University of Pittsburgh

Algorithms and Data Structures 1
CS 0445

Fall 2022

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(Slides adapted from slides of Textbook, Dr. Ramirez's CS 0445, and Dr. Farnan's CS 1501)

Announcements

- Upcoming Deadlines
  - Homework 11 and Lab 11: next Monday 12/12
  - Assignment 3: Friday 12/16 @ 11:59 pm
  - Assignment 4: Friday 12/16 @ 11:59 pm
  - Lab 12 and Homework 12: Monday 12/19

Bonus Opportunities

- Bonus Lab (1%) and homework (2%) due on 12/19
- Assignment 5 is bonus (4%) and is due on 12/19
- 1 bonus point for entire class when OMETs response rate >= 80%
  - Currently at 23%
  - Deadline is Sunday 12/11

Final Exam

- Same format as midterm
- Non-cumulative
- Date, time and location on PeopleSoft
  - Thursday 12/15 8-9:50 am (coffee served!)
- Same classroom as lectures
- Study guide and practice test to be posted soon

Previous Lecture …

- Hashing!
  - Handling collisions
    - Open addressing
      - Double hashing
- Code walkthrough of Hash Table implementation
- String matching
  - brute-force algorithm

This Lecture …

- Handling collisions in Hash Tables using closed addressing
- String matching
  - Brute-force
  - Boyer Moore
  - Rabin Karp
- ADT Queue

Muddiest Points

- **Q: When we use double hashing do we tend to keep the same secondary hash function or is it protocol to change it?**
- Secondary hash function is different from primary hash functions. Within the same application, it is rare to change the hash function(s). If you have to, you need to rehash the existing items with the new hash function.
- **Q: Are lab 12 and homework 12 for extra credit? Or will there be a 13th lab and homework? And do we still get 2 drops?**
- The bonus lab/homework is the 13^{th}. We still get 2 drops for labs and homework assignments and 1 drop for programming assignments
- **Q: Can you please explain again why iterators are important and how exactly they work.**
- We need iterators when we want the client to traverse items inside a container without revealing the implementation and without giving direct access to internal instance variables
- **Q: when will midterm regrade be done?**
- I am hoping to finish them before the final exam
- **Q: Can you go over some applications of hashing?**
- We will see on application today to solve the String Matching problem

Closed addressing

- i.e., if a pigeon's hole is taken, it lives with a roommate
- Most commonly done with **separate chaining**
  - Create **a linked-list** of keys at each index in the table
  - Similar to Assignment 2!
    - array of linked lists

Closed addressing

- Performance depends on chain length
  - Which is determined by the load factor $\alpha = n/m$ and the quality of the hash function
  - With a good hash function, on average, n/m keys per chain
- In closed addressing, number of keys n > table size m
  - not possible with open addressing

In general...

- Closed-addressing hash tables are fast and efficient for many applications
- Where would open addressing be preferable?
  - Strict memory limits
  - Lack of dynamic memory allocation
    - needed to allocating nodes in the linked lists in separate chaining

String Matching

- Have a pattern string p of length m
- Have a text string t of length n
- Can we find an index i of string t such that each of the m characters in the substring of t starting at i matches each character in p
  - Example: can we find the pattern "fox" in the text "the quick brown fox jumps over the lazy dog"?
    - Yes! At index 16 of the text string!

Simple approach

- BRUTE FORCE
  - Start at the beginning of both pattern and text
  - Compare characters left to right
  - Mismatch?
  - Start again at the 2nd character of the text and the beginning of the pattern...

Brute force code

```
public static int bf_search(String pat, String txt) {
    int m = pat.length();
    int n = txt.length();
    for (int i = 0; i <= n - m; i++) {
        int j;
        for (j = 0; j < m; j++) {
            if (txt.charAt(i + j) != pat.charAt(j))
                break;
        }
        if (j == m)
            return i; // found at offset i
    }
    return n; // not found
}
```

Alternate implementation of Brute-Force Algorithm

```
public static int bf_search(String pat, String txt)
{
   int j, m = pat.length();
   int i, n = txt.length();
   for (i = 0, j = 0; i < n && j < m; i++) {
     if (txt.charAt(i) == pat.charAt(j))
          j++;
     else { i -= j; j = 0; }
   }
   if (j == m)
        return i - m; // found at offset i
   else return n; // not found
}
```

Tracing Brute force Algorithm

i:	0							
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:	0							

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
              j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:	0	1						
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:	0	1						

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:		1	2					
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:		1	2					

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:			2	3				
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:			2	3				

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:				3	4			
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:				3	4			

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:					4	5		
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:					4	5		

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:	0					5		
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:	0					5		

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
              j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:	0	1						
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:	0							

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
              j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:		1						
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:	0							

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
              j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:		1	2					
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:	0							

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
              j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:			2	3				
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:	0	1						

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:				3	4			
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:		1	2					

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:					4	5		
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:			2	3				

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:						5	6	
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:				3	4			

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:							6	7
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:					4	5		

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:								7	8
text:	A	B	A	B	A	B	A	C	
pattern:	A	B	A	B	A	C			
j:						5	6		

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
             j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

i:								8
text:	A	B	A	B	A	B	A	C
pattern:	A	B	A	B	A	C		
j:							6	

```
public static int bf_search(String pat, String txt)
{
    int j, m = pat.length();
    int i, n = txt.length();
    for (i = 0, j = 0; i < n && j < m; i++) {
       if (txt.charAt(i) == pat.charAt(j))
              j++;
       else { i -= j; j = 0; }
    }
    if (j == m)
          return i - m; // found at offset i
    else return n; // not found
}
```

Brute force analysis

- Runtime?
  - What does the worst case look like?
    - t = XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXY
    - p = XXXXY
  - m (n - m + 1)
    - O(nm) if n >> m
  - Is the average case runtime any better?
    - Assume we mostly mismatch on the first pattern character
    - O(n + m)
      - O(n) if n >> m

Where do we improve?

- Improve worst case
  - Theoretically very interesting
  - Practically doesn't come up that often for human language
- Improve average case
  - Much more practically helpful
    - Especially if we anticipate searching through large files

Improve Average Case: Boyer Moore

- What if we compare starting at the end of the pattern?
  - t = ABCDVABCDWABCDXABCDYABCDZ
  - p = ABCDE
  - V does not match E
    - Further V is nowhere in the pattern…
    - So skip ahead m positions with 1 comparison!
      - Runtime?
        - In the best case, n/m
- When searching through text with a large alphabet, will often come across characters not in the pattern.
  - One of Boyer Moore's heuristics takes advantage of this fact
    - Mismatched character heuristic

Mismatched character heuristic

- How well it works depends on the pattern and text at hand
  - What do we do in the general case after a mismatch?
    - Consider:
      - `t = XYXYXYZXXXXXXXXXXXXXX`
      - `p = XYXYZ`
    - If mismatched character *does* appear in p, need to "slide" to the right to the next occurrence of that character in p
      - Requires us to pre-process the pattern
        - Create a right array

Pattern: A B C D E

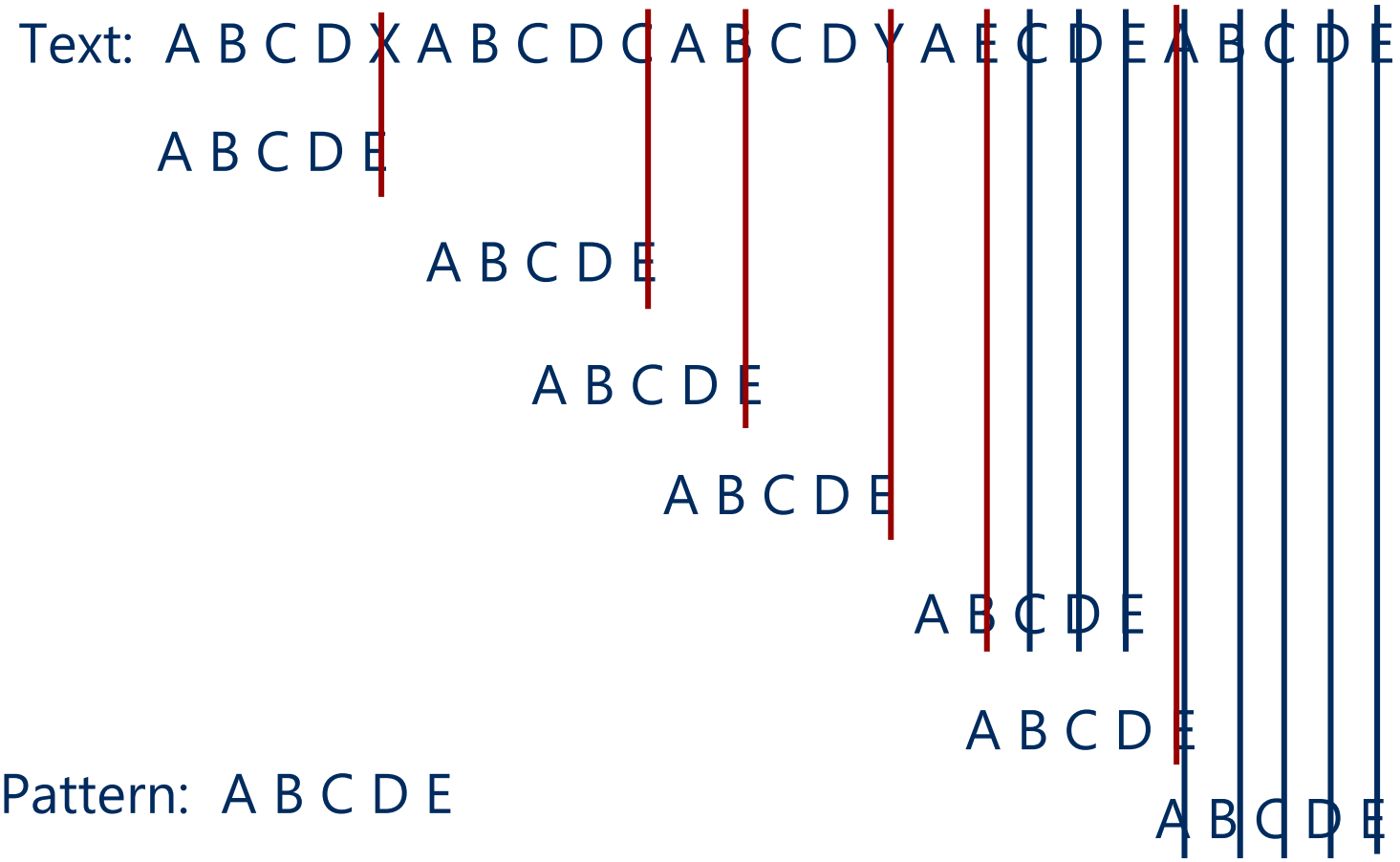
right = [0, 1, 2, 3, 4, -1, -1, …]

```
for (int i = 0; i < R; i++)
   right[i] = -1;
for (int j = 0; j < m; j++)
   right[p.charAt(j)] = j;
```

Mismatched character Procedure

- Let j be the index in the pattern currently under comparison
- At mismatch, slide pattern to the right by
  - j - right[mismatched_text_char] positions
  - If < 1, slide 1

Mismatched character heuristic example



Pattern: A B C D E

right = [0, 1, 2, 3, 4, -1, -1, …]

Runtime for mismatched character

- What does the worst case look like?
  - t = XXXXXXXXXXXXXXXXXXXXXXXXXXXXX

    p = YXXXXX
  - Runtime:
    - O(nm)
      - Same as brute force!
- This is why mismatched character is only one of Boyer Moore's heuristics
- See BoyerMoore.java

Let's use hashing!

- Hashing was cool, let's try using that

```
public static int hash_search(String pat, String txt) {
    int m = pat.length();
    int n = txt.length();
    int pat_hash = h(pat);
    for (int i = 0; i <= n - m; i++) {
            if (h(txt.substring(i, i + m)) == pat_hash)
                return i; // found!
    }
    return n; // not found
}
```

Well that was simple

- Is it efficient?
  - Nope! Practically worse than brute force
    - Instead of nm character comparisons, we perform n hashes of m character strings
- Can we make an efficient pattern matching algorithm based on hashing?

Horner's method

- Brought up during the hashing lecture

```
public long horners_hash(String key, int m) {
        long h = 0;
        for (int j = 0; j < m; j++)
                h = (R * h + key.charAt(j)) % Q;
        return h;
}
```

Can we compute the hash of the next m characters using the hash of the previous m characters in O(1) time?

Efficient hash-based pattern matching

```
text = "abcdefg"
pattern = "defg"
```

```
m = pattern.length
ph = horners_hash(text, m)
th = horners_hash(pattern, m)
i = 0
while ph != th:
      th -= pattern[i] * R^m
      th *= R
      th = (th + pattern[i + m]) % Q
      i++
```

- This is Rabin-Karp

What about collisions?

- Note that we're not storing any values in a hash table…
  - So increasing Q doesn't affect memory utilization!
    - Make Q really big and the chance of a collision becomes really small!
      - But not 0…
- OK, so do a character by character comparison on a hash match just to be sure
  - Worst case runtime?
    - Back to brute force esque runtime…

Assorted casinos

- Two options:
  - Do a character by character comparison after hash match
    - Guaranteed correct
    - Probably fast

    Las Vegas
  - Assume a hash match means a substring match
    - Guaranteed fast
    - Probably correct

    Monte Carlo

ADT Queue

- Queue
  - Data is **added to the end** and **removed from the front**
  - Logically the items other than the front item cannot be accessed
    - Think of a bowling ball return lane
      - Balls are put in at the end and removed from the front, and you can only see / remove the front ball
  - Fundamental Operations
    - enqueue an item to the end of the queue
    - dequeue an item from the front of the queue
    - front – look at the top item without disturbing it

Queues

- A Queue organizes data by First In First Out, or FIFO (or LILO – Last In Last Out)
- Like a Stack, a Queue is a simple but powerful data structure
  - Used extensively for simulations
    - Many real life situations are organized in FIFO, and Queues can be used to simulate these
    - Allows problems to be modeled and analyzed on the computer, saving time and money