University of Pittsburgh

Algorithms and Data Structures 1
CS 0445

Fall 2022

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Upcoming Deadlines:
  - Homework 5: this Friday @ 11:59 pm
  - Lab 4: next Monday @ 11:59 pm
  - Programming Assignment 1: ~~Friday Oct. 7th~~ Monday Oct. 10th
- **Live Remote Support Session** for Assignment 1
  - Recording and slides on Canvas
- **Student Support Hours** of the teaching team are posted on the Syllabus page

Previous Lecture …

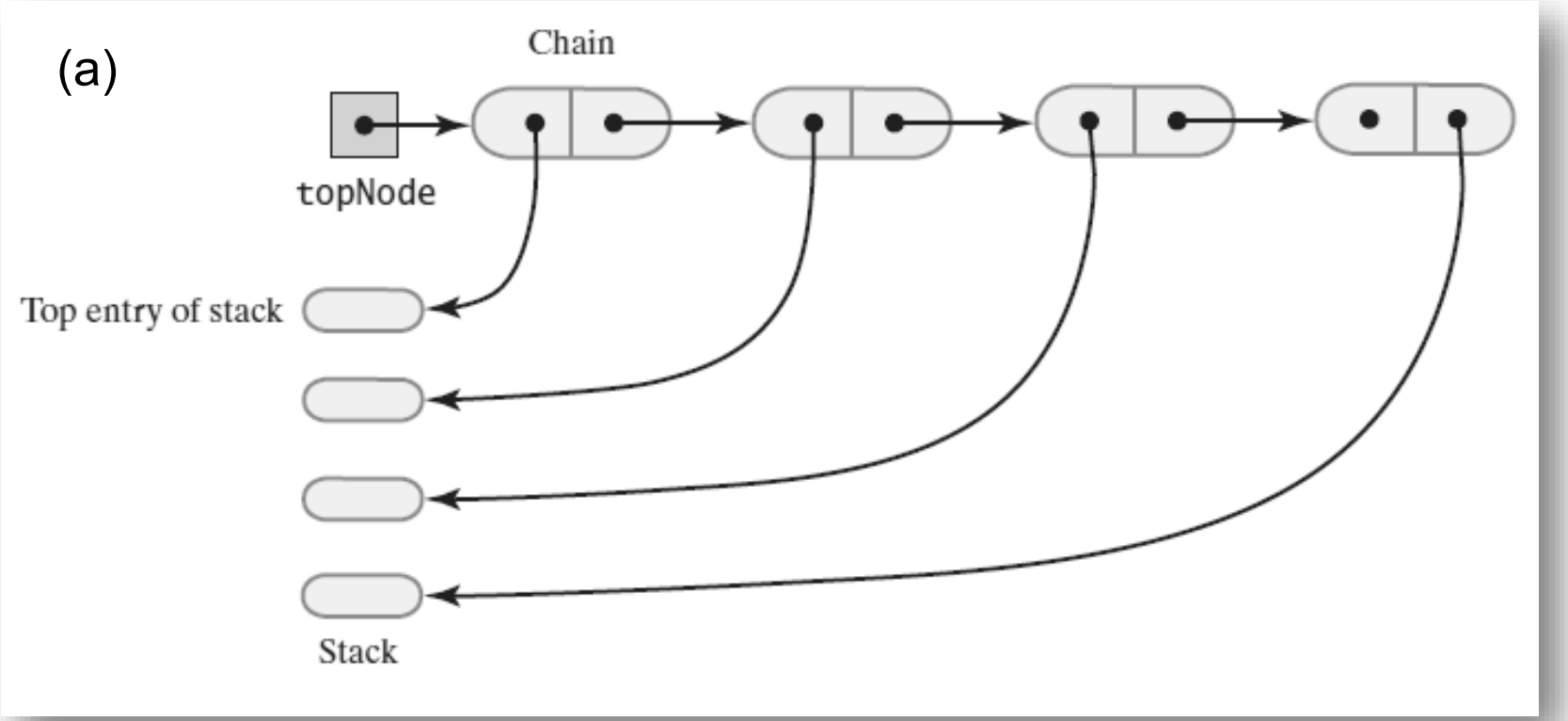
- ADT List
  - Refined Linked implementation with head and tail references
- ADT Stack
  - Array-based implementation
  - Linked implementation

Today …

- ADT Stack
  - Linked implementation
  - Implementation using ADT List
  - Application: Building a simple parser of Algebraic expressions

Linked Implementation

The stack before the
first node in the chain is deleted



Linked Implementation

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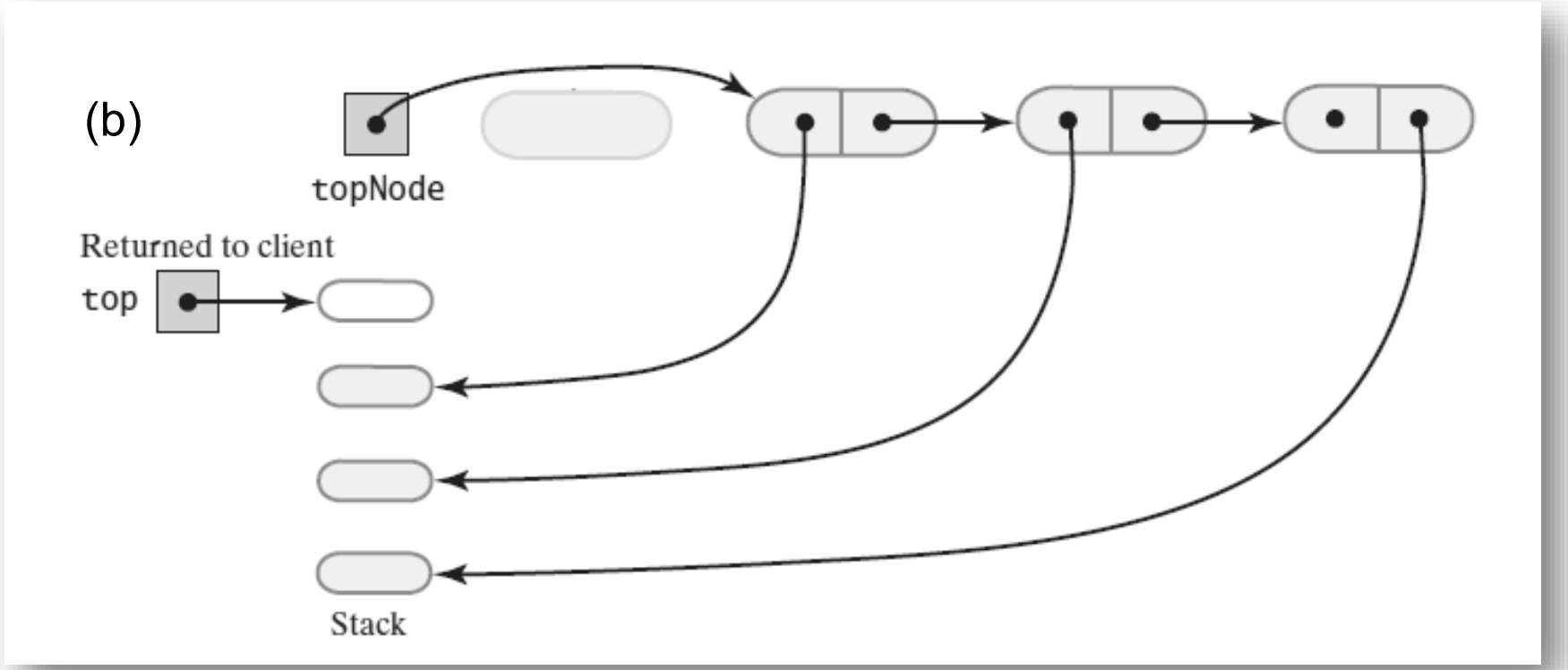


Linked Implementation

- Definition of `pop`

```
public T pop()
{
   T top = peek(); // Might throw EmptyStackException
   assert topNode != null;
   topNode = topNode.getNextNode();
   return top;
} // end pop
```

Linked Implementation

- Definition of rest of class.

```
public boolean isEmpty()
{
   return topNode == null;
} // end isEmpty

public void clear()
{
   topNode = null;
} // end clear
```

ADT Stack Application

Let's use the ADT Stack to design and implement a simple parser of Algebraic Expressions

Processing Algebraic Expressions

- Algebraic expressions can take different forms:
  - Infix: each binary operator appears between its operands $a + b$
  - Prefix: each binary operator appears before its operands $+ a b$
  - Postfix*:* each binary operator appears after its operands $a b +$
- Prefix and Postfix forms are easy to evaluate
  - no parentheses needed
  - no need for operator precedence rules while evaluating the Postfix expression
- But we have to make sure first that the expressions is balanced
  - parentheses paired correctly

Our Plan

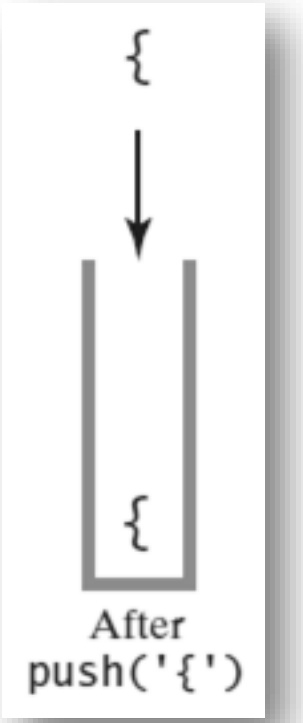
1. Check if input infix expression is balanced
2. Convert the expression from infix to postfix
3. Evaluate the postfix expression

Step 1: Balance Checking an Algebraic Expression

- Let's use a stack!
- initialize an empty Stack
- for each character in the input infix expressions
  - if an open parenthesis
    - push to Stack
  - if a closing parenthesis
    - pop from stack and compare
    - if a matching pair, continue
    - else, report unbalanced and stop
  - if the stack is not empty
    - report unbalanced and stop
  - report balanced

Balance Checking an Algebraic Expression

The contents of a stack during the scan of an expression that contains the balanced delimiters { [( )] }



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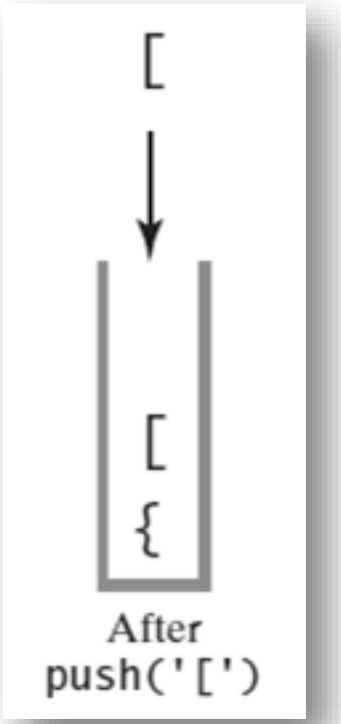

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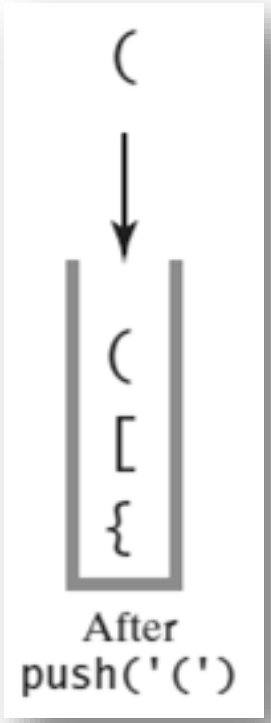

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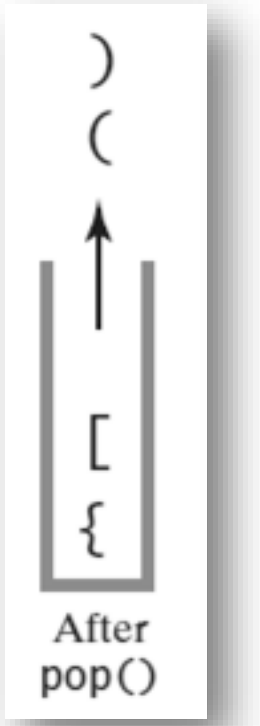

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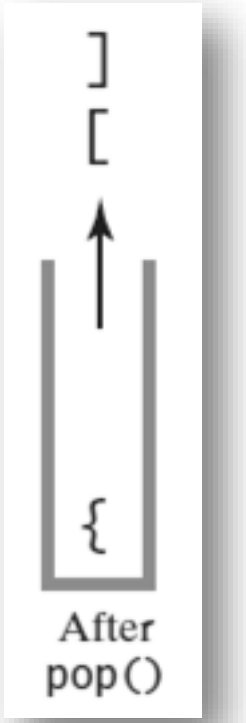

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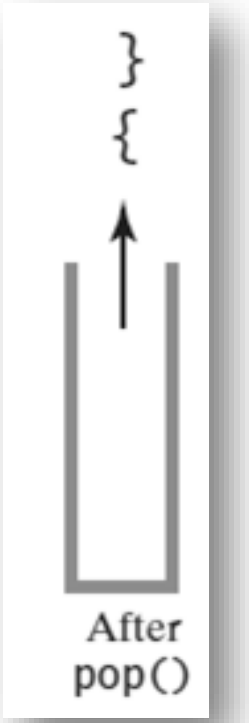

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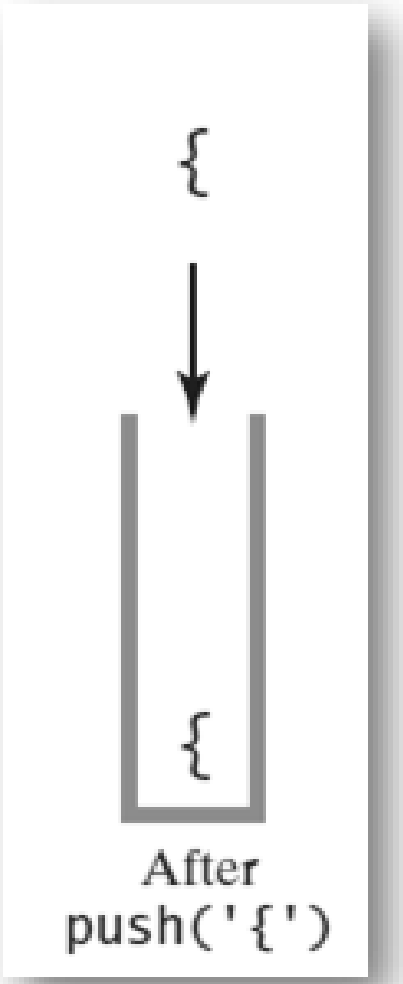
Unbalanced Expression: Case 1

The contents of a stack during the scan of an expression that contains the unbalanced delimiters { [( ]) }



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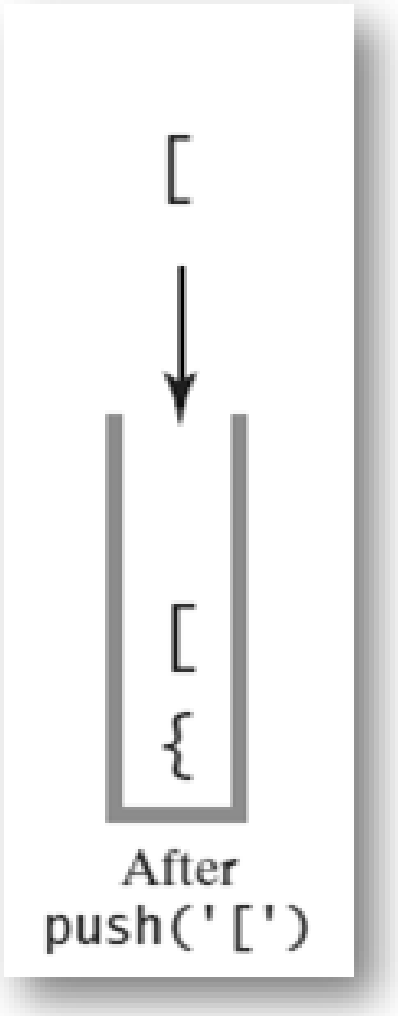

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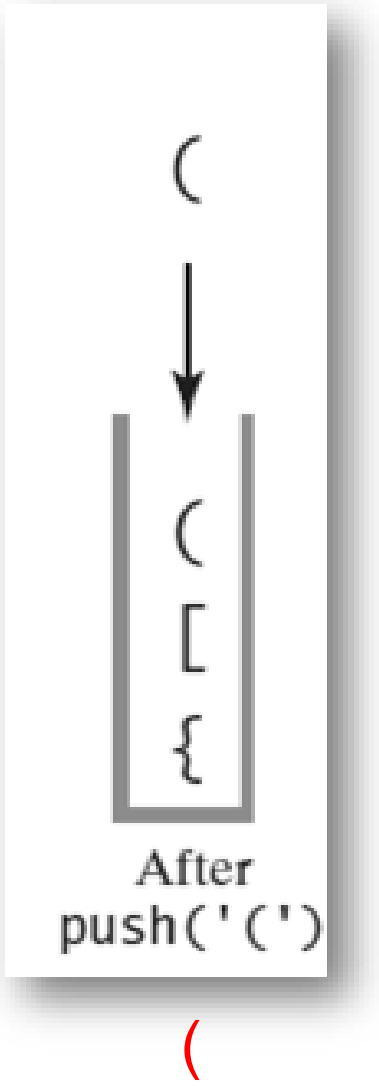

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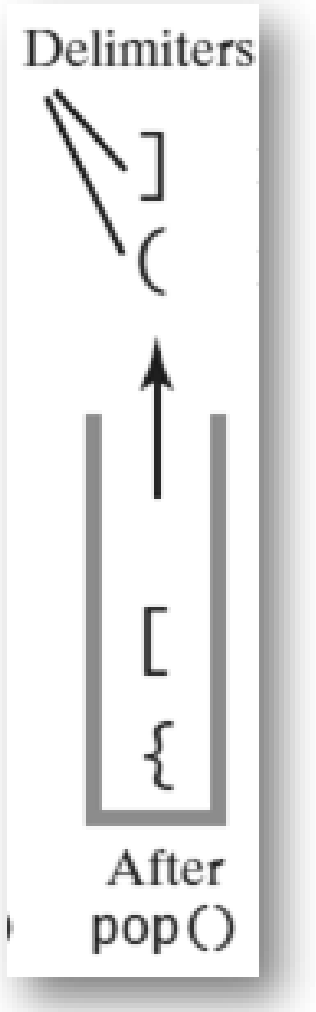

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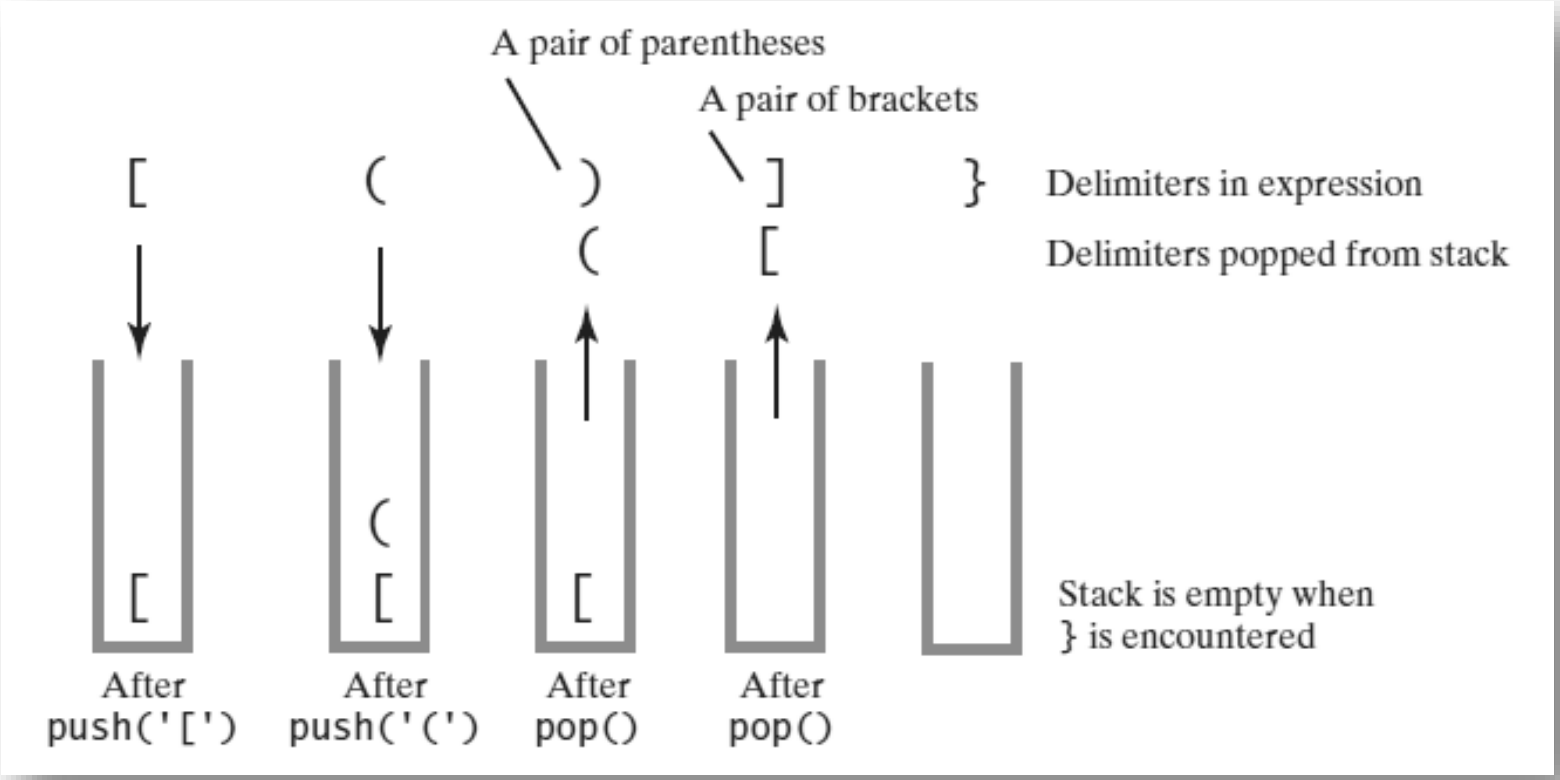
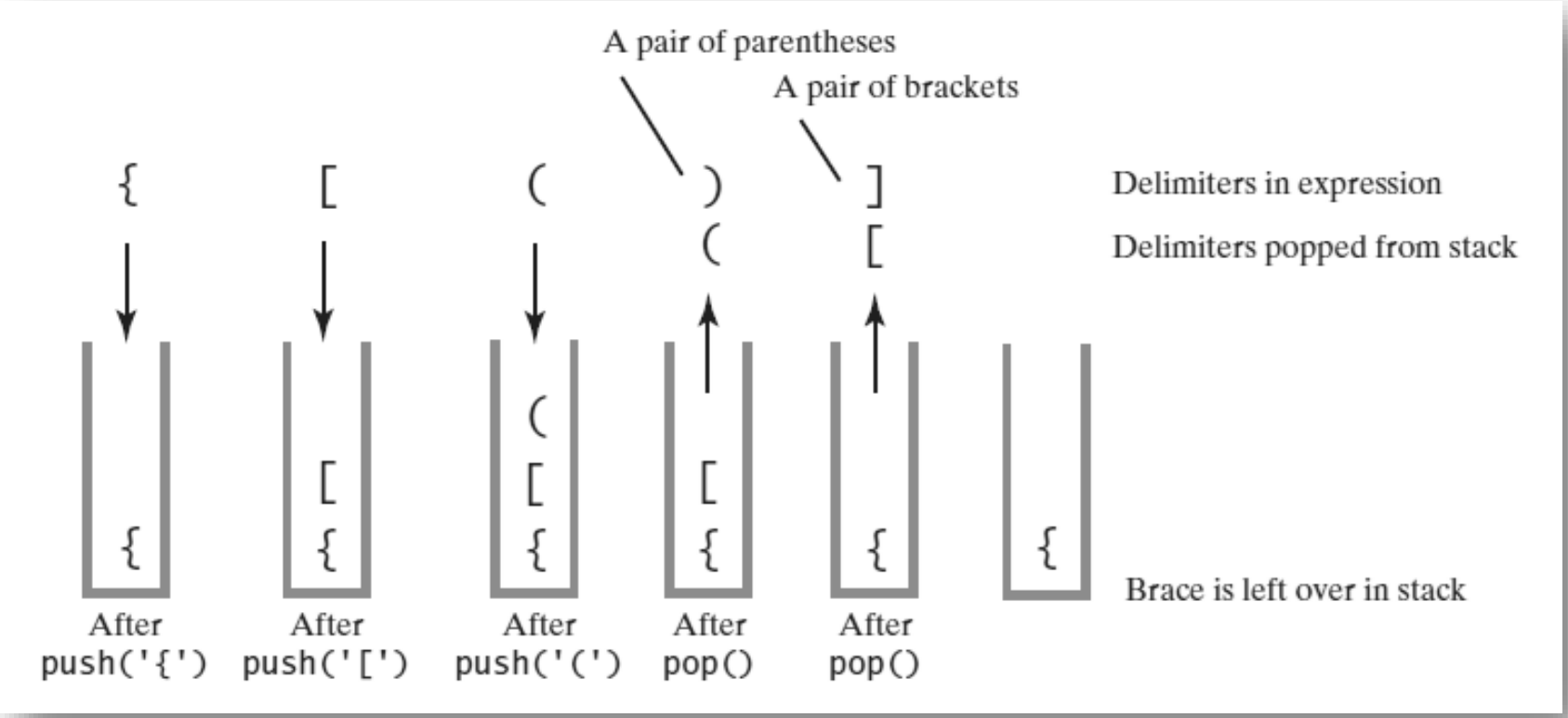
Unbalanced Expression: Case 2

The contents of a stack during the scan of an expression that contains the unbalanced delimiters [( )] }



Unbalanced Expression: Case 3

The contents of a stack during the scan of an expression that contains the unbalanced delimiters { [( )]



Processing Algebraic Expressions

Algorithm to check for balanced expression

```
Algorithm checkBalance(expression)
// Returns true if the parentheses, brackets, and braces in an expression are paired correctly.

isBalanced = true
while ((isBalanced == true) and not at end of expression)
{
   nextCharacter = next character in expression
   switch (nextCharacter)
   {
      case '(': case '[': case '{':
         Push nextCharacter onto stack
         break

      case ')': case ']': case '}':
         if (stack is empty)
            isBalanced = false
         else
```

Processing Algebraic Expressions

```
            case ')' : case ']' : case '}' :
               if (stack is empty)
                  isBalanced = false
               else
               {
                  openDelimiter = top entry of stack
                  Pop stack
                  isBalanced = true or false according to whether openDelimiter and
                               nextCharacter are a pair of delimiters
               }
               break
         }
      }

      if (stack is not empty)
         isBalanced = false
      return isBalanced
```

Java Implementation

```
3    /** Decides whether the parentheses, brackets, and braces
4        in a string occur in left/right pairs.
5        @param expression  A string to be checked.
6        @return  True if the delimiters are paired correctly. */
7    public static boolean checkBalance(String expression)
8    {
9       StackInterface<Character> openDelimiterStack = new OurStack<>();
10
11      int characterCount = expression.length();
12      boolean isBalanced = true;
13      int index = 0;
14      char nextCharacter = ' ';
15
16      while (isBalanced && (index < characterCount))
17      {
18         nextCharacter = expression.charAt(index);
19         switch (nextCharacter)
20         {
21            case '(': case '[': case '{':
```

Java Implementation

```
16        while (isBalanced && (index < characterCount))
17        {
18           nextCharacter = expression.charAt(index);
19           switch (nextCharacter)
20           {
21              case '(': case '[': case '{':
22                 openDelimiterStack.push(nextCharacter);
23                 break;
24              case ')': case ']': case '}':
25                 if (openDelimiterStack.isEmpty())
26                    isBalanced = false;
27                 else
28                 {
29                    char openDelimiter = openDelimiterStack.pop();
30                    isBalanced = isPaired(openDelimiter, nextCharacter);
31                 } // end if
```

Java Implementation

```
32                  break;
33               default: break; // Ignore unexpected characters
34            } // end switch
35            index++;
36         } // end while
37
38         if (!openDelimiterStack.isEmpty())
39            isBalanced = false;
40         return isBalanced;
41      } // end checkBalance
42
43      // Returns true if the given characters, open and close, form a pair
44      // of parentheses, brackets, or braces.
45      private static boolean isPaired(char open, char close)
46      {
47         return (open == '(' && close == ')') ||
48                (open == '[' && close == ']') ||
49                (open == '{' && close == '}');
50      } // end isPaired
51   } // end BalanceChecker
```

Step 2: Infix-to-postfix Conversion Algorithm

- for each character in the input expression

• Operand	Append each operand to the end of the output expression.

Infix-to-postfix Conversion Algorithm

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• Operand	Append each operand to the end of the output expression.
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• Open parenthesis	Push (onto the stack.

Infix-to-postfix Conversion Algorithm

- for each character in the input expression

• Operand	Append each operand to the end of the output expression.
• Operator ^	Push ^ onto the stack.
• Operator +, -, *, or /	Pop operators from the stack, appending them to the output expression, until the stack is empty or its top entry has a lower precedence than the new operator. Then push the new operator onto the stack.
• Open parenthesis	Push (onto the stack.
• Close parenthesis	Pop operators from the stack and append them to the output expression until an open parenthesis is popped. Discard both parentheses.

Infix to Postfix: Example 1

Converting the infix expression *a* + *b* * *c* to postfix form

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	→

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+	*a*	+
b	*a b*	+

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b	*a b*	+
*	*a b*	+ *

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b	*a b*	+
*	*a b*	+ *
c	*a b c*	+ *

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c	*a b c*	+ *
	a b c *	+

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c	*a b c*	+ *
	a b c *	+
	a b c * +	

Example with Successive Operators with Same Precedence

Converting an infix expression
to postfix form: *a - b + c*

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a	*a*	
–	*a*	–
b	*a b*	–
+	*a b* –	

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a	*a*	
–	*a*	–
b	*a b*	–
+	*a b* –	
	a b –	+

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Converting an infix expression to postfix form: *a - b + c*

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	a	
$-$	a	$-$
b	$a\ b$	$-$
$+$	$a\ b\ -$	
	$a\ b\ -$	$+$
c	$a\ b\ -\ c$	$+$

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$-$	a	$-$
b	$a\ b$	$-$
$+$	$a\ b\ -$	
	$a\ b\ -$	$+$
c	$a\ b\ -\ c$	$+$
	$a\ b\ -\ c\ +$	

Another Example with Successive Operators with Same Precedence

Converting an infix expression
to postfix form: *a* ^ *b* ^ *c*

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
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^	*a*	^

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^	*a*	^
b	*a b*	^
^	*a b*	^ ^

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^	*a*	^
b	*a b*	^
^	*a b*	^ ^
c	*a b c*	^ ^

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b	*a b*	^
^	*a b*	^ ^
c	*a b c*	^ ^
	a b c ^	^

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b	*a b*	^
^	*a b*	^ ^
c	*a b c*	^ ^
	a b c ^	^
	a b c ^ ^	

Infix to Postfix: Larger Example

The steps in converting the infix expression
*a / b * (c + (d - e))* to postfix form

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Infix to Postfix: Larger Example

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+	*a b / c*	* (+
(	*a b / c*	* (+ (
d	*a b / c d*	* (+ (

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c	*a b / c*	* (
+	*a b / c*	* (+
(	*a b / c*	* (+ (
d	*a b / c d*	* (+ (
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Infix to Postfix: Larger Example

The steps in converting the infix expression
*a / b * (c + (d - e))* to postfix form

Next Character from Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
/	*a*	/
b	*a b*	/
*	*a b /*	
	a b /	*
(	*a b /*	* (
c	*a b / c*	* (
+	*a b / c*	* (+
(	*a b / c*	* (+ (
d	*a b / c d*	* (+ (
−	*a b / c d*	* (+ (−
e	*a b / c d e*	* (+ (−

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Infix-to-postfix Algorithm

```
Algorithm convertToPostfix(infix)
// Converts an infix expression to an equivalent postfix expression.

operatorStack = a new empty stack
postfix = a new empty string
while (infix has characters left to parse)
{
    nextCharacter = next nonblank character of infix
    switch (nextCharacter)
    {
        case variable:
            Append nextCharacter to postfix
            break

        case '^' :
            operatorStack.push(nextCharacter)
            break

        case '+' : case '-' : case '*' : case '/' :
```

Infix-to-postfix Algorithm

```
case '+' : case '-' : case '*' : case '/' :
   while (!operatorStack.isEmpty() and
          precedence of nextCharacter <= precedence of operatorStack.peek())
   {
      Append operatorStack.peek() to postfix
      operatorStack.pop()
   }
   operatorStack.push(nextCharacter)
   break
case '( ' :
   operatorStack.push(nextCharacter)
   break
case ')' : // Stack is not empty if infix expression is valid
   topOperator = operatorStack.pop()
   while (topOperator != '(')
   {
```

Infix-to-postfix Algorithm

```
                Append topOperator to postfix
                topOperator = operatorStack.pop()
            }
            break
        default: break // Ignore unexpected characters
    }
}
while (!operatorStack.isEmpty())
{
    topOperator = operatorStack.pop()
    Append topOperator to postfix
}
return postfix
```