MATH H104: Homework 9

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59.

Theorem 1. *If $\sum a_n$ converges and $a_n \geq 0$, then show $\sum \sqrt{a_n}/n$ converges.*

Proof. Let $x = (\sqrt{a_n})_n$, $y = \left(\frac{1}{n}\right)_n$. Clearly $y \in \ell_1$, and since $\sum a_n \to c$, $a_n \to 0$ implies that $\sqrt{a_n} \to 0$. Therefore, $x \in \ell_1$. Since ℓ_1 is an inner product space, the cauchy schwartz inequality gives,

$$0 \leq \sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n} = \langle x, y \rangle \leq |x||y| = \sqrt{\sum_{n=1}^{\infty} a_n} \sqrt{\sum_{n=1}^{\infty} \frac{1}{n^2}} = \sqrt{\frac{c}{6}}\pi$$

and so the series is bounded and therefore converges. □

61. Consider the following $\{a_n\} \in \ell_1$. We say that $a_n = 1/4^n$ if n odd and $a_n = 1/2^n$ otherwise. Clearly

$$0 < \sum_{n \in \mathbb{N}} a_n = \sum_{n \text{ odd}} \frac{1}{4^n} + \sum_{n \text{ even}} \frac{1}{2^n} < \sum \frac{1}{2^n} < \sum \frac{1}{n^2} = \frac{\pi^2}{6}.$$

So the series converges. Let $\rho_N = \sup_{n>N} |a_{n+1}|/|a_n| = \sup_{n>N} 2^n = \infty$. So clearly $\rho = \lim \rho_N = \infty$, and yet the series converges. If we were to suppose that $\lambda =$ then the test would be wrong since $\lambda > 1$ implies divergence. So it must be the case that the test is inconclusive when ≥ 1.

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