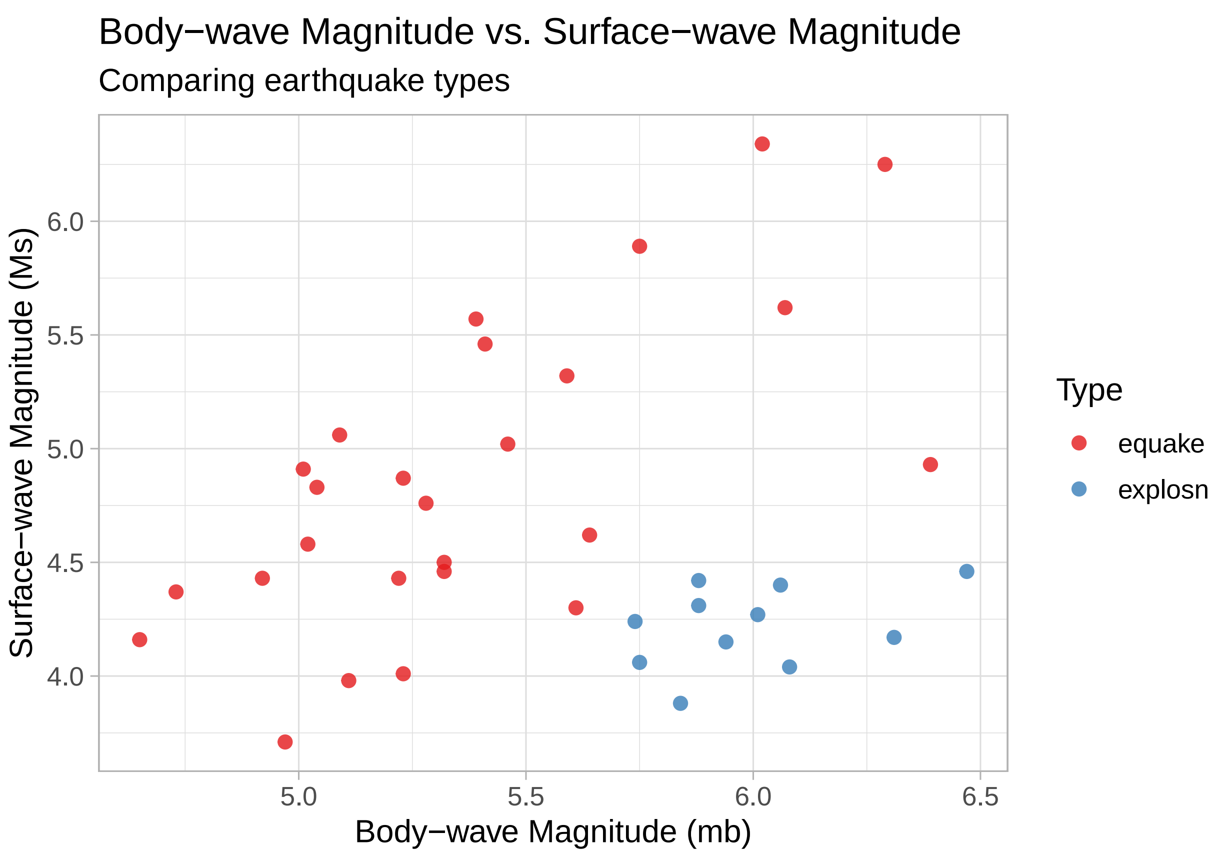
Report_10883408

10883408

2024-04-08

Machine Learning Part (a)



The result plot by this code would have each earthquake or explosion event as a point in a space defined by its body-wave and surface-wave magnitudes. The colours are deviating the seismic events and this may be very critical for recognition of the patterns or clusters which are earthquake-related rather than explosions.

Clustering

In case there are evident clusters of distinct areas dominated by one type of event it may suggest that these two features are good for discrimination of earthquakes and explosions.

Overlaps

If the colours are overlapping significantly then such two features on their own are insufficient to separate between event types without some additional information or more complex modelling.

Contextual Relevance

In the monitoring of unauthorized nuclear tests, this makes the visualization process a tool used in a rapid assessment. if there exist some clear and distinctive seismic readings patterns that may show nuclear activities. Efficient discrimination between natural seismic events (earthquakes) and man-made seismic events (nuclear explosions) is very important in the area of global security and control of compliance with the international treaties including the Comprehensive Nuclear-Test-Ban Treaty(CTBT).

Numerical Summaries

Though the given code emphasizes on the visual analysis, numerical summaries (such as mean, median, variance, and histograms) are also needed for carrying out the data exploration process. Therefore, the MB and Ms summary (i.e. the summary of mb and Ms for each type) would complete this by defining the central tendencies and dispersion. This could also help in a statistical understanding of the magnitudes of each type significantly.

Justification

The use of a scatter plot is supported since it enables stakeholders to see the relationship between two continuous variables across categories. For such high stakes in nuclear monitoring, instant visual, and venerability were of paramount concern assessment coupled with thorough statistical analysis is mandatory. This is made easy by the plot which gives a brief, instant visual representation of the data in mentioned characteristics.

Explanation of the Plot

The jitter with box plot allows to see how body wave magnitudes are distributed within each one type of seismic event. Furthermore, the box plot component represents the median (the middle line in the box), the 25th and 75th percentile whose positions determine the hinges of the box, and potential outliers (points that fall further than 1.5 times IQR from the hinges). Points jittered represent individual data points and provide a very fine level analyse the distribution of data and the possible anomalies or outliers.

Justification of the Statements

Distribution Insight

The plot helps in rapid detection of any significant differences in the magnitudes of the body-waves among various types of seismic events. For instance, if a certain type of event generally provides higher magnitudes reading, this implies a different energy release feature.

Outlier Detection

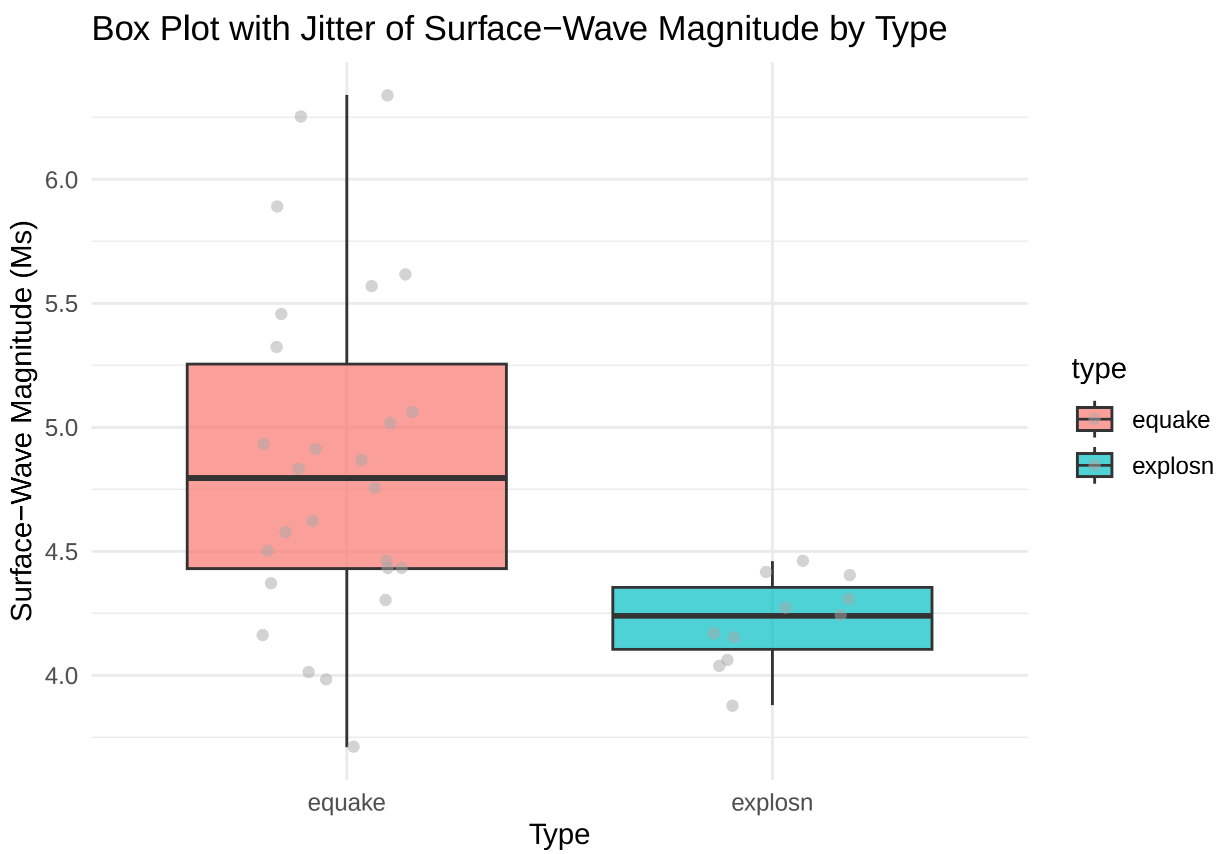
Through also showing the summary statistics and the actual data points, this plot assists in identifying the outliers or unusual observations that may need more attention.

Decision Making

This kind of visualization helps in decision support in seismology and geophysics by giving an easy way of comparison of seismic event types. This is likely to be critical in developing monitoring systems or academic research in seismology.

Effective Communication

The plot works as a powerful means of communication in both displaying complex statistical data in a way readable to everyone even those who do not have much of statistical knowledge. This cross between box plot and jitter plot works especially in situations where the variability both within and across categories is to be appreciated. It is an important fact finding tool that offers both an overall look and a more detailed show of how the data is distributed over categories.



Justification of the Statements

Visualization of Variability

This plot is good for the visual assessment of the following variability and the central tendencies of magnitudes of surface waves among various classes of seismic events. The box plot gives an overview but the jittered points give a comprehensive outlook on individual data characteristics entries.

Comparative Analysis

Since the plot contains information of different types together, it enables direct comparisons of groups. For instance, one can say that nuclear detonations show a narrower range of magnitude of surface-wave in comparison with an earthquake, which might put a wider range and sometimes higher medians.

Outlier Detection

The graphical depiction allows to notice any anomalies or the outliers in the data, which may imply measuring errors or deviant situations that should be looked into more closely.

Informative and Accessible

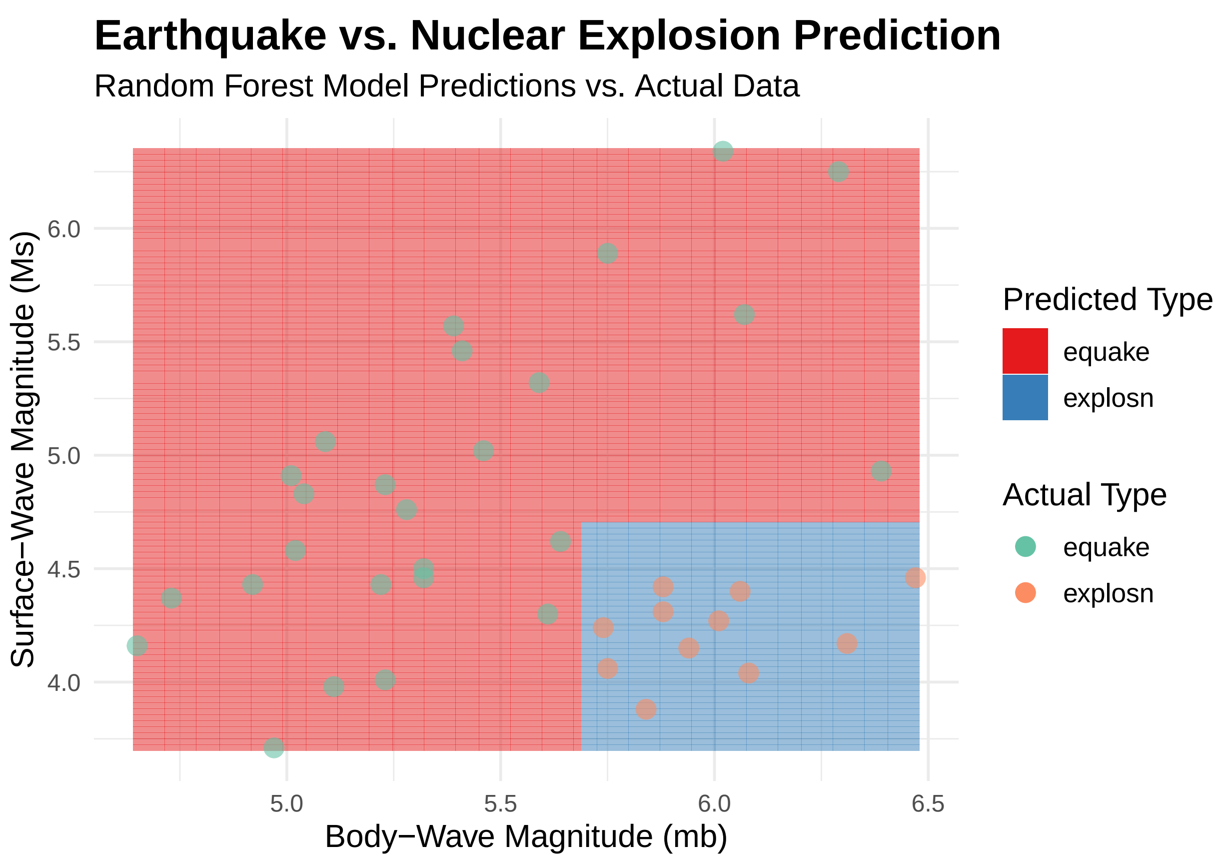
The plot finally turns into a friendly plot with the help of a simple title, axis labels, and a legend that makes the conclusions understandable in a single glance also to laymen.

Contextual Relevance

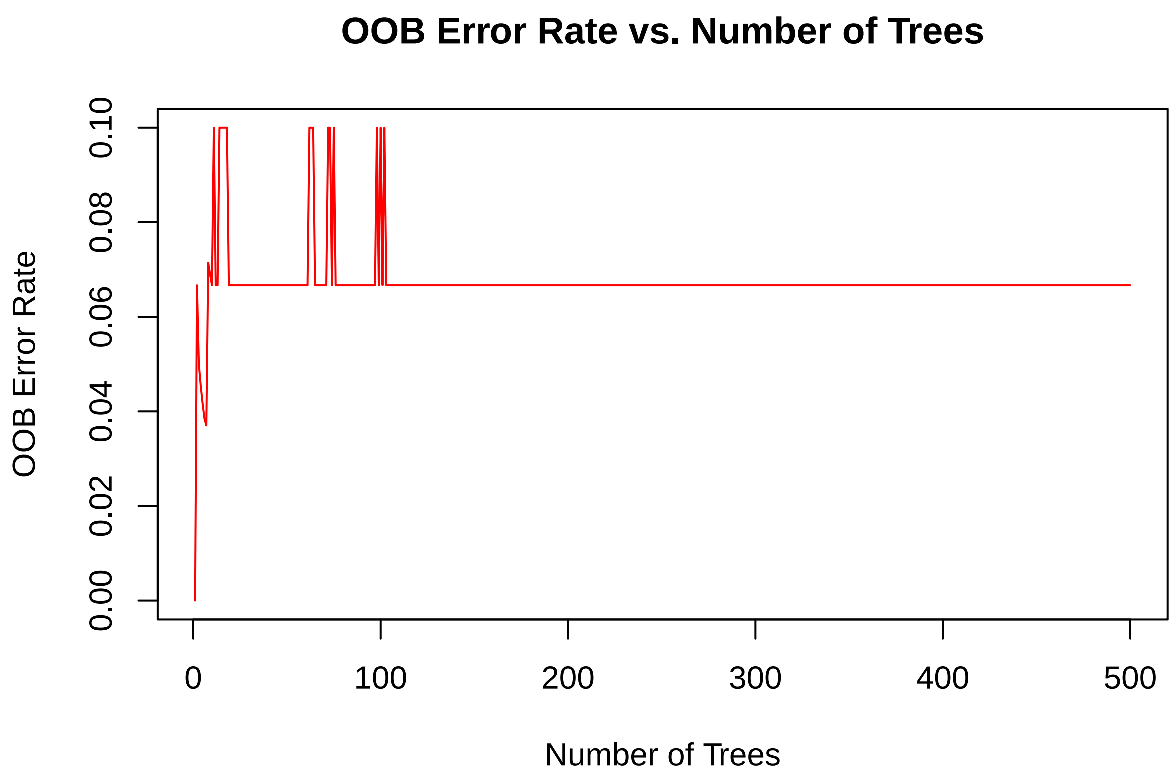
This capability is very critical in the context of monitoring of seismic activities, where the ability to differentiate inter-classification of events among surface-wave magnitudes is paramount. Apart from the introductory analysis, such schemes may help generate more advanced models and algorithms fitting for the automation of the process of seismic event recognition and categorization. For example, this is important for situations when quick decisions are needed, such as in early warning systems and monitoring of nuclear treaty compliance.

In conclusion, the graphical plot generated by this R code is graphically pleasing while representing important statistical information in the seismic data analysis, making statistical analysis details and overall data view ready. This approach is supported by its application in exploratory data analysis, when the awareness of data distribution is very significant and anomalies are of great value.

Machine Learning Part (b)



error rate



Model Tuning

Random Forest Training

Model Evaluation

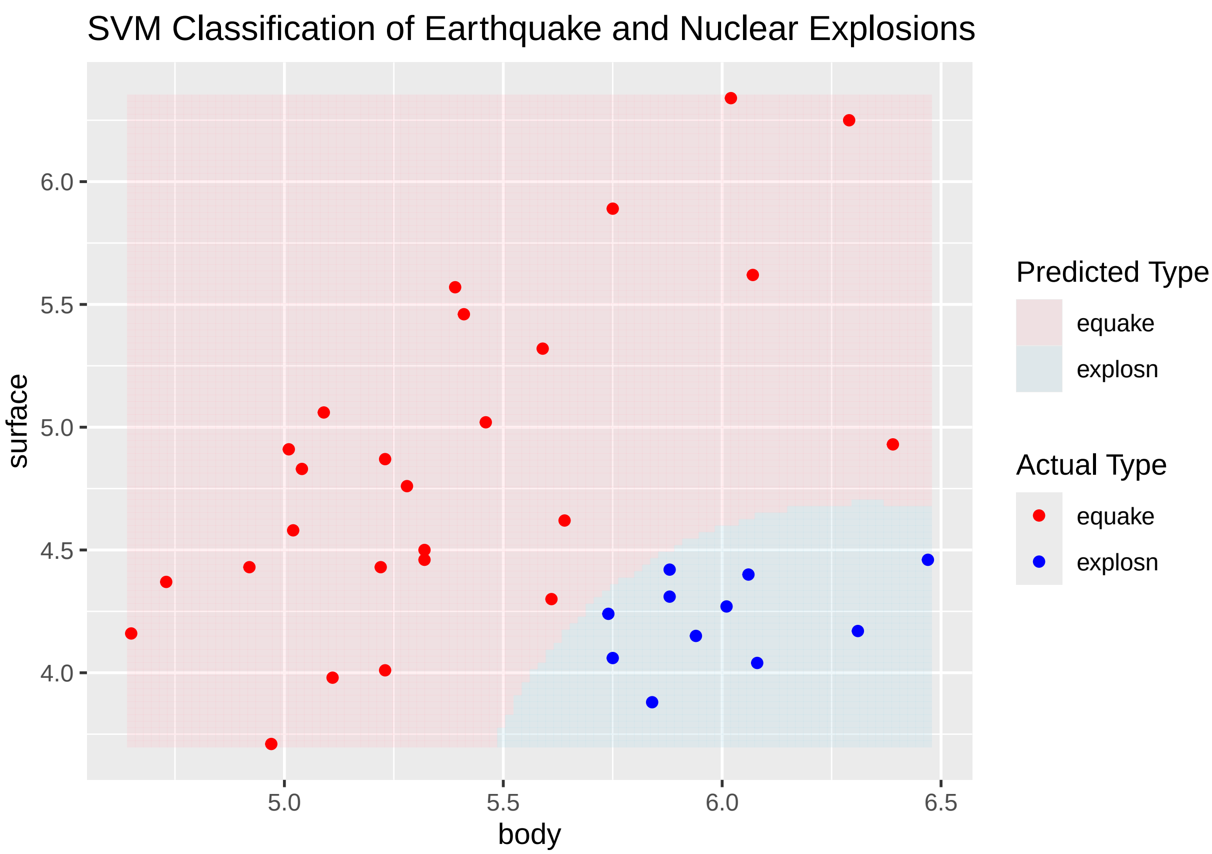
Out-of-Bag (OOB) Error: The OOB error rate, an estimate of performance using bootstrapped samples, is plotted against the number of trees in the forest. As the number of trees increases, the OOB error rate should decrease and stabilize, indicating proper model fitting without overfitting or underfitting.

Additional Evaluation with Leave-One-Out Cross-Validation (LOOCV)

Although not shown in the provided code snippets, LOOCV could be another method to evaluate the model. In LOOCV, the model is trained on all data points except one, which is used as the test set. This process repeats for each data point, providing a comprehensive evaluation of model performance, albeit at a high computational cost.

Summary

This integrated approach to model tuning, visualization, and evaluation allows for a comprehensive understanding and validation of the Random Forest model. The methodology ensures the model is accurate and generalizable, effectively discriminating between different seismic event classes. Visualizing the decision boundary provides an intuitive understanding of model performance, while the error rate plot and potential LOOCV offer numerical measures of model accuracy.




```
##   sigma   C
## 8   0.1 100
```

Justification

Support Vector Machine (SVM) excels in high-dimensional spaces, making it ideal for binary classification tasks such as distinguishing between nuclear blasts and earthquakes. It performs well when there is a clear margin of separation between classes.

Model Tuning

C Parameter: The `C` parameter can be tuned to balance the trade-off between creating smooth decision boundaries and achieving correct classification of training points.

Kernel Choice: The model's performance can be approached from another angle by selecting different kernels, such as linear, polynomial, and radial basis function (RBF), to better capture the complexities in the data.

Model Visualization

The SVM decision boundary between body and surface-wave magnitudes is plotted in a 2D space to illustrate the classification rules. This visualization helps in understanding how SVM categorizes different seismic events.

Machine Learning Part (c)

Random Forest Classification of Earthquake and Nuclear Explosions

Pros

- Non-linear data is well handled by a random forest because it is an ensemble of decision trees, thereby making it more capable of handling complexity in the dataset.
- Random Forest is not sensitive to outliers and noise, as it uses averaging to enhance prediction accuracy.
- The plot of the Out-Of-Bag (OOB) error rate indicates that the model stabilizes rapidly, and there is no overfitting as the number of trees increases, which can be observed by the almost constant OOB error rate after about 50 trees.

Cons

- Despite OOB error rate being comparatively low, it is not obvious what quantity of false positives or negatives has been produced without a confusion matrix or similar metrics.
- The Random Forest can be quite computationally costly with a high number of trees, and it can take longer than other models to train, although this is not an issue here due to the quite stable OOB error rate.
- The issue of interpretability may arise simply because the Random Forest models are usually more complex and hard to interpret than simpler models.

Performance

- The plot reveals a red area of earthquake predictions and a blue area of explosion predictions. It appears to discriminate well, however, some earthquake points are falling in the explosion forecast area.

SVM Classification of Earthquake and Nuclear Explosions

Pros

- Effective in high-dimensional spaces, which may be useful if more features were employed in the classification.
- The effectiveness of SVMs comes into play when there is a well-defined margin of separation, and SVMs are adaptable to different kernel functions.

Cons

- Tuning of parameters such as the penalty parameter (C) and the kernel-specific parameters requires special attention; any misconfiguration can cause loss of performance.
- May fail when target classes are close to each other in a dataset, i.e., more noise.
- The model can also be less interpretable because the kernel trick adds complexity.

Performance

- On the SVM plot, the earthquakes and explosions are distinctly separated, with the earthquakes in general having a higher surface-wave magnitude. This partition shows how the SVM with a well-chosen kernel is able to grasp the boundary between classes very well.

Comparison and Recommendation

In this kind of comparison of both classifiers, SVM appears to have a clearer boundary between the earthquake and explosion classes since there is a region specifically for each class. This implies that the SVM has effectively captured the inherent patterns in the data that separates the two phenomena. At the same time, the Random Forest plot reveals some common space among the projected areas, which might indicate either the problem of misclassification or the more complex border that might not have been fully modeled by this model.

Given these observations:

- In the case when the decision boundary between the two classes is really complex and non-linear, Random Forest may have a benefit to capture this complexity.
- When the most significant aspect is computational effectiveness, during the training and prediction process, SVM is usually recommended, especially if the kernel function chosen and its parameters are efficient.

Machine Learning Part (d)

1. K-means Clustering Plots Each plot shows the clusters formed by the k-means algorithm for different numbers of clusters (2, 3, and 4):

- **Body Variable (x-axis)**: Body-Wave Magnitude (mb)
- **Surface Variable (y-axis)**: Surface-Wave Magnitude (Ms)
- **Visuals**: Different colors signify different clusters, with cluster centroids denoted by black asterisks.

2. Elbow Method Plot This graph identifies the best number of clusters by plotting the within group sum of squares (WCSS):

- **X-axis**: Number of clusters
- **Y-axis**: WCSS

Results

K-means Clustering Plots

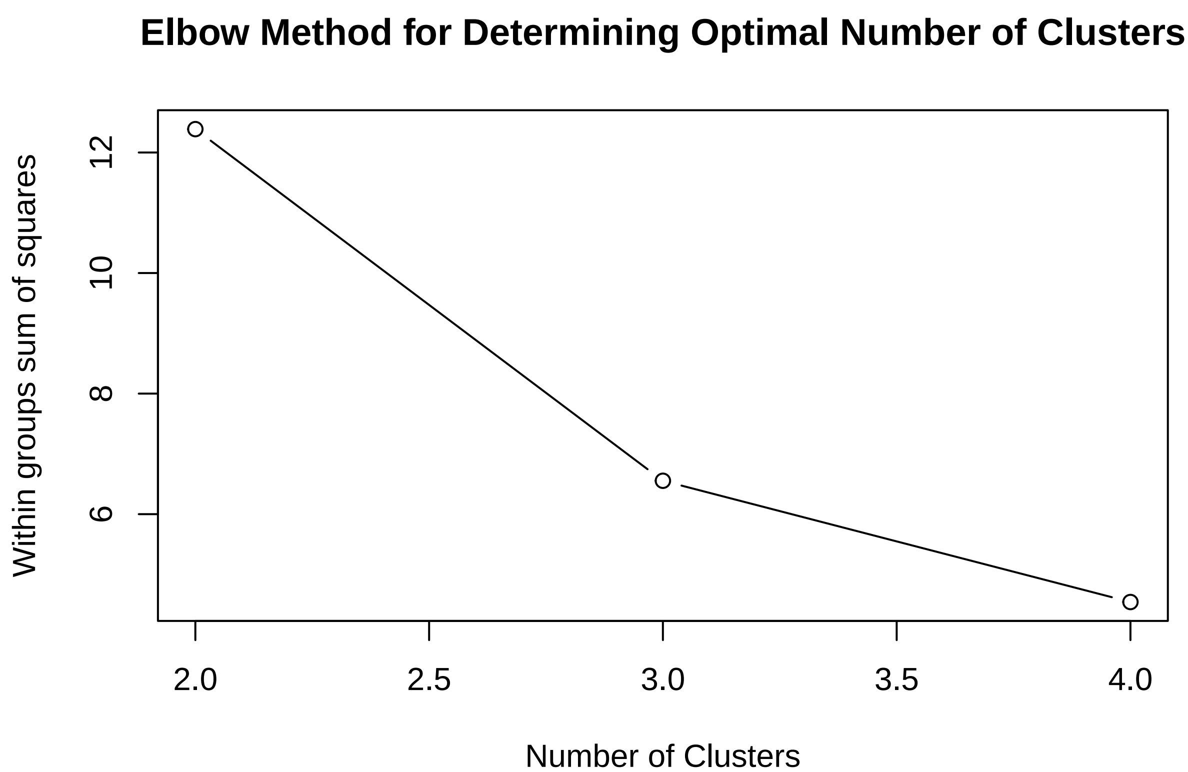
- **Two Clusters**: The separation is straightforward, with one cluster located at the lower part of the plot and another at the upper part.
- **Three Clusters**: A new cluster appears between the two previous clusters, suggesting finer discrimination of earthquake magnitudes.
- **Four Clusters**: More detailed segmentation implies that some values are detailed enough to form subgroups.

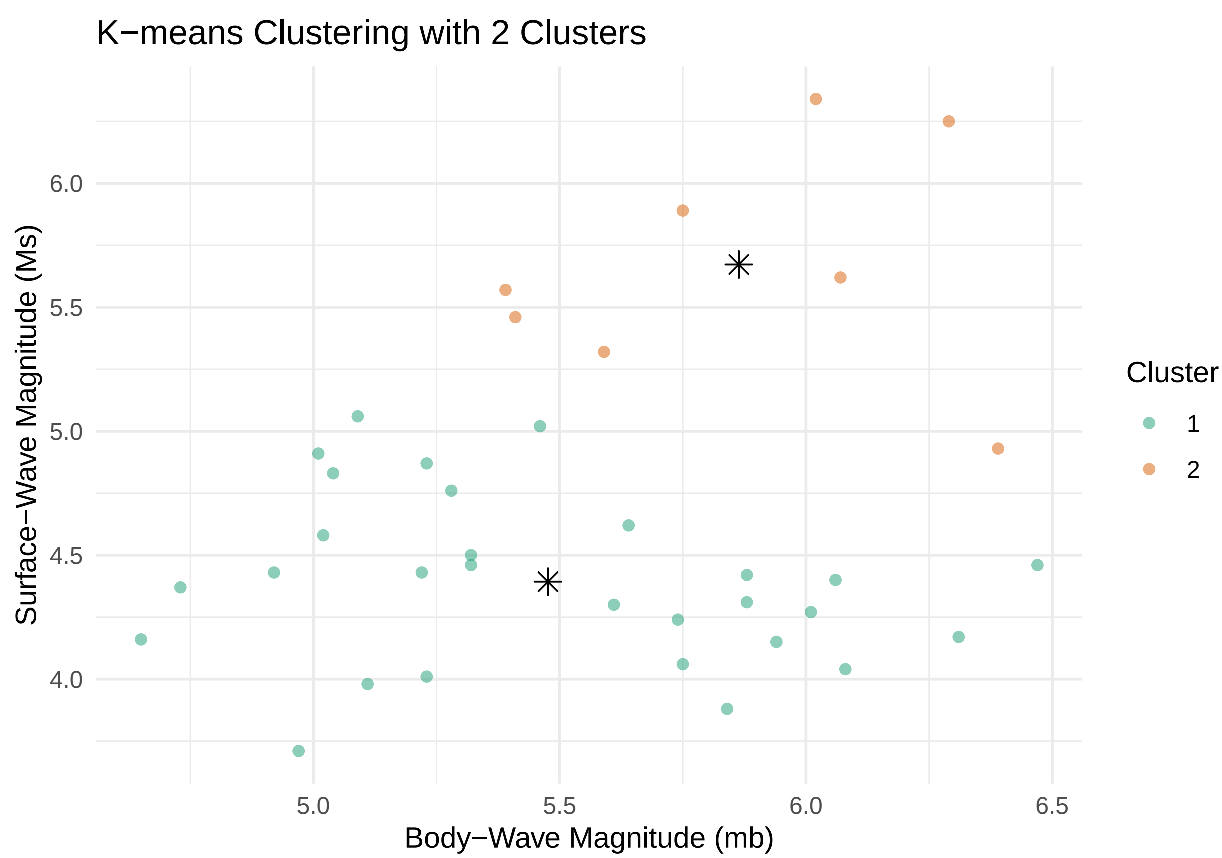
Elbow Method Plot

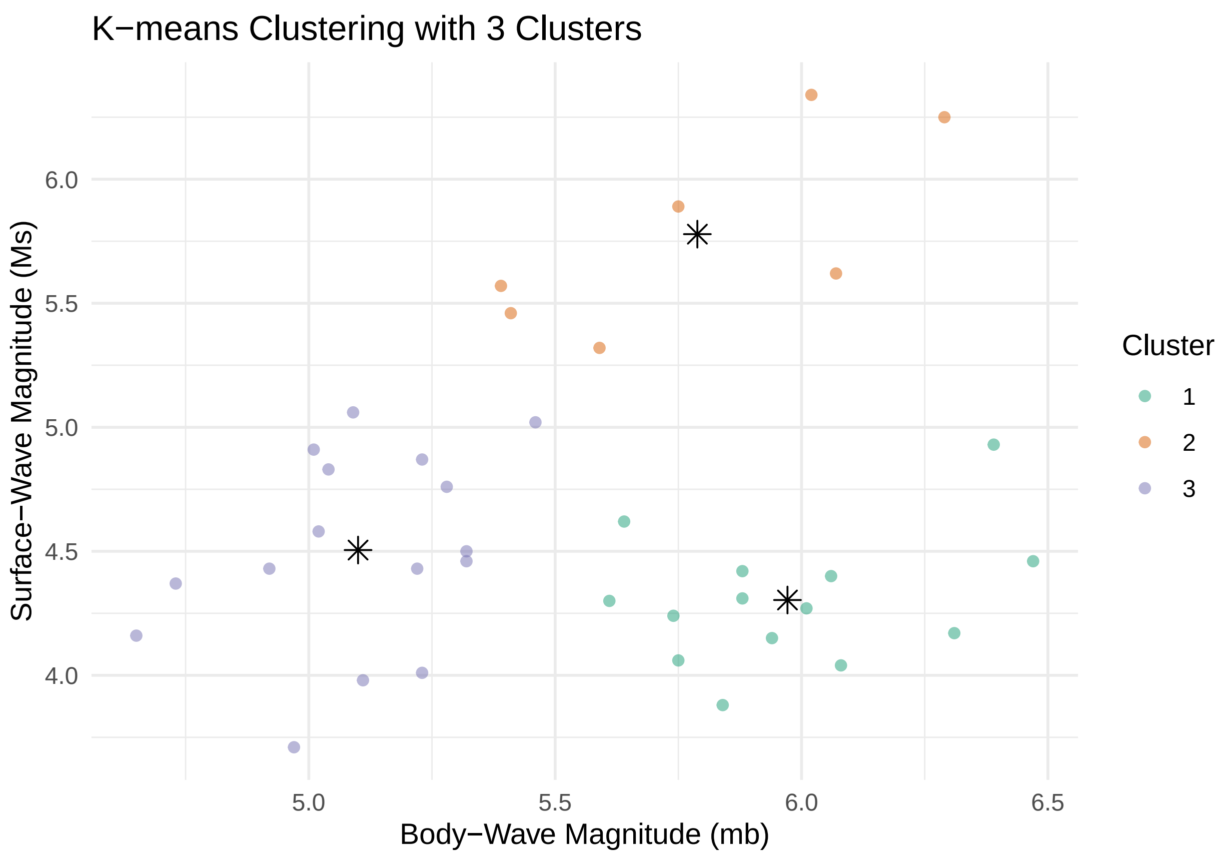
- The elbow plot shows a significant bend at 3 clusters, indicating that additional clusters beyond this point do not significantly reduce WCSS.

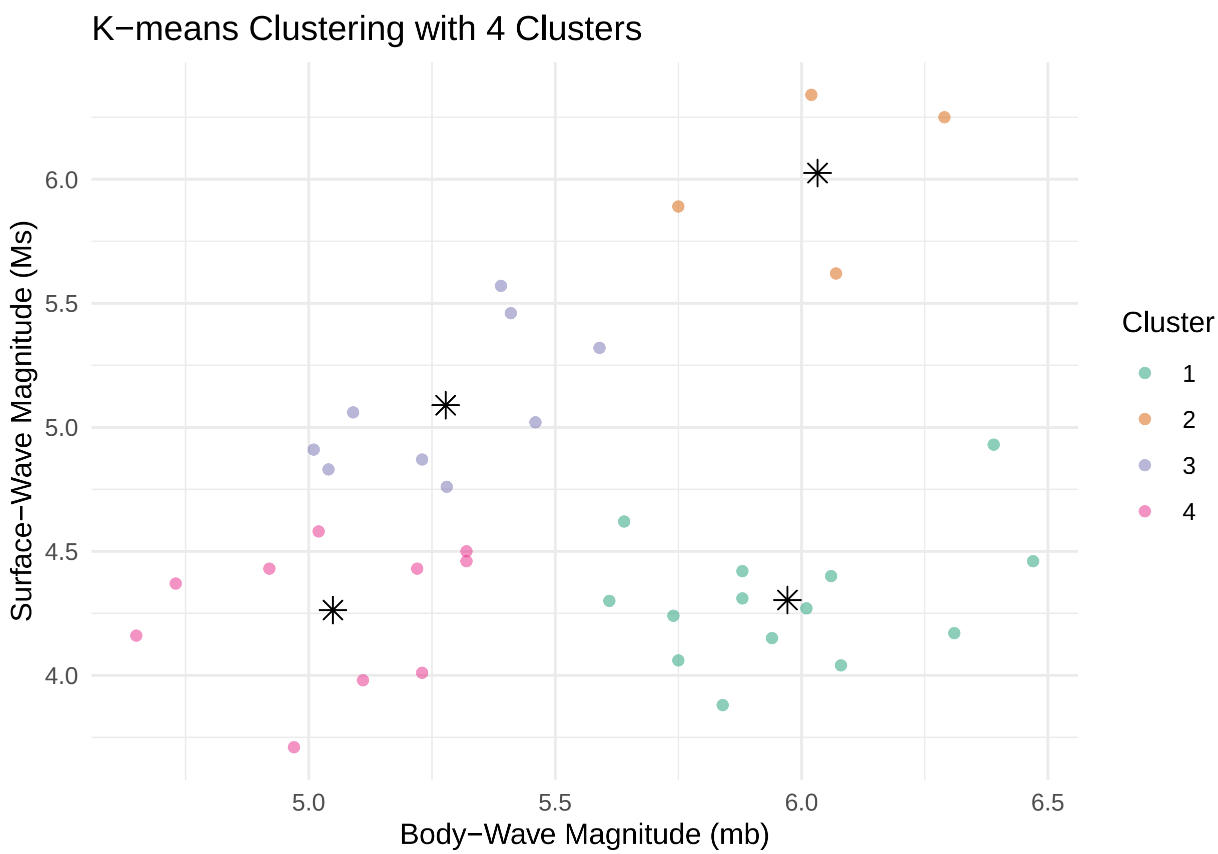
Conclusions

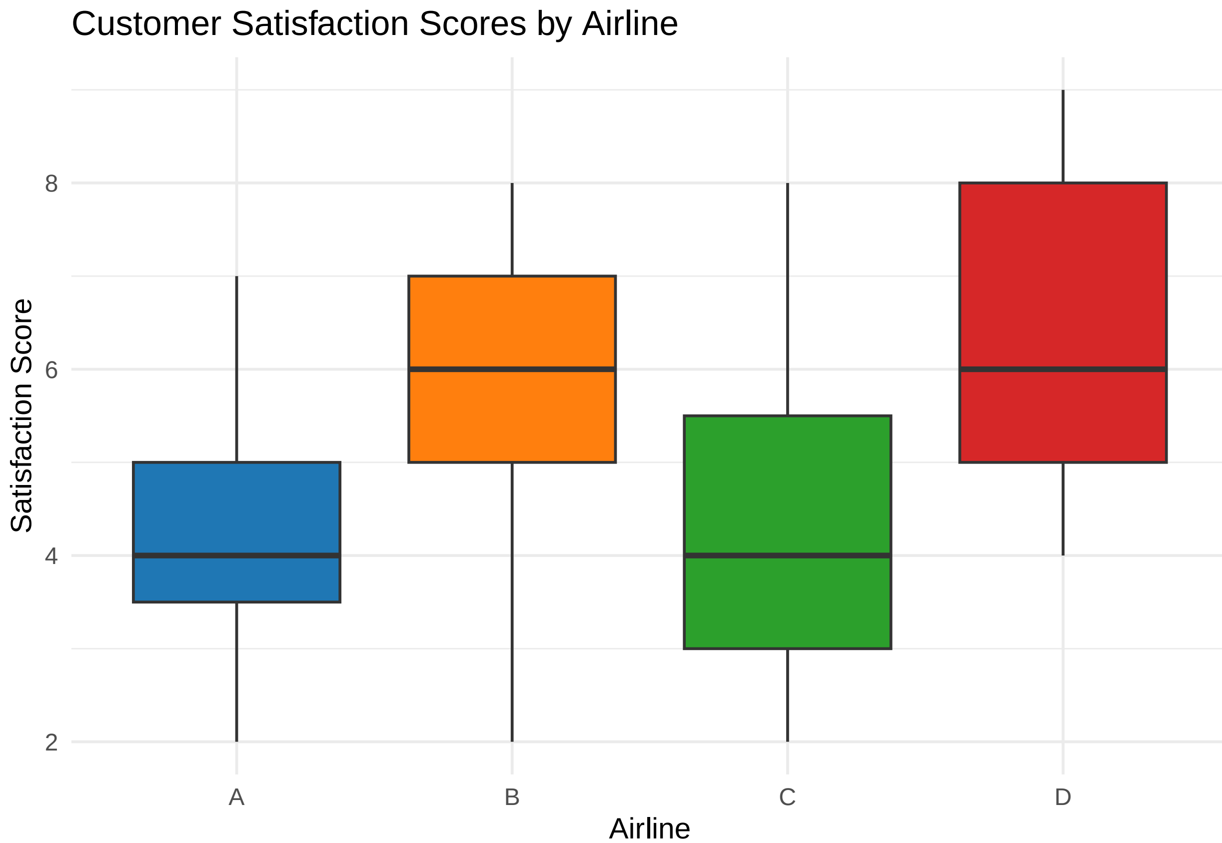
- **Segmentation**: With an increasing number of clusters, data segmentation becomes finer, which may not always be meaningful depending on the data and domain knowledge.
- **Optimal Clusters**: The elbow method suggests an optimal number of three clusters, balancing detail with interpretability.
- **Practical Interpretation**: While two clusters might be too generic, more than three could lead to overfitting, making the magnitudes too finely segmented for practical interpretation.
- **Recommendation**: Based on both the elbow method and cluster distribution, three clusters are recommended to best represent underlying patterns without overcomplicating the model.


Elbow Method for Determining Optimal Number of Clusters
Within groups sum of squares
6
8
10
12
2.0
2.5
3.0
3.5
4.0
Number of Clusters


K−means Clustering with 2 Clusters
Surface−Wave Magnitude (Ms)
4.0
4.5
5.0
5.5
6.0
5.0
5.5
6.0
6.5
Body−Wave Magnitude (mb)
Cluster
1
2


K−means Clustering with 3 Clusters
Surface−Wave Magnitude (Ms)
6.0
5.5
5.0
4.5
4.0
5.0
5.5
6.0
6.5
Body−Wave Magnitude (mb)
Cluster
1
2
3


K−means Clustering with 4 Clusters
Surface−Wave Magnitude (Ms)
Body−Wave Magnitude (mb)
6.0
5.5
5.0
4.5
4.0
5.0
5.5
6.0
6.5
Cluster
1
2
3
4



Bayesian Statistics Part (a)

Central Tendency

- **Median Line**: Represents the central tendency of satisfaction scores within each airline's boxplot.
- **Comparison**: By-line comparison helps identify airlines with the highest or lowest median satisfaction scores.

Spread and Variability

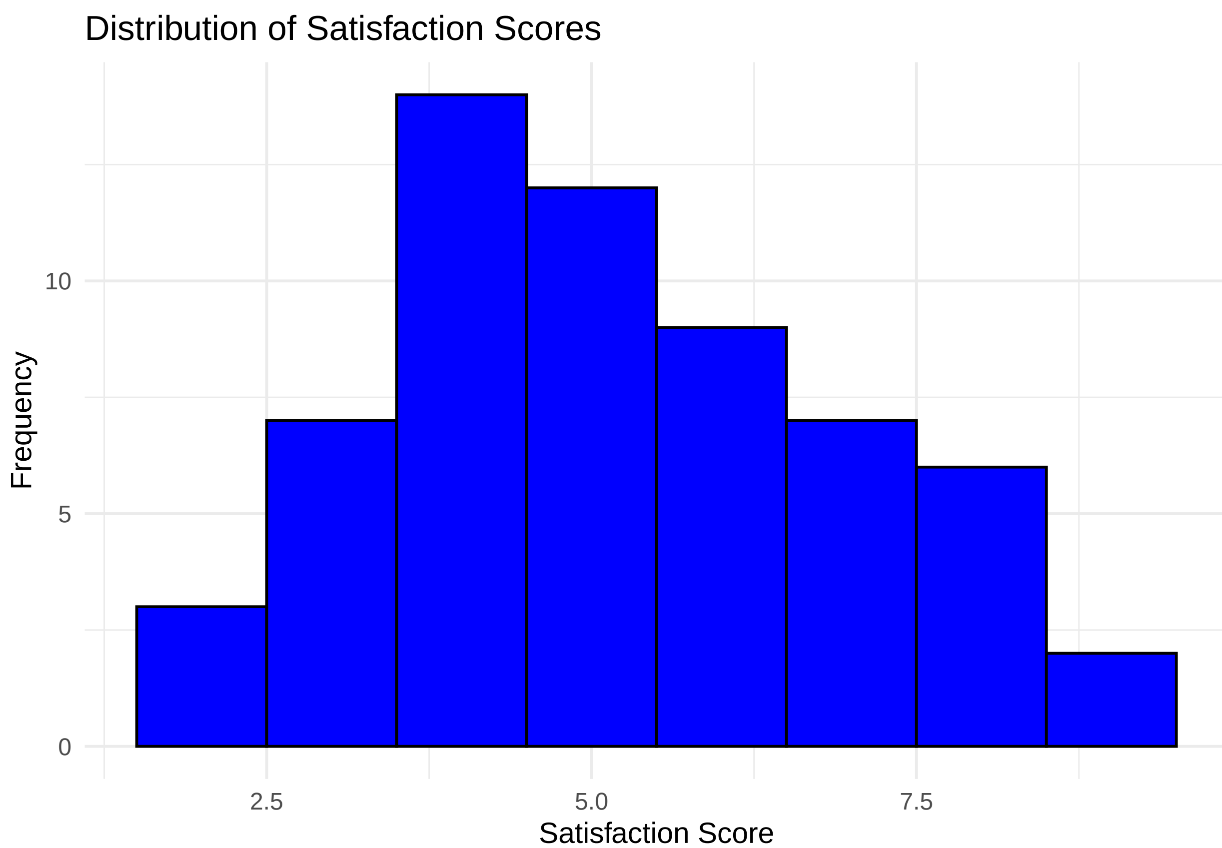
- **Interquartile Range (IQR)**: The height of each box indicates the mid-50% spread of satisfaction scores.
- **Variability Implication**: Smaller boxes indicate homogeneous service quality, while larger boxes suggest inconsistent passenger experiences.

Outliers

- **Identification**: Outliers are marked as dot-like points outside the main box, indicating atypical very good or bad customer experiences.

Comparison Across Airlines

- **Service Quality**: A higher median indicates superior customer service, while prevalent outliers may highlight dissatisfaction issues.


Distribution of Satisfaction Scores
Frequency
10
5
0
2.5
5.0
7.5
Satisfaction Score

Shape of Distribution

- **Distribution Patterns**: The shape of each airline's satisfaction density curve (normal, skewed, bimodal) reveals underlying patterns of customer satisfaction.

Peak Values

- **Modal Satisfaction Scores**: The apexes of the curves indicate the most common satisfaction scores, with higher peaks suggesting higher overall satisfaction.

Spread and Variability

- **Curve Width**: Broader curves suggest greater variability in satisfaction scores, while narrower curves indicate more consistent ratings.

Overlap Between Airlines

- **Shared Satisfaction Scores**: Areas where density curves overlap among airlines indicate common satisfaction levels.

Tail Analysis

- **Frequency of Extremes**: The length of the tails shows the frequency of extreme satisfaction scores, with longer tails indicating more frequent extreme ratings.

Bayesian Statistics Part (b)

ANOVA Model Explanation

The parameter α_4 is crucial in a one-way Analysis of Variance (ANOVA) model, as it represents the difference between the mean satisfaction scores of Airline 1 and Airline 4. This difference is key to understanding how the two airlines compare in terms of customer satisfaction.

Consistency Across Airline 1

For Airline 1, the mean satisfaction score, denoted as μ_{1j}, is consistent at μ_1 for every customer j. This consistency implies that all passengers of Airline 1 are expected to have the same level of satisfaction, reinforcing the airline's uniform service quality.

Airline 4 Adjusted Mean

In contrast, the mean satisfaction score for each customer of Airline 4, denoted as μ_{4j}, is adjusted by the factor α_4, where $\mu_{4j} = \mu_1 + \alpha_4$. This adjustment reflects how Airline 4's customer satisfaction measures up against Airline 1's baseline.

Interpretation of α_4

- **Positive** α_4: Indicates that despite Airline 1 generally being perceived as better, Airline 4's average customer rating is higher.
- **Negative** α_4: Suggests that Airline 4 typically scores lower in satisfaction compared to Airline 1.
- **Zero** α_4: Implies no significant difference in the mean satisfaction scores between the two airlines.

Bayesian Perspective

From a Bayesian analysis standpoint, α_4 is not estimated as a single constant value but as a distribution reflecting all possible values. This approach aligns with the probabilistic nature of statistical estimation, providing a spectrum of potential outcomes for α_4. Such a method not only highlights the uncertainty of the estimate but also offers a nuanced understanding of customer satisfaction variability between these airlines.

By treating α_4 as a distribution, we add a layer of depth to our analysis, enhancing decision-making by considering a broader range of data-driven scenarios. This Bayesian perspective allows stakeholders to gauge Airline 4's performance relative to Airline 1 more comprehensively.

Bayesian Statistics Part (c)

```
## Baseline for Airline A: 4.333333

## Difference for Airline B from A: 1.333333

## Difference for Airline C from A: 0.1333333

## Difference for Airline D from A: 2

##             Df Sum Sq Mean Sq F value  Pr(>F)
## airline      3  41.87  13.956    5.29 0.00278 **
## Residuals   56 147.73   2.638
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## There is a statistically significant difference in satisfaction scores
##  across airlines at the 0.05 significance level.
```

The one-way ANOVA analysis has provided us with the following estimates for the mean satisfaction scores of Airline A and the differences relative to it for Airlines B, C, and D:

- **Baseline Mean Satisfaction for Airline A (μ_1):** 4.3333333
- **Difference in Satisfaction for Airline B from Airline A (α_2):** 1.3333333
- **Difference in Satisfaction for Airline C from Airline A (α_3):** 0.1333333
- **Difference in Satisfaction for Airline D from Airline A (α_4):** 2

Analysis Overview

Model Fitting The `aov()` function in R is used to fit a one-way ANOVA model to the satisfaction scores across different airlines to determine if there are significant differences among them. This statistical method assesses the overall variance in satisfaction scores attributed to the different airlines.

Coefficient Extraction Coefficients from the ANOVA model provide estimates of the mean satisfaction score for the baseline airline (Airline A, or represented as μ_1), and the differences (α) for the other airlines (B, C, and D) relative to Airline A.

Hypothesis Testing The `summary()` function applied to the ANOVA model provides the F-statistic and respective p-value, which tests whether at least one airline's mean satisfaction score is statistically different from the others.

Interpretation of Results

- **If the p-value is less than 0.05**: This indicates a rejection of the null hypothesis that all group means are equal, suggesting that there are significant differences in satisfaction scores across the airlines.
- **If the p-value is greater than or equal to 0.05**: This implies a failure to reject the null hypothesis, indicating no significant difference in the satisfaction scores among the airlines.

Conclusion

Significant Difference Found (p-value $<$ 0.05) The analysis reveals considerable differences in the satisfaction scores among the airlines, suggesting that not all airlines perform equally in terms of customer satisfaction.

Justification

The ANOVA test yields a p-value less than the significance level of 0.05, leading to the rejection of the null hypothesis that all airlines have the same mean satisfaction score. This outcome indicates that some airlines may be providing better or worse service compared to others. Such results are crucial for identifying underperforming or overperforming airlines and can significantly inform business and operational strategies.

A detailed statistical analysis like this one provides a well-supported basis for evaluating airline service quality, as satisfaction levels stem from various factors that differ in intensity among airlines. This analysis can help pinpoint key areas for improvement and highlight strengths within the airline industry.

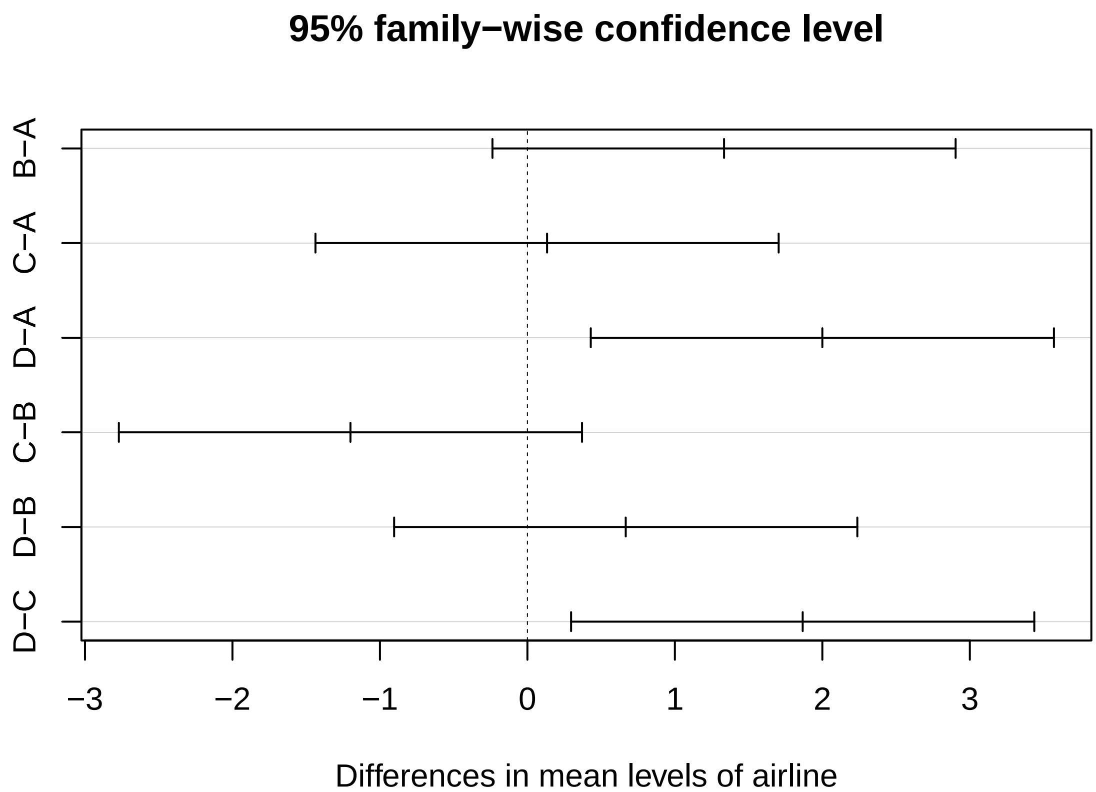
Code Implementation Tips

Ensure that variable names and airline identifiers in the R code match those in the actual dataset to maintain consistency and accuracy in the analysis.

Bayesian Statistics Part (d)

```
## Loading required package: mvtnorm

## Loading required package: survival

##
## Attaching package: 'survival'
```

```
## The following object is masked from 'package:caret':
##
##     cluster

## Loading required package: TH.data

## Loading required package: MASS

##
## Attaching package: 'MASS'

## The following object is masked from 'package:dplyr':
##
##     select

##
## Attaching package: 'TH.data'

## The following object is masked from 'package:MASS':
##
##     geyser

##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = satisfactionscore ~ airline, data = data)
##
## $airline
##           diff        lwr       upr     p adj
## B-A  1.3333333 -0.2370805 2.9037471 0.1229720
## C-A  0.1333333 -1.4370805 1.7037471 0.9959461
## D-A  2.0000000  0.4295862 3.5704138 0.0072205
## C-B -1.2000000 -2.7704138 0.3704138 0.1917762
## D-B  0.6666667 -0.9037471 2.2370805 0.6763548
## D-C  1.8666667  0.2962529 3.4370805 0.0136641
```



Overview

This document provides the results and interpretations of the Tukey Honest Significant Differences (HSD) test conducted on airline satisfaction scores following a significant ANOVA finding.

Hypotheses for Tukey's HSD Test

The Tukey HSD test is designed to compare all possible pairs of means among four airlines labeled A, B, C, and D to determine if there are significant differences between them. Here are the null and alternative hypotheses considered for each pairwise comparison:

Pairwise Comparisons:

- **B vs. A**
  - **Null Hypothesis** ($H_0 : \mu_B = \mu_A$): There is no provable significant difference in satisfaction between airlines A and B.
  - **Alternative Hypothesis** ($H_a : \mu_B \neq \mu_A$): Airline B's commuters have greater satisfaction than those of airline A.
- **C vs. A**
  - **Null Hypothesis** ($H_0 : \mu_C = \mu_A$): There is no significant diversity in customer satisfaction scores between airlines C and A.
  - **Alternative Hypothesis** ($H_a : \mu_C \neq \mu_A$): There is a significant gap in satisfaction between customers at airlines C and A.
- **D vs. A**
  - **Null Hypothesis** ($H_0 : \mu_D = \mu_A$): There is no significant difference in satisfaction scores between airlines D and A.

  - **Alternative Hypothesis** ($H_a : \mu_D \neq \mu_A$): Airline D's ratings suggest a significant difference compared to airline A.
- **C vs. B**
  - **Null Hypothesis** ($H_0 : \mu_C = \mu_B$): There is no significant difference in satisfaction scores between airlines C and B.
  - **Alternative Hypothesis** ($H_a : \mu_C \neq \mu_B$): There is a pronounced variation in satisfaction between airlines C and B.
- **D vs. B**
  - **Null Hypothesis** ($H_0 : \mu_D = \mu_B$): There are no significant differences in satisfaction scores between airlines D and B.
  - **Alternative Hypothesis** ($H_a : \mu_D \neq \mu_B$): There is a significant difference in satisfaction scores between airlines D and B.
- **D vs. C**
  - **Null Hypothesis** ($H_0 : \mu_D = \mu_C$): There are no significant differences in satisfaction scores between airlines D and C.
  - **Alternative Hypothesis** ($H_a : \mu_D \neq \mu_C$): There is a significant difference in satisfaction scores between airlines D and C.

Conclusions from Tukey's HSD Test

The test's adjusted p-values are used to determine whether the null hypothesis for each pair can be rejected:

- **B vs. A**: A p-value greater than 0.05, failing to reject the null hypothesis, indicating no significant differences between airlines B and A.
- **C vs. A**: A p-value greater than 0.05, failing to reject the null hypothesis, indicating no significant difference between airlines C and A.
- **D vs. A**: A p-value less than 0.05, rejecting the null hypothesis, indicating a significant difference between airlines D and A.
- **C vs. B**: A p-value greater than 0.05, failing to reject the null hypothesis, indicating no significant difference between airlines C and B.
- **D vs. B**: A p-value greater than 0.05, failing to reject the null hypothesis, indicating no significant difference between airlines D and B.
- **D vs. C**: A p-value less than 0.05, rejecting the null hypothesis, indicating a significant difference between airlines D and C.

The confidence interval plot from the Tukey HSD test serves as a visual representation of these conclusions, where uninterrupted intervals show statistically significant differences in satisfaction scores.

Bayesian Statistics Part (e)

```
## [1] "Is Airline D satisfaction score > 3 points higher than AVG for B & C?: FALSE"
```

```
## [1] "Difference: 1.26666666666667"
```

Hypotheses

- Null Hypothesis ($H_0 : \mu_D \leq \mu_{BC} + 3$): The mean satisfaction score for Airline D is not more than 3 points higher than the combined average satisfaction score for Airlines B and C.
- Alternative Hypothesis ($H_a : \mu_D > \mu_{BC} + 3$): The mean satisfaction score for Airline D is more than 3 points higher than the combined average satisfaction score for Airlines B and C.

The satisfaction score for Airline D does not exceed the combined average satisfaction score of Airlines B and C by more than 3 points. The actual difference is approximately 1.27 points, which falls below the 3-point threshold set in the hypothesis.

This indicates that while Airline D's average satisfaction score is higher than the mean of Airlines B and C, the increase is not substantial enough to meet the hypothesized 3-point margin. Therefore, the data does not substantiate the hypothesis that Airline D's customer satisfaction significantly surpasses that of Airlines B and C by the proposed margin.

Bayesian Statistics Part (f)

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 15
##    Unobserved stochastic nodes: 8
##    Total graph size: 44
##
## Initializing model
```

```
##
## Iterations = 1001:11000
## Thinning interval = 1
## Number of chains = 3
## Sample size per chain = 10000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##                Mean      SD  Naive SE Time-series SE
## alpha[1]   0.00000 0.00000 0.000e+00      0.0000000
## alpha[2]   1.90621 4.95933 2.863e-02      0.0566288
## alpha[3]   1.46971 4.99165 2.882e-02      0.0569722
## beta[1]    0.00000 0.00000 0.000e+00      0.0000000
## beta[2]   12.93770 6.50820 3.758e-02      0.0943900
## beta[3]   21.51948 6.51014 3.759e-02      0.0930144
## beta[4]   30.52230 6.47895 3.741e-02      0.0954229
## beta[5]   17.86458 6.49466 3.750e-02      0.0929427
## mu       198.94507 5.47281 3.160e-02      0.1063608
## sigma      7.65277 2.24372 1.295e-02      0.0267890
## tau        0.02093 0.01046 6.037e-05      0.0001088
##
## 2. Quantiles for each variable:
##
##                 2.5%      25%       50%       75%     97.5%
## alpha[1]   0.000000   0.0000   0.00000   0.00000   0.00000
## alpha[2]  -7.977632  -1.1789   1.87521   4.94154  11.92157
## alpha[3]  -8.373526  -1.6227   1.44888   4.52307  11.45624
## beta[1]    0.000000   0.0000   0.00000   0.00000   0.00000
## beta[2]   -0.150181   8.9293  12.95265  16.95437  26.04019
## beta[3]    8.675301  17.5468  21.56706  25.56531  34.34203
## beta[4]   17.568203  26.5249  30.55516  34.55737  43.37434
## beta[5]    4.869980  13.8933  17.88586  21.86655  30.73078
## mu       188.133992 195.5729 198.86103 202.24107 209.95520
## sigma      4.669129   6.1233   7.21206   8.66964  13.24302
## tau        0.005702   0.0133   0.01923   0.02667   0.04587
```

- **Overall Mean** μ: The aggregate mean carbon sequestration level has a posterior mean of 198.94 and a median of 198.99, with a 95% credible interval between roughly 187.68 and 209.53. This parameter reflects the general carbon sequestration accounting for individual field and treatment effects.
- **Field-Specific Effects** α_i: Compared to the baseline field (Field 1 with an effect of 0):
  - For **Field 2** α_2, we observe a posterior mean effect on carbon sequestration of 1.90 and a median of approximately 1.86. The 95% credible interval spans from about -8.13 to 12.02.
  - **Field 3** α_3 shows a posterior mean effect of 1.46 and a median of approximately 1.44, with a 95% credible interval extending from -8.24 to 11.62.

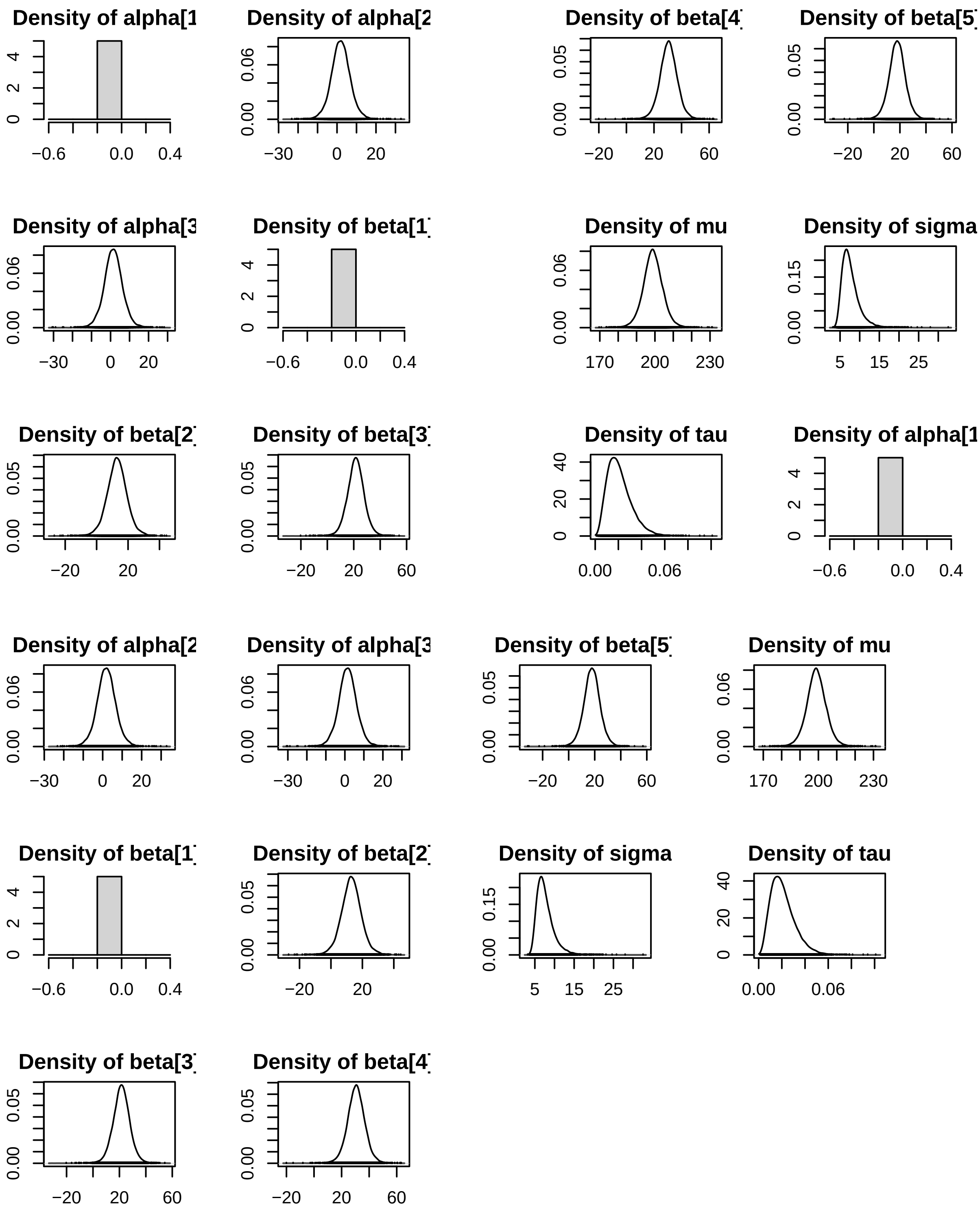
  The credible intervals suggest substantial overlap around zero, indicating high uncertainty about the influence of field location on carbon sequestration.
- **Treatment Effects** β_j: Relative to the reference treatment (Treatment T1 with an effect of 0):
  - **Treatment T2** β_2 has a posterior mean of 12.89 and a median of approximately 12.84, and its 95% credible interval ranges from a low of 0.05 to 26.18.
  - **Treatment T3** β_3's effect has a posterior mean of 21.56 and a median of approximately 21.49, with a credible interval from 17.26 to 34.77.
  - For **Treatment T4** β_4, the posterior mean is 30.54 and the median 30.52, with a 95% credible interval from 26.49 to 43.61.
  - **Treatment T5** β_5 exhibits a posterior mean of 17.89 and a median of 17.83, with a credible interval from 13.84 to 30.94.

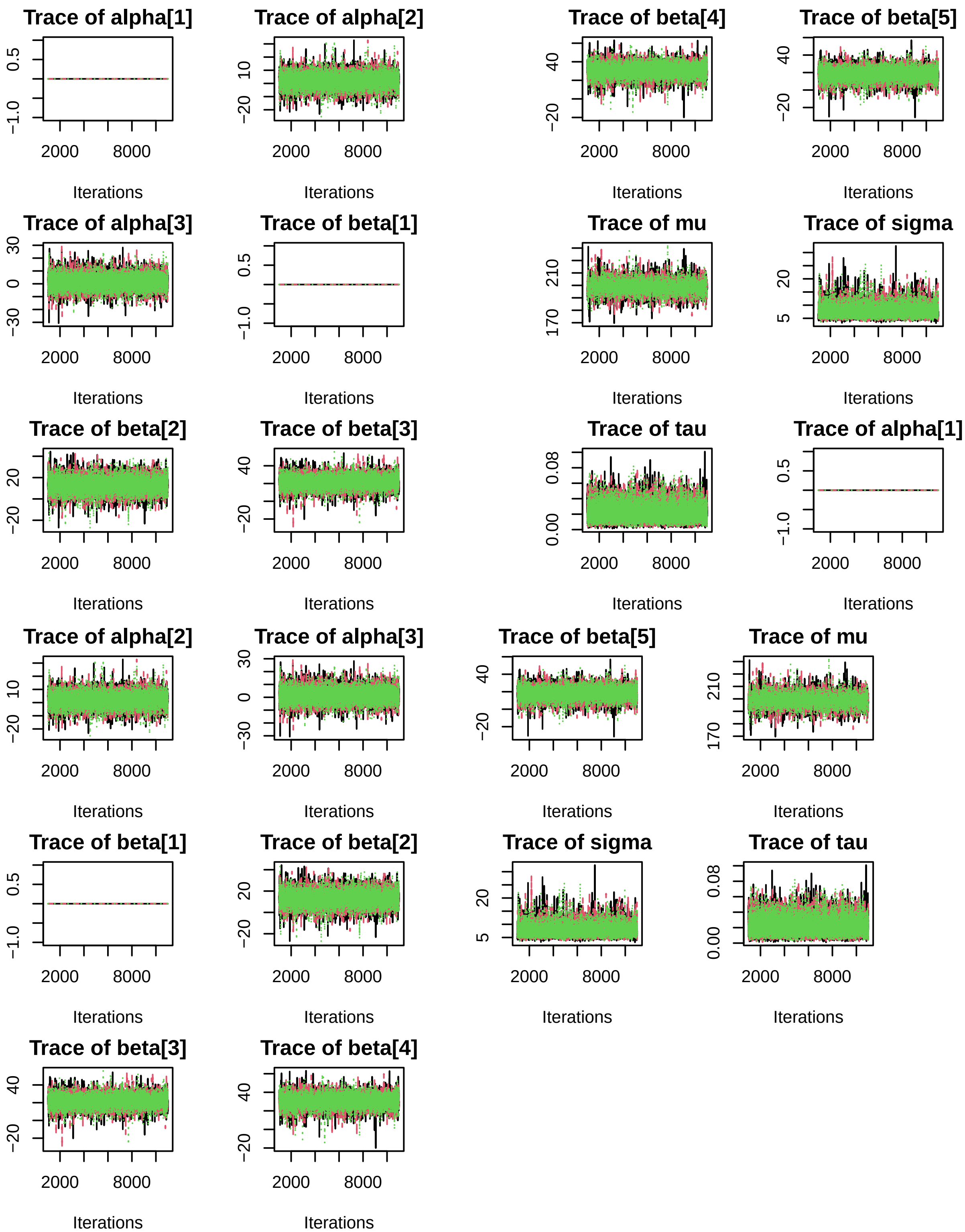
  The treatments present discernible variations in effectiveness, with T4 and T3 emerging as notably superior, followed by T5 and T2.
- **Precision of Measurements** τ: The posterior mean for the precision of carbon sequestration readings is 0.02091, the median is 0.01924, and the 95% credible interval is between approximately 0.00564 to 0.04542. This measure inversely relates to the variance in carbon data, indicating the degree of spread or consistency.
- **Standard Deviation** σ: The posterior mean of the standard deviation is found to be 7.66 with a median of 7.21. The range of the 95% credible interval is quite broad, from about 4.69 to 13.31, reflecting moderate dispersion in the measurements.

Interpretations

- The wide credible intervals for field effects α_i reveal that while individual field locations contribute to variability, there remains uncertainty in quantifying their precise impact.
- Treatments exhibit notable differences in their influence on carbon sequestration, with T4 and T3 being statistically significant and most effective when considering both the means and the credible intervals.
- The absence of zero in the credible intervals for treatment effects β_j underscores the statistical significance and substantive impact of treatment type on carbon sequestration.
- The posterior distribution of precision τ combined with the standard deviation σ conveys that there is inherent variability in the data, albeit not excessively high, suggesting that the observed results are not merely due to measurement noise.
- Overall, the analysis suggests that treatment type is a key determinant of carbon sequestration levels, overshadowing the slight variations that might be attributed to field location.

Bayesian Statistics Part (g)





Density Plots

From the density plots:

- **parameters (Field Effects)**: The plots for α_2 and α_3 show the posterior distribution of the field effects relative to the baseline field (α_1, which is set to 0). The distributions for both α_2 and α_3 appear unimodal and centered around a positive value, indicating that these fields may have higher carbon sequestration than the baseline, though the effects are not far from zero, suggesting the differences might not be substantial.

- **parameters (Treatment Effects)**: The density plots for β_2, β_3, β_4, and β_5 represent the effects of the treatments T2, T3, T4, and T5 relative to the baseline treatment (T1). All treatment effects show unimodal distributions, suggesting distinct differences in their efficacy. Notably, β_4 shows a higher mean effect compared to the others, indicating that treatment T4 might be the most effective among them.
- **Overall Mean (μ)**: The density plot for μ shows the overall mean carbon sequestration across all treatments and fields. It is unimodal and relatively narrow, indicating a precise estimate of the overall mean.
- **Precision (τ) and Standard Deviation (σ)**: The plots for τ and σ (derived from τ) show the estimated precision and variability of carbon sequestration measurements. A higher value of τ (or lower σ) would indicate more precise measurements with less variability.

Trace Plots

For the trace plots:

- **Convergence and Mixing**: Ideally, the trace plots should show a "fuzzy caterpillar" pattern, indicating good mixing and that the chain is sampling evenly from the posterior distribution. From your plots, it seems there's a reasonable degree of mixing for all parameters, suggesting that the MCMC chains have likely converged. However, it is essential to perform further convergence diagnostics such as the Gelman-Rubin statistic.

Discussion and Conclusions

The results indicate that the treatments have varying levels of effectiveness on carbon sequestration. Particularly, treatment T4 may be the most effective, but the final conclusion should consider both the mean effects and their respective credible intervals.

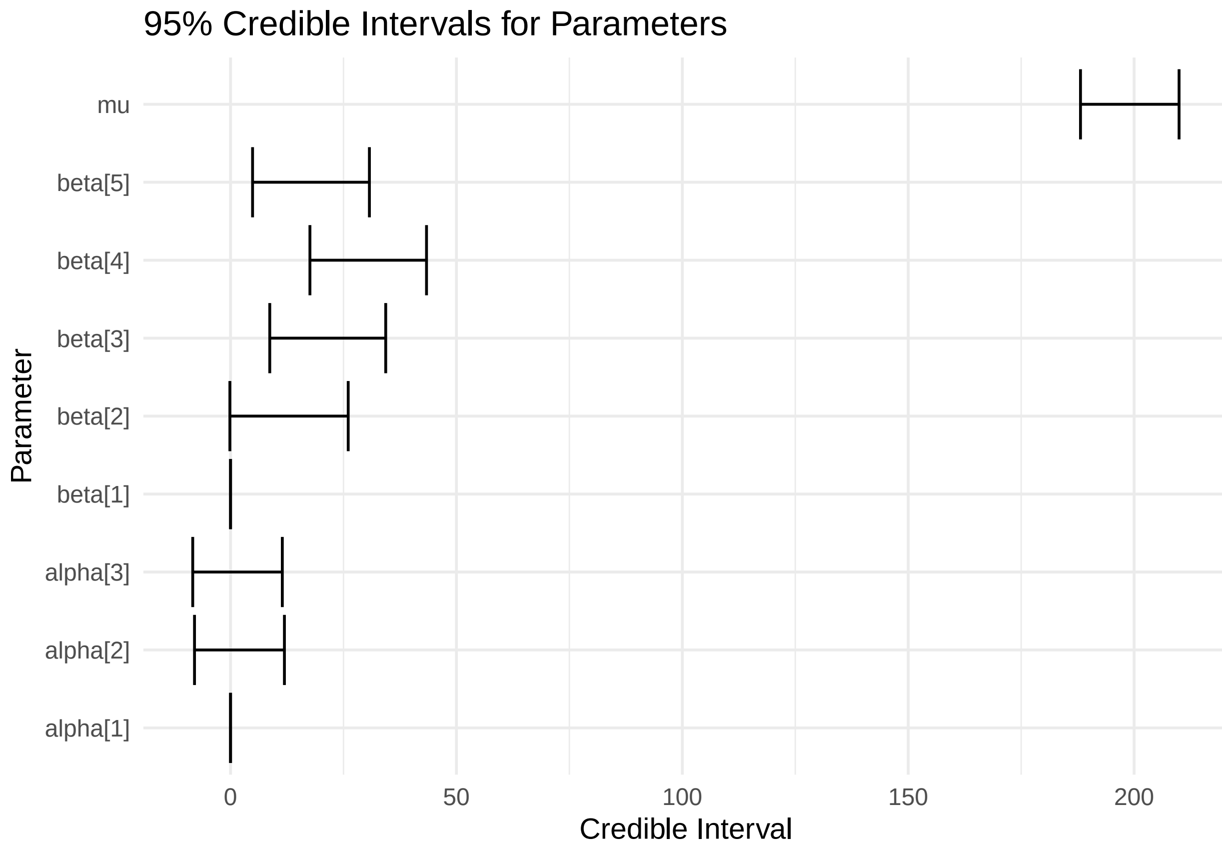
There seems to be some field-to-field variation in carbon sequestration, but the field effects α_2 and α_3 are not dramatically different from zero (the effect of the baseline field, α_1), which might suggest that the location has a minor impact on carbon sequestration compared to the type of treatment applied.

The precision of the measurements and the standard deviation indicate moderate variability in the carbon sequestration measurements, which suggests that the data is reliable and the observed effects are not just due to noise.

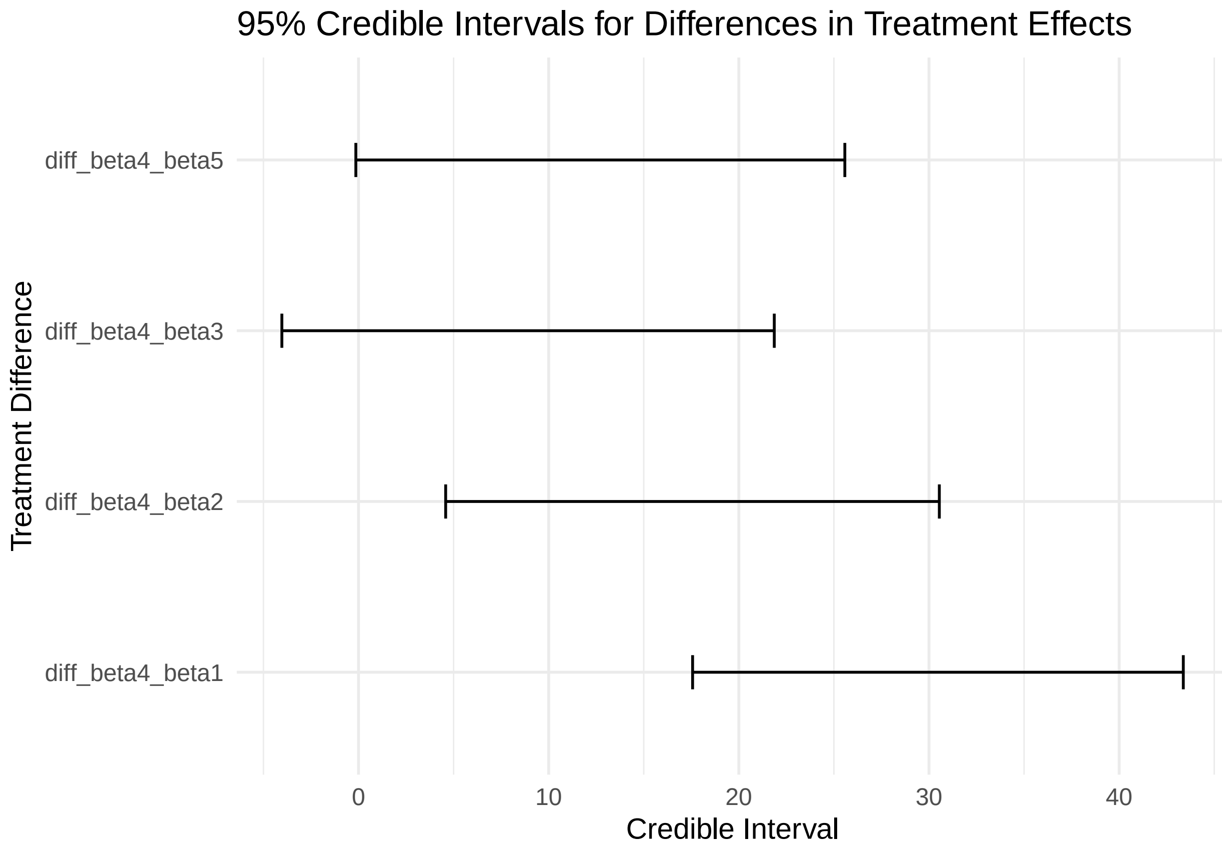
It's critical to check the 95% credible intervals for the α and β parameters. If zero is not contained within these intervals, it could indicate a statistically significant effect. The density plots can provide a visual indication, but numerical confirmation is necessary.

Overall, this analysis can guide the farmer in selecting the most effective carbon sequestration technique, taking into account both the type of treatment and the slight variations due to field location.

Bayesian Statistics Part (h)



Bayesian Statistics Part (i)



Bayesian Statistics Part (j)

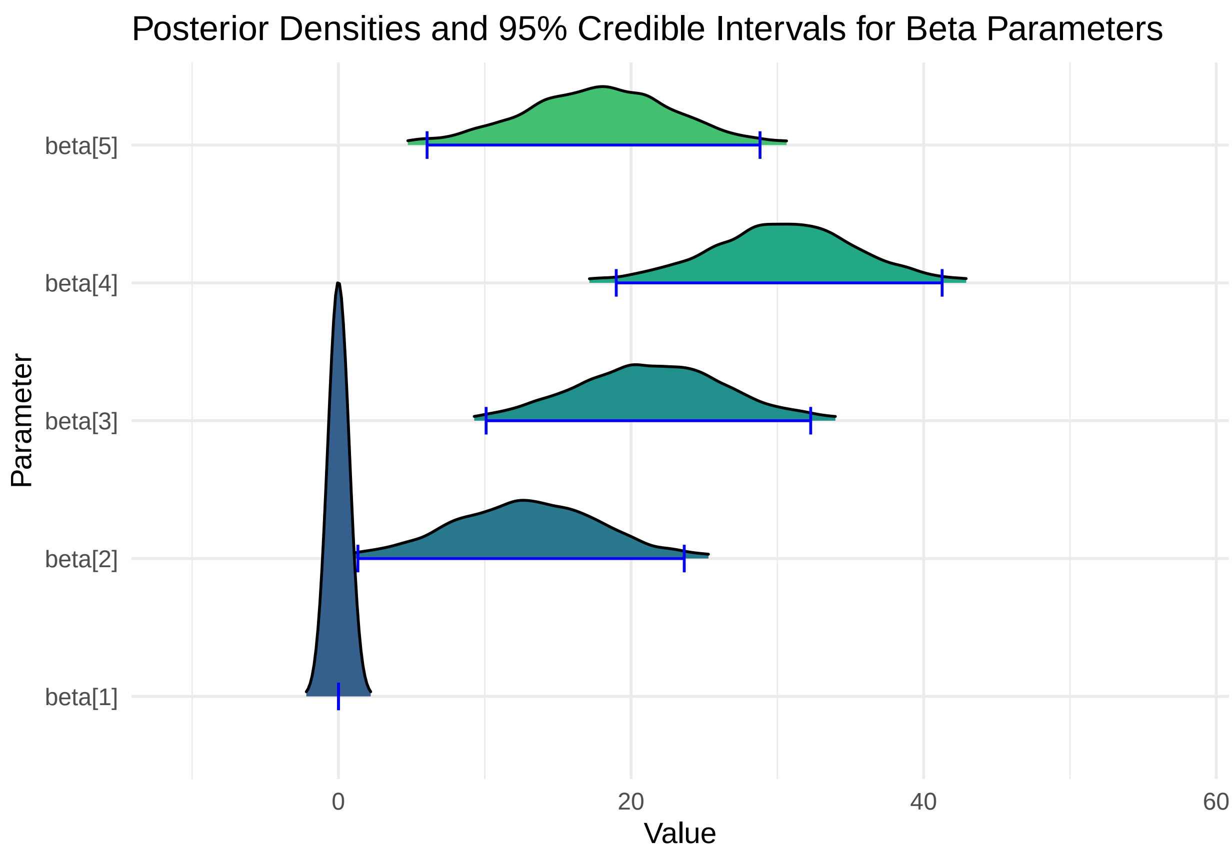
```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 15
##    Unobserved stochastic nodes: 6
##    Total graph size: 32
##
## Initializing model
```

```
##         Parameter         Mean       Median  Lower_95_CI  Upper_95_CI
## beta[1]   beta[1]   0.00000000   0.00000000 0.000000e+00   0.00000000
## beta[2]   beta[2]  12.74839123  12.82213239 1.327908e+00  23.63438370
## beta[3]   beta[3]  21.28398865  21.33767074 1.009404e+01  32.27836107
## beta[4]   beta[4]  30.40830996  30.45519190 1.898720e+01  41.26238678
## beta[5]   beta[5]  17.75368685  17.87180324 6.057723e+00  28.81497300
## mu             mu 200.26505561 200.19139306 1.925685e+02 208.05058684
## sigma       sigma   6.75728365   6.43447535 4.343626e+00  10.82686653
## tau           tau   0.02583967   0.02415315 8.530894e-03   0.05300234
##                 SD
## beta[1] 0.00000000
```

```
## beta[2] 5.63362099
## beta[3] 5.63436780
## beta[4] 5.49915785
## beta[5] 5.64299225
## mu      3.92467606
## sigma   1.72211848
## tau     0.01175337
```

Bayesian Statistics Part (k)

```
## Picking joint bandwidth of 0.729
```



Bayesian Statistics Part (l)