MATH H105: Homework 2

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15.

Theorem 1. *Let* $f : \mathbb{R}^2 \to \mathbb{R}$ *be a function so that*

$$\begin{aligned} (x, y) &\mapsto \frac{xy}{x^2 + y^2} \\ (0, 0) &\mapsto 0. \end{aligned} \tag{1}$$

Then, f *has partial derivatives at* $(0, 0)$ *but is not differentiable there.*

Proof. By definition we take the partial derivative to be the limit

$$\begin{aligned} \frac{\partial f}{\partial x} &= \lim_{t \to 0} \frac{f(x + h, y) - f(x, y)}{h} \\ &= \lim_{t \to 0} \frac{(h)^2}{h} \\ &= 0. \end{aligned} \tag{2}$$

Since the closed form for f is identical, we have the same definition for $\partial f / \partial y$.

However, if f is differentiable, then it is continuous at $(0, 0)$. But the limik along $y = x$, does not exist unabiguously

$$\lim_{x \to 0} \frac{x^2}{x^2 + x^2} = \frac{1}{2} \neq 0.$$

Therefore it could not possibly differentiable at $(0, 0)$. □

16. Yass!!!!!!!!!!!!!!!!!!!!!!!!

We build the matrix of partials accordingly! Using partial differnetiation we get

$$(Df)_p = \begin{bmatrix} 1 & 0 \\ \cos 1 & 0 \\ \sin 1 & 0 \end{bmatrix}. \tag{3}$$

$$(Dg)_q = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}. \tag{4}$$

$$(Dg \circ f)_p = (Dg)_q \circ (Df)_p = 0. \tag{5}$$

We get that

$$g \circ f = w(s,t) = (st)(s\cos t) + (s\cos t)(s\sin t) + (s\sin t)(st) \tag{6}$$

and so the pial derivatives at least contain s in every term:

$$\begin{aligned} D_s w &= 2st(\cos t) + 2s\cos t\sin t + 2st\sin t \\ D_t w &= s^2(\cos t - t\sin t) + s^2(\cos t\cos t - \sin t\sin t) + s^2(\sin t + t\cos t) \end{aligned} \tag{7}$$

These partials evaluate to 0 and so are 0.

The statement of multivariable chain rule for functions $g : \mathbb{R} \to \mathbb{R}^m, f : \mathbb{R}^m \to \mathbb{R}$ is that $d/dt f \circ g = \sum \partial f/\partial g_i \partial g_i/\partial t$ which is the row vector matrix Df with the column vector Dg.

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