University of Pittsburgh

Algorithms and Data Structures 1
CS 0445

Fall 2022

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(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS 0445 slides.)

Announcements

- Upcoming Deadlines:
  - **Assignment 2: Late Deadline 11/9 @ 11:59 pm**
  - Lab 8: next Monday 11/14 @ 11:59 pm
  - Midterm reattempts: Thursday 11/10 @ 11:59 pm
- Live Support Session for Assignment 2
  - Video and slides available on Canvas
- QA Session on Piazza every Friday 4:30-5:30 pm

Today …

- Sorting Algorithms

Muddiest Points

- **Q: I have not seen stringBuilder before. Could you explain that?**
- StringBuilder is mutable, whereas String is mutable
- Internally, a resizable array is used
- Appending to StringBuilder is O(1), whereas appending to String is O(n)
  - StringBuilder sb = new StringBuilder(“Hello”);
  - sb.append(“!”); //O(1)
  - String s = new String(“Hello”);
  - s = s + “!”; //O(n)

Muddiest Points

- **Q: Could you show us how to input the file under debug mode? I can not follow to cwd and json part.**
- You need to edit launch.json to add an “args” field to the run configuration

Sorting

- We have seen a few container data structures
  - Bag, Stack, List
- Sorting Problem: arrange items in a List such that *entry 1* ≤ *entry 2* ≤ . . . ≤ *entry n*
- Efficiency of a sorting algorithm is significant
- Sorting an array is usually easier than sorting a chain of linked nodes

Sorting Algorithms

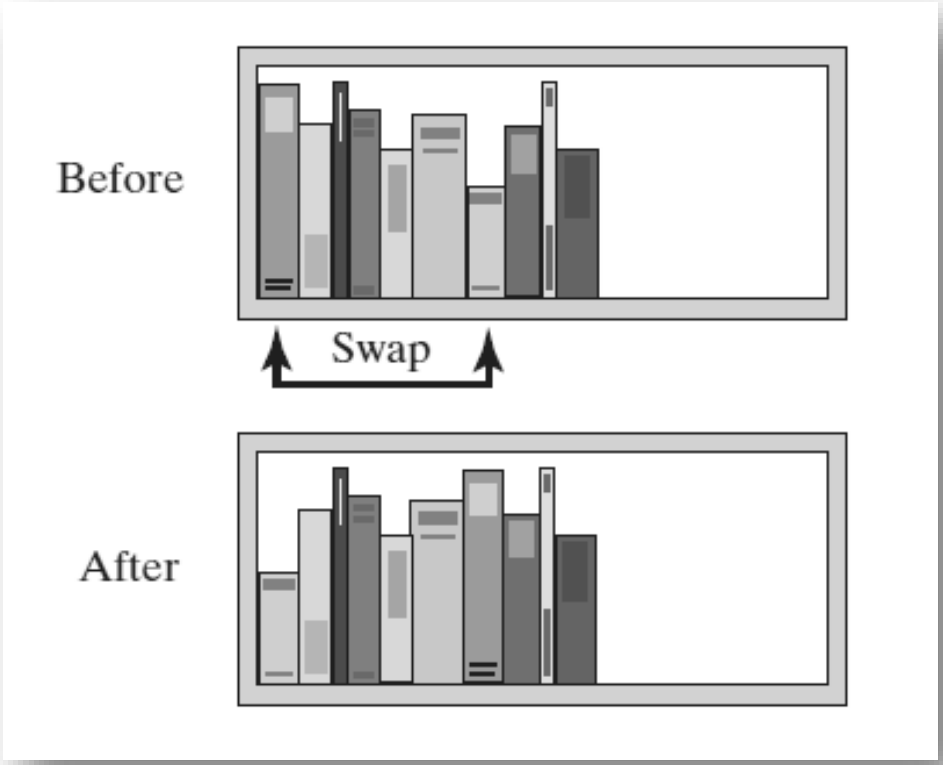
- $O(n^2)$
  - Selection Sort
  - Insertion Sort
  - Shell Sort
- O(n log n)
  - Merge Sort
  - Quick Sort
- O(1) Sorting
  - Radix Sort

Sorting Algorithms

- For each algorithm
  - understand the main concept using an example
  - implement the algorithm
    - on an Array
      - iterative
      - recursive
    - on a linked list
      - iterative
      - recursive

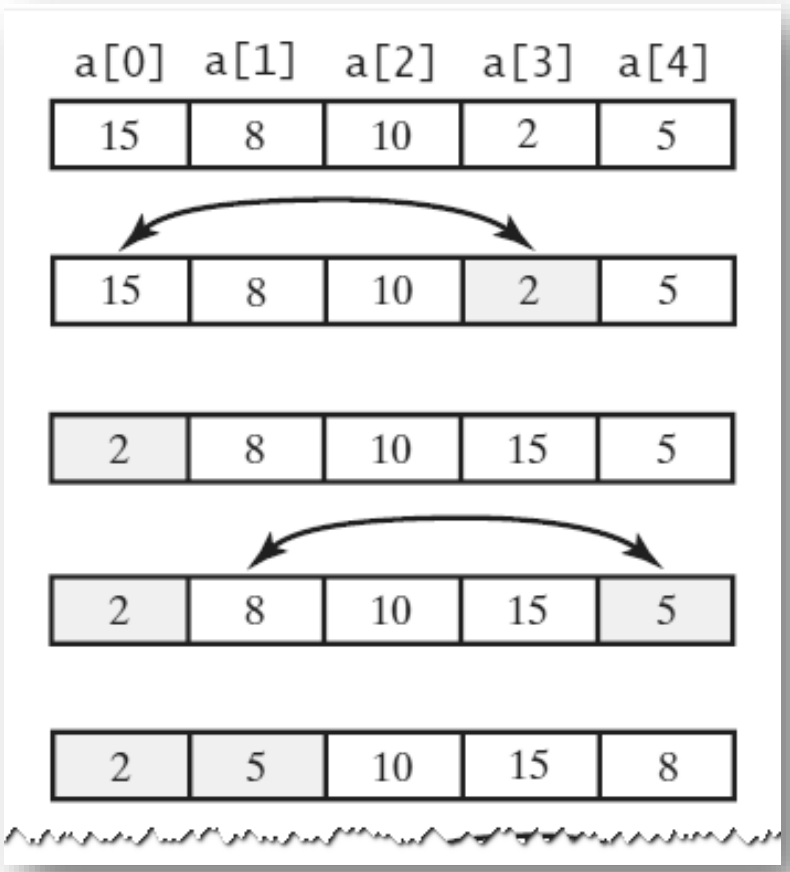
Selection Sort

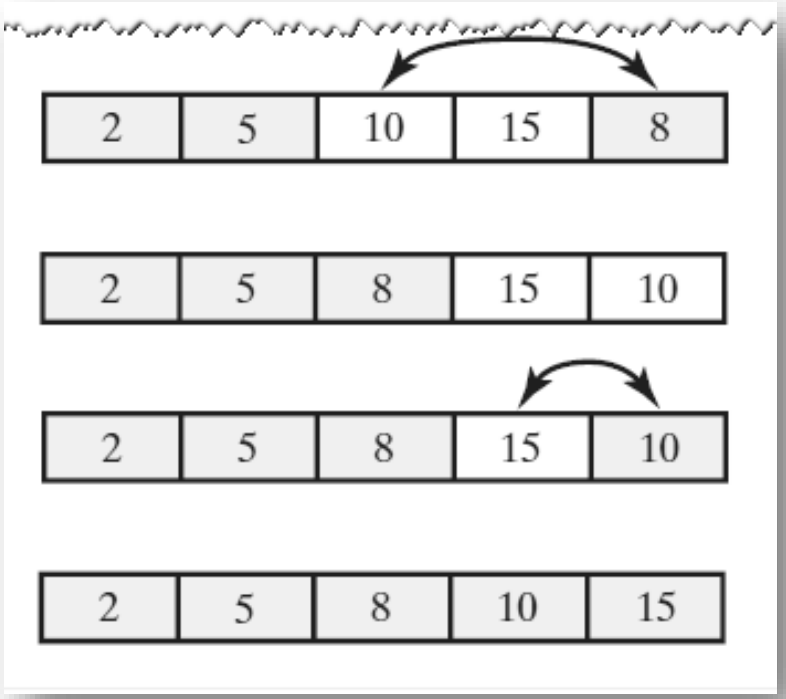
- Before and after exchanging the shortest book and the first book



Selection Sort

- A selection sort of an array of integers into ascending order





Iterative Selection Sort

- This pseudocode describes an iterative algorithm for the selection sort

```
Algorithm selectionSort(a, n)
// Sorts the first n entries of an array a.

for (index = 0; index < n - 1; index++)
{
   indexOfNextSmallest = the index of the smallest value among
                         a[index], a[index + 1], . . . , a[n - 1]
   Interchange the values of a[index] and a[indexOfNextSmallest]
   // Assertion: a[0] ≤ a[1] ≤ . . . ≤ a[index], and these are the smallest
   // of the original array entries. The remaining array entries begin at a[index + 1].
}
```

Iterative Selection Sort

- LISTING 8-1 A class for sorting an array using selection sort

```
1  /**
2     Class for sorting an array of Comparable objects from smallest to largest.
3  */
4  public class SortArray
5  {
6     /** Sorts the first n objects in an array into ascending order.
7         @param a  An array of Comparable objects.
8         @param n  An integer > 0. */
9     public static <T extends Comparable<? super T>>
10           void selectionSort(T[] a, int n)
11    {
12       for (int index = 0; index < n - 1; index++)
13       {
14          int indexOfNextSmallest = getIndexOfSmallest(a, index, n - 1);
15          swap(a, index, indexOfNextSmallest);
16          // Assertion: a[0] <= a[1] <= . . . <= a[index] <= all other a[i].
17       } // end for
18    } // end selectionSort
```

Iterative Selection Sort

- LISTING 8-1 A class for sorting an array using selection sort

```
14          int indexOfNextSmallest = getIndexOfSmallest(a, index, n - 1);
15          swap(a, index, indexOfNextSmallest);
16          // Assertion: a[0] <= a[1] <= . . . <= a[index] <= all other a[i].
17       } // end for
18    } // end selectionSort
19
20    // Finds the index of the smallest value in a portion of an array a.
21    // Precondition: a.length > last >= first >= 0.
22    // Returns the index of the smallest value among
23    // a[first], a[first + 1], . . . , a[last].
24    private static <T extends Comparable<? super T>>
25            int getIndexOfSmallest(T[] a, int first, int last)
26    {
27       T min = a[first];
```

Iterative Selection Sort

- LISTING 8-1 A class for sorting an array using selection sort

```
28        int indexOfMin = first;
29        for (int index = first + 1; index <= last; index++)
30        {
31           if (a[index].compareTo(min) < 0)
32           {
33              min = a[index];
34              indexOfMin = index;
35           } // end if
36           // Assertion: min is the smallest of a[first] through a[index].
37        } // end for
38        return indexOfMin;
```

Iterative Selection Sort

- LISTING 8-1 A class for sorting an array using selection sort

```
36            // Assertion: min is the smallest of a[first] through a[index].
37         } // end for
38         return indexOfMin;
39
40      } // end getIndexOfSmallest
41      // Swaps the array entries a[i] and a[j].
42
43      private static void swap(Object[] a, int i, int j)
44      {
45         Object temp = a[i];
46         a[i] = a[j];
47         a[j] = temp;
48      } // end swap
49   } // end SortArray
```

Recursive Selection Sort

- Recursive selection sort algorithm

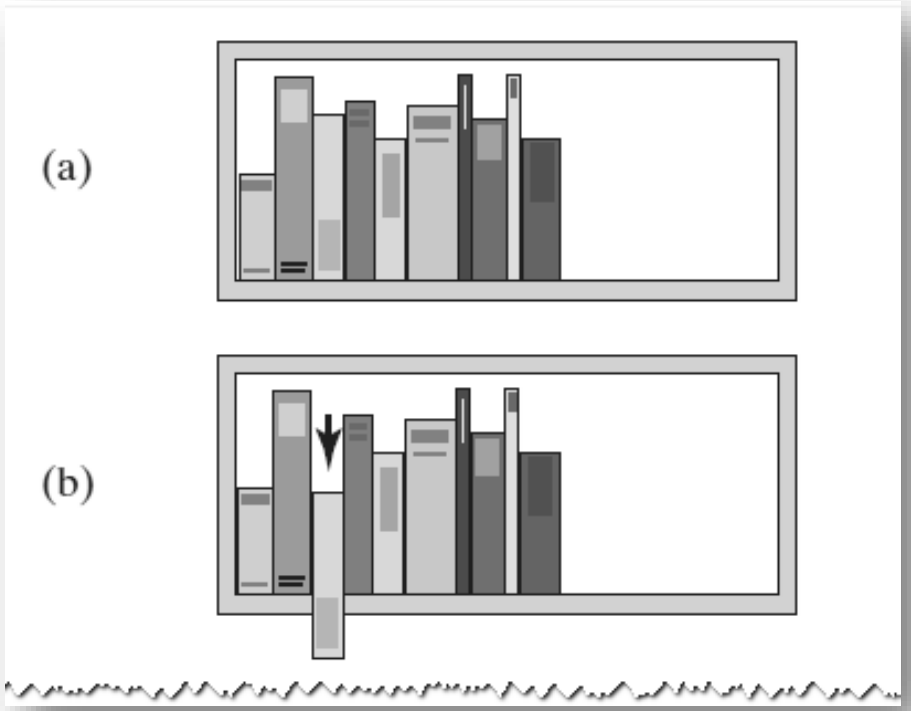
```
Algorithm selectionSort(a, first, last)
// Sorts the array entries a[first] through a[last] recursively.

if (first < last)
{
   indexOfNextSmallest = the index of the smallest value among
                         a[first], a[first + 1], . . . , a[last]
   Interchange the values of a[first] and a[indexOfNextSmallest]
   // Assertion: a[0] ≤ a[1] ≤ . . . ≤ a[first] and these are the smallest
   // of the original array entries. The remaining array entries begin at a[first + 1].
   selectionSort(a, first + 1, last)
}
```

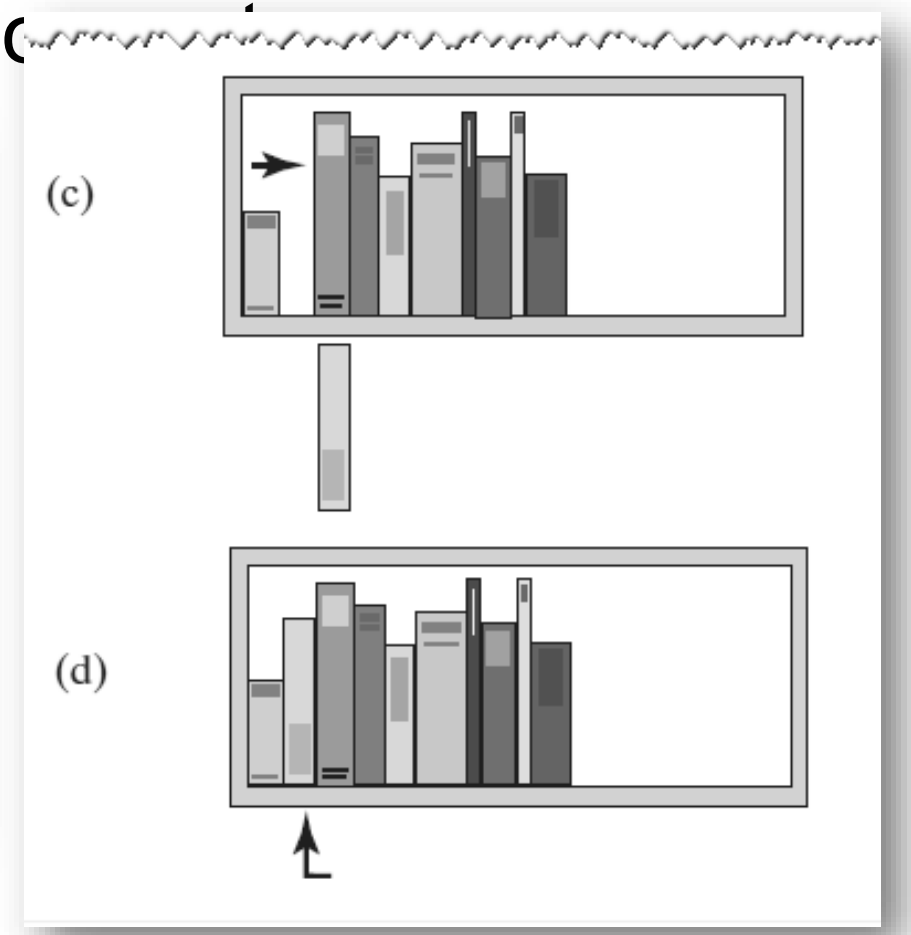
Efficiency of Selection Sort

- Selection sort is $O(n^2)$ regardless of the initial order of the entries.
  - Requires $O(n^2)$ comparisons
  - Does only $O(n)$ swaps

Insertion Sort

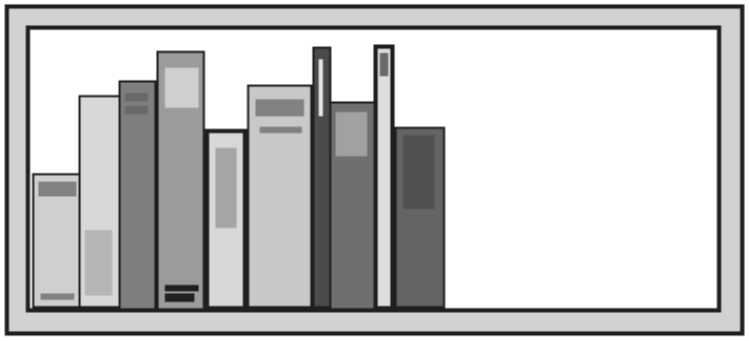
- FIGURE 8-3 The placement of the third book during an insertio





Insertion Sort

- FIGURE 8-4 An insertion sort of books



1. Remove the next unsorted book.
2. Slide the sorted books to the right one by one until you find the right spot for the removed book.
3. Insert the book into its new position.

Iterative Insertion Sort

- Iterative algorithm describes an insertion sort of the entries at indices **`first`** through **`last`** of the array **`a`**

```
Algorithm insertionSort(a, first, last)
// Sorts the array entries a[first] through a[last] iteratively.

for (unsorted = first + 1 through last)
{
   nextToInsert = a[unsorted]
   insertInOrder(nextToInsert, a, first, unsorted - 1)
}
```

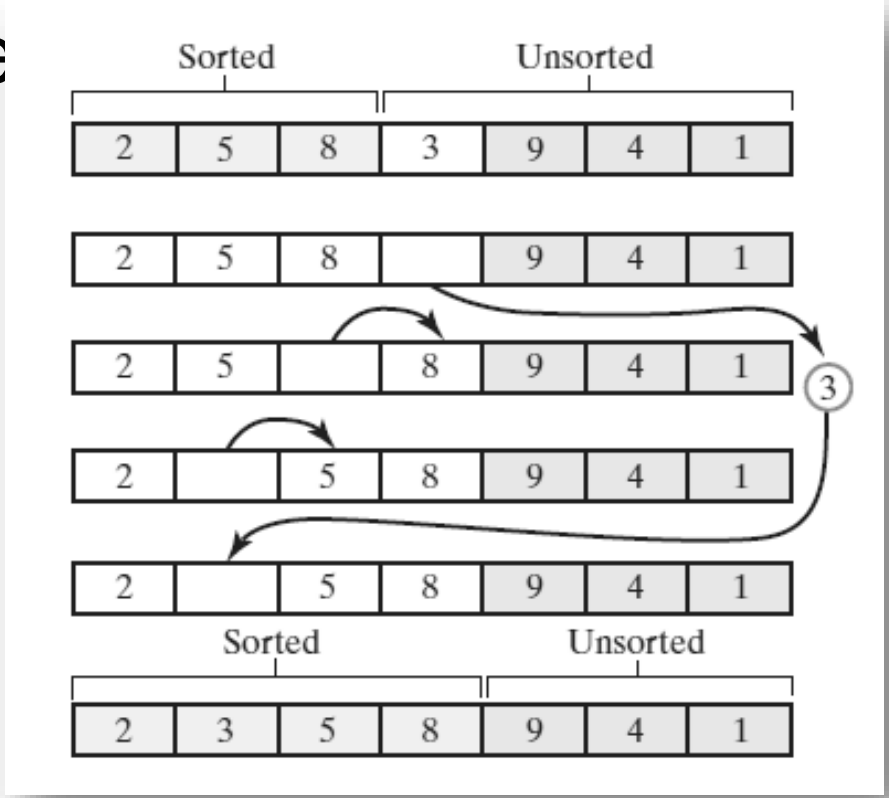
Iterative Insertion Sort

- Pseudocode of method, **`insertInOrder`**, to perform the insertions.

```
Algorithm insertInOrder(anEntry, a, begin, end)
// Inserts anEntry into the sorted entries a[begin] through a[end].

index = end                    // Index of last entry in the sorted portion
// Make room, if needed, in sorted portion for another entry
while ( (index >= begin) and (anEntry < a[index]) )
{
   a[index + 1] = a[index] // Make room
   index--
}
// Assertion: a[index + 1] is available.

a[index + 1] = anEntry      // Insert
```

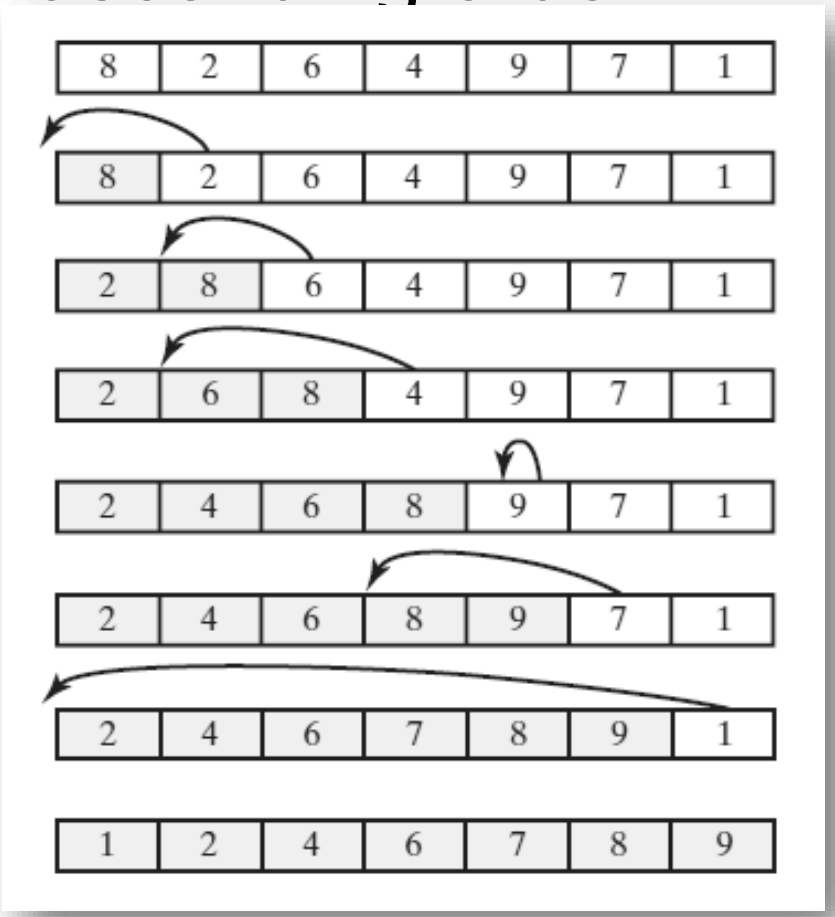
Iterative Insertion Sort

- FIGURE 8-5 Inserting the next unsorted entry into its proper location within the sorted portion of an array during an inse



Iterative Insertion Sort

- FIGURE 8-6 An insertion sort of an array of integers into ascending order



Recursive Insertion Sort

- This pseudocode describes a recursive insertion sort.

```
Algorithm insertionSort(a, first, last)
// Sorts the array entries a[first] through a[last] recursively.

if (the array contains more than one entry)
{
    Sort the array entries a[first] through a[last - 1]
    Insert the last entry a[last] into its correct sorted position within the rest of the array
}
```

Recursive Insertion Sort

- Implementing the algorithm in Java

```
public static <T extends Comparable<? super T>>
       void insertionSort(T[] a, int first, int last)
{
   if (first < last)
   {
      // Sort all but the last entry
      insertionSort(a, first, last - 1);

      // Insert the last entry in sorted order
      insertInOrder(a[last], a, first, last - 1);
   } // end if
} // end insertionSort
```

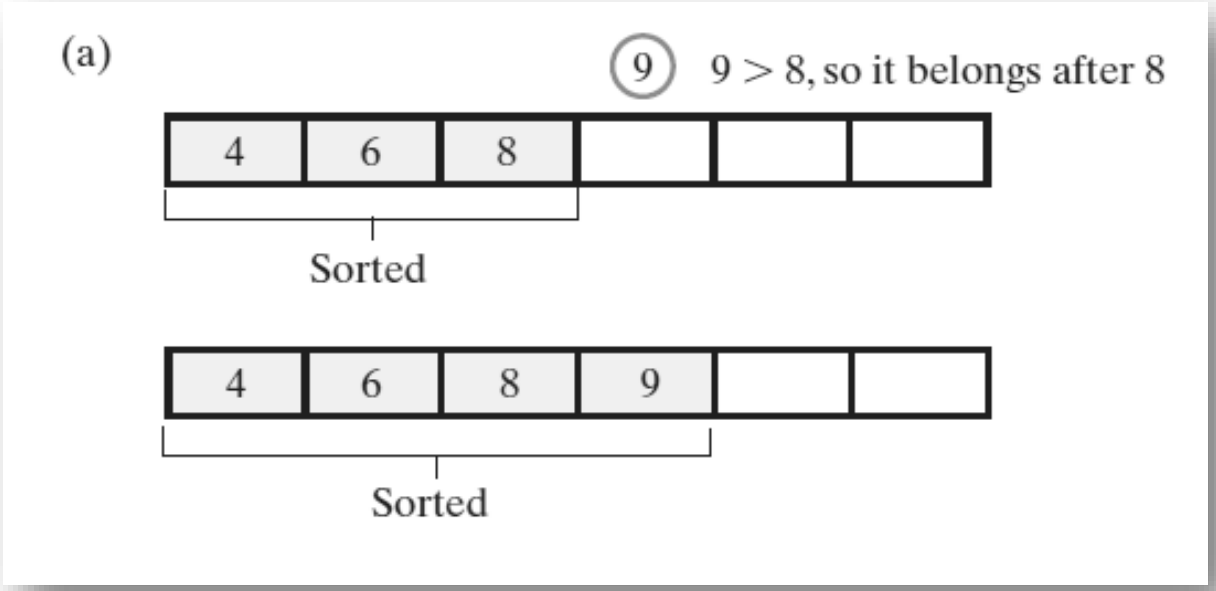
Recursive Insertion Sort

- First draft of **`insertInOrder`** algorithm.

```
Algorithm insertInOrder(anEntry, a, begin, end)
// Inserts anEntry into the sorted array entries a[begin] through a[end].
// First draft.

if (anEntry >= a[end])
   a[end + 1] = anEntry
else
{
   a[end + 1] = a[end]
   insertInOrder(anEntry, a, begin, end - 1)
}
```

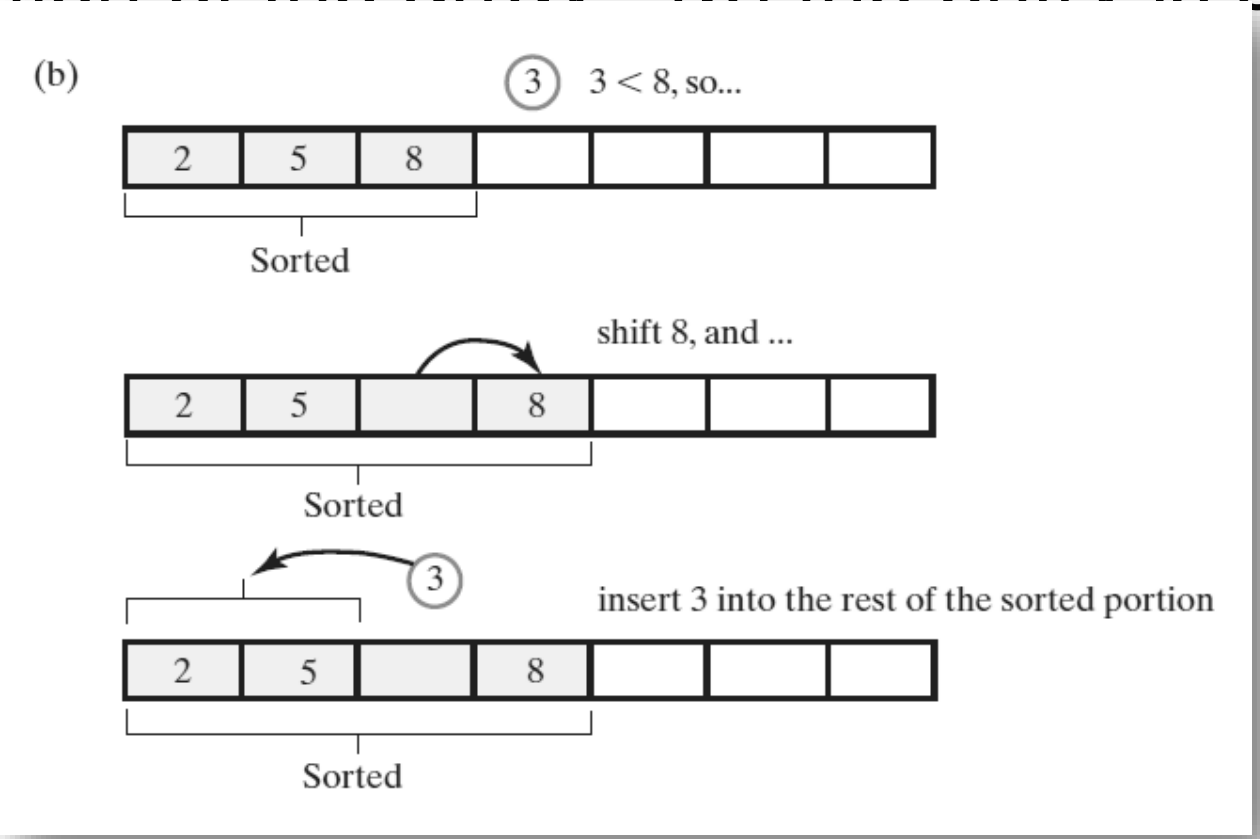
Recursive Insertion Sort

- FIGURE 8-8 Inserting the first unsorted entry into the sorted portion of the array. (a) The entry is greater than or equal to the last sorted entry



Recursive Insertion Sort

- FIGURE 8-8 Inserting the first unsorted entry into the sorted portion of the array (b) the entry is smaller than the la



Recursive Insertion Sort

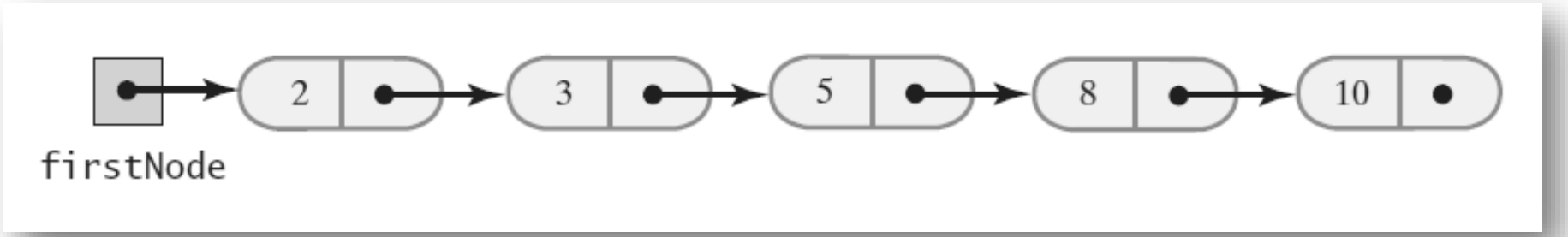
- The algorithm **`insertInOrder`**: final draft.

Note: insertion sort efficiency (worst case) is $O(n^2)$

```
Algorithm insertInOrder(anEntry, a, begin, end)
// Inserts anEntry into the sorted array entries a[begin] through a[end].
// Revised draft.

if (anEntry >= a[end])
   a[end + 1] = anEntry

   else if (begin < end)
   {
      a[end + 1] = a[end]
      insertInOrder(anEntry, a, begin, end - 1)
   }
   else // begin == end and anEntry < a[end]
   {
      a[end + 1] = a[end]
      a[end] = anEntry
   }
```

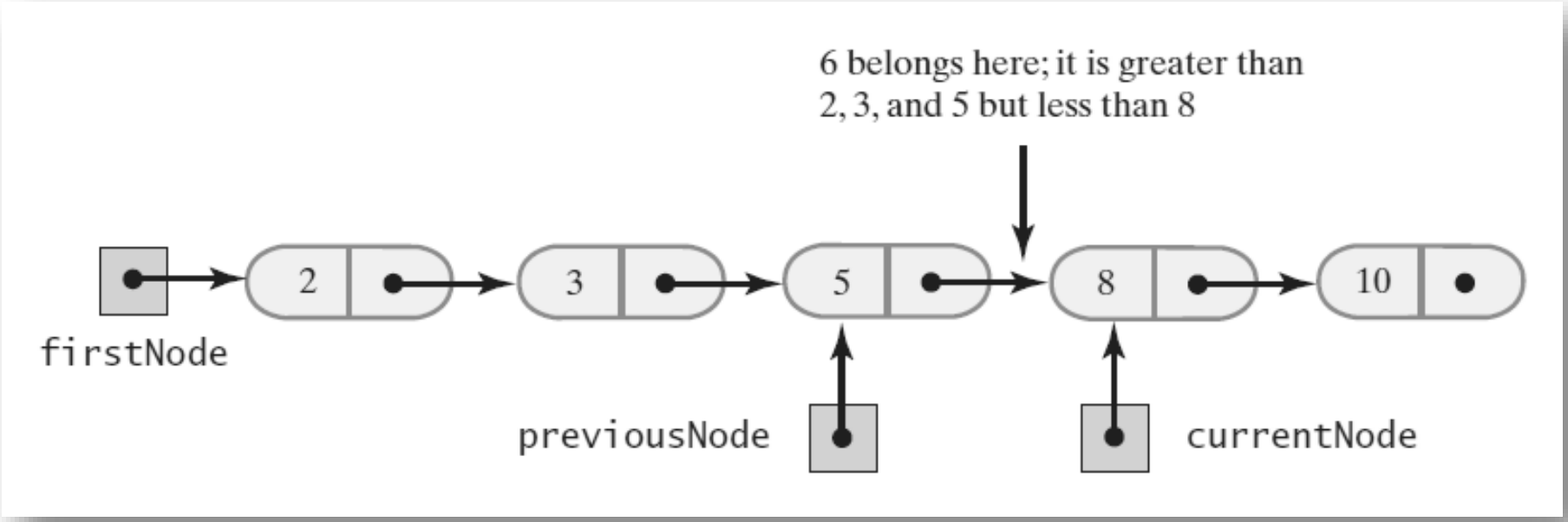
Insertion Sort of a Chain of Linked Nodes

- FIGURE 8-8 A chain of integers sorted into ascending order



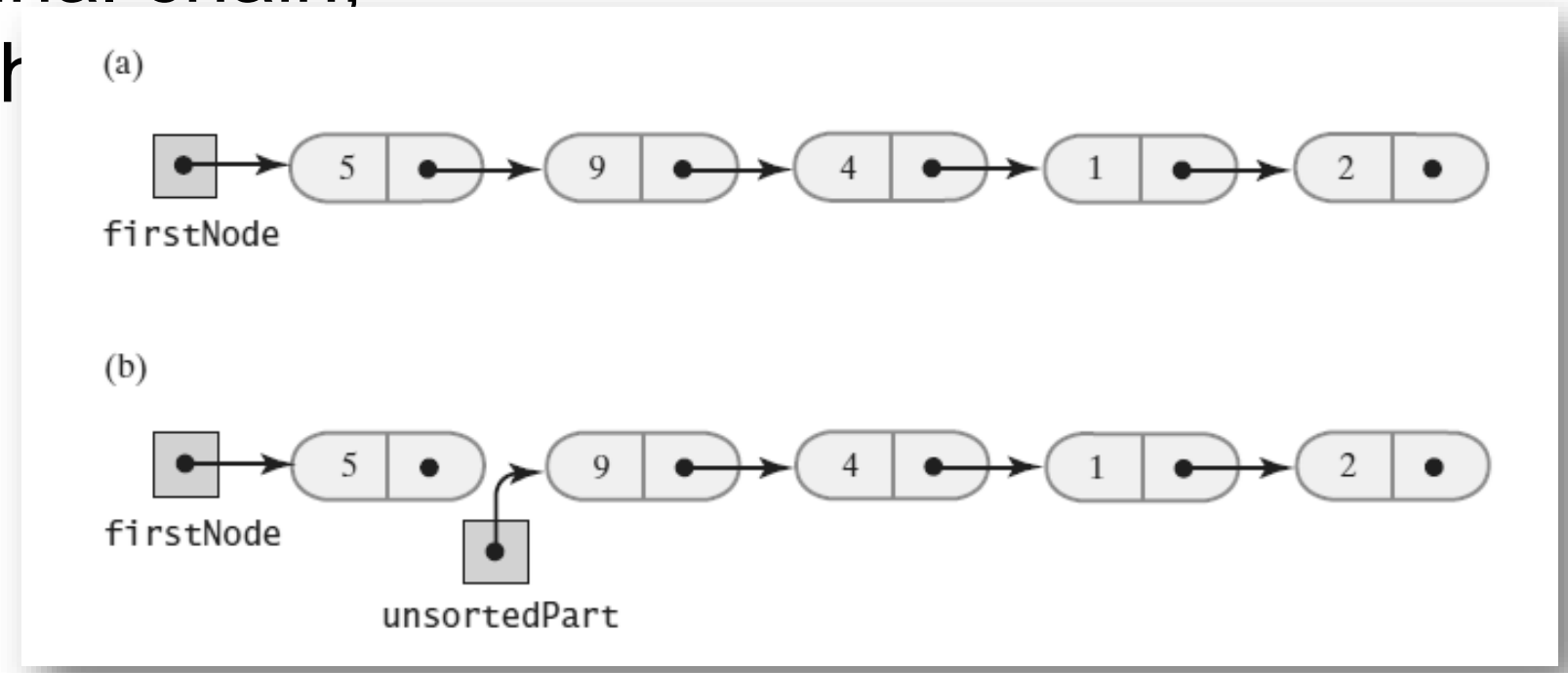
Insertion Sort of a Chain of Linked Nodes

- FIGURE 8-9 During the traversal of a chain to locate the insertion point, save a reference to the node before the current one



Insertion Sort of a Chain of Linked Nodes

- FIGURE 8-10 Breaking a chain of nodes into two pieces as the first step in an insertion sort: (a) the original chain;
(b) th



Insertion Sort of a Chain of Linked Nodes

- Add a sort method to a class **`LinkedGroup`**
- that uses a linked chain to represent a certain collection

```
public class LinkedGroup<T extends Comparable<? super T>>
{
   private Node firstNode;
   int length; // Number of objects in the group
   . . .
```

Insertion Sort of a Chain of Linked Nodes

- This class has an inner class **`Node`** that has set and get methods

```
private void insertInOrder(Node nodeToInsert)
{
   T item = nodeToInsert.getData();
   Node currentNode = firstNode;
   Node previousNode = null;

   // Locate insertion point
   while ( (currentNode != null) &&
           (item.compareTo(currentNode.getData()) > 0) )
   {
      previousNode = currentNode;
      currentNode = currentNode.getNextNode();
   } // end while

   // Make the insertion
```

Insertion Sort of a Chain of Linked Nodes

- This class has an inner class **`Node`** that has set and get methods

```
    } // end while

    // Make the insertion
    if (previousNode != null)
    { // Insert between previousNode and currentNode
       previousNode.setNextNode(nodeToInsert);
       nodeToInsert.setNextNode(currentNode);
    }
    else // Insert at beginning
    {
       nodeToInsert.setNextNode(firstNode);
       firstNode = nodeToInsert;
    } // end if
} // end insertInOrder
```

Insertion Sort of a Chain of Linked Nodes

- The method to perform the insertion sort.

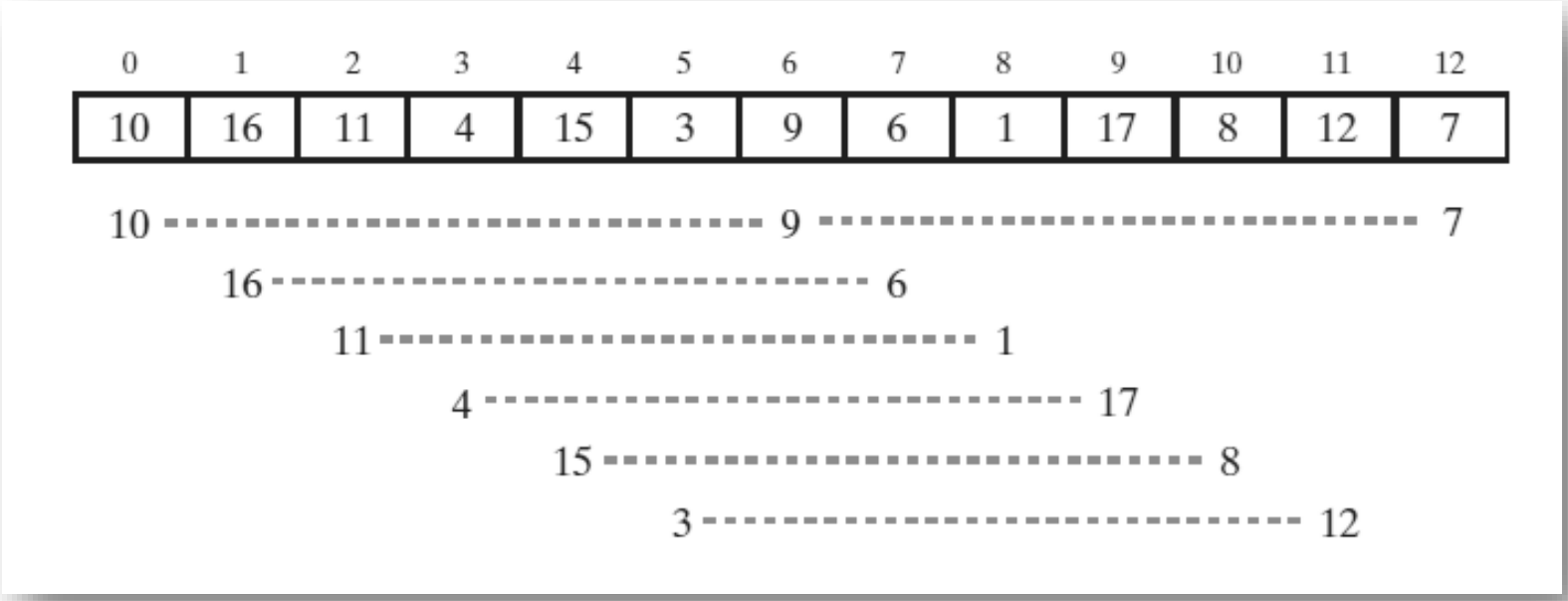
```
public void insertionSort()
{
   // If zero or one item is in the chain, there is nothing to do
   if (length > 1)
   {
       assert firstNode != null;
       // Break chain into 2 pieces: sorted and unsorted
       Node unsortedPart = firstNode.getNextNode();
       assert unsortedPart != null;
       firstNode.setNextNode(null);

       while (unsortedPart != null)
       {
          Node nodeToInsert = unsortedPart;
          unsortedPart = unsortedPart.getNextNode();
          insertInOrder(nodeToInsert);
       } // end while
    } // end if
} // end insertionSort
```

Shell Sort

- Algorithms seen so far are simple but inefficient for large arrays at $O(n^2)$
- Note, the more sorted an array is, the less work `insertInOrder` must do
- Improved insertion sort developed by Donald Shell

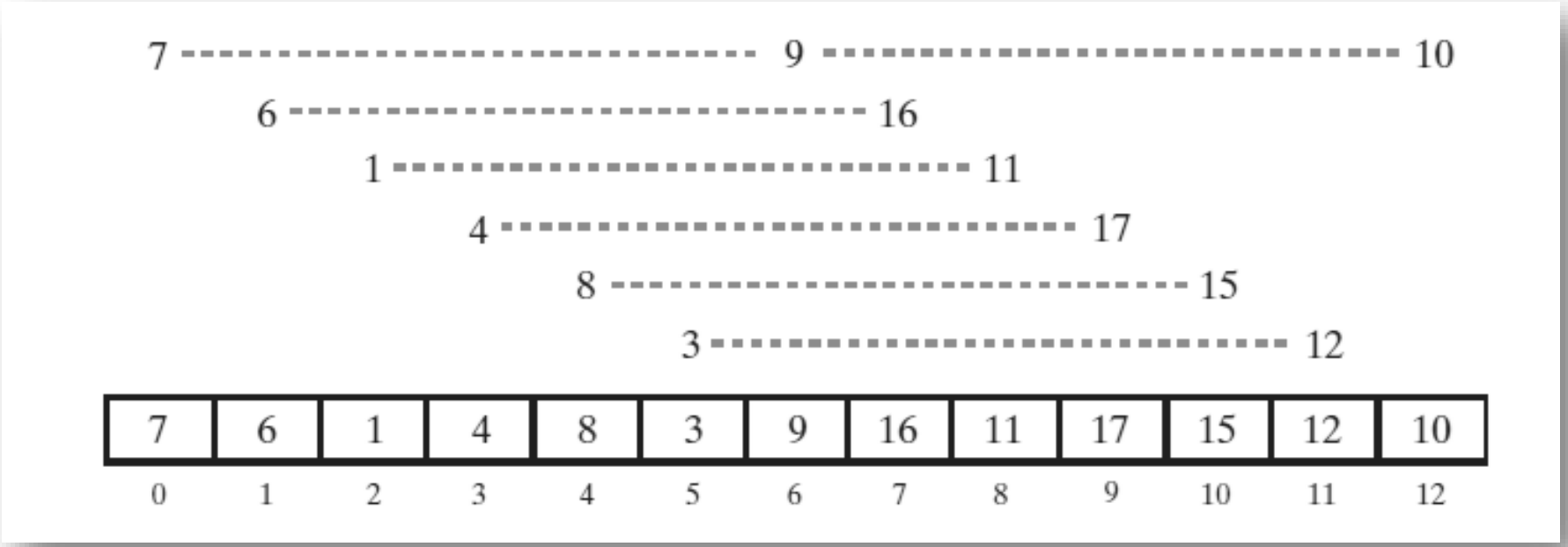
Shell Sort

- FIGURE 8-11 An array and the subarrays formed by grouping entries whose indices are 6 apart.



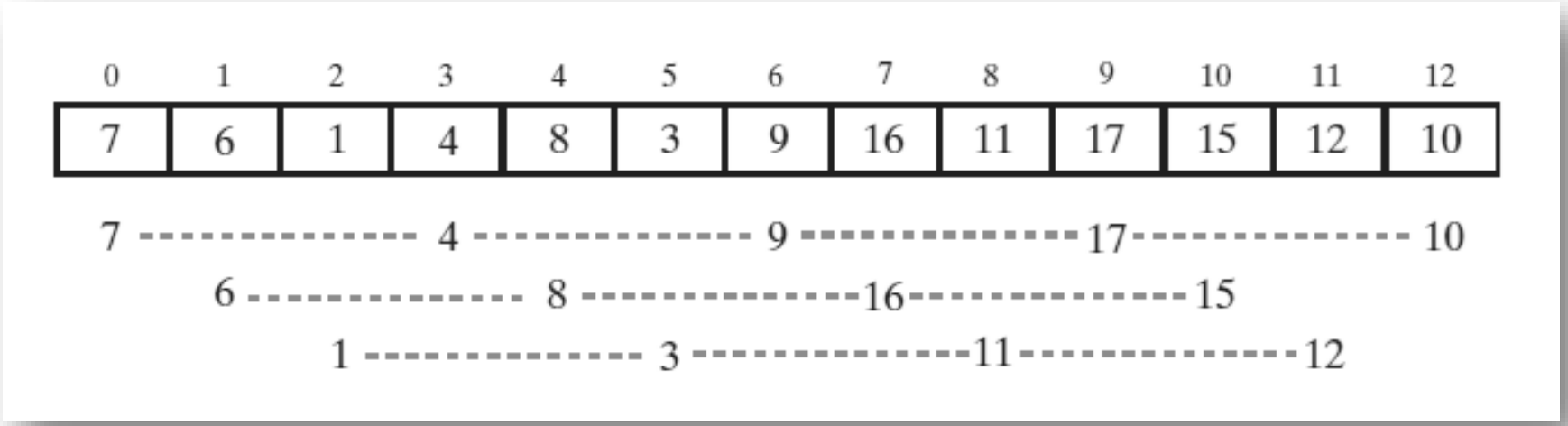
Shell Sort

- FIGURE 8-12 The subarrays of Figure 8-11 after each is sorted, and the array that contains them



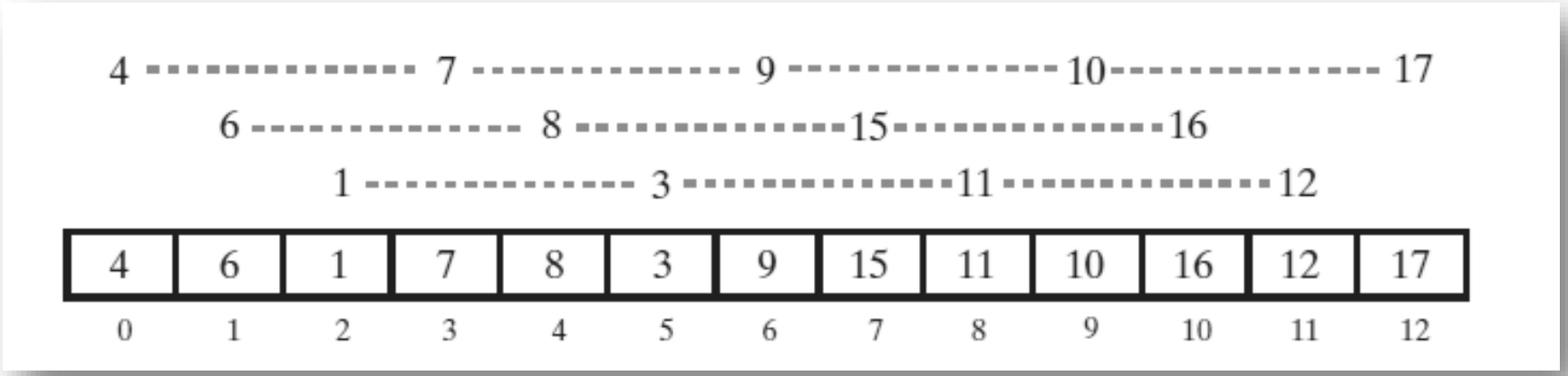
Shell Sort

- FIGURE 8-13 The subarrays of the array in Figure 8-12 formed by grouping entries whose indices are 3 apart



Shell Sort

- FIGURE 8-14 The subarrays of Figure 8-13 after each is sorted, and the array that contains them



Shell Sort

- Algorithm that sorts array entries whose indices are separated by an increment of **space**.

```
Algorithm incrementalInsertionSort(a, first, last, space)
// Sorts equally spaced entries of an array a[first..last] into ascending order.
// first >= 0 and < a.length; last >= first and < a.length;
// space is the difference between the indices of the entries to sort.

   for (unsorted = first + space through last at increments of space)
   {
      nextToInsert = a[unsorted]
      index = unsorted - space
      while ( (index >= first) and (nextToInsert.compareTo(a[index]) < 0) )
      {
         a[index + space] = a[index]
         index = index - space
      }
      a[index + space] = nextToInsert
   }
```

Shell Sort

- Algorithm to perform a Shell sort will invoke **`incrementalInsertionSort`** and supply any sequenc t) can be O(n[1]

```
Algorithm shellSort(a, first, last)
// Sorts the entries of an array a[first..last] into ascending order.
// first >= 0 and < a.length; last >= first and < a.length.

   n = number of array entries
   space = n / 2
   while (space > 0)
   {
      for (begin = first through first + space - 1)
      {
         incrementalInsertionSort(a, begin, last, space)
      }
      space = space / 2
   }
```

Comparing the Algorithms

- FIGURE 8-16 The time efficiencies of three sorting algorithms, expressed in Big Oh notation

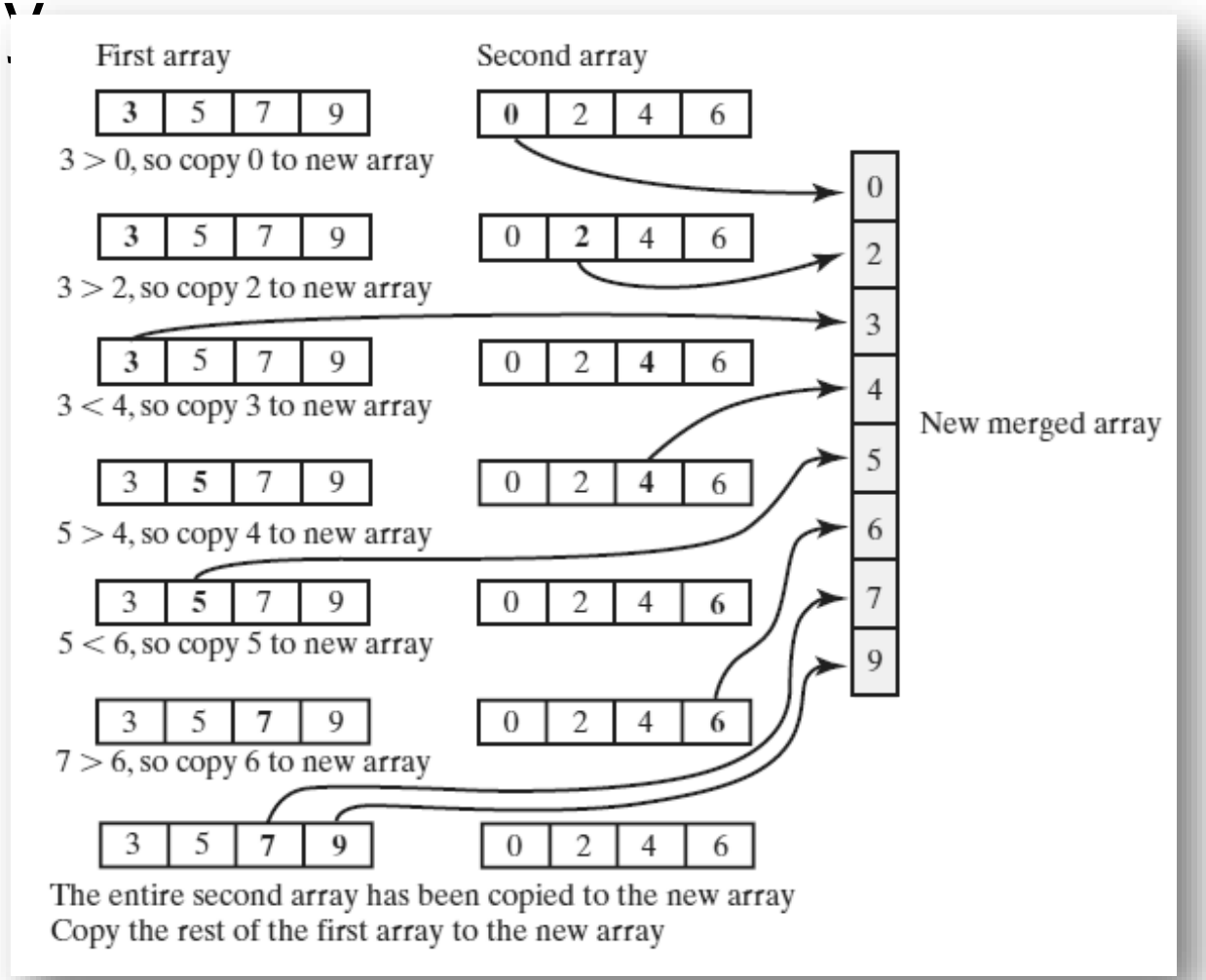
	Best Case	Average Case	Worst Case
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$
Shell sort	$O(n)$	$O(n^{1.5})$	$O(n^2)$ or $O(n^{1.5})$

Merge Sort

- Divides an array into halves
- Sorts the two halves,
  - Then merges them into one sorted array.
- The algorithm for merge sort is usually stated recursively.
- Major programming effort is in the merge process

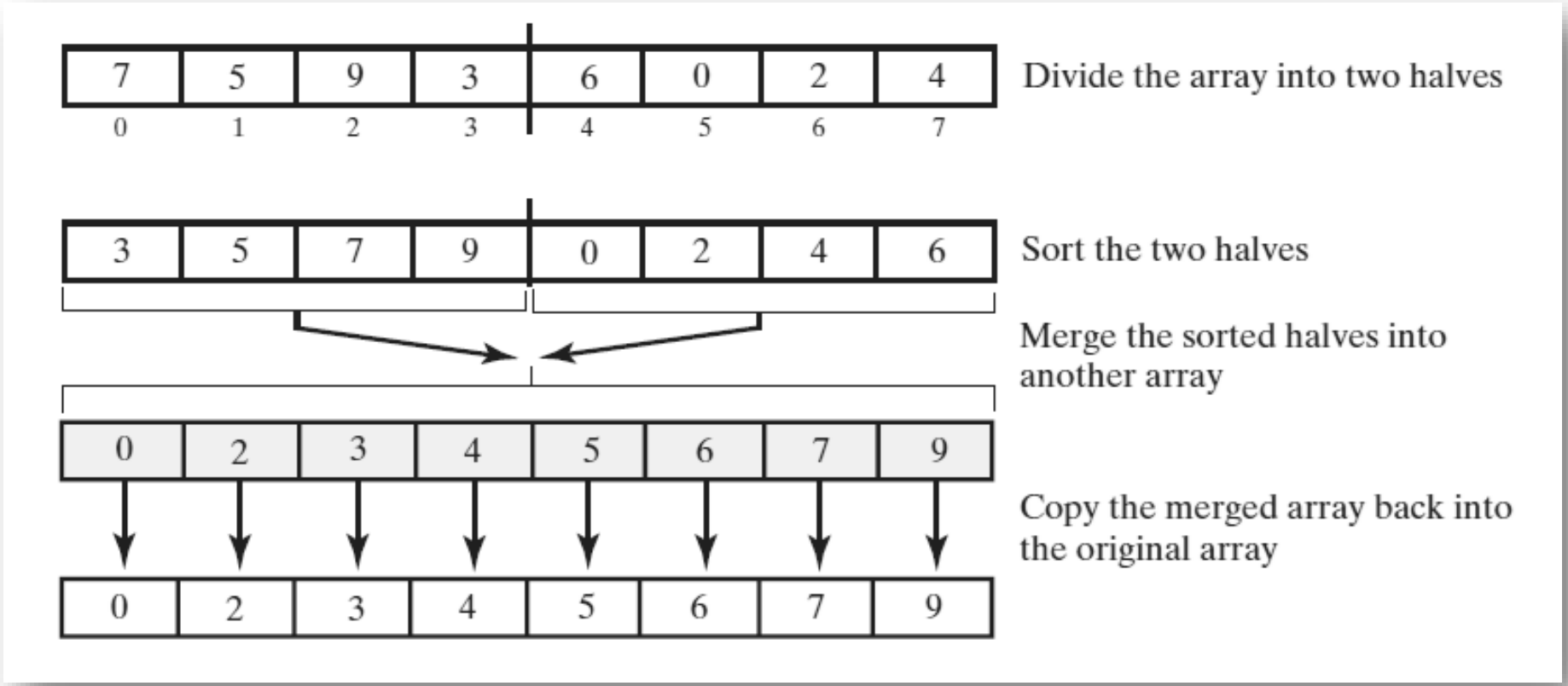
Merging Arrays

- FIGURE 9-1 Merging two sorted arrays into one sorted array



Recursive Merge Sort

- FIGURE 9-2 The major steps in a merge sort



Recursive Merge Sort

- Recursive algorithm for merge sort.

```
Algorithm mergeSort(a, tempArray, first, last)
// Sorts the array entries a[first] through a[last] recursively.
if (first < last)
{
   mid = approximate midpoint between first and last
   mergeSort(a, tempArray, first, mid)
   mergeSort(a, tempArray, mid + 1,last)
   Merge the sorted halves a[first..mid] and a[mid + 1..last] using the array tempArray
}
```

Merge Sort

- Divides an array into halves
- Sorts the two halves,
  - Then merges them into one sorted array.
- The algorithm for merge sort is usually stated recursively.
- Major programming effort is in the merge process

Merging Arrays

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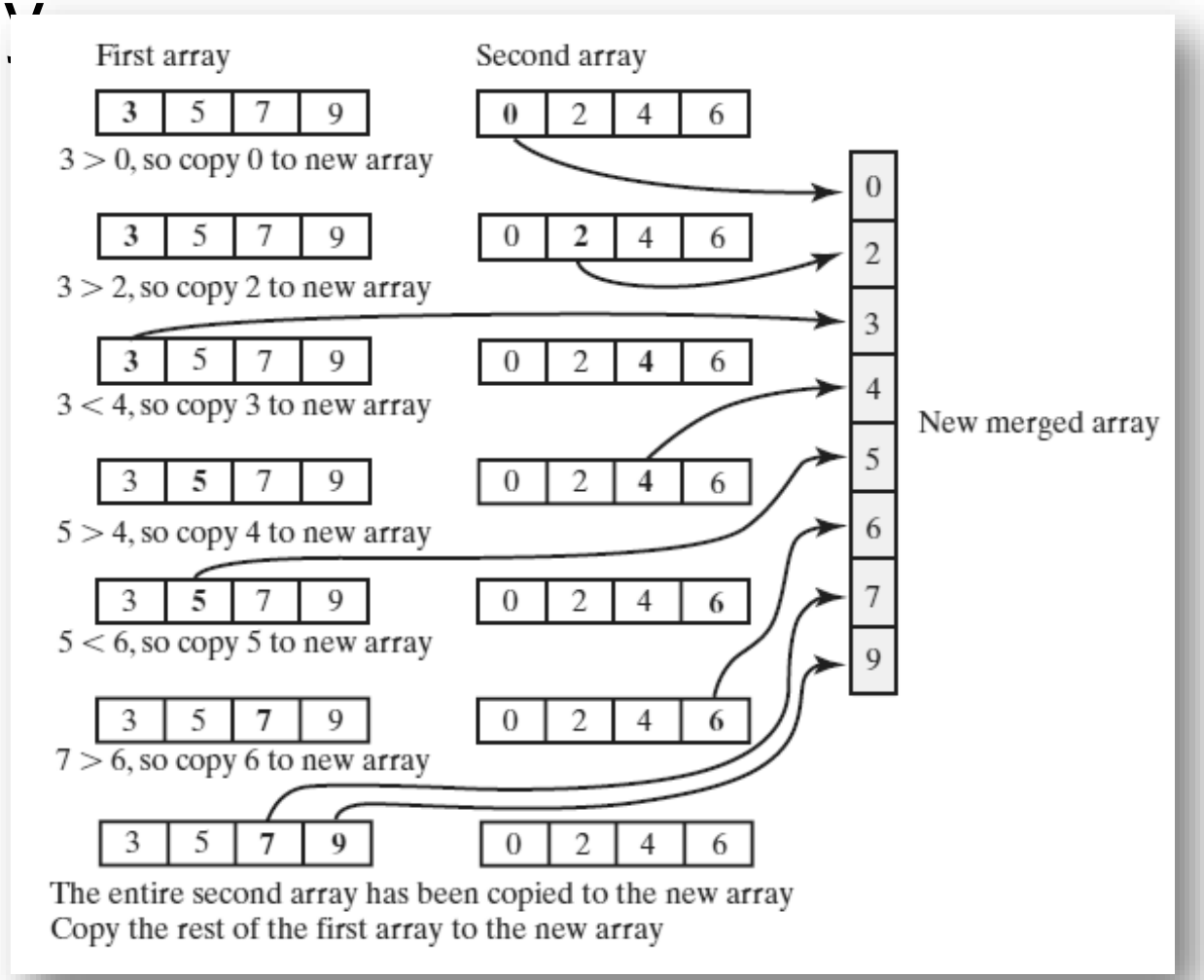
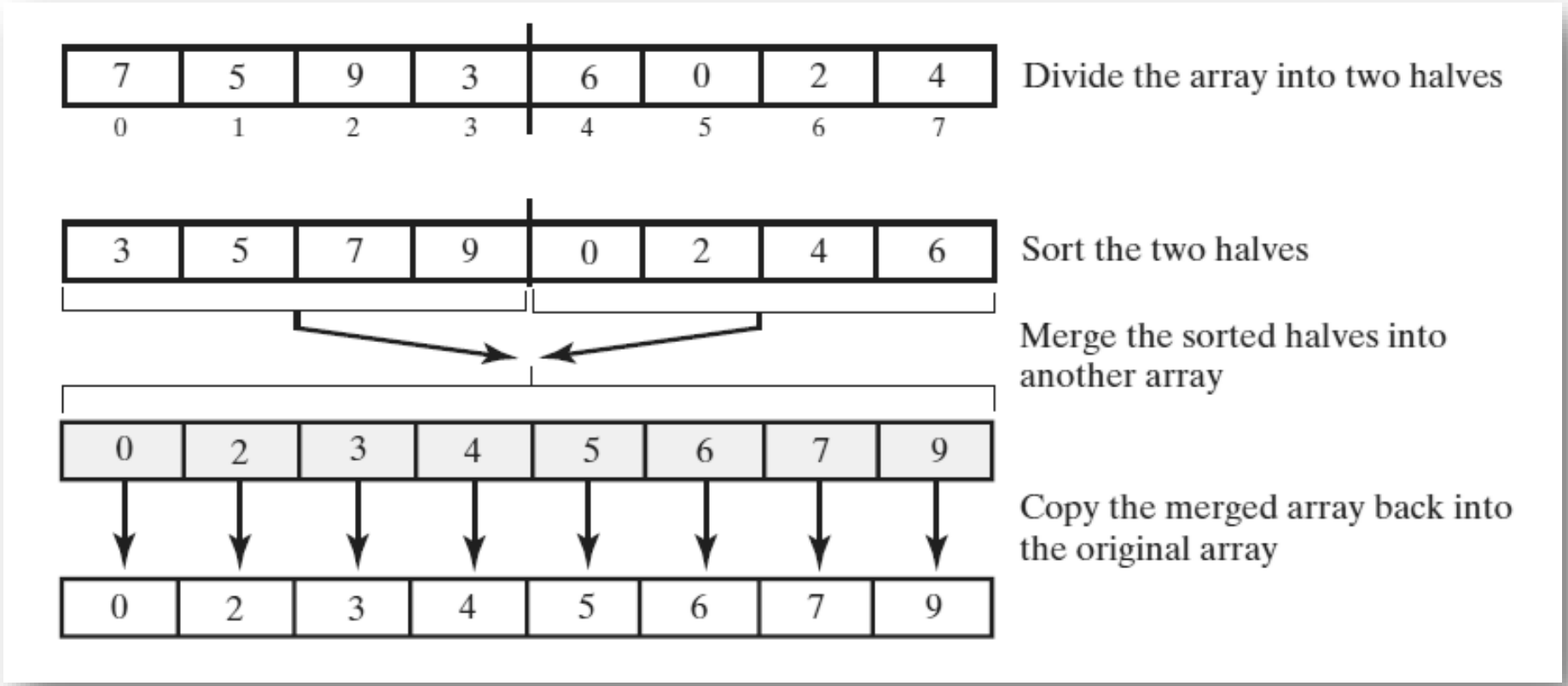


Recursive Merge Sort

- FIGURE 9-2 The major steps in a merge sort



Recursive Merge Sort

- Recursive algorithm for merge sort.

```
Algorithm mergeSort(a, tempArray, first, last)
// Sorts the array entries a[first] through a[last] recursively.
if (first < last)
{
   mid = approximate midpoint between first and last
   mergeSort(a, tempArray, first, mid)
   mergeSort(a, tempArray, mid + 1,last)
   Merge the sorted halves a[first..mid] and a[mid + 1..last] using the array tempArray
}
```

Recursive Merge Sort

- Pseudocode which describes the merge step.

```
Algorithm merge(a, tempArray, first, mid, last)
// Merges the adjacent subarrays a[first..mid] and a[mid + 1..last].
beginHalf1 = first
endHalf1 = mid
beginHalf2 = mid + 1
endHalf2 = last
// While both subarrays are not empty, compare an entry in one subarray with
// an entry in the other; then copy the smaller item into the temporary array
index = 0 // Next available location in tempArray
while ( (beginHalf1 <= endHalf1) and (beginHalf2 <= endHalf2) )
{
   if (a[beginHalf1] <= a[beginHalf2])
   {
      tempArray[index] = a[beginHalf1]
      beginHalf1++
   }
```

Recursive Merge Sort

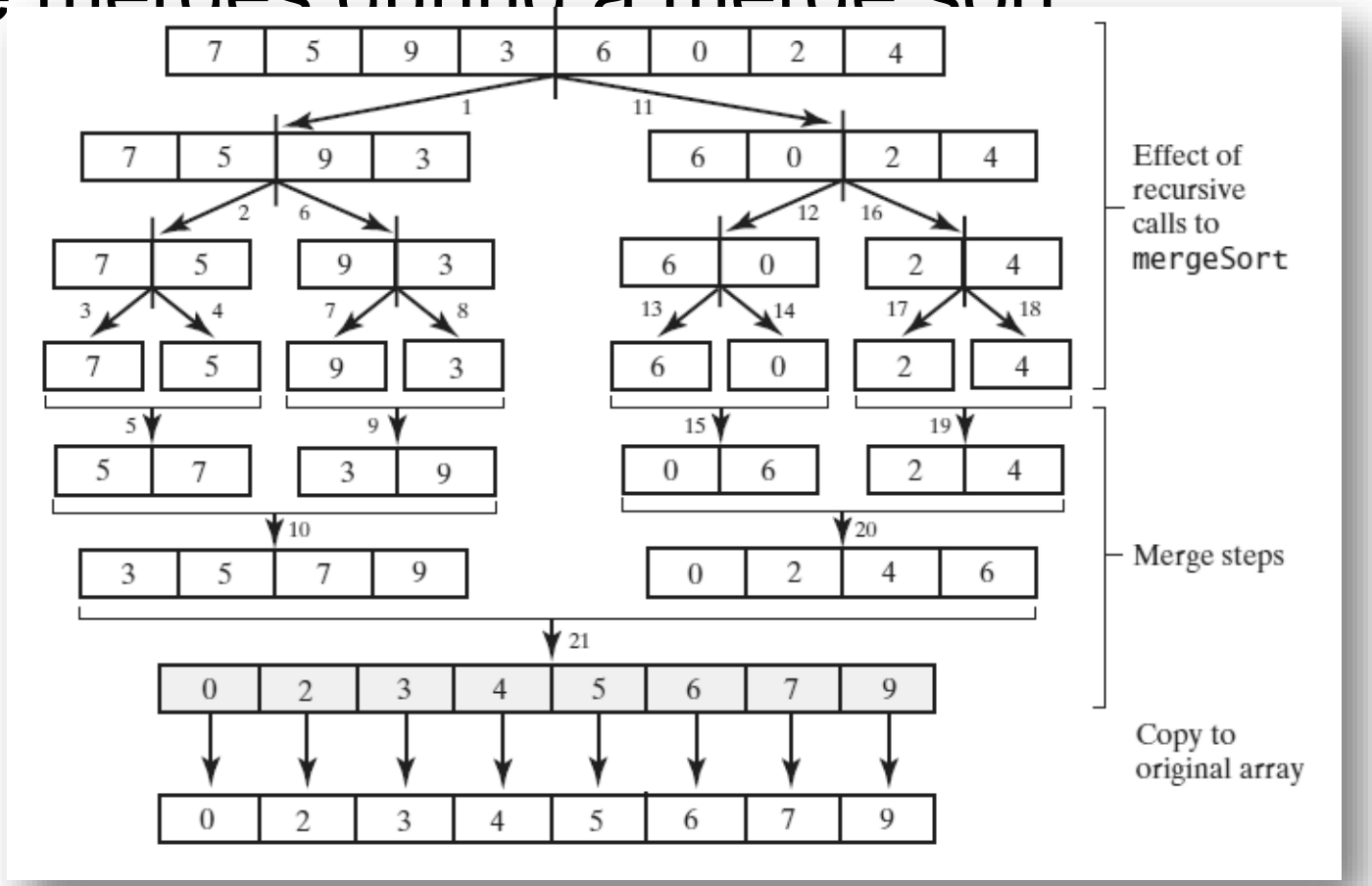
- Pseudocode which describes the merge step.

```
        tempArray[index] = a[beginHalf1]
        beginHalf1++
    }
    else
    {
        tempArray[index] = a[beginHalf2]
        beginHalf2++
    }
    index++
}
// Assertion: One subarray has been completely copied to tempArray.

Copy remaining entries from other subarray to tempArray
Copy entries from tempArray to array a
```

Recursive Merge Sort

- FIGURE 9-3 The effect of the recursive calls and the merges during a merge sort



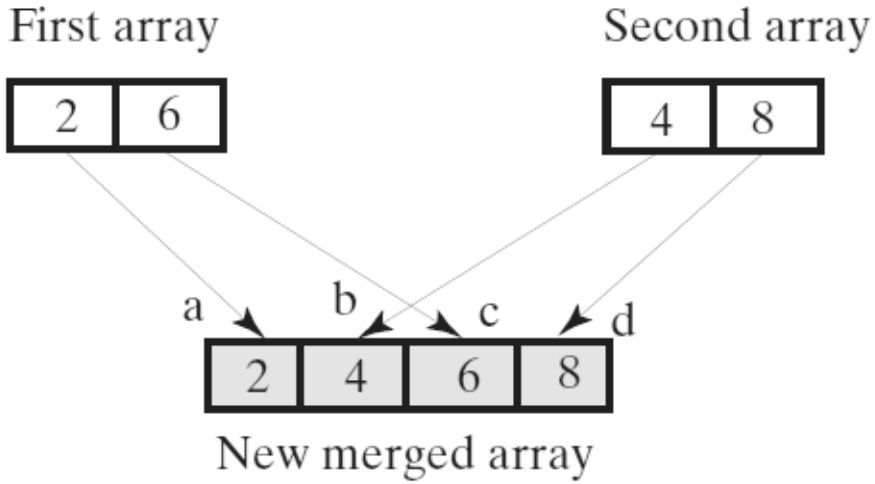
Recursive Merge Sort

- Be careful to allocate the temporary array only once.

```
public static <T extends Comparable<? super T>>
       void mergeSort(T[] a, int first, int last)
{
   // The cast is safe because the new array contains null entries
   @SuppressWarnings("unchecked")
   T[] tempArray = (T[])new Comparable<?>[a.length]; // Unchecked cast
   mergeSort(a, tempArray, first, last);
} // end mergeSort
```

Efficiency of Merge Sort

- FIGURE 9-4 A worst-case merge of two sorted arrays.
- Efficiency is $O(n \log n)$.



a. $2 < 4$, so copy 2 to new array

b. $6 > 4$, so copy 4 to new array

c. $6 < 8$, so copy 6 to new array

d. Copy 8 to new array

Iterative Merge Sort

- Less simple than recursive version.
  - Need to control the merges.
- Will be more efficient of both time and space.
  - But, trickier to code without error.

Iterative Merge Sort

- Starts at beginning of array
  - Merges pairs of individual entries to form two-entry subarrays
- Returns to the beginning of array and merges pairs of the two-entry subarrays to form four-entry subarrays
  - And so on
- After merging all pairs of subarrays of a particular length, might have entries left over.

Merge Sort in the Java Class Library

- Class **`Arrays`** in the package **`java.util`** defines versions of a static method **`sort`**

```
public static void sort(Object[] a)

public static void sort(Object[] a, int first, int after)
```

Quick Sort

- Divides an array into two pieces
  - Pieces are not necessarily halves of the array
  - Chooses one entry in the array—called the pivot
- Partitions the array

Quick Sort

- When pivot chosen, array rearranged such that:
  - Pivot is in position that it will occupy in final sorted array
  - Entries in positions before pivot are less than or equal to pivot
  - Entries in positions after pivot are greater than or equal to pivot

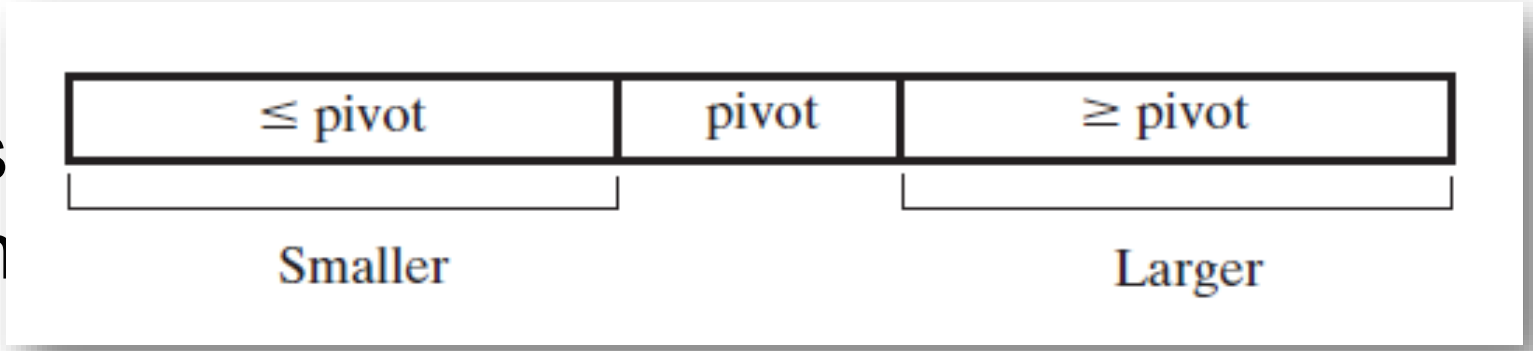
Quick Sort

- Algorithm that describes our sorting strategy:

```
Algorithm quickSort(a, first, last)
// Sorts the array entries a[first] through a[last] recursively.
if (first < last)
{
    Choose a pivot
    Partition the array about the pivot
    pivotIndex = index of pivot
    quickSort(a, first, pivotIndex - 1) // Sort Smaller
    quickSort(a, pivotIndex + 1, last)  // Sort Larger
}
```

Quick Sort

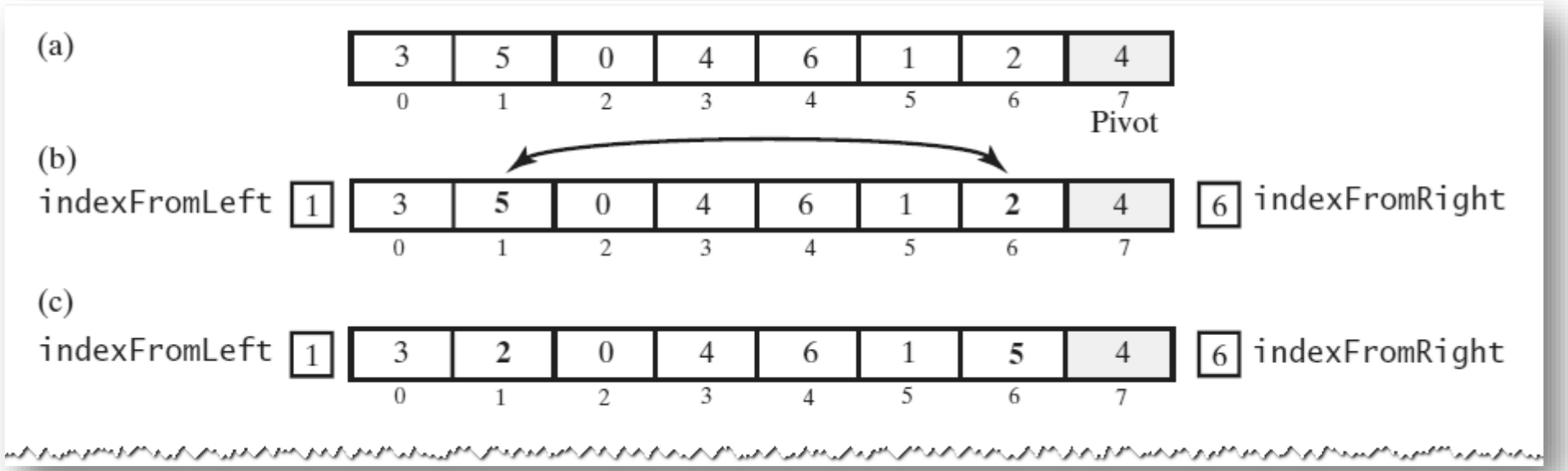
- FIGURE 9-5 A partition of an array during a quick sort



- Quick s
  $O(n^2)$ in
- Choice of pivots affects behavior

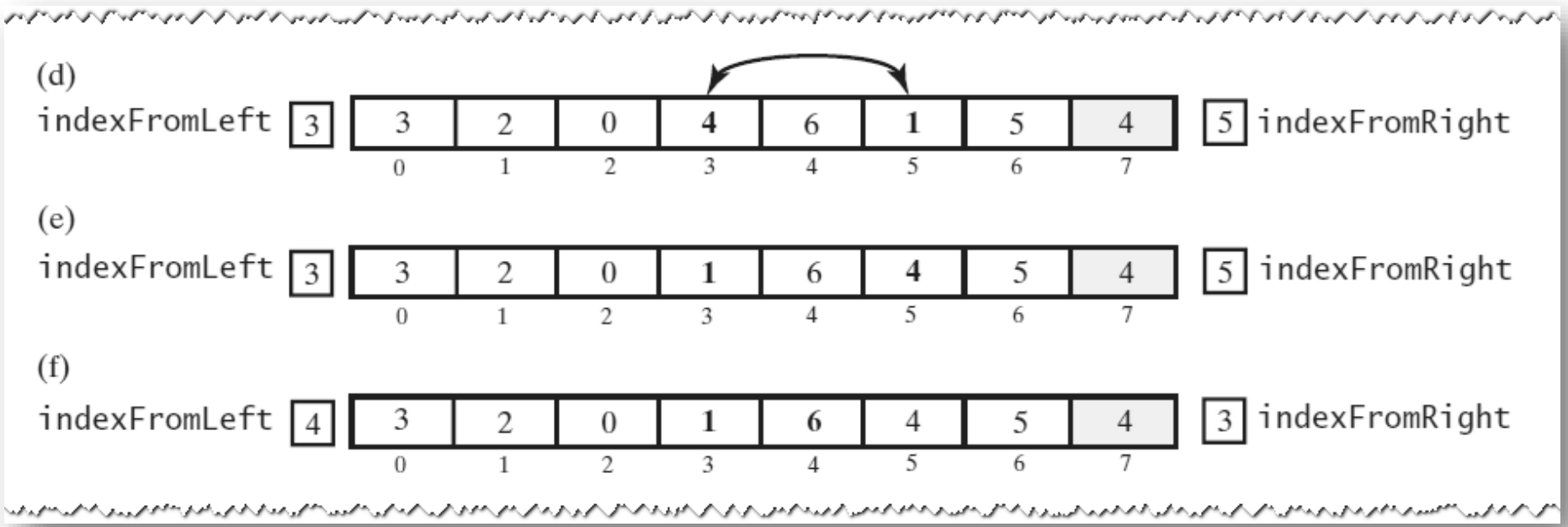
Creating the Partition

- FIGURE 9-6 A partitioning strategy for quick sort



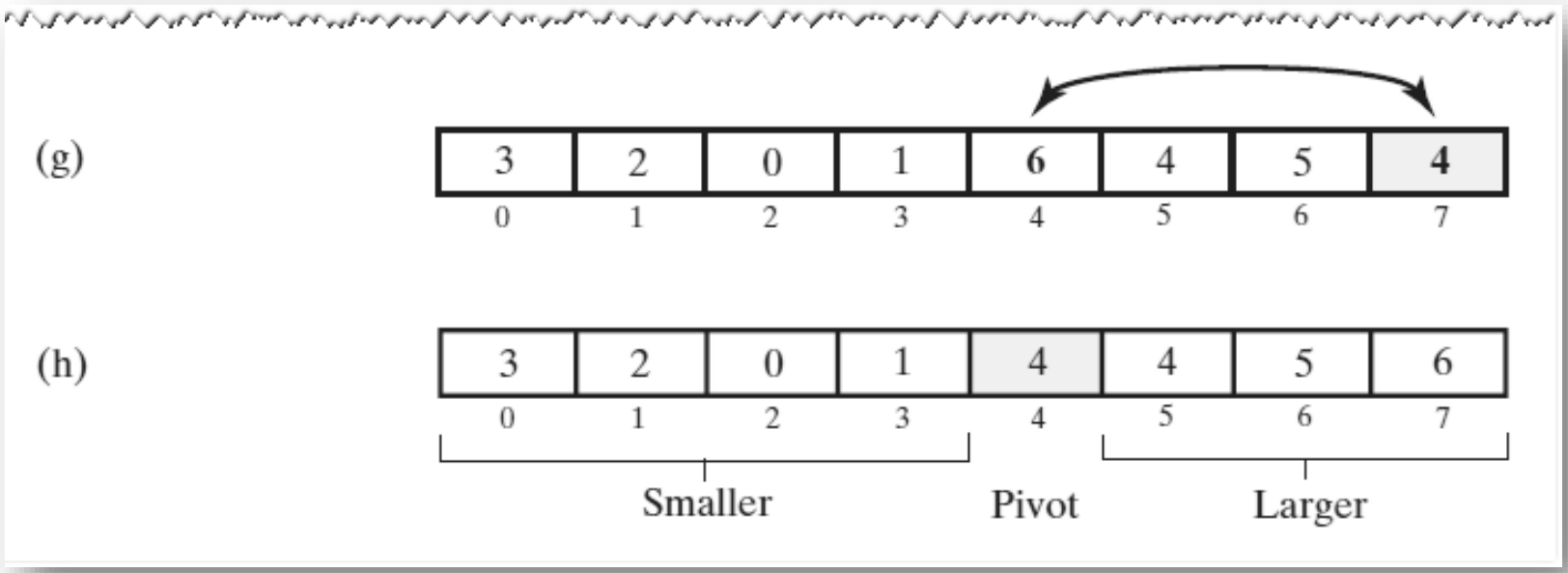
Creating the Partition

- FIGURE 9-6 A partitioning strategy for quick sort



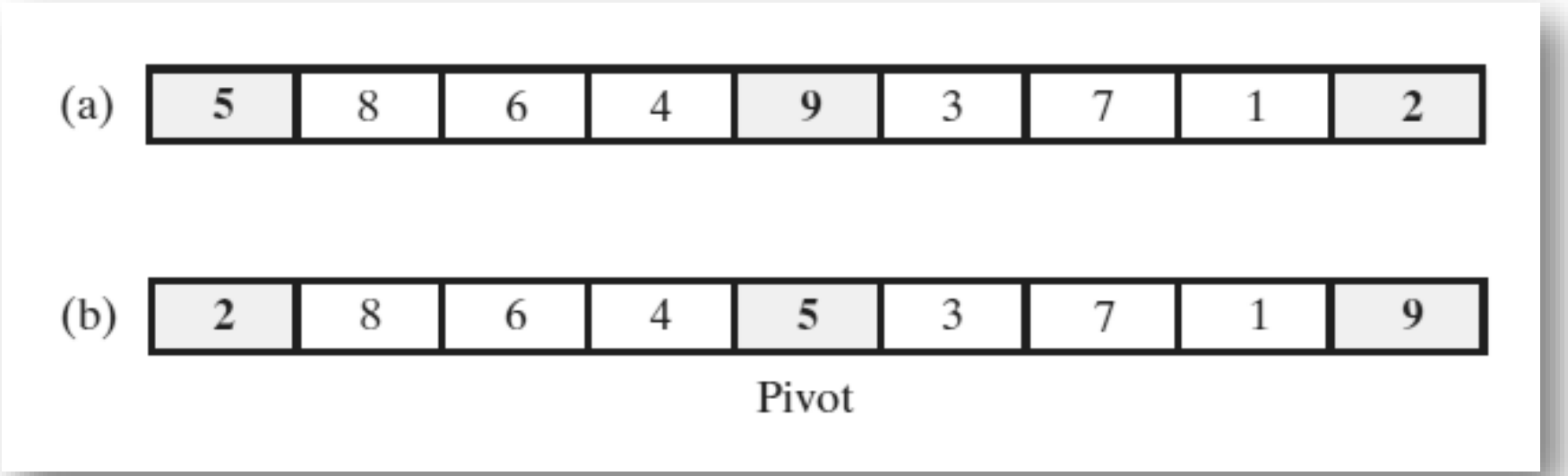
Creating the Partition

- FIGURE 9-6 A partitioning strategy for quick sort



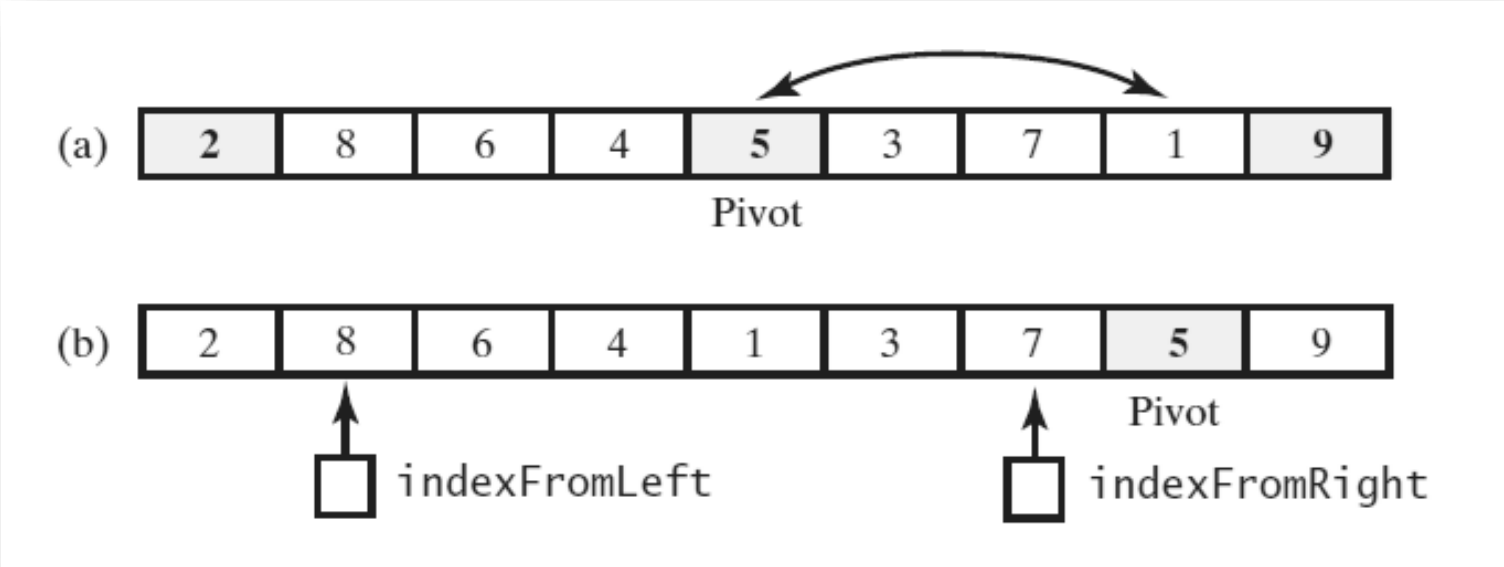
Creating the Partition

- FIGURE 9-7 Median-of-three pivot selection: (a) The original array; (b) the array with its first, middle, and last entries sorted



Adjusting the Partition Algorithm

- FIGURE 9-8 (a) The array with its first, middle, and last entries sorted; (b) the array after positioning the pivot and just before partitioning



Adjusting the Partition Algorithm

```
Algorithm partition(a, first, last)
// Partitions an array a[first..last] as part of quick sort into two subarrays named
// Smaller and Larger that are separated by a single entry—the pivot— named pivotValue.
// Entries in Smaller are <= pivotValue and appear before pivotValue in the array.
// Entries in Larger are >= pivotValue and appear after pivotValue in the array.
// first >= 0; first < a.length; last - first >= 3; last < a.length.
// Returns the index of the pivot.

   mid = index of the array's middle entry
   sortFirstMiddleLast(a, first, mid, last)
   // Assertion: a[mid] is the pivot, that is, pivotValue;
   // a[first] <= pivotValue and a[last] >= pivotValue, so do not compare these two
   // array entries with pivotValue.

   // Move pivotValue to next-to-last position in array
```

Adjusting the Partition Algorithm

```
// Move pivotValue to next-to-last position in array
Exchange a[mid] and a[last - 1]
pivotIndex = last - 1
pivotValue = a[pivotIndex]

// Determine two subarrays:
//    Smaller = a[first..endSmaller] and
//    Larger  = a[endSmaller+1..last-1]
// such that entries in Smaller are <= pivotValue and
// entries in Larger are >= pivotValue.
// Initially, these subarrays are empty.
indexFromLeft = first + 1
indexFromRight = last - 2
done = false
while (!done)
```

Adjusting the Partition Algorithm

```
while (!done)
{
   // Starting at the beginning of the array, leave entries that are < pivotValue and
   // locate the first entry that is >= pivotValue. You will find one, since the last
   // entry is >= pivotValue.
   while (a[indexFromLeft] < pivotValue)
      indexFromLeft++

   // Starting at the end of the array, leave entries that are > pivotValue and
   // locate the first entry that is <= pivotValue. You will find one, since the first
   // entry is <= pivotValue.
   while (a[indexFromRight] > pivotValue)
      indexFromRight--

   // Assertion: a[indexFromLeft] >= pivotValue and
   //            a[indexFromRight] <= pivotValue
   if (indexFromLeft < indexFromRight)
```

Adjusting the Partition Algorithm

```
    //          a[indexFromRight] <= pivotValue
    if (indexFromLeft < indexFromRight)
    {
       Exchange a[indexFromLeft] and a[indexFromRight]
       indexFromLeft++
       indexFromRight--

    }
    else
       done = true
 }

Exchange a[pivotIndex] and a[indexFromLeft]
pivotIndex = indexFromLeft

// Assertion: Smaller = a[first..pivotIndex-1]
//            pivotValue = a[pivotIndex]
//            Larger = a[pivotIndex+1..last]
return pivotIndex
```

The Quick Sort Method

- Above method implements quick sort.

```
/** Sorts an array into ascending order. Uses quick sort with
    median-of-three pivot selection for arrays of at least
    MIN_SIZE entries, and uses insertion sort for smaller arrays. */
public static <T extends Comparable<? super T>>
       void quickSort(T[] a, int first, int last)
{
   if (last - first + 1 < MIN_SIZE)
   {
      insertionSort(a, first, last);
   }
   else
   {
       // Create the partition: Smaller | Pivot | Larger
       int pivotIndex = partition(a, first, last);

       // Sort subarrays Smaller and Larger
       quickSort(a, first, pivotIndex - 1);
       quickSort(a, pivotIndex + 1, last);
   } // end if
} // end quickSort
```

QuickSort in the Java Class Library

- Class **`Arrays`** in the package **`java.util`** uses a quick sort to sort arrays of primitive types into ascending order

```
public static void sort(type[] a)

public static void sort(type[] a, int first, int after)
```