Math 113 — Problem Set 10 — William Guss

(P174. 6) (P175. 12)

(P182. 2) Solve the equation $3x = 2$ in the field
(a) $\mathbb{Z}_7$.

Proof. We must find x so that $x \mod 7 = 2$ and $3|x$. First $3 \times 1 = 3 \mod 7 = 3$, then $3 \times 3 = 9 \mod 7 = 2$, thus $x = 2$ in $\mathbb{Z}_7$. $\square$

(b) $\mathbb{Z}_2 3$

Proof. We msut find x so that $x \mod 23 = 2$ and $3|x$. Take $x = 16$, then $3x = 48$. Finally $23 \times 2 = 46$ so $48 \mod 46 = 2$ and $3x = 2$. We could have found this by showing that $3y = 1$ if $y = 3^{-1}$ and thus $3 \times 8 = 24 \mod 23 = 1$ so $y = 8$. Then $3x \equiv 2$ is solved by $x \equiv y3x \equiv y \times 2 = 16$. $\square$

(P182. 3) Find all solutions of the equation $x^2 + 2x + 2 = 0$ in $\mathbb{Z}_6$.

Proof. First $\mathbb{Z}_6$ is not a field since $2 \times 3 = 6 \equiv 0$ so $\mathbb{Z}_6$ is not an integral domain. We factor the polynomial however $\square$

(P182. 14)