University of Pittsburgh

Algorithms and Data Structures 1
CS 0445

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(Slides adapted from slides of Textbook, Dr. Ramirez's CS 0445, and Dr. Farnan's CS 1501)

Announcements

- Upcoming Deadlines:
  - Homework 9 and Lab 9
    - reopened until this Friday 12/2 @ 11:59 pm
  - Homework 10 (to be posted soon) and Lab 11
    - next Monday 12/5 @ 11:59 pm
  - Assignment 3 and 4: both due on Friday 12/9 @ 11:59 pm
    - very small amount of work!

This Lecture …

- Hashing!

Muddiest Points

- **Q:** I could not quite follow the last step of the average runtime for QuickSort. Could you explain the math a bit more?
- **Q:** Will we go through the code of Radix method?
- **Q:** Lab 10 was difficult. I understood the logic of how to do it but had trouble implementing it

Wouldn't it be wonderful if...

- Search through a collection could be accomplished in $\Theta(1)$ with relatively small memory needs?
- Lets try this:
  - Assume we have an array of length m (call it HT)
  - Assume we have a function h(x) that maps from our key space to {0, 1, 2, …, m-1}
    - E.g., $\mathbb{Z} \rightarrow \{0, 1, 2, \ldots, m-1\}$ for integer keys
    - Let's also assume h(x) is efficient to compute
- This is the basic premise of *hash tables*

How do we search/insert with a hash map?

- Insert:

```
i = h(x)
HT[i] = x
```

- Search:

```
i = h(x)
if (HT[i] == x) return true;
else return false;
```

- This is a very general, simple approach to a hash table implementation
  - Where will it run into problems?

What do we do if h(x) == h(y) where x != y?

- Called a *collision*

Consider an example

- Company has 500 employees
- Stores records using a hashmap with 1000 entries
- Employee SSNs are hashed to store records in the hashmap
    - Keys are SSNs, so |keyspace| == 10^9
- Specifically what keys are needed can't be known in advance
    - Due to employee turnover
- What if one employee (with SSN x) is fired and replacement has an SSN of y?
    - Can we design a hash function that guarantees h(y) does not collide with the 499 other employees' hashed SSNs?

Can we ever guarantee collisions will not occur?

- Yes, if our keyspace is smaller than our hashmap
  - If |keyspace| <= m, *perfect hashing* can be used
    - i.e., a hash function that maps every key to a distinct integer < m
    - Note it can also be used if n < m and the keys to be inserted are known in advance
      - E.g., hashing the keywords of a programming language during compilation
- If |keyspace| > m, collisions cannot be avoided

Handling collisions

- Can we reduce the number of collisions?
    - Using a good hash function is a start
        - What makes a good hash function?
            1. Utilize the entire key
            2. Exploit differences between keys
            3. Uniform distribution of hash values should be produced

Examples

- Hash list of classmates by phone number
  - Bad?
    - Use first 3 digits
  - Better?
    - Consider it a single int
    - Take that value modulo m
- Hash words
  - Bad?
    - Add up the ASCII values
  - Better?
    - Use Horner’s method to do modular hashing again
      - See Section 3.4 of the text

The madness behind Horner's method

- Base 10
  - 12345
  - $= 1 * 10^4 + 2 * 10^3 + 3 * 10^2 + 4 * 10^1 + 5 * 10^0$
- Base 2
  - 10100
  - $= 1 * 2^4 + 0 * 2^3 + 1 * 2^2 + 0 * 2^1 + 0 * 2^0$
- Base 16
  - BEEF3
  - $= 11 * 16^4 + 14 * 16^3 + 14 * 16^2 + 15 * 16^1 + 3 * 16^0$
- ASCII Strings
  - HELLO
  - $= \text{'H'} * 256^4 + \text{'E'} * 256^3 + \text{'L'} * 256^2 + \text{'L'} * 256^1 + \text{'O'} * 256^0$
  - $= 72 * 256^4 + 69 * 256^3 + 76 * 256^2 + 76 * 256^1 + 79 * 256^0$

Modular hashing

- Overall a good simple, general approach to implement a hash map
- Basic formula:
  - $h(x) = c(x) \bmod m$
    - Where $c(x)$ converts x into a (possibly) large integer
- Generally want m to be a prime number
  - Consider $m = 100$
  - Only the least significant digits matter
    - $h(1) = h(401) = h(4372901)$

Back to collisions

- We've done what we can to cut down the number of collisions, but we still need to deal with them
- Collision resolution: two main approaches
  - Open Addressing
  - Closed Addressing

Open Addressing

- I.e., if a pigeon's hole is taken, it has to find another
- If h(x) == h(y) == i
  - And x is stored at index i in an example hash table
  - If we want to insert y, we must try alternative indices
    - This means y will not be stored at HT[h(y)]
      - We must select alternatives in a consistent and predictable way so that they can be located later

Linear probing

- Insert:
  - If we cannot store a key at index i due to collision
    - Attempt to insert the key at index i+1
    - Then i+2 …
    - And so on …
    - mod m
    - Until an open space is found
- Search:
  - If another key is stored at index i
    - Check i+1, i+2, i+3 … until
      - Key is found
      - Empty location is found
      - We circle through the buffer back to i

Linear probing example

- h(x) = x mod 11
- Insert 14, 17, 25, 37, 34, 16, 26

0	1	2	3	4	5	6	7	8	9	10
	34		14	25	37	17	16	26		

- How would deletes be handled?
  - What happens if key 17 is removed?

Alright! We solved collisions!

- Well, not quite...
- Consider the *load factor* $\alpha = n/m$
- As α increases, what happens to hash table performance?

- Consider an empty table using a good hash function
  - What is the probability that a key x will be inserted into any one of the indices in the hash table?

- Consider a table that has a cluster of c consecutive indices occupied
  - What is the probability that a key x will be inserted into the index directly after the cluster?

Avoiding clustering

- We must make sure that even *after* a collision, all of the indices of the hash table are possible for a key
    - Probability of filled locations need to be distributed throughout the table

Double hashing

- After a collision, instead of attempting to place the key x in i+1 mod m, look at i+h2(x) mod m
  - h2() is a second, different hash function
    - Should still follow the same general rules as h() to be considered good, but needs to be different from h()
      - h(x) == h(y) AND h2(x) == h2(y) should be very unlikely
        - Hence, it should be unlikely for two keys to use the same increment

Double hashing

- h(x) = x mod 11
- h2(x) = (x mod 7) +1
- Insert 14, 17, 25, 37, 34, 16, 26

0	1	2	3	4	5	6	7	8	9	10
	34		14	37	16	17		25		26

- Why could we not use h2(x) = x mod 7?
  - Try to insert 2401

A few extra rules for h2()

- Second hash function cannot map a value to 0
- You should try all indices once before trying one twice

- Were either of these issues for linear probing?

As $\alpha \rightarrow 1$...

- Meaning n approaches m...
- Both linear probing and double hashing degrade to $\Theta(n)$
  - How?
    - Multiple collisions will occur in both schemes
    - Consider inserts and misses...
      - Both continue until an empty index is found
        - With few indices available, close to m probes will need to be performed
          - $\Theta(m)$
        - n is approaching m, so this turns out to be $\Theta(n)$

Open addressing issues

- Must keep a portion of the table empty to maintain respectable performance
  - For linear hashing ½ is a good rule of thumb
    - Can go higher with double hashing

Closed addressing

- I.e., if a pigeon's hole is taken, it lives with a roommate
- Most commonly done with separate chaining
  - Create a linked-list of keys at each index in the table
    - As with DLBs, performance depends on chain length
      - Which is determined by α and the quality of the hash function

In general...

- Closed-addressing hash tables are fast and efficient for a large number of applications