# MATH 105: Homework 5

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65. The winding (w)one form.

**Definition 1.** *We denote the winding one form, $d\theta$, such that*

$$d\theta = \frac{-y}{r^2}dx + \frac{x}{r^2}dy. \tag{1}$$

**Theorem 1.** *The winding one form is closed but not exact.*

*Proof.* We take the exterior derivative of $d\theta$. Using $d(f dx) = df \wedge dx$ we have the following:

$$\begin{aligned}
d(d\theta) &= d\left(\frac{-y}{r^2}dx + \frac{x}{r^2}dy\right)\\
&= d\left(\frac{-y}{r^2}dx\right) + d\left(\frac{x}{r^2}dy\right)\\
&= d\left(\frac{-y}{r^2}\right)\wedge dx + d\left(\frac{x}{r^2}\right)\wedge dy\\
&= \left(\frac{\partial}{\partial x}\frac{-y}{x^2+y^2}dx\frac{\partial}{\partial y}\frac{-y}{x^2+y^2}dy+\right)\wedge dx + d\left(\frac{x}{r^2}\right)\wedge dy\\
&= \frac{y^2-x^2}{r^4}dy\wedge dx + d\left(\frac{x}{r^2}\right)\wedge dy\\
&= \frac{y^2-x^2}{r^4}dy\wedge dx + \left(\frac{\partial}{\partial x}\frac{x}{r^2}dx + \frac{\partial}{\partial y}\frac{x}{r^2}dy\right)\wedge dy\\
&= \frac{y^2-x^2}{r^4}dy\wedge dx + \frac{\partial}{\partial x}\frac{x}{r^2}dx\wedge dy\\
&= \frac{y^2-x^2}{r^4}dy\wedge dx + \frac{y^2-x^2}{r^4}dx\wedge dy = 0
\end{aligned} \tag{2}$$

Substitution of $r\cos\theta$ for $x$ and similar for $y$ yields that the form is infact $d\theta$, ($dx = \cos\theta dr - r\sin\theta d\theta$). If the form were exact then its anti-exterior derivative should be $\theta$, therefore its evaluation along a 1 cell should be its net change along its end points.

Consider the curve which takes the unit circle counter clockwise around the origin.

$$d\theta(c) = \int_0^\tau -\sin t dx + \cos t dy = \int_0^\tau dt = \tau. \tag{3}$$

And $\tau = 2\pi \neq 0$, but the net change in $\theta = 0$ So it could not be that this form is exact. □

The name $d\theta$ is totally misleading, it implies that $d\theta = d(\theta)$ which is false.

68. Closedness of scalar multiplication.

**Theorem 2.** *If $\omega$ is closed then $f\omega$ is not necisarrily closed.*

*Proof.* Take the exterior derivative of the expression and get

$$d(f\omega) = df \wedge \omega + f \wedge d\omega = df \wedge \omega. \tag{4}$$

So if the differential of $f$ is 0 then $f\omega$ is closed. Otherwise, no. □