DEFINITION *Differentiable at x*	THEOREM *Mean Value Theorem*
PROOF *Mean Value Theorem*	DEFINITION *Lipschitz Condition*
THEOREM *Ratio Mean Value Theorem*	EXAMPLE *A function satisfying the lipschitz condition*
EXAMPLE *Discontinuity of the second kind*	DEFINITION *r-th order differenitable at x.*
DEFINITION *Darboux continuous*	THEOREM *Continuity of the derivative of a differentiable function*

A continuous function $f : [a, b] \to \mathbb{R}$ which is differentiable on (a, b) has the mean value property: there exists a $\theta \in (a, b)$ such that

$$f(b) - f(a) = f'(\theta)(b - a).$$

The function $f : (a, b) \to \mathbb{R}$ is differentiable at x iff

$$\lim_{t \to x} \frac{f(t) - f(x)}{t - x} = L$$

exists.

$f : M \to N$ satisfies the lipschitz condition if and only if there exists a K such that

$$d(fx, fy) \leq K d(x, y)$$

Take $f(x) - \frac{f(b)-f(a)}{b-a}(x - a) = g(x)$. Then $g(x)$ attains a maximum or a minimum on $[a, b]$ by its continuity. At either the min or max, $\theta \in (a, b)$. Then $g'(\theta) = 0$. Therefore $f'(\theta) = S$.
Draw the secant line!

$$f(x) = Kx.$$
$$|f(x)'| \leq K$$

Let $f, g : [a, b] \to \mathbb{R}$ be continuous functions. Then there exists a $\theta \in (a, b)$ such that

$$\frac{f'(\theta)}{g'(\theta)} = \frac{\Delta f}{\Delta g}$$

The function f is r-th order differentiable at x if and only if it is differentiable up to r and $f^{(r-1)}$ is continuous.

$$f(x) = x^2 \sin\left(\frac{1}{x}\right), f(0) = 0$$

If f is differentiable on (a, b) then its derivative is Darboux continuous.

A function which posesses the intermediate value property.

DEFINITION *Smooth function*	DEFINITION *Analytic function*
EXAMPLE *Nonanalytic Smooth Function*	

$f : (a, b) \to \mathbb{R}$ is analytic if for each $x \in (a, b)$ there is a power series

$$\sum a_r h^r$$

and a $\delta > 0$ such that if $|h| < \delta$ then

$$f(x + h) = \sum_{r=0}^{\infty} a_r h^r$$

A function $f : (a, b) \to \mathbb{R}$ is smooth if and only if it is infiniteley differentiable.