MATH 113: Homework 2

William Guss
26793499
wguss@berkeley.edu

September 1, 2016

32. Let $\mathcal{R}$ be the following relation. We say that $x\mathcal{R}y \in \mathbb{R}$ if and only if $|x-y| \leq 3$. We claim that the relation is not an equivalence relation.

    Proof. Let $x = 0$, $y = 3$, $z = 6$. It is obvious that $x\mathcal{R}y$ and $y\mathcal{R}z$, but since $|z-x| = 6 \not\leq 3$ so it is not the case that $x\mathcal{R}z$. Therefore the relation is not transitive by counter example, and thereby is not an equivalence relation. $\square$

4. We compute the result of $(-i)^{35}$.

$$(-i)^{35} = (-1)^{35} i^{35} = -i^3 = -i^2 i = i.$$

8. We compute the result of $(i+1)^3$ by first establishing the coefficients of pascals triangle as follows.

$$\begin{array}{lccccc} (a+b)^0 & & & 1 & & \\ (a+b)^1 & & 1 & & 1 & \\ (a+b)^2 & & 1 & 2 & 1 & \\ (a+b)^3 & 1 & 3 & & 3 & 1 \\ (a+b)^4 & 1 & 4 & 6 & 4 & 1 \end{array}$$

Therefore we apply the rule to our equation and yield

$$(i+1)^3 = 1 + 3i + 3i^2 + i^3 = -2 + 3i - i = -2 + 2i.$$

19. We find all solutions to $z^3 = -27i$. First let $z = re^{i\theta}$. Then $r^3 e^{i3\theta} = 27e^{-i\pi/2}$. Therefore $r = 3$ and $\theta = -\pi/2$ gives the principle solution $\theta^* = -\pi/6$. It also follows however that $\theta = -\pi/6 + 2k\pi/3$ are all valid solutions since in the cube

$$e^{i\theta^3} = e^{-i\pi/6 + i2k\pi/3^3} = e^{-i\pi/2 + i2k\pi} = e^{-i\pi/2}.$$

Hence the following set satisfies $z^3 = -27i$

$$S = \left\{ 3\exp\left(i\pi\left(\frac{2k}{3} - \frac{1}{6}\right)\right) \;\middle|\; k \in \mathbb{Z}. \right\}.$$

21. We find all solutions to $z^6 = -64$ using the same logic as before. Observe that $2^6 = 64$, and $e^{i\pi} + 1 = 0$ (*magic!!!*). Then

$$S = \left\{ 2\exp\left(i\pi \frac{1+2k}{6} \right) \;\middle|\; k \in \mathbb{Z}. \right\} \tag{1}$$

satisfies $z \in S \implies z^6 = -64$.