Algorithms and Data Structures 1
CS 0445

Fall 2022

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS 0445 slides.)

Announcements

- Upcoming Deadlines:
  - **Assignment 2: Monday 11/7 @ 11:59 pm**
  - Lab 7: next Monday 11/7 @ 11:59 pm
  - Midterm reattempts: Thursday 11/10 @ 11:59 pm
- Live Support Session for Assignment 2
  - Video and slides available on Canvas
- QA Session on Piazza every Friday 4:30-5:30 pm

Today …

- Sorting Algorithms

Muddiest Points

- **Q: Could you explain the backtracking of word search?**
- Sure!

Muddiest Points

- **Q: I am confused on assignment 2 runtime of push, to find the index where to push you need a loop which breaks O(1)**
- The loop is over the alphabet array. The alphabet size is assumed to be constant in this assignment

Muddiest Points

- **Q: When would you use proof by induction vs. a recursion tree to find the runtime of a recursive method?**
- You can either technique in most of the cases. I personally prefer the recursion tree approach.

Muddiest Points

- **Q: Towers of Hanoi is very confusing**
- Let’s reiterate the problem

Muddiest Points

- **Since today was a muddiest point review a lot of things were clarified, but I would also like a refresher on proof by induction.**
- Please check the RecursionTimeComplexity.pdf on Canvas for more examples

Sorting

- We have seen a few container data structures
  - Bag, Stack, List
- Sorting Problem: arrange items in a List such that *entry 1 ≤ entry 2 ≤ . . . ≤ entry n*
- Efficiency of a sorting algorithm is significant
- Sorting an array is usually easier than sorting a chain of linked nodes

Sorting Algorithms

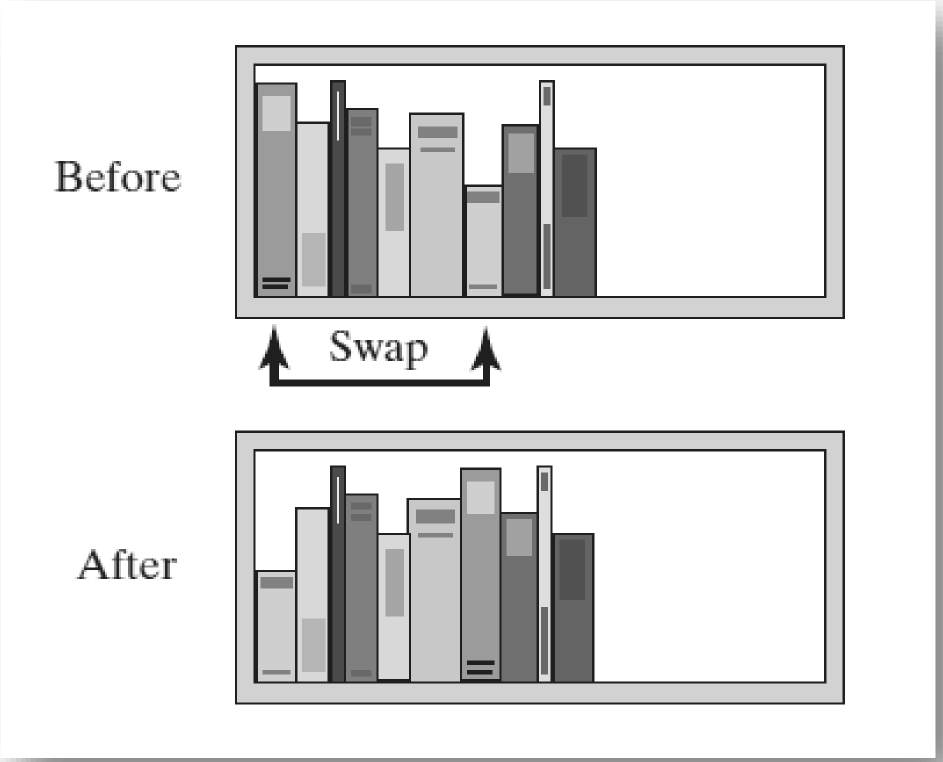
- $O(n^2)$
  - Selection Sort
  - Insertion Sort
  - Shell Sort
- $O(n \log n)$
  - Merge Sort
  - Quick Sort
- O(1) Sorting
  - Radix Sort

Sorting Algorithms

- For each algorithm
  - understand the main concept
  - implement the algorithm
    - on an Array
      - iterative
      - recursive
    - on a linked list
      - iterative
      - recursive

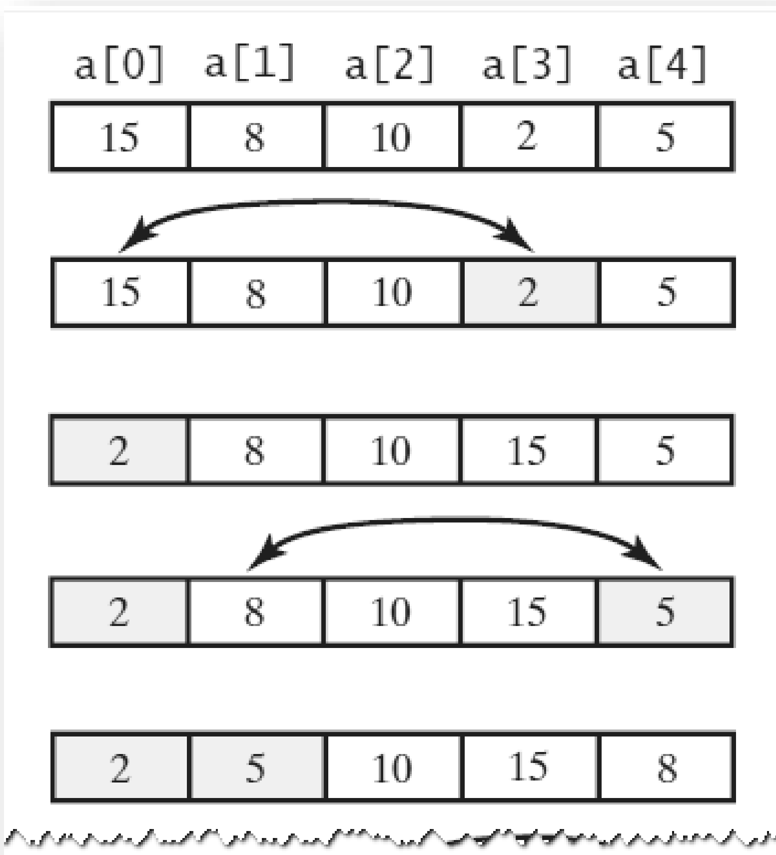
Selection Sort

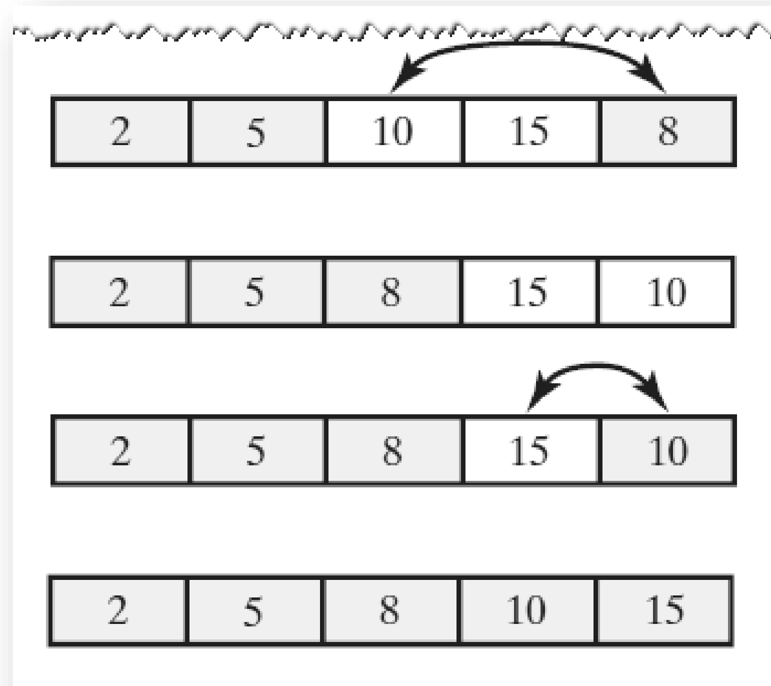
- Before and after exchanging the shortest book and the first book



Selection Sort

- A selection sort of an array of integers into ascending order





Iterative Selection Sort

- This pseudocode describes an iterative algorithm for the selection sort

```
Algorithm selectionSort(a, n)
// Sorts the first n entries of an array a.

for (index = 0; index < n - 1; index++)
{
    indexOfNextSmallest = the index of the smallest value among
                          a[index], a[index + 1], . . . , a[n - 1]
    Interchange the values of a[index] and a[indexOfNextSmallest]
    // Assertion: a[0] ≤ a[1] ≤ . . . ≤ a[index], and these are the smallest
    // of the original array entries. The remaining array entries begin at a[index + 1].
}
```

Iterative Selection Sort

- LISTING 8-1 A class for sorting an array using selection sort

```java
/**
   Class for sorting an array of Comparable objects from smallest to largest.
*/
public class SortArray
{
   /** Sorts the first n objects in an array into ascending order.
       @param a  An array of Comparable objects.
       @param n  An integer > 0. */
   public static <T extends Comparable<? super T>>
          void selectionSort(T[] a, int n)
   {
      for (int index = 0; index < n - 1; index++)
      {
         int indexOfNextSmallest = getIndexOfSmallest(a, index, n - 1);
         swap(a, index, indexOfNextSmallest);
         // Assertion: a[0] <= a[1] <= . . . <= a[index] <= all other a[i].
      } // end for
   } // end selectionSort
```

Iterative Selection Sort

- LISTING 8-1 A class for sorting an array using selection sort

```
         int indexOfNextSmallest = getIndexOfSmallest(a, index, n - 1);
         swap(a, index, indexOfNextSmallest);
         // Assertion: a[0] <= a[1] <= . . . <= a[index] <= all other a[i].
      } // end for
   } // end selectionSort

   // Finds the index of the smallest value in a portion of an array a.
   // Precondition: a.length > last >= first >= 0.
   // Returns the index of the smallest value among
   // a[first], a[first + 1], . . . , a[last].
   private static <T extends Comparable<? super T>>
           int getIndexOfSmallest(T[] a, int first, int last)
   {
      T min = a[first];
```

Iterative Selection Sort

- LISTING 8-1 A class for sorting an array using selection sort

```
      int indexOfMin = first;
      for (int index = first + 1; index <= last; index++)
      {
         if (a[index].compareTo(min) < 0)
         {
            min = a[index];
            indexOfMin = index;
         } // end if
         // Assertion: min is the smallest of a[first] through a[index].
      } // end for
      return indexOfMin;
```

Iterative Selection Sort

- LISTING 8-1 A class for sorting an array using selection sort

```
            // Assertion: min is the smallest of a[first] through a[index].
        } // end for
        return indexOfMin;

    } // end getIndexOfSmallest
    // Swaps the array entries a[i] and a[j].

    private static void swap(Object[] a, int i, int j)
    {
        Object temp = a[i];
        a[i] = a[j];
        a[j] = temp;
    } // end swap
} // end SortArray
```

Recursive Selection Sort

- Recursive selection sort algorithm

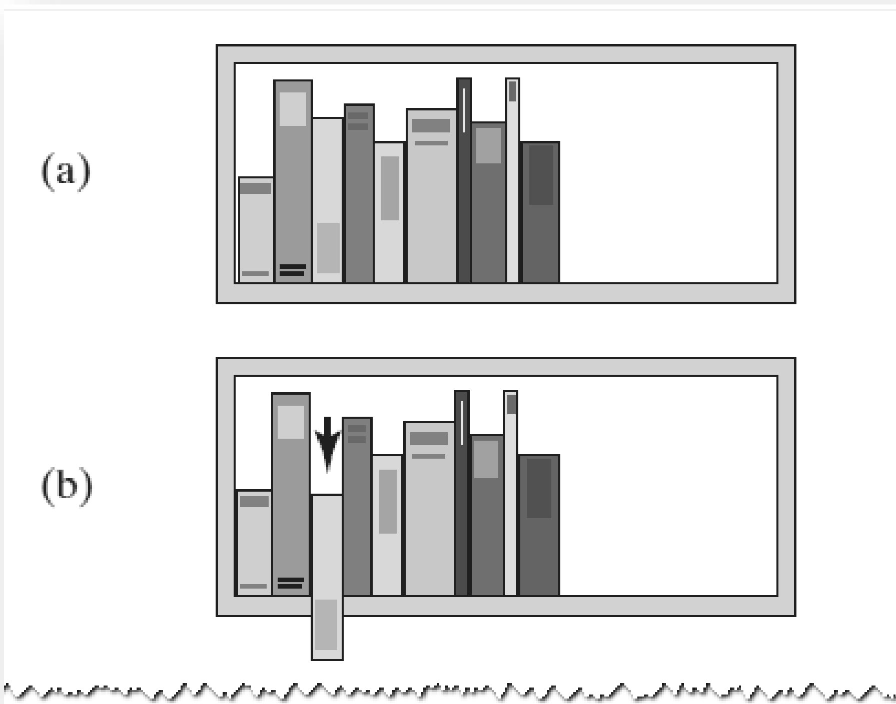
```
Algorithm selectionSort(a, first, last)
// Sorts the array entries a[first] through a[last] recursively.

if (first < last)
{
   indexOfNextSmallest = the index of the smallest value among
                         a[first], a[first + 1], . . . , a[last]
   Interchange the values of a[first] and a[indexOfNextSmallest]
   // Assertion: a[0] ≤ a[1] ≤ . . . ≤ a[first] and these are the smallest
   // of the original array entries. The remaining array entries begin at a[first + 1].
   selectionSort(a, first + 1, last)
}
```

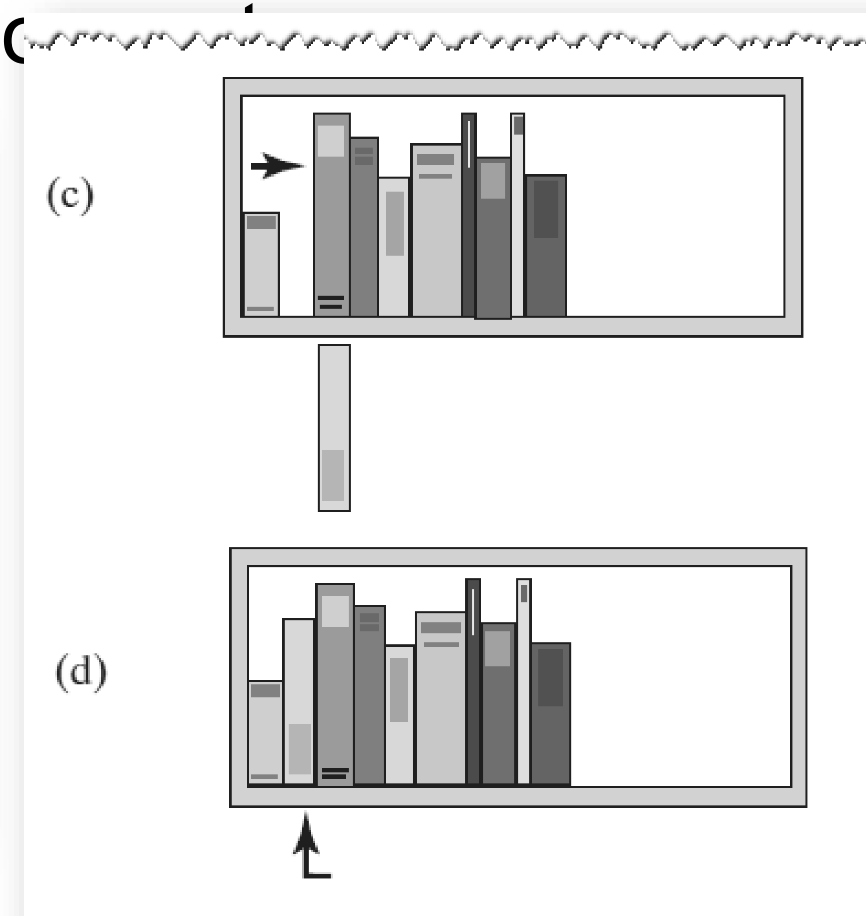
Efficiency of Selection Sort

- Selection sort is $O(n^2)$ regardless of the initial order of the entries.
  - Requires $O(n^2)$ comparisons
  - Does only $O(n)$ swaps

Insertion Sort

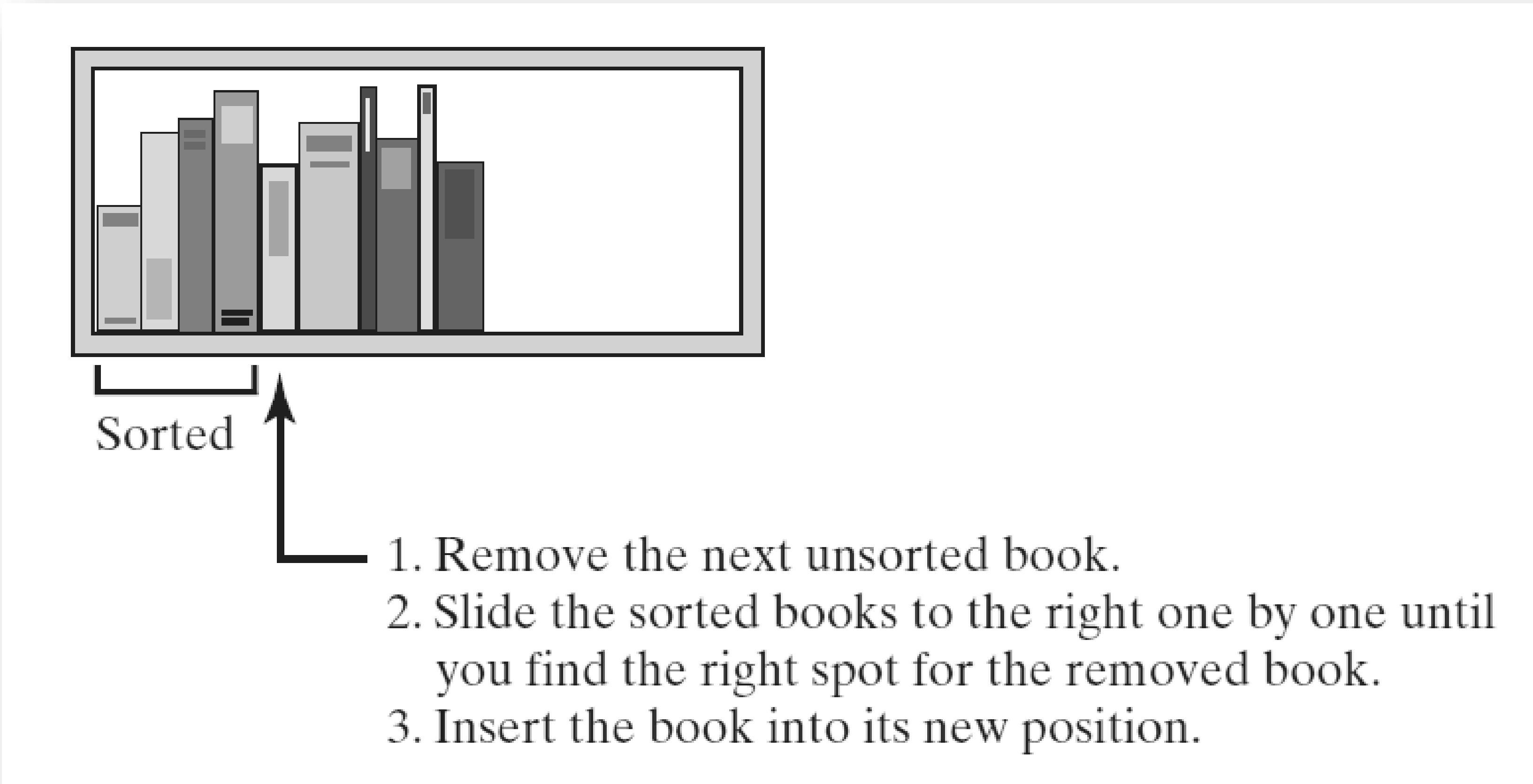
- FIGURE 8-3 The placement of the third book during an insertio





Insertion Sort

- FIGURE 8-4 An insertion sort of books



Iterative Insertion Sort

- Iterative algorithm describes an insertion sort of the entries at indices `first` through `last` of the array `a`

```
Algorithm insertionSort(a, first, last)
// Sorts the array entries a[first] through a[last] iteratively.

for (unsorted = first + 1 through last)
{
   nextToInsert = a[unsorted]
   insertInOrder(nextToInsert, a, first, unsorted - 1)
}
```

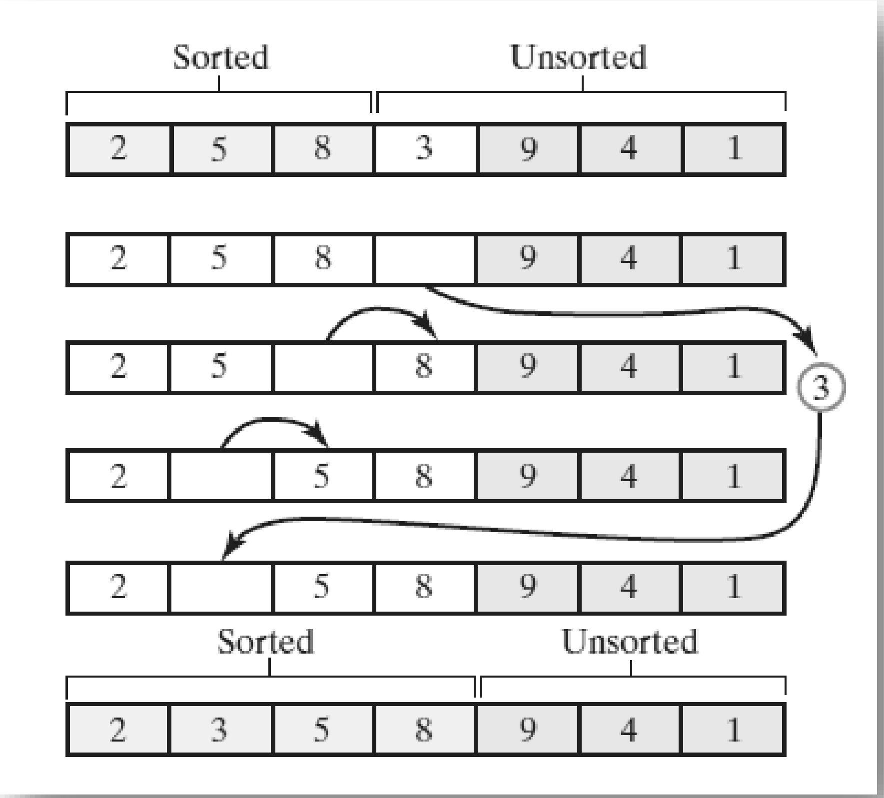
Iterative Insertion Sort

- Pseudocode of method, **`insertInOrder`**, to perform the insertions.

```
Algorithm insertInOrder(anEntry, a, begin, end)
// Inserts anEntry into the sorted entries a[begin] through a[end].
index = end                    // Index of last entry in the sorted portion
// Make room, if needed, in sorted portion for another entry
while ( (index >= begin) and (anEntry < a[index]) )
{
   a[index + 1] = a[index] // Make room
   index--
}
// Assertion: a[index + 1] is available.
a[index + 1] = anEntry      // Insert
```

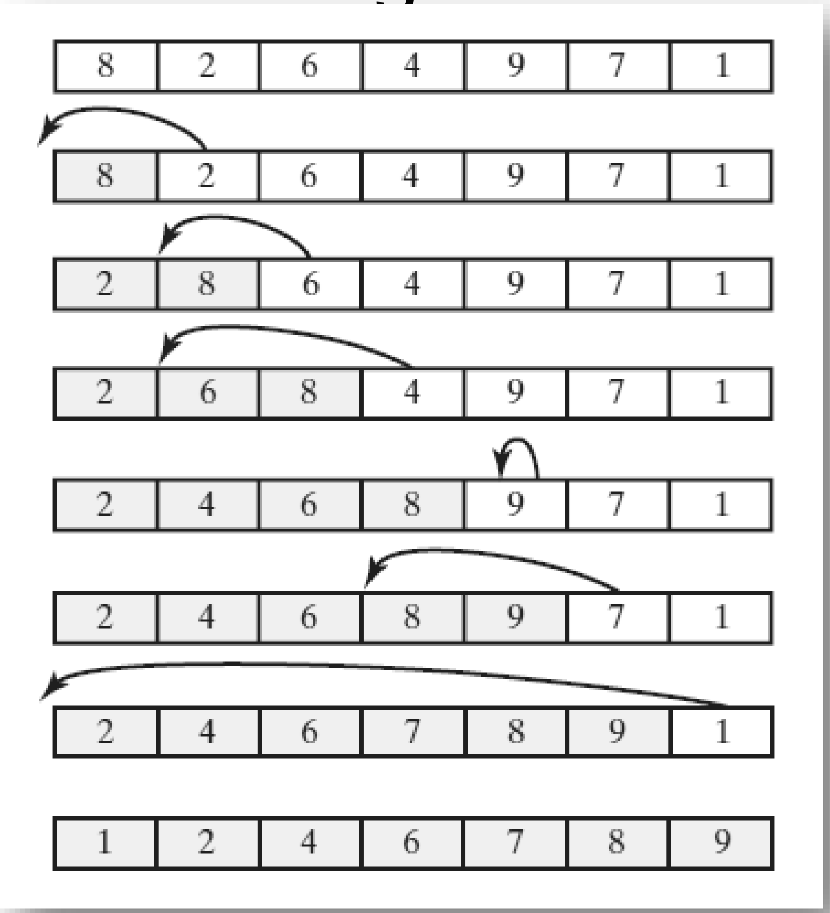
Iterative Insertion Sort

- FIGURE 8-5 Inserting the next unsorted entry into its proper location within the sorted portion of an array during an inse



Iterative Insertion Sort

- FIGURE 8-6 An insertion sort of an array of integers into ascending order



Recursive Insertion Sort

- This pseudocode describes a recursive insertion sort.

```
Algorithm insertionSort(a, first, last)
// Sorts the array entries a[first] through a[last] recursively.

if (the array contains more than one entry)
{
    Sort the array entries a[first] through a[last - 1]
    Insert the last entry a[last] into its correct sorted position within the rest of the array
}
```

Recursive Insertion Sort

- Implementing the algorithm in Java

```java
public static <T extends Comparable<? super T>>
        void insertionSort(T[] a, int first, int last)
{
    if (first < last)
    {
        // Sort all but the last entry
        insertionSort(a, first, last - 1);

        // Insert the last entry in sorted order
        insertInOrder(a[last], a, first, last - 1);
    } // end if
} // end insertionSort
```

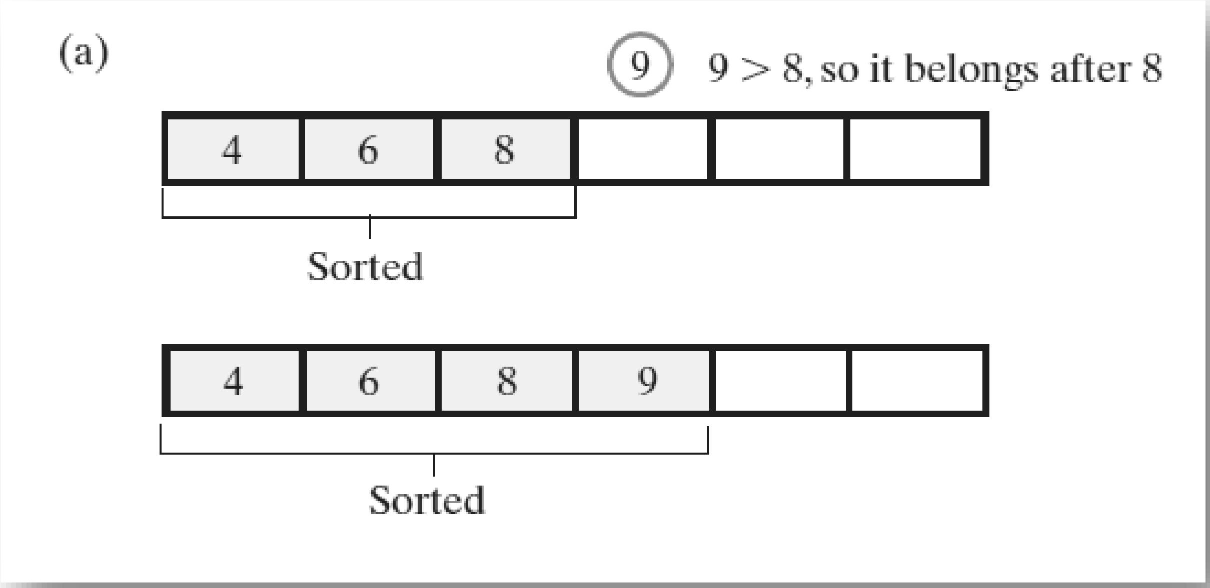
Recursive Insertion Sort

- First draft of **`insertInOrder`** algorithm.

```
Algorithm insertInOrder(anEntry, a, begin, end)
// Inserts anEntry into the sorted array entries a[begin] through a[end].
// First draft.

if (anEntry >= a[end])
   a[end + 1] = anEntry
else
{
   a[end + 1] = a[end]
   insertInOrder(anEntry, a, begin, end - 1)
}
```

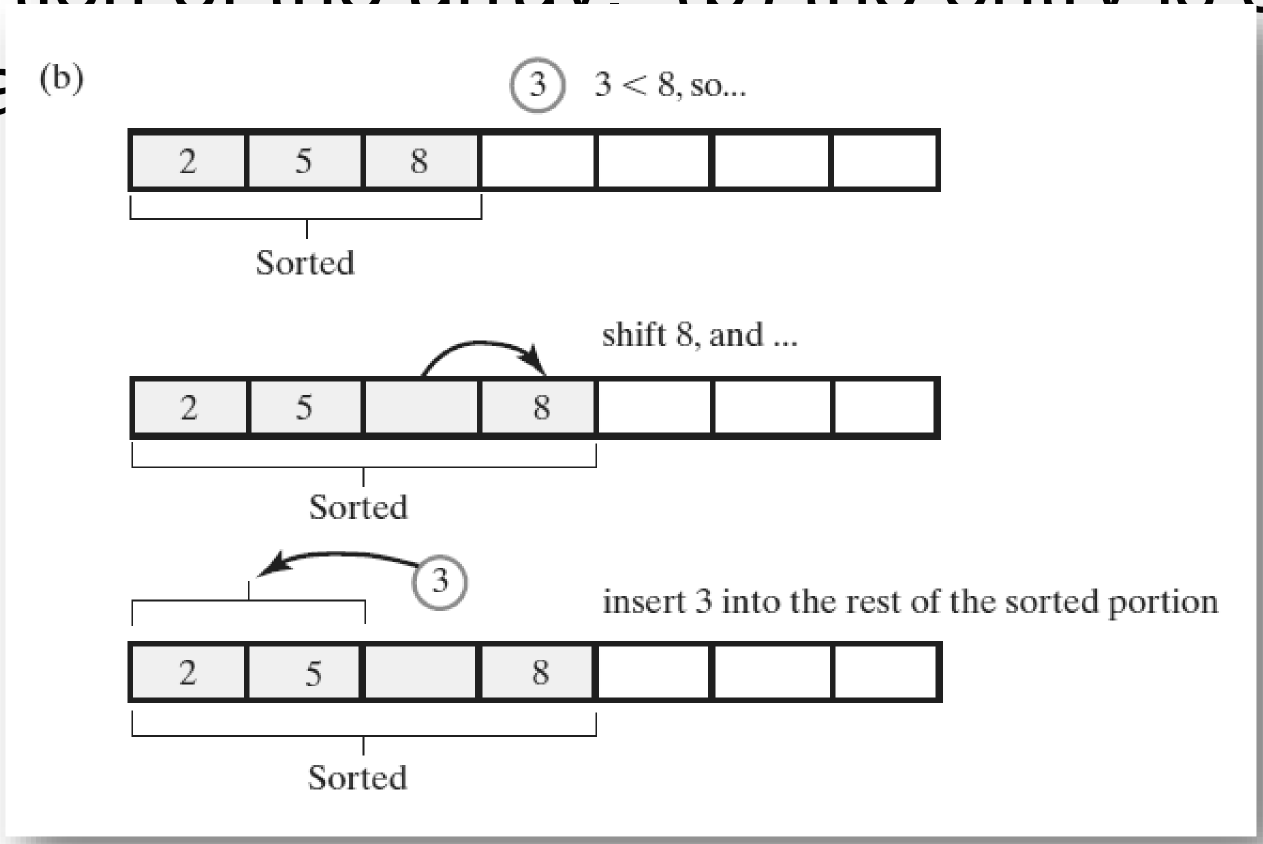
Recursive Insertion Sort

- FIGURE 8-8 Inserting the first unsorted entry into the sorted portion of the array. (a) The entry is greater than or equal to the last sorted entry



Recursive Insertion Sort

- FIGURE 8-8 Inserting the first unsorted entry into the sorted portion of the array. (b) the entry is smaller than the la



Recursive Insertion Sort

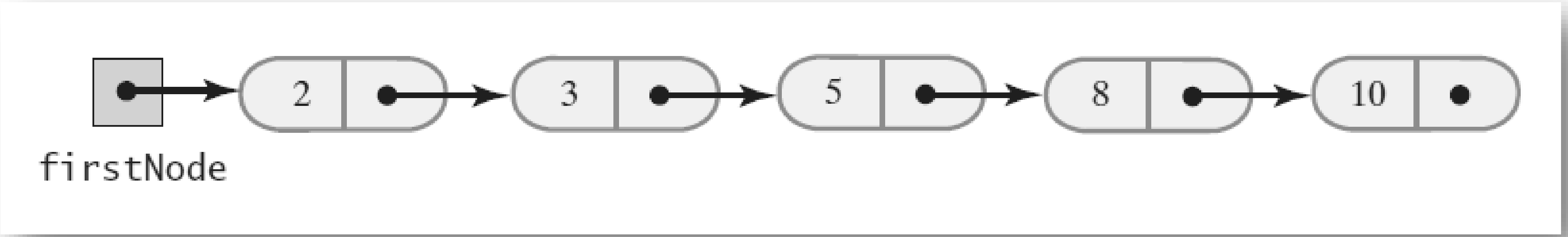
- The algorithm `insertInOrder`: final draft.

  Note: insertion sort efficiency (worst case) is $O(n^2)$

```
Algorithm insertInOrder(anEntry, a, begin, end)
// Inserts anEntry into the sorted array entries a[begin] through a[end].
// Revised draft.

if (anEntry >= a[end])
   a[end + 1] = anEntry

   else if (begin < end)
   {
      a[end + 1] = a[end]
      insertInOrder(anEntry, a, begin, end - 1)
   }
   else // begin == end and anEntry < a[end]
   {
      a[end + 1] = a[end]
      a[end] = anEntry
   }
```

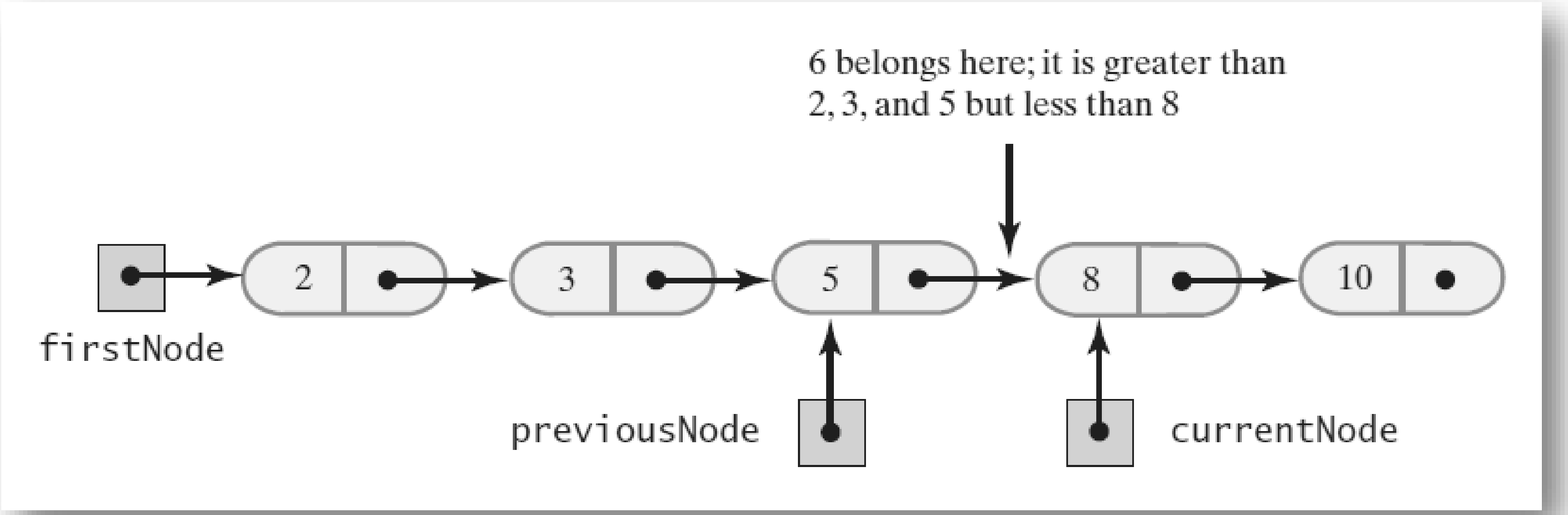
Insertion Sort of a Chain of Linked Nodes

- FIGURE 8-8 A chain of integers sorted into ascending order



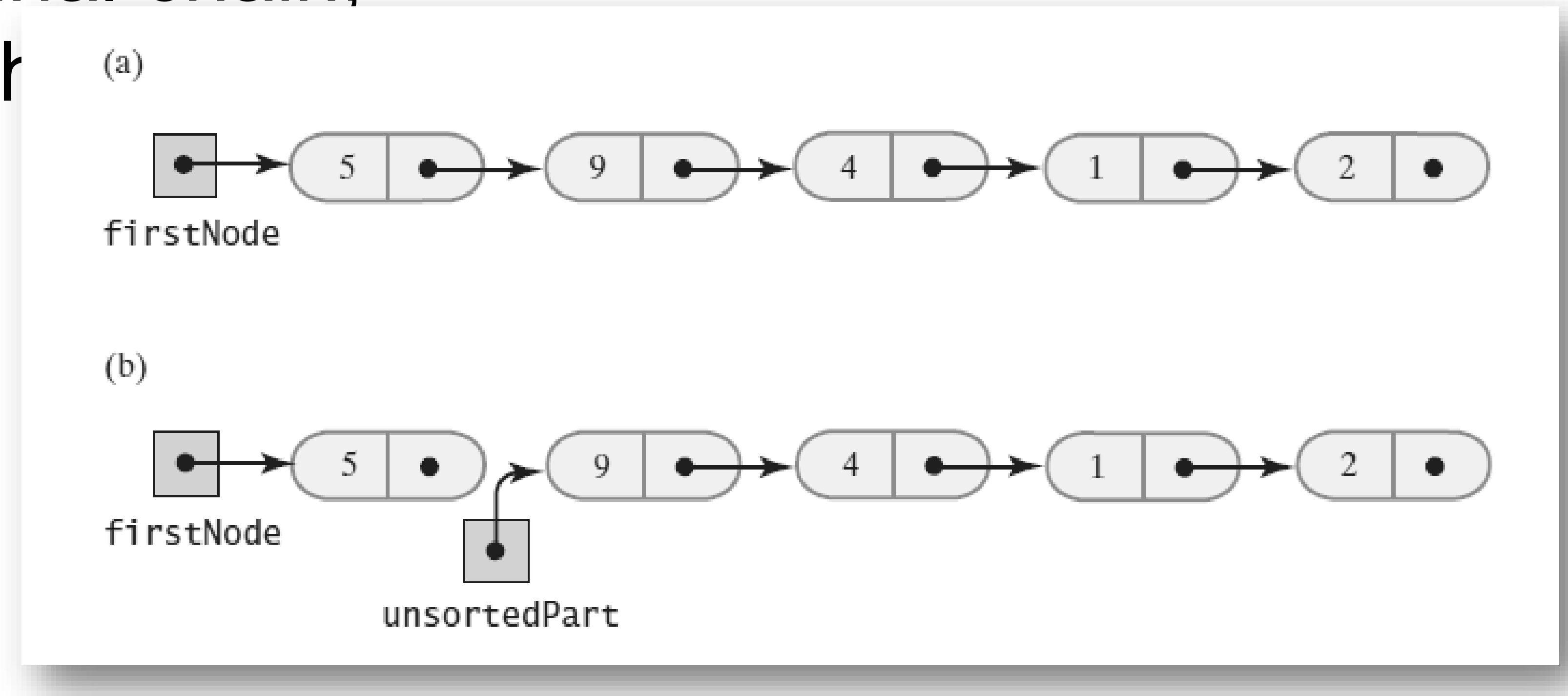
Insertion Sort of a Chain of Linked Nodes

- FIGURE 8-9 During the traversal of a chain to locate the insertion point, save a reference to the node before the current one



Insertion Sort of a Chain of Linked Nodes

- FIGURE 8-10 Breaking a chain of nodes into two pieces as the first step in an insertion sort: (a) the original chain;
  (b) th



Insertion Sort of a Chain of Linked Nodes

- Add a sort method to a class **`LinkedGroup`**
- that uses a linked chain to represent a certain collection

```
public class LinkedGroup<T extends Comparable<? super T>>
{
   private Node firstNode;
   int length; // Number of objects in the group
   . . .
```

Insertion Sort of a Chain of Linked Nodes

- This class has an inner class **Node** that has set and get methods

```
private void insertInOrder(Node nodeToInsert)
{
   T item = nodeToInsert.getData();
   Node currentNode = firstNode;
   Node previousNode = null;

   // Locate insertion point
   while ( (currentNode != null) &&
           (item.compareTo(currentNode.getData()) > 0) )
   {
      previousNode = currentNode;
      currentNode = currentNode.getNextNode();
   } // end while

   // Make the insertion
```

Insertion Sort of a Chain of Linked Nodes

- This class has an inner class **Node** that has set and get methods

```
    } // end while
    // Make the insertion
    if (previousNode != null)
    { // Insert between previousNode and currentNode
       previousNode.setNextNode(nodeToInsert);
       nodeToInsert.setNextNode(currentNode);
    }
    else // Insert at beginning
    {
       nodeToInsert.setNextNode(firstNode);
       firstNode = nodeToInsert;
    } // end if
 } // end insertInOrder
```

Insertion Sort of a Chain of Linked Nodes

- The method to perform the insertion sort.

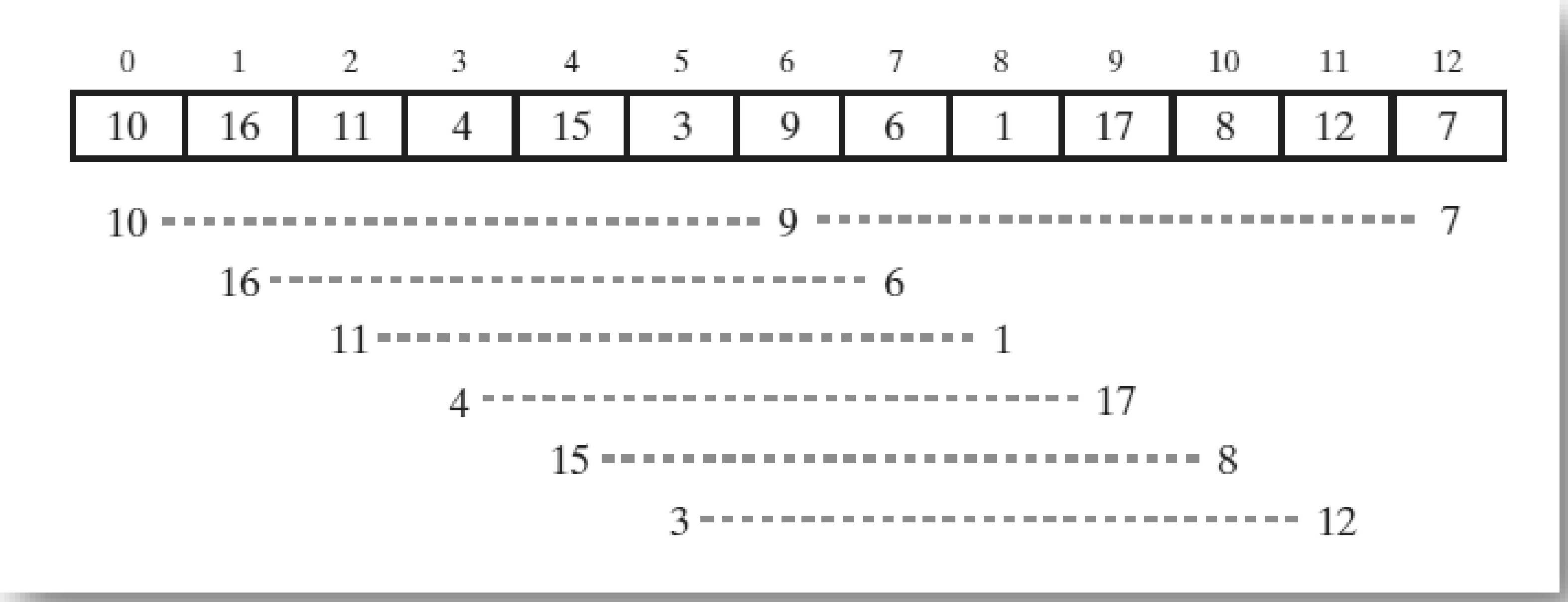
```
public void insertionSort()
{
   // If zero or one item is in the chain, there is nothing to do
   if (length > 1)
   {
       assert firstNode != null;
       // Break chain into 2 pieces: sorted and unsorted
       Node unsortedPart = firstNode.getNextNode();
       assert unsortedPart != null;
       firstNode.setNextNode(null);

       while (unsortedPart != null)
       {
          Node nodeToInsert = unsortedPart;
          unsortedPart = unsortedPart.getNextNode();
          insertInOrder(nodeToInsert);
       } // end while
    } // end if
} // end insertionSort
```

Shell Sort

- Algorithms seen so far are simple but inefficient for large arrays at $O(n^2)$
- Note, the more sorted an array is, the less work `insertInOrder` must do
- Improved insertion sort developed by Donald Shell

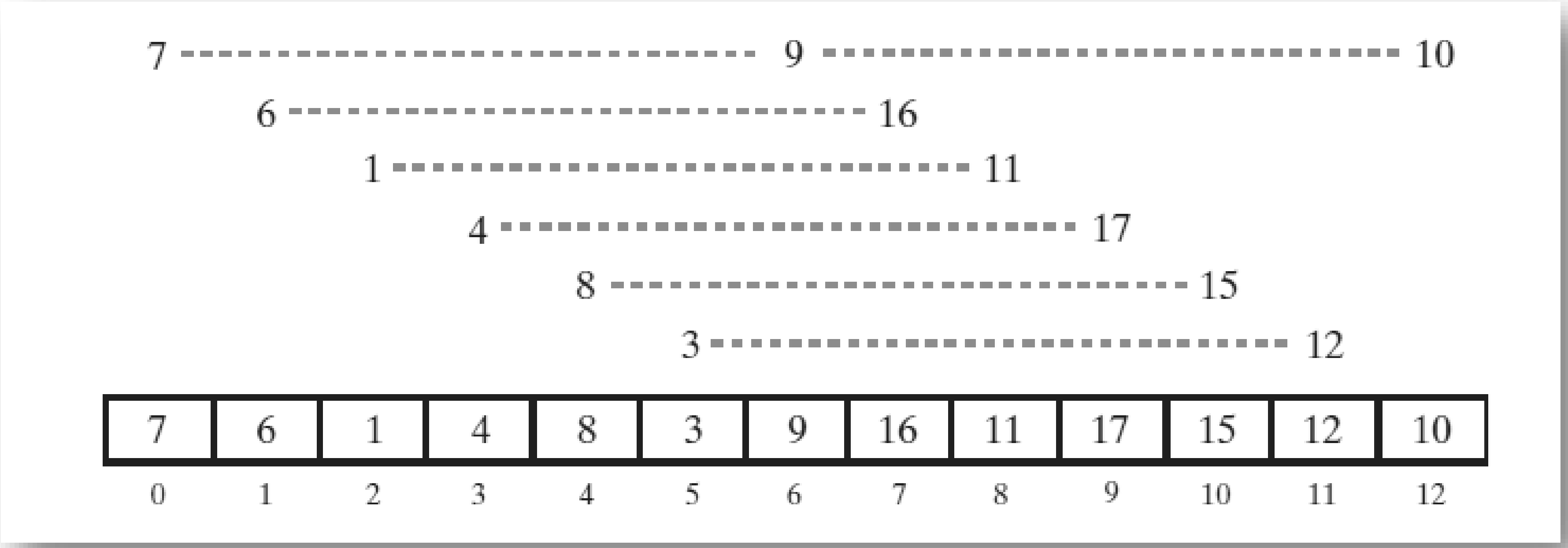
Shell Sort

- FIGURE 8-11 An array and the subarrays formed by grouping entries whose indices are 6 apart.



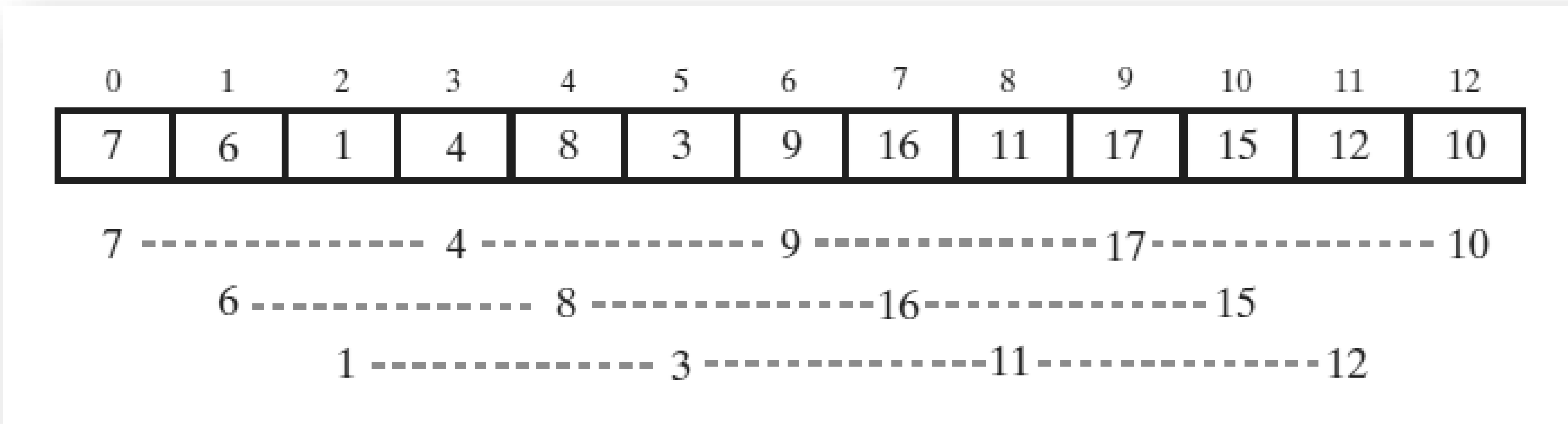
Shell Sort

- FIGURE 8-12 The subarrays of Figure 8-11 after each is sorted, and the array that contains them



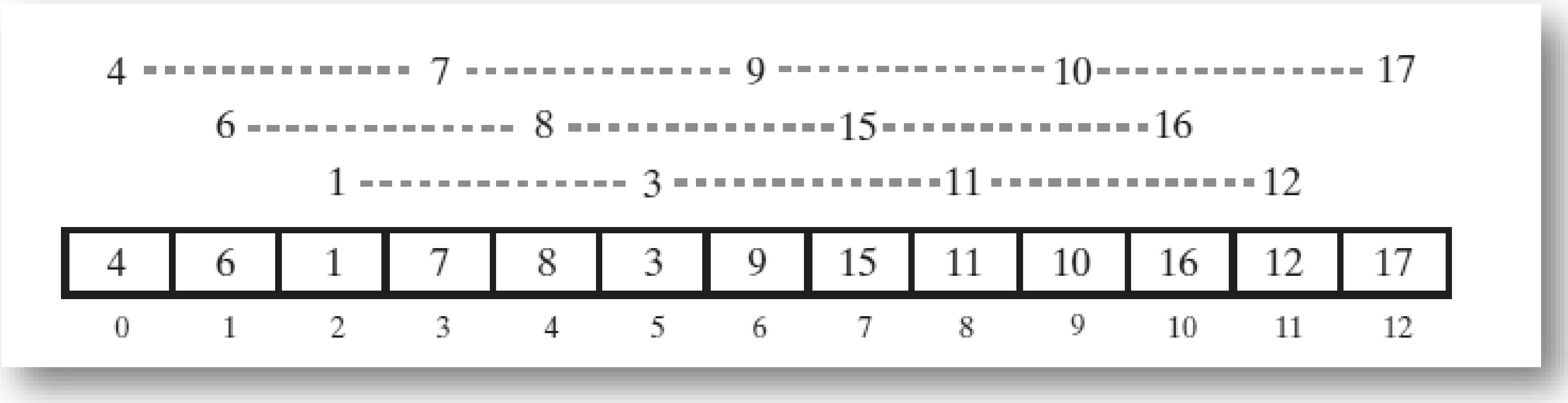
Shell Sort

- FIGURE 8-13 The subarrays of the array in Figure 8-12 formed by grouping entries whose indices are 3 apart



Shell Sort

- FIGURE 8-14 The subarrays of Figure 8-13 after each is sorted, and the array that contains them



Shell Sort

- Algorithm that sorts array entries whose indices are separated by an increment of **`space`**.

```
Algorithm incrementalInsertionSort(a, first, last, space)
// Sorts equally spaced entries of an array a[first..last] into ascending order.
// first >= 0 and < a.length; last >= first and < a.length;
// space is the difference between the indices of the entries to sort.

   for (unsorted = first + space through last at increments of space)
   {
      nextToInsert = a[unsorted]
      index = unsorted - space
      while ( (index >= first) and (nextToInsert.compareTo(a[index]) < 0) )
      {
         a[index + space] = a[index]
         index = index - space
      }
      a[index + space] = nextToInsert
   }
```

Shell Sort

- Algorithm to perform a Shell sort will invoke **`incrementalInsertionSort`** and supply any sequenc… t) can be O(n[1]…

```
Algorithm shellSort(a, first, last)
// Sorts the entries of an array a[first..last] into ascending order.
// first >= 0 and < a.length; last >= first and < a.length.

   n = number of array entries
   space = n / 2
   while (space > 0)
   {
      for (begin = first through first + space - 1)
      {
         incrementalInsertionSort(a, begin, last, space)
      }
      space = space / 2
   }
```

Comparing the Algorithms

- FIGURE 8-16 The time efficiencies of three sorting algorithms, expressed in Big Oh notation

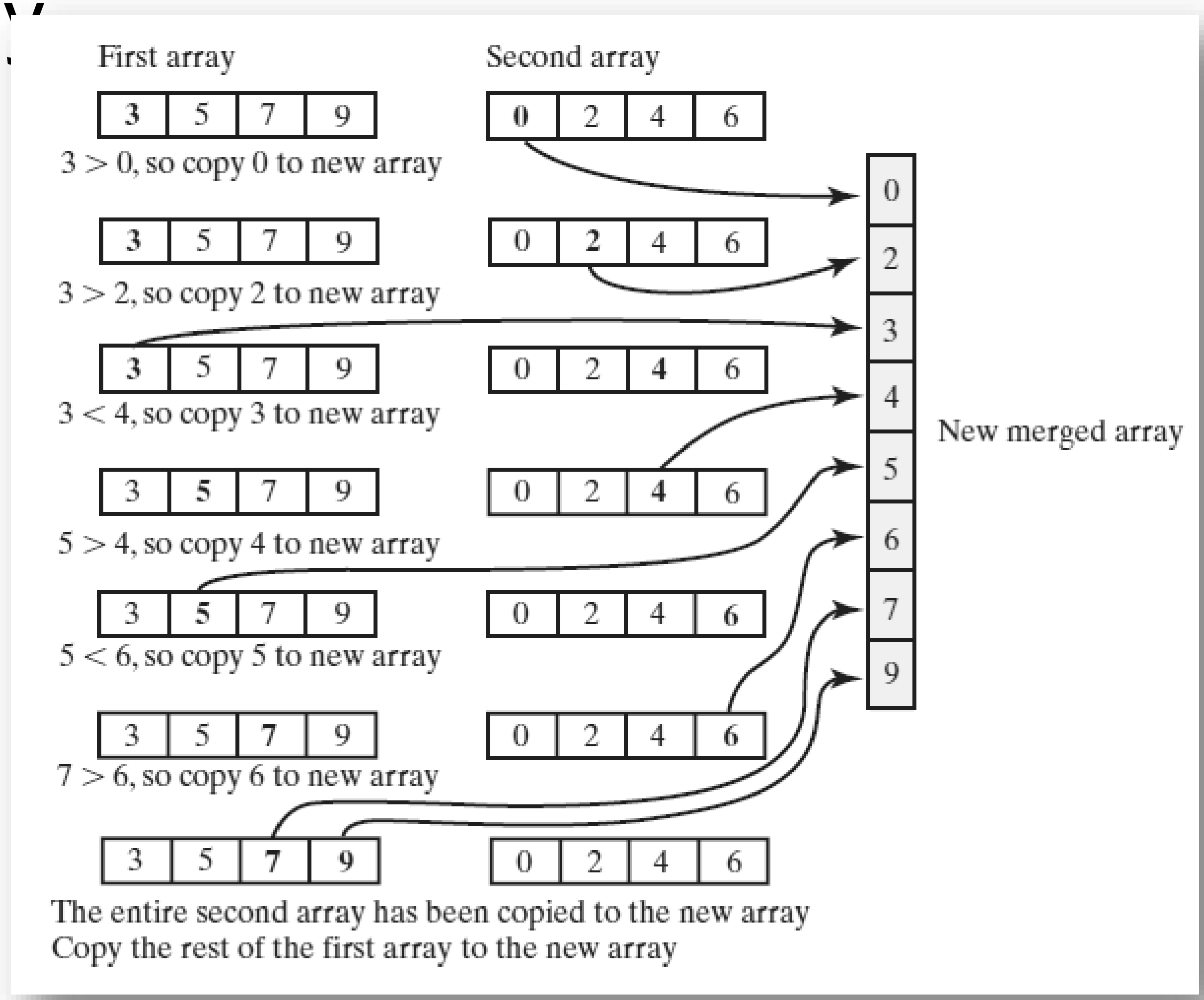
	Best Case	Average Case	Worst Case
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$
Shell sort	$O(n)$	$O(n^{1.5})$	$O(n^2)$ or $O(n^{1.5})$

Merge Sort

- Divides an array into halves
- Sorts the two halves,
  - Then merges them into one sorted array.
- The algorithm for merge sort is usually stated recursively.
- Major programming effort is in the merge process

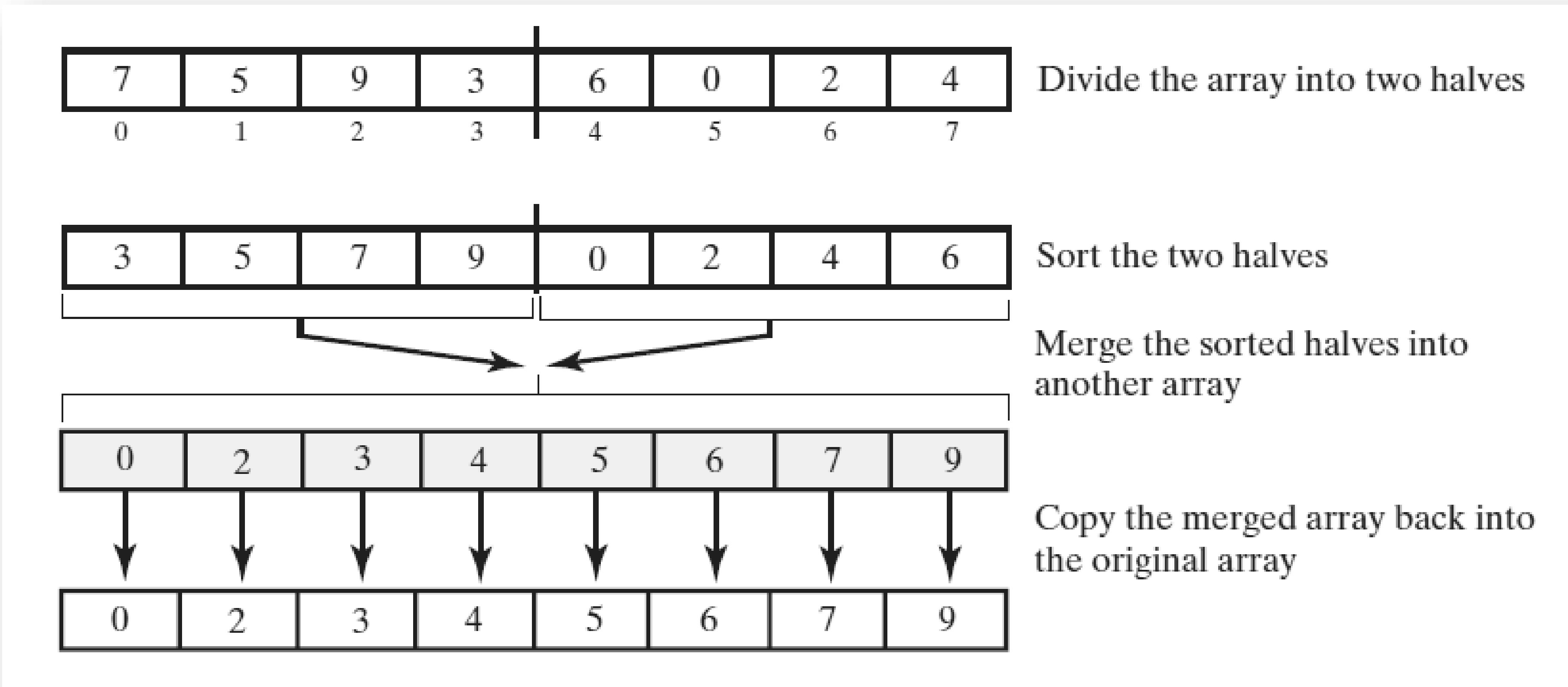
Merging Arrays

- FIGURE 9-1 Merging two sorted arrays into one sorted array



Recursive Merge Sort

- FIGURE 9-2 The major steps in a merge sort



Recursive Merge Sort

- Recursive algorithm for merge sort.

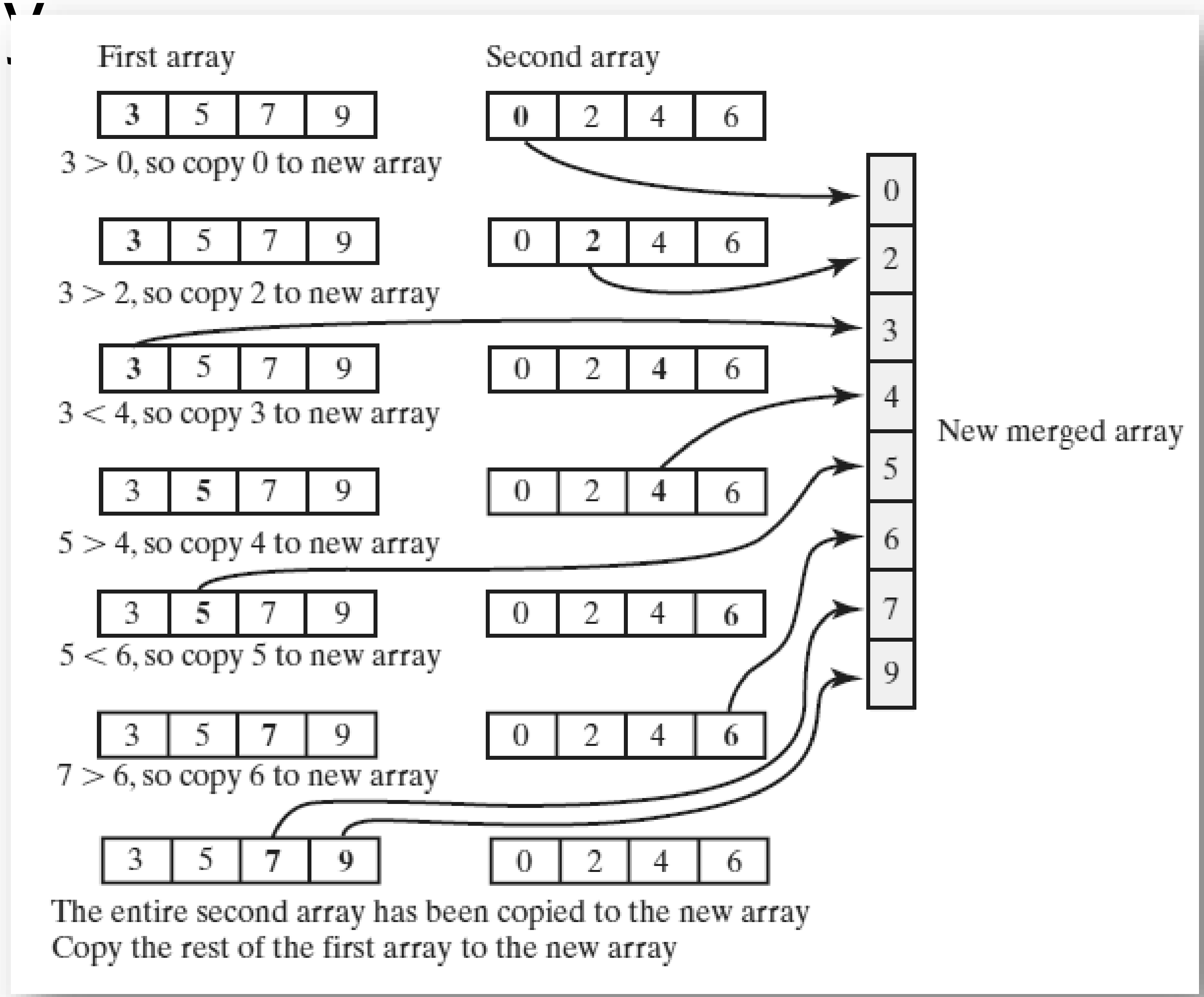
```
Algorithm mergeSort(a, tempArray, first, last)
// Sorts the array entries a[first] through a[last] recursively.

if (first < last)
{
   mid = approximate midpoint between first and last
   mergeSort(a, tempArray, first, mid)
   mergeSort(a, tempArray, mid + 1,last)
   Merge the sorted halves a[first..mid] and a[mid + 1..last] using the array tempArray
}
```

Merge Sort

- Divides an array into halves
- Sorts the two halves,
  - Then merges them into one sorted array.
- The algorithm for merge sort is usually stated recursively.
- Major programming effort is in the merge process

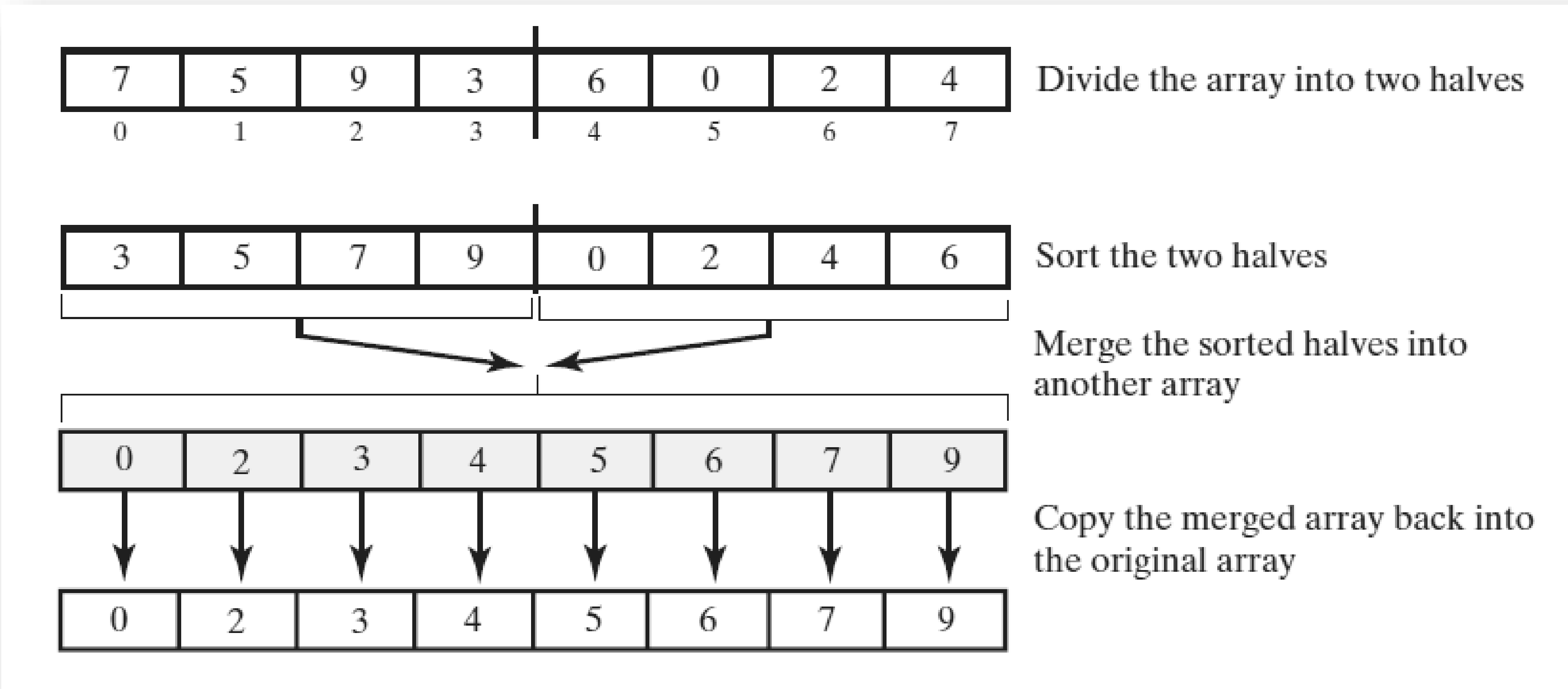
Merging Arrays

- FIGURE 9-1 Merging two sorted arrays into one sorted array



Recursive Merge Sort

- FIGURE 9-2 The major steps in a merge sort



Recursive Merge Sort

- Recursive algorithm for merge sort.

```
Algorithm mergeSort(a, tempArray, first, last)
// Sorts the array entries a[first] through a[last] recursively.

if (first < last)
{
   mid = approximate midpoint between first and last
   mergeSort(a, tempArray, first, mid)
   mergeSort(a, tempArray, mid + 1,last)
   Merge the sorted halves a[first..mid] and a[mid + 1..last] using the array tempArray
}
```

Recursive Merge Sort

- Pseudocode which describes the merge step.

```
Algorithm merge(a, tempArray, first, mid, last)
// Merges the adjacent subarrays a[first..mid] and a[mid + 1..last].
beginHalf1 = first
endHalf1 = mid
beginHalf2 = mid + 1
endHalf2 = last
// While both subarrays are not empty, compare an entry in one subarray with
// an entry in the other; then copy the smaller item into the temporary array
index = 0 // Next available location in tempArray
while ( (beginHalf1 <= endHalf1) and (beginHalf2 <= endHalf2) )
{
   if (a[beginHalf1] <= a[beginHalf2])
   {
      tempArray[index] = a[beginHalf1]
      beginHalf1++
   }
```

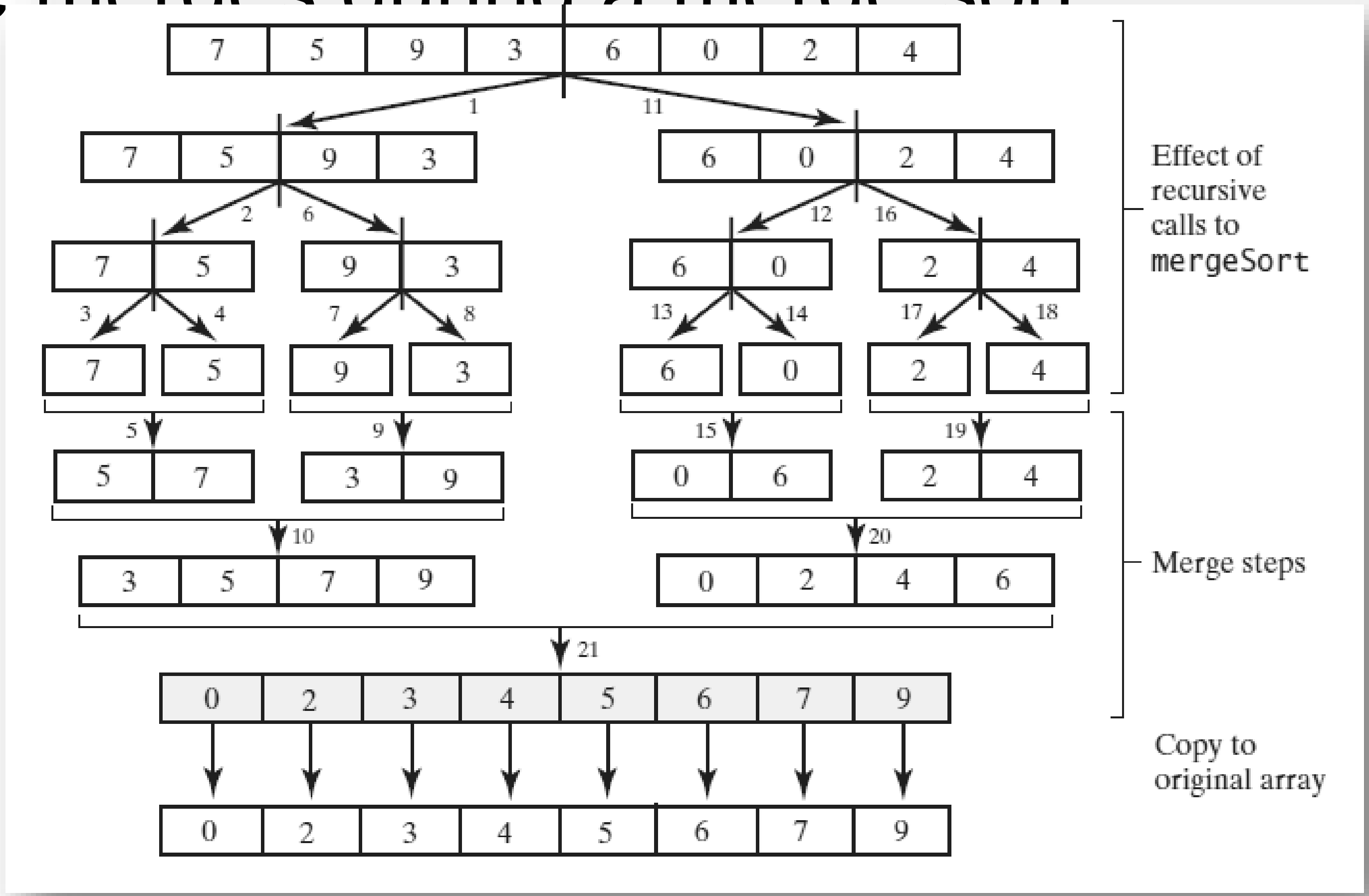
Recursive Merge Sort

- Pseudocode which describes the merge step.

```
        tempArray[index] = a[beginHalf1]
        beginHalf1++
    }
    else
    {
        tempArray[index] = a[beginHalf2]
        beginHalf2++
    }
    index++
}
// Assertion: One subarray has been completely copied to tempArray.

Copy remaining entries from other subarray to tempArray
Copy entries from tempArray to array a
```

Recursive Merge Sort

- FIGURE 9-3 The effect of the recursive calls and the merges during a merge sort



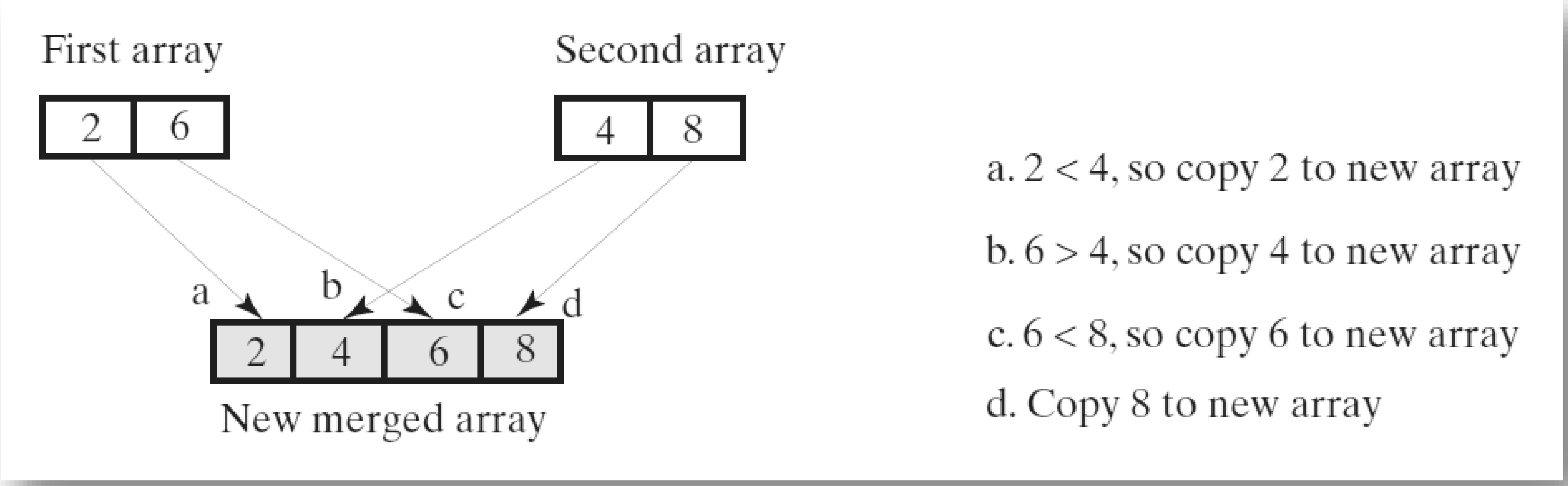
Recursive Merge Sort

- Be careful to allocate the temporary array only once.

```java
public static <T extends Comparable<? super T>>
       void mergeSort(T[] a, int first, int last)
{
   // The cast is safe because the new array contains null entries
   @SuppressWarnings("unchecked")
   T[] tempArray = (T[])new Comparable<?>[a.length]; // Unchecked cast
   mergeSort(a, tempArray, first, last);
} // end mergeSort
```

Efficiency of Merge Sort

- FIGURE 9-4 A worst-case merge of two sorted arrays.
- Efficiency is O(n log n).



Iterative Merge Sort

- Less simple than recursive version.
    - Need to control the merges.
- Will be more efficient of both time and space.
    - But, trickier to code without error.

Iterative Merge Sort

- Starts at beginning of array
  - Merges pairs of individual entries to form two-entry subarrays
- Returns to the beginning of array and merges pairs of the two-entry subarrays to form four-entry subarrays
  - And so on
- After merging all pairs of subarrays of a particular length, might have entries left over.

Merge Sort in the Java Class Library

- Class **`Arrays`** in the package **`java.util`** defines versions of a static method **`sort`**

```
public static void sort(Object[] a)

public static void sort(Object[] a, int first, int after)
```

Quick Sort

- Divides an array into two pieces
  - Pieces are not necessarily halves of the array
  - Chooses one entry in the array—called the pivot
- Partitions the array

Quick Sort

- When pivot chosen, array rearranged such that:
  - Pivot is in position that it will occupy in final sorted array
  - Entries in positions before pivot are less than or equal to pivot
  - Entries in positions after pivot are greater than or equal to pivot

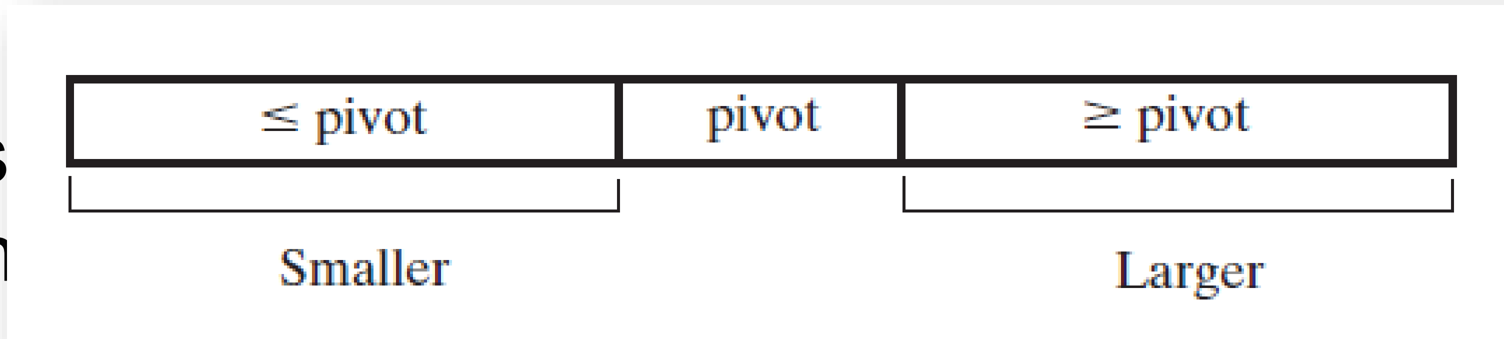
Quick Sort

- Algorithm that describes our sorting strategy:

```
Algorithm quickSort(a, first, last)
// Sorts the array entries a[first] through a[last] recursively.
if (first < last)
{
    Choose a pivot
    Partition the array about the pivot
    pivotIndex = index of pivot
    quickSort(a, first, pivotIndex - 1)  // Sort Smaller
    quickSort(a, pivotIndex + 1, last)   // Sort Larger
}
```

Quick Sort

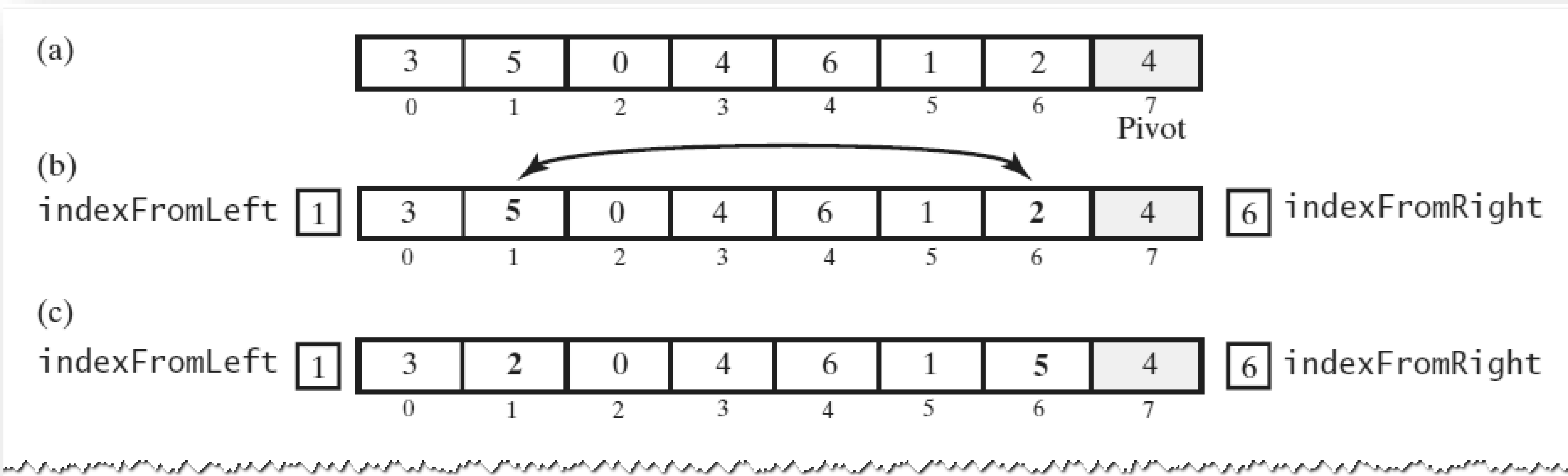
- FIGURE 9-5 A partition of an array during a quick sort



- Quick s
  O(n^2) in
- Choice of pivots affects behavior

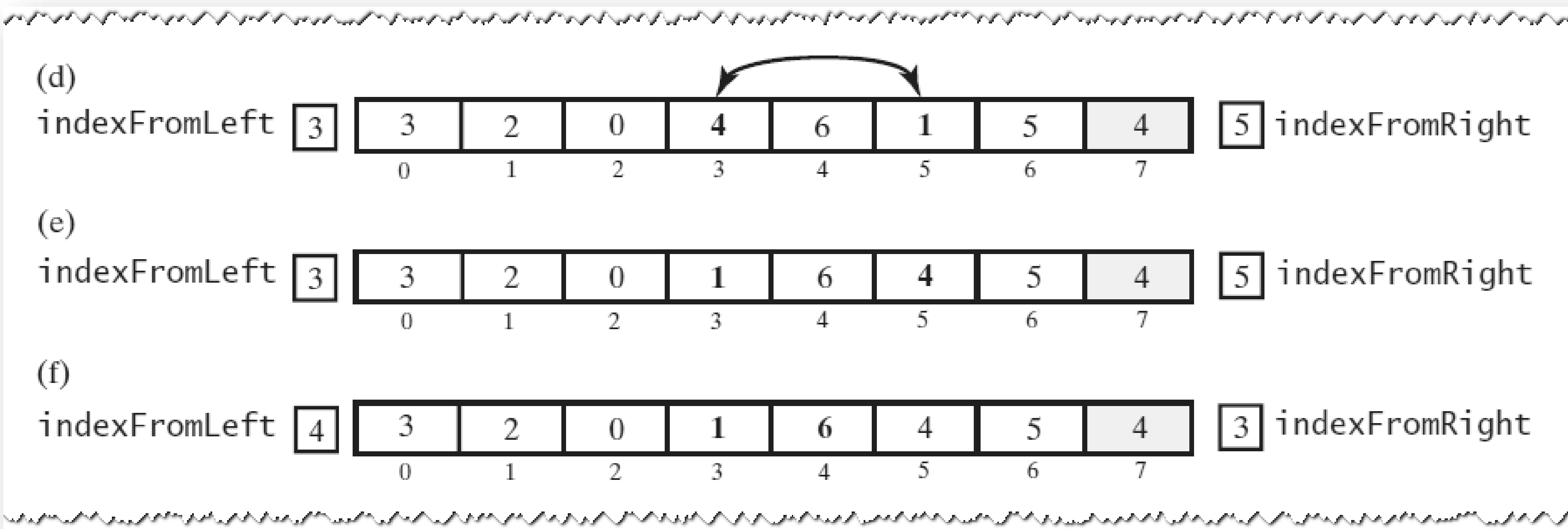
Creating the Partition

- FIGURE 9-6 A partitioning strategy for quick sort



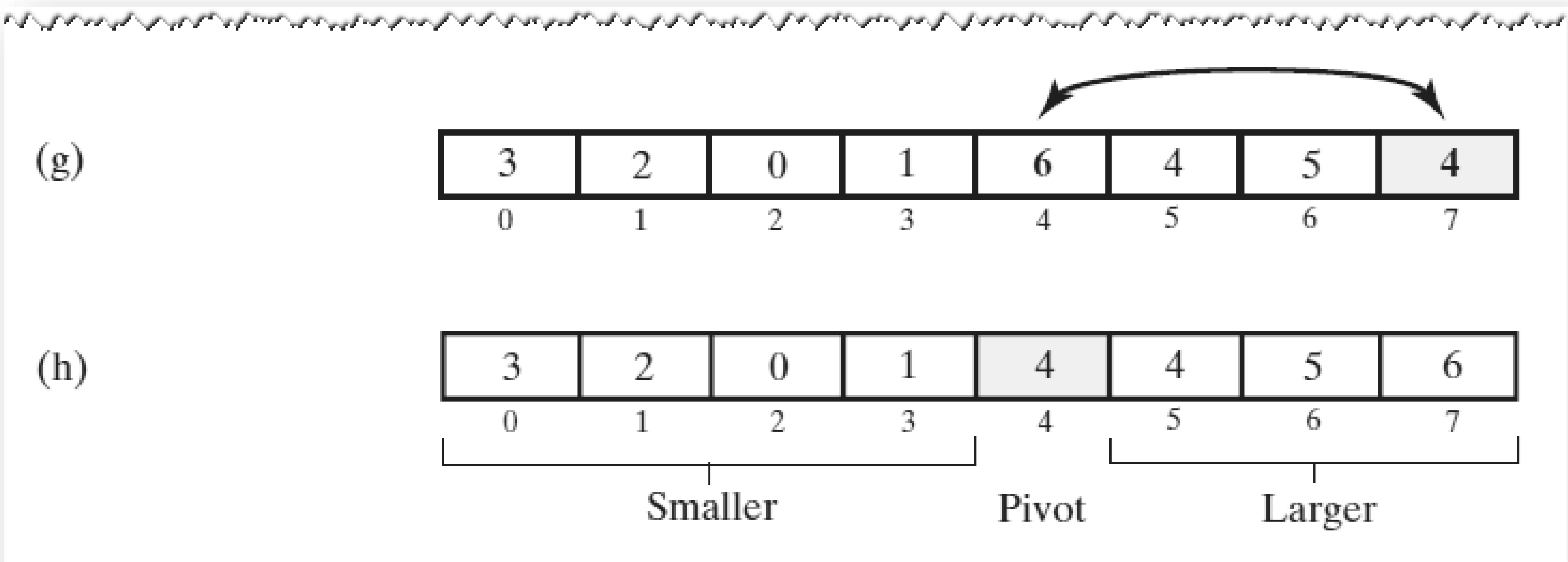
Creating the Partition

- FIGURE 9-6 A partitioning strategy for quick sort



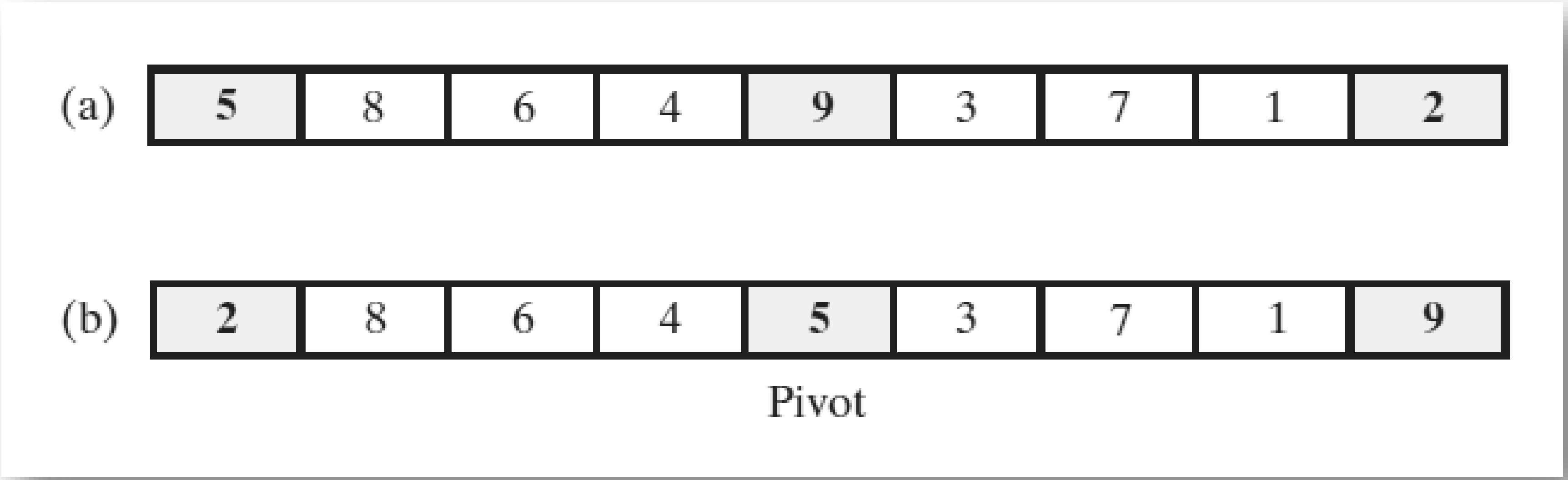
Creating the Partition

- FIGURE 9-6 A partitioning strategy for quick sort



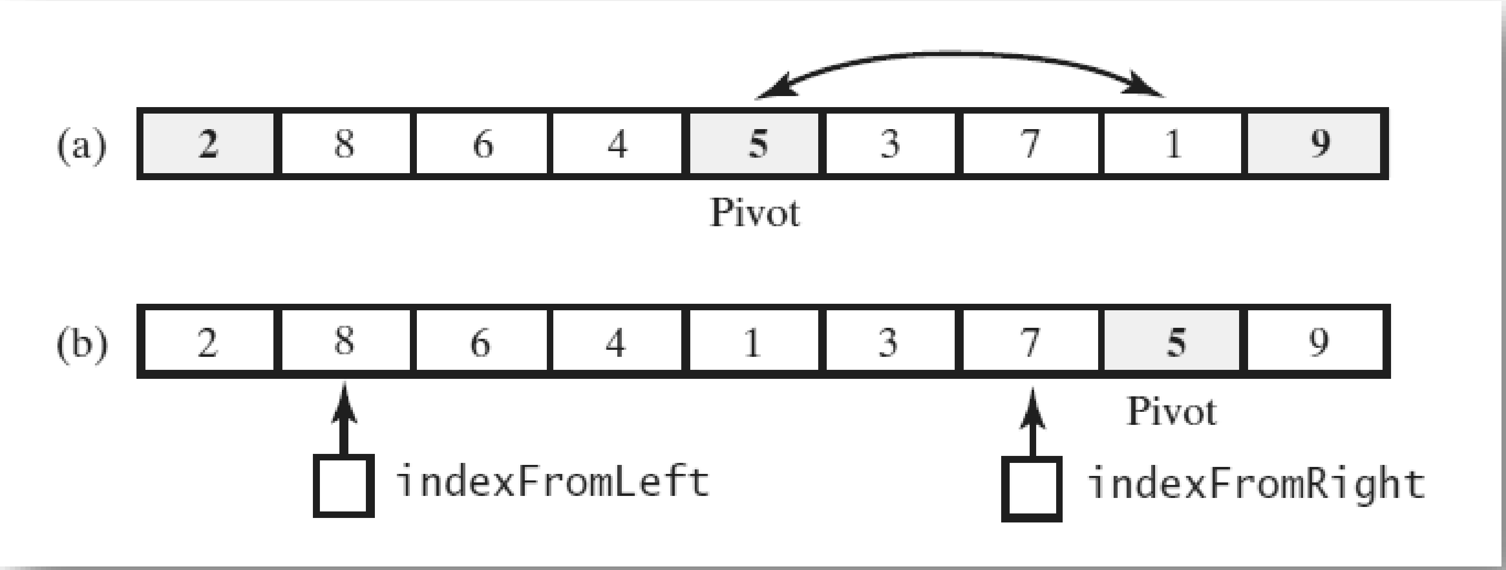
Creating the Partition

- FIGURE 9-7 Median-of-three pivot selection: (a) The original array; (b) the array with its first, middle, and last entries sorted



Adjusting the Partition Algorithm

- FIGURE 9-8 (a) The array with its first, middle, and last entries sorted; (b) the array after positioning the pivot and just before partitioning



Adjusting the Partition Algorithm

```
Algorithm partition(a, first, last)
// Partitions an array a[first..last] as part of quick sort into two subarrays named
// Smaller and Larger that are separated by a single entry—the pivot— named pivotValue.
// Entries in Smaller are <= pivotValue and appear before pivotValue in the array.
// Entries in Larger are >= pivotValue and appear after pivotValue in the array.
// first >= 0; first < a.length; last - first >= 3; last < a.length.
// Returns the index of the pivot.

   mid = index of the array's middle entry
   sortFirstMiddleLast(a, first, mid, last)
   // Assertion: a[mid] is the pivot, that is, pivotValue;
   // a[first] <= pivotValue and a[last] >= pivotValue, so do not compare these two
   // array entries with pivotValue.

   // Move pivotValue to next-to-last position in array
```

Adjusting the Partition Algorithm

```
// Move pivotValue to next-to-last position in array
Exchange a[mid] and a[last - 1]
pivotIndex = last - 1
pivotValue = a[pivotIndex]

// Determine two subarrays:
//    Smaller = a[first..endSmaller] and
//    Larger  = a[endSmaller+1..last-1]
// such that entries in Smaller are <= pivotValue and
// entries in Larger are >= pivotValue.
// Initially, these subarrays are empty.
indexFromLeft = first + 1
indexFromRight = last - 2
done = false
while (!done)
```

Adjusting the Partition Algorithm

```
while (!done)
{
    // Starting at the beginning of the array, leave entries that are < pivotValue and
    // locate the first entry that is >= pivotValue. You will find one, since the last
    // entry is >= pivotValue.
    while (a[indexFromLeft] < pivotValue)
        indexFromLeft++

    // Starting at the end of the array, leave entries that are > pivotValue and
    // locate the first entry that is <= pivotValue. You will find one, since the first
    // entry is <= pivotValue.
    while (a[indexFromRight] > pivotValue)
        indexFromRight--

    // Assertion: a[indexFromLeft] >= pivotValue and
    //            a[indexFromRight] <= pivotValue
    if (indexFromLeft < indexFromRight)
```

Adjusting the Partition Algorithm

```
    //         a[indexFromRight] <= pivotValue
    if (indexFromLeft < indexFromRight)
    {
        Exchange a[indexFromLeft] and a[indexFromRight]
        indexFromLeft++
        indexFromRight--

    }
    else
        done = true
}

Exchange a[pivotIndex] and a[indexFromLeft]
pivotIndex = indexFromLeft

// Assertion: Smaller = a[first..pivotIndex-1]
//            pivotValue = a[pivotIndex]
//            Larger = a[pivotIndex+1..last]
return pivotIndex
```

The Quick Sort Method

- Above method implements quick sort.

```
/** Sorts an array into ascending order. Uses quick sort with
    median-of-three pivot selection for arrays of at least
    MIN_SIZE entries, and uses insertion sort for smaller arrays. */
public static <T extends Comparable<? super T>>
       void quickSort(T[] a, int first, int last)
{
   if (last - first + 1 < MIN_SIZE)
   {
      insertionSort(a, first, last);
   }
   else
   {
      // Create the partition: Smaller | Pivot | Larger
      int pivotIndex = partition(a, first, last);

      // Sort subarrays Smaller and Larger
      quickSort(a, first, pivotIndex - 1);
      quickSort(a, pivotIndex + 1, last);
   } // end if
} // end quickSort
```

Quick Sort in the Java Class Library

- Class **`Arrays`** in the package **`java.util`** uses a quick sort to sort arrays of primitive types into ascending order

```
public static void sort(type[] a)

public static void sort(type[] a, int first, int after)
```