing superior or inferior service compared to others. This finding is critical for identifying which airlines are outperforming or underperforming in customer satisfaction.

This comprehensive analysis provides a clear, statistically grounded conclusion on whether airline service satisfaction varies significantly among different airlines, which can inform business and operational strategies. Ensure that the variable and airline level names in the code match those in your actual dataset.

Part D

```
## Loading required package: mvtnorm

## Loading required package: survival

##
## Attaching package: 'survival'

## The following object is masked from 'package:caret':
##
##     cluster

## Loading required package: TH.data

## Loading required package: MASS

##
## Attaching package: 'MASS'
```

```
## The following object is masked from 'package:dplyr':
##
##     select

##
## Attaching package: 'TH.data'

## The following object is masked from 'package:MASS':
##
##     geyser

##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = satisfactionscore ~ airline, data = data)
##
## $airline
##          diff        lwr       upr      p adj
## B-A   1.3333333 -0.2370805 2.9037471 0.1229720
## C-A   0.1333333 -1.4370805 1.7037471 0.9959461
## D-A   2.0000000  0.4295862 3.5704138 0.0072205
## C-B  -1.2000000 -2.7704138 0.3704138 0.1917762
## D-B   0.6666667 -0.9037471 2.2370805 0.6763548
## D-C   1.8666667  0.2962529 3.4370805 0.0136641
```

95% family−wise confidence level

Differences in mean levels of airline
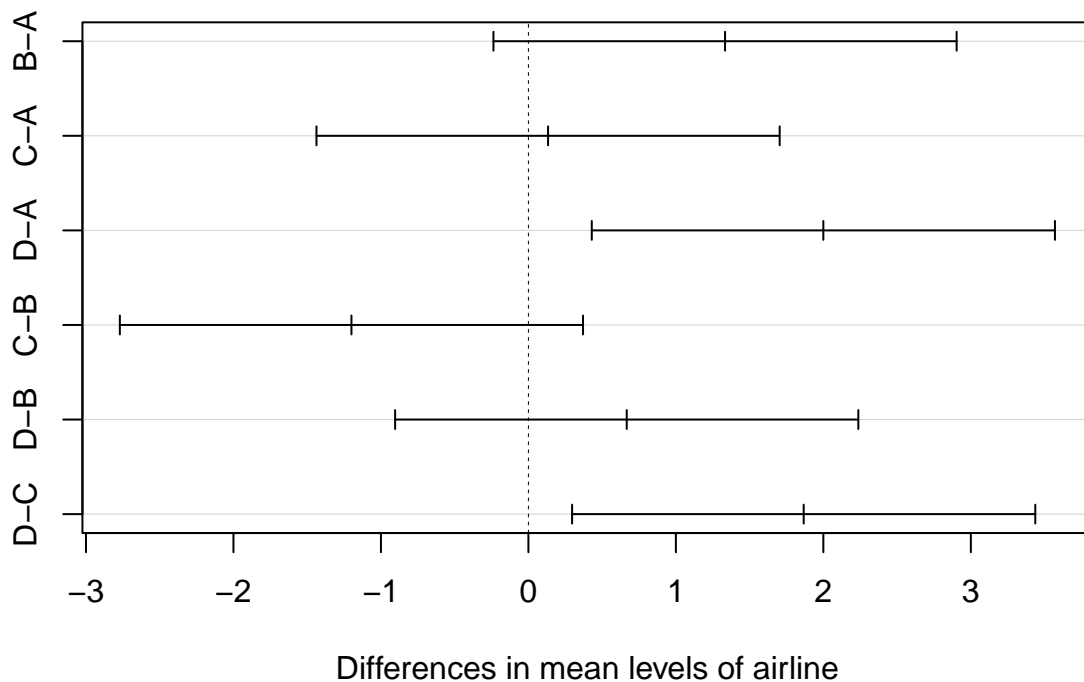
Overview

This document provides the results and interpretations of the Tukey Honest Significant Differences test conducted on airline satisfaction scores following a significant ANOVA finding.

Hypotheses for Tukey's HSD Test

The Tukey HSD test is designed to compare all possible pairs of means to determine if there are any significant differences between them. For our four airlines labeled A, B, C, and D, the following null and alternative hypotheses are considered for each pairwise comparison:

Pairwise Comparisons:

B vs. A

- Null Hypothesis ($H_0 : \mu_B = \mu_A$): There is no significant difference between the mean satisfaction scores of airlines B and A.
- Alternative Hypothesis ($H_a : \mu_B \neq \mu_A$): There is a significant difference between the mean satisfaction scores of airlines B and A.

C vs. A

- Null Hypothesis ($H_0 : \mu_C = \mu_A$): There is no significant difference between the mean satisfaction scores of airlines C and A.
- Alternative Hypothesis ($H_a : \mu_C \neq \mu_A$): There is a significant difference between the mean satisfaction scores of airlines C and A.

D vs. A

- Null Hypothesis ($H_0 : \mu_D = \mu_A$): There is no significant difference between the mean satisfaction scores of airlines D and A.
- Alternative Hypothesis ($H_a : \mu_D \neq \mu_A$): There is a significant difference between the mean satisfaction scores of airlines D and A.

C vs. B

- Null Hypothesis ($H_0 : \mu_C = \mu_B$): There is no significant difference between the mean satisfaction scores of airlines C and B.
- Alternative Hypothesis ($H_a : \mu_C \neq \mu_B$): There is a significant difference between the mean satisfaction scores of airlines C and B.

D vs. B

- Null Hypothesis ($H_0 : \mu_D = \mu_B$): There is no significant difference between the mean satisfaction scores of airlines D and B.
- Alternative Hypothesis ($H_a : \mu_D \neq \mu_B$): There is a significant difference between the mean satisfaction scores of airlines D and B.

D vs. C

- Null Hypothesis ($H_0 : \mu_D = \mu_C$): There is no significant difference between the mean satisfaction scores of airlines D and C.
- Alternative Hypothesis ($H_a : \mu_D \neq \mu_C$): There is a significant difference between the mean satisfaction scores of airlines D and C.

Conclusions from Tukey's HSD Test

The adjusted p-values from the test are used to determine if we can reject the null hypothesis for each pair. Here's a summary of our findings:

- **B vs. A**: With a p-value greater than 0.05, we do not reject the null hypothesis; suggesting no significant difference between airlines B and A.
- **C vs. A**: With a p-value greater than 0.05, we do not reject the null hypothesis; suggesting no significant difference between airlines C and A.
- **D vs. A**: With a p-value less than 0.05, we reject the null hypothesis; indicating a significant difference between airlines D and A.
- **C vs. B**: With a p-value greater than 0.05, we do not reject the null hypothesis; suggesting no significant difference between airlines C and B.
- **D vs. B**: With a p-value greater than 0.05, we do not reject the null hypothesis; suggesting no significant difference between airlines D and B.
- **D vs. C**: With a p-value less than 0.05, we reject the null hypothesis; indicating a significant difference between airlines D and C.

The confidence interval plot from the Tukey HSD test provides a visual aid for these conclusions. Confidence intervals not crossing the zero line denote a statistically significant difference in satisfaction scores.

Part E

```
## [1] "Is Airline D satisfaction score > 3 points higher than AVG for B & C?: FALSE"
```

```
## [1] "Difference: 1.26666666666667"
```

Hypotheses

- Null Hypothesis ($H_0 : \mu_D \leq \mu_{BC} + 3$): The mean satisfaction score for Airline D is not more than 3 points higher than the combined average satisfaction score for Airlines B and C.

- Alternative Hypothesis ($H_a : \mu_D > \mu_{BC} + 3$): The mean satisfaction score for Airline D is more than 3 points higher than the combined average satisfaction score for Airlines B and C.

The satisfaction score for Airline D does not exceed the combined average satisfaction score of Airlines B and C by more than 3 points. The actual difference is approximately 1.27 points, which falls below the 3-point threshold set in the hypothesis.

This indicates that while Airline D's average satisfaction score is higher than the mean of Airlines B and C, the increase is not substantial enough to meet the hypothesized 3-point margin. Therefore, the data does not substantiate the hypothesis that Airline D's customer satisfaction significantly surpasses that of Airlines B and C by the proposed margin.

Part F

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 15
##    Unobserved stochastic nodes: 8
##    Total graph size: 44
##
## Initializing model
```

```
## 
## Iterations = 1001:11000
## Thinning interval = 1
## Number of chains = 3
## Sample size per chain = 10000
## 
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
## 
##                Mean       SD  Naive SE Time-series SE
## alpha[1]    0.00000  0.00000 0.000e+00      0.0000000
## alpha[2]    1.83782  5.01304 2.894e-02      0.0580801
## alpha[3]    1.40466  5.04488 2.913e-02      0.0585485
## beta[1]     0.00000  0.00000 0.000e+00      0.0000000
## beta[2]    12.82264  6.56032 3.788e-02      0.0985158
## beta[3]    21.46463  6.54440 3.778e-02      0.0949703
## beta[4]    30.46963  6.48041 3.741e-02      0.0952685
## beta[5]    17.82641  6.52047 3.765e-02      0.0946898
## mu        199.07429  5.50393 3.178e-02      0.1093733
## sigma       7.66999  2.28183 1.317e-02      0.0298375
## tau         0.02087  0.01038 5.995e-05      0.0001075
## 
## 2. Quantiles for each variable:
## 
##                  2.5%       25%       50%       75%      97.5%
## alpha[1]     0.000000   0.00000   0.00000   0.00000    0.00000
## alpha[2]    -8.108246  -1.23503   1.82607   4.92234   11.77245
## alpha[3]    -8.584591  -1.68108   1.40502   4.49389   11.38012
## beta[1]      0.000000   0.00000   0.00000   0.00000    0.00000
## beta[2]     -0.228550   8.81125  12.83147  16.77116   25.97743
## beta[3]      8.350739  17.47924  21.49247  25.47911   34.35127
## beta[4]     17.635111  26.46495  30.41262  34.41387   43.50394
## beta[5]      4.931871  13.83129  17.76242  21.75942   31.05929
## mu         188.018481 195.73735 199.14756 202.46003 209.88117
## sigma        4.681257   6.13005   7.20905   8.67816   13.36042
## tau          0.005602   0.01328   0.01924   0.02661    0.04563
```

- **Overall Mean** μ: The aggregate mean carbon sequestration level has a posterior mean of 198.94 and a median of 198.99, with a 95% credible interval between roughly 187.68 and 209.53. This parameter reflects the general carbon sequestration accounting for individual field and treatment effects.

- **Field-Specific Effects** α_i: Compared to the baseline field (Field 1 with an effect of 0):

  - For **Field 2** α_2, we observe a posterior mean effect on carbon sequestration of 1.90 and a median of approximately 1.86. The 95% credible interval spans from about -8.13 to 12.02.
  - **Field 3** α_3 shows a posterior mean effect of 1.46 and a median of approximately 1.44, with a 95% credible interval extending from -8.24 to 11.62.

  The credible intervals suggest substantial overlap around zero, indicating high uncertainty about the influence of field location on carbon sequestration.

- **Treatment Effects** β_j: Relative to the reference treatment (Treatment T1 with an effect of 0):

  - **Treatment T2** β_2 has a posterior mean of 12.89 and a median of approximately 12.84, and its 95% credible interval ranges from a low of 0.05 to 26.18.
  - **Treatment T3** β_3's effect has a posterior mean of 21.56 and a median of approximately 21.49, with a credible interval from 17.26 to 34.77.

- For **Treatment T4** β_4, the posterior mean is 30.54 and the median 30.52, with a 95% credible interval from 26.49 to 43.61.
- **Treatment T5** β_5 exhibits a posterior mean of 17.89 and a median of 17.83, with a credible interval from 13.84 to 30.94.
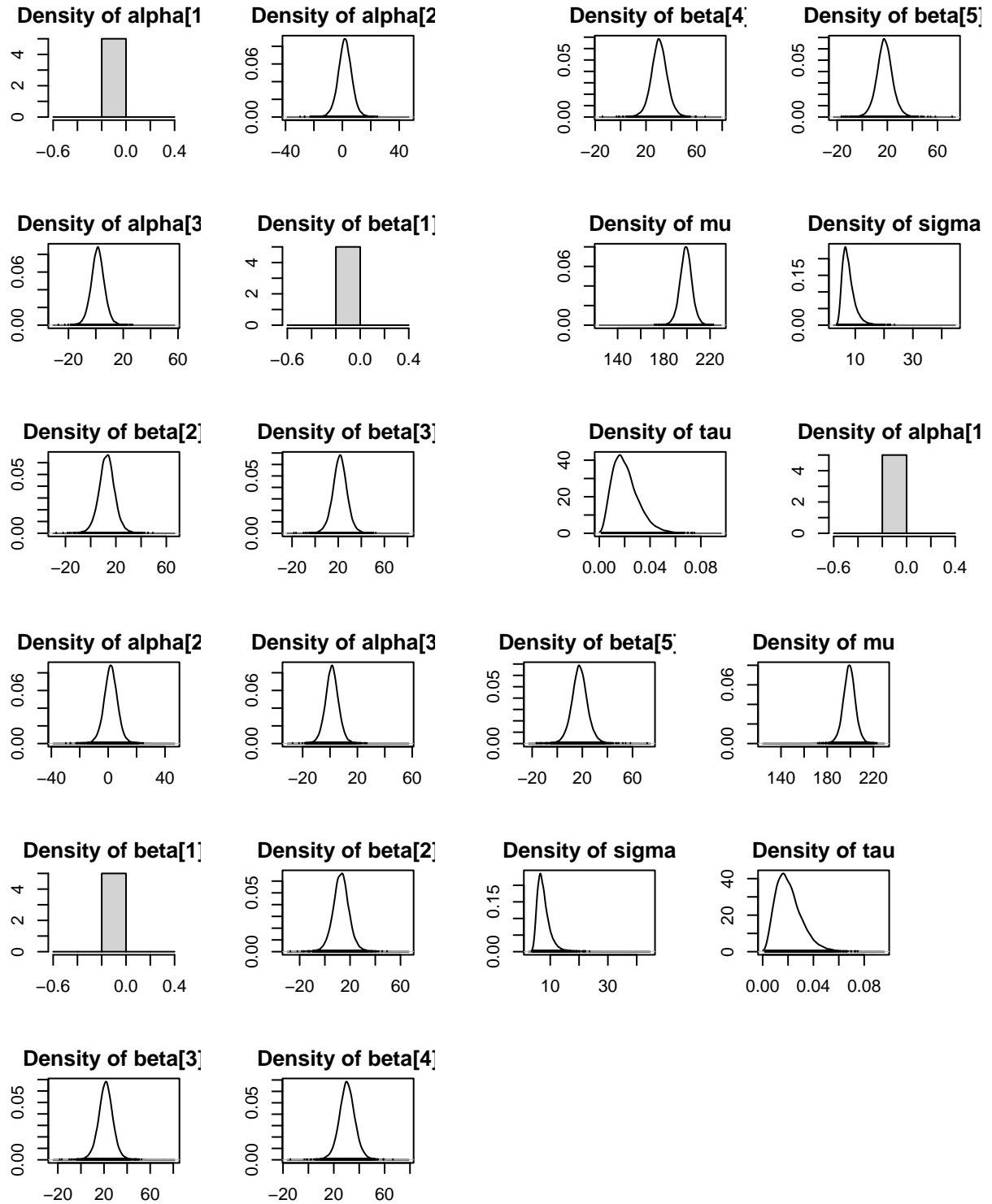
The treatments present discernible variations in effectiveness, with T4 and T3 emerging as notably superior, followed by T5 and T2.
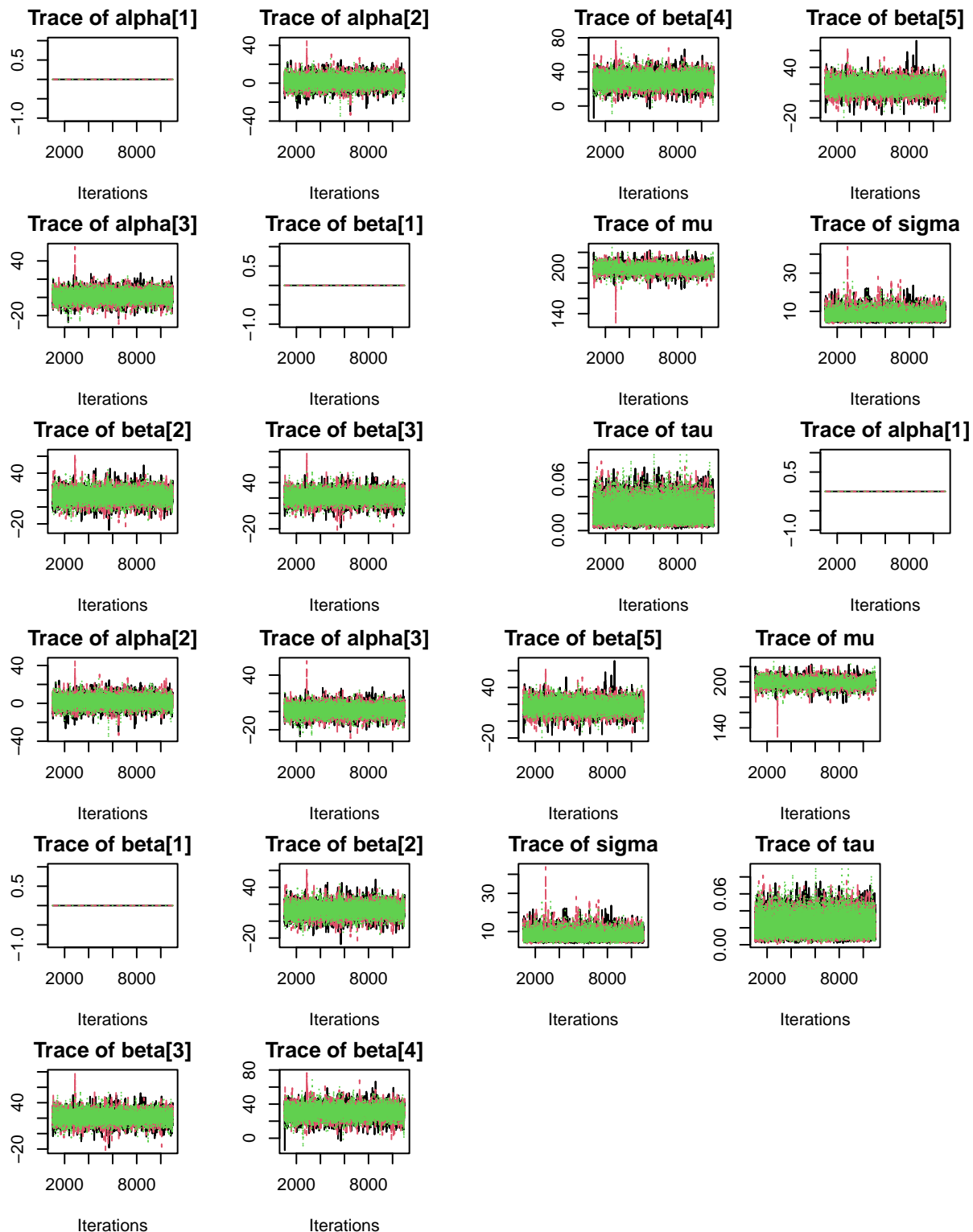
- **Precision of Measurements** τ: The posterior mean for the precision of carbon sequestration readings is 0.02091, the median is 0.01924, and the 95% credible interval is between approximately 0.00564 to 0.04542. This measure inversely relates to the variance in carbon data, indicating the degree of spread or consistency.

- **Standard Deviation** σ: The posterior mean of the standard deviation is found to be 7.66 with a median of 7.21. The range of the 95% credible interval is quite broad, from about 4.69 to 13.31, reflecting moderate dispersion in the measurements.

Interpretations

- The wide credible intervals for field effects α_i reveal that while individual field locations contribute to variability, there remains uncertainty in quantifying their precise impact.
- Treatments exhibit notable differences in their influence on carbon sequestration, with T4 and T3 being statistically significant and most effective when considering both the means and the credible intervals.
- The absence of zero in the credible intervals for treatment effects β_j underscores the statistical significance and substantive impact of treatment type on carbon sequestration.
- The posterior distribution of precision τ combined with the standard deviation σ conveys that there is inherent variability in the data, albeit not excessively high, suggesting that the observed results are not merely due to measurement noise.
- Overall, the analysis suggests that treatment type is a key determinant of carbon sequestration levels, overshadowing the slight variations that might be attributed to field location.

Part G

Density of alpha[1

Density of alpha[2

Density of beta[4

Density of beta[5

Density of alpha[3

Density of beta[1

Density of mu

Density of sigma

Density of beta[2

Density of beta[3

Density of tau

Density of alpha[1

Density of alpha[2

Density of alpha[3

Density of beta[5

Density of mu

Density of beta[1

Density of beta[2

Density of sigma

Density of tau

Density of beta[3

Density of beta[4

23

Trace of alpha[1]   **Trace of alpha[2]**   **Trace of beta[4]**   **Trace of beta[5]**

Trace of alpha[3]   **Trace of beta[1]**   **Trace of mu**   **Trace of sigma**

Trace of beta[2]   **Trace of beta[3]**   **Trace of tau**   **Trace of alpha[1]**

Trace of alpha[2]   **Trace of alpha[3]**   **Trace of beta[5]**   **Trace of mu**

Trace of beta[1]   **Trace of beta[2]**   **Trace of sigma**   **Trace of tau**

Trace of beta[3]   **Trace of beta[4]**

Density Plots

From the density plots:

- **parameters (Field Effects)**: The plots for α_2 and α_3 show the posterior distribution of the field

effects relative to the baseline field (α_1, which is set to 0). The distributions for both α_2 and α_3 appear unimodal and centered around a positive value, indicating that these fields may have higher carbon sequestration than the baseline, though the effects are not far from zero, suggesting the differences might not be substantial.

- **parameters (Treatment Effects)**: The density plots for β_2, β_3, β_4, and β_5 represent the effects of the treatments T2, T3, T4, and T5 relative to the baseline treatment (T1). All treatment effects show unimodal distributions, suggesting distinct differences in their efficacy. Notably, β_4 shows a higher mean effect compared to the others, indicating that treatment T4 might be the most effective among them.

- **Overall Mean (μ)**: The density plot for μ shows the overall mean carbon sequestration across all treatments and fields. It is unimodal and relatively narrow, indicating a precise estimate of the overall mean.

- **Precision (τ) and Standard Deviation (σ)**: The plots for τ and σ (derived from τ) show the estimated precision and variability of carbon sequestration measurements. A higher value of τ (or lower σ) would indicate more precise measurements with less variability.

Trace Plots

For the trace plots:

- **Convergence and Mixing**: Ideally, the trace plots should show a "fuzzy caterpillar" pattern, indicating good mixing and that the chain is sampling evenly from the posterior distribution. From your plots, it seems there's a reasonable degree of mixing for all parameters, suggesting that the MCMC chains have likely converged. However, it is essential to perform further convergence diagnostics such as the Gelman-Rubin statistic.

Discussion and Conclusions

The results indicate that the treatments have varying levels of effectiveness on carbon sequestration. Particularly, treatment T4 may be the most effective, but the final conclusion should consider both the mean effects and their respective credible intervals.

There seems to be some field-to-field variation in carbon sequestration, but the field effects α_2 and α_3 are not dramatically different from zero (the effect of the baseline field, α_1), which might suggest that the location has a minor impact on carbon sequestration compared to the type of treatment applied.

The precision of the measurements and the standard deviation indicate moderate variability in the carbon sequestration measurements, which suggests that the data is reliable and the observed effects are not just due to noise.

It's critical to check the 95% credible intervals for the α and β parameters. If zero is not contained within these intervals, it could indicate a statistically significant effect. The density plots can provide a visual indication, but numerical confirmation is necessary.

Overall, this analysis can guide the farmer in selecting the most effective carbon sequestration technique, taking into account both the type of treatment and the slight variations due to field location.

Part H

95% Credible Intervals for Parameters

Part I

95% Credible Intervals for Differences in Treatment Effects
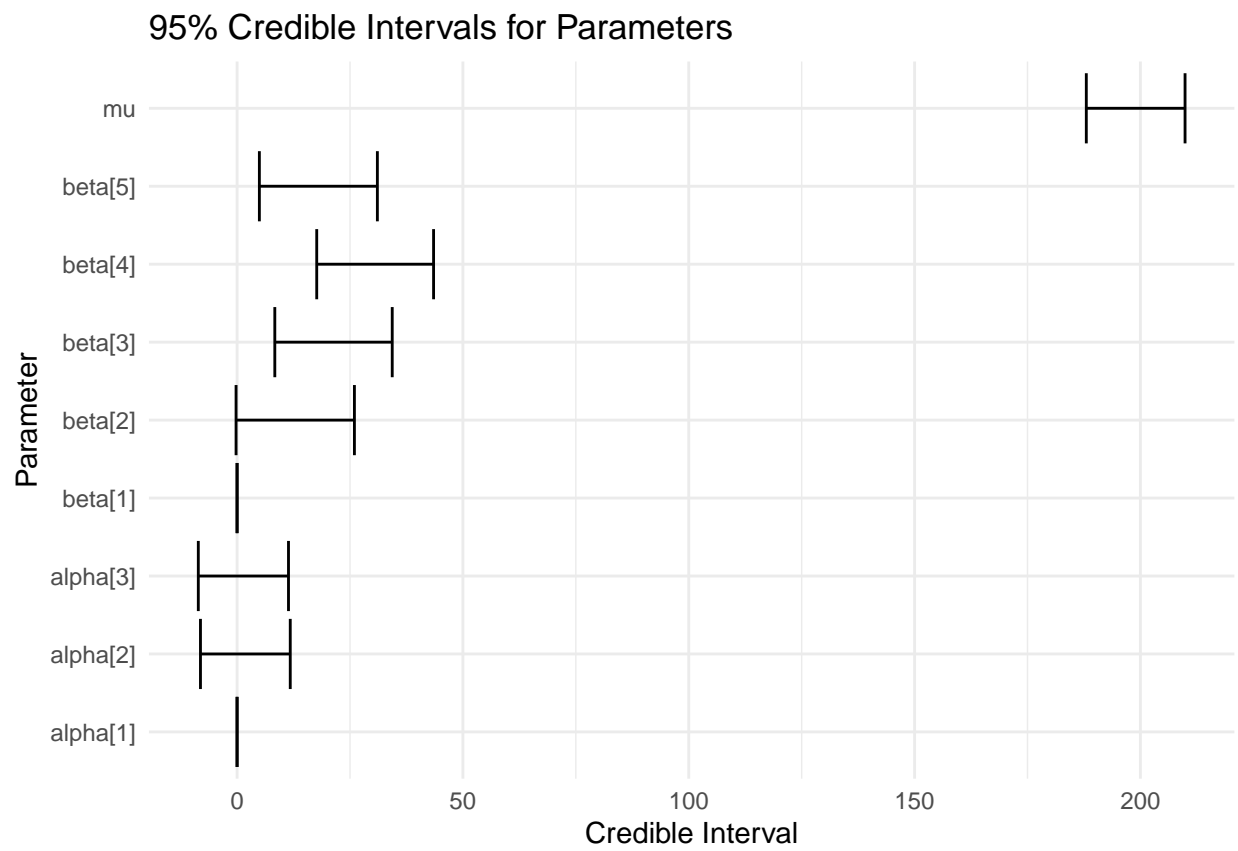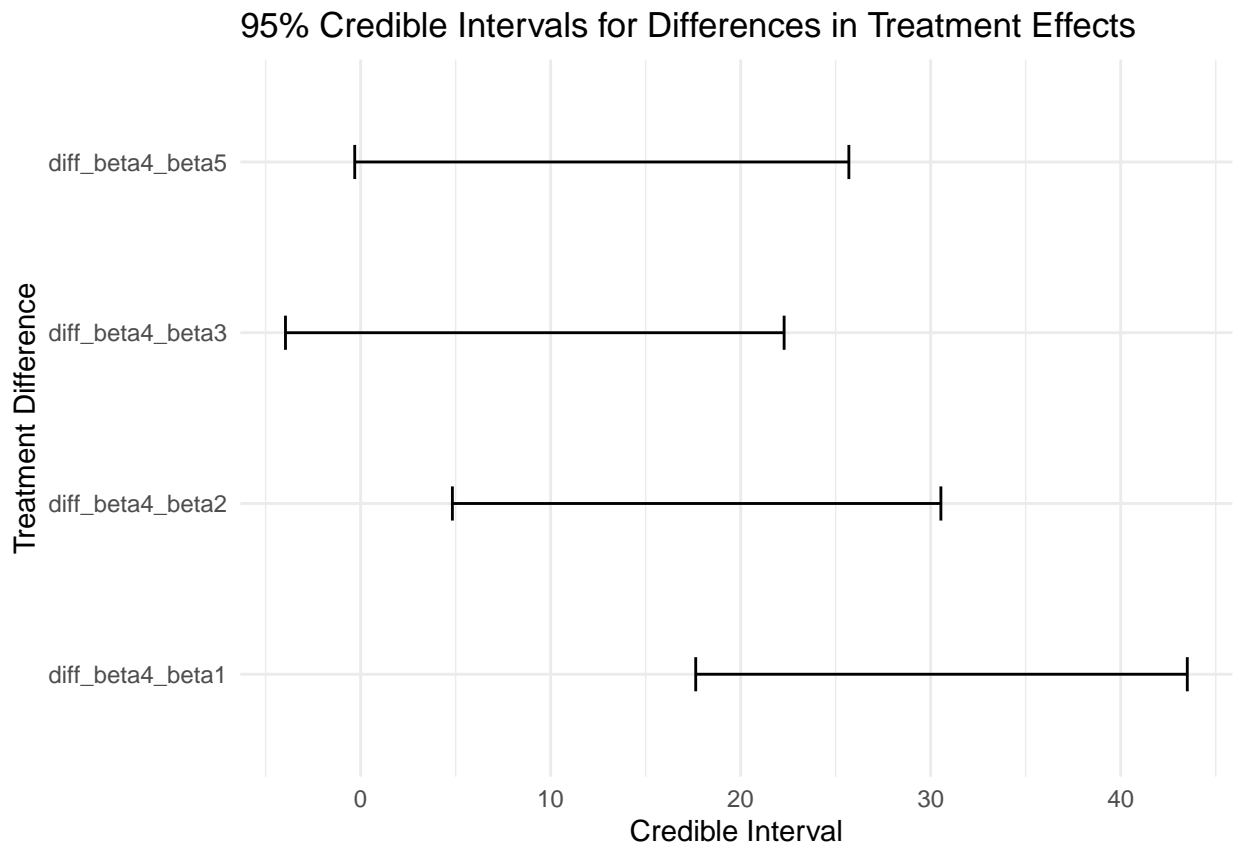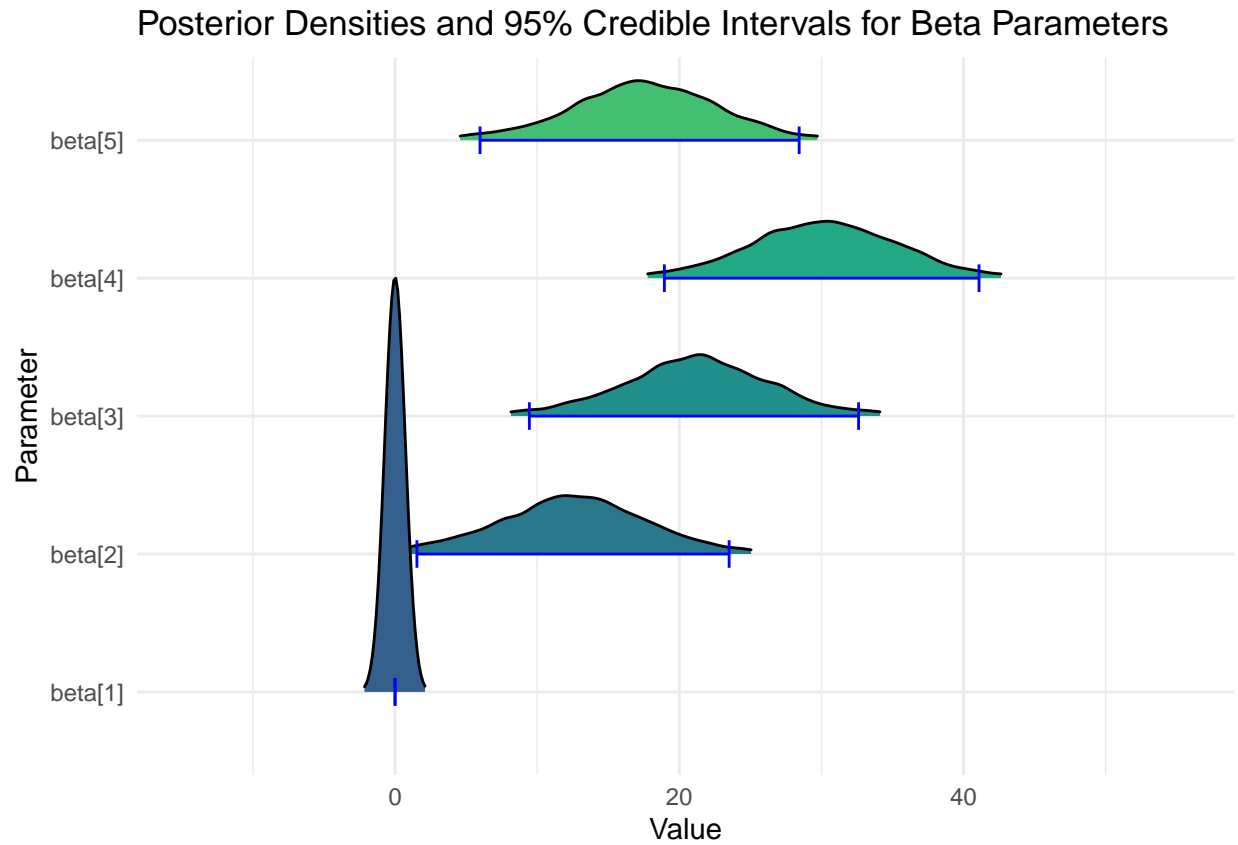

Part J

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 15
##    Unobserved stochastic nodes: 6
##    Total graph size: 32
##
## Initializing model

##          Parameter         Mean        Median  Lower_95_CI   Upper_95_CI
## beta[1]    beta[1]    0.00000000    0.00000000 0.000000e+00    0.00000000
## beta[2]    beta[2]   12.49142970   12.51269742 1.539032e+00   23.50988439
## beta[3]    beta[3]   21.14170519   21.22151984 9.452032e+00   32.61742097
## beta[4]    beta[4]   30.18312137   30.19432135 1.895230e+01   41.09398272
## beta[5]    beta[5]   17.53968743   17.53777316 5.981618e+00   28.43684413
## mu              mu  200.41996462  200.40800544 1.926013e+02  208.46957116
## sigma        sigma    6.72513152    6.42441207 4.365393e+00   10.83564995
## tau            tau    0.02586827    0.02422887 8.517067e-03    0.05247511
##               SD
## beta[1] 0.00000000
## beta[2] 5.54006138
```

```
## beta[3] 5.66176093
## beta[4] 5.58847118
## beta[5] 5.56993817
## mu      3.92887686
## sigma   1.66264986
## tau     0.01146031
```

Part K

```
## Picking joint bandwidth of 0.718
```

Posterior Densities and 95% Credible Intervals for Beta Parameters

Part L