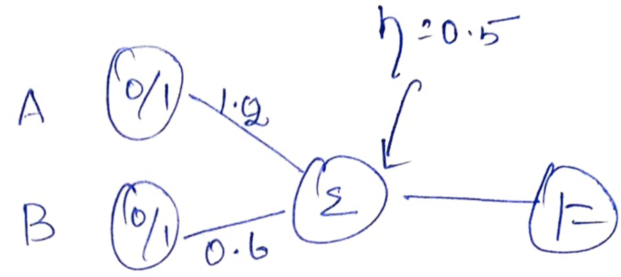
$W_1 = 1.2$ $W_2 = 0.6$ Threshold $= 1$

Learning rate $\eta = 0.5$

| A | B | A ∧ B |
|---|---|---|
| 0 | 0 | 0 ✓ |
| 0 | 1 | 0 ✓ |
| 1 | 0 | 0 |
| 1 | 1 | 1 |




① $A = 0$ $B = 0$

$w_i x_i \Rightarrow 0 * 1.2 + 0 * 0.6 = 0$ ✓

② $A = 0$ $B = 1$

$w_i x_i = 0 * 01.2 + 1 * 0.6$

$= 0.6 < 1 = 0$ ✓

3. $A = 1$ $B = 0$ $T = 0$

$w_i x_1 = 1 * 1.2 + 0 * 0.6$

$= 1.2 > 1 \rightarrow$ ①

$w_i = w_i + \eta (t - 0) x_i$

* $w_1 = 1.2 + 0.5(0 - 1) 1 = 1.2 - 0.5 = 0.7$

* $w_2 = 0.6 + \underline{0.5(0 - 1) 0} = 0.6$

Modified values.

Learned Parameters of AND gate.

$$\boxed{W_1 = 0.7, \quad W_2 = 0.6 \quad T = 1 \quad \eta = 0.5}$$

i) $A = 0, B = 0$ & Tar $= 0$

$w_i x_i = 0 \times 0.7 + 0 \times 0.6 = 0$ ✓

ii) $A = 0, B = \underline{1}$ Tar $= 0$

$w_i x_i = 0.6 < 1 = 0$

iii) $A = 1, B = 0$ Targ $= 0$

$1 \times 0.7 + 0 \times 0.6 = 0.7 < 1$ ✓

iv) $A = 1 \quad B = 1$

$w_i x_i = 1 \times 0.7 + 1 \times 0.6 = \underline{1.3} > \underline{1}$

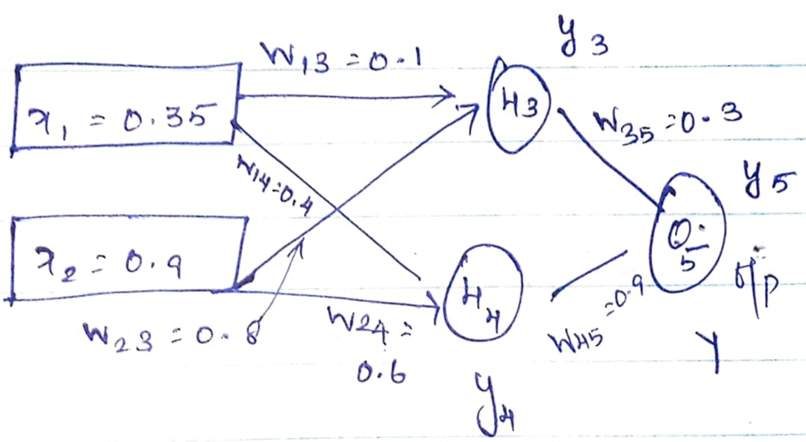
1 ✓

# Back Propagation

Neurons have a sigmoid activation fn, perform fwd pass and a backward pass on the n/w.

Assume that the actual o/p of y is $\boxed{0.5}$ and learning rate : 1

perform another forward pass.




forward pass :

Compute $y_3$, $y_4$ and $y_4$

$$a_j = \sum (w_i * x_i) \qquad y_j = f(a_j) = \frac{1}{1+e^{-a_j}}$$

Summation fn

activation fn

Sigmoid activation fn.

$$a_1 = W_{13} \times x_1 + W_{23} \times x_2$$

$$= (0.1 \times 0.35) + (0.8 \times 0.9) = 0.755$$

$$y_3 = f(a_1) = \frac{1}{1 + e^{-0.755}} = 0.68$$

$$a_2 = W_{14} \times \overset{(x_1)}{0.35} + W_{24} \times x_2$$

$$= (0.4 \times 0.35) + (0.6 \times 0.9) = 0.68$$

$$y_4 = f(a_2) = 0.6637$$

$$a_3 = (W_{35} \times y_3) + (W_{45} \times y_4)$$

$$= (0.3 \times 0.68) + (0.9 \times 0.66)$$

$$= 0.801$$

$$y_5 = f(a_3) = 1 / (1 + e^{-0.801})$$

$$\boxed{= 0.69}$$ (N/w o/p)

Error = tar − Predicted

0.5 − 0.69

$\boxed{\text{Error} = -0.19}$

Each weight changed by

$$\boxed{\Delta W_{ji} = \eta \, \delta_j \, O_i}$$

Error at $j^{th}$

Prediction o/p at $i^{th}$

(1) $\delta_j = O_j(1-O_j)(t_j - O_j)$ if $j$ is an o/p Unit

(2) $\delta_j = O_j(1-O_j)\left(\sum_k \delta_k W_{kj}\right)$ if $j$ is a hidden unit

$k \to$ no. of o/p Units.

* $\eta \to$ learning rate

$t_j \to$ Correct (teacher) o/p for unit $j$.

$\delta_j \to$ is error measure for unit $j$.

$y_3 = 0.68$

$y_4 = 0.6637$ $\quad y_5 = 0.69$

* Backward Pass :

Compute $\delta_3, \delta_4, \delta_5$

for o/p unit :

$$\delta_5 = y(1-y)(y_{tar} - y)$$

$$= 0.69(1-0.69) * (0.5 - 0.69)$$

$$= -0.0406 \checkmark$$

for hidden Unit :

$$\delta_3 = y_3(1-y_3)\ W_{35} * \delta_5$$

$$\Rightarrow 0.68(1-0.68) * (0.3 * -0.0406)$$

$$= -0.00265 \checkmark$$

$$\delta_4 = y_4(1-y_4)\ W_{45} * \delta_5$$

$$\Rightarrow 0.6637 * (1 - 0.6637) * (0.9 * -0.0406)$$

$$= -0.0082 \checkmark$$

Compute new weights :

$$\Delta W_{ji} = \eta \delta_j O_i$$

(1)

$$\Delta W_{45} = \eta \delta_5 y_4$$

$$= 1 * -0.0406 * 0.6637 = -0.0269$$

$$W_{45}(new) = \Delta W_{45} + W_{45}(old)$$

$$-0.0269 + 0.9 = 0.8731$$

(2)

$$\Delta W_{14} = \eta \delta_4 x_1 = 1 * -0.0082 * 0.35$$

$$= -0.00287$$

$$W_{14}(new) = \Delta W_{14} + W_{14}(old)$$

$$= -0.00287 + 0.4 = 0.3971$$

ex:

$$\Delta W_{35} = \eta \delta_5 y_3$$

$$= 1 * -0.0406 * 0.68 \Rightarrow -0.027608$$

$$W_{35}\ new = \Delta W_{35} + W_{35}(old)$$

$$-0.027608 + 0.3 = 0.27239$$

Similarly update all other weights

| $i$ | $j$ | $w_{ij}$ | $\delta_j$ | $x_i$ | $\eta$ | Updated $w_{ij}$ |
|---|---|---|---|---|---|---|
| 1 | 3 | 0.1 | -0.00265 | 0.35 | 1 | 0.0991 |
| 2 | 3 | 0.8 | -0.00265 | 0.9 | 1 | 0.7976 |
| 1 | 4 | 0.4 | -0.0082 | 0.35 | 1 | 0.3971 |
| 2 | 4 | 0.6 | -0.0082 | 0.9 | 1 | 0.5926 |
| 3 | 5 | 0.3 | -0.0406 | 0.68 | 1 | 0.2724 |
| 4 | 5 | 0.9 | -0.0406 | 0.6637 | 1 | 0.8731 |

Forward Pass: Compute → $y_3$ $y_4$ $y_5$

$a_1 = 0.7525$ $\quad y_3 = 0.6797$

$a_2 = 0.6723$ $\quad y_4 = 0.6620$

$a_3 = 0.7631$ $\quad \boxed{y_5 = 0.6820}$

N/w O/P

Error

$0.5 - 0.6820$

$\Rightarrow -0.182$

* Back Propagation Alg:

when error occurs, we go in backward direction

i) o/p → hidden → i/p layer

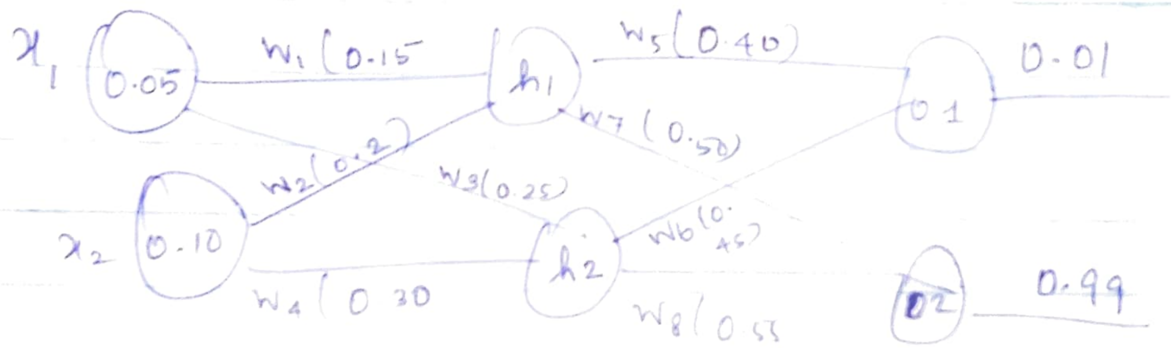
Part 1:

Calculate forward propagation error. (in and out)

$h_1(in) = w_1 x_1 + w_2 x_2 + b_1$

$= 0.15 \times 0.05 + 0.2 \times 0.1 + 0.35$

$= 0.377.$




$b_1 = 0.35$ $b_2 = 0.60$

$h_1(out) = \dfrac{1}{1 + e^{-h_1(in)}} = \dfrac{1}{1 + e^{-0.377}}$

$= 0.5932$

ii) Calculate $h_2$ (in & out)

$$h_2(in) = x_1 w_3 + x_2 w_4 + b_1$$

$$= 0.3925$$

$$h_2(out) = 0.5968$$

iii) Calculate $O_1$ (in & out)

$$O_1(in) = h_1(out) * w_5 + h_2(out) * w_6 + b_2$$

$$= 0.593 \times 0.4 + 0.596 \times 0.45 + 0.6$$

$$= 1.105$$

$$O_1(out) = \frac{1}{1+e^{-O_1(in)}} = 0.7513$$

iv) Calculate $O_2$ (in & out)

$$O_2(in) = h_1(out) \times w_7 + h_2(out) * w_8 + b_2$$

$$= 1.22484$$

$$O_2(out) = 0.7729$$

v) Calculate $E_{total}$

$$E_{total} = \frac{1}{2}\left(t - O(P)\right)^2$$

$$= E_{o1} + E_{o2}$$

$$= \frac{1}{2}(0.01 - 0.7513)^2 + \frac{1}{2}(0.99 - 0.7729)^2$$

$$= 0.29837 \text{ (approximately)}$$

Update the weight to minimize the error.

Part-2: Calculating backward Propagation error.

ie O/P layer (to) $\longrightarrow$ hidden layer.

~~$w_5$, $w_6$, $w_7$, and $w_8$~~

~~First let us adjust $w_5$~~