MATH H105: Homework 1

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55. Take the following dx_{321}, dx_{546}. See their wedge product,

$$\begin{aligned} dx_{321} \wedge dx_{546} & \\ &= dx_3 \wedge dx_2 \wedge dx_1 \wedge dx_5 \wedge dx_4 \wedge dx_6 \\ &= -dx_2 \wedge dx_3 \wedge dx_1 \wedge dx_5 \wedge dx_4 \wedge dx_6 \\ &= dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_5 \wedge dx_4 \wedge dx_6 \\ &= dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge dx_5 \wedge dx_6 \\ &= dx_{12} \wedge dx_3 \wedge dx_4 \wedge dx_5 \wedge dx_6 \\ &= dx_{12} \wedge dx_{34} \wedge dx_5 \wedge dx_6 \\ &= dx_{12} \wedge dx_{34} \wedge dx_{56} \\ &= dx_{12} \wedge dx_{3456} \\ &= dx_{123456}. \end{aligned} \tag{1}$$

56. False. Observe that $a = 0$ if and only if $a = -a$. Take ω to be a k-form. Then observe that $\omega \wedge \omega = (-1)^{k^2} \omega \wedge \omega$ if we rearrange the wedge product. So for k odd the square is odd so $\omega \wedge \omega = 0$ except for when $k = 2n$, $\omega \wedge \omega \neq 0$.

57. Consider the forms $\alpha + \beta$. Then, $d(\alpha + \beta) = d(f dx_I + g dx_I) = d((f + g) \wedge dx_I) = d(f + g) \wedge dx_I = (df + dg) \wedge dx_I = df \wedge dx_I + dg \wedge dx_I = d(\alpha) + d(\beta)$.

58.

59.

60.

61. See 56.