MATH 185: Homework 2

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1.

Definition 1. *A set* $S \subset \mathbb{C}$ *is bounded if and only if there exists* $z \in \mathbb{C}$ *such that for every* $s \in S$, $|s| \leq |z|$

Definition 2. *Alternativeley, a set* $S \subset \mathbb{C}$ *is bounded if and only if there is an* r *such that* $S \subset B_r(0)$, *where* $B_s(z)$ *is the ball of radius* s *with center* z.

Theorem 1. *If* $(z_n)_{n=1}^{\infty}$ *is a convergent sequence of compex numbers, then the sequence is bounded.*

Proof. Take the value set $S = \{z_n\}$. Then suppose there were no r such that $S \subset B_r(0)$. If this is the case, the countability of S implies that for every n, $S \cap B_n(0)$ is finite. Since $z_n \to z$, take $N \in \mathbb{N}$ such that $N > |z|$. Such an n exists by the archimedian principle of $\mathbb{R}$. Then $S \cap N$ must be finite.

Take $\epsilon = N - |z|$, then there is an M such that for all $m > M$, $d(z_n, z) < \epsilon$. That is there are infinite elements within ϵ of z, and thereby there are infinite elements in $S \cap B_N(0)$. This is a conradiction to its finiteness.

Therefore it must be that the value set is contained within the N ball, and therefore, (z_n) is bounded. $\square$

2. Exercise II.1.11

Theorem 2. *The function* $Arg : \mathbb{C} \to \mathbb{R}$ *is continuous except for along the line* $L = \{z : z = 0, Re(z) < 0\}$.

Proof. A function is continuous if and only if it preserves limits. Specifically, if $\lim_{h \to x} f(h) = f(x)$ implies that f is continuous at h. Consider the restricted Arg function, say $A : \mathbb{C} \backslash L \to \mathbb{R}$. Then it is clear that $\lim_{\mathbb{C} \backslash L} A(h) = (-\pi, \pi)$, since if a point is within an ϵ neighborhood of another point, its gradial distance is proportionate to $\sin^{-1}$ of its ϵ distance, (a continuous function).

However consider any $z \in L$ Such that $h \to z$ approaches from the upper half plane and $g \to z$ from the lower. Clearly $Arg(h) \to \pi$ and $Arg(g) \to -\pi$, so no limit exists and the function is not continuous at z. This completes the proof. $\square$