The Dynamics of Viral Marketing

JURE LESKOVEC
Carnegie Mellon University
LADA A. ADAMIC
University of Michigan
and
BERNARDO A. HUBERMAN
HP Labs

We present an analysis of a person-to-person recommendation network, consisting of 4 million people who made 16 million recommendations on half a million products. We observe the propagation of recommendations and the cascade sizes, which we explain by a simple stochastic model. We analyze how user behavior varies within user communities defined by a recommendation network. Product purchases follow a 'long tail' where a significant share of purchases belongs to rarely sold items. We establish how the recommendation network grows over time and how effective it is from the viewpoint of the sender and receiver of the recommendations. While on average recommendations are not very effective at inducing purchases and do not spread very far, we present a model that successfully identifies communities, product, and pricing categories for which viral marketing seems to be very effective.

Categories and Subject Descriptors: J.4 [**Social and Behavioral Sciences**]: *Economics*

General Terms: Economics

Additional Key Words and Phrases: Viral marketing, word-of-mouth, e-commerce, long tail, recommender systems, network analysis

ACM Reference Format:

Leskovec, J., Adamic, L. A., and Huberman, B. A. 2007. The dynamics of viral marketing. ACM Trans. Web, 1, 1, Article 5 (May 2007), 39 pages. DOI = 10.1145/1232722.1232727 http://doi.acm.org/ 10.1145/1232722.1232727

This work was partially supported by the National Science Foundation under grants SENSOR-0329549 IIS-0326322 IIS-0534205. This work is also supported in part by the Pennsylvania Infrastructure Technology Alliance (PITA). Additional funding was provided by a generous gift from Hewlett-Packard. Jure Leskovec was partially supported by a Microsoft Research Graduate Fellowship.

This is an extended version of the paper that appeared in Proceedings of the 7th ACM Conference on Electronic Commerce.

Author's address: J. Leskovec, School of Computer Science, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh Pa 15213; email: jure@cs.cmu.edu.

1. INTRODUCTION

With consumers showing increasing resistance to traditional forms of advertising such as TV or newspaper ads, marketers have turned to alternate strategies, including viral marketing. Viral marketing exploits existing social networks by encouraging customers to share product information with their friends. Previously, a few in-depth studies have shown that social networks affect the adoption of individual innovations and products (for a review see Rogers [1995] or Strang and Soule [1998]). But until recently, it has been difficult to measure how influential person-to-person recommendations actually are over a wide range of products. Moreover, Subramani and Rajagopalan [2003] noted that "there needs to be a greater understanding of the contexts in which viral marketing strategy works and the characteristics of products and services for which it is most effective. This is particularly important because the inappropriate use of viral marketing can be counterproductive by creating unfavorable attitudes towards products. What is missing is an analysis of viral marketing that highlights systematic patterns in the nature of knowledge-sharing and persuasion by influencers and responses by recipients in online social networks."

Here we were able to study in detail the mentioned problem. We were able to directly measure and model the effectiveness of recommendations by studying one online retailer's incentivized viral marketing program. The Web site gave discounts to customers recommending any of its products to others, and then tracked the resulting purchases and additional recommendations.

Although word-of-mouth can be a powerful factor influencing purchasing decisions, it can be tricky for advertisers to tap into. Some services used by individuals to communicate are natural candidates for viral marketing because the product can be observed or advertised as part of the communication. Email services such as Hotmail and Yahoo had very fast adoption curves because every email sent through them contained an advertisement for the service and because they were free. Hotmail spent a mere $50,000 on traditional marketing and still grew from zero to 12 million users in 18 months [Jurvetson 2000]. The Hotmail user base grew faster than any media company in history—faster than CNN, faster than AOL, even faster than Seinfeld's audience. By mid-2000, Hotmail had over 66 million users with 270,000 new accounts established each day [Bronson 1998]. Google's Gmail also captured a significant part of market share in spite of the fact that the only way to sign up for the service was through a referral.

Most products cannot be advertised in such a direct way. At the same time, the choice of products available to consumers has increased manyfold thanks to online retailers who can supply a much wider variety of products than traditional brick-and-mortar stores. Not only is the variety of products larger, but one observes a fat-tail phenomenon where a large fraction of purchases are of relatively obscure items. On Amazon.com, somewhere between 20 to 40 percent of unit sales fall outside of its top-100,000 ranked products [Brynjolfsson et al. 2003]. Rhapsody, a streaming-music service, streams more tracks outside than inside its top-10,000 tunes [Anonymous 2005]. Some argue that the presence of the long tail indicates that niche products with low sales are contributing significantly to overall sales online.

We find that product purchases that result from recommendations are not far from the usual 80-20 rule. The rule states that the top twenty percent of the products account for 80 percent of the sales. In our case, the top 20% of the products contribute to about half the sales.

Effectively advertising these niche products using traditional advertising approaches is impractical. Therefore using more targeted marketing approaches is advantageous both to the merchant and the consumer who would benefit from learning about new products.

The problem is partly addressed by the advent of online product and merchant reviews, both at retail sites such as EBay and Amazon, and specialized product comparison sites such as Epinions and CNET. Of further help to the consumer are collaborative filtering recommendations of the form "people who bought x also bought y" feature [Linden et al. 2003]. These refinements help consumers discover new products and receive more accurate evaluations, but they cannot completely substitute personalized recommendations that one receives from a friend or relative. It is human nature to be more interested in what a friend buys than what an anonymous person buys and to be more likely to trust their opinion and be more influenced by their actions. As one would expect, our friends are also acquainted with our needs and tastes and can make appropriate recommendations. A Lucid Marketing survey found that 68% of individuals consulted friends and relatives before purchasing home electronics, more than the half who used search engines to find product information [Burke 2003].

In our study we are able to directly observe the effectiveness of person-to-person word-of-mouth advertising for hundreds of thousands of products for the first time. We find that most recommendation chains do not grow very large, often terminating with the initial purchase of a product. However, occasionally a product will propagate through a very active recommendation network. We propose a simple stochastic model that seems to explain the propagation of recommendations.

Moreover, the characteristics of recommendation networks influence the purchase patterns of their members. For example, an individual's likelihood of purchasing a product initially increases as they receive additional recommendations for it, but a saturation point is quickly reached. Interestingly, as more recommendations are sent between the same two individuals, the likelihood that they will be heeded decreases.

We find that communities (automatically found by graph theoretic community finding algorithm) were usually centered around a product group such as books, music, or DVDs, but almost all of them shared recommendations for all types of products. We also find patterns of homophily, the tendency of like to associate with like, with communities of customers recommending types of products reflecting their common interests.

We propose models to identify products for which viral marketing is effective: We find that the category and price of a product plays a role, with recommendations for expensive products of interest to small, well-connected communities resulting in a purchase more often. We also observe patterns in the timing of recommendations and purchases corresponding to times of day when people are likely to be shopping online or reading email.

We report on these and other findings in the following sections. We first survey related work in Section 2. We then describe the characteristics of the incentivized recommendations program and the dataset in Section 3. Section 4 studies the temporal and static characteristics of the recommendation network. We investigate the propagation of recommendations and model the cascading behavior in Section 5. Next, we concentrate on the various aspects of the recommendation success from the viewpoint of the sender and the recipient of the recommendation in Section 6. The timing and the time lag between the recommendations and purchases is studied in Section 7. We study network communities, product characteristics, and purchasing behavior in Section 8. Last, in Section 9, we present a model that relates product characteristics and the surrounding recommendation network to predict the product recommendation success. We discuss the implications of our findings and conclude in Section 10.

2. RELATED WORK

Viral marketing can be thought of as a diffusion of information about the product and its adoption over the network. Primarily in social sciences there is a long history of the research on the influence of social networks on innovation and product diffusion. However, such studies have been usually limited to small networks and usually a single product or service. For example, Brown and Reingen [1987] interviewed the families of students being instructed by three piano teachers in order to find out the network of referrals. They found that strong ties, those between family or friends, were more likely to be activated for information flow and were also more influential than weak ties [Granovetter 1973] between acquaintances. Similar observations were also made by DeBruyn and Lilien in [2004] in the context of electronic referrals. They found that characteristics of the social tie influenced recipients behavior but had different effects at different stages of the decision-making process: tie strength facilitates awareness, perceptual affinity triggers recipients interest, and demographic similarity had a negative influence on each stage of the decision-making process.

Social networks can be composed by using various information, that is, geographic similarity, age, similar interests, and so on. Yang and Allenby [2003] showed that the geographically defined network of consumers is more useful than the demographic network for explaining consumer behavior in purchasing Japanese cars. A recent study by Hill et al. [2006] found that adding network information, specifically whether a potential customer was already talking to an existing customer, was predictive of the chances of adoption of a new phone service option. For the customers linked to a prior customer, the adoption rate was 3–5 times greater than the baseline.

Factors that influence customer willingness to actively share the information with others via word-of-mouth have also been studied. Frenzen and

Nakamoto [1993] surveyed a group of people and found that the stronger the moral hazard presented by the information, the stronger the ties must be to foster information propagation. Also, the network structure and information characteristics interact when individuals form decisions about transmitting information. Bowman and Narayandas [2001] found that self-reported loyal customers were more likely to talk to others about the products when they were dissatisfied, but, interestingly, they were not more likely to talk to others when they were satisfied.

In the context of the Internet, word-of-mouth advertising is not restricted to pairwise or small-group interactions between individuals. Rather, customers can share their experiences and opinions regarding a product with everyone. Quantitative marketing techniques have been proposed [Montgomery 2001] to describe product information flow online, and the rating of products and merchants has been shown to effect the likelihood that an item will be bought [Resnick and Zeckhauser 2002; Chevalier and Mayzlin 2006]. More sophisticated online recommendation systems allow users to rate the reviews of others, or directly rate other reviewers to implicitly form a trusted reviewer network that may have very little overlap with a person's actual social circle. Richardson and Domingos [2002] used Epinions' trusted reviewer network to construct an algorithm to maximize viral marketing efficiency, assuming that an individual's probability of purchasing a product depends on the opinions on the trusted peers in their network. Kempe et al. [2003] have followed up on Richardson and Domingos' challenge of maximizing the spread of viral information by evaluating several algorithms, given various models of adoption that we discuss next.

Most of the previous research on the flow of information and influence through the networks has been done in the context of epidemiology and the spread of diseases over the network. See the works of Bailey [1975] and Anderson and May [2002] for reviews in this area. The classical disease propagation models are based on the stages of a disease in a host: a person is first susceptible to a disease, then if she is exposed to an infectious contact she can become infected, and thus infectious. After the disease ceases the person is recovered or removed. The person is then immune for some period. The immunity can wear off, and the person becomes susceptible again. Thus SIR (susceptible/infected/recovered) models diseases where a recovered person never again becomes susceptible, while SIRS (SIS, susceptible/infected/(recovered)/susceptible) models a population in which recovered host can become susceptible again. Given a network and a set of infected nodes, the *epidemic threshold* is studied, that is, conditions under which the disease will either dominate or die out. In our case, a SIR model would correspond to the case where a set of initially infected nodes corresponds to people who purchased a product without first receiving the recommendations. A node can purchase a product only once, and then tries to infect its neighbors with a purchase by sending out the recommendations. The SIS model corresponds to a less realistic case where a person can purchase a product multiple times as a result of multiple recommendations. The problem with these types of models is that they assume a known social network over which the diseases (product recommendations) are spreading and usually a single parameter which specifies the infectiousness of the disease. In our context, this would mean that the whole population is equally susceptible to recommendations of a particular product.

There are numerous other models of influence spread in social networks. One of the first and most influential diffusion models was proposed by Bass [1969]. The model of product diffusion predicts the number of people who will adopt an innovation over time. It does not explicitly account for the structure of the social network but rather it assumes that the rate of adoption is a function of the current proportion of the population who have already adopted (purchased a product in our case). The diffusion equation models the cumulative proportion of adopters in the population as a function of the intrinsic adoption rate and the measure of social contagion. The model describes an S-shaped curve, where adoption is slow at first, takes off exponentially, and flattens at the end. It can effectively model word-of-mouth product diffusion at the aggregate level but not at the level of an individual person, which is one of the topics we explore in this article.

Diffusion models that try to model the process of adoption of an idea or a product can generally be divided into two groups.

—*Threshold model* [Granovetter 1978] where each node in the network has a threshold $t \in [0, 1]$, typically drawn from some probability distribution. We also assign *connection weights* $w_{u,v}$ on the edges of the network. A node adopts the behavior if a sum of the connection weights of its neighbors that already adopted the behavior (purchased a product in our case) is greater than the threshold, $t \leq \sum_{\text{adopters}(u)} w_{u,v}$.
—*Cascade model* [Goldenberg et al. 2001] where whenever a neighbor v of node u adopts, then node u also adopts with probability $p_{u,v}$. In other words, every time a neighbor of u purchases a product, there is a chance that u will decide to purchase as well.

In the independent cascade model, Goldenberg et al. [2001] simulated the spread of information on an artificially generated network topology that consisted both of strong ties within groups of spatially proximate nodes and weak ties between the groups. They found that weak ties were important to the rate of information diffusion. Centola and Macy [2005] modeled product adoption on small world topologies when a person's chance of adoption is dependent on having more than one contact who had previously adopted. Wu and Huberman [2004] modeled opinion formation on different network topologies and found that, if highly connected nodes were seeded with a particular opinion, this would proportionally effect the long-term distribution of opinions in the network. Holme and Newman [2006] introduced a model where individuals' preferences are shaped by their social networks, but their choices of whom to include in their social network are also influenced by their preferences.

While these models address the question of how influence spreads in a network, they are based on assumed rather than measured influence effects. In contrast, our study tracks the actual diffusion of recommendations through email, allowing us to quantify the importance of factors such as the presence of highly-connected individuals or the effect of receiving recommendations from multiple contacts. Compared to previous empirical studies which tracked the adoption of a single innovation or product, our data encompasses over half a million different products, allowing us to model a product's suitability for viral marketing in terms of both the properties of the network and the product itself.

3. THE RECOMMENDATION NETWORK

3.1 Recommendation Program and Dataset Description

Our analysis focuses on the recommendation referral program run by a large retailer. The program rules were as follows. Each time a person purchases a book, music, or a movie he or she is given the option of sending emails recommending the item to friends. The first person to purchase the same item through a referral link in the email gets a 10% discount. When this happens, the sender of the recommendation receives a 10% credit on their purchase.

The following information is recorded for each recommendation

(1) sender customer ID (shadowed)
(2) receiver customer ID (shadowed)
(3) date sent
(4) purchase flag (buy-bit)
(5) purchase date (error-prone due to asynchrony in the servers)
(6) product identifier
(7) price

The recommendation dataset consists of 15,646,121 recommendations made among 3,943,084 distinct users. The data was collected from June 5, 2001, to May 16, 2003. In total, 548,523 products were recommended, 99% of them belonging to 4 main product groups: books, DVDs, music and videos. In addition to recommendation data, we also crawled the retailer's Web site to obtain product categories, reviews, and ratings for all products. Of the products in our data set, 5,813 (1%) were discontinued (the retailer no longer provided any information about them).

Although the data gives us a detailed and accurate view of recommendation dynamics, it does have its limitations. The only indication of the success of a recommendation is the observation of the recipient purchasing the product through the same vendor. We have no way of knowing if the person had decided instead to purchase elsewhere, borrow, or otherwise obtain the product. The delivery of the recommendation is also somewhat different from one person simply telling another about a product they enjoy, possibly in the context of a broader discussion of similar products. The recommendation is received as a form email including information about the discount program. Someone reading the email might consider it spam, or at least deem it less important than a recommendation given in the context of a conversation. The recipient might also doubt whether the friend is recommending the product because they think the recipient might enjoy it or if that are simply trying to get a discount for themselves. Finally, because the recommendation takes place before the recommender receives the product, it might not be based on a direct observation of the product. Nevertheless, we believe that these recommendation networks are reflective of the nature of word-of-mouth advertising and give us key insights into the influence of social networks on purchasing decisions.

3.2 Identifying Successful Recommendations

For each recommendation, the dataset includes information about the recommended product, sender, receive of the recommendation, and most importantly, the success of recommendation. See Section 3.1 for more details.

We represent this dataset as a directed multigraph. The nodes represent customers, and a directed edge contains all the information about the recommendation. The edge (i, j, p, t) indicates that i recommended product p to customer j at time t. Note that because there can be multiple recommendations between people (even on the same product), there can be multiple edges between two nodes.

The typical process generating edges in the recommendation network is as follows. A node i first buys a product p at time t, and then it recommends it to nodes $j_1, \ldots, j_n$. The j nodes can then buy the product and further recommend it. The only way for a node to recommend a product is to first buy it. Note that even if all nodes j buy a product, only the edge to the node j_k that first made the purchase (within a week after the recommendation) will be marked by a buy-bit. Because the buy-bit is set only for the first person who acts on a recommendation, we identify additional purchases by the presence of outgoing recommendations for a person, since all recommendations must be preceded by a purchase. We call this type of evidence of purchase a buy-edge. Note that buy-edges provide only a lower bound on the total number of purchases without discounts. It is possible for a customer not to be the first to act on a recommendation and also not to recommend the product to others. Unfortunately, this was not recorded in the dataset. We consider, however, the buy-bits and buy-edges as proxies for the total number of purchases through recommendations.

As mentioned previously, the first buyer only gets a discount (the buy-bit is turned on) if the purchase is made within one week of the recommendation. In order to account for as many purchases as possible, we consider all purchases where the recommendation preceded the purchase (buy-edge) regardless of the time difference between the two events.

To avoid confusion, we will refer to edges in a multigraph as recommendations (or multi-edges); there can be more than one recommendation between a pair of nodes. We will use the term edge (or unique edge) to refer to edges in the usual sense, that is, there is only one edge between a pair of people. And, to get from recommendations to edges, we create an edge between a pair of people if they exchanged at least one recommendation.

Table I. Product Group Recommendation Statistics
p: number of products, n: number of nodes, r: number of recommendations,
e: number of edges, b_b: number of buy bits, b_e: number of buy edges.

Group	p	n	r	e	b_b	b_e
Book	103,161	2,863,977	5,741,611	2,097,809	65,344	17,769
DVD	19,829	805,285	8,180,393	962,341	17,232	58,189
Music	393,598	794,148	1,443,847	585,738	7,837	2,739
Video	26,131	239,583	280,270	160,683	909	467
Full network	542,719	3,943,084	15,646,121	3,153,676	91,322	79,164

4. THE RECOMMENDATION NETWORK

For each product group, we took recommendations on all products from the group and created a network. Table I shows the sizes of various product group recommendation networks with p the total number of products in the product group, n the total number of nodes spanned by the group recommendation network, and r the number of recommendations (there can be multiple recommendations between two nodes). Column e shows the number of (unique) edges disregarding multiple recommendations between the same source and recipient (i.e., number of pairs of people that exchanged at least one recommendation).

In terms of the number of different items, music CDs are the largest group by far, followed by books and videos. There is a surprisingly small number of DVD titles. On the other hand, DVDs account for more than half of all recommendations in the dataset. The DVD network is also the most dense, with about 10 recommendations per node, while books and music have about 2 recommendations per node and videos have only a bit more than 1 recommendation per node.

Music recommendations reached about the same number of people as DVDs but used more than 5 times fewer recommendations to achieve the same coverage of the nodes. Book recommendations reached by far the most people, 2.8 million. Notice that all networks have a very small number of unique edges. For books, videos and music, the number of unique edges is smaller than the number of nodes. This suggests that the networks are highly disconnected [Erdös and Rényi 1960].

Back to Table I: given the total number of recommendations r and purchases $(b_b + b_e)$ influenced by recommendations, we can estimate how many recommendations need to be independently sent over the network to induce a new purchase. Using this metric, books have the most influential recommendations, followed by DVDs and music. For books, one out of 69 recommendations resulted in a purchase. For DVDs, it increases to 108 recommendations per purchase and further increases to 136 for music and 203 for video.

Table II gives more insight into the structure of the largest connected component of each product group's recommendation network. We performed the same measurements as in Table I except that we did not use the whole network, only its largest weakly connected component. The table shows the number of nodes n, the number of recommendations r_c, and the number of (unique) edges e_c in the largest component. The last two columns (b_{bc} and b_{ec}) show the number of purchases resulting in a discount (buy-bit, b_{bc}) and the number of purchases through buy-edges (b_{ec}) in the largest connected component.

Table II. Statistics for the Largest Connected Component of Each Product Group
n_c: number of nodes in largest connected component, r_c: number recommendations in the component, e_c: number of edges in the component, b_{bc}: number of buy bits, b_{ec}: number of buy edges in the largest connected component, and b_{bc} and b_{ec} are the number of purchase through a buy-bit and a buy-edge, respectively.

Group	n_c	r_c	e_c	b_{bc}	b_{ec}
Book	53,681	933,988	184,188	1,919	1,921
DVD	39,699	6,903,087	442,747	6,199	41,744
Music	22,044	295,543	82,844	348	456
Video	4,964	23,555	15,331	2	74
Full network	100,460	8,283,753	521,803	8,468	44,195

First, notice that the largest connected components are very small. DVDs have the largest, containing 4.9% of the nodes; books have the smallest at 1.78%. One would also expect that the fraction of the recommendations in the largest component would be proportional to its size. We notice that this is not the case. For example, the largest component in the full recommendation network contains 2.54% of the nodes and 52.9% of all recommendations, which is the result of heavy bias in DVD recommendations. Breaking this down by product categories, we see that for DVDs 84.3% of the recommendations are in the largest component (which contains 4.9% of all DVD nodes) vs. 16.3% for book recommendations (component size 1.79%), 20.5% for music recommendations (component size 2.77%), and 8.4% for video recommendations (component size 2.1%). This shows that the dynamic in the largest component is very much different from the rest of the network. Especially for DVDs, we can see that a very small fraction of users generated most of the recommendations.

4.1 Recommendation Network Over Time

The recommendations that occurred were exchanged over an existing underlying social network. In the real world, it is estimated that any two people on the globe are connected via a short chain of acquaintances, popularly known as the small-world phenomenon [Travers and Milgram 1969]. We examined whether the edges formed by aggregating recommendations over all products would similarly yield a small-world network even though they represent only a small fraction of a person's complete social network. We measured the growth of the largest weakly connected component over time, shown in Figure 1. Within the weakly connected component, any node can be reached from any other node by traversing (undirected) edges. For example, if u recommended product x to v, and w recommended product y to v, then u and w are linked through one intermediary and thus belong to the same weakly connected component. Note that connected components do not necessarily correspond to communities (clusters) which we often think of as densely linked parts of the networks. Nodes belong to same component if they can reach each other via an undirected path regardless of how densely they are linked.

Figure 1 shows the size of the largest connected component as a fraction of the total network. The largest component is very small over all time. Even though we compose the network using all the recommendations in the dataset,

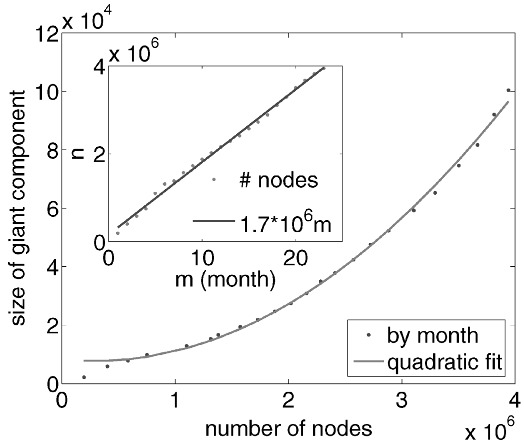


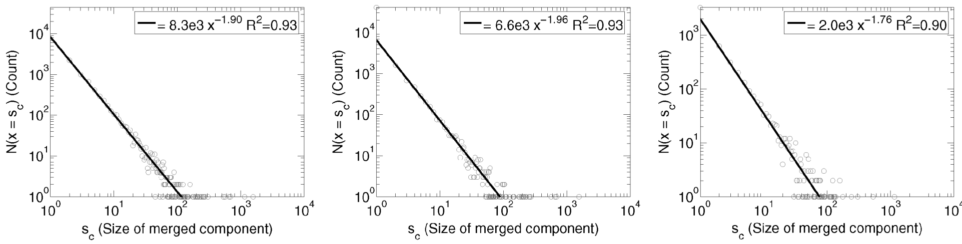
Fig. 1. (a) The size of the largest connected component of customers over time. The inset shows the linear growth in the number of customers n over time.

the largest connected component contains less than 2.5% (100,420) of the nodes, and the second largest component has only 600 nodes. Still, some smaller communities, numbering in the tens of thousands of purchasers of DVDs in categories such as westerns, classics, and Japanese animated films (anime), had connected components spanning about 20% of their members.

The insert in Figure 1 shows the growth of the customer base over time. Surprisingly it was linear, adding on average of 165,000 new users each month, which is an indication that the service itself was not spreading epidemically. Further evidence of nonviral spread is provided by the relatively high percentage (94%) of users who made their first recommendation without having previously received one.

4.1.1 *Growth of the Largest Connected Component.* Next, we examine the growth of the largest connected component (LCC). In Figure 1, we saw that the largest component seems to grow quadratically over time, but at the end of the data collection period is still very small, that is, only 2.5% of the nodes belong to largest weakly connected component. Here we are not interested in how fast the largest component grows over time but rather how big other components are when they get merged into the largest component. Also, since our graph is directed, we are interested in determining whether smaller components become attached to the largest component by a recommendation sent from inside of the largest component. One can think of these recommendations as being tentacles reaching out of the largest component to attach to smaller components. The other possibility is that the recommendation comes from a node outside the component to a member of the largest component, and thus the initiative to attach comes from outside the largest component.

We look at whether the largest component grows gradually, adding nodes one-by-one as the members send out more recommendations or whether a new recommendation might act as a bridge to a component consisting of several nodes that are already linked by their previous recommendations. To this end,



(a) LCC growth (b) Sender in LCC (c) Sender outside LCC

Fig. 2. Growth of the largest connected component (LCC). (a) The distribution of sizes of components when they are merged into the largest connected component. (b) The same as (a), but restricted to cases when a member of the LCC sends a recommendation to someone outside the largest component. (c) A sender outside the largest component sends a recommendation to a member of the component.

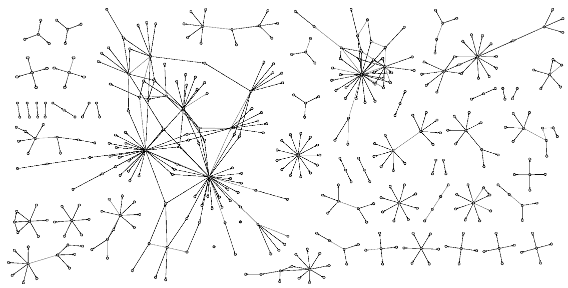
we measure the distribution of a component's size when it gets merged to the largest weakly connected component.

We operate under the following setting. Recommendations are arriving over time one-by-one creating edges between the nodes of the network. As more edges are added, the size of the largest connected component grows. We keep track of the currently largest component and measure how big the separate components are when they get attached to the largest component.

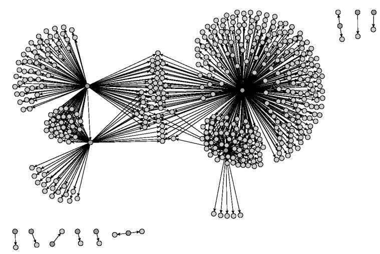
Figure 2(a) shows the distribution of merged connected component (CC) sizes. On the x-axis, we plot the component size (number of nodes N) and on the y-axis, the number of components of size N that were merged over time with the largest component. We see that, majority of the time, a single node (component of size 1) merged with the currently largest component. On the other extreme is the case when a component of $1,568$ nodes merged with the largest component.

Interestingly, out of all merged components, in 77% of the cases, the source of the recommendation comes from inside the largest component, while in the remaining 23% of the cases, it is the smaller component that attaches itself to the largest one. Figure 2(b) shows the distribution of component sizes only for the case when the sender of the recommendation was a member of the largest component, that is, the small component was attached from the largest component. Last, Figure 2(c) shows the distribution for the opposite case when the sender of the recommendation was not a member of the largest component, that is, the small component attached itself to the largest.

Also notice that in all cases the distribution of merged component sizes follows a heavy-tailed distribution. We fit a power-law distribution and note the power-law exponent of 1.90 (Figure 2(a)) when considering all merged components. Limiting the analysis to the cases where the source of the edge that attached a small component to the largest is in the largest component, we obtain a power-law exponent of 1.96 (Figure 2(b)), and when the edge originated from the small component that attached it to the largest, the power-law exponent is 1.76. This shows that even though in most cases the LCC absorbs the small component, we see that components that attach themselves to the



(a) Medical book



(b) Japanese graphic novel

Fig. 3. Examples of two product recommendation networks: (a) First-aid study guide *First Aid for the USMLE Step*, (b) Japanese graphic novel (manga) *Oh My Goddess!: Mara Strikes Back*.

LCC tend to be larger (smaller power-law exponent) than those attracted by the LCC. This means that the component sometimes grows a bit before it attaches itself to the largest component. Intuitively, an individual node can get attached to the largest component simply by passively receiving a recommendation. But if it is the outside node that sends a recommendation to someone in the giant component, it is already an active recommender and could therefore have recommended to several others previously, thus forming a slightly bigger component that is then merged.

From these experiments, we see that the largest component is very active, adding smaller components by generating new recommendations. Most of the time, these newly merged components are quite small, but occasionally sizable components are attached (see Figure 3).

4.2 Preliminary Observations and Discussion

Even with these simple counts and experiments, we can already make a few observations. It seems that some people got quite heavily involved in the recommendation program and that they tended to recommend a large number of products to the same set of friends (since the number of unique edges is so small as shown on Table I). This means that people tend to buy more DVDs and also like to recommend them to their friends, while they seem to be more conservative with books. One possible reason is that a book is a bigger time investment than a DVD: one usually needs several days to read a book, while a DVD can be viewed in a single evening. Another factor may be how informed the customer is about the product. DVDs, while fewer in number, are more heavily advertised on TV, billboards, and movie theater previews. Furthermore, it is possible that a customer has already watched a movie and is adding the DVD to their collection. This could make them more confident in sending recommendations before viewing the purchased DVD.

One external factor which may be affecting the recommendation patterns for DVDs is the existence of referral Web sites (www.dvdtalk.com). On these Web sites people who want to buy a DVD and get a discount would ask for recommendations. This way there would be recommendations made between

Table III. Fraction of People Who Purchase and Also Recommend Forward
Purchases: number of nodes that purchased as a result of receiving a recommendation.
Forward: nodes that purchased and then also recommended the product to others.

	Number of nodes		
Group	Purchases	Forward	Percent
Book	65,391	15,769	24.2
DVD	16,459	7,336	44.6
Music	7,843	1,824	23.3
Video	909	250	27.6
Total	90,602	25,179	27.8

people who don't really know each other but rather have an economic incentive to cooperate.

In effect, the viral marketing program is altering, albeit briefly and most likely unintentionally, the structure of the social network on which it is spreading. We were not able to find similar referral-sharing sites for books or CDs.

5. PROPAGATION OF RECOMMENDATIONS

5.1 Forward Recommendations

Not all people who accept a recommendation by making a purchase decide to give recommendations. In estimating what fraction of people who purchase and then decide to recommend forward, we can only use the nodes with purchases that resulted in a discount. Table III shows that only about a third of the people who purchase also recommend the product forward. The ratio of forward recommendations is much higher for DVDs than for other kinds of products. Videos also have a higher ratio of forward recommendations, while books have the lowest. This shows that people are most keen on recommending movies, possibly for the previously mentioned reasons, while they are more conservative when recommending books and music.

Figure 4 shows the cumulative out-degree distribution, that is, the number of people who sent out at least k_p recommendations, for a product. We fit a power-law to all but the tail of the distribution. Also notice the exponential decay in the tail of the distribution which could be, among other reasons, attributed to the finite time horizon of our dataset.

Figure 4 shows that the deeper an individual is in the cascade, if they choose to make recommendations, they tend to recommend to a greater number of people on average (the fitted line has a smaller slope γ, that is, the distribution has higher variance). This effect is probably characteristic of Table IV only very heavily recommended products producing large enough cascades to reach a certain depth. We also observe, as is shown in Table IV, that the probability of an individual making a recommendation at all (which can only occur if they make a purchase), declines after an initial increase as one gets deeper into the cascade.

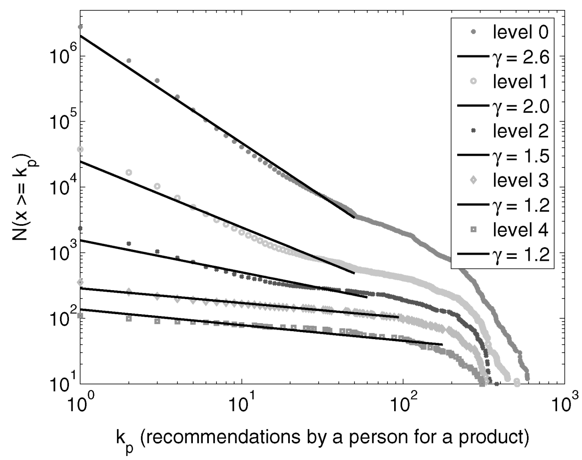


Fig. 4. The number of recommendations sent by a user with each curve representing a different depth of the user in the recommendation chain. A power-law exponent γ is fitted to all but the tail, which shows an exponential drop-off at around 100 recommendations sent). This drop-off is consistent across all depth levels and may reflect either a natural disinclination to send recommendation to over a hundred people or a technical issue that might have made it more inconvenient to do so. The fitted lines follow the order of the level number (i.e., top line corresponds to level 0 and the bottom one to level 4).

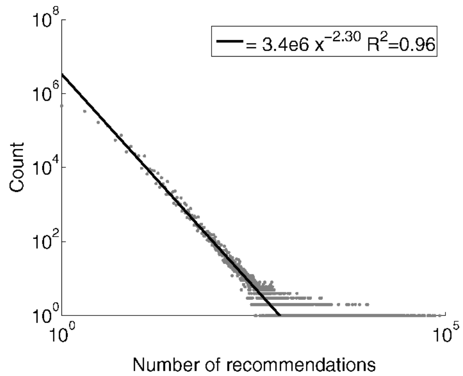
Table IV. Statistics about Individuals at Different Levels of the Cascade

level	prob. buy & forward	average out-degree
0	N/A	1.99
1	0.0069	5.34
2	0.0149	24.43
3	0.0115	72.79
4	0.0082	111.75

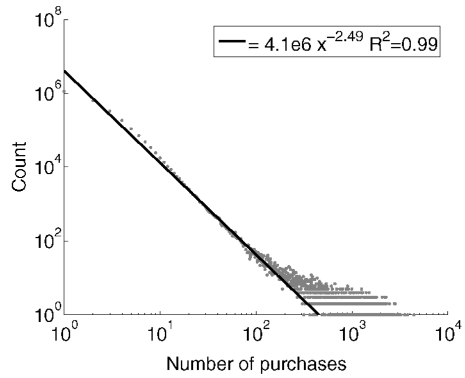
5.2 Identifying Cascades

As customers continue forwarding recommendations, they contribute to the formation of cascades. In order to identify cascades, that is, the causal propagation of recommendations, we track *successful recommendations* that influence purchases and further recommendations. We define a recommendation to be successful if it reached a node before its first purchase. We consider only the first purchase of an item because there are many cases when a person made multiple purchases of the same product, and in between those purchases, she may have received new recommendations. In this case, one cannot conclude that recommendations following the first purchase influenced the later purchases.

Each cascade is a network consisting of customers (nodes) who purchased the same product as a result of each other's recommendations (edges). We delete *late recommendations*—all incoming recommendations that happened after the first purchase of the product. This way we make the network *time increasing* or *causal* for each node, all incoming edges (recommendations) occurred before



(a) Recommendations



(b) Purchases

Fig. 5. Distribution of the number of recommendations and number of purchases made by a customer.

all outgoing edges. Now each connected component represents a time-obeying propagation of recommendations.

Figure 3 shows two typical product recommendation networks: (a) a medical study guide and (b) a Japanese graphic novel. Throughout the dataset, we observe very similar patterns. Most product recommendation networks consist of a large number of small disconnected components where we do not observe cascades. Then there is usually a small number of relatively small components with recommendations successfully propagating. This observation is reflected in the heavy-tailed distribution of cascade sizes (see Figure 6), having a power-law exponent close to 1 for DVDs in particular. We determined the power-law exponent by fitting a line on log-log scales using the least squares method.

We also notice bursts of recommendations (Figure 3(b)). Some nodes recommend to many friends, forming a star-like pattern. Figure 5 shows the distribution of the recommendations and purchases made by a single node in the recommendation network. Notice the power-law distributions and long flat tails. The most active customer made 83,729 recommendations and purchased 4,416 different items. Finally, we also sometimes observe collisions, where nodes receive recommendations from two or more sources. A detailed enumeration and analysis of observed topological cascade patterns for this dataset is made in Leskovec et al. [2006].

Last, we examine the number of exchanged recommendations between a pair of people in Figure 7. Overall, 39% of pairs of people exchanged just a single recommendation. This number decreases for DVDs to 37% and increases for books to 45%. The distribution of the number of exchanged recommendations follows a heavy-tailed distribution. To get a better understanding of the distributions, we show the power-law decay lines. Notice that one gets a much stronger decay exponent (distribution has a weaker tail) of -2.7 for books and a very shallow power-law exponent of -1.5 for DVDs. This means that even a pair of people exchanges more DVD than book recommendations.

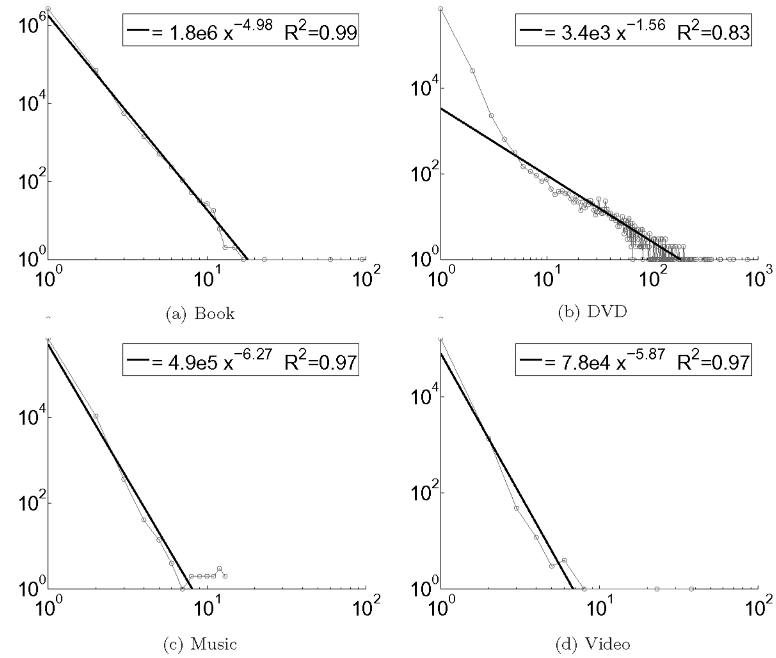


Fig. 6. Size distribution of cascades (size of cascade vs. count). The bold line presents a power fit.

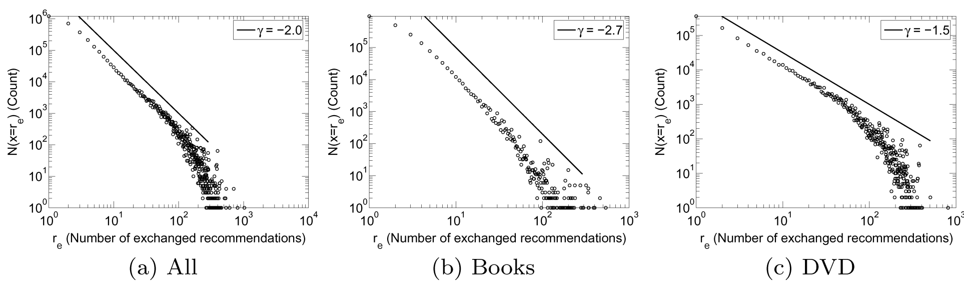


Fig. 7. Distribution of the number of exchanged recommendations between pairs of people.

5.3 The Recommendation Propagation Model

A simple model can help explain how the wide variance we observe in the number of recommendations made by individuals can lead to power-laws in cascade sizes (Figure 6). The model assumes that each recipient of a recommendation will forward it to others if its value exceeds an arbitrary threshold that the individual sets for herself. Since exceeding this value is a probabilistic event, let's call p_t the probability that at time step t the recommendation exceeds the threshold. In this case, the number of recommendations N_{t+1} at time $(t+1)$ is given in terms of the number of recommendations at an earlier time by

$$N_{t+1} = p_t N_t, \tag{1}$$

where the probability p_t is defined over the unit interval.

Notice that, because the probabilistic nature of the threshold is exceeded, one can only compute the final distribution of recommendation chain lengths, which we now proceed to do.

Subtracting from both sides of this equation the term N_t and diving by it, we obtain

$$\frac{N_{(t+1)} - N_t}{N_t} = p_t - 1. \tag{2}$$

Summing both sides from the initial time to some very large time T and assuming that for long times the numerator is smaller than the denominator (a reasonable assumption), we get, up to a unit constant

$$\frac{dN}{N} = \sum p_t. \tag{3}$$

The left-hand integral is just $\ln(N)$, and the right-hand side is a sum of random variables, which in the limit of a very large uncorrelated number of recommendations is normally distributed (central limit theorem).

This means that the logarithm of the number of messages is normally distributed, or equivalently, that the number of messages passed is log-normally distributed. In other words, the probability density for N is given by

$$P(N) = \frac{1}{N\sqrt{2\pi\sigma^2}} \exp \frac{-(\ln(N) - \mu)^2}{2\sigma^2}, \tag{4}$$

which, for large variances, describes a behavior whereby the typical number of recommendations is small (the mode of the distribution) but there are unlikely events of large chains of recommendations which are also observable.

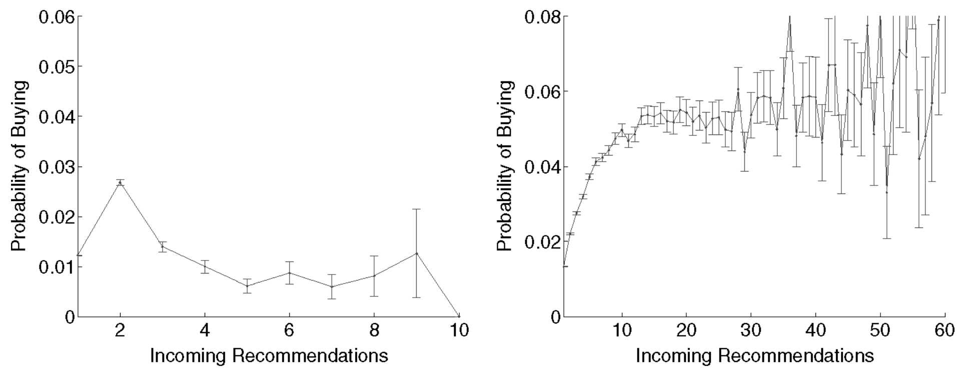
Furthermore, for large variances, the lognormal distribution can behave like a power-law for a range of values. In order to see this, take the logarithms on both sides of the equation (equivalent to a log-log plot) and one obtains

$$\ln(P(N)) = -\ln(N) - \ln(\sqrt{2\pi\sigma^2}) - \frac{(\ln(N) - \mu)^2}{2\sigma^2}. \tag{5}$$

So, for large σ, the last term of the right-hand side goes to zero, and, since the second term is a constant, one obtains a power-law behavior with exponent value of minus one. There are other models which produce power-law distributions of cascade sizes, but we present ours for its simplicity since it does not depend on network topology [Gruhl et al. 2004] or critical thresholds in the probability of a recommendation being accepted [Watts 2002].

6. SUCCESS OF RECOMMENDATIONS

So far, we only looked into the aggregate statistics of the recommendation network. Next, we ask questions about the effectiveness of recommendations in the recommendation network itself. First, we analyze the probability of purchasing as one gets more and more recommendations. Next, we measure recommendation effectiveness as two people exchange more and more recommendations. Last, we observe the recommendation network from the perspective of the sender of the recommendation. Does a node that makes more recommendations also influence more purchases?



(a) Books (b) DVD

Fig. 8. Probability of buying a book (DVD) given a number of incoming recommendations.

6.1 Probability of Buying versus Number of Incoming Recommendations

First, we examine how the probability of purchasing changes as one gets more and more recommendations. One would expect that a person is more likely to buy a product if she gets more recommendations. On the other hand, one would also think that there is a saturation point; if a person hasn't bought a product after a number of recommendations, they are not likely to change their minds after receiving even more recommendations. So, how many recommendations are too many?

Figure 8 shows the probability of purchasing a product as a function of the number of incoming recommendations on the product. Because we exclude late recommendations, that is, those that were received after the purchase, an individual is the recipient of three recommendations only if they did not make a purchase after the first two, and they either purchased or did not receive further recommendations after receiving the third one. As we move to higher numbers of incoming recommendations, the number of observations drops rapidly. For example, there were 5 million cases with 1 incoming recommendation on a book, and only 58 cases where a person got 20 incoming recommendations on a particular book. The maximum was 30 incoming recommendations. For these reasons we cut off the plot when the number of observations becomes too small and the error bars too large.

We calculate the purchase probabilities and the standard errors of the estimates which we use to plot the error bars in the following way. We regard each point as a binomial random variable. Given the number of observations n, let m be the number of successes, and $k (k = n - m)$ the number of failures. In our case, m is the number of people that first purchased a product after receiving r recommendations on it, and k is the number of people that received the total of r recommendations on a product (till the end of the dataset) but did purchase it. Then the estimated probability of purchasing is $\hat{p} = m/n$. The standard error $s_{\hat{p}}$ of estimate $\hat{p}$ is $s_{\hat{p}} = \sqrt{p(1-p)/n}$.

Figure 8(a) shows that overall book recommendations are rarely followed. Even more surprisingly, as more and more recommendations are received, their success decreases. We observe a peak in probability of buying at 2 incoming recommendations and then a slow drop. This implies that if a person doesn't buy a book after the first recommendation, but receives another, they are more likely to be persuaded by the second recommendation. But thereafter, they are less likely to respond to additional recommendations, possibly because they perceive them as spam, are less susceptible to others' opinions, have a strong opinion on the particular product, or have a different means of accessing it.

For DVDs (Figure 8(b)), we observe a saturation at around 10 incoming recommendations. This means that with each additional recommendation, a person is more and more likely to be persuaded, up to a point. After a person gets 10 recommendations on a particular DVD, their probability of buying does not increase anymore. The number of observations is 2.5 million at 1 incoming recommendation and 100 at 60 incoming recommendations. The maximum number of received recommendations is 172 (and that person did not buy), but someone purchased a DVD after receiving 169 recommendations. The different patterns between book and DVD recommendations may be a result of the recommendation exchange Web sites for DVDs. Someone receiving many DVD recommendations may have signed up to receive them for a product they intended to purchase, and hence a greater number of received recommendations corresponds to a higher likelihood of purchase (up to a point).

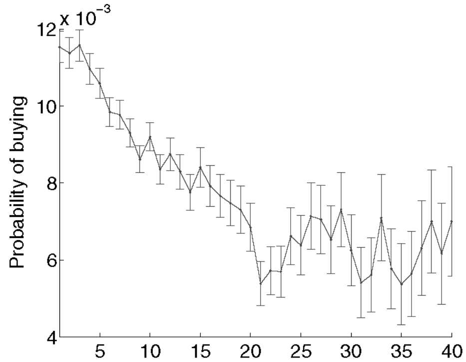
6.2 Success of Subsequent Recommendations

Next, we analyze how the effectiveness of recommendations changes as one receives more and more recommendations from the same person. A large number of exchanged recommendations can be a sign of trust and influence, but a sender of too many recommendations can be perceived as a spammer. A person who recommends only a few products will have her friends' attention, but one who floods her friends with all sorts of recommendations will start to loose her influence.

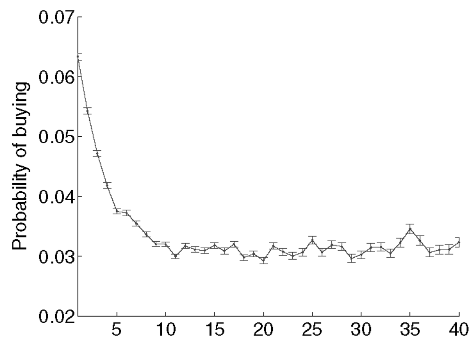
We measure the effectiveness of recommendations as a function of the total number of previously received recommendations from a particular node. We thus measure how spending changes over time, where time is measured in the number of received recommendations.

We construct the experiment in the following way. For every recommendation r on some product p between nodes u and v, we first determine how many recommendations node u received from v before getting r. Then we check whether v, the recipient of recommendation, purchased p after the recommendation r arrived. If so, we count the recommendation as successful since it influenced the purchase. In this way, we can calculate the recommendation success rate as more recommendations were exchanged. For the experiment, we consider only node pairs (u, v) where there were at least a total of 10 recommendations sent from u to v. We perform the experiment using only recommendations from the same product group.

We decided to set a lower limit on the number of exchanged recommendations so that we can measure how the effectiveness of recommendations changes as the same two people exchange more and more recommendations. Considering all pairs of people would heavily bias our findings since most pairs exchange



(a) Books



(b) DVD

Fig. 9. The effectiveness of recommendations with the number of received recommendations.

just a few or even just a single recommendation. Using the data from Figure 7, we see that 91% of pairs of people that exchange at least 1 recommendation exchange less than 10. For books, this number increases to 96%, and for DVDs, it is even smaller (81%). In the DVD network, there are 182 thousand pairs that exchanged more than 10 recommendations, and 70 thousand for the book network.

Figure 9 shows the probability of buying as a function of the total number of received recommendations from a particular person up to that point. One can think of the x-axis as measuring the time where the unit is the number of received recommendations from a particular person.

For books, we observe that the effectiveness of recommendation remains about constant up to 3 exchanged recommendations. As the number of exchanged recommendations increases, the probability of buying starts to decrease to about half of the original value and then levels off. For DVDs, we observe an immediate and consistent drop. We performed the experiment also for video and music, but the number of observations was too low and the measurements were noisy. This experiment shows that recommendations start to lose effect after more than two or three are passed between two people. Also, notice that the effectiveness of book recommendations decays much more slowly than that of DVD recommendations, flattening out at around 20 recommendations compared to around 10 DVD exchanged recommendations.

The result has important implications for viral marketing because providing too much incentive for people to recommend to one another can weaken the very social network links that the marketer is intending to exploit.

6.3 Success of Outgoing Recommendations

In previous sections, we examined the data from the viewpoint of the receiver of the recommendation. Now we look from the viewpoint of the sender. The two interesting questions are: (1) how does the probability of getting a 10% credit change with the number of outgoing recommendations, and (2) given a number of outgoing recommendations, how many purchases will they influence?

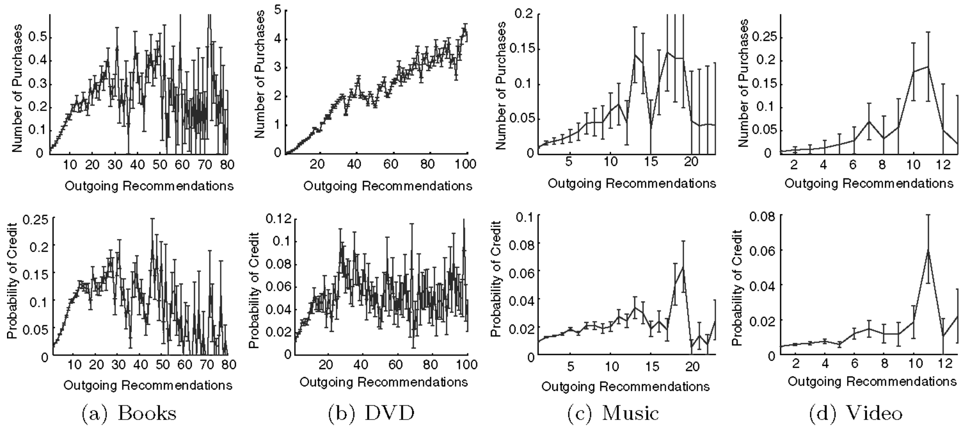


Fig. 10. Top row: number of resulting purchases given a number of outgoing recommendations. Bottom row: probability of getting a credit given a number of outgoing recommendations.

One would expect that recommendations would be the most effective when recommended to the relevant subset of friends. If one is very selective and recommends to too few friends, then the chances of success are slim. One the other hand, recommending to everyone and spamming them with recommendations may have limited returns as well.

The top row of Figure 10 shows how the average number of purchases changes with the number of outgoing recommendations. For books, music, and videos, the number of purchases soon saturates: it grows fast up to around 10 outgoing recommendations and then the trend either slows or starts to drop. DVDs exhibit different behavior, with the expected number of purchases increasing throughout.

These results are even more interesting since the receiver of the recommendation does not know how many other people also received the recommendation. Thus the plots of Figure 10 show that there are interesting dependencies between the product characteristics and the recommender that manifest through the number of recommendations sent. It could be the case that widely recommended products are not suitable for viral marketing (we find something similar in Section 9.2), or that the recommender did not put too much thought into who to send the recommendation to, or simply that people soon start to ignore mass recommenders.

Plotting the probability of getting a 10% credit as a function of the number of outgoing recommendations, as in the bottom row of Figure 10, we see that the success of DVD recommendations saturates as well, while books, videos, and music have qualitatively similar trends. The difference in the curves for DVD recommendations points to the presence of collisions in the dense DVD network, which has 10 recommendations per node and around 400 per product, which is an order of magnitude more than other product groups. This means that many different individuals are recommending to the same person, and after that person makes a purchase, even though all of them made a 'successful recommendation' by our definition, only one of them receives a credit.

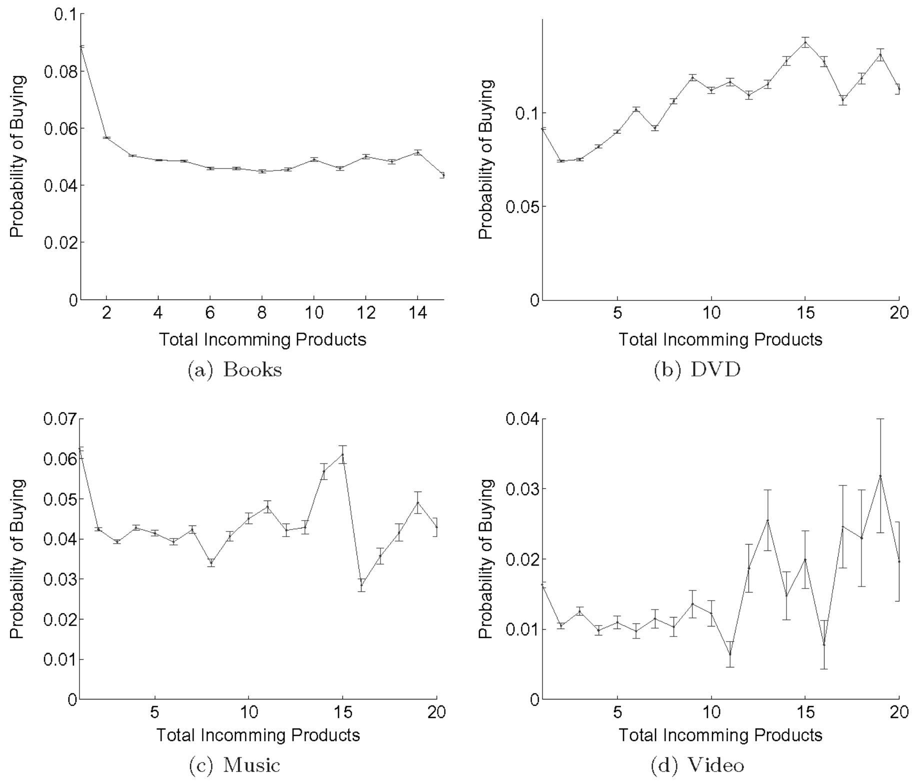


Fig. 11. The probability of buying a product given a number of different products on which a node got recommendations.

6.4 Probability of Buying Given the Total Number of Incoming Recommendations

The collisions of recommendations are a dominant feature of the DVD recommendation network. Book recommendations have the highest chance of getting a credit, but DVD recommendations result in the most purchases. So far, it seems people are very keen on recommending various DVDs, while very conservative on recommending books. But how does the behavior of customers change as they get more involved in the recommendation network? We would expect that most of the people are not heavily involved so their probability of buying is not high. In the extreme case, we expect to find people who buy almost everything they get a recommendations for.

There are two ways to measure the level of involvement of a person in the network, for instance, by the total number of incoming recommendations (on all products) or the total number of different products recommended to them. For every purchase of a book at time t, we count the number of different books (DVDs, etc.) the person received a recommendations for before time t. As in all previous experiments, we delete late recommendations, that is, recommendations that arrived after the first purchase of a product.

We show the probability of buying as a function of the number of different products recommended in Figure 11. Figure 12 plots the same data but with

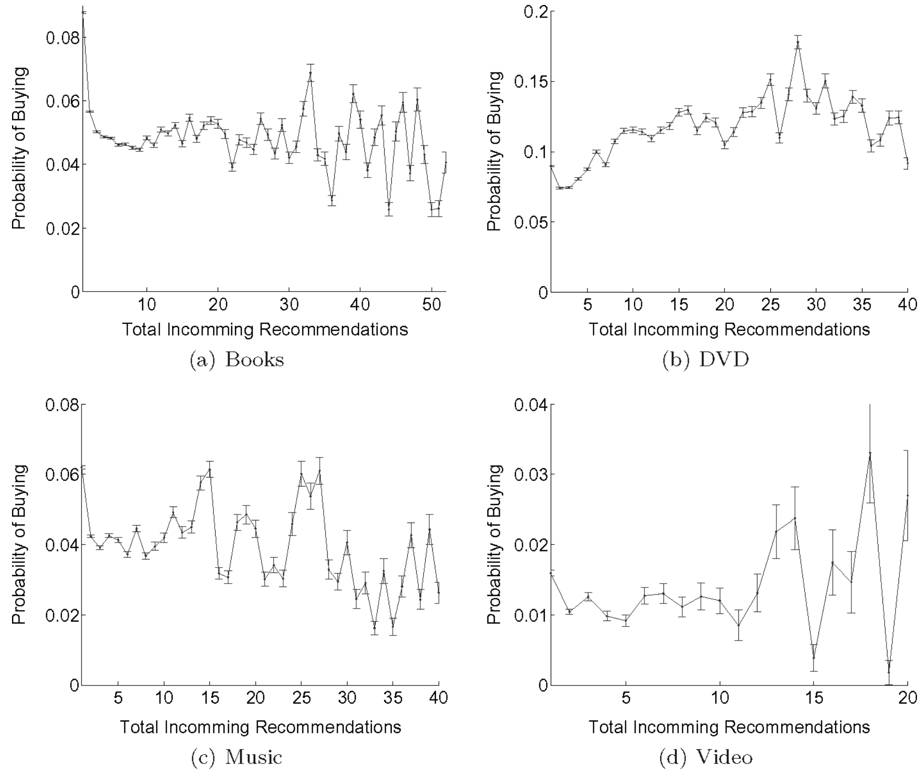
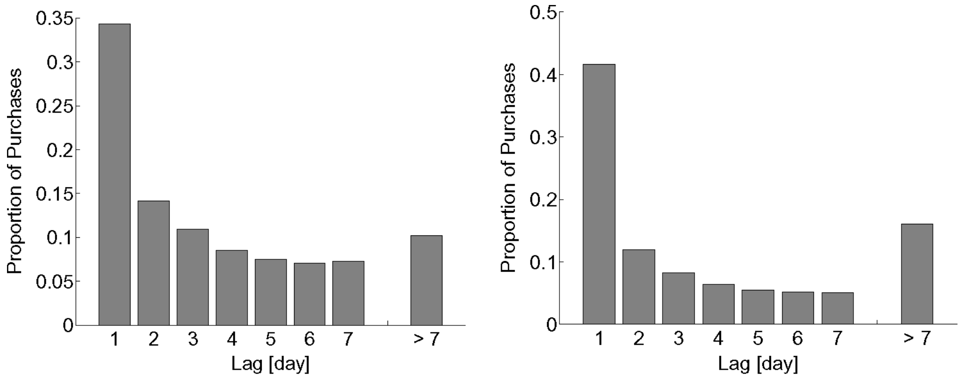


Fig. 12. Probability of buying a product given a total number of incoming recommendations on all products.

the total number of incoming recommendations on the x-axis. We calculate the error bars as described in Section 6.1. The number of observations is large enough (error bars are sufficiently small) to draw conclusions about the trends observed in the Figures. For example, there are more than 15, 000 observations (users) who had 15 incoming DVD recommendations. Notice that trends are quite similar regardless of whether we measure how involved the user is in the network by counting the number of products recommended (Figure 11) or the number of incoming recommendations (Figure 12).

We observe two distinct trends. For books and music (Figures 11 and 12, (a) and (c)) the probability of buying is the highest when a person got recommendations on just 1 item; as the number of incoming recommended products increases to 2 or more, the probability of buying quickly decreases, and then flattens.

Movies (DVDs and videos) exhibit different behavior (Figure 11 and 12, (b) and (d)). A person is more likely to buy, the more recommendations she gets. For DVDs the peak is at around 15 incoming products, while for videos there is no such peak, and the probability remains fairly level. Interestingly, for DVDs, the distribution reaches its low at 2 and 3 items, while for videos it lies somewhere between 3 and 8 items. The results suggest that book and music buyers tend



(a) Books (b) DVD

Fig. 13. The time between the recommendation and the actual purchase. We use all purchases.

to be conservative and focused. On the other hand, there are people who like to buy movies in general. One could hypothesize that buying a book is a larger investment of time and effort than buying a movie. One can finish a movie in an evening, while reading a book requires more time. There are also many more book and music titles than movie titles.

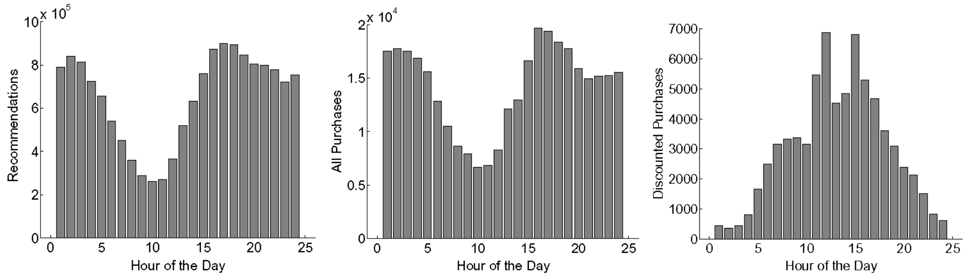
The other difference between the book and music recommendations in comparison to movies are the recommendation referral Web sites where people could go to get recommendations. One could see these Web sites as recommendation subscription services, for example, posting one's email on a list results in a higher number of incoming recommendations. Movies, people with a high number of incoming recommendations subscribed to them and thus expected/wanted the recommendations. On the other hand people with high numbers of incoming book or music recommendations did not sign up for them so they may perceive recommendations as spam and thus the influence of recommendations drops.

Another evidence of the existence of recommendation referral Web sites includes the DVD recommendation network degree distribution. The DVDs follow a power-law degree distribution with the exception of a peak at out-degree 50. Other plots of DVD recommendation behavior also exhibited abnormalities at around 50 recommendations. We believe these can be attributed to the recommendation referral Web sites.

7. TIMING OF RECOMMENDATIONS AND PURCHASES

The recommendation referral program encourages people to purchase as soon as possible after they get a recommendation since this maximizes the probability of getting a discount. We study the time lag between the recommendation and the purchase of different product groups, effectively how long it takes a person to receive a recommendation, consider it, and act on it.

We present the histograms of the *thinking time*, that is, the difference between the time of purchase and the time the last recommendation was received for the product prior to the purchase (Figure 13). We use a bin size of 1 day. Around 35%-40% of book and DVD purchases occurred within a day after the



(a) Recommendations (b) Purchases (c) Purchases with Discount

Fig. 14. Time of day for purchases and recommendations. (a) shows the distribution of recommendations over the day, (b) shows all purchases and (c) shows only purchases, that resulted in a discount.

last recommendation was received. For DVDs, 16% of purchases occur more than a week after the last recommendation, while this drops to 10% for books. In contrast, if we consider the lag between the purchase and the first recommendation, only 23% of DVD purchases are made within a day, while the proportion stays the same for books. This reflects a greater likelihood for a person to receive multiple recommendations for a DVD than for a book. At the same time, DVD recommenders tend to send out many more recommendations only one of which can result in a discount. Individuals then often miss their chance of a discount, which is reflected in the high ratio (78%) of recommended DVD purchases that did not a get discount (see Table I, columns b_b and b_e). In contrast, for books, only 21% of purchases through recommendations did not receive a discount.

We also measure the variation in intensity by time of day for three different activities in the recommendation system: recommendations (Figure 14(a)), all purchases (Figure 14(b)), and finally just the purchases which resulted in a discount (Figure 14(c)). Each is given as a total count by hour of day.

The recommendations and purchases follow the same pattern. The only small difference is that purchases reach a sharper peak in the afternoon (after 3pm Pacific Time, 6pm Eastern time). This means that the willingness to recommend does not change with time since about a constant fraction of purchases also result in recommendations sent (plots 14(a) and (b) follow the same shape).

The purchases that resulted in a discount (Figure 14(c)) look like a negative image of the first two Figures. If recommendations had no effect, then plot (c) should follow the same shape as (a) and (b), since a fraction of people who buy would become first buyers, that is, the more recommendations sent, the more first buyers, and thus discounts. However, this does not seem to be the case. The number of purchases with discount is the highest when the number of purchases is small. This means that most of discounted purchases happened in the morning when the traffic (number of purchases/recommendations) on the retailer's Web site was low. This makes sense since most of the recommendations happened during the day, and if the person wanted to get the discount by being the first one to purchase, she had the highest chances when the traffic on the Web site was the lowest.

There are also other factors that come into play here. Assuming that recommendations are sent to people's personal (non-work) email addresses, then people probably check these email accounts for new email less regularly while at work. So checking personal email while at work and reacting to a recommendation would mean higher chances of getting a discount. Second, there are also network effects, that is, the more recommendations sent, the higher chance of recommendation collision, the lower chance of getting a discount since one competes with the larger set of people.

8. RECOMMENDATIONS AND COMMUNITIES OF INTEREST

Social networks are a product of the contexts that bring people together. The context can be a shared interest in a particular topic or kind of a book. Sometimes there are circumstances, such as a specific job or religious affiliation, that make people more likely to be interested in the same type of book or DVD. We first apply a community discovery algorithm to automatically detect communities of individuals who exchange recommendations with one another and to identify the kinds of products each community prefers. We then compare the effectiveness of recommendations across book categories, showing that books on different subjects have varying success rates.

8.1 Communities and Purchases

In aggregating all recommendations between any two individuals in Section 4.1, we showed that the network consists of one large component, containing a little over 100,000 customers, and many smaller components, the largest of which has 634 customers. However, knowing that a hundred-thousand customers are linked together in a large network does not reveal whether a product in a particular category is likely to diffuse through it. Consider, for example, a new science fiction book one would like to market by word-of-mouth. If science fiction fans are scattered throughout the network with very few recommendations shared between them, then recommendations about the new book are unlikely to diffuse. If, on the other hand, one finds one or more science fiction communities where sci-fi fans are close together in the network because they exchange recommendations with one another, then the book recommendation has a chance of spreading by word-of-mouth.

In the following analysis, we use a community-finding algorithm [Clauset et al. 2004] in order to discover the types of products that link customers and so define a community. The algorithm breaks up the component into parts such that the modularity Q, where

$$Q = \text{(number of edges within communities)} - \text{(expected number of such edges)}, \tag{6}$$

is maximized. In other words, the algorithm identifies communities such that individuals within those communities tend to preferentially exchange recommendations with one another.

The results of the community-finding analysis, while primarily descriptive, illustrate both the presence of communities whose members are linked by their common interests and the presence of cross-cutting interests between

Table V. A Sample of the Medium-Sized Communities Present in the Largest Component

# nodes	# senders	topics
735	74	books: American literature, poetry
710	179	sci-fi books, TV series DVDs, alternative rock music
667	181	music: dance, indie
653	121	discounted DVDs
541	112	books: art & photography, web development, graphical design, sci-fi
502	104	books: sci-fi and other
388	77	books: Christianity and Catholicism
309	81	books: business and investing, computers, Harry Potter
192	30	books: parenting, women's health, pregnancy
163	48	books: comparative religion, Egypt's history, new age, role playing games

communities. Applying the algorithm to the largest component, we identify many small communities and a few larger ones. The largest contains 21,000 nodes, 5,000 of which are senders of a relatively modest 335,000 recommendations. More interesting than simply observing the size of communities is discovering what interests bring them together. We identify those interests by observing product categories where the number of recommendations within the community is significantly higher than it is for the overall customer population. Let p_c be the proportion of all recommendations that fall within a particular product category c. Then for a set of individuals sending x_g recommendations, we would expect by chance that $x_g * p_c \pm \sqrt{x_g * p_c * (1 - p_c)}$ would fall within category c. We note the product categories for which the observed number of recommendations in the community is many standard deviations higher than expected. For example, compared to the background population, the largest community is focused on a wide variety of books and music. In contrast, the second largest community, involving 10,412 individuals (4,205 of whom are sending over 3 million recommendations), is predominantly focused on DVDs from many different genres, with no particular emphasis on anime. The anime community itself emerges as a highly unusual group of 1,874 users who exchanged over 3 million recommendations.

Perhaps the most interesting are the medium-sized communities, some of which are listed in Table V, having between 100 and 1000 members and often reflecting specific interests. Among the hundred or so medium-sized communities, we found, for example, several communities focusing on Christianity. While some of the Christian communities also shared an interest in children's books, broadway musicals, and travel to Italy, others focused on prayer and bibles, still others also enjoyed DVDs of the Simpsons TV series, why others took an interest in Catholicism, occult spirituality and kabbalah.

Communities were usually centered around a product group such as books, music, or DVDs, but almost all of them shared recommendations for all types of products. The DVD communities ranged from bargain shoppers purchasing discounted comedy and action DVDs to smaller anime or independent movie communities to a group of customers purchasing predominantly children's movies. One community focused heavily on indie music, imported dance, and club music. Another seemed to center around intellectual pursuits, including reading books on sociology, politics, artificial intelligence, mathematics, and media culture, listening to classical music, and watching neo-noir film. Several communities centered around business and investment books and frequently also recommended books on computing. One business and investment community included fans of the Harry Potter fiction series, while another enjoyed science fiction and adventure DVDs. One of the communities with the most particular interests recommended not only business and investing books to one another, but also an unusual number of books on terrorism, bacteriology, and military history. A community of what one can presume are Web designers recommended books to one another on art and photography, Web development, graphical design, and Ray Bradbury's science fiction novels. Several sci-fi TV series such as Buffy the Vampire Slayer and Star Trek appeared prominently in a few communities, while Stephen King and Douglas Clegg featured in a community recommending horror, sci-fi, and thrillers to one another. One community focused predominantly on parenting, women's health and pregnancy, while another recommended a variety of books but especially a collection of cookie-baking recipes.

Going back to components in the network that were disconnected from the largest component, we find similar patterns of homophily, the tendency of like to associate with like. Two of the components recommended technical books about medicine, one focused on dance music, while some others predominantly purchased books on business and investing. Given more time, it is quite possible that one of the customers in one of these disconnected components would have received a recommendation from a customer within the largest component, and the two components would have merged. For example, a disconnected component of medical students purchasing medical textbooks might have sent or received a recommendation from the medical community within the largest component. However, the medical community may also become linked to other parts of the network through a different interest of one of its members. At the very least, many communities, no matter what their focus, will have recommendations for children's books or movies since children are a focus for a great many people. The community-finding algorithm, on the other hand, is able to break up the larger social network to automatically identify groups of individuals with a particular focus or a set of related interests. Now that we have shown that communities of customers recommend types of products reflecting their interests, we will examine whether these different kinds of products tend to have different success rates in their recommendations.

8.2 Recommendation Effectiveness by Book Category

Some contexts result in social ties that are more effective at inducing an action. For example, in small-world experiments where participants attempt to reach a target individual through their chain of acquaintances, profession trumped geography, which in turn was more useful in locating a target than attributes such as religion or hobbies [Killworth and Bernard 1978; Travers and Milgram 1969]. In the context of product recommendations, we can ask whether a recommendation for a work of fiction, which may be made by any friend or neighbor, is more or less influential than a recommendation for a technical book, which may be made by a colleague at work or school.

Table VI. Statistics by Book Category
n_p:number of products in category, n number of customers, cc percentage of customers in the largest connected component, r_{p1} avg. # reviews in 2001 – 2003, r_{p2} avg. # reviews 1st 6 months 2005, v_{av} average star rating, c_{av} average number of people recommending product, c_{av}/r_{p1} ratio of recommenders to reviewers, p_m median price, b ratio of the number of purchases resulting from a recommendation to the number of recommenders. The symbol ** denotes statistical significance at the 0.01 level, * at the 0.05 level.

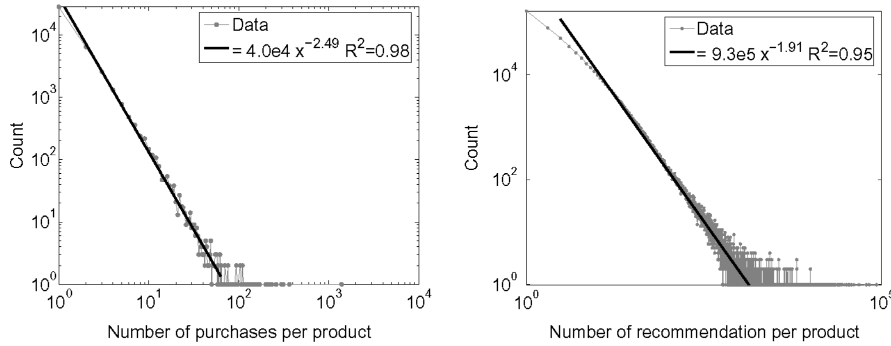
Category	n_p	n	cc	r_{p1}	v_{av}	c_{av}/r_{p1}	p_m	$b*100$
Books general	370230	2,860,714	1.87	5.28	4.32	1.41	14.95	3.12
Fiction								
Children	46,451	390,283	2.82	6.44	4.52	1.12	8.76	2.06**
Literature	41,682	502,179	3.06	13.09	4.30	0.57	11.87	2.82*
Mystery	10,734	123,392	6.03	20.14	4.08	0.36	9.60	2.40**
Science fiction	10,008	175,168	6.17	19.90	4.15	0.64	10.39	2.34**
Romance	6,317	60,902	5.65	12.81	4.17	0.52	6.99	1.78**
Teens	5,857	81,260	5.72	20.52	4.36	0.41	9.56	1.94**
Comics	3,565	46,564	11.70	4.76	4.36	2.03	10.47	2.30*
Horror	2,773	48,321	9.35	21.26	4.16	0.44	9.60	1.81**
Personal								
Religion	43,423	441,263	1.89	3.87	4.45	1.73	9.99	3.13
Health/Body	33,751	572,704	1.54	4.34	4.41	2.39	13.96	3.04
History	28,458	28,3406	2.74	4.34	4.30	1.27	18.00	2.84
Home/Garden	19,024	180,009	2.91	1.78	4.31	3.48	15.37	2.26**
Entertainment	18,724	258,142	3.65	3.48	4.29	2.26	13.97	2.66*
Arts/Photo	17,153	179,074	3.49	1.56	4.42	3.85	20.95	2.87
Travel	12,670	113,939	3.91	2.74	4.26	1.87	13.27	2.39**
Sports	10,183	120,103	1.74	3.36	4.34	1.99	13.97	2.26**
Parenting	8,324	182,792	0.73	4.71	4.42	2.57	11.87	2.81
Cooking	7,655	146,522	3.02	3.14	4.45	3.49	13.97	2.38*
Outdoors	6,413	59,764	2.23	1.93	4.42	2.50	15.00	3.05
Professional								
Professional	41,794	459,889	1.72	1.91	4.30	3.22	32.50	4.54**
Business	29,002	476,542	1.55	3.61	4.22	2.94	20.99	3.62**
Science	25,697	271,391	2.64	2.41	4.30	2.42	28.00	3.90**
Computers	18,941	375,712	2.22	4.51	3.98	3.10	34.95	3.61**
Medicine	16,047	175,520	1.08	1.41	4.40	4.19	39.95	5.68**
Engineering	10,312	107,255	1.30	1.43	4.14	3.85	59.95	4.10**
Law	5,176	53,182	2.64	1.89	4.25	2.67	24.95	3.66*
Other								
Nonfiction	55,868	560,552	2.03	3.13	4.29	1.89	18.95	3.28**
Reference	26,834	371,959	1.94	2.49	4.19	3.04	17.47	3.21
Biographies	18,233	277,356	2.80	7.65	4.34	0.90	14.00	2.96

Table VI shows recommendation trends for all top-level book categories by subject. For clarity, we group the results by 4 different category types: fiction, personal/leisure, professional/technical, and nonfiction/other. Fiction encompasses categories such as Sci-Fi and Romance, as well as children's and young adult books. Personal/Leisure encompasses everything from gardening, photography and cooking to health and religion.

First, we compare the relative number of recommendations to reviews posted on the site (column c_{av}/r_{p1} of Table VI). Surprisingly, we find that the number of people making personal recommendations was only a few times greater than the number of people posting a public review on the Web site. We observe that fiction books have relatively few recommendations compared to the number of reviews, while professional and technical books have more recommendations than reviews. This could reflect several factors. One is that people feel more confident reviewing fiction than technical books. Another is that they hesitate to recommend a work of fiction before reading it themselves since the recommendation must be made at the point of purchase. Yet another explanation is that the median price of a work of fiction is lower than that of a technical book. This means that the discount received for successfully recommending a mystery novel or thriller is lower, and hence people have less incentive to send recommendations.

Next, we measure the per-category efficacy of recommendations by observing the ratio of the number of purchases occurring within a week following a recommendation to the number of recommenders for each book subject category (column *b* of Table VI). On average, only 2% of the recommenders of a book received a discount because their recommendation was accepted, and another 1% made a recommendation that resulted in a purchase but not a discount. We observe marked differences in the response to recommendation for different categories of books. Fiction, in general, is not very effectively recommended with only around 2% of recommenders succeeding. The efficacy was a bit higher (around 3%) for non-fiction books dealing with personal and leisure pursuits. Perhaps people generally know what their friends' leisure interests are, or even have gotten to know them through those shared interests. On the other hand, they may not know as much about each others' tastes in fiction. Recommendation success is highest in the professional and technical category. Medical books have nearly double the average rate of recommendation acceptance. This could be in part attributed to the higher median price of medical books and technical books in general. As we will see in Section 9.2, a higher product price increases the chance that a recommendation will be accepted.

Recommendations are also more likely to be accepted for certain religious categories: 4.3% for Christian living and theology and 4.8% for Bibles. In contrast, books not tied to organized religions, such as ones on the subject of new age (2.5%) and occult (2.2%) spirituality, have lower recommendation effectiveness. These results raise the interesting possibility that individuals have greater influence over one another in an organized context, for example, through a professional contact or a religious one. There are exceptions, of course. For example, Japanese anime DVDs have a strong following in the US, and this is reflected in their frequency and success in recommendations. Another example is that of gardening. In general, recommendations for books relating to gardening have only a modest chance of being accepted which agrees with the individual prerogative that accompanies this hobby. At the same time, orchid cultivation can be a highly organized and social activity with frequent shows and online communities devoted entirely to orchids. Perhaps because of this, the rate of acceptance of orchid book recommendations is twice as high as those for books on growing vegetables or tomatos.



(a) Purchases (b) Recommendations

Fig. 15. Distribution of number of purchases and recommendations of a product. (a) shows the number of purchases that resulted in a discount per product, and (b) shows the distribution of the number of recommendations per product.

9. PRODUCTS AND RECOMMENDATIONS

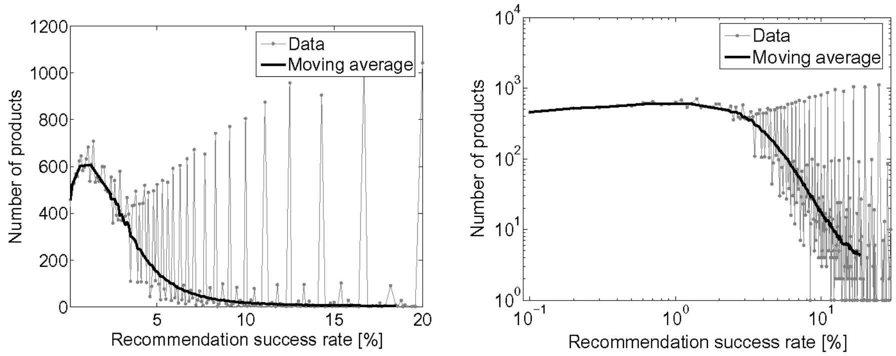
We have examined the properties of the recommendation network in relation to viral marketing. Now we focus on the products themselves and their characteristics that determine the success of recommendations.

9.1 How Long is the Long Tail?

Recently a long-tail phenomenon has been observed where a large fraction of purchases are of relatively obscure items, and each of them sells in very low numbers but there are many of those items. On Amazon.com, somewhere between 20 to 40% of unit sales fall outside of its top 100,000-ranked products [Brynjolfsson et al. 2003]. Considering that a typical brick-and-mortar store holds around 100,000 books, this presents a significant share. A streaming-music service streams more tracks outside than inside its top-10,000 tunes [Anonymous 2005].

We performed a similar experiment using our data. Since we do not have direct sales data, we used the number of successful recommendations as a proxy to the number of purchases. Figure 15 plots the distribution of the number of purchases and the number of recommendations per product. Notice that both the number of recommendations and the number of purchases per product follow a heavy-tailed distribution and that the distribution of recommendations has a heavier tail.

Interestingly, Figure 15(a) shows that just the top-100 products account for 11.4% of the all sales (purchases with discount), and the top-1000 products amount to 27% of total sales through the recommendation system. On the other hand, 67% of the products have only a single purchase, and they account for 30% of all sales. This shows that a significant portion of sales come from products that sell very few times. Recently there has been some debate about the long tail [Gomes 2006; Anderson 2006]. Some argue that the presence of the long tail indicates that niche products with low sales are contributing significantly



(a) Success rate (linear scale)

(b) Success rate (log scale)

Fig. 16. Distribution of product recommendation success rates. Both plots show the same data: (a) on a linear (lin-lin) scale, and (b) on a logarithmic (log-log) scale. The bold line presents the moving average smoothing.

to overall sales online. We also find that the tail is a bit longer than the usual 80-20 rule, with the top 20% of the products contributing to about half the sales. It is important to note, however, that our observations do not reflect the total sales of the products on the Web site since they include only successful recommendations that resulted in a discount. This incorporates both a bias in the kind of product that is likely to be recommended, and in the probability that a recommendation for that kind of product is accepted.

If we look at the distribution in the number of recommendations per product, shown in Figure 15(b), we observe an even more skewed distribution. 30% of the products have only a single recommendation and the top 56,000 most recommended products (top 10%) account for 84% of all recommendations. This is consistent with our previous observations some that types of products, for example, anime DVDs, are more heavily recommended than others.

Next we examine the distribution of the product recommendation success rate. Out of more than half-million products, we took all the products with at least a single purchase, of which there are 41,000 (7%). Figure 16 shows the success rate (purchases/recommendations). Notice that the distribution is not heavy tailed and has a mode at around 1.3% recommendation success rate. 55% of the products have a success rate below 5%, and there are around 14% of the products that have a recommendation success rate higher than 20%.

9.2 Modeling the Product Recommendation Success

So far, we have seen that some products generate many recommendations and some have a better return than others on those recommendations, but one question still remains: what determines the product's viral marketing success? We present a model which characterizes product categories for which recommendations are more likely to be accepted. We use a regression of the following product attributes to correlate them with recommendation success:

—n: number of nodes in the social network (number of unique senders and receivers)
—n_s: number of senders of recommendations
—n_r: number of recipients of recommendations
—r: number of recommendations
—e: number of edges in the social network (number of unique (sender, receiver) pairs)
—p: price of the product
—v: number of reviews of the product
—t: average product rating

From the original set of the half-million products, we compute a success rate s for the 8,192 DVDs and 50,631 books that had at least 10 recommendation senders and for which a price was given. In Section 8.2, we defined recommendation success rate s as the ratio of the total number of purchases made through recommendations and the number of senders of the recommendations. We decided to use this kind of normalization rather than normalizing by the total number of recommendations sent in order not to penalize communities where a few individuals send out many recommendations (Figure 3(b)). Note that, in general, s could be greater than 1, but, in practice, this happens extremely rarely (there are only 107 products where $s > 1$ which were discarded for the purposes of this analysis).

Since the variables follow a heavy-tailed distribution, we use the following model:

$$s = \exp\left(\sum_i \beta_i \log(x_i) + \epsilon_i\right), \tag{7}$$

where x_i are the product attributes (as described on previously), and ϵ_i is random error.

We fit the model using least squares and obtain the coefficients β_i shown in Table VIII. With the exception of the average rating, they are all significant, but just the number of recommendations alone accounts for 15% of the variance (taking all eight variables into consideration yields an R^2 of 0.30 for books and 0.81 for DVDs). We should also note that the variables in our model are highly collinear as can be seen from the pairwise correlation matrix (Table VII). For example, the number of recommendations r has a high negative correlation with the dependent variable ($\ln(s)$) but, in the regression model, it exhibits a positive influence on the dependent variable. This is probably due to the fact that the number of recommendations is naturally dependent on the number of senders and number of recipients, but it is the high number of recommendations relative to the number of senders that is of importance.

To illustrate the dependencies between the variables, we train a Bayesian dependency network [Chickering 2003] and show the learned structure for the combined (books and DVD) data in Figure 17. A directed acyclic graph where nodes are variables and directed edges indicate that the distribution of a child

Table VII. Pairwise Correlation Matrix of the Books and DVD Product Attributes (ln(s): log recommendation success rate, ln(n): log number of nodes, ln(n_s): log number of senders of recommendations, ln(n_r): log number of receivers, ln(r): log number of recommendations, ln(e): log number of edges, ln(p): log price, ln(v): log number of reviews, ln(t): log average rating.)

	ln(s)	ln(n)	ln(n_s)	ln(n_e)	ln(r)	ln(e)	ln(p)	ln(v)	ln(t)
ln(s)	1								
ln(n)	0.275	1							
ln(n_s)	0.103	0.907	1						
ln(n_r)	0.310	0.994	0.864	1.000					
ln(r)	0.396	0.979	0.828	0.988	1				
ln(e)	0.392	0.981	0.831	0.990	0.999	1			
ln(p)	0.185	0.098	0.088	0.098	0.107	0.106	1		
ln(v)	−0.050	0.465	0.490	0.449	0.421	0.423	−0.053	1	
ln(t)	−0.031	0.064	0.071	0.061	0.056	0.056	−0.019	0.269	1

Table VIII. Regression Using the Log of the Recommendation Success Rate log(s) as the Dependent Variable for Books and DVDs Separately. (For each coefficient we provide the standard error and the statistical significance level (**:0.001, *:0.1). We fit separate models of Books and DVDs.)

Variable	Books Coefficient β_i	DVD Coefficient β_i
const	1.317 (0.0038)**	0.929 (0.0100)**
n	−0.579 (0.0060)**	0.171 (0.0124)**
n_s	0.144 (0.0018)**	−0.070 (0.0023)**
n_r	−0.006 (0.0064)	−0.360 (0.0104)**
r	0.062 (0.0084)**	−0.002 (0.0083)
e	0.383 (0.0106)**	0.251 (0.0088)**
p	0.013 (0.0003)**	0.007 (0.0016)**
v	−0.003 (0.0001)**	−0.003 (0.0006)**
t	−0.001 (0.0006)*	0.000 (0.0009)
R^2	0.30	0.81

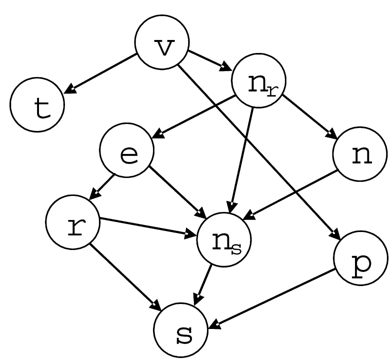


Fig. 17. A Bayesian network showing the dependencies between the variables. s: recommendation success rate, n: number of nodes, n_s: number of senders of recommendations, n_r: log number of receivers, r: number of recommendations, e: number of edges, p: price, v: number of reviews, t: average rating.

depends on the values taken in the parent variables. Notice that the average rating (t) is not predictive of the recommendation success rate (s). It is no surprise that the number of recommendations r is predictive of number of senders n_s. Similarly, the number of edges e is predictive of number of senders n_s. Interestingly, price p is only related to the number of reviews v, Number of recommendations r, number of senders n_s and price p, are directly predictive of the recommendation success rate s.

Returning to our regression model, we find that the numbers of nodes and receivers have negative coefficients, showing that successfully recommended products are actually more likely to be not so widely popular. The only attributes with positive coefficients are the number of recommendations r, number of edges e, and price p. This shows that more expensive and more recommended products have a higher success rate. These recommendations should occur between a small number of senders and receivers, which suggests a very dense recommendation network where lots of recommendations are exchanged between a small community of people. These insights could be of use to marketers—personal recommendations are most effective in small, densely-connected communities enjoying expensive products.

10. DISCUSSION AND CONCLUSION

Although the retailer may hope to boost its revenues through viral marketing, the additional purchases that result from recommendations are just a drop in the bucket of sales that occur through the Web site. Nevertheless, we were able to obtain a number of interesting insights into how viral marketing works that challenge common assumptions made in epidemic and rumor propagation modeling.

First, it is frequently assumed in epidemic models (e.g., SIRS type of models) that individuals have an equal probability of being infected every time they interact [Anderson and May 2002; Bailey 1975]. Contrary to this, we observe that the probability of infection decreases with repeated interaction. Marketers should take heed that providing excessive incentives for customers to recommend products could backfire by weakening the credibility of the very same links they are trying to take advantage of.

Traditional epidemic and innovation diffusion models also often assume that individuals either have a constant probability of 'converting' every time they interact with an infected individual [Goldenberg et al. 2001] or that they convert once the fraction of their contacts who are infected exceeds a threshold [Granovetter 1978]. In both cases, an increasing number of infected contacts results in an increased likelihood of infection. Instead, we find that the probability of purchasing a product increases with the number of recommendations received but quickly saturates to a constant and relatively low probability. This means individuals are often impervious to the recommendations of their friends, and resist buying items that they do not want.

In network-based epidemic models, extremely highly-connected individuals play a very important role. For example, in needle-sharing and sexual contact networks, these nodes become the super-spreaders by infecting a large number of people. But these models assume that a high-degree node has as much of a probability of infecting each of its neighbors as a low-degree node does. In contrast, we find that there are limits to how influential high-degree nodes are in the recommendation network. As a person sends out more and more recommendations past a certain number for a product, the success per recommendation declines. This would seem to indicate that individuals have influence over a few of their friends, but not everybody they know.

We also presented, a simple stochastic model that allows for the presence of relatively large cascades for a few products, but reflects well the general tendency of recommendation chains to terminate after just a short number of steps. Aggregating such cascades over all the products, we obtain a highly disconnected network where the largest component grows over time by aggregating typically very small but occasionally fairly large components. We observed that the most popular categories of items recommended within communities in the largest component reflect differing interests between these communities. We presented a model which shows that these smaller and more tightly knit groups tend to be more conducive to viral marketing.

We saw that the characteristics of product reviews and the effectiveness of recommendations vary by category and price with more successful recommendations made on technical or religious books, which presumably are placed in the social context of a school, workplace, or place of worship. A small fraction of the products accounts for a large proportion of the recommendations. Although not quite as extreme in proportion, the number of successful recommendations also varies widely by product. Still, a sizeable portion of successful recommendations were for a product with only one such sale, hinting at a long-tail phenomenon.

Since viral marketing was found to be in general not as epidemic as one might have hoped, marketers who want to develop normative strategies for word-of-mouth advertising should analyze the topology and interests of the social network of their customers. Our study has provided a number of new insights which we hope will have general applicability to marketing strategies and to future models of the spread of viral information.

ACKNOWLEDGMENTS

We thank the anonymous reviewers for their insightful comments.

REFERENCES

ANDERSON, C. 2006. *The Long Tail: Why the Future of Business Is Selling Less of More*. Hyperion.

ANDERSON, R. M. AND MAY, R. M. 2002. *Infectious Diseases of Humans: Dynamics and Control*. Oxford University Press.

ANONYMOUS. 2005. Profiting from obscurity: What the long tail means for the economics of e-commerce. *Economist*.

BAILEY, N. 1975. *The Mathematical Theory of Infectious Diseases and its Applications*. Griffin, London, UK.

BASS, F. 1969. A new product growth for model consumer durables. *Manage. Sci. 15*, 5, 215–227.

BOWMAN, D. AND NARAYANDAS, D. 2001. Managing customerinitiated contacts with manufacturers: The impact on share of category requirements and word-of-mouth behavior. *J. Market. Resear. 38*, 3 (Aug.), 281–297.

BRONSON, P. 1998. Hotmale. *Wired Mag. 6*, 12.

BROWN, J. J. AND REINGEN, P. H. 1987. Social ties and word-of-mouth referral behavior. *J. Consum. Resear. 14*, 3, 350–362.

BRYNJOLFSSON, E., HU, Y., AND SMITH, M. D. 2003. Consumer surplus in the digital economy: Estimating the value of increased product variety at online booksellers. *Manage. Sci. 49*, 11, 1580–1596.

BURKE, K. 2003. As consumer attitudes shift, so must marketing strategies.

CENTOLA, D. AND MACY, M. 2005. Complex contagion and the weakness of long ties. ftp://hive.soc.cornell.edu/mwm14/webpage/WLT.pdf.

CHEVALIER, J. AND MAYZLIN, D. 2006. The effect of word-of-mouth on sales: Online book reviews. *J. Market. Resear. 43*, 3, 345.

CHICKERING, D. M. 2003. Optimal structure identification with greedy search. *J. Machine Learn. Resear. 3*, 507–554.

CLAUSET, A., NEWMAN, M. E. J., AND MOORE, C. 2004. Finding community structure in very large networks. *Physical Rev. E 70*, 066111.

DEBRUYN, A. AND LILIEN, G. 2004. A multi-stage model of word-of-mouth through electronic referrals.

ERDÖS, P. AND RÉNYI, A. 1960. On the evolution of random graphs. *Publ. Math. Inst. Hung. Acad. Sci. 5*, 17–61.

FRENZEN, J. AND NAKAMOTO, K. 1993. Structure, cooperation, and the flow of market information. *J. Consum. Resear. 20*, 3 (Dec.), 360–375.

GOLDENBERG, J., LIBAI, B., AND MULLER, E. 2001. Talk of the network: A complex systems look at the underlying process of word-of-mouth. *Market. Lett. 3*, 12, 211–223.

GOMES, L. 2006. It may be a long time before the long tail is wagging the web. *The Wall Street Jounal*. July 26 2006.

GRANOVETTER, M. 1978. Threshold models of collective behavior. *Ameri. J. Sociol. 83*, 6, 1420–1443.

GRANOVETTER, M. S. 1973. The strength of weak ties. *Ameri. J. Sociol. 78*, 1360–1380.

GRUHL, D., GUHA, R., LIBEN-NOWELL, D., AND TOMKINS, A. 2004. Information diffusion through blogspace. In *World Wide Web Conference* 2004.

HILL, S., PROVOST, F., AND VOLINSKY, C. 2006. Network-based marketing: Identifying likely adopters via consumer networks. *Statist. Sci. 21*, 2, 256–276.

HOLME, P. AND NEWMAN, M. E. J. 2006. Nonequilibrium phase transition in the coevolution of networks and opinions. *Physical Rev. E 74*, 056108.

JURVETSON, S. 2000. What exactly is viral marketing? *Red Herring 78*, 110–112.

KEMPE, D., KLEINBERG, J., AND TARDOS, E. 2003. Maximizing the spread of infuence in a social network. In *ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD)*.

KILLWORTH, P. AND BERNARD, H. 1978. Reverse small world experiment. *Social Netw. 1*, 159–192.

LESKOVEC, J., ADAMIC, L. A., AND HUBERMAN, B. A. 2006. The dynamics of viral marketing. In *Proceedings of the ACM Conference on Electronic Commerce*. 228–237.

LESKOVEC, J., SINGH, A., AND KLEINBERG, J. 2006. Patterns of influence in a recommendation network. In *Pacific-Asia Conference on Knowledge Discovery and Data Mining (PAKDD)*.

LINDEN, G., SMITH, B., AND YORK, J. 2003. Amazon.com recommendations: item-to-item collaborative filtering. *IEEE Internet Comput. 7*, 1, 76–80.

MONTGOMERY, A. L. 2001. Applying quantitative marketing techniques to the internet. *Interfaces 30*, 90–108.

RESNICK, P. AND ZECKHAUSER, R. 2002. Trust among strangers in internet transactions: Empirical analysis of ebays reputation system. In *The Economics of the Internet and E-Commerce*. Elsevier Science.

RICHARDSON, M. AND DOMINGOS, P. 2002. Mining knowledge-sharing sites for viral marketing. In *ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD)*.

ROGERS, E. M. 1995. *Diffusion of Innovations*, Fourth ed. Free Press, New York, NY.

STRANG, D. AND SOULE, S. A. 1998. Diffusion in organizations and social movements: From hybrid corn to poison pills. *Ann. Rev. Sociol. 24*, 265–290.

SUBRAMANI, M. R. AND RAJAGOPALAN, B. 2003. Knowledge-sharing and influence in online social networks via viral marketing. *Comm. ACM 46*, 12, 300–307.

TRAVERS, J. AND MILGRAM, S. 1969. An experimental study of the small world problem. *Sociometry 32*, 425–443.

WATTS, D. 2002. A simple model of global cascades on random networks. In *Proceedings of the National Academy of Science 99*, 9 (April), 4766–5771.

WU, F. AND HUBERMAN, B. A. 2004. Social structure and opinion formation. Available at http://ideas.repec.org/p/wpa/wuwpco/0407002.html.

YANG, S. AND ALLENBY, G. M. 2003. Modeling interdependent consumer preferences. *J. Market. Resear. 40*, 3 (Aug.), 282–294.

Received September 2006; revised February 2007; accepted February 2007